Neutrosophic Sets and Systems

Volume 40 Article 6

2-10-2021

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Neutrosophic Sets and Systems, Vol. 40, 2021
University of New Mexico

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Abstract: In this paper, we develop a new method of decision-making algorithm with Hausdorff distance and Hausdorff similarity measures based on generalized set-valued neutrosophic quadruple numbers. To establish the algorithm, we define Hausdorff distance measure and Hausdorff similarity measure on generalized set-valued neutrosophic quadruple. Next, we give a new method of decision-making application for impact of online learning on the learner. Also, we obtain different result from some previous applications (based on neutrosophic sets) for decision making algorithm. Thanks to our decision-making algorithm and similarity measure, researchers can obtain new applications for other decision making problems.

Keywords: Generalized set – valued neutrosophic quadruple sets, Hausdorff measures, decision making applications, adequacy of online education application

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1 Introduction

The rapid population growth experienced in the world at the end of the twentieth century and the inadequacy of classical learning-teaching (education-training) activities and methods in this respect led to new searches in the field of education. As a result of these searches, online education programs have been developed. Online education program is the name given to the study carried out with the curriculum prepared by educational institutions in a certain order to help students practice education alone. In the most general sense, we can define online education as the education practices that are structured on environments where teachers and students are separated from each other in terms of time and space. In this study, we will define a new similarity measure for generalized set-valued neutrosophic quadruple numbers to assess the competence of online education and remove uncertainties and provide a more objective assessment, and show the requirements for the similarity measure. Some of the environmental factors that affect the competence of online education are infrastructure, course material, and course hours. The difference of the similarity measure we will define from other similarity measures is that we add set operations on it. These set operations caused the result of the similarity measure to be seen more clearly. Similarities between human beings, a medicine or a new law to be exemplified can be examples of the assets we are talking about. In this report, some criteria will be selected to evaluate the adequacy of online education and the weight values of these criteria will be determined. A community of experts will then be created and an ideal (I) student template will be prepared for the assessment of online education, using generalized set-valued neutrosophic quadruples and numbers. Then, experts will be able to evaluate other students' criteria as generalized set - valued neutrosophic quadruple sets and numbers with the help of this ideal student. The evaluation result of each student will be handled separately and evaluation results of each will be obtained. Thus, an objective assessment will be made.

Smarandache defined neutrosophic logic and neutrosophic sets [1] in 1998. In terms of neutrosophic logic and neutrosophic sets, there is a membership degree (T), an indeterminacy degree (I) and a non-membership degree (F). These degrees are defined independently. A neutrosophic value is in the form (T, I, F). In other words, in explaining an event or finding a solution to a problem, a condition is handled according to its accuracy, inaccuracy and uncertainty. Therefore, neutrosophic logic and the neutrosophic sets help us find solutions to many uncertainties around us and in explaining complexity. Also, the distance measures and similarity measures are useful for decision making applications in neutrosophic theory. Therefore, many researchers studied neutrosophic theory [2-25] and decision making for neutrosophic theory [25-31]. Recently, Uluçay et al. [6] introduced neutrosophic multi-groups and applications; Uluçay [7] introduced a new similarity function of trapezoidal fuzzy multiple numbers based on multiple criteria decision making; Şahin et al. [8] obtained some weighted arithmetic operators and geometric operators with SVNSs and their application to multi-criteria decision making problems; Şahin et al. [9] studied some new operations of (α, β, γ) interval cut set of interval valued neutrosophic sets; Şahin et al. [10] obtained refined neutrosophic hierarchical clustering methods; Sahin et al. [11] studied extension principle based on neutrosophic multi-fuzzy sets and algebraic operations; Şahin et al. [12] introduced neutrosophic triplet partial g-metric space; Şahin et al. [13] introduced neutrosophic triplet normed ring space; Şahin et al. [14] studied neutrosophic quadruple theory; Broumi et al. [15] obtained Hausdroff distance and similarity measure for neutrosophic set and numbers; Şahin et al. [16] studied combined classic–neutrosophic sets and double neutrosophic sets; Şahin et al.[17] obtained decision-making applications in professional proficiencies in neutrosophic theory; Uluçay et al. [18] introduced decision-making method based on neutrosophic soft expert graphs; Ulucay et al. [19] studied an outranking approach for MCDM-problems with neutrosophic multi-sets; Hassan et al. [32] studied Q-neutrosophic soft expert set and its application in decision making; Bakbak et al. [33] obtained a theoretic approach to decision making problems in architecture with neutrosophic soft set; Şahin et al. [34] introduced neutrosophic triplet metric topology; Aslan et al. [35] introduced neutrosophic modeling of Talcott Parsons's action; Şahin et al. [36] studied an outperforming approach for multi-criteria decision-making problems with interval-valued bipolar neutrosophic sets; Abdel-Basset et al. studied a new hybrid multi-criteria decision- making approach for location selection of sustainable offshore wind energy stations [37]; Abdel-Basset et al. introduced neutrosophic theory based security approach for fog and mobile-edge computing [38]; Abdel-Basset et al. studied a model for the effective COVID-19 identification in uncertainty environment using primary symptoms and CT scans [39]; Abdel-Basset et al. introduced evaluation of sustainable hydrogen production options using an advanced hybrid MCDM approach [40].

Smarandache [20] discussed the neutrosophic quadruple set and the neutrosophic quadruple number. Neutrosophic quadruple sets are a generalized form of neutrosophic set. A neutrosophic quadruple set is represented by {(k, lT, mI, nF): k, l, m, n $\in \mathbb{R}$ or $\mathbb{C}$}. Here k is named as the known part, (lT, mI, nF) is named as the unknown part and T, I, F have the usual neutrosophic logic tools. Also, Şahin et al. [21] introduced generalized set-valued neutrosophic quadruple sets. Unlike neutrosophic quadruple set and number, in a generalized set-valued neutrosophic set and numbers; k, l, m and n are sets and T, I and F are not fixed. Thus, generalized set-valued neutrosophic set and numbers are more useful for decision making applications.

The organization of this paper is as follows: In section 2, some basis conception of the neutrosophic sets [1, 4], Hausdorff measures [15], the concept of neutrosophic quadruple sets [20, 21], Euclid measures [23] and Dice measures [22]. By adding set operations to the known Hausdorff distance measurement, we will obtain a larger set point Hausdorff distance measurement based on generalized set-valued neutrosophic quadruple numbers so that we can more clearly deal with the problems we encounter in section 3. In section 4, we will write an algorithm that we can use on sets of neutrosophic quadruple. Later, we will show the operability of Hausdorff's distance measurement, which we developed, by writing a numerical example with a neutrosophic quadruple structure. The example we gave in section 4 was calculated with other distance measurements in Seciton 5. and then, as a result of this calculation, we will comparison that the distance measurement we developed gives different results. Section 6 presents final conclusions and further research.

2 Preliminaries

Definition 2.1: [1] Let E be the universal set. For $\forall x \in E, 0^{-} \leq T_A(x) + I_A(x) + F_A(x) \leq 3^{+}$, by the help of the functions $T_A: E \to]^{-}0, 1^{+}[$, $I_A: E \to]^{-}0, 1^{+}[$ and $F_A: E \to]^{-}0, 1^{+}[$ a neutrosophic set A on E is defined by

$$A = \{\langle x, T_A(x), I_A(x), F_A(x) \rangle : x \in E\} .$$

Here, $T_A(x), I_A(x)$ and $F_A(x)$ are the degrees of trueness, indeterminacy and falsity of $x \in E$ respectively. Where, $^{-}0 = 0 - \varepsilon$ and $1^{+} = 1 + \varepsilon$.

Definition 2.2: [4] Let E be the universal set. For $\forall x \in E, 0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$, using the functions $T_A: E \to [0,1]$, $I_A: E \to [0,1]$ and $F_A: E \to [0,1]$, a single-valued neutrosophic set A on E is defined by

$$A = \{\langle x, T_A(x), I_A(x), F_A(x) \rangle : x \in E\} .$$

Here, $T_A(x), I_A(x)$ and $F_A(x)$ are the degrees of trueness, indeterminacy and falsity of $x \in E$, respectively.

Definition 2.3: [15] Let A_1 = <$T_{A_1}(x)$, $I_{A_1}(x)$, $F_{A_1}(x)$> and A_2 = <$T_{A_2}(x)$, $I_{A_2}(x)$, $F_{A_2}(x)$> be two single – valued neutrosophic numbers. The Hausdorff distance measure between A_1 and A_2, which is shown as $d_h(A_1, A_2)$ is defined as

$$d_h = (A_1, A_2) = max\{|T_{A_1}(x) - T_{A_2}(x)|, |I_{A_1}(x) - I_{A_2}(x)|, |F_{A_1}(x) - F_{A_2}(x)|\} .$$

Also, the Hausdorff similarity measure between A_1 and A_2, which is shown as $S_h(A_1, A_2)$ is defined as

$$S_h = (A_1, A_2) = 1 - max\{|T_{A_1}(x) - T_{A_2}(x)|, |I_{A_1}(x) - I_{A_2}(x)|, |F_{A_1}(x) - F_{A_2}(x)|\}.$$

Theorem 2.4: [15] Let X_1, X_2 and X_3 be three single – valued neutrosophic sets, d_h be Hausdorff distance measure. Then the following properties hold.

i. $0 \leq d_H(X_1, X_2) \leq 1$
ii. $X_1 = X_2$ if and only if $d_H(X_1, X_2) = 0$
iii. $d_H(X_1, X_2) = d_H(X_2, X_1)$
iv. If $X_1 \sqsubseteq X_2 \sqsubseteq X_3$, then $d_H(X_1, X_2) \leq d_H(X_1, X_3)$ and $d_H(X_1, X_3) \leq d_H(X_2, X_3)$.

Theorem 2.5: [15]: Let A_1, A_2 and A_3 be three single – valued neutrosophic sets, S_h be Hausdorff similarity measure. Then the following properties hold.

i. $0 \leq S_h(A_1, A_2) \leq 1$
ii. $S_h(A_1, A_2) = 1 \Leftrightarrow A_1 = A_2$
iii. $S_h(A_1, A_2) = S_h(A_2, A_1)$
iv. If $A_1 \subseteq A_2 \subseteq A_3 \in E$, then $S_h(A_1, A_3) \leq S_h(A_1, A_2)$ and $S_h(A_1, A_3) \leq S_h(A_2, A_3)$.

Definition 2.6: [20] NQN is a number of the form (k, lT, mI, nF). Here, T, I and F are used as the ordinary neutrosophic logical tools and k, l, m, n ∈ ℝ or ℂ. NQ = {(k, lT, mI, nF): k, l, m, n ∈ ℝ or ℂ} is defined by neutrosophic quadruple set.

For a neutrosophic quadruple number (k, lT, mI, nF), k is named the known part and (lT, mI, nF) is named the unknown part where k represents any asset such as a number, an idea, an object, etc.

Definition 2.7: [21] Let X be a set and P(X) be power set of X. A generalized set – valued neutrosophic quadruple set is a set of the form $G_{s_i} = \{(A_{s_i}, B_{s_i}T_{s_i}, C_{s_i}I_{s_i}, D_{s_i}F_{s_i})$: $A_{s_i}, B_{s_i}, C_{s_i}, D_{s_i} \in$ P(X); i = 1, 2, 3, … , n}.

Where T_i, I_i and F_i have their usual neutrosophic logic means and generalized set – valued neutrosophic quadruple number defined by

$$G_{N_i} = (A_{s_i}, B_{s_i}T_{s_i}, C_{s_i}I_{s_i}, D_{s_i}F_{s_i}).$$

As in neutrosophic quadruple number, for a generalized set – valued neutrosophic quadruple number $(A_{s_i}, B_{s_i}T_{s_i}, C_{s_i}I_{s_i}, D_{s_i}F_{s_i})$ representing any entity which may be a number, an idea, an object, etc.; A_{s_i} is called the known part and $(B_{s_i}T_{s_i}, C_{s_i}I_{s_i}, D_{s_i}F_{s_i})$ is called the unknown part.

Definition 2.8: [20] Let $G_{N_i} = (A_{s_i}, B_{s_i}T_{s_i}, C_{s_i}I_{s_i}, D_{s_i}F_{s_i})$ and $G_{N_j} = (A_{s_j}, B_{s_j}T_{s_j}, C_{s_j}I_{s_j}, D_{s_j}F_{s_j})$ be two generalized set – valued neutrosophic quadruple numbers. $A_{s_i} \subseteq A_{s_j}$, $A_{s_i} \subseteq A_{s_j}$, $A_{s_i} \subseteq A_{s_j}$, $A_{s_i} \subseteq A_{s_j}$ and $T_{s_i} \leq T_{s_j}, I_{s_i} \leq I_{s_j}, F_{s_i} \leq F_{s_j}$, then we say G_{N_i} is a subset of G_{N_J} and denote it by $G_{N_i} \subseteq G_{N_J}$.

Definition 2.9: [23] Let A_1 = <$T_{A_1}(x)$, $I_{A_1}(x)$, $F_{A_1}(x)$> and A_2 = <$T_{A_2}(x)$, $I_{A_2}(x)$, $F_{A_2}(x)$> be two single – valued neutrosophic numbers. The Euclid similarity measure between A_1 and A_2, which is shown as $d_E(A_1, A_2)$ is defined as

$$d_E(A_1, A_2) = 1 - \frac{1}{3}\sum_{j=1}^{n}\sqrt{\left(T_{A_1}(x) - T_{A_2}(x)\right)^2 + \left(I_{A_1}(x) - I_{A_2}(x)\right)^2 + \left(F_{A_1}(x) - F_{A_2}(x)\right)^2}.$$

Definition 2.10: [22] Let A_1 = <$T_{A_1}(x)$, $I_{A_1}(x)$, $F_{A_1}(x)$> and A_2 = <$T_{A_2}(x)$, $I_{A_2}(x)$, $F_{A_2}(x)$> be two single – valued neutrosophic numbers. The Dice similarity measure between A_1 and A_2, which is shown as $d_E(A_1, A_2)$ is defined as

$$S_{D1}(A_1, A_2) = 1 - \frac{2[\left(T_{A_1}(x).T_{A_2}(x) + I_{A_1}(x).I_{A_2}(x) + F_{A_1}(x).F_{A_2}(x)\right)]}{\left(\left(T_{A_1}(x)\right)^2 + \left(I_{A_1}(x)\right)^2 + \left(F_{A_1}(x)\right)^2\right) + \left(\left(T_{A_2}(x)\right)^2 + \left(I_{A_2}(x)\right)^2 + \left(F_{A_2}(x)\right)^2\right)}$$

3 Hausdorff Measures Based on Generalized Set-Valued Neutrosophic Quadruple Numbers and Sets

In this paper, we take T, I, F ∈ [0, 1] like single valued neutrosophic numbers in Definition 2.2.

Definition 3.1: $G_{N_1} = (A_{s_1}, B_{s_1}T_{s_1}, C_{s_1}I_{s_1}, D_{s_1}F_{s_1})$ and $G_{N_2} = (A_{s_2}, B_{s_2}T_{s_2}, C_{s_2}I_{s_2}, D_{s_2}F_{s_2})$ be two generalized set – valued neutrosophic quadruple number. We define a function $d_{QHN}: G_{N_1} \times G_{N_2} \rightarrow [0,1]$ such that

$$d_{QHN}(G_{N_1}, G_{N_2}) = d_{QHN}\left((A_{s_1}, B_{s_1}T_{s_1}, C_{s_1}I_{s_1}, D_{s_1}F_{s_1}), (A_{s_2}, B_{s_2}T_{s_2}, C_{s_2}I_{s_2}, D_{s_2}F_{s_2})\right)$$

$$= \frac{1}{2}\left[max\{|T_{s_1} - T_{s_2}|, |I_{s_1} - I_{s_2}|, |F_{s_1} - F_{s_2}|\} + \frac{1}{4}\left(\frac{max\{s(A_{s_1} \setminus A_{s_2}), s(A_{s_2} \setminus A_{s_1})\}}{max\{s(A_{s_1}), s(A_{s_2}), 1\}} + \frac{max\{s(B_{s_1} \setminus B_{s_2}), s(B_{s_2} \setminus B_{s_1})\}}{max\{s(B_{s_1}), s(B_{s_2}), 1\}} + \frac{max\{s(C_{s_1} \setminus C_{s_2}), s(C_{s_2} \setminus C_{s_1})\}}{max\{s(C_{s_1}), s(C_{s_2}), 1\}} + \frac{max\{s(D_{s_1} \setminus D_{s_2}), s(D_{s_2} \setminus D_{s_1})\}}{max\{s(D_{s_1}), s(D_{s_2}), 1\}}\right)\right]$$

Then, d_{QHN} is called a Hausdorff distance measure on generalized set-valued neutrosophic quadruple numbers.

Where, s(A) is number of element of set A.

Also, we generalized Hausdorff distance measure for generalized set-valued neutrosophic quadruple numbers in Definition 3.1.

Theorem 3.2: Let $G_{N_1} = (A_{s_1}, B_{s_1}T_{s_1}, C_{s_1}I_{s_1}, D_{s_1}F_{s_1})$, $G_{N_2} = (A_{s_2}, B_{s_2}T_{s_2}, C_{s_2}I_{s_2}, D_{s_2}F_{s_2})$ and $G_{N_3} = (A_{s_3}, B_{s_3}T_{s_3}, C_{s_3}I_{s_3}, D_{s_3}F_{s_3})$ be two generalized set – valued neutrosophic quadruple numbers. Then, d_{QHN} satisfies the below conditions.

i) $d_{QHN}(G_{N_1}, G_{N_2}) \in [0,1]$

ii) $d_{QHN}(G_{N_1}, G_{N_2}) = 0 \Leftrightarrow G_{N_1} = G_{N_2}$

iii) $d_{QHN}(G_{N_1}, G_{N_2}) = d_{QHN}(G_{N_2}, G_{N_1})$

iv) If $G_{N_1} \subset G_{N_2} \subset G_{N_3}$, then

$d_{QHN}(G_{N_1}, G_{N_2}) \leq d_{QHN}(G_{N_1}, G_{N_3})$ and $d_{QHN}(G_{N_2}, G_{N_3}) \leq d_{QHN}(G_{N_1}, G_{N_3})$.

Proof:

i)

Let $G_{N_1} = G_{N_2}$. From Definition 2.8,

$A_{s_1} = A_{s_2}$, $B_{s_1} = B_{s_2}$, $C_{s_1} = C_{s_2}$, $D_{s_1} = D_{s_2}$, $T_{s_1} = T_{s_2}$, $I_{s_1} = I_{s_2}$ and $F_{s_1} = F_{s_2}$ .Thus, we have

$$\begin{aligned} d_{QHN}(G_{N_1}, G_{N_1}) &= \frac{1}{2}\Bigg[max\{|T_{s_1} - T_{s_1}|, |I_{s_1} - I_{s_1}|, |F_{s_1} - F_{s_1}|\} \\ &+ \frac{1}{4}\Bigg(\frac{max\{s(A_{s_1} \setminus A_{s_1}), s(A_{s_1} \setminus A_{s_1})\}}{max\{s(A_{s_1}), s(A_{s_1}), 1\}} + \frac{max\{s(B_{s_1} \setminus B_{s_1}), s(B_{s_1} \setminus B_{s_1})\}}{max\{s(B_{s_1}), s(B_{s_1}), 1\}} \\ &+ \frac{max\{s(C_{s_1} \setminus C_{s_1}), s(C_{s_1} \setminus C_{s_1})\}}{max\{s(C_{s_1}), s(C_{s_1}), 1\}} + \frac{max\{s(D_{s_1} \setminus D_{s_1}), s(D_{s_1} \setminus D_{s_1})\}}{max\{s(D_{s_1}), s(D_{s_1}), 1\}}\Bigg)\Bigg] \\ &= \frac{1}{2}\left[0 + \frac{1}{4}\left(\frac{0}{s(A_{s_1})} + \frac{0}{s(B_{s_1})} + \frac{0}{s(C_{s_1})} + \frac{0}{s(D_{s_1})}\right)\right] = 0 \end{aligned}$$

Let $G_{N_1} \neq G_{N_2}$. We have $A_{s_1} \neq A_{s_2}$, $B_{s_1} \neq B_{s_2}$, $C_{s_1} \neq C_{s_2}$, $D_{s_1} \neq D_{s_2}$, $T_{s_1} \neq T_{s_2}$, $I_{s_1} \neq I_{s_2}$, $F_{s_1} \neq F_{s_2}$.

In this case, $d_{QHN}(G_{N_1}, G_{N_2}) > 0$.

Let $G_{N_1} \neq \emptyset$ and $G_{N_2} = \emptyset$. So,

$G_{N_1} = (A_{s_1}, B_{s_1}T_{s_1}, C_{s_1}I_{s_1}, D_{s_1}F_{s_1})$, $G_{N_2} = \emptyset = (\emptyset, \emptyset T_{s_2}, \emptyset I_{s_2}, \emptyset F_{s_2})$. Since we are looking for the highest value of the result, we take $T_{s_1} = I_{s_1} = F_{s_1} = 1$ and $T_{s_2} = I_{s_2} = F_{s_2} = 0$.

$$\begin{aligned} d_{QHN}(G_{N_1}, G_{N_2}) &= \frac{1}{2}\Bigg[max\{|T_{s_1} - 0|, |I_{s_1} - 0|, |F_{s_1} - 0|\} \\ &+ \frac{1}{4}\Bigg(\frac{max\{s(A_{s_1} \setminus \emptyset), s(\emptyset \setminus A_{s_1})\}}{max\{s(A_{s_1}), 0, 1\}} + \frac{max\{s(B_{s_1} \setminus \emptyset), s(\emptyset \setminus B_{s_1})\}}{max\{s(B_{s_1}), 0, 1\}} \\ &+ \frac{max\{s(C_{s_1} \setminus \emptyset), s(\emptyset \setminus C_{s_1})\}}{max\{s(C_{s_1}), 0, 1\}} + \frac{max\{s(D_{s_1} \setminus \emptyset), s(\emptyset \setminus D_{s_1})\}}{max\{s(D_{s_1}), 0, 1\}}\Bigg)\Bigg] \\ &= \frac{1}{2}\left[1 + \frac{1}{4}(1 + 1 + 1 + 1)\right] = 1 \end{aligned}$$

As the highest value of $d_{QHN}(G_{N_1}, G_{N_2})$ is 1 and the lowest value is 0, $d_{QHN}(G_{N_1}, G_{N_2}) \in [0, 1]$.

ii) $d_{QHN}(G_{N_1}, G_{N_2}) = 0 \Leftrightarrow G_{N_1} = G_{N_2}$

(⇒): If $d_{QHN}(G_{N_1}, G_{N_2}) = \frac{1}{2}\Big[max\{|T_{s_1} - T_{s_2}|, |I_{s_1} - I_{s_2}|, |F_{s_1} - F_{s_2}|\} + \frac{1}{4}\Big(\frac{max\{s(A_{s_1}\setminus A_{s_2}), s(A_{s_2}\setminus A_{s_1})\}}{max\{s(A_{s_1}), s(A_{s_2}), 1\}} + \frac{max\{s(B_{s_1}\setminus B_{s_2}), s(B_{s_2}\setminus B_{s_1})\}}{max\{s(B_{s_1}), s(B_{s_2}), 1\}} + \frac{max\{s(C_{s_1}\setminus C_{s_2}), s(C_{s_2}\setminus C_{s_1})\}}{max\{s(C_{s_1}), s(C_{s_2}), 1\}} + \frac{max\{s(D_{s_1}\setminus D_{s_2}), s(D_{s_2}\setminus D_{s_1})\}}{max\{s(D_{s_1}), s(D_{s_2}), 1\}}\Big)\Big]$

$= 0$,

then

$max\{|T_{s_1} - T_{s_2}|, |I_{s_1} - I_{s_2}|, |F_{s_1} - F_{s_2}|\} = 0$ and

$\frac{1}{4}\Big(\frac{max\{s(A_{s_1}\setminus A_{s_2}), s(A_{s_2}\setminus A_{s_1})\}}{max\{s(A_{s_1}), s(A_{s_2}), 1\}} + \frac{max\{s(B_{s_1}\setminus B_{s_2}), s(B_{s_2}\setminus B_{s_1})\}}{max\{s(B_{s_1}), s(B_{s_2}), 1\}} + \frac{max\{s(C_{s_1}\setminus C_{s_2}), s(C_{s_2}\setminus C_{s_1})\}}{max\{s(C_{s_1}), s(C_{s_2}), 1\}} + \frac{max\{s(D_{s_1}\setminus D_{s_2}), s(D_{s_2}\setminus D_{s_1})\}}{max\{s(D_{s_1}), s(D_{s_2}), 1\}}\Big) = 0$

If $|T_{s_1} - T_{s_2}| = 0$, then $T_{s_1} = T_{s_2}$; if $|I_{s_1} - I_{s_2}| = 0$, then $I_{s_1} = I_{s_2}$; if $|F_{s_1} - F_{s_2}| = 0$, then $F_{s_1} = F_{s_2}$ and if

$$\frac{max\{s(A_{s_1} \setminus A_{s_2}), s(A_{s_2} \setminus A_{s_1})\}}{max\{s(A_{s_1}), s(A_{s_2}), 1\}} = 0, \frac{max\{s(B_{s_1} \setminus B_{s_2}), s(B_{s_2} \setminus B_{s_1})\}}{max\{s(B_{s_1}), s(B_{s_2}), 1\}} = 0,$$

$$\frac{max\{s(C_{s_1} \setminus C_{s_2}), s(C_{s_2} \setminus C_{s_1})\}}{max\{s(C_{s_1}), s(C_{s_2}), 1\}} = 0, \frac{max\{s(D_{s_1} \setminus D_{s_2}), s(D_{s_2} \setminus D_{s_1})\}}{max\{s(D_{s_1}), s(D_{s_2}), 1\}} = 0,$$

then

$A_{s_1} = A_{s_2}$, $B_{s_1} = B_{s_2}$, $C_{s_1} = C_{s_2}$, $D_{s_1} = D_{s_2}$.

Then, from Definition 2.8, we obtain that $G_{N_1} = G_{N_2}$.

(⟸):

Let $G_{N_1} = G_{N_2}$. From i, we have $d_{QHN}(G_{N_1}, G_{N_1}) = 0$.

iii)

$$\begin{aligned} d_{QHN}(G_{N_1}, G_{N_2}) = {} & \frac{1}{2}\Bigg[max\{|T_{s_1} - T_{s_2}|, |I_{s_1} - I_{s_2}|, |F_{s_1} - F_{s_2}|\} \\ & + \frac{1}{4}\Bigg(\frac{max\{s(A_{s_1} \setminus A_{s_2}), s(A_{s_2} \setminus A_{s_1})\}}{max\{s(A_{s_1}), s(A_{s_2}), 1\}} + \frac{max\{s(B_{s_1} \setminus B_{s_2}), s(B_{s_2} \setminus B_{s_1})\}}{max\{s(B_{s_1}), s(B_{s_2}), 1\}} \\ & + \frac{max\{s(C_{s_1} \setminus C_{s_2}), s(C_{s_2} \setminus C_{s_1})\}}{max\{s(C_{s_1}), s(C_{s_2}), 1\}} + \frac{max\{s(D_{s_1} \setminus D_{s_2}), s(D_{s_2} \setminus D_{s_1})\}}{max\{s(D_{s_1}), s(D_{s_2}), 1\}}\Bigg)\Bigg] \\ = {} & \frac{1}{2}\Bigg[max\{|T_{s_2} - T_{s_1}|, |I_{s_2} - I_{s_1}|, |F_{s_2} - F_{s_1}|\} \\ & + \frac{1}{4}\Bigg(\frac{max\{s(A_{s_2} \setminus A_{s_1}), s(A_{s_1} \setminus A_{s_2})\}}{max\{s(A_{s_2}), s(A_{s_1}), 1\}} + \frac{max\{s(B_{s_2} \setminus B_{s_1}), s(B_{s_1} \setminus B_{s_2})\}}{max\{s(B_{s_2}), s(B_{s_1}), 1\}} \\ & + \frac{max\{s(C_{s_2} \setminus C_{s_1}), s(C_{s_1} \setminus C_{s_2})\}}{max\{s(C_{s_2}), s(C_{s_1}), 1\}} + \frac{max\{s(D_{s_2} \setminus D_{s_1}), s(D_{s_1} \setminus D_{s_2})\}}{max\{s(D_{s_2}), s(D_{s_1}), 1\}}\Bigg)\Bigg] = d_{QHN}(G_{N_2}, G_{N_1}). \end{aligned}$$

iv) Let $G_{N_1} \subset G_{N_2} \subset G_{N_3}$. From Definition 2.8, we obtain $A_{s_1} \subset A_{s_2} \subset A_{s_3}$, $B_{s_1} \subset B_{s_2} \subset B_{s_3}$, $C_{s_1} \subset C_{s_2} \subset C_{s_3}$, $D_{s_1} \subset D_{s_2} \subset D_{s_3}$. Also, we have

$s(A_{s_1}) \le s(A_{s_2}) \le s(A_{s_3})$, $s(B_{s_1}) \le s(B_{s_2}) \le s(B_{s_3})$, $s(C_{s_1}) \le s(C_{s_2}) \le s(C_{s_3})$, $s(D_{s_1}) \le s(D_{s_2}) \le s(D_{s_3})$, and

$$\frac{max\{s(A_{s_1}\backslash A_{s_2}),s(A_{s_2}\backslash A_{s_1})\}}{max\{s(A_{s_1}),s(A_{s_2}),1\}}+\frac{max\{s(B_{s_1}\backslash B_{s_2}),s(B_{s_2}\backslash B_{s_1})\}}{max\{s(B_{s_1}),s(B_{s_2}),1\}}+\frac{max\{s(C_{s_1}\backslash C_{s_2}),s(C_{s_2}\backslash C_{s_1})\}}{max\{s(C_{s_1}),s(C_{s_2}),1\}}+\frac{max\{s(D_{s_1}\backslash D_{s_2}),s(D_{s_2}\backslash D_{s_1})\}}{max\{s(D_{s_1}),s(D_{s_2}),1\}}\leq \frac{max\{s(A_{s_1}\backslash A_{s_3}),s(A_{s_3}\backslash A_{s_1})\}}{max\{s(A_{s_1}),s(A_{s_3}),1\}}+\frac{max\{s(B_{s_1}\backslash B_{s_3}),s(B_{s_3}\backslash B_{s_1})\}}{max\{s(B_{s_1}),s(B_{s_3}),1\}}+\frac{max\{s(C_{s_1}\backslash C_{s_3}),s(C_{s_3}\backslash C_{s_1})\}}{max\{s(C_{s_1}),s(C_{s_3}),1\}}+\frac{max\{s(D_{s_1}\backslash D_{s_3}),s(D_{s_3}\backslash D_{s_1})\}}{max\{s(D_{s_1}),s(D_{s_3}),1\}}$$

since

$s(A_{s_1} \setminus A_{s_2}) = s(A_{s_2} \setminus A_{s_3}) = s(A_{s_1} \setminus A_{s_3}) = \emptyset$

$s(B_{s_1} \setminus B_{s_2}) = s(B_{s_2} \setminus B_{s_3}) = s(B_{s_1} \setminus B_{s_3}) = \emptyset$

$s(C_{s_1} \setminus C_{s_2}) = s(C_{s_2} \setminus C_{s_3}) = s(C_{s_1} \setminus C_{s_3}) = \emptyset$

$s(D_{s_1} \setminus D_{s_2}) = s(D_{s_2} \setminus D_{s_3}) = s(D_{s_1} \setminus D_{s_3}) = \emptyset$

$s(A_{s_2} \setminus A_{s_1}) \leq s(A_{s_3} \setminus A_{s_1})$, $s(B_{s_2} \setminus B_{s_1}) \leq s(B_{s_3} \setminus B_{s_1})$, $s(C_{s_2} \setminus C_{s_1}) \leq s(C_{s_3} \setminus C_{s_1})$, $s(D_{s_2} \setminus D_{s_1}) \leq s(D_{s_3} \setminus D_{s_1})$,

$s(A_{s_3} \setminus A_{s_2}) \leq s(A_{s_3} \setminus A_{s_1})$, $s(B_{s_3} \setminus B_{s_2}) \leq s(B_{s_3} \setminus B_{s_1})$, $s(C_{s_3} \setminus C_{s_2}) \leq s(C_{s_3} \setminus C_{s_1})$, $s(D_{s_3} \setminus D_{s_2}) \leq s(D_{s_3} \setminus D_{s_1})$.

Also, from Definition 2.8, we obtain

$|T_{s_1} - T_{s_2}| \leq |T_{s_1} - T_{s_3}|$, $|I_{s_1} - I_{s_2}| \leq |I_{s_1} - I_{s_3}|$, $|F_{s_1} - F_{s_2}| \leq |F_{s_1} - F_{s_3}|$,

$|T_{s_2} - T_{s_3}| \leq |T_{s_1} - T_{s_3}|$, $|I_{s_2} - I_{s_3}| \leq |I_{s_1} - I_{s_3}|$, $|F_{s_2} - F_{s_3}| \leq |F_{s_1} - F_{s_3}|$.

Thus, we have $d_{QHN}(G_{N_1}, G_{N_2}) \leq d_{QHN}(G_{N_1}, G_{N_3})$.

Where, $d_{QHN}(G_{N_2}, G_{N_3}) \leq d_{QHN}(G_{N_1}, G_{N_3})$ can be shown similar to $d_{QHN}(G_{N_1}, G_{N_2}) \leq d_{QHN}(G_{N_1}, G_{N_3})$.

Definition 3.3: Let $G_{N_1} = (A_{s_1}, B_{s_1}T_{s_1}, C_{s_1}I_{s_1}, D_{s_1}F_{s_1})$ and $G_{N_2} = (A_{s_2}, B_{s_2}T_{s_2}, C_{s_2}I_{s_2}, D_{s_2}F_{s_2})$ be two generalized set – valued neutrosophic quadruple numbers. We define a function $d_{HD}: G_{N_1} \times G_{N_2} \to [0, 1]$ such that

$$S_{QHN}: (G_{N_1}, G_{N_2}) = S_{QHN}\ ((A_{s_1}, B_{s_1}T_{s_1}, C_{s_1}I_{s_1}, D_{s_1}F_{s_1}), (A_{s_2}, B_{s_2}T_{s_2}, C_{s_2}I_{s_2}, D_{s_2}F_{s_2}))$$
$$= 1 - \frac{1}{2}\Bigg[max\{|T_{s_1} - T_{s_2}|, |I_{s_1} - I_{s_2}|, |F_{s_1} - F_{s_2}|\} + \frac{1}{4}\Bigg(\frac{max\{s(A_{s_1} \setminus A_{s_2}), s(A_{s_2} \setminus A_{s_1})\}}{max\{s(A_{s_1}), s(A_{s_2}), 1\}} + \frac{max\{s(B_{s_1} \setminus B_{s_2}), s(B_{s_2} \setminus B_{s_1})\}}{max\{s(B_{s_1}), s(B_{s_2}), 1\}} + \frac{max\{s(C_{s_1} \setminus C_{s_2}), s(C_{s_2} \setminus C_{s_1})\}}{max\{s(C_{s_1}), s(C_{s_2}), 1\}} + \frac{max\{s(D_{s_1} \setminus D_{s_2}), s(D_{s_2} \setminus D_{s_1})\}}{max\{s(D_{s_1}), s(D_{s_2}), 1\}}\Bigg)\Bigg]$$

Then, S_{QHN} is called a Hausdorff similarity measure on generalized set - valued neutrosophic quadruple numbers.

Where, s(A) is number of element of set A.

Also, we generalized Hausdorff similarity measure for generalized set - valued neutrosophic quadruple numbers in Definition 3.3.

Theorem 3.4: Let $G_{N_1} = \left(A_{s_1}, B_{s_1}T_{s_1}, C_{s_1}I_{s_1}, D_{s_1}F_{s_1}\right)$, $G_{N_2} = (A_{s_2}, B_{s_2}T_{s_2}, C_{s_2}I_{s_2}, D_{s_2}F_{s_2})$ and $G_{N_3} = (A_{s_3}, B_{s_3}T_{s_3}, C_{s_3}I_{s_3}, D_{s_3}F_{s_3})$ be three generalized set – valued neutrosophic quadruple numbers. Then, S_{QHN} satisfies the below conditions.

i) $S_{QHN}(G_{N_1}, G_{N_2}) \in [0,1]$

ii) $S_{QHN}(G_{N_1}, G_{N_2}) = 1 \Leftrightarrow G_{N_1} = G_{N_2}$

iii) $S_{QHN}(G_{N_1}, G_{N_2}) = S_{QHN}(G_{N_2}, G_{N_1})$

iv) If $G_{N_1} \subset G_{N_2} \subset G_{N_3}$, then

$S_{QHN}(G_{N_1}, G_{N_3}) \leq S_{QHN}(G_{N_1}, G_{N_2})$ and $S_{QHN}\left(G_{N_1}, G_{N_3}\right) \leq S_{QHN}\left(G_{N_2}, G_{N_3}\right)$.

Proof:

i) From Theorem 3.2,

when $d_{QHN}(G_{N_1}, G_{N_1}) = 0$, $S_{QHN}(G_{N_1}, G_{N_1}) = 1 - d_{QHN}(G_{N_1}, G_{N_1}) = 1 - 0 = 1$.

when $d_{QHN}(G_{N_1}, G_{N_1}) = 1$, $S_{QHN}(G_{N_1}, G_{N_1}) = 1 - d_{QHN}(G_{N_1}, G_{N_1}) = 1 - 1 = 0$.

Then, $S_{QHN}(G_{N_1}, G_{N_2}) \in [0,1]$.

ii)

From Theorem 3.2,

(⇒): If $S_{QHN}(G_{N_1}, G_{N_2}) = 1$, then $S_{QHN}(G_{N_1}, G_{N_2}) = 1 - d_{QHN}(G_{N_1}, G_{N_2})$

$$d_{QHN}(G_{N_1}, G_{N_2}) = 1 - S_{QHN}(G_{N_1}, G_{N_2})$$

$$d_{QHN}(G_{N_1}, G_{N_2}) = 1 - 1 = 0.$$

From Theorem 3.2,

(⇐): If $G_{N_1} = G_{N_2}$, then $d_{QHN}(G_{N_1}, G_{N_2}) = 0$.

Since $S_{QHN}(G_{N_1}, G_{N_2}) = 1 - d_{QHN}(G_{N_1}, G_{N_2}) = 1 - 0 = 1$, one can write $S_{QHN}(G_{N_1}, G_{N_2}) = 1$.

iii) From Theorem 3.2,

Since $S_{QHN}(G_{N_1}, G_{N_2}) = 1 - d_{QHN}(G_{N_1}, G_{N_2})$ and $d_{QHN}(G_{N_1}, G_{N_2}) = d_{QHN}(G_{N_2}, G_{N_1})$,

$S_{QHN}(G_{N_1}, G_{N_2}) = 1 - d_{QHN}(G_{N_1}, G_{N_2}) = 1 - d_{QHN}(G_{N_2}, G_{N_1}) = S_{QHN}(G_{N_2}, G_{N_1})$.

iv) Let $G_{N_1} \subset G_{N_2} \subset G_{N_3}$.

From Theorem 3.2, if $A_{s_1} \subset A_{s_2} \subset A_{s_3}$, $B_{s_1} \subset B_{s_2} \subset B_{s_3}$, $C_{s_1} \subset C_{s_2} \subset C_{s_3}$, $D_{s_1} \subset D_{s_2} \subset D_{s_3}$, then $d_{QHN}\ (G_{N_1}, G_{N_2}) \leq d_{QHN}\ (G_{N_1}, G_{N_3})$ and $d_{QHN}\ (G_{N_2}, G_{N_3}) \leq d_{QHN}\ (G_{N_1}, G_{N_3})$.

$$d_{QHN}\ (G_{N_1}, G_{N_2}) \leq d_{QHN}\ (G_{N_1}, G_{N_3})$$

$$-d_{QHN}\ (G_{N_1}, G_{N_2}) \geq -d_{QHN}\ (G_{N_1}, G_{N_3})$$

$$1 - d_{QHN}\ (G_{N_1}, G_{N_2}) \geq 1 - d_{QHN}\ (G_{N_1}, G_{N_3})$$

$$S_{QHN}\ (G_{N_1}, G_{N_2}) \geq S_{QHN}\ (G_{N_1}, G_{N_3}).$$

Also, $S_{QHN}\ (G_{N_1}, G_{N_3}) \leq S_{QHN}\ (G_{N_2}, G_{N_3})$ can be shown similar to $S_{QHN}\ (G_{N_1}, G_{N_2}) \geq S_{QHN}\ (G_{N_1}, G_{N_3})$.

Example 3.5: Let $X = (\{x_2, x_3, x_5, x_6\}, \{x_1, x_6, x_8\}(1), \emptyset(0), \emptyset(0))$ and

$X_1 = \left(\{x_1, x_3, x_5, x_7, x_9\}, \{x_2, x_4, x_5, x_6, x_7\}(0{,}4), \{x_2, x_3, x_7\}(0{,}1), \{x_4, x_5\}(0{,}2)\right)$ be two generalized set-valued neutrosophic quadruple numbers.

We calculate $d_{QHN}(X, X_1)$, namely the distance between X and X_1.

$$d_{QHN}(X, X_1) = \frac{1}{2}\Bigg[max\{|\mathrm{T} - T_1|, |\mathrm{I} - I_1|, |\mathrm{F} - F_1|\} + \frac{1}{4}\left(\frac{max\{s(\mathrm{A} \setminus A_1), s(A_1 \setminus \mathrm{A})\}}{max\{s(\mathrm{A}), s(A_1), 1\}} + \frac{max\{s(\mathrm{B} \setminus B_1), s(B_1 \setminus \mathrm{B})\}}{max\{s(\mathrm{B}), s(B_1), 1\}} + \frac{max\{s(\mathrm{C} \setminus C_1), s(C_1 \setminus \mathrm{C})\}}{max\{s(\mathrm{C}), s(C_1), 1\}} + \frac{max\{s(\mathrm{D} \setminus D_1), s(D_1 \setminus \mathrm{D})\}}{max\{s(\mathrm{D}), s(D_1), 1\}}\right)\Bigg].$$

$$d_{QHN}(X, X_1) = \frac{1}{2}\Bigg[max\{|1 - 0{,}4|, |0 - 0{,}1|, |0 - 0{,}2|\} + \frac{1}{4}\left(\frac{max\{s(\{x_2, x_3, x_5, x_6\} \setminus \{x_1, x_3, x_5, x_7, x_9\}), s(\{x_1, x_3, x_5, x_7, x_9\} \setminus \{x_2, x_3, x_5, x_6\})\}}{max\{s(\{x_2, x_3, x_5, x_6\}), s(\{x_1, x_3, x_4\}), 1\}} + \frac{max\{s(\{x_1, x_6, x_8\} \setminus \{x_2, x_4, x_5, x_6, x_7\}), s(\{x_2, x_4, x_5, x_6, x_7\} \setminus \{x_1, x_6, x_8\})\}}{max\{s(\{x_1, x_6, x_8\}), s(\{x_2, x_4, x_5, x_6, x_7\}), 1\}} + \frac{max\{s(\emptyset \setminus \{x_2, x_3, x_7\}), s(\{x_2, x_3, x_7\} \setminus \emptyset)\}}{max\{s(\{\}), s(\{x_2, x_3, x_7\}), 1\}} + \frac{max\{s(\emptyset \setminus \{x_4, x_5\}), s(\{x_4, x_5\} \setminus \emptyset)\}}{max\{s(\{\}, s(\{x_4, x_5\}), 1\}}\right)\Bigg]$$

$$d_{QHN}(X, X_1) = \frac{1}{2}\left[max\{0.6, 0.1, 0.2\} + \frac{1}{4}\left(\frac{max\{2{,}3\}}{max\{4{,}3{,}1\}} + \frac{max\{2{,}4\}}{max\{3{,}5{,}1\}} + \frac{max\{0{,}3\}}{max\{0{,}3{,}1\}} + \frac{max\{0{,}2\}}{max\{0{,}2{,}1\}}\right)\right]$$

$$= \frac{1}{2}\left[0.6 + \frac{1}{4}\left(\frac{3}{4} + \frac{4}{5} + \frac{3}{3} + \frac{2}{2}\right)\right] = 0.74375.$$

As $d_{QHN}(X, X_1) = 0.74375$, $S_{QHN}\ (X, X_1) = 1 - d_{QHN}(X, X_1) = 1 - 0.74375 = 0.25625$.

4 Decision Making Applications for Adequacy of Online Education

Now, we give an algorithm based on the generalized set-valued neutrosophic quadruple numbers and Hausdroff measures on the generalized set-valued neutrosophic quadruple numbers for multi-criteria decision making method applications.

Algorithm 4.1:

Step 1: The criteria are determined. The criteria set get K.

$$K = \{k_1, k_2, \dots, k_n\}\ (n \in \mathbb{N})$$

The weight values of the criteria determined to $W = \{w_1, w_2, \dots, w_n\}$ $(n \in \mathbb{N})$ and $\sum_{i=1}^{n} w_i = 1$, $w_i \in \mathbb{N}$. Where,

w_1 is the weight of criterion k_1,

w_2 is the weight of criterion k_2,

w_3 is the weight of criterion k_3,

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w_n is the weight of criterion k_n.

Step 2: Let I be the ideal status. For the generalized set – valued neutrosophic quadruple numbers, we define I such that

$$I = \{k_1: (P(X),\ P(X)T_1,\ \emptyset I_1,\ \emptyset F_1),\ k_2: (P(Y),\ P(Y)T_2,\ \emptyset I_2,\ \emptyset F_2), \dots,\ k_n: (P(Z),\ P(Z)T_n,\ \emptyset I_n,\ \emptyset F_{n_i})\}.$$

Where,

$$T_1 = T_2 = \dots = T_n = 1$$

$$I_1 = I_2 = \cdots = I_n = 0$$

$$F_1 = F_2 = \cdots = F_n = 0 .$$

Step 3: The adequacy of the efficiency of the criteria should be assessed by samples references according to each criterion and each status should be identified as a generalized set valued neutrosophic quadruple numbers.

Let the sets of the samples references be

$$A_1 = \{k_1: (X_{1_1}, X_{1_2}T_{1_1}, X_{1_3}I_{1_1}, X_{1_4}F_{1_1}), k_2: (Y_{1_1}, Y_{1_2}T_{1_2}, Y_{1_3}I_{1_2}, Y_{1_4}F_{1_2}), \dots , \quad k_n: (Z_{1_1}, Z_{1_2}T_{1_n}, Z_{1_3}I_{1_n}, Z_{1_4}F_{1_n})\}$$

$$A_2 = \{k_1: (X_{2_1}, X_{2_2}T_{2_1}, X_{2_3}I_{2_1}, X_{2_4}F_{2_1}), k_2: (Y_{2_1}, Y_{2_2}T_{2_2}, Y_{2_3}I_{2_2}, Y_{2_4}F_{2_2}), \dots, \quad k_n: (Z_{2_1}, Z_{2_2}T_{2_n}, Z_{2_3}I_{2_n}, Z_{2_4}F_{2_n})\}$$

.

.

.

$$A_n = \{k_1: (X_{n_1}, X_{n_2}T_{n_1}, X_{n_3}I_{n_1}, X_{n_4}F_{n_1}), k_2: (Y_{n_1}, Y_{n_2}T_{n_2}, Y_{n_3}I_{n_2}, Y_{n_4}F_{n_2}), \dots, \quad k_m: (Z_{n_1}, Z_{n_2}T_{n_n}, Z_{n_3}I_{n_n}, Z_{n_4}F_{n_n})\}$$

and each samples reference is evaluated according to each criterion. Here;

$X_{i_j} \in P(X)$, $Y_{i_j} \in P(Y)$, … , $Z_{i_j} \in P(Z)$ (i = 1, 2, 3, …,n) (j = 1, 2, … , n)

Step 4: The sample reference criteria are given as generalized set valued neutrosophic quadruple numbers in Step 4. Now show them in Table 1.

Table 1. Example reference criterion table

	k_1	k_2	…	k_n
A_1	$(X_{1_1}, X_{1_2}T_{1_1}, X_{1_3}I_{1_1}, X_{1_4}F_{1_1})$	$(Y_{1_1}, Y_{1_2}T_{1_2}, Y_{1_3}I_{1_2}, Y_{1_4}F_{1_2})$	…	$(Z_{1_1}, Z_{1_2}T_{1_n}, Z_{1_3}I_{1_n}, Z_{1_4}F_{1_n})$
A_2	$(X_{2_1}, X_{2_2}T_{2_1}, X_{2_3}I_{2_1}, X_{2_4}F_{2_1})$	$(Y_{2_1}, Y_{2_2}T_{2_2}, Y_{2_3}I_{2_2}, Y_{2_4}F_{2_2})$	…	$(Z_{2_1}, Z_{2_2}T_{2_n}, Z_{2_3}I_{2_n}, Z_{2_4}F_{2_n})$

.	.	.	…	.
.	.	.	…	.
.	.	.	…	.
A_n	$(X_{n_1}, X_{n_2}T_{n_1}, X_{n_3}I_{n_1}, X_{n_4}F_{n_1})$	$(Y_{n_1}, Y_{n_2}T_{n_2}, Y_{n_3}I_{n_2}, Y_{n_4}F_{n_2})$	…	$(Z_{n_1}, Z_{n_2}T_{n_n}, Z_{n_3}I_{n_n}, Z_{n_4}F_{n_n})$

Step 5: Let's calculate the similarity values of the sample references with the I ideal criterion. While doing this, calculate $S_{QHN}\left(I_{k_j}, A_{i_{k_j}}\right)$ in Table 2.

Table 2. The I ideal criterion and the similarity values of the sample references

	k_1	k_2	…	k_n
A_1	$S_{QHN}(I_{k_1}, A_{1_{k_1}})$	$S_{QHN}(I_{k_2}, A_{1_{k_2}})$	…	$S_{QHN}(I_{k_n}, A_{1_{k_n}})$
A_2	$S_{QHN}(I_{k_1}, A_{2_{k_1}})$	$S_{QHN}(I_{k_2}, A_{2_{k_2}})$	…	$S_{QHN}(I_{k_n}, A_{2_{k_n}})$
.	.	.	…	.
.	.	.	.	.
.	.	.	.	.
A_n	$S_{QHN}(I_{k_1}, A_{n_{k_1}})$	$S_{QHN}(I_{k_2}, A_{n_{k_2}})$	…	$S_{QHN}(I_{k_n}, A_{n_{k_n}})$

Step 6: In this last step ın the similarity found, it is multiplied by the weight value of a criterion. For this, use the k-th weight value for each of the similarity values in the k-th column (k = 1, 2, …, n). Thus,

get the weighted similarity table in Table 3. The sum of A_i in Table 3 will be given as ${S_{QHN}}^i$ similarity value. (i = 1, 2, … n) ($n \in \mathbb{N}$)

Table 3. Weighted similarity table

	$w_1 k_1$	$w_2 k_2$	…	$w_n k_n$	$\sum_{i=1}^{n} w_i k_i = {S_{QHN}}^i$
A_1	$w_1.S_{QHN}(I_{k_1}, A_{1_{k_1}})$	$w_2.S_{QHN}(I_{k_2}, A_{1_{k_2}})$	…	$w_n.S_{QHN}(I_{k_n}, A_{1_{k_n}})$	${S_{QHN}}^1$
A_2	$w_1.S_{QHN}(I_{k_1}, A_{2_{k_1}})$	$w_2.S_{QHN}(I_{k_2}, A_{2_{k_2}})$	…	$w_n.S_{QHN}(I_{k_n}, A_{2_{k_n}})$	${S_{QHN}}^2$
. . .	. . .	. . .	… . .	. . .	. . .
A_n	$w_1.S_{QHN}(I_{k_1}, A_{n_{k_1}})$	$w_2.S_{QHN}(I_{k_2}, A_{n_{k_2}})$	…	$w_n.S_{QHN}(I_{k_n}, A_{n_{k_n}})$	${S_{QHN}}^n$

Example 4.2: The similarity measure is an important mathematical tool to deal with the problems we encounter in daily life. One of the bad consequences of the epidemic that affects the whole world is that we have to stop education. Therefore, education and training institutions have temporarily started online education practices so that students do not stay away from education. Of course, it has been seen that future online education does not have the same effect on students. Some of the factors that negatively affect students in this process are the environment, internet infrastructure, and the materials used in the course. In this section, the new similarity measure is applied to an online education problem. The generalized set-valued neutrosophic quadruple number is just a tool to deal with such cases, and for each evaluations for an alternative under the criterias can be considered as a generalized set-valued

neutrosophic quadruple number. Now, In the example below, 4 criteria and weight values of these criteria are determined in the first step. In step 2, the ideal set I to be referenced is written. In step 3, how to determine the efficiency of online courses, 10 student sets will be determined and these sets will be written as generalized set-valued neutrosophic quadruple number. The similarity values of these student sets with ideal set I are calculated and the results are multiplied by the weight values of the criteria. The similarity values of each criterion are added and the student with the best result is determined by finding the similarity values of each student separately.

Step 1: Let the set of criteria to be considered in evaluating the students' efficiency in online education be K.

$$K = \{k_1, k_2, k_3, k_4\}.$$

k_1 : Communication. The criterion weight values w_1= 0.4

k_2 : Lesson plan. The criterion weight values $w_2 = 0.2$

k_3 : Attendance. The criterion weight values $w_3 = 0.1$

k_4 : Source of Knowledge. The criterion weight values $w_4 = 0.3$

Step 2: For the I ideal student, in the generalized set valued neutrosophic quadruple set

$I = \{\, k_1\!: \big(\{x_1,\ x_2,\ x_3,\ x_4,\ x_5\}, \{x_1,\ x_2,\ x_3,\ x_4,\ x_5\}(1),\ \emptyset(0),\ \emptyset(0)\big),$

$k_2\!: \big(\{y_1,\ y_2,\ y_3,\ y_4,\ y_5\}, \{y_1,\ y_2,\ y_3,\ y_4,\ y_5\}(1),\ \emptyset(0),\ \emptyset(0)\big),$

$k_3\!: \big(\{z_1,\ z_2,\ z_3,\ z_4,\ z_5\}, \{z_1,\ z_2,\ z_3,\ z_4,\ z_5\}(1),\ \emptyset(0),\ \emptyset(0)\big),$

$k_4\!: \big(\{t_1,\ t_2,\ t_3,\ t_4,\ t_5\}, \{t_1,\ t_2,\ t_3,\ t_4,\ t_5\}(1),\ \emptyset(0),\ \emptyset(0)\big)\},$

Step 3: Each student whose adequacy of the efficiency of the online lessons will be evaluated according to each criterion and each student is determined as a generalized set valued neutrosophic quadruple number.

Let the set of the students be A = $\{A_1,\ A_2,\ A_3,\ A_4,\ A_5,\ A_6,\ A_7,\ A_8,\ A_9,\ A_{10}\}$.

$A_1 = \{k_1: (\{x_1, x_2, x_4, x_5\}, \{x_2, x_4\}(0.4), \{x_1, x_2, x_4\}(0.2), \{x_1, x_2\}(0.3)), k_2: (\{y_1, y_2, y_3, y_5\},$

$\{y_1, y_2, y_3\}(0.5), \{y_2, y_3, y_5\}(0.2), \{y_1, y_5\}(0.4)), k_3: (\{z_2, z_3, z_4, z_5\}, \{z_2, z_4, z_5\}(0.3), \{z_2\}(0.2),$

$\{z_2, z_3, z_5\}(0.4)), k_4: (\{t_1, t_2, t_3, t_4, t_5\}, \{t_1\}(0.2), \{t_4, t_5\}(0.3), \{t_4\}(0.2))\}$

$A_2 = \{k_1: (\{x_2, x_3\}, \{x_3\}(0.5), \{x_2, x_3\}(0.3), \{x_3\}(0.1)), k_2: (\{y_1, y_3, y_4, y_5\}, \{y_1, y_4, y_5\}(0.7), \{y_3\}(0.1),$
$\{y_1\}(0.1)), k_3: (\{z_5\}, \{z_5\}(0.6), \{z_5\}(0.3), \emptyset(0.1)), k_4: (\{t_1, t_2, t_3\}, \{t_1, t_2\}(0.5), \{t_2, t_3\}(0.2),$

$\{t_1, t_3\}(0.2))\}$

$A_3 = \{k_1: (\{x_1, x_2, x_3, x_4\}, \{x_1, x_2, x_3\}(0.6), \{x_1, x_2, x_3, x_4\}(0.2), \{x_1, x_3, x_4\}(0.3)), k_2: (\{y_1, y_2, y_3, y_4\},$

$\{y_1, y_2, y_3, y_4\}(0.09), \{y_1, y_2, y_3\}(0.05), \{y_1, y_2, y_3, y_4\}(0.01)), k_3: (\{z_5\}, \{z_5\}(0.4), \emptyset(0.1), \emptyset(0.3)),$

$k_4: (\{t_1, t_2, t_3, t_4\}, \{t_1, t_2, t_3, t_4\}0.7), \emptyset(0.7), \emptyset(0.7))\}$

$A_4 = \{k_1: (\{x_1, x_2, x_3, x_4\}, \emptyset(0.5), \emptyset(0.1), \emptyset(0.1)), k_2: (\{y_1, y_2, y_4, y_5\}, \{y_1, y_4, y_5\}(0.4), \emptyset(0.6),$

$\{y_1, y_2\}(0.8)), k_3: (\{z_1, z_2, z_3, z_4\}, \{z_1, z_2\}(0.8), \emptyset(0.7), \{z_3\}(0.3)), k_4: (\{t_1, t_2, t_3, t_4, t_5\},$

$\{t_1, t_4\}(0.6), \{t_1, t_3, t_4, t_5\}(0.1), \{t_1, t_4\}(0.2))\}$

$A_5 = \{k_1: (\{x_1, x_2, x_3\}, \{x_2, x_3\}(0.9), \{x_1, x_2, x_3\}(0.02), \{x_1, x_3\}(0.1)), k_2: (\{y_2, y_3\}, \{y_2, y_3\}(0.7),$

$\{y_3\}(0.6), \{y_2, y_3\}(0.4)), k_3: (\{z_3, z_4, z_5\}, \{z_3\}(0.1), \{z_5\}(0.1), \{z_3, z_5\}(0.2)), k_4: (\{t_1, t_2, t_5\}, \emptyset(0.8),$

$\{t_1, t_5\}(0.9), \{t_5\}(0.9))\}$

$A_6 = \{k_1: (\{x_1, x_3\}, \{x_5\}(0.04), \{x_3, x_4\}(0,06), \emptyset(0.003)), k_2: (\{y_2, y_3, y_4, y_5\},$

$\emptyset(0.07), \emptyset(0.02), \{y_2\}(0.01)), k_3: (\{z_2\}, \{z_2, z_3, z_4\}(0.4),$

$\{z_2, z_3, z_4, z_5\}(0.02), \emptyset(0.02)), k_4: (\emptyset, \{t_1, t_5\}(0.004), \{t_3, t_4\}(0.02),$

$\{t_5\}(0.5))\}$

$A_7 = \{k_1: (\{x_1, x_3, x_4, x_5\}, \{x_5\}(0.9), \{x_5\}(0.8), \emptyset(0.08)), k_2: (\{y_1, y_2, y_3, y_4, y_5\},$

$\{y_1, y_2, y_3, y_4, y_5\}(0.1), \emptyset(0.1), \emptyset(0.1)), k_3: (\emptyset, \{z_4, z_5\}(0.6), \emptyset(0.3), \{z_4\}(0.9)),$

$k_4: (\emptyset, \emptyset(0.9), \emptyset\ (0.9), \emptyset(0.1))\}$

$A_8 \ = \ \{k_1: (\{x_1, x_2, x_3, x_4, x_5\}, \{x_1, x_2, x_3, x_4, x_5\}(0.3), \{x_1, x_2, x_3, x_4, x_5\}(0.5), \{x_1, x_2, x_3, x_4, x_5\}(0.2)),$

$k_2: (\{y_4\}, \emptyset(0.8), \{y_4\}(0.3), \emptyset(0.1)), k_3: (\{z_2\}, \emptyset(0.2), \emptyset(0.02), \emptyset(0.1)),$

$k_4: (\{t_1, t_5\}, \emptyset(0.9), \emptyset(0.1), \{t_5\}(0.1))\}$

$A_9 = \ \{k_1: (\emptyset, \emptyset(0.2), \emptyset(0.2), \emptyset(0.1)), k_2: (\{y_1, y_3, y_4\}, \{y_3, y_4\}(0.6), \{y_4\}(0.03), \{z_1, z_3, y_4\}(0.09)),$

$k_3: (\{z_2, z_3, z_4, z_5\}, \{z_2\}(0.1), \{z_2, z_3, z_4, z_5\}(0.7), \{z_3, z_4, z_5\}(0.7)),$

$k_4: (\{t_1, t_2, t_3, t_4, t_5\}, \{t_4\}(0.6), \{t_3\}(0.9), \{t_2\}(0.9))\}$

$A_{10} = \{k_1: (\emptyset, \emptyset(0.01), \emptyset(0.02), \emptyset(0.02)), k_2: (\{y_1, y_2, y_3, y_4, y_5\}, \{y_1, y_2, y_3, y_4, y_5\}(0.4),$

$\emptyset(0.1), \{y_3\}(0.1)), k_3: (\emptyset, \emptyset(0.01), \emptyset(0.01), \emptyset(0.01)), k_4: (\emptyset, \emptyset(0.05), \emptyset(0.05), \emptyset(0.1))\}$

Step 4: We **s**how the criteria of the students which were given as neutrosophic quadruple sets in Table 4.

Table 4. Student criteria table

	k_1	k_2	k_3	k_4
A_1	$(\{x_1, x_2, x_4, x_5\},$ $\{x_2, x_4\}(0.4),$ $\{x_1, x_2, x_4\}(0.2),$ $\{x_1, x_2\}(0.3))$	$(\{y_1, y_2, y_3, y_5\},$ $\{y_1, y_2, y_3\}(0.5),$ $\{y_2, y_3, y_5\}(0.2),$ $\{y_1, y_5\}(0.4))$	$(\{z_2, z_3, z_4, z_5\},$ $\{z_2, z_4, z_5\}(0.3),$ $\{z_2\}(0.2),$ $\{z_2, z_3, z_5\}(0.4))$	$(\{t_1, t_2, t_3, t_4, t_5\}$ $\{t_1\}(0.2),$ $\{t_4, t_5\}(0.3),$ $\{t_4\}(0.2))$

A_2	$(\{x_2, x_3\},$ $\{x_3\}(0.5),$ $\{x_2, x_3\}(0.3),$ $\{x_3\}(0.1))$	$(\{y_1, y_3, y_4, y_5\},$ $\{y_1, y_4, y_5\}(0.7),$ $\{y_3\}(0.1),$ $\{y_1\}(0.1))$	$(\{z_5\},$ $\{z_5\}(0.6),$ $\{z_5\}(0.3),$ $\emptyset(0.1))$	$(\{t_1, t_2, t_3\},$ $\{t_1, t_2\}(0.5),$ $\{t_2, t_3\}(0.2),$ $\{t_1, t_3\}(0.2))$
A_3	$(\{x_1, x_2, x_3, x_4\},$ $\{x_1, x_2, x_3\}(0.6),$ $\{x_1, x_2, x_3, x_4\}(0.2),$ $\{x_1, x_3, x_4\}(0.3))$	$(\{y_1, y_2, y_3, y_4\},$ $\{y_1, y_2, y_3, y_4\}(0.09),$ $\{y_1, y_2, y_3\}(0.05),$ $\{y_1, y_2, y_3, y_4\}(0.01))$	$(\{z_5\},$ $\{z_5\}(0.4),$ $\emptyset(0.1),$ $\emptyset(0.3))$	$(\{t_1, t_2, t_3, t_4\},$ $\{t_1, t_2, t_3, t_4\}0.7),$ $\emptyset(0.7),$ $\emptyset(0.7))$
A_4	$(\{x_1, x_2, x_3, x_4\},$ $\emptyset(0.5),$ $\emptyset(0.1),$ $\emptyset(0.1))$	$(\{y_1, y_2, y_4, y_5\},$ $\{y_1, y_4, y_5\}(0.4),$ $\emptyset(0.6),$ $\emptyset(0.8))$	$(\{z_1, z_2, z_3, z_4\},$ $\{z_1, z_2\}(0.8),$ $\emptyset(0.7),$ $\{z_3\}(0.3))$	$(\{t_1, t_2, t_3, t_4, t_5\},$ $\{t_1, t_4\}(0.6),$ $\{t_1, t_3, t_4, t_5\}(0.1),$ $\{t_1, t_4\}(0.2))$
A_5	$(\{x_1, x_2, x_3\},$ $\{x_2, x_3\}(0.9),$ $\{x_1, x_2, x_3\}(0.02),$ $\{x_1, x_3\}(0.1))$	$(\{y_2, y_3\},$ $\{y_2, y_3\}(0.7),$ $\{y_3\}(0.6),$ $\{y_2, y_3\}(0.4))$	$(\{z_3, z_4, z_5\},$ $\{z_3\}(0.1),$ $\{z_5\}(0.1),$ $\{z_3, z_5\}(0.2))$	$(\{t_1, t_2, t_5\},$ $\emptyset(0.8),$ $\{t_1, t_5\}(0.9),$ $\{t_5\}(0.9))$

A_6	$(\{x_1, x_3\}$, $\{x_5\}(0.04)$, $\{x_3, x_4\}(0.06)$, $\emptyset(0.003))$	$(\{y_2, y_3, y_4, y_5\}$, $\emptyset(0.07)$, $\emptyset(0.02)$, $\{y_2\}(0.01))$	$(\{z_2\}$, $\{z_2, z_3, z_4\}(0.4)$, $\{z_2, z_3, z_4,$ $z_5\}(0.02)$, $\emptyset(0.02))$	$\emptyset$, $\{t_1, t_5\}(0.004)$, $\{t_3, t_4\}(0.02)$, $\{t_5\}(0.5))$
A_7	$(\{x_1, x_3, x_4, x_5\}$, $\{x_5\}(0.9)$, $\{x_5\}(0.8)$, $\emptyset(0.08))$	$(\{y_1, y_2, y_3, y_4,$ $y_5\}$, $\{y_1, y_2, y_3, y_4,$ $y_5\}(0.1)$, $\emptyset(0.1)$, $\emptyset(0.1))$	$(\emptyset$, $\{z_4, z_5\}(0.6)$, $\emptyset(0.3)$, $\{z_4\}(0.9))$	$(\emptyset$, $\emptyset(0.9)$, $\emptyset(0.9)$, $\emptyset(0.1))$
A_8	$(\{x_1, x_2,$ $x_3, x_4, x_5\}$, $\{x_1, x_2,$ $x_3, x_4, x_5\}(0.3)$, $\{x_1, x_2,$ $x_3, x_4, x_5\}(0.5)$, $\{x_1, x_2,$ $x_3, x_4, x_5\}(0.2))$	$(\{y_4\}$, $\emptyset(0.8)$, $\{y_4\}(0.3)$, $\emptyset(0.01))$	$(\{z_3\}$, $\emptyset(0.2)$, $\emptyset(0.02)$, $\emptyset(0.1))$	$(\{t_1, t_5\}$, $\emptyset(0.9)$, $\emptyset(0.1)$, $\{t_5\}(0.1))$
A_9	$(\emptyset$, $\emptyset(0.2)$, $\emptyset$ (0.2),	$(\{y_1, y_3, y_4\}$, $\{y_3, y_4\}(0.6)$, $\{y_4\}(0.03)$,	$(\{z_2, z_3, z_4, z_5\}$, $\{z_2\}(0.1)$, $\{z_2, z_3, z_4,$	$(\{t_1, t_2, t_3, t_4, t_5\}$, $\{t_5\}(0.6)$, $\{t_3\}(0.9)$,

	∅(0.1))	$\{y_1, y_3, y_4\}$(0.09))	$z_5\}$(0.7), $\{z_3, z_4, z_5\}$(0.7))	$\{t_2\}$(0.9))
A_{10}	(∅, ∅(0.01), ∅(0.02), ∅(0.02))	($\{y_1, y_2, y_3, y_4, y_5\}$, $\{y_1, y_2, y_3, y_4, y_5\}$(0.4), ∅(0.1), $\{y_3\}$(0.1))	(∅, ∅(0.01), ∅(0.01), ∅(0.01))	(∅, ∅(0.05), ∅(0.05), ∅(0.1))

Step 5: We calculate the individual evaluation values of the students given in Table 4 with respect to the criteria values of the I ideal student given in Step 3, one by one, using the measure of similarity. Thus, we obtain Table 5.

Table 5. Similarity table

	k_1	k_2	k_3	k_4
A_1	0.4750	0.3500	0.5250	0.4625
A_2	0.6750	0.6175	0.7000	0.5125
A_3	0.7250	0.9125	0.4750	0.5000
A_4	0,7750	0.4375	0.4875	0.7125
A_5	0.8250	0.5375	0.3875	0.3750
A_6	0.1270	0.1600	0.400	0.127
A_7	0.9500	0.8000	0.9750	0.5500

A_8	0.6000	0.6325	0.3125	0.7375
A_9	0.3000	0.6500	0.4500	0.4000
A_{10}	0.3050	0.5875	0.2550	0.2750

Step 6: We multiply the weights of all criteria in Step 2 with each of the similarity values in Table 6. The sum of the similarity values of each criterion is given as the similarity value of our student set.

Table 6. Weighted similarity table

	$0,4 * k_1$	$0,2 * k_2$	$0,1 * k_3$	$0,3 * k_4$	$\sum_{i=1}^{4} w_i k_i = S_{QHN}{}^i$
A_1	0.19000	0.07000	0.05250	0.13875	$S_{QHN}{}^1(\mathrm{I}, A_1) = 0.45125$
A_2	0.27000	0.12350	0.00700	0.15375	$S_{QHN}{}^2(\mathrm{I}, A_2) = 0.555425$
A_3	0.29000	0.18250	0.04750	0.15000	$S_{QHN}{}^3(\mathrm{I}, A_3) = 0.67000$
A_4	0.31000	0.08750	0.04875	0.21375	$S_{QHN}{}^4(\mathrm{I}, A_4) = 0.66000$
A_5	0.33000	0.10750	0.03875	0.11250	$S_{QHN}{}^5(\mathrm{I}, A_5) = 0.58875$
A_6	0.05080	0.03200	0.04000	0.03810	$S_{QHN}{}^6(\mathrm{I}, A_6) = 0.1609$
A_7	0.38000	0.16000	0.09750	0.16500	$S_{QHN}{}^7(\mathrm{I}, A_7) = 0.80250$
A_8	0.24000	0.12650	0.03125	0.22125	$S_{QHN}{}^8(\mathrm{I}, A_8) = 0.62025$
A_9	0.12000	0.13000	0.04500	0.12000	$S_{QHN}{}^9(\mathrm{I}, A_9) = 0.41500$
A_{10}	0.12200	0.11750	0.02550	0.08250	$S_{QHN}{}^{10}(\mathrm{I}, A_{10}) = 0.34750$

$$A_7 > A_3 > A_4 > A_8 > A_5 > A_2 > A_1 > A_9 > A_{10} > A_6$$

Similarity values of each student were calculated. According to the results, the most efficient student in online education is A_7 student with similarity value of 0.80250.

5 Numerical Comparison Analysis

In this section, we will compare the results of Euclid similarity measure [23], Dice similarity measure [22] and Hausdorff similarity measure [15] using the only values (T, I, F) for which we calculate the similarity value with the Hausdorff measures based on generalized set-valued neutrosophic quadruple numbers.

i) The result of calculating the similarity value of the students calculated in 4.2 with Hausdorff similarity measure [15] in Table 7.

Table 7. Result according to Hausdorff similarity measure [15]

A_1	0,650
A_2	0.450
A_3	0.628
A_4	0,550
A_5	0,520
A_6	0,228
A_7	0,860
A_8	0,450
A_9	0,760
A_{10}	0,283

$$A_7 > A_9 > A_1 > A_3 > A_4 > A_5 > A_2 = A_8 > A_{10} > A_6$$

ii) The result of calculating the similarity value of the students calculated in 4.2 with Euclid similarity measure [23] in Table 8.

Table 8. Result according to Euclid similarity measure [23]

A_1	0,7463
A_2	0.8244
A_3	0.7407
A_4	0,7827
A_5	0,7689
A_6	0,5308
A_7	0,6981
A_8	0,8129
A_9	0,6836
A_{10}	0,6980

$$A_2 > A_8 > A_4 > A_5 > A_1 > A_3 > A_7 > A_{10} > A_9 > A_6$$

iii) The result of calculating the similarity value of the students calculated in 4.2 with Dice similarity measure [22] in Table 9.

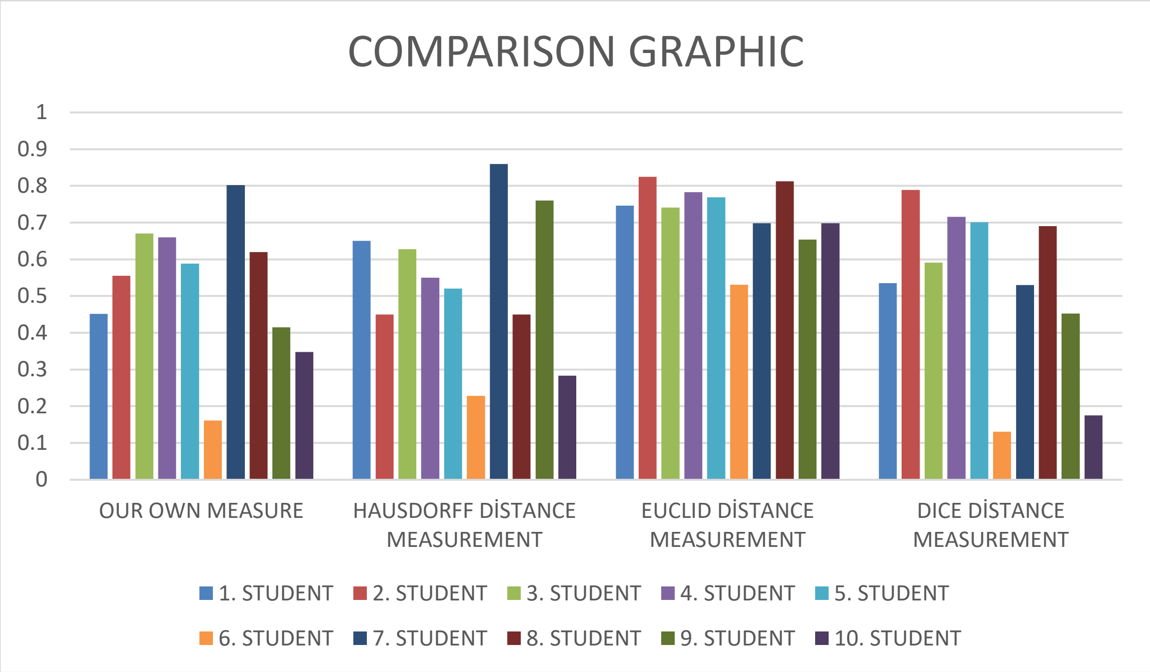
Table 9. Result according to Dice similarity measure [22]

A_1	0,5349
A_2	0.7892
A_3	0.5911
A_4	0,7162
A_5	0,7008
A_6	0,1303
A_7	0,5301
A_8	0,6908
A_9	0,4528
A_{10}	0,1748

$$A_2 > A_4 > A_5 > A_8 > A_3 > A_1 > A_7 > A_9 > A_{10} > A_6$$

From i, ii, iii; we obtain Graphic 1.

Graphic 1: Comparison of similarity measures



6 Conclusions

In this study, a new decision making application based on generalized set – valued neutrosophic quadruple numbers has been developed to calculate the efficiency of students participating in online education, which is applied to students who have to take a break from their education. We define some measures for generalized set-valued neutrosophic quadruple sets. We proved that this similarity measures satisfies the similarity conditions. Using this similarity measure, we developed an algorithm to evaluate the adequacy of online education applied to ensure that students' education is not interrupted by the epidemic, and we gave an example through this algorithm. In the developed algorithm and in the example given, we determined the highest efficiency student among the students taking courses with online education by using the generalized set-valued neutrosophic quadruple numbers. Also, we obtain different result from some previous applications (based on neutrosophic sets) for decision making algorithm. In future, we will discuss the following integration of the related topics;

1) This measure and algorithm we have obtained can be used not only for online education, but also to evaluate the competence of any newly designed application, the competence of the people who will enter the profession and its effect on a law.

2) For proposed method the effect of a drug on a particular disease.

3) For proposed method more than one expert opinion can be obtained and different weight values can be created for each expert.

4) In addition, criteria and criterion weights can be selected as desired in proposed method.

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Received: Sep 5, 2020. Accepted: Feb 10, 2021