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Neutrosophic Pre-α, Semi-α & Pre- β Irresolute Functions

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Abstract: Smarandache introduced and developed interesting concepts Neutrosophic set from the Intuitionistic fuzzy sets. A.A. Salama introduced NTSs and continuity. Aim of this paper is we introduce and study the concepts Neutrosophic Pre-α, Semi- α & Pre- β Irresolute Functions and its Properties are discussed details.

Keywords: Neutrosophic Irresolute Functions, Neutrosophic Pre-α, Neutrosophic Semi- α, Neutrosophic Pre- β Irresolute Functions.

1. Introduction

Neutrosophic concepts have wide range of applications in the area of decision making Artificial Intelligence, Information Systems, Computer Science, Medicine, Applied Mathematics, Mechanics, Electrical & Electronic and, Management Science, etc,. In 1980s the international movement called paradoxism based on contradictions in science and literature, was founded by Smarandache[15,16], who then extended it to neutrosophy, based on contradictions and their neutrals. The mapping is the one of the important concept in topology. Neutrosophic sets have three kind like T Truth, F -Falsehood, I- Indeterminacy. Neutrosophic topological spaces (N-T-S) introduced by Salama [27,28]etal., by using Smarandache neutrosophy set. In this Paper new type of functions called as Neutrosophic Pre-α irresolute functions, Neutrosophic Pre-α, Semi- α and Pre-β Irresolute Functions. Also the interrelationships of these functions with the other existing functions are established. Several characterizations and some interesting properties of these classes of functions are given

2. Preliminaries

In this section, we provide basic definition and operation of Neutrosophic sets and its Results

Definition 2.1 [15,16] Let $\mathcal{X}_{\mathcal{N}}$ be a non-empty fixed set. A Neutrosophic set $\mathcal{E}_1^*$ is a object having the form

$$\mathcal{E}_1^* = \{< x, \mu_{\mathcal{E}_1^*}(x), \sigma_{\mathcal{E}_1^*}(x), \gamma_{\mathcal{E}_1^*}(x) >: x \in \mathcal{X}_{\mathcal{N}}\},$$

$\mu_{\mathcal{E}_1^*}(x)$- membership function

$\sigma_{\mathcal{E}_1^*}(x)$- indeterminacy and then

$\gamma_{\mathcal{E}_1^*}(x)$- non-membership function

Definition 2.2 [15,16]. Neutrosophic set $\mathcal{E}_1^* = \{< x, \mu_{\mathcal{E}_1^*}(x), \sigma_{\mathcal{E}_1^*}(x), \gamma_{\mathcal{E}_1^*}(x) >: x \in \mathcal{X}_{\mathcal{N}}\}$, on $\mathcal{X}_{\mathcal{N}}$ and

$\forall x \in \mathcal{X}_{\mathcal{N}}$

$\mathcal{E}_2^* = \{< x, \mu_{\mathcal{E}_2^*}(x), \sigma_{\mathcal{E}_2^*}(x), \gamma_{\mathcal{E}_2^*}(x) >: x \in \mathcal{X}_{\mathcal{N}}\}$

1. $\mathcal{E}_1^* \cap \mathcal{E}_2^* = \{< x, \mu_{\mathcal{E}_1^*}(x) \cap \mu_{\mathcal{E}_2^*}(x), \sigma_{\mathcal{E}_1^*}(x) \cap \sigma_{\mathcal{E}_2^*}(x), \gamma_{\mathcal{E}_1^*}(x) \cup \gamma_{\mathcal{E}_2^*}(x) >: x \in \mathcal{X}_{\mathcal{N}}\}$

2. $\mathcal{E}_1^* \cup \mathcal{E}_2^* = \{< x, \mu_{\mathcal{E}_1^*}(x) \cup \mu_{\mathcal{E}_2^*}(x), \sigma_{\mathcal{E}_1^*}(x) \cup \sigma_{\mathcal{E}_2^*}(x), \gamma_{\mathcal{E}_1^*}(x) \cap \gamma_{\mathcal{E}_2^*}(x) >: x \in \mathcal{X}_{\mathcal{N}}\}$

3. $\mathcal{E}_1^* \subseteq \mathcal{E}_2^* \Leftrightarrow \mu_{\mathcal{E}_1^*}(x) \leq \mu_{\mathcal{E}_2^*}(x), \sigma_{\mathcal{E}_1^*}(x) \leq \sigma_{\mathcal{E}_2^*}(x) \& \gamma_{\mathcal{E}_1^*}(x) \geq \gamma_{\mathcal{E}_2^*}(x)\}$

4. the complement of $\mathcal{E}_1^*$ is $\mathcal{E}_1^{*C} = \{< x, \gamma_{\mathcal{E}_1^*}(x), 1 - \sigma_{\mathcal{E}_1^*}(x), \mu_{\mathcal{E}_1^*}(x) >: x \in \mathcal{X}_{\mathcal{N}}\}$

Definition 2.3 [28]. Let $\mathcal{X}_{\mathcal{N}}$ be non-empty set and τ_N be the collection of Neutrosophic subsets of $\mathcal{X}_{\mathcal{N}}$ satisfying the following properties:

1.$0_N, 1_N \in \tau_N$

3.$T_1 \cap T_2 \in \tau_N$ for any $T_1, T_2 \in \tau_N$

4. $\cup T_i \in \tau_N$ for every $\{T_i : i \in j\} \subseteq \tau_N$

Then the space $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ is called a Neutrosophic topological spaces (N-T-S).

The element of τ_N are called Ne.OS (Neutrosophic open set)

and its complement is Ne.CS(Neutrosophic closed set)

Example 2.4. Let $\mathcal{X}_{\mathcal{N}}$ ={x} and $\forall x \in \mathcal{X}_{\mathcal{N}}$

$A_1 = \langle x, \frac{6}{10}, \frac{6}{10}, \frac{5}{10} \rangle$, $A_2 = \langle x, \frac{5}{10}, \frac{7}{10}, \frac{9}{10} \rangle$

$A_3 = \langle x, \frac{6}{10}, \frac{7}{10}, \frac{5}{10} \rangle$, $A_4 = \langle x, \frac{5}{10}, \frac{6}{10}, \frac{9}{10} \rangle$

Then the collection $\tau_N = \{0_N, A_1, A_2, A_3, A_4, 1_N\}$ is called a N-T-S on $\mathcal{X}_{\mathcal{N}}$.

Definition 2.5. Let $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ be a N-T-S and $\mathcal{E}_1^* = \{< x, \mu_{\mathcal{E}_1^*}(x), \sigma_{\mathcal{E}_1^*}(x), \gamma_{\mathcal{E}_1^*}(x) >: x \in \mathcal{X}_{\mathcal{N}}\}$ be a

Neutrosophic set in $\mathcal{X}_{\mathcal{N}}$. Then $\mathcal{E}_1^*$ is named as

1. Neutrosophic b closed set [20] (Ne.bCS) if Ne.cl(Ne.int($\mathcal{E}_1^*$))∩Ne.int(Ne.cl($\mathcal{E}_1^*$))⊆ $\mathcal{E}_1^*$,

3. Neutrosophic α-closed set [7] (Ne. αCS) if Ne.cl(Ne.int(Ne.cl($\mathcal{E}_1^*$)))⊆ $\mathcal{E}_1^*$,

4. Neutrosophic pre-closed set [30] (Ne.Pre-CS) if Ne.cl(Ne.int($\mathcal{E}_1^*$))⊆ $\mathcal{E}_1^*$,

5. Neutrosophic regular closed set [7] (Ne.RCS) if Ne.cl(Ne.int($\mathcal{E}_1^*$))=$\mathcal{E}_1^*$,

5. Neutrosophic semi closed set [17] (Ne.SCS) if Ne.int(Ne.cl($\mathcal{E}_1^*$))⊆ $\mathcal{E}_1^*$,

Definition 2.6.[9] $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ be a N-T-S and $\mathcal{E}_1^* = \{< x, \mu_{\mathcal{E}_1^*}(x), \sigma_{\mathcal{E}_1^*}(x), \gamma_{\mathcal{E}_1^*}(x) >: x \in \mathcal{X}_{\mathcal{N}}\}$ be a

Neutrosophic set in $\mathcal{X}_{\mathcal{N}}$.Then

Neutrosophic closure of $\mathcal{E}_1^*$ is Ne.Cl($\mathcal{E}_1^*$)=∩{H:H is a Ne.CS in $\mathcal{X}_{\mathcal{N}}$ and $\mathcal{E}_1^*$ ⊆H}

Neutrosophic interior of $\mathcal{E}_1^*$ is Ne.Int($\mathcal{E}_1^*$)=∪{M:M is a Ne.OS in $\mathcal{X}_{\mathcal{N}}$ and M⊆ $\mathcal{E}_1^*$}.

Definition 2.7. Let $(\mathcal{X}_{\mathcal{N}}, \mathcal{T}_{\mathcal{N}})$ be an NTS and be an NS in $\mathcal{X}_{\mathcal{N}}$.

The Neutrosophic β-closure &β-interior of A are defined by

(i) $\mathcal{N}\beta cl(\mathcal{E}_1^*) = \cap\{\mathcal{E}_3^* : \mathcal{E}_3^*$ is a βCS in $\mathcal{X}_{\mathcal{N}}$ and $\mathcal{E}_3^* \supseteq \mathcal{E}_1^*\}$;

(ii) $\mathcal{N}\beta\mathrm{int}(\mathcal{E}_1^*) = \cup\{\mathcal{E}_4^*: \mathcal{E}_4^*$ is a $\mathcal{N}\beta OS$ in $\mathcal{X}_\mathcal{N}$ and $\mathcal{E}_4^* \subseteq \mathcal{E}_1^*\}$.

Lemma 2.8.

Let $\mathcal{E}_1^*$ be an NS in NTS $(\mathcal{X}_\mathcal{N},\mathcal{T}_\mathcal{N})$.Then

(i) $\mathrm{Nint}(\mathcal{E}_1^*) \subseteq \mathrm{NP\,int}(\mathcal{E}_1^*) \subseteq \mathcal{E}_1^* \subseteq \mathrm{NPcl}(\mathcal{E}_1^*) \subseteq \mathrm{Ncl}(\mathcal{E}_1^*)$

(ii) $\mathrm{Nint}(\mathcal{E}_1^*) \subseteq \mathrm{N}\alpha\mathrm{int}(\mathcal{E}_1^*) \subseteq \mathcal{E}_1^* \subseteq \mathrm{N}\alpha\mathrm{cl}(\mathcal{E}_1^*) \subseteq \mathrm{Ncl}(\mathcal{E}_1^*)$

(iii) $\mathrm{Nint}(\mathcal{E}_1^*)\subseteq\mathrm{NSint}(\mathcal{E}_1^*)\subseteq\mathcal{E}_1^*\subseteq\mathrm{NScl}(\mathcal{E}_1^*)\subseteq\mathrm{Ncl}(\mathcal{E}_1^*)$

(iv) $\mathrm{Nint}(\mathcal{E}_1^*)\subseteq\mathrm{N}\beta\mathrm{int}(\mathcal{E}_1^*)\subseteq \mathcal{E}_1^* \subseteq \mathrm{N}\beta\mathrm{cl}(\mathcal{E}_1^*)\subseteq\mathrm{Ncl}(\mathcal{E}_1^*)$.

Proof: It is easy to prove.

3.Neutrosophic Pre-α, Semi- α & Pre- β Irresolute Functions

In this section Neutrosophic pre-α-irresolute, semi-α-irresolute, Neutrosophic pre-β-irresolute functions are defined. Also, the relationships of these functions with the other existing functions are studied.

Definition 3.1.

A function $\ddot{f}: (\mathcal{X}_\mathcal{N},\mathcal{T}_\mathcal{N}) \longrightarrow (\mathcal{Y}_\mathcal{N},\mathcal{G}_\mathcal{N})$ from an NTS $(\mathcal{X}_\mathcal{N},\mathcal{T}_\mathcal{N})$ to another NTS $(\mathcal{Y}_\mathcal{N},\mathcal{G}_\mathcal{N})$ is named as Neutrosophic β-irresolute if $\ddot{f}^{-1}(\mathcal{E}_2^*)$is a $\mathcal{N}\beta OS$ in $(\mathcal{X}_\mathcal{N},\mathcal{T}_\mathcal{N})$ for each $\mathcal{N}\beta OS$ $\mathcal{E}_2^*$ in $(\mathcal{Y}_\mathcal{N},\mathcal{G}_\mathcal{N})$.

Definition 3.2 A function$\ddot{f}: (\mathcal{X}_\mathcal{N},\mathcal{T}_\mathcal{N}) \longrightarrow (\mathcal{Y}_\mathcal{N},\mathcal{G}_\mathcal{N})$ from an NTS $(\mathcal{X}_\mathcal{N},\mathcal{T}_\mathcal{N})$ to another NTS $(\mathcal{Y}_\mathcal{N},\mathcal{G}_\mathcal{N})$ is named as Neutrosophic pre-α-irresolute if $\ddot{f}^{-1}(\mathcal{E}_2^*)$ is an NPOS in $(\mathcal{X}_\mathcal{N},\mathcal{T}_\mathcal{N})$ for each NαOS $\mathcal{E}_2^*$ in $(\mathcal{Y}_\mathcal{N},\mathcal{G}_\mathcal{N})$.

Definition 3.3 A function $\ddot{f}: (\mathcal{X}_\mathcal{N},\mathcal{T}_\mathcal{N}) \longrightarrow (\mathcal{Y}_\mathcal{N},\mathcal{G}_\mathcal{N})$ from an NTS $(\mathcal{X}_\mathcal{N},\mathcal{T}_\mathcal{N})$ to another NTS $(\mathcal{Y}_\mathcal{N},\mathcal{G}_\mathcal{N})$ is named as Neutrosophic α-irresolute if $\ddot{f}^{-1}(\mathcal{E}_2^*))$ is a NαOS in $(\mathcal{X}_\mathcal{N},\mathcal{T}_\mathcal{N})$ for each NαOS $\mathcal{E}_2^*$ in $(\mathcal{Y}_\mathcal{N},\mathcal{G}_\mathcal{N})$.

Definition 3.4 A function $\ddot{f}: (\mathcal{X}_\mathcal{N},\mathcal{T}_\mathcal{N}) \longrightarrow (\mathcal{Y}_\mathcal{N},\mathcal{G}_\mathcal{N})$ from an NTS $(\mathcal{X}_\mathcal{N},\mathcal{T}_\mathcal{N})$ to another NTS $(\mathcal{Y}_\mathcal{N},\mathcal{G}_\mathcal{N})$ is named as Neutrosophic semi-α-irresolute if $\ddot{f}^{-1}(\mathcal{E}_2^*))$ is an NSOS in $(\mathcal{X}_\mathcal{N},\mathcal{T}_\mathcal{N})$ for each NαOS $\mathcal{E}_2^*$ in $(\mathcal{Y}_\mathcal{N},\mathcal{G}_\mathcal{N})$.

Definition 3.5 A function $\ddot{f}: (\mathcal{X}_\mathcal{N},\mathcal{T}_\mathcal{N}) \longrightarrow (\mathcal{Y}_\mathcal{N},\mathcal{G}_\mathcal{N})$ from an NTS $(\mathcal{X}_\mathcal{N},\mathcal{T}_\mathcal{N})$ to another NTS $(\mathcal{Y}_\mathcal{N},\mathcal{G}_\mathcal{N})$ is named as Neutrosophic pre-β-irresolute if $\ddot{f}^{-1}(\mathcal{E}_2^*))$ is a NPOS in $(\mathcal{X}_\mathcal{N},\mathcal{T}_\mathcal{N})$ for each $\mathcal{N}\beta OS$ $\mathcal{E}_2^*$ i$\mathcal{X}_\mathcal{N}$n $(\mathcal{Y}_\mathcal{N},\mathcal{G}_\mathcal{N})$.

Proposition 3.6 Every Nα-irresolute function is Npre-α (NSemi-α,resp.)-irresolute function.

Proof: Let $\ddot{f}: (\mathcal{X}_\mathcal{N},\mathcal{T}_\mathcal{N}) \longrightarrow (\mathcal{Y}_\mathcal{N},\mathcal{G}_\mathcal{N})$ be Nα-irresolute function from NTS$(\mathcal{X}_\mathcal{N},\mathcal{T}_\mathcal{N})$ to another NTS $(\mathcal{Y}_\mathcal{N},\mathcal{G}_\mathcal{N})$. Let $\mathcal{E}_2^*$ be NαOS in $\mathcal{Y}_\mathcal{N}$. Since $\ddot{f}$ is Nα-irresolute function,$\ddot{f}^{-1}(\mathcal{E}_2^*))$ is NαOS in $\mathcal{X}_\mathcal{N}$. Every NαOS is NPOS (NSOS, resp.). So $\ddot{f}^{-1}(\mathcal{E}_2^*))$ is NPOS (NSOS, resp.) in $\mathcal{X}_\mathcal{N}$. Hence $\ddot{f}$ is Npre-α (NSemi-α, resp.)-irresolute function.

Proposition 3.7 Every Npre-β-irresolute function is Npre-α-(Npre, resp.) irresolute function.

Proof: Let $\ddot{f}: (\mathcal{X}_\mathcal{N},\mathcal{T}_\mathcal{N}) \longrightarrow (\mathcal{Y}_\mathcal{N},\mathcal{G}_\mathcal{N})$ be Npre-β irresolute function from NTS $(\mathcal{X}_\mathcal{N},\mathcal{T}_\mathcal{N})$ to another NTS $(\mathcal{Y}_\mathcal{N},\mathcal{G}_\mathcal{N})$. Let $\mathcal{E}_2^*$ be NαOS (NPOS resp.) in $\mathcal{Y}_\mathcal{N}$. Every NαOS (NPOS, resp.) is $\mathcal{N}\beta OS$. Since $\ddot{f}$ is Npre-β-irresolute function,$\ddot{f}^{-1}(\mathcal{E}_2^*))$ is NPOS in $\mathcal{X}_\mathcal{N}$. Hence $\ddot{f}$ is Npre-α-(Npre, resp.) irresolute function.

Proposition 3.8 Every Npre-β-irresolute function is $\mathcal{N}\beta$-irresolute function.

Proof: Let $\ddot{f}: (\mathcal{X}_\mathcal{N},\mathcal{T}_\mathcal{N}) \longrightarrow (\mathcal{Y}_\mathcal{N},\mathcal{G}_\mathcal{N})$ be Npre-β irresolute function from NTS $(\mathcal{X}_\mathcal{N},\mathcal{T}_\mathcal{N})$ to another NTS$(\mathcal{Y}_\mathcal{N},\mathcal{G}_\mathcal{N})$. Let $\mathcal{E}_2^*$ be βOS . Since $\ddot{f}$ is Npre-β- irresolute function, $\ddot{f}^{-1}(\mathcal{E}_2^*))$ is NPOS in $\mathcal{X}_\mathcal{N}$. As every NPOS is $\mathcal{N}\beta OS$, $\ddot{f}^{-1}(\mathcal{E}_2^*))$ is $\mathcal{N}\beta OS$ in $\mathcal{X}_\mathcal{N}$. Hence $\ddot{f}$ is $\mathcal{N}\beta$ -irresolute function.

Proposition 3.9 Every Nirresolute function is NS-α-irresolute function.

Proof: Let $\ddot{f}$: $(\mathcal{X}_{\mathcal{N}},\mathcal{T}_{\mathcal{N}}) \longrightarrow (\mathcal{Y}_{\mathcal{N}},\mathcal{G}_{\mathcal{N}})$ be Nirresolute function from NTS$(\mathcal{X}_{\mathcal{N}},\mathcal{T}_{\mathcal{N}})$ to another NTS $(\mathcal{Y}_{\mathcal{N}},\mathcal{G}_{\mathcal{N}})$. Let $\mathcal{E}_2^*$ be NαOS in $\mathcal{Y}_{\mathcal{N}}$. Every N$\alpha$OS is NSOS. Since $\ddot{f}$ is Nirresolute function, $\ddot{f}^{-1}(\mathcal{E}_2^*)$) is NSOS in $\mathcal{X}_{\mathcal{N}}$. Hence $\ddot{f}$ is NS-α- irresolute function.

Example 3.10 Let $\mathcal{X}_{\mathcal{N}}$={a,b} $\mathcal{Y}_{\mathcal{N}}$={c,d} and $\mathcal{T}_{\mathcal{N}}$= {0, $\mathcal{E}_1^*$, 1}, $\Gamma_{\mathcal{N}}$= {0, $\mathcal{E}_2^*$, 1},are NTS on $\mathcal{X}_{\mathcal{N}}$ and $\mathcal{Y}_{\mathcal{N}}$ respectively where

$$\mathcal{E}_1^* = \langle \mathrm{x}, \left(\frac{6}{10},\frac{5}{10},\frac{4}{10}\right), \left(\frac{5}{10},\frac{5}{10},\frac{5}{10}\right)\rangle.$$

$$\mathcal{E}_2^* = \langle \mathrm{y}, \left(\frac{2}{10},\frac{5}{10},\frac{6}{10}\right), \left(\frac{4}{10},\frac{5}{10},\frac{5}{10}\right)\rangle$$

Define an Neutrosophic function$\ddot{f}$: $(\mathcal{X}_{\mathcal{N}},\mathcal{T}_{\mathcal{N}}) \longrightarrow (\mathcal{Y}_{\mathcal{N}},\mathcal{G}_{\mathcal{N}})$. By $\ddot{f}$ (a)=d, $\ddot{f}$ (b)=c $\mathcal{E}_2^*$ is a NOS in $(\mathcal{Y}_{\mathcal{N}},\mathcal{G}_{\mathcal{N}})$. So $\mathcal{E}_2^*$ is NαOS, NPOS, and $\mathcal{N}\beta OS$ in $\mathcal{Y}_{\mathcal{N}}$.

, since$\ddot{f}^{-1}(\mathcal{E}_2^*) = \langle \mathrm{x}, \left(\frac{4}{10},\frac{5}{10},\frac{5}{10}\right), \left(\frac{2}{10},\frac{5}{10},\frac{6}{10}\right)\rangle$ is an NPOS in $\mathcal{X}_{\mathcal{N}}$

$$\ddot{f}^{-1}(\mathcal{E}_2^*) \subseteq Nint(Ncl\left(\ddot{f}^{-1}(\mathcal{E}_2^*)\right)) = 1_N$$

Also $\ddot{f}^{-1}(\mathcal{E}_2^*) \subseteq Ncl(Nint(Ncl\left(\ddot{f}^{-1}(\mathcal{E}_2^*)\right))) = 1_N$

So $\ddot{f}^{-1}(\mathcal{E}_2^*)$ is a $\mathcal{N}\beta OS$ in $\mathcal{X}_{\mathcal{N}}$. Thus $\ddot{f}$ is Npre-β-irresolute, Npre irresolute function, Npre-α-irresolute function and $\mathcal{N}\beta$ -irresolute function.Also $\ddot{f}$ is a N precontinuous and $\mathcal{N}\beta$ -continuous. As $Nint(Ncl(Nint\left(\ddot{f}^{-1}(\mathcal{E}_2^*)\right))) = 0_N$,$\ddot{f}^{-1}(\mathcal{E}_2^*) \nsubseteq Ncl(Nint(Ncl\left(\ddot{f}^{-1}(\mathcal{E}_2^*)\right)))$

$\ddot{f}^{-1}(\mathcal{E}_2^*)$ is not NαOS in $\mathcal{X}_{\mathcal{N}}$. Also $\ddot{f}^{-1}(\mathcal{E}_2^*) \nsubseteq Ncl(Nint\left(\ddot{f}^{-1}(\mathcal{E}_2^*)\right)$=$0_N$.implies $\ddot{f}^{-1}(\mathcal{E}_2^*)$ is not NSOS in $\mathcal{X}_{\mathcal{N}}$. Thus $\ddot{f}$ is not Nα-irresolute function, not NSemi-α-irresolute function, not Nα-continuous, not NSemi continuous, and not Nirresolute function.

Example 3.11

Let $\mathcal{X}_{\mathcal{N}}$={a,b} $\mathcal{Y}_{\mathcal{N}}$={c,d} and $\mathcal{T}_{\mathcal{N}}$= {0, $\mathcal{E}_1^*$, 1}, $\Gamma_{\mathcal{N}}$= {0, $\mathcal{E}_2^*$, 1},are NTS on $\mathcal{X}_{\mathcal{N}}$ and $\mathcal{Y}_{\mathcal{N}}$ respectively is a NS in $\mathcal{Y}_{\mathcal{N}}$.

$$\mathcal{E}_1^* = \langle \mathrm{x}, \left(\frac{2}{10},\frac{5}{10},\frac{3}{10}\right), \left(\frac{4}{10},\frac{5}{10},\frac{4}{10}\right)\rangle.$$

$$\mathcal{E}_2^* = \langle \mathrm{y}, \left(\frac{4}{10},\frac{5}{10},\frac{4}{10}\right), \left(\frac{2}{10},\frac{5}{10},\frac{3}{10}\right)\rangle$$

$$\mathcal{E}_3^* = \langle \mathrm{y}, \left(\frac{4}{10},\frac{5}{10},\frac{4}{10}\right), \left(\frac{3}{10},\frac{5}{10},\frac{2}{10}\right)\rangle$$

is a NS in $\mathcal{Y}_{\mathcal{N}}$.

Define a Neutrosophic function $\ddot{f}$: $(\mathcal{X}_{\mathcal{N}},\mathcal{T}_{\mathcal{N}}) \longrightarrow (\mathcal{Y}_{\mathcal{N}},\mathcal{G}_{\mathcal{N}})$ by $\ddot{f}$ (a)=d, $\ddot{f}$ (b)=c $\mathcal{E}_2^*$ is a NOS in $(\mathcal{Y}_{\mathcal{N}},\mathcal{G}_{\mathcal{N}})$. Also $\mathcal{E}_2^*$ is NαOS, NPOS and NSOS in $\mathcal{Y}_{\mathcal{N}}$.

$$\ddot{f}^{-1}(\mathcal{E}_2^*) = \langle \mathrm{x}, \left(\frac{2}{10},\frac{5}{10},\frac{3}{10}\right), \left(\frac{4}{10},\frac{5}{10},\frac{4}{10}\right)\rangle$$

and $\ddot{f}^{-1}(\mathcal{E}_2^*) \subseteq Nint(Ncl\left(\ddot{f}^{-1}(\mathcal{E}_2^*)\right)) = \mathcal{E}_1^*$= . So $\ddot{f}^{-1}(\mathcal{E}_2^*) \subseteq Nint(Ncl(Nint\left(\ddot{f}^{-1}(\mathcal{E}_2^*)\right))$ This implies $\ddot{f}^{-1}(\mathcal{E}_2^*)$is a N$\alpha$OS in $\mathcal{X}_{\mathcal{N}}$. Also $\ddot{f}^{-1}(\mathcal{E}_2^*)$ is NPOS and NSOS in $\mathcal{X}_{\mathcal{N}}$. Hence $\ddot{f}$ is a Nα-irresolute function, NS-α-irresolute function, Npre-α-irresolute function, Nα-continuous, NSemicontinuous, and Nprecontinuous.$\mathcal{E}_3^* \subseteq \mathrm{Ncl}(\mathrm{Nint}(\mathcal{E}_3^*) = \overline{\mathcal{E}_2^*}$. So $\mathcal{E}_3^*$ is a NSOS in $\mathcal{Y}_{\mathcal{N}}$.

Also $\ddot{f}^{-1}(\mathcal{E}_3^*) = \langle \mathrm{x}, \left(\frac{3}{10}, \frac{5}{10}, \frac{2}{10}\right), \left(\frac{4}{10}, \frac{5}{10}, \frac{4}{10}\right)\rangle$.Then$\ddot{f}^{-1}(\mathcal{E}_3^*) \nsubseteq Nint(Ncl(Nint\left(\ddot{f}^{-1}(\mathcal{E}_3^*)\right))) = \mathcal{E}_1^*$

.Hence$\ddot{f}^{-1}(\mathcal{E}_3^*)$ is not NαOS in $\mathcal{X}_{\mathcal{N}}$. Thus $\ddot{f}$ is not Nstrongly α-continuous.

Example 3.12 Let $\mathcal{X}_{\mathcal{N}}$={a,b} $\mathcal{Y}_{\mathcal{N}}$={c,d} and $\mathcal{T}_{\mathcal{N}}$= {0, $\mathcal{E}_1^*$, 1}, $\Gamma_{\mathcal{N}}$= {0, $\mathcal{E}_2^*$, 1},are NTS on $\mathcal{X}_{\mathcal{N}}$ and $\mathcal{Y}_{\mathcal{N}}$ respectively, where

$$\mathcal{E}_1^* = \langle \mathrm{x}, \left(\frac{6}{10}, \frac{5}{10}, \frac{4}{10}\right), \left(\frac{5}{10}, \frac{5}{10}, \frac{5}{10}\right)\rangle.$$

$$\mathcal{E}_2^* = \langle \mathrm{y}, \left(\frac{2}{10}, \frac{5}{10}, \frac{6}{10}\right), \left(\frac{4}{10}, \frac{5}{10}, \frac{5}{10}\right)\rangle$$

Define a Neutrosophic function$\ddot{f}$: $(\mathcal{X}_{\mathcal{N}},\mathcal{T}_{\mathcal{N}}) \longrightarrow (\mathcal{Y}_{\mathcal{N}},\mathcal{G}_{\mathcal{N}})$. By $\ddot{f}$ (a)=d, $\ddot{f}$ (b)=c. $\mathcal{E}_2^*$ is a NOS in $\mathcal{G}_{\mathcal{N}}$ .Hence $\mathcal{E}_2^*$ is NαOS, NPOS, NSOS and $\mathcal{N}\beta OS$ in $(\mathcal{Y}_{\mathcal{N}},\mathcal{G}_{\mathcal{N}})$.

$$\ddot{f}^{-1}(\mathcal{E}_2^*) = \langle \mathrm{x}, \left(\frac{5}{10}, \frac{5}{10}, \frac{5}{10}\right), \left(\frac{4}{10}, \frac{5}{10}, \frac{5}{10}\right)\rangle$$

$\ddot{f}^{-1}(\mathcal{E}_2^*)\subseteq$ Ncl(Nint($\ddot{f}^{-1}(\mathcal{E}_2^*)$)) $= \overline{\mathcal{E}_1^*}$ implies$\ddot{f}^{-1}(\mathcal{E}_2^*)$ is a NSOS in $\mathcal{X}_{\mathcal{N}}$. Also $\ddot{f}^{-1}(\mathcal{E}_2^*)$is a $\mathcal{N}\beta OS$ in $\mathcal{X}_{\mathcal{N}}$, since $\ddot{f}^{-1}(\mathcal{E}_2^*) \subseteq \mathrm{Ncl}(\mathrm{Nint}\left(\ddot{f}^{-1}(\mathcal{E}_2^*)\right) = \overline{\mathcal{E}_1^*}$.Hence $\ddot{f}$ is Nirresolute function, NS-α-irresolute function, NSemi continuous and $\mathcal{N}\beta$ -continuous. Nint(Ncl (Nint($\ddot{f}^{-1}(\mathcal{E}_2^*)$)) = $\mathcal{E}_1^*$. So $\ddot{f}^{-1}(\mathcal{E}_2^*) \nsubseteq$ Nint(Ncl (Nint($\ddot{f}^{-1}(\mathcal{E}_2^*)$)). Hence $\ddot{f}^{-1}(\mathcal{E}_2^*)$ is not NαOS in $\mathcal{X}_{\mathcal{N}}$. Also $\ddot{f}^{-1}(\mathcal{E}_2^*)$ $\nsubseteq$Nint(Ncl($\ddot{f}^{-1}(\mathcal{E}_2^*)$)). Hence $\ddot{f}^{-1}(\mathcal{E}_2^*)$ is not NPOS in $\mathcal{X}_{\mathcal{N}}$. Thus $\ddot{f}$ is not Nα- irresolute function, not Npre-α-irresolute function, not Npre irresolute function, not Npre-β-irresolute function, not Nα-continuous and not Npre continuous.

Example 3.13

Let $\mathcal{X}_{\mathcal{N}}$ = {a,b,c} = $\mathcal{Y}_{\mathcal{N}}$ and $\mathcal{T}_{\mathcal{N}}$ = {0_N, 1_N , $\mathcal{E}_1^*$, $\mathcal{E}_1^* \cup \mathcal{E}_2^*$, $\mathcal{E}_1^* \cap \mathcal{E}_2^*$},$\Gamma_{\mathcal{N}}$ = {0_N, 1_N, $\mathcal{E}_3^*$} are NTS on $\mathcal{X}_{\mathcal{N}}$ and $\mathcal{Y}_{\mathcal{N}}$ where

$$\mathcal{E}_1^* = \langle \mathrm{x}, \left(\frac{5}{10}, \frac{5}{10}, \frac{5}{10}\right), \left(\frac{3}{10}, \frac{5}{10}, \frac{7}{10}\right), \left(\frac{6}{10}, \frac{5}{10}, \frac{4}{10}\right)\rangle.$$

$$\mathcal{E}_2^* = \langle \mathrm{x}, \left(\frac{2}{10}, \frac{5}{10}, \frac{7}{10}\right), \left(\frac{4}{10}, \frac{5}{10}, \frac{6}{10}\right), \left(\frac{3}{10}, \frac{5}{10}, \frac{7}{10}\right)\rangle$$

$$\mathcal{E}_3^* = \langle \mathrm{y}, \left(\frac{5}{10}, \frac{5}{10}, \frac{5}{10}\right), \left(\frac{4}{10}, \frac{5}{10}, \frac{6}{10}\right), \left(\frac{6}{10}, \frac{5}{10}, \frac{4}{10}\right)\rangle$$

$$\mathcal{E}_4^* = \langle \mathrm{y}, \left(\frac{5}{10}, \frac{5}{10}, \frac{4}{10}\right), \left(\frac{4}{10}, \frac{5}{10}, \frac{5}{10}\right), \left(\frac{6}{10}, \frac{5}{10}, \frac{4}{10}\right)\rangle$$

is a NS in $\mathcal{Y}_{\mathcal{N}}$. Define an identity Neutrosophic function $\ddot{f}$: $(\mathcal{X}_{\mathcal{N}},\mathcal{T}_{\mathcal{N}}) \longrightarrow (\mathcal{Y}_{\mathcal{N}},\mathcal{G}_{\mathcal{N}})$. $\mathcal{E}_3^*$ is a NOS in $\mathcal{Y}_{\mathcal{N}}$.and $\ddot{f}^{-1}(\mathcal{E}_3^*) = \langle \mathrm{x}, \left(\frac{5}{10}, \frac{5}{10}, \frac{5}{10}\right), \left(\frac{4}{10}, \frac{5}{10}, \frac{6}{10}\right), \left(\frac{6}{10}, \frac{5}{10}, \frac{4}{10}\right)\rangle$

Nint(Ncl(Nint($\ddot{f}^{-1}(\mathcal{E}_3^*)$)) = $\mathcal{E}_1^* \cup \mathcal{E}_2^*$.Thus $\ddot{f}^{-1}(\mathcal{E}_3^*)$. ⊆ Nint(Ncl(Nint($\ddot{f}^{-1}(\mathcal{E}_3^*)$))Hence $\ddot{f}^{-1}(\mathcal{E}_3^*)$is a N$\alpha$OS in $(\mathcal{X}_{\mathcal{N}},\mathcal{T}_{\mathcal{N}})$. Also $\ddot{f}^{-1}(\mathcal{E}_3^*)$ is NPOS, NSOS and $\mathcal{N}\beta OS$ in $\mathcal{X}_{\mathcal{N}}$. Therefore $\ddot{f}$ is Nα-continuous, Npre continuous, NSemicontinuous and $\mathcal{N}\beta$ -continuous. $\mathcal{E}_4^*$ is a NS in $\mathcal{Y}_{\mathcal{N}}$ and

$\mathcal{E}_4^* \subseteq \mathrm{Nint}(\mathrm{Ncl}(\mathrm{Nint}(\mathcal{E}_4^*) = 1_N$. Hence $\mathcal{E}_4^*$ is a NαOS in $\mathcal{Y}_{\mathcal{N}}$. Also $\mathcal{E}_4^*$ is NPOS, NSOS and $\mathcal{N}\beta OS$ in $\mathcal{Y}_{\mathcal{N}}$.

$$\ddot{f}^{-1}(\mathcal{E}_4^*) = \langle x, \left(\frac{5}{10}, \frac{5}{10}, \frac{4}{10}\right), \left(\frac{4}{10}, \frac{5}{10}, \frac{5}{10}\right), \left(\frac{6}{10}, \frac{5}{10}, \frac{4}{10}\right)\rangle$$

And $\ddot{f}^{-1}(\mathcal{E}_4^*) \subseteq \mathrm{Ncl}(\mathrm{Nint}\left(\ddot{f}^{-1}(\mathcal{E}_4^*)\right) = \overline{\mathcal{E}_2^*}$.Hence$\ddot{f}^{-1}(\mathcal{E}_4^*)$. Hence $\ddot{f}^{-1}(\mathcal{E}_4^*)$is NSOS and also $\mathcal{N}\beta OS$ in $\mathcal{X}_{\mathcal{N}}$. So $\ddot{f}$ is Nirresolute function, NS-α-irresolute function and $\mathcal{N}\beta$ –irresolute function. Since $\ddot{f}^{-1}(\mathcal{E}_4^*) \nsubseteq$Nint(Ncl Nint($\ddot{f}^{-1}(\mathcal{E}_4^*)$) $= \mathcal{E}_1^* \cup \mathcal{E}_2^*$, $\ddot{f}^{-1}(\mathcal{E}_4^*)$ is not NαOS in $\mathcal{X}_{\mathcal{N}}$ and $\ddot{f}^{-1}(\mathcal{E}_4^*) \nsubseteq$ Nint(Ncl($\ddot{f}^{-1}(\mathcal{E}_4^*) = \mathcal{E}_1^* \cup \mathcal{E}_2^*$, $\ddot{f}^{-1}(\mathcal{E}_4^*)$ is not NPOS in $\mathcal{X}_{\mathcal{N}}$. Thus $\ddot{f}$ is not Nα-irresolute function, not Npre-α-irresolute function and not Npre-β-irresolute function.

Example 3.14

Let $\mathcal{X}_{\mathcal{N}}$={a,b} $\mathcal{Y}_{\mathcal{N}}$={c,d} and $\mathcal{T}_{\mathcal{N}}$= {0, $\mathcal{E}_1^*$, 1}, $\Gamma_{\mathcal{N}}$= {0, $\mathcal{E}_2^*$, 1},are NTS on $\mathcal{X}_{\mathcal{N}}$ and $\mathcal{Y}_{\mathcal{N}}$ respectively where

$$\mathcal{E}_1^* = \langle x, \left(\frac{3}{10}, \frac{5}{10}, \frac{6}{10}\right), \left(\frac{4}{10}, \frac{5}{10}, \frac{5}{10}\right)\rangle$$

$$\mathcal{E}_2^* = \langle y, \left(\frac{4}{10}, \frac{5}{10}, \frac{5}{10}\right), \left(\frac{5}{10}, \frac{5}{10}, \frac{3}{10}\right)\rangle$$

$$\text{And } \mathcal{E}_3^* = \langle y, \left(\frac{4}{10}, \frac{5}{10}, \frac{5}{10}\right), \left(\frac{6}{10}, \frac{5}{10}, \frac{2}{10}\right)\rangle$$

is a NS in $\mathcal{Y}_{\mathcal{N}}$ Define a Neutrosophic function $\ddot{f}$: ($\mathcal{X}_{\mathcal{N}}$,$\mathcal{T}_{\mathcal{N}}$) $\longrightarrow$ ($\mathcal{Y}_{\mathcal{N}}$,$\mathcal{G}_{\mathcal{N}}$). By $\ddot{f}$ (a)=d, $\ddot{f}$ (b)=c $\mathcal{E}_2^*$ is a NOS in ($\mathcal{Y}_{\mathcal{N}}$,$\mathcal{G}_{\mathcal{N}}$). And $\ddot{f}^{-1}(\mathcal{E}_2^*) = \langle y, \left(\frac{5}{10}, \frac{5}{10}, \frac{3}{10}\right), \left(\frac{4}{10}, \frac{5}{10}, \frac{5}{10}\right)\rangle$ and

$\mathrm{Ncl}(\mathrm{Nint}((\ddot{f}^{-1}(\mathcal{E}_2^*)))) = \overline{\mathcal{E}_1^*}$. Thus $\ddot{f}^{-1}(\mathcal{E}_2^*) \subseteq \mathrm{Ncl}(\mathrm{Nint}((\ddot{f}^{-1}(\mathcal{E}_2^*))))$. Hence $\ddot{f}^{-1}(\mathcal{E}_2^*)$ is an NSOS in $\mathcal{X}_{\mathcal{N}}$, which implies $\ddot{f}$ is NSemi continuous and also $\ddot{f}$ is $\mathcal{N}\beta$ -continuous. $\mathcal{E}_3^*$ is a NS in $\mathcal{Y}_{\mathcal{N}}$. Also $\mathcal{E}_3^*$⊆Nint(Ncl(Nint($\mathcal{E}_3^*$)= 1_Nwhich implies $\mathcal{E}_3^*$ is a NαOS in $\mathcal{Y}_{\mathcal{N}}$. Hence $\mathcal{E}_3^*$ is NPOS, NSOS and $\mathcal{N}\beta OS$ in $\mathcal{Y}_{\mathcal{N}}$.$\ddot{f}^{-1}(\mathcal{E}_3^*) = \langle y, \left(\frac{5}{10}, \frac{5}{10}, \frac{3}{10}\right), \left(\frac{4}{10}, \frac{5}{10}, \frac{5}{10}\right)\rangle$ So $\ddot{f}^{-1}(\mathcal{E}_3^*)$is a NPOS and $\mathcal{N}\beta OS$ in $\mathcal{X}_{\mathcal{N}}$.Thus $\ddot{f}$ is Npre-α-irresolute function, Npre-β-irresolute function and $\mathcal{N}\beta$ -irresolute function. Since $\ddot{f}^{-1}(\mathcal{E}_3^*) \nsubseteq$ Nint(Ncl (Nint($\ddot{f}^{-1}(\mathcal{E}_3^*)$= $\mathcal{E}_1^*$, $\ddot{f}^{-1}(\mathcal{E}_3^*)$ is not NαOS in $\mathcal{X}_{\mathcal{N}}$. Also $\ddot{f}^{-1}(\mathcal{E}_3^*) \nsubseteq$ Ncl(Nint($\ddot{f}^{-1}(\mathcal{E}_3^*)$ =$\overline{\mathcal{E}_1^*}$. So $\ddot{f}^{-1}(\mathcal{E}_3^*)$is not NSOS in $\mathcal{X}_{\mathcal{N}}$. Hence $\ddot{f}$ is not Nα-irresolute function, not Nirresolute function,and not NS-α-irresolute function.

Example 3.15

Let $\mathcal{X}_{\mathcal{N}}$ = {a,b} = $\mathcal{Y}_{\mathcal{N}}$ and

$\mathcal{T}_{\mathcal{N}}$ = {0_N, 1_N , $\mathcal{E}_1^*$, $\mathcal{E}_1^* \cup \mathcal{E}_2^*$, $\mathcal{E}_1^* \cap \mathcal{E}_2^*$}

,$\Gamma_{\mathcal{N}}$ = {0_N, 1_N, $\mathcal{E}_3^*$} are NTS on $\mathcal{X}_{\mathcal{N}}$ and $\mathcal{Y}_{\mathcal{N}}$ where

$$\mathcal{E}_1^* = \langle x, \left(\frac{2}{10}, \frac{5}{10}, \frac{5}{10}\right), \left(\frac{2}{10}, \frac{5}{10}, \frac{4}{10}\right)\rangle.$$

$$\mathcal{E}_2^* = \langle x, \left(\frac{4}{10}, \frac{5}{10}, \frac{6}{10}\right), \left(\frac{4}{10}, \frac{5}{10}, \frac{6}{10}\right)\rangle$$

$$\mathcal{E}_3^* = \langle y, \left(\frac{2}{10}, \frac{5}{10}, \frac{6}{10}\right), \left(\frac{4}{10}, \frac{5}{10}, \frac{6}{10}\right)\rangle$$

$$\mathcal{E}_4^* = \langle y, \left(\frac{3}{10}, \frac{5}{10}, \frac{3}{10}\right), \left(\frac{4}{10}, \frac{5}{10}, \frac{5}{10}\right), \rangle$$

is a NS in $\mathcal{Y}_{\mathcal{N}}$. Define an identity Neutrosophic function $\ddot{f}$: ($\mathcal{X}_{\mathcal{N}}$,$\mathcal{T}_{\mathcal{N}}$) $\longrightarrow$ ($\mathcal{Y}_{\mathcal{N}}$,$\mathcal{G}_{\mathcal{N}}$). $\mathcal{E}_3^*$ is a NOS in $\mathcal{Y}_{\mathcal{N}}$.. $\mathcal{E}_3^*$ is a NOS, NαOS, NPOS in ($\mathcal{Y}_{\mathcal{N}}$,$\mathcal{G}_{\mathcal{N}}$). $\ddot{f}^{-1}(\mathcal{E}_3^*)$ = ⟨y, $\left(\frac{2}{10},\frac{5}{10},\frac{6}{10}\right),\left(\frac{4}{10},\frac{5}{10},\frac{6}{10}\right)$⟩

So $\ddot{f}^{-1}(\mathcal{E}_3^*)$⊆Nint(Ncl(Nint($\ddot{f}^{-1}(\mathcal{E}_3^*)$ = $\mathcal{E}_1^*$∪$\mathcal{E}_2^*$. Thus $\ddot{f}^{-1}(\mathcal{E}_3^*)$is a N$\alpha$OS in $\mathcal{X}_{\mathcal{N}}$.. Hence $\ddot{f}^{-1}(\mathcal{E}_3^*)$is NPOS and NSOS in $\mathcal{X}_{\mathcal{N}}$. Thus $\ddot{f}$ is Nα-irresolute, Nsemi-α-irresolute and Npre-α-irresolute function, Nα-continuous, Nprecontinuous and NSemi continuous. $\mathcal{E}_4^*$ is a NS in $\mathcal{Y}_{\mathcal{N}}$ and Ncl(Nint($\ddot{f}^{-1}(\mathcal{E}_4^*)$)) = $\overline{\mathcal{E}_3^*}$.Hence $\mathcal{E}_4^*$ ⊆Ncl (Nint($\mathcal{E}_4^*$). Thus $\mathcal{E}_4^*$ is a NSOS in $\mathcal{Y}_{\mathcal{N}}$.

$$\ddot{f}^{-1}(\mathcal{E}_4^*) = \langle x, \left(\frac{3}{10},\frac{5}{10},\frac{3}{10}\right),\left(\frac{4}{10},\frac{5}{10},\frac{5}{10}\right)\rangle$$

and Ncl(Nint($\ddot{f}^{-1}(\mathcal{E}_4^*)$)) = $\overline{\mathcal{E}_1^* \cup \mathcal{E}_2^*}$

. So $\ddot{f}^{-1}(\mathcal{E}_4^*)$ $\nsubseteq$ Ncl(Nint($\ddot{f}^{-1}(\mathcal{E}_4^*)$)). Thus $\ddot{f}^{-1}(\mathcal{E}_4^*)$is not NSOS in $\mathcal{X}_{\mathcal{N}}$.Hence $\ddot{f}$ is not Nirresolute function.

Example 3.16

Let $\mathcal{X}_{\mathcal{N}}$ = {a,b,c} = $\mathcal{Y}_{\mathcal{N}}$ and $\mathcal{T}_{\mathcal{N}}$ = {0N, 1N , $\mathcal{E}_1^*$},$\Gamma_{\mathcal{N}}$ = {0N, 1N, $\mathcal{E}_2^*$} are NTS on $\mathcal{X}_{\mathcal{N}}$ and $\mathcal{Y}_{\mathcal{N}}$ where

$$\mathcal{E}_1^* = \langle x, \left(\frac{7}{10},\frac{5}{10},\frac{1}{10}\right),\left(\frac{5}{10},\frac{5}{10},\frac{4}{10}\right),\left(\frac{3}{10},\frac{5}{10},\frac{2}{10}\right)\rangle.$$

$$\mathcal{E}_2^* = \langle x, \left(\frac{1}{10},\frac{5}{10},\frac{2}{10}\right),\left(\frac{3}{10},\frac{5}{10},\frac{4}{10}\right),\left(\frac{4}{10},\frac{5}{10},\frac{6}{10}\right)\rangle.$$

Define a Neutrosophic function $\ddot{f}$: ($\mathcal{X}_{\mathcal{N}}$,$\mathcal{T}_{\mathcal{N}}$) $\longrightarrow$ ($\mathcal{Y}_{\mathcal{N}}$,$\mathcal{G}_{\mathcal{N}}$) by $\ddot{f}$ (a) = b, $\ddot{f}$ (b) = c, $\ddot{f}$ (c) = a.
$\mathcal{E}_2^*$ is a NOS in ($\mathcal{Y}_{\mathcal{N}}$,$\mathcal{G}_{\mathcal{N}}$). Also $\mathcal{E}_2^*$ is NαOS, NPOS, NSOS and $\mathcal{N}\beta OS$ in $\mathcal{Y}_{\mathcal{N}}$ and

$$\ddot{f}^{-1}(\mathcal{E}_2^*) = \langle x, \left(\frac{3}{10},\frac{5}{10},\frac{4}{10}\right),\left(\frac{4}{10},\frac{5}{10},\frac{6}{10}\right),\left(\frac{1}{10},\frac{5}{10},\frac{2}{10}\right)\rangle$$

Nint(Ncl($\ddot{f}^{-1}(\mathcal{E}_2^*)$)) = 1_N. Since $\ddot{f}^{-1}(\mathcal{E}_2^*)$ ⊆ Nint(Ncl($\ddot{f}^{-1}(\mathcal{E}_2^*)$)) $\ddot{f}^{-1}(\mathcal{E}_2^*)$,is a NPOS in ($\mathcal{X}_{\mathcal{N}}$,$\mathcal{T}_{\mathcal{N}}$) and also $\ddot{f}^{-1}(\mathcal{E}_2^*)$ is $\mathcal{N}\beta OS$ in $\mathcal{X}_{\mathcal{N}}$. Thus $\ddot{f}$ is a Npre irresolute function, Npre-α-irresolute function, Npre continuous and $\mathcal{N}\beta$ -continuous. Now $\ddot{f}^{-1}(\mathcal{E}_2^*)$ $\nsubseteq$ Ncl(Nint($\ddot{f}^{-1}(\mathcal{E}_2^*)$)) = 0_N. So $\ddot{f}^{-1}(\mathcal{E}_2^*)$ is not NSOS in $\mathcal{X}_{\mathcal{N}}$. Also $\ddot{f}^{-1}(\mathcal{E}_2^*)$ $\nsubseteq$ Nint(Ncl(Ncl(($\ddot{f}^{-1}(\mathcal{E}_2^*)$))))= 0_N. Hence $\ddot{f}^{-1}(\mathcal{E}_2^*)$ is not NαOS in $\mathcal{X}_{\mathcal{N}}$.Thus $\ddot{f}$ is not Nα-irresolute function, not NS-α-irresolute function and not Nα-continuous and not NSemi continuous.

Example 3.17 Let $\mathcal{X}_{\mathcal{N}}$={a,b} $\mathcal{Y}_{\mathcal{N}}$={c,d} and $\mathcal{T}_{\mathcal{N}}$= {0, $\mathcal{E}_1^*$, 1}, $\Gamma_{\mathcal{N}}$= {0, $\mathcal{E}_2^*$, 1},are NTS on $\mathcal{X}_{\mathcal{N}}$ and $\mathcal{Y}_{\mathcal{N}}$ respectively where

$$\mathcal{E}_1^* = \langle x, \left(\frac{4}{10},\frac{5}{10},\frac{4}{10}\right),\left(\frac{5}{10},\frac{5}{10},\frac{2}{10}\right)\rangle$$

$$\mathcal{E}_2^* = \langle y, \left(\frac{4}{10},\frac{5}{10},\frac{5}{10}\right),\left(\frac{5}{10},\frac{5}{10},\frac{5}{10}\right)\rangle$$

$$\mathcal{E}_3^* = \langle y, \left(\frac{2}{10},\frac{5}{10},\frac{2}{10}\right),\left(\frac{4}{10},\frac{5}{10},\frac{5}{10}\right)\rangle$$

is a NS in $\mathcal{Y}_{\mathcal{N}}$. Define an Neutrosophic function $\ddot{f}$: ($\mathcal{X}_{\mathcal{N}}$,$\mathcal{T}_{\mathcal{N}}$) $\longrightarrow$ ($\mathcal{Y}_{\mathcal{N}}$,$\mathcal{G}_{\mathcal{N}}$). by $\ddot{f}$(a) =c, $\ddot{f}$(b)=d. $\mathcal{E}_2^*$ is a NOS in ($\mathcal{Y}_{\mathcal{N}}$,$\mathcal{G}_{\mathcal{N}}$). Also $\mathcal{E}_2^*$ is NαOS, NPOS in $\mathcal{Y}_{\mathcal{N}}$.

$$\ddot{f}^{-1}(\mathcal{E}_2^*) = \langle x, \left(\frac{4}{10},\frac{5}{10},\frac{2}{10}\right),\left(\frac{3}{10},\frac{5}{10},\frac{5}{10}\right)\rangle$$

and$\ddot{f}^{-1}(\mathcal{E}_2^*)$ ⊆ Nint(Ncl($\ddot{f}^{-1}(\mathcal{E}_2^*)$)= 1_N.

Thus $\ddot{f}^{-1}(\mathcal{E}_2^*)$ ⊆ Nint(Ncl ($\ddot{f}^{-1}(\mathcal{E}_2^*)$)) Hence $\ddot{f}^{-1}(\mathcal{E}_2^*)$ is a NPOS in $\mathcal{X}_{\mathcal{N}}$.

Now $\mathcal{E}_3^* \subseteq$ Nint(Ncl($\mathcal{E}_3^*$) = 1_NTherefore $\mathcal{E}_3^*$ is an NPOS in $\mathcal{Y}_\mathcal{N}$. Also $\mathcal{E}_3^*$ is an NβOS in $\mathcal{Y}_\mathcal{N}$

$$\ddot{f}^{-1}(\mathcal{E}_3^*) = \langle x, \left(\frac{4}{10}, \frac{5}{10}, \frac{6}{10}\right), \left(\frac{4}{10}, \frac{5}{10}, \frac{5}{10}\right)\rangle$$

Nint(Ncl($\ddot{f}^{-1}(\mathcal{E}_3^*)$)= 0_N.Thus $\ddot{f}^{-1}(\mathcal{E}_3^*) \subseteq$ Nint(Ncl($\ddot{f}^{-1}(\mathcal{E}_3^*)$)).
Hence $\ddot{f}^{-1}(\mathcal{E}_3^*)$ is not an NPOS in $\mathcal{X}_\mathcal{N}$.So $\ddot{f}$ is not Npre-β-irresolute function and $\ddot{f}$ is not Npre irresolute function. Since $\ddot{f}^{-1}(\mathcal{E}_3^*) \nsubseteq Ncl\left(\text{Nint}\left(\text{Ncl}(\mathcal{E}_3^*)\right)\right) = 0_N$.

$\ddot{f}^{-1}(\mathcal{E}_3^*)$is not N$\beta$OS in $\mathcal{X}_\mathcal{N}$. So $\ddot{f}$ is not Nβ-irresolute function.

Example 3.18 Let $\mathcal{X}_\mathcal{N}$={a,b} $\mathcal{Y}_\mathcal{N}$={c,d} and $\mathcal{T}_\mathcal{N}$= {0, $\mathcal{E}_1^*$, 1}, $\Gamma_\mathcal{N}$= {0, $\mathcal{E}_2^*$, 1},are NTS on $\mathcal{X}_\mathcal{N}$ and $\mathcal{Y}_\mathcal{N}$ respectively where

$$\mathcal{E}_1^* = \langle x, \left(\frac{4}{10}, \frac{5}{10}, \frac{4}{10}\right), \left(\frac{5}{10}, \frac{5}{10}, \frac{2}{10}\right)\rangle$$

$$\mathcal{E}_2^* = \langle y, \left(\frac{4}{10}, \frac{5}{10}, \frac{5}{10}\right), \left(\frac{5}{10}, \frac{5}{10}, \frac{5}{10}\right)\rangle$$

$$\mathcal{E}_3^* = \langle y, \left(\frac{2}{10}, \frac{5}{10}, \frac{4}{10}\right), \left(\frac{2}{10}, \frac{5}{10}, \frac{5}{10}\right)\rangle$$

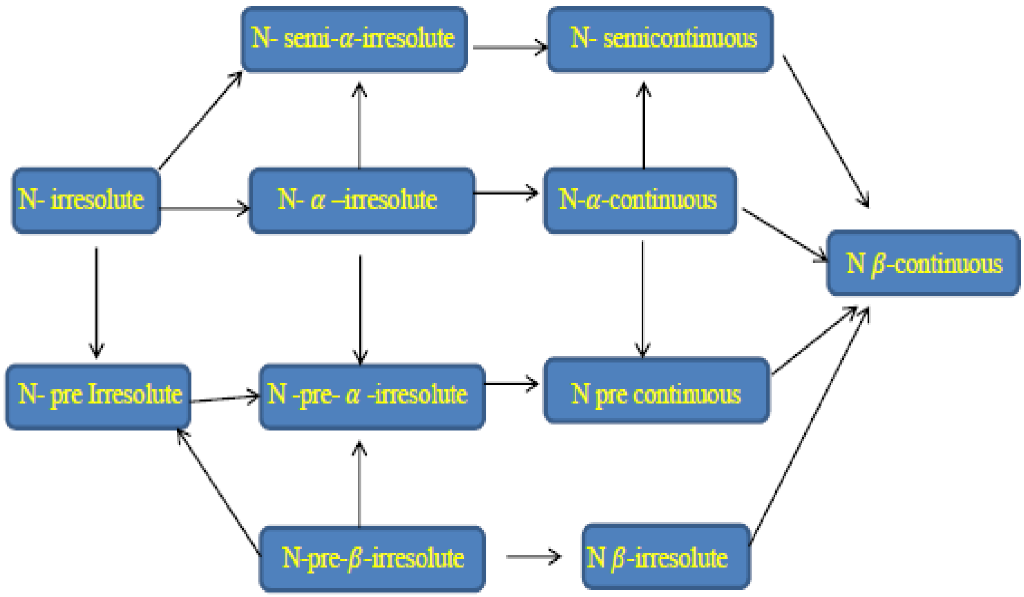
is a NS in $\mathcal{Y}_\mathcal{N}$. Define an Neutrosophic function $\ddot{f}$: ($\mathcal{X}_\mathcal{N}$,$\mathcal{T}_\mathcal{N}$) $\longrightarrow$ ($\mathcal{Y}_\mathcal{N}$,$\mathcal{G}_\mathcal{N}$) by $\ddot{f}$ (a) =c, $\ddot{f}$ (b)=d. $\mathcal{E}_2^*$ is a NOS in ($\mathcal{Y}_\mathcal{N}$,$\mathcal{G}_\mathcal{N}$). Also $\mathcal{E}_2^*$ is NαOS, NPOS in $\mathcal{Y}_\mathcal{N}$.

$$\ddot{f}^{-1}(\mathcal{E}_2^*) = \langle x, \left(\frac{4}{10}, \frac{5}{10}, \frac{5}{10}\right), \left(\frac{5}{10}, \frac{5}{10}, \frac{5}{10}\right)\rangle$$

and Nint(Ncl $\ddot{f}^{-1}(\mathcal{E}_2^*)$) = 1_N .Thus $\ddot{f}^{-1}(\mathcal{E}_2^*) \subseteq$Nint(Ncl $\ddot{f}^{-1}(\mathcal{E}_2^*)$). Hence $\ddot{f}^{-1}(\mathcal{E}_2^*)$ is a NPOS in $\mathcal{X}_\mathcal{N}$.Therefore $\ddot{f}$ is a Npre irresolute, Npre-α-irresolute and Npre continuous.
$\mathcal{E}_3^*$ is a NS in $\mathcal{Y}_\mathcal{N}$ and $\mathcal{E}_3^* \subseteq$Ncl(Nint($\ddot{f}^{-1}(\mathcal{E}_3^*)$=$\overline{\mathcal{E}_2^*}$. Hence $\mathcal{E}_3^*$ is a $\mathcal{N}\beta OS$ in $\mathcal{Y}_\mathcal{N}$.

$$\ddot{f}^{-1}(\mathcal{E}_3^*) = \langle x, \left(\frac{2}{10}, \frac{5}{10}, \frac{4}{10}\right), \left(\frac{2}{10}, \frac{5}{10}, \frac{5}{10}\right)\rangle$$

and Nint(Ncl($\ddot{f}^{-1}(\mathcal{E}_3^*)$) = 0_N.Thus $\ddot{f}^{-1}(\mathcal{E}_3^*) \nsubseteq$ Nint(Ncl($\ddot{f}^{-1}(\mathcal{E}_3^*)$). So $\ddot{f}^{-1}(\mathcal{E}_3^*)$ is not an NPOS in $\mathcal{X}_\mathcal{N}$. Hence $\ddot{f}$ is not Npre-β-irresolute function.
Diagram: I



4.PROPERTIES

Theorem 4.1 If a function $\ddot{f}$: $(\mathcal{X}_{\mathcal{N}},\mathcal{T}_{\mathcal{N}}) \longrightarrow (\mathcal{Y}_{\mathcal{N}},\mathcal{G}_{\mathcal{N}})$ is Npre-α-irresolute (Nα-irresolute and NS-α-irresolute, resp.) then$\ddot{f}^{-1}(\mathcal{E}_1^*)$ is $\mathcal{NPCS}$ (Nα-closed and NSemiclosed, resp.) in $\mathcal{X}_{\mathcal{N}}$ for any N nowhere dense set $\mathcal{E}_1^*$ of $\mathcal{Y}_{\mathcal{N}}$.

Proof:

Let $\mathcal{E}_1^*$ be an N nowhere dense set in $\mathcal{Y}_{\mathcal{N}}$. Then Nint(Ncl($\mathcal{E}_1^*$))= 0_N. Now, $\overline{\text{Nint(Ncl}(\mathcal{E}_1^*))} = 1_N \Rightarrow \overline{\text{Ncl(Ncl}(\mathcal{E}_1^*))} = 1_N$ which implies Ncl(Nint($\overline{\mathcal{E}_1^*}$)) $= 1_N$. Since Nint 1_N $=$ 1_N .Hence $\overline{\mathcal{E}_1^*} \subseteq$ Nint(Ncl(Nint($\overline{\mathcal{E}_1^*}$) Then $\overline{\mathcal{E}_1^*}$is a N$\alpha$OS in $\mathcal{Y}_{\mathcal{N}}$. Since $\ddot{f}$ is Npre-α-irresolute (Nα-irresolute and N semi-α-irresolute, resp.), $\ddot{f}^{-1}(\overline{\mathcal{E}_1^*})$) is a NPOS (N$\alpha$OS and NSOS, resp.) in $\mathcal{X}_{\mathcal{N}}$.Hence $\ddot{f}^{-1}(\mathcal{E}_1^*)$) is a NPCS (N$\alpha$CS and NSCS, resp.) in $\mathcal{X}_{\mathcal{N}}$.

Theorem 4.2 If a function $\ddot{f}$: $(\mathcal{X}_{\mathcal{N}},\mathcal{T}_{\mathcal{N}}) \longrightarrow (\mathcal{Y}_{\mathcal{N}},\mathcal{G}_{\mathcal{N}})$ is Npre-β-irresolute, then $\ddot{f}^{-1}(\mathcal{E}_1^*)$ is $\mathcal{NPCS}$ in $\mathcal{X}_{\mathcal{N}}$ for any Nnowheredense set $\mathcal{E}_1^*$ of $\mathcal{Y}_{\mathcal{N}}$.

Proof: Let $\mathcal{E}_1^*$ be an Nnowhere dense set in $\mathcal{Y}_{\mathcal{N}}$. Then Nint(Ncl($\mathcal{E}_1^*$)) = 0_N. Now,$\overline{\text{Nint(Ncl}(\mathcal{E}_1^*))} = 1_N$. $\Rightarrow$ $\overline{\text{Ncl(Ncl}(\mathcal{E}_1^*))} = 1_N$ which implies Ncl(Nint($\overline{\mathcal{E}_1^*}$)) = 1_N Since Nint1_N = $1_N$and Ncl(Nint($\overline{\mathcal{E}_1^*}$) $\subseteq$ Ncl(Nint(Nint($\overline{\mathcal{E}_1^*}$)). Hence $\overline{\mathcal{E}_1^*}\subseteq$ 1_N =Ncl(Nint($\overline{\mathcal{E}_1^*}$) $\subseteq$Ncl(Nint(Ncl($\overline{\mathcal{E}_1^*}$)). Then $\overline{\mathcal{E}_1^*}$ is a $\mathcal{N}\beta\mathcal{OS}$ in $\mathcal{Y}_{\mathcal{N}}$. Since $\ddot{f}$ is Npre-β-irresolute, $\ddot{f}^{-1}(\mathcal{E}_1^*)$is a NPOS in $\mathcal{X}_{\mathcal{N}}$. Hence $\ddot{f}^{-1}(\mathcal{E}_1^*)$ is a NPCS in $\mathcal{X}_{\mathcal{N}}$.

Theorem 4.3 A function $\ddot{f}$: $(\mathcal{X}_{\mathcal{N}},\mathcal{T}_{\mathcal{N}}) \longrightarrow (\mathcal{Y}_{\mathcal{N}},\mathcal{G}_{\mathcal{N}})$ from an NTS $\mathcal{X}_{\mathcal{N}}$ into an NTS $\mathcal{Y}_{\mathcal{N}}$ is Npre-α-irresolute if and only if for each NP $\wp(\alpha,\beta)$) in $\mathcal{X}_{\mathcal{N}}$ and NαOS $\mathcal{E}_2^*$ in $\mathcal{Y}_{\mathcal{N}}$ such that $\ddot{f}$ ($\wp(\alpha,\beta)$) $\mathcal{E}_2^*$, there exists an NPOS $\mathcal{E}_1^*$ in $\mathcal{X}_{\mathcal{N}}$ such that $\wp(\alpha,\beta)$)$\in$ $\mathcal{E}_1^*$ and $\ddot{f}$ ($\mathcal{E}_1^*$)) $\subseteq$ $\mathcal{E}_2^*$.

Proof: Let $\ddot{f}$ be any Npre-α-irresolute function. $\wp(\alpha,\beta)$) be an NP in $\mathcal{X}_{\mathcal{N}}$ and $\mathcal{E}_2^*$ be any NαOS in $\mathcal{Y}_{\mathcal{N}}$ such that $\ddot{f}$ ($\wp(\alpha,\beta)$) $\in \mathcal{E}_2^*$. Then $\ddot{f}^{-1}(\mathcal{E}_2^*)$. Let $\mathcal{E}_1^*$=$\ddot{f}^{-1}(\mathcal{E}_2^*)$.Then $\mathcal{E}_1^*$ is a NPOS in $\mathcal{X}_{\mathcal{N}}$ which containing NP $\wp(\alpha,\beta)$) and $\ddot{f}$ ($\mathcal{E}_1^*$)))=$\ddot{f}^{-1}(\mathcal{E}_2^*)\subseteq\mathcal{E}_2^*$.Conversely, let $\mathcal{E}_2^*$ be a NαOS in $\mathcal{Y}_{\mathcal{N}}$ and $\wp(\alpha,\beta)$) be an NP in $\mathcal{X}_{\mathcal{N}}$ such that $\wp(\alpha,\beta) \in \ddot{f}^{-1}(\mathcal{E}_2^*)$. According to an assumption, there exists an NPOS $\mathcal{E}_1^*$ in $\mathcal{X}_{\mathcal{N}}$ such that $\wp(\alpha,\beta)$)$\in$ $\mathcal{E}_1^*$ and $\ddot{f}(\mathcal{E}_1^*)$)$\subseteq\mathcal{E}_2^*$. Hence $\wp(\alpha,\beta)$) $\in$ $\mathcal{E}_1^*$ $\subseteq$ $\ddot{f}^{-1}(\mathcal{E}_2^*)$. Also $\wp(\alpha,\beta)$) $\in$ $\mathcal{E}_1^*\subseteq$Nint((Ncl($\mathcal{E}_1^*$) $\subseteq$ Nint(Ncl$\ddot{f}^{-1}(\mathcal{E}_2^*)$. Therefore, $\ddot{f}^{-1}(\mathcal{E}_2^*)\subseteq$Nint(Ncl($\ddot{f}^{-1}(\mathcal{E}_2^*)$)is NPOS in $\mathcal{X}_{\mathcal{N}}$. Thus, $\ddot{f}$ is a Npre-α-irresolute function.

Theorem 4.4.A function $\ddot{f}: (\mathcal{X}_{\mathcal{N}}, \mathcal{T}_{\mathcal{N}}) \longrightarrow (\mathcal{Y}_{\mathcal{N}}, \mathcal{G}_{\mathcal{N}})$ from an NTS $\mathcal{X}_{\mathcal{N}}$ into an NTS $\mathcal{Y}_{\mathcal{N}}$ is -α-irresolute if and only if for each NP $p(\alpha,\beta)$ in $\mathcal{X}_{\mathcal{N}}$ and NαOS $\mathcal{E}_2^*$ in $\mathcal{Y}_{\mathcal{N}}$ such that $(p(\alpha,\beta))\ \mathcal{E}_2^*$, there exists an N$\alpha$OS $\mathcal{E}_1^*$ in $\mathcal{X}_{\mathcal{N}}$ such that $p(\alpha,\beta))\in \mathcal{E}_1^*$ and $\ddot{f}\ (\mathcal{E}_1^*)) \subseteq \mathcal{E}_2^*$.

Proof: Let $\ddot{f}$ be any Nα-irresolute function. $p(\alpha,\beta))$ be an NP in $\mathcal{X}_{\mathcal{N}}$ and $\mathcal{E}_2^*$ be any NαOS in $\mathcal{Y}_{\mathcal{N}}$ such that $\ddot{f}\ (p(\alpha,\beta))\in \mathcal{E}_2^*$. Then $p(\alpha,\beta) \in \ddot{f}^{-1}(\mathcal{E}_2^*)$ =Nαint $\ddot{f}^{-1}(\mathcal{E}_2^*)$.Let $\mathcal{E}_1^*$= Nαint $\ddot{f}^{-1}(\mathcal{E}_2^*)$.Then $\mathcal{E}_1^*$ is a NαOS in $\mathcal{X}_{\mathcal{N}}$ which containing NP $p(\alpha,\beta))$ and $\ddot{f}(\mathcal{E}_1^*) = \ddot{f}(\text{N}\alpha\text{int}\,\ddot{f}^{-1}(\mathcal{E}_2^*) = f(\ddot{f}^{-1}(\mathcal{E}_2^*)) \subseteq \mathcal{E}_2^*$ .Conversely, let $\mathcal{E}_2^*$ be an NαOS in $\mathcal{Y}_{\mathcal{N}}$ and $p(\alpha,\beta))$ be an NP in $\mathcal{X}_{\mathcal{N}}$ such that $p(\alpha,\beta))\in \ddot{f}^{-1}(\mathcal{E}_2^*)$. According to an assumption, there exists an NαOS $\in \mathcal{E}_1^*$ in $\mathcal{X}_{\mathcal{N}}$ such that $p(\alpha,\beta)) \in \mathcal{E}_1^*$ and $\ddot{f}\ (\mathcal{E}_1^*)\subseteq \mathcal{E}_2^*$.Hence $p(\alpha,\beta))\in \mathcal{E}_1^* \subseteq \ddot{f}^{-1}(\mathcal{E}_2^*))$ and $p(\alpha,\beta))\in \mathcal{E}_1^* = \ddot{f}^{-1}(\mathcal{E}_2^*)$= N$\alpha$int$A \subseteq$ Nαint$\ddot{f}^{-1}(\mathcal{E}_2^*)$. Since $p(\alpha,\beta))$ be an arbitrary NP and $\ddot{f}^{-1}(\mathcal{E}_2^*)$ is union of all NPs containing in $\ddot{f}^{-1}(\mathcal{E}_2^*)$,which gives that $\ddot{f}^{-1}(\mathcal{E}_2^*)\subseteq$ Nint$\ddot{f}^{-1}(\mathcal{E}_2^*)$is N$\alpha$OS in $\mathcal{X}_{\mathcal{N}}$. Hence $\ddot{f}$ is a Nα-irresolute function.

Theorem 4.5 $\mathcal{E}_1^*$ function $\ddot{f}: (\mathcal{X}_{\mathcal{N}},\mathcal{T}_{\mathcal{N}}) \longrightarrow (\mathcal{Y}_{\mathcal{N}},\mathcal{G}_{\mathcal{N}})$ from an NTS $\mathcal{X}_{\mathcal{N}}$ into an NTS $\mathcal{Y}_{\mathcal{N}}$ is N semi-α-irresolute if and only if for each NP $p(\alpha,\beta))$ in $\mathcal{X}_{\mathcal{N}}$ and NαOS $\mathcal{E}_2^*$ in $\mathcal{Y}_{\mathcal{N}}$ such that $\ddot{f}$ $(p(\alpha,\beta))\in \mathcal{E}_2^*$, there exists an NSOS $\mathcal{E}_1^*$ in $\mathcal{X}_{\mathcal{N}}$ such that $p(\alpha,\beta))\in \mathcal{E}_1^*$ and $\ddot{f}\ (\mathcal{E}_1^*)) \subseteq \mathcal{E}_2^*$.

Proof: Let $\ddot{f}$ be any NS-α-irresolute function, $p(\alpha,\beta))$ be an NP in $\mathcal{X}_{\mathcal{N}}$ and $\mathcal{E}_2^*$ be any NαOS in $\mathcal{Y}_{\mathcal{N}}$ such that $\ddot{f}\ (p(\alpha,\beta))\in \mathcal{E}_2^*$. Then $p(\alpha,\beta) \in (\mathcal{E}_2^*)$. Let $\mathcal{E}_1^*$=$\ddot{f}^{-1}(\mathcal{E}_2^*)$.Then $\mathcal{E}_1^*$ is a NSOS in $\mathcal{X}_{\mathcal{N}}$ which containing NP $p(\alpha,\beta))$ and $\ddot{f}\ (\mathcal{E}_1^*)$=$\ddot{f}(\ \ddot{f}^{-1}(\mathcal{E}_2^*))\subseteq \mathcal{E}_2^*$

Conversely, let $\mathcal{E}_2^*$ be an NαOS in $\mathcal{Y}_{\mathcal{N}}$ and $p(\alpha,\beta))$ be an NP in $\mathcal{X}_{\mathcal{N}}$ such that $p(\alpha,\beta)\in \ddot{f}^{-1}(\mathcal{E}_2^*)$. According to an assumption, there exists an NSOS $\mathcal{E}_1^*$ in $\mathcal{X}_{\mathcal{N}}$ such that $p(\alpha,\beta))\ \mathcal{E}_1^*$ and $\ddot{f}\ (\mathcal{E}_1^*)) \subseteq \mathcal{E}_2^*$.Hence $p(\alpha,\beta))\in \mathcal{E}_1^* \subseteq \ddot{f}^{-1}(\mathcal{E}_2^*)$. Also $p(\alpha,\beta) \in\mathcal{E}_1^* \subseteq$ Ncl(Nint($\mathcal{E}_1^*$) $\subseteq$ Ncl(Nint($\ddot{f}^{-1}(\mathcal{E}_2^*)$)Therefore, $\ddot{f}^{-1}(\mathcal{E}_2^*)\subseteq$ Ncl(Nint($\ddot{f}^{-1}(\mathcal{E}_2^*)$)is NSOS in $\mathcal{X}_{\mathcal{N}}$. Hence $\ddot{f}$ is a NS-α-irresolute function

Theorem 4.6 A function $\ddot{f}: (\mathcal{X}_{\mathcal{N}},\mathcal{T}_{\mathcal{N}}) \longrightarrow (\mathcal{Y}_{\mathcal{N}},\mathcal{G}_{\mathcal{N}})$ from an NTS $\mathcal{X}_{\mathcal{N}}$ into an NTS $\mathcal{Y}_{\mathcal{N}}$ is N pre-β-irresolute if and only if for each NP $p(\alpha,\beta)$ in $\mathcal{X}_{\mathcal{N}}$ and $\mathcal{N}\beta OS$ $\mathcal{E}_2^*$ in $\mathcal{Y}_{\mathcal{N}}$ such that $\ddot{f}$ $(p(\alpha,\beta))\in\mathcal{E}_2^*$, there exists an NPOS $\mathcal{E}_1^*$ in $\mathcal{X}_{\mathcal{N}}$ such that $p(\alpha,\beta))\in \mathcal{E}_1^*$ and $\ddot{f}\ (\mathcal{E}_1^*)) \subseteq \mathcal{E}_2^*$.

Proof: Let $\ddot{f}$ be any Npre-β-irresolute mapping. $p(\alpha,\beta))$ be an NP in $\mathcal{X}_{\mathcal{N}}$ and $\mathcal{E}_2^*$ be any $\mathcal{N}\beta OS$ in $\mathcal{Y}_{\mathcal{N}}$ such that $\ddot{f}\ (p(\alpha,\beta))\in \mathcal{E}_2^*$. Then $(p(\alpha,\beta))\in \ddot{f}^{-1}(\mathcal{E}_2^*)$. Let $\mathcal{E}_1^*$=$\ddot{f}^{-1}(\mathcal{E}_2^*)$.Then $\mathcal{E}_1^*$ is a NPOS in $\mathcal{X}_{\mathcal{N}}$ which containing NP $p(\alpha,\beta))$ and $\ddot{f}\ (\mathcal{E}_1^*)))\ f(\ddot{f}^{-1}(\mathcal{E}_2^*))\subseteq \mathcal{E}_2^*$.

Conversely, let $\mathcal{E}_2^*$ be an $\mathcal{N}\beta OS$ in $\mathcal{Y}_{\mathcal{N}}$ and $p(\alpha,\beta))$ be an NP in $\mathcal{X}_{\mathcal{N}}$ such that $p(\alpha,\beta))\in \ddot{f}^{-1}(\mathcal{E}_2^*)$. According to an assumption, there exists an NPOS $\mathcal{E}_1^*$ in $\mathcal{X}_{\mathcal{N}}$ such that $p(\alpha,\beta))\in \mathcal{E}_1^*$ and $\ddot{f}\ (\mathcal{E}_1^*)\subseteq \mathcal{E}_2^*$. Hence $p(\alpha,\beta) \in\mathcal{E}_1^* \subseteq \ddot{f}^{-1}(\mathcal{E}_2^*)$.Also $p(\alpha,\beta) \in\mathcal{E}_1^* \subseteq$ Nint((Ncl($\mathcal{E}_1^*$)) $\subseteq$ Nint(Ncl$\ddot{f}^{-1}(\mathcal{E}_2^*)$). Therefore, $\ddot{f}^{-1}(\mathcal{E}_2^*) \subseteq$ Nint(Ncl$\ddot{f}^{-1}(\mathcal{E}_2^*)$). is NPOS in $\mathcal{X}_{\mathcal{N}}$. Hence $\ddot{f}$ is a Npre-β-irresolute function.

Theorem 4.7 A function $\ddot{f}: (\mathcal{X}_{\mathcal{N}},\mathcal{T}_{\mathcal{N}}) \longrightarrow (\mathcal{Y}_{\mathcal{N}},\mathcal{G}_{\mathcal{N}})$ from an NTS $\mathcal{X}_{\mathcal{N}}$ into an NTS $\mathcal{Y}_{\mathcal{N}}$ is N pre-α-irresolute if and only if for each NP $p(\alpha,\beta))$ in $\mathcal{X}_{\mathcal{N}}$ and NαOS $\mathcal{E}_2^*$ in $\mathcal{Y}_{\mathcal{N}}$ such that $\ddot{f}$ $(p(\alpha,\beta))\in \mathcal{E}_2^*$,N*cl*($\ddot{f}^{-1}(\mathcal{E}_2^*)$)is a NN of NP $p(\alpha,\beta))$ in $\mathcal{X}_{\mathcal{N}}$.

Proof: Let $\ddot{f}$ be any Npre-α-irresolute function. $p(\alpha,\beta))$ be an NP in $\mathcal{X}_{\mathcal{N}}$ and $\mathcal{E}_2^*$ be any NαOS in $\mathcal{Y}_{\mathcal{N}}$ such that $\ddot{f}\ (p(\alpha,\beta))\in \mathcal{E}_2^*$. Then $p(\alpha,\beta))\in \ddot{f}^{-1}(\mathcal{E}_2^*)\subseteq$Nint(Ncl ($\ddot{f}^{-1}(\mathcal{E}_2^*)$)). Hence Ncl($\ddot{f}^{-1}(\mathcal{E}_2^*)$) is IFN of $p(\alpha,\beta))$ in $\mathcal{X}_{\mathcal{N}}$.

Conversely, let $\mathcal{E}_2^*$ be a NαOS in $\mathcal{Y}_{\mathcal{N}}$ and $p(\alpha,\beta))$ be an NP in $\mathcal{X}_{\mathcal{N}}$ such that $\ddot{f}\ (p(\alpha,\beta))\in \mathcal{E}_2^*$. Then $p(\alpha,\beta))\in \ddot{f}^{-1}(\mathcal{E}_2^*)$ According to an assumption,*Ncl(*$\ddot{f}^{-1}(\mathcal{E}_2^*)$)is NN of NP $p(\alpha,\beta))$ in $\mathcal{X}_{\mathcal{N}}$. So $p(\alpha,\beta))\in$Nint(Ncl($\ddot{f}^{-1}(\mathcal{E}_2^*)$). Thus $\ddot{f}^{-1}(\mathcal{E}_2^*)\subseteq$ Nint(Ncl $\ddot{f}^{-1}(\mathcal{E}_2^*)$). Hence $\ddot{f}^{-1}(\mathcal{E}_2^*)$is a NPOS in $\mathcal{X}_{\mathcal{N}}$. Therefore $\ddot{f}$ is a Npre-α-irresolute function.

Theorem 4.8:

A function $\ddot{f}: (\mathcal{X}_{\mathcal{N}}, \mathcal{T}_{\mathcal{N}}) \longrightarrow (\mathcal{Y}_{\mathcal{N}}, \mathcal{G}_{\mathcal{N}})$ from an NTS $\mathcal{X}_{\mathcal{N}}$ into an NTS $\mathcal{Y}_{\mathcal{N}}$ is N pre-β-irresolute if and only if for each NP $p(\alpha,\beta))$ in $\mathcal{X}_{\mathcal{N}}$ and $\mathcal{N}\beta OS$ $\mathcal{E}_2^*$ in $\mathcal{Y}_{\mathcal{N}}$ such that $\ddot{f}$ $p(\alpha,\beta))\in \mathcal{E}_2^*$,N*cl(*$\ddot{f}^{-1}(\mathcal{E}_2^*)$), is a NN of NP $p(\alpha,\beta))$ in $\mathcal{X}_{\mathcal{N}}$.

Proof: Let $\ddot{f}$ be any Npre-β-irresolute function. $p(\alpha,\beta))$ be an NP in $\mathcal{X}_{\mathcal{N}}$ and $\mathcal{E}_2^*$ be any $\mathcal{N}\beta OS$ in $\mathcal{Y}_{\mathcal{N}}$ such that $\ddot{f}$ $(p(\alpha,\beta))\in \mathcal{E}_2^*$. Then $p(\alpha,\beta))\in \ddot{f}^{-1}(\mathcal{E}_2^*)\subseteq$Nint(Ncl($\ddot{f}^{-1}(\mathcal{E}_2^*)\subseteq$ Ncl(Nint($\ddot{f}^{-1}(\mathcal{E}_2^*)$). Hence Ncl(Nint$\ddot{f}^{-1}(\mathcal{E}_2^*)$) is IFN of $p(\alpha,\beta))$ in $\mathcal{X}_{\mathcal{N}}$.

Conversely, let $\mathcal{E}_2^*$ be an $\mathcal{N}\beta OS$ in $\mathcal{Y}_{\mathcal{N}}$ and $p(\alpha,\beta))$ be an NP in $\mathcal{X}_{\mathcal{N}}$ such that $\ddot{f}(p(\alpha,\beta))\in \mathcal{E}_2^*$. Then $p(\alpha,\beta))\in \ddot{f}^{-1}(\mathcal{E}_2^*)$ According to an assumption,N*cl(Nint* $\ddot{f}^{-1}(\mathcal{E}_2^*)$) is nN of NP$p(\alpha,\beta))$ in $\mathcal{X}_{\mathcal{N}}$. Thus $p(\alpha,\beta))\in$ Nint(Ncl($\ddot{f}^{-1}(\mathcal{E}_2^*)$), so $\ddot{f}^{-1}(\mathcal{E}_2^*)\subseteq$Nint(Ncl($\ddot{f}^{-1}(\mathcal{E}_2^*)$). Hence $\ddot{f}^{-1}(\mathcal{E}_2^*)$is a NPOS in $\mathcal{X}_{\mathcal{N}}$. Therefore $\ddot{f}$ is a Npre-β-irresolute function.

Theorem 4.9

The following hold for functions $\ddot{f}: \mathcal{X}_{\mathcal{N}} \longrightarrow \mathcal{Y}_{\mathcal{N}}$ and $\ddot{g} : \mathcal{Y}_{\mathcal{N}} \longrightarrow \mathcal{Z}_{\mathcal{N}}$

i) If $\ddot{f}$ is Npre irresolute and g is Npre-α-irresolute (Npre-β-irresolute, resp.), then $\ddot{g} \circ \ddot{f}$ is N pre-α-irresolute (Npre-β-irresolute, resp.) function.

ii) If $\ddot{f}$ is Npre-α- irresolute (Npre-β-irresolute, resp.), and g is Nα-continuous ($\mathcal{N}\beta$ – continuous, resp.), then $\ddot{g} \circ \ddot{f}$ is N pre continuous.

iii) If $\ddot{f}$ is Npre-α-irresolute (Npre-β-irresolute, resp.) and g is Nα-irresolute ($\mathcal{N}\beta$ – irresolute, resp.), then $\ddot{g} \circ \ddot{f}$ is Npre-α-irresolute (Npre-β-irresolute, resp.).

iv) If $\ddot{f}$ is NS-α-irresolute (Nα-irresolute, resp.) and g is IFα-continuous, then $\ddot{g} \circ \ddot{f}$ is N Semi continuous (Nα-continuous, resp.).

v) If $\ddot{f}$ is NS-α-irresolute (Nα-irresolute, resp.) and g is IFα-irresolute, then $\ddot{g} \circ \ddot{f}$ is NS-α-irresolute (Nα-irresolute, resp.).

vi) If $\ddot{f}$ is Nirresolute and g is NS-α-irresolute, then $\ddot{g} \circ \ddot{f}$ is NS-α-irresolute.

vii) If $\ddot{f}$ is Nα-irresolute and g is Nstrongly α-continuous, then $\ddot{g} \circ \ddot{f}$ is Nstrongly α-continuous.

Proof:

(i) Let $\mathcal{E}_2^*$ be an NαOS ($\mathcal{N}\beta OS$, resp.) in Z. Since g is Npre-α-irresolute (Npre-β-irresolute, resp.) $\ddot{g}^{-1}$ $(\mathcal{E}_2^*)$is a NPOS in $\mathcal{Y}_{\mathcal{N}}$.Now $(\ddot{g} \circ \ddot{f})^{-1}(\mathcal{E}_2^*) = \ddot{f}^{-1}(\ddot{g}^{-1}(\mathcal{E}_2^*))$. Since $\ddot{f}$ is Npre irresolute,$\ddot{f}^{-1}(\ddot{g}^{-1}(\mathcal{E}_2^*))$ is a NPOS in $\mathcal{X}_{\mathcal{N}}$. Hence $\ddot{g} \circ \ddot{f}$ is Npre-α-irresolute (Npre-β-irresolute, resp.).

(ii) Let $\mathcal{E}_2^*$ be an NOS in Z. Since g is Nα-continuous ($\mathcal{N}\beta$ -continuous, resp.), $\ddot{g}^{-1}$ $(\mathcal{E}_2^*)$ is a NαOS ($\mathcal{N}\beta OS$, resp.) in $\mathcal{Y}_{\mathcal{N}}$.Now $(\ddot{g} \circ \ddot{f})^{-1}(\mathcal{E}_2^*)= \ddot{f}^{-1}(\ddot{g}^{-1}(\mathcal{E}_2^*))$. Since $\ddot{f}$ is Npre-α-irresolute (Npre-β-irresolute, resp.), $\ddot{f}^{-1}(\ddot{g}^{-1}(\mathcal{E}_2^*))$is a NPOS in $\mathcal{X}_{\mathcal{N}}$. Hence $\ddot{g} \circ \ddot{f}$ is Npre continuous.

(iii) Let $\mathcal{E}_2^*$ be an NαOS ($\mathcal{N}\beta OS$, resp.) in Z. Since g is Nα-irresolute ($\mathcal{N}\beta$ -irresolute, resp.), $\ddot{g}^{-1}$ $(\mathcal{E}_2^*)$ is a NαOS ($\mathcal{N}\beta OS$ resp.) in $\mathcal{Y}_{\mathcal{N}}$. Now $(\ddot{g} \circ \ddot{f})^{-1}(\mathcal{E}_2^*)= \ddot{f}^{-1}(\ddot{g}^{-1}(\mathcal{E}_2^*))$. Since $\ddot{f}$ is Npre-α-irresolute (Npre-β-irresolute, resp.), $\ddot{f}^{-1}(\ddot{g}^{-1}(\mathcal{E}_2^*))$ is a NPOS in $\mathcal{X}_{\mathcal{N}}$. Hence $\ddot{g} \circ \ddot{f}$ is Npre-α-irresolute (Npre-β-irresolute, resp.).

(iv) Let $\mathcal{E}_2^*$ be an NOS in Z. Since g is Nα-continuous, $\ddot{g}^{-1}$ $(\mathcal{E}_2^*)$ is an NαOS in $\mathcal{Y}_{\mathcal{N}}$. Now $(\ddot{g} \circ \ddot{f})^{-1}(\mathcal{E}_2^*)$ $\ddot{f}^{-1}(\ddot{g}^{-1}(\mathcal{E}_2^*))$. Since $\ddot{f}$ isNS-α-irresolute (Nα-irresolute, resp.), $\ddot{f}^{-1}(\ddot{g}^{-1}(\mathcal{E}_2^*))$ is a NSOS (NαOS, resp.) in $\mathcal{X}_{\mathcal{N}}$. Hence $\ddot{g} \circ \ddot{f}$ is NSemi continuous (Nα-continuous, resp.).

(v) Let $\mathcal{E}_2^*$ be an NαOS in Z. Since g is Nα-irresolute, $\ddot{g}^{-1}(\mathcal{E}_2^*)$ is an NαOS in $\mathcal{Y}_{\mathcal{N}}$. Now Since $\ddot{f}$ is NS-α-irresolute (Nα- irresolute, resp.), $\ddot{f}^{-1}(\ddot{g}^{-1}(\mathcal{E}_2^*))$ is a NSOS (NαOS, resp.) in $\mathcal{X}_{\mathcal{N}}$. Hence $\ddot{g} \circ \ddot{f}$ is NS-α-irresolute (Nα-irresolute, resp.).

(vi) Let $\mathcal{E}_2^*$ be an NαOS in Z. Since g is NS-α-irresolute, $\ddot{g}^{-1}(\mathcal{E}_2^*)$ is a NSOS in $\mathcal{Y}_{\mathcal{N}}$. Now $(\ddot{g} \circ \ddot{f})^{-1}(\mathcal{E}_2^*)= \ddot{f}^{-1}(\ddot{g}^{-1}(\mathcal{E}_2^*))$. Since $\ddot{f}$ is Nirresolute, $\ddot{f}^{-1}(\ddot{g}^{-1}(\mathcal{E}_2^*))$is a NSOS in $\mathcal{X}_{\mathcal{N}}$. Hence $\ddot{g} \circ \ddot{f}$ is NS-α-irresolute.

(vii) Let $\mathcal{E}_2^*$ be an NSOS in Z. Since g is Nstrongly α-continuous $\ddot{g}^{-1}(\mathcal{E}_2^*)$ is a NαOS in $\mathcal{Y}_{\mathcal{N}}$. Now $(\ddot{g} \circ \ddot{f})^{-1}(\mathcal{E}_2^*)\ \ddot{f}^{-1}(\ddot{g}^{-1}(\mathcal{E}_2^*))$. Since $\ddot{f}$ is Nα-irresolute, $\ddot{f}^{-1}(\ddot{g}^{-1}(\mathcal{E}_2^*))$ is a NαOS in $\mathcal{X}_{\mathcal{N}}$. Hence $\ddot{g} \circ \ddot{f}$ is Nstrongly α-continuous.

5 . CHARACTERIZATIONS

In this section, several characterizations of Neutrosophic pre-α-irresolute functions, Neutrosophic α-irresolute functions, Neutrosophic semi-α-irresolute functions and Neutrosophic pre-β-irresolute functions are established

Theorem 5.1 If $\ddot{f}$ is a function from an NTS $(\mathcal{X}_{\mathcal{N}},\mathcal{T}_{\mathcal{N}})$ to another NTS $(\mathcal{Y}_{\mathcal{N}},\mathcal{G}_{\mathcal{N}})$,
then the following are equivalent.

(a) $\ddot{f}$ is a Npre-α-irresolute.

(b) $\ddot{f}^{-1}(\mathcal{E}_2^*) \subseteq$ int $(cl\ (\ddot{f}^{-1}(\mathcal{E}_2^*)))$ for every NαOS $\mathcal{E}_2^*$ in $\mathcal{Y}_{\mathcal{N}}$.

(c) $\ddot{f}^{-1}(\mathcal{E}_3^*)$ is $\mathcal{NPCS}$ in $\mathcal{X}_{\mathcal{N}}$ for every NαCS $\mathcal{E}_3^*$ in $\mathcal{Y}_{\mathcal{N}}$.

(d) Ncl (Nint $(\ddot{f}^{-1}(\mathcal{E}_4^*)))\subseteq\ddot{f}^{-1}(N\alpha cl(\mathcal{E}_4^*)$ for every NS $\mathcal{E}_4^*$ of $\mathcal{Y}_{\mathcal{N}}$.

(e) $\ddot{f}$ (Ncl (Nint $\mathcal{E}_5^*$)) $\subseteq$N$\alpha clf(\mathcal{E}_5^*)$ for every NS $\mathcal{E}_5^*$ of $\mathcal{X}_{\mathcal{N}}$.

Proof:

(a)⇒ (b): Let $\mathcal{E}_2^*$ be an NαOS in $\mathcal{Y}_{\mathcal{N}}$. By (a), $\ddot{f}^{-1}(\mathcal{E}_2^*)$ is $\mathcal{NPOS}$ in $\mathcal{X}_{\mathcal{N}}$. $\ddot{f}^{-1}(\mathcal{E}_2^*)\subseteq$ Nint(Ncl($\ddot{f}^{-1}(\mathcal{E}_2^*)$)). Hence (a)⇒ (b) is proved.

(b)⇒ (c): Let $\mathcal{E}_3^*$ be any NαCS in $\mathcal{Y}_{\mathcal{N}}$. Then $\overline{\mathcal{E}_3^*}$ is NαOS in $\mathcal{Y}_{\mathcal{N}}$. By (b),$(\ddot{f}^{-1}(\mathcal{E}_3^*)\ \overline{\mathcal{E}_3^*}$ $\subseteq$Nint(Ncl $\ddot{f}^{-1}(\overline{\mathcal{E}_3^*})$). But $\overline{\ddot{f}^{-1}(\mathcal{E}_3^*)}\subseteq$ Nint(Ncl($\overline{\ddot{f}^{-1}(\mathcal{E}_3^*)}$)))= Nint(Nint($\overline{\ddot{f}^{-1}(\mathcal{E}_3^*)}$)))= $\overline{Ncl(Nint(\ddot{f}^{-1}(\mathcal{E}_3^*))}$
This implies $\overline{\ddot{f}^{-1}(\mathcal{E}_3^*)}\subseteq$ $\overline{Ncl(Nint(\ddot{f}^{-1}(\mathcal{E}_3^*))}$ $\Rightarrow Ncl(Nint(\ddot{f}^{-1}(\mathcal{E}_3^*) \subseteq \ddot{f}^{-1}(\mathcal{E}_3^*) \Rightarrow \ddot{f}^{-1}(\mathcal{E}_3^*)$ is $\mathcal{NPCS}$ in $\mathcal{X}_{\mathcal{N}}$. Hence(b) ⇒ (c) is proved.

(c) ⇒ (d): Let $\mathcal{E}_4^*$ be an $\mathcal{NS}$ in $\mathcal{Y}_{\mathcal{N}}$. Then N$\alpha$cl($\mathcal{E}_4^*$)) is N$\alpha$-closed in $\mathcal{Y}_{\mathcal{N}}$. ⇒ $\ddot{f}^{-1}$(Nαcl($\mathcal{E}_4^*$))) is $\mathcal{NPCS}$ in $\mathcal{X}_{\mathcal{N}}$. Then Ncl(Nint($\ddot{f}^{-1}$(N$\alpha$cl($\mathcal{E}_4^*$)))⊆(Nαcl($\mathcal{E}_4^*$))) Hence (c)⇒ (d) is proved.

(d)⇒ (e): Let $\mathcal{E}_5^*$ be an NS in $\mathcal{X}_{\mathcal{N}}$. Then Ncl(Nint($\mathcal{E}_5^*$) ⊆ Ncl(Nint$\ddot{f}^{-1}\big(\ddot{f}(\mathcal{E}_5^*)\big)$
⊆ Ncl(Nint($\ddot{f}^{-1}\big($Nαcl$\ddot{f}(\mathcal{E}_5^*)\big)\big)$⊆ N$\alpha$cl$\ddot{f}(\mathcal{E}_5^*)$⊆ Then Ncl(Nint(($\mathcal{E}_5^*$)) ⊆$\ddot{f}^{-1}\big(N\alpha$cl$\ddot{f}(\mathcal{E}_5^*)\big)\big)$.
Thus $\ddot{f}(Ncl(Nint((\mathcal{E}_5^*))\subseteq$N$\alpha$cl$\ddot{f}(\mathcal{E}_5^*)$). Hence (d) ⇒ (e) isproved.

(e)⇒ (a): Let $\mathcal{E}_2^*$ be an NαOS in $\mathcal{Y}_{\mathcal{N}}$. Then $\ddot{f}^{-1}(\mathcal{E}_2^*) = \overline{\ddot{f}^{-1}(\mathcal{E}_2^*)}$ = Nint(Ncl($\ddot{f}^{-1}(\mathcal{E}_2^*)$))is a NS in $\mathcal{X}_{\mathcal{N}}$. By (e),

$$\ddot{f}(Ncl(Nint(\ddot{f}^{-1}(\mathcal{E}_2^*))) \subseteq N\alpha cl(\ddot{f}\left(\ddot{f}^{-1}(\mathcal{E}_2^*)\right) \subseteq N\alpha cl\big(\overline{\mathcal{E}_2^*}\big) = \overline{N\alpha int(\mathcal{E}_2^*)} = \overline{\mathcal{E}_2^*}$$

Thus, $\ddot{f}(Ncl(Nint(\ddot{f}^{-1}(\mathcal{E}_2^*))) \subseteq N\alpha cl(\ddot{f}\left(\ddot{f}^{-1}(\mathcal{E}_2^*)\right)\subseteq \overline{\mathcal{E}_2^*}$. ------(1)

Consider

$\overline{Nint(Ncl(\ddot{f}^{-1}(\mathcal{E}_2^*))} = Ncl\overline{(Ncl(\ddot{f}^{-1}(\mathcal{E}_2^*))}$ =Ncl(Nint$\overline{(\ddot{f}^{-1}(\mathcal{E}_2^*))}$ ⊆ $\ddot{f}^{-1}\ddot{f}(Ncl\left(Nint(\ddot{f}^{-1}(\overline{\mathcal{E}_2^*}))\right)$------(2)

By (1) and (2), $\overline{Nint(Ncl(\ddot{f}^{-1}(\mathcal{E}_2^*))} \subseteq \ddot{f}^{-1}\ddot{f}(Ncl\left(Nint(\ddot{f}^{-1}(\overline{\mathcal{E}_2^*}))\right)) \subseteq$Nint(Ncl($\ddot{f}^{-1}(\mathcal{E}_2^*)$)) $\Rightarrow \ddot{f}^{-1}(\overline{\mathcal{E}_2^*}) = \overline{\ddot{f}^{-1}(\mathcal{E}_2^*)} \Rightarrow \ddot{f}^{-1}(\mathcal{E}_2^*) \subseteq$ Nint((Ncl($\ddot{f}^{-1}(\mathcal{E}_2^*)$))$\Rightarrow \ddot{f}^{-1}(\mathcal{E}_2^*)is\mathcal{N}POS$ in $\mathcal{X}_{\mathcal{N}}$. Thus $\ddot{f}$ is Npre-α-irresolute. Hence (e)$\Rightarrow$ (a) is proved.

Theorem 5.2 If $\ddot{f}$: $(\mathcal{X}_{\mathcal{N}},\mathcal{T}_{\mathcal{N}}) \longrightarrow (\mathcal{Y}_{\mathcal{N}},\mathcal{G}_{\mathcal{N}})$ be a mapping from NTS $\mathcal{X}_{\mathcal{N}}$ into NTS $\mathcal{Y}_{\mathcal{N}}$.
Then the following are equivalent.

(a) $\ddot{f}$ is Nα-irresolute.

(b) $\ddot{f}^{-1}(\mathcal{E}_2^*)$ is NαCS in $\mathcal{X}_{\mathcal{N}}$ for each NαCS $\mathcal{E}_2^*$ in $\mathcal{Y}_{\mathcal{N}}$.

(c) $\ddot{f}$ (NαclA) $\subseteq$Nαclf $(\mathcal{E}_1^*)$)) for each NS $\mathcal{E}_1^*$ in $\mathcal{X}_{\mathcal{N}}$.

(d) Nαcl ($\ddot{f}^{-1}$ $(\mathcal{E}_2^*)$) $\subseteq \ddot{f}^{-1}$(N$\alpha cl(\mathcal{E}_2^*)$for each NS $\mathcal{E}_2^*$ in $\mathcal{Y}_{\mathcal{N}}$.

(e) $\ddot{f}^{-1}$(Nα int $\mathcal{E}_2^*$) $\subseteq$Nα int $\ddot{f}^{-1}(\mathcal{E}_2^*)$ *f*or each NS $\mathcal{E}_2^*$ in $\mathcal{Y}_{\mathcal{N}}$.

Proof:

(a)⇒(b): Let $\mathcal{E}_2^*$ be NαCS in $\mathcal{Y}_{\mathcal{N}}$. Then $\overline{\mathcal{E}_2^*}$ is NαOS in $\mathcal{Y}_{\mathcal{N}}$. Since $\ddot{f}$ is Nα-irresolute, $\ddot{f}^{-1}(\mathcal{E}_2^*) = \overline{\ddot{f}^{-1}(\mathcal{E}_2^*)}$is N$\alpha$OS in $\mathcal{X}_{\mathcal{N}}$. Hence $\ddot{f}^{-1}(\mathcal{E}_2^*)$ is NαCS in $\mathcal{X}_{\mathcal{N}}$.Thus (a)⇒(b) is proved.

(b)⇒(c): Let $\mathcal{E}_1^*$ be NS in $\mathcal{X}_{\mathcal{N}}$. Then $\mathcal{E}_1^* \subseteq \ddot{f}^{-1}(\ddot{f}(\mathcal{E}_1^*)) \subseteq \ddot{f}^{-1}(N\alpha cl(\ddot{f}(\mathcal{E}_1^*))$ .As $N\alpha cl(\ddot{f}(\mathcal{E}_1^*))$ is NαCS in $\mathcal{Y}_{\mathcal{N}}$, by(b), $\ddot{f}^{-1}(N\alpha cl(\ddot{f}(\mathcal{E}_1^*))$ is a NαCS in $\mathcal{X}_{\mathcal{N}}$. $N\alpha cl(\mathcal{E}_1^*) \subseteq N\alpha cl((N\alpha cl\left(\ddot{f}(\mathcal{E}_1^*)\right) = N\alpha cl\left(\ddot{f}(\mathcal{E}_1^*)\right)$Hence (b)⇒(c) is proved.

(c)⇒(d): For any NS $\mathcal{E}_2^*$ in $\mathcal{Y}_{\mathcal{N}}$, let $\ddot{f}^{-1}(\mathcal{E}_2^*) = \mathcal{E}_1^*$ By (c), $\ddot{f}(N\alpha cl\left(\ddot{f}^{-1}(\mathcal{E}_2^*)\right) \subseteq N\alpha cl\ddot{f}\left(\ddot{f}^{-1}(\mathcal{E}_2^*)\right) \subseteq N\alpha cl(\mathcal{E}_2^*)$.and $\alpha cl\left(\ddot{f}^{-1}(\mathcal{E}_2^*)\right) \subseteq \ddot{f}^{-1}\left(\ddot{f}(N\alpha cl\left(\ddot{f}^{-1}(\mathcal{E}_2^*)\right)\right) \subseteq \ddot{f}^{-1}\left(\ddot{f}(N\alpha cl\left(\ddot{f}^{-1}(\mathcal{E}_2^*)\right)\right) \subseteq \ddot{f}^{-1}(N\alpha cl(\mathcal{E}_2^*))$

Thus $N\alpha cl\ddot{f}^{-1}(\mathcal{E}_2^*) \subseteq \ddot{f}^{-1}(N\alpha cl(\mathcal{E}_2^*))$ Hence (c)⇒(d) is proved.

(d)⇒(e): For any NS $\mathcal{E}_2^*$ in $\mathcal{Y}_{\mathcal{N}}$, $N\alpha int(\mathcal{E}_2^*) = \overline{N\alpha cl(\overline{\mathcal{E}_2^*})}$.Now$\ddot{f}^{-1}(N\alpha int(\mathcal{E}_2^*)) = \ddot{f}^{-1}\left(\overline{N\alpha cl(\overline{\mathcal{E}_2^*})}\right) = \overline{\ddot{f}^{-1}\left(N\alpha cl(\overline{\mathcal{E}_2^*})\right)} = \overline{N\alpha cl\ddot{f}^{-1}(\overline{\mathcal{E}_2^*})} = \overline{N\alpha int\overline{\ddot{f}^{-1}(\mathcal{E}_2^*)}} \subseteq N\alpha int\ddot{f}^{-1}(\mathcal{E}_2^*)$

(e)⇒(a): Let $\mathcal{E}_2^*$ be NαOS in $\mathcal{Y}_{\mathcal{N}}$. Then $\mathcal{E}_2^* = N\alpha int(\mathcal{E}_2^*)$ and $\ddot{f}^{-1}(\mathcal{E}_2^*) = \ddot{f}^{-1}(\mathcal{E}_2^*) \subseteq N\alpha int(\mathcal{E}_2^*)$. By definition $\ddot{f}^{-1}(\mathcal{E}_2^*) \supseteq N\alpha int$ $(\ddot{f}^{-1}(\mathcal{E}_2^*))$.So $\ddot{f}^{-1}(\mathcal{E}_2^*)$=$N\alpha int$ $(\ddot{f}^{-1}(\mathcal{E}_2^*))$.Thus $\ddot{f}^{-1}(\mathcal{E}_2^*)$ is a NαOS in $\mathcal{X}_{\mathcal{N}}$ which implies $\ddot{f}$ is Nα-irresolute. Thus (e)⇒(a) is proved.

Theorem 5.3 If $\ddot{f}$ is a function from an $\mathcal{NTS}(\mathcal{X}_{\mathcal{N}},\mathcal{T}_{\mathcal{N}})$ to another NTS $(\mathcal{Y}_{\mathcal{N}},\mathcal{G}_{\mathcal{N}})$,
then the following are equivalent.

(a) $\ddot{f}$ is a NS-α-irresolute.

(b) $\ddot{f}^{-1}(\mathcal{E}_2^*)$ $\subseteq$ N*cl* (Nint ($\ddot{f}^{-1}$ $(\mathcal{E}_2^*)$)) for every NαOS $\mathcal{E}_2^*$ in $\mathcal{Y}_{\mathcal{N}}$.

(c) $\ddot{f}^{-1}$ $(\mathcal{E}_3^*)$ is NSemiclosed in $\mathcal{X}_{\mathcal{N}}$ for every Nα-closed set $\mathcal{E}_3^*$ in $\mathcal{Y}_{\mathcal{N}}$.

(d) Nint (N*cl* ($\ddot{f}^{-1}(\mathcal{E}_4^*)$)) $\subseteq \ddot{f}^{-1}$(Nαcl $\mathcal{E}_4^*$) for every NS $\mathcal{E}_4^*$ of $\mathcal{Y}_{\mathcal{N}}$.

(e) $\ddot{f}$ (Nint (N*clE*)) $\subseteq$Nαclf $(\mathcal{E}_5^*)$for every NS $\mathcal{E}_5^*$ of $\mathcal{X}_{\mathcal{N}}$.

Proof: (a)⇒ (b):

Let $\mathcal{E}_2^*$ be an NαOS in $\mathcal{Y}_{\mathcal{N}}$. By (a), $\ddot{f}^{-1}(\mathcal{E}_2^*)$ is $\mathcal{N}SOS$ in $\mathcal{X}_{\mathcal{N}}$. $\Rightarrow \ddot{f}^{-1}(\mathcal{E}_2^*)$ $\subseteq$ Ncl(Nint $\ddot{f}^{-1}(\mathcal{E}_2^*)$. Hence (a)⇒ (b) is proved.

(b)⇒ (c): Let $\mathcal{E}_3^*$ be any NαCS in $\mathcal{Y}_{\mathcal{N}}$. Then $\overline{\mathcal{E}_3^*}$ is a NαOS in $\mathcal{Y}_{\mathcal{N}}$. By (b), $\ddot{f}^{-1}(\overline{\mathcal{E}_3^*}) \subseteq$ Ncl(Nint($\ddot{f}^{-1}(\overline{\mathcal{E}_3^*})$. But $\overline{\ddot{f}^{-1}(\mathcal{E}_3^*)} \subseteq$ Ncl(Nint($\ddot{f}^{-1}(\overline{\mathcal{E}_3^*})$) = $\overline{\text{Ncl(Ncl}(\ddot{f}^{-1}(\mathcal{E}_3^*))}$ = Nint(Ncl($\ddot{f}^{-1}(\mathcal{E}_3^*)$) ⊆ $\ddot{f}^{-1}(\mathcal{E}_3^*) \Rightarrow \ddot{f}^{-1}(\mathcal{E}_3^*)$ is NSemiclosed in $\mathcal{X}_{\mathcal{N}}$. Hence (b)⇒ (c) is proved.

(c)⇒ (d): Let $\mathcal{E}_4^*$ be an NS in $\mathcal{Y}_{\mathcal{N}}$. Then Nα-cl($\mathcal{E}_4^*$) is Nα-closed in $\mathcal{Y}_{\mathcal{N}}$. By (c) $\ddot{f}^{-1}$(Nαcl($\mathcal{E}_4^*$))) is NSemiclosed in $\mathcal{X}_{\mathcal{N}}$. Then Nint(Ncl$\ddot{f}^{-1}$(Nαcl($\mathcal{E}_4^*$))) ⊆$\ddot{f}^{-1}$(Nαcl($\mathcal{E}_4^*$))). Hence (c)⇒ (d) is proved.

(d)⇒ (e): Let $\mathcal{E}_5^*$ be an NS in $\mathcal{X}_{\mathcal{N}}$. Then Nint(Ncl($\mathcal{E}_5^*$) ⊆ Nint(Ncl$\ddot{f}^{-1}(\ddot{f}(\mathcal{E}_5^*$)))⊆ Nint(Ncl$\ddot{f}^{-1}(N\alpha cl\ddot{f}(\mathcal{E}_5^*$)))Therefore Nint(Ncl($\mathcal{E}_5^*$)⊆ $\ddot{f}^{-1}(N\alpha cl\ddot{f}(\mathcal{E}_5^*$))).

Consequently $\ddot{f}(Nint(Ncl(\mathcal{E}_5^*) \subseteq N\alpha cl\ddot{f}(\mathcal{E}_5^*)$. Hence (d) ⇒(e) is proved.

(e)⇒ (a): Let $\mathcal{E}_2^*$ be an NαOS in $\mathcal{Y}_{\mathcal{N}}$. Then $\ddot{f}^{-1}(\overline{\mathcal{E}_2^*})$= $\overline{\ddot{f}^{-1}(\mathcal{E}_2^*)}$))is a NS in $\mathcal{X}_{\mathcal{N}}$.

By (e), $\ddot{f}(Nint(Ncl(\ddot{f}^{-1}(\overline{\mathcal{E}_2^*})) \subseteq N\alpha cl\ddot{f}(\ddot{f}^{-1}(\overline{\mathcal{E}_2^*})) \subseteq N\alpha cl(\overline{\mathcal{E}_2^*}) = \overline{N\alpha int(\mathcal{E}_2^*)} = \overline{\mathcal{E}_2^*}$

Thus, $\ddot{f}(Nint(Ncl(\ddot{f}^{-1}(\overline{\mathcal{E}_2^*})) \subseteq \overline{\mathcal{E}_2^*}$. ------(1)

Consider $Ncl(Nint\left(\ddot{f}^{-1}(\mathcal{E}_2^*)\right) = Nint(Nint\left(\ddot{f}^{-1}(\mathcal{E}_2^*)\right) = Nint(Ncl\left(\ddot{f}^{-1}(\overline{\mathcal{E}_2^*})\right)$

$Nint(Ncl(\ddot{f}^{-1}(\mathcal{E}_2^*)) \subseteq \ddot{f}^{-1}(\ddot{f}(Nint(Ncl(\ddot{f}^{-1}(\mathcal{E}_2^*))))$)-----(2)

By (1) and (2),

$\overline{Ncl(Nint\left(\ddot{f}^{-1}(\mathcal{E}_2^*)\right)} \subseteq \ddot{f}^{-1}(\ddot{f}(Nint(Ncl(\ddot{f}^{-1}(\mathcal{E}_2^*))))$)

$\subseteq \ddot{f}^{-1}(\overline{\mathcal{E}_2^*}) = \overline{\ddot{f}^{-1}(\overline{\mathcal{E}_2^*})} \Rightarrow \ddot{f}^{-1}(\mathcal{E}_2^*) \subseteq Ncl(Nint(\ddot{f}^{-1}(\mathcal{E}_2^*)) \Rightarrow \ddot{f}^{-1}(\mathcal{E}_2^*)$ is$\mathcal{N}SOS$ in $\mathcal{X}_{\mathcal{N}}$.

Thus $\ddot{f}$ is NS-α-irresolute. Hence (e)⇒ (a) is proved.

Theorem 5.4 If $\ddot{f}$ is a function from an NTS ($\mathcal{X}_{\mathcal{N}}$,$\mathcal{T}_{\mathcal{N}}$) to another NTS ($\mathcal{Y}_{\mathcal{N}}$,$\mathcal{G}_{\mathcal{N}}$), then the following are equivalent.

(a) $\ddot{f}$ is a Npre-β-irresolute.

(b) $\ddot{f}^{-1}(\mathcal{E}_2^*) \subseteq$ int (*cl* ($\ddot{f}^{-1}$ ($\mathcal{E}_2^*$))) for every $\mathcal{N}\beta OS$ $\mathcal{E}_2^*$ in $\mathcal{Y}_{\mathcal{N}}$.

(c) $\ddot{f}^{-1}$ ($\mathcal{E}_3^*$) is $\mathcal{N}PCS$ in $\mathcal{X}_{\mathcal{N}}$ for every $\mathcal{N}\beta$ -closed set $\mathcal{E}_3^*$ in $\mathcal{Y}_{\mathcal{N}}$.

(d) *cl* (int ($\ddot{f}^{-1}(\mathcal{E}_4^*)$)) ⊆ $\ddot{f}^{-1}(N\beta clD)$for every NS $\mathcal{E}_4^*$ of $\mathcal{Y}_{\mathcal{N}}$.

(e) $\ddot{f}$ (*cl* (int $\mathcal{E}_5^*$)) ⊆ $N\beta cl\ddot{f}$ ($\mathcal{E}_5^*$)for every NS $\mathcal{E}_5^*$ of $\mathcal{X}_{\mathcal{N}}$.

Proof: (a)⇒ (b): Let $\mathcal{E}_2^*$ be an $\mathcal{N}\beta OS$ in $\mathcal{Y}_{\mathcal{N}}$. By (a), $\ddot{f}^{-1}(\mathcal{E}_2^*)$ is $\mathcal{N}POS$ in $\mathcal{X}_{\mathcal{N}} \Rightarrow \ddot{f}^{-1}(\mathcal{E}_2^*) \subseteq Nint(Ncl(\ddot{f}^{-1}(\mathcal{E}_2^*))$. Hence (a)⇒ (b) is proved.

(b)⇒ (c): Let $\mathcal{E}_3^*$ be any $\mathcal{N}\beta CS$ in $\mathcal{Y}_{\mathcal{N}}$. Then $\overline{\mathcal{E}_3^*}$ is $\mathcal{N}\beta OS$ in $\mathcal{Y}_{\mathcal{N}}$. By (b), $\ddot{f}^{-1}(\overline{\mathcal{E}_3^*}) \subseteq Nint(Ncl(\ddot{f}^{-1}(\overline{\mathcal{E}_3^*}))$. But $\overline{\ddot{f}^{-1}(\mathcal{E}_3^*)} \subseteq$ Nint(N(cl($\overline{\ddot{f}^{-1}(\mathcal{E}_3^*)}$) = Nint(N(int($\overline{\ddot{f}^{-1}(\mathcal{E}_3^*)}$) =$\overline{\text{Ncl(Nint}(\ddot{f}^{-1}(\mathcal{E}_3^*)))}$ ⇒ $\overline{\ddot{f}^{-1}(\mathcal{E}_3^*)} \subseteq \overline{\text{Ncl(Nint}(\ddot{f}^{-1}(\mathcal{E}_3^*)))}$.

This implies Ncl(Nint($\ddot{f}^{-1}(\mathcal{E}_3^*)$)) ⊆ $\ddot{f}^{-1}(\mathcal{E}_3^*) \Rightarrow \ddot{f}^{-1}(\mathcal{E}_3^*)$is $\mathcal{N}PCS$ in $\mathcal{X}_{\mathcal{N}}$.

Hence (b) ⇒ (c) is proved.

(c)⇒ (d): Let $\mathcal{E}_4^*$ be an NS in $\mathcal{Y}_{\mathcal{N}}$. Then $N\beta cl(\mathcal{E}_4^*)$ is $\mathcal{N}\beta$ -closed in $\mathcal{Y}_{\mathcal{N}}$. By (c),$\overline{\ddot{f}^{-1}(N\beta cl(\mathcal{E}_4^*)}$)is $\mathcal{N}PCS$ in $\mathcal{X}_{\mathcal{N}}$. Then ($\ddot{f}^{-1}(N\beta cl(\mathcal{E}_4^*)) \subseteq \ddot{f}^{-1}(N\beta cl(\mathcal{E}_4^*))$

Thus Ncl(Nint($\overline{\ddot{f}^{-1}(N\beta cl(\mathcal{E}_4^*))} \subseteq \ddot{f}^{-1}(N\beta cl(\mathcal{E}_4^*))$

Hence (c) ⇒ (d) is proved.

(d)⇒ (e):

Let $\mathcal{E}_5^*$ be an NS in $\mathcal{X}_{\mathcal{N}}$. Then Ncl(Nint($\mathcal{E}_5^*$)) ⊆ Ncl(Nint$\ddot{f}^{-1}(\ddot{f}(\mathcal{E}_5^*$))) ⊆ Ncl(Nint($\ddot{f}^{-1}(N\beta cl(\ddot{f}(\mathcal{E}_5^*))))$ ⊆ $\ddot{f}^{-1}(N\beta cl(\ddot{f}(\mathcal{E}_5^*))))$ then Ncl(Nint($\mathcal{E}_5^*$)) ⊆$\ddot{f}^{-1}N\beta cl(\ddot{f}(\mathcal{E}_5^*))))$.This implies $\ddot{f}(Ncl(Nint(\mathcal{E}_5^*))) \subseteq N\beta cl(\ddot{f}(\mathcal{E}_5^*))))$. Hence (d)⇒ (e) is proved.

(e)⇒ (a)**:** Let $\mathcal{E}_2^*$ be an $\mathcal{N}\beta OS$ in $\mathcal{Y}_{\mathcal{N}}$. Then $\ddot{f}^{-1}$ ($\overline{\mathcal{E}_2^*}$)= $\overline{\ddot{f}^{-1}(\mathcal{E}_2^*)}$is a NS in $\mathcal{X}_{\mathcal{N}}$.

By (e), $\ddot{f}(Ncl(Nint\left(\ddot{f}^{-1}\left(\overline{\mathcal{E}_2^*}\right)\right)) \subseteq N\beta cl\ddot{f}(\ddot{f}^{-1}((\overline{\mathcal{E}_2^*}))) \subseteq \overline{\mathcal{E}_2^*}$------(1)Consider $\overline{\mathrm{Nint}(\mathrm{Ncl}(\ddot{f}^{-1}(\mathcal{E}_2^*))}$

$= \overline{\mathrm{Ncl}(\mathrm{Ncl}(\ddot{f}^{-1}(\mathcal{E}_2^*))} = \mathrm{Nint}(\mathrm{Ncl}(\overline{\ddot{f}^{-1}(\mathcal{E}_2^*)}) \subseteq Ncl(Nint\left(\ddot{f}^{-1}\left(\overline{\mathcal{E}_2^*}\right)\right) \subseteq \ddot{f}^{-1}(\ddot{f}(Ncl(Nint(\ddot{f}^{-1}(\overline{\mathcal{E}_2^*}))$------(2)

By (1) and (2), $\overline{\mathrm{Nint}(\mathrm{Ncl}(\ddot{f}^{-1}(\mathcal{E}_2^*))} \subseteq \ddot{f}^{-1}(\ddot{f}(Ncl(Nint(\ddot{f}^{-1}(\overline{\mathcal{E}_2^*})) \subseteq \ddot{f}^{-1}(\overline{\mathcal{E}_2^*}) = \overline{\ddot{f}^{-1}(\mathcal{E}_2^*)}$ This implies $\ddot{f}^{-1}(\mathcal{E}_2^*) = \subseteq \mathrm{Nint}((\mathrm{Ncl}(\ddot{f}^{-1}(\mathcal{E}_2^*))$ which proves $\ddot{f}^{-1}(\mathcal{E}_2^*)$ is $\mathcal{N}POS$ in $\mathcal{X}_{\mathcal{N}}$. Thus $\ddot{f}$ is pre-β-irresolute. Hence (e)⇒ (a) is proved.

References

[1] Abdel-Basset, M., Gamal, A., Son, L. H., & Smarandache, F. (2020). A Bipolar Neutrosophic Multi Criteria Decision Making Framework for Professional Selection. Applied Sciences, 10(4), 1203.

[2] Abdel-Basset, M., Mohamed, R., Zaied, A. E. N. H., Gamal, A., & Smarandache, F. (2020). Solving the supply chain problem using the best-worst method based on a novel Plithogenic model. In Optimization Theory Based on Neutrosophic and Plithogenic Sets (pp. 1-19). Academic Press.

[3] Abdel-Basset, Mohamed, et al. "An integrated plithogenic MCDM approach for financial performance evaluation of manufacturing industries." Risk Management (2020): 1-27.

[4] Abdel-Basst, M., Mohamed, R., & Elhoseny, M. (2020). A novel framework to evaluate innovation value proposition for smart product-service systems. Environmental Technology & Innovation, 101036.

[5] Abdel-Basst, Mohamed, Rehab Mohamed, and Mohamed Elhoseny. "< covid19> A model for the effective COVID-19 identification in uncertainty environment using primary symptoms and CT scans." Health Informatics Journal (2020): 1460458220952918.

[6] K.Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems 20,87-95. (1986)

[7] I. Arokiarani, R. Dhavaseelan, S. Jafari, M. Parimala: On Some New Notions and Functions in Neutrosophic topological spaces , *Neutrosophic Sets and Systems*, Vol. 16 **(2017),** pp. 16-19. doi.org/10.5281/zenodo.831915.

[8] A. Atkinswestley S. Chandrasekar, Neutrosophic Weakly G^*-closed sets, Advances in Mathematics: Scientific Journal 9 (2020), no.5, 2853–2861.

[9] A. Atkinswestley S. Chandrasekar, Neutrosophic g#S closed sets in Neutrosophic topological spaces, Malaya Journal of Matematik, Vol. 8, No. 4, 1786-1791, 2020

[10] A. Atkinswestley S. Chandrasekar, Neutrosophic g*-Closed Sets and its maps, Neutrosophic Sets and Systems, Vol. 36, 2020,96-107

[11] V. Banupriya S.Chandrasekar: Neutrosophic αgs Continuity and Neutrosophic αgs Irresolute Maps, *Neutrosophic Sets and Systems,* vol. 28, **2019,** pp. 162-170. DOI: 10.5281/zenodo.3382531

[12] V. Banu Priya,,S. Chandrasekar and M. Suresh, Neutrosophic α-generalized semi homeomorphisms, Malaya Journal of Matematik, Vol. 8, No. 4, 1824-1829, 2020

[13] Charles Le, Preamble to Neutrosophy and Neutrosophic Logic, Multiple-Valued Logic / An International Journal, Taylor & Francis, UK & USA, Vol. 8, No. 3, 285-295, June 2002.

[14] R.Dhavaseelan, S.Jafari, and Hanifpage.md.: Neutrosophic generalized α-contra-continuity, *creat. math. inform.* 27, no.2, 133 - 139,(2018)

[15] FlorentinSmarandache.:, Neutrosophic and NeutrosophicLogic, *First International Conference* On Neutrosophic, Neutrosophic Logic, Set, Probability, and Statistics University of New Mexico, Gallup, NM 87301, USA, *smarand@unm.edu*,(2002)

[16] FloretinSmarandache.:, Neutrosophic Set: - A Generalization of Intuitionistic Fuzzy set, *Journal of Defense Resources Management*1,107-114,(2010).

[17] P.Ishwarya, and K.Bageerathi., On Neutrosophicsemiopen sets in NTSs, *International Jour. Of Math. Trends and Tech.* , 214-223,(2016).

[18] D.Jayanthi,αGeneralized closed Sets in NTSs, *International Journal of Mathematics Trends and Technology (IJMTT)*- Special Issue ICRMIT March (2018).

[19] A.Mary Margaret, and M.Trinita Pricilla.,Neutrosophic Vague Generalized Pre-closed Sets in Neutrosophic Vague Topological Spaces,*International Journal of Mathematics And its Applications,*Volume 5, Issue 4-E, 747-759.(2017).

[20] C.Maheswari, M.Sathyabama, S.Chandrasekar.,:,Neutrosophic generalized b-closed Sets In Neutrosophic topological spaces,*Journal of physics Conf. Series* 1139 (2018) 012065.doi:10.1088/1742-6596/1139/1/012065

[21] C.Maheswari, S. Chandrasekar , Neutrosophic gb-closed Sets and Neutrosophic gb-Continuity, Neutrosophic Sets and Systems, Vol. 29, 2019,89-99

[22] C.Maheswari, S. Chandrasekar , Neutrosophic bg-closed Sets and its Continuity, NeutrosophicSets and Systems, Vol. 36, 2020,108-120.

[23] T Rajesh kannan , S.Chandrasekar,Neutosophic $\omega\alpha$-closed sets in Neutrosophic topological spaces, Journal Of Computer And Mathematical Sciences,vol.9(10),1400-1408 Octobe2018.

[24] T Rajesh kannan ,S.Chandrasekar, Neutosophic α-continuous multifunction in Neutrosophic topological spaces, The International Journal of Analytical and Experimental Modal Analysis, Volume XI,IssueIX,September 2019,1360-9367

[25] T.RajeshKannan, and S.Chandrasekar, Neutrosophic α-Irresolute Multifunction In Neutrosophic topological spaces, " Neutrosophic Sets and Systems 32, 1 (2020),390-400. https://digitalrepository.unm.edu/ nss_journal/vol32/iss1/25

[26] T. Rajesh Kannan and S. Chandrasekar ,Neutrosophic PRE $-\alpha$, SEMI $-\alpha$ and PRE $-\beta$ irresolute open and closed mappings in Neutrosophic topological spaces, Malaya Journal of Matematik, Vol. 8, No. 4, 1795-1806, 2020

[27] A.A.Salama and S.A. Alblowi., Generalized Neutrosophic Set and Generalized NTSs, *Journal computer Sci. Engineering*, Vol.(ii),No.(7)(2012).

[28] A.A.Salama, and S.A.Alblowi.,Neutrosophic set and NTS, *ISOR J.mathematics,*Vol.(iii),Issue(4),.pp-31-35,(2012).

[29] V.K.Shanthi.,S.Chandrasekar, K.SafinaBegam, Neutrosophic Generalized Semi-closed Sets In NTSs, *International Journal of Research in Advent Technology,* Vol.(ii),6, No.7, , 1739-1743, July (2018)

[30] V.VenkateswaraRao.,Y.SrinivasaRao.,Neutrosophic Pre-open Sets and Pre-closed Sets in Neutrosophic Topology, *International Journal of ChemTech Research*, Vol.(10),No.10, pp 449-458, (2017)

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