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Neutrosophic Pre-\(\alpha\), Semi- \(\alpha\) & Pre- \(\beta\) Irresolute Functions

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Abstract: Smarandache introduced and developed interesting concepts Neutrosophic set from the Intuitionistic fuzzy sets. A.A. Salama introduced NTSs and continuity. Aim of this paper is we introduce and study the concepts Neutrosophic Pre-\(\alpha\), Semi- \(\alpha\) & Pre- \(\beta\) Irresolute Functions and its Properties are discussed details.

Keywords: Neutrosophic Irresolute Functions, Neutrosophic Pre-\(\alpha\), Neutrosophic Semi- \(\alpha\), Neutrosophic Pre- \(\beta\) Irresolute Functions.

1. Introduction

Neutrosophic concepts have wide range of applications in the area of decision making Artificial Intelligence, Information Systems, Computer Science, Medicine, Applied Mathematics, Mechanics, Electrical & Electronic and, Management Science, etc. In 1980s the international movement called paradoxism based on contradictions in science and literature, was founded by Smarandache[15,16], who then extended it to neutrosophy, based on contradictions and their neutrals. The mapping is the one of the important concept in topology. Neutrosophic sets have three kind like T Truth, F -Falsehood, I -Indeterminacy. Neutrosophic topological spaces (N-T-S) introduced by Salama [27,28] etal., by using Smarandache neutrosophy set. In this Paper new type of functions called as Neutrosophic Pre-\(\alpha\) irresolute functions, Neutrosophic Pre-\(\alpha\), Semi- \(\alpha\) and Pre- \(\beta\) Irresolute Functions. Also the interrelationships of these functions with the other existing functions are established. Several characterizations and some interesting properties of these classes of functions are given.

2. Preliminaries

In this section, we provide basic definition and operation of Neutrosophic sets and its Results

**Definition 2.1 [15,16]** Let \(X_N\) be a non-empty fixed set. A Neutrosophic set \(E^*_1\) is a object having the form

\[ E^*_1 = \{ x, \mu_{E^*_1}(x), \sigma_{E^*_1}(x), \gamma_{E^*_1}(x) : x \in X_N \}, \]

\(\mu_{E^*_1}(x)\) - membership function
\(\sigma_{E^*_1}(x)\) - indeterminacy and then
\(\gamma_{E^*_1}(x)\) - non-membership function
Definition 2.2 [15,16]. Neutrosophic set $E'_1 = \{< x, \mu_{E'_1}(x), \sigma_{E'_1}(x), \gamma_{E'_1}(x) >: x \in X_N \}$, on $X_N$ and

$\forall x \in X_N$

$E'_2 = \{< x, \mu_{E'_2}(x), \sigma_{E'_2}(x), \gamma_{E'_2}(x) >= x \in X_N \}$

1. $E'_1 \cap E'_2 = \{< x, \mu_{E'_1}(x) \cap \mu_{E'_2}(x), \sigma_{E'_1}(x) \cap \sigma_{E'_2}(x), \gamma_{E'_1}(x) \cup \gamma_{E'_2}(x) >= x \in X_N \}$

2. $E'_1 \cup E'_2 = \{< x, \mu_{E'_1}(x) \cup \mu_{E'_2}(x), \sigma_{E'_1}(x) \cup \sigma_{E'_2}(x), \gamma_{E'_1}(x) \cap \gamma_{E'_2}(x) >= x \in X_N \}$

3. $E'_1 \subseteq E'_2 \Leftrightarrow \mu_{E'_1}(x) \leq \mu_{E'_2}(x), \sigma_{E'_1}(x) \leq \sigma_{E'_2}(x) \& \gamma_{E'_1}(x) \geq \gamma_{E'_2}(x)$

4. The complement of $E'_1$ is $E'_1^c = \{< x, \gamma_{E'_1}(x), 1 - \sigma_{E'_1}(x), \mu_{E'_1}(x) >: x \in X_N \}$

Definition 2.3 [28]. Let $X_N$ be non-empty set and $\tau_N$ be the collection of Neutrosophic subsets of $X_N$ satisfying the following properties:

1. $\emptyset, X_N \in \tau_N$

2. $T_1 \cap T_2 \in \tau_N$ for any $T_1, T_2 \in \tau_N$

3. $\cup T_i \in \tau_N$ for every $\{T_i : i \in I\} \subseteq \tau_N$

Then the space $(X_N, \tau_N)$ is called a Neutrosophic topological spaces (N-T-S).

The element of $\tau_N$ are called Ne.OS (Neutrosophic open set) and its complement is Ne.CS (Neutrosophic closed set).

Example 2.4. Let $X_N = \{x\}$ and $\forall x \in X_N$

$A_1 = (x, 6_{10}^6, 6_{10}^6, 5_{10}^5), A_2 = (x, 5_{10}^5, 7_{10}^7, 9_{10}^9), A_3 = (x, 6_{10}^6, 7_{10}^7, 5_{10}^5), A_4 = (x, 5_{10}^5, 6_{10}^6, 9_{10}^9)$

Then the collection $\tau_N = \{0_N, A_1, A_2, A_3, A_4, 1_N\}$ is called a N-T-S on $X_N$.

Definition 2.5. Let $(X_N, \tau_N)$ be a N-T-S and $E'_1 = \{< x, \mu_{E'_1}(x), \sigma_{E'_1}(x), \gamma_{E'_1}(x) >= x \in X_N \}$ be a Neutrosophic set in $X_N$. Then $E'_1$ is named as

1. Neutrosophic b closed set [20] (Ne.bCS) if $\text{Ne.cl}(\text{Ne.int}(E'_1)) \cap \text{Ne.int}(\text{Ne.cl}(E'_1)) \subseteq E'_1$.

2. Neutrosophic a-closed set [7] (Ne.aCS) if $\text{Ne.cl}(\text{Ne.int}(\text{Ne.cl}(E'_1))) \subseteq E'_1$.

3. Neutrosophic pre-closed set [30] (Ne.Pre-CS) if $\text{Ne.cl}(\text{Ne.int}(E'_1)) \subseteq E'_1$.

4. Neutrosophic regular closed set [7] (Ne.RCS) if $\text{Ne.cl}(\text{Ne.int}(E'_1)) = E'_1$.

5. Neutrosophic semi closed set [17] (Ne.SCS) if $\text{Ne.int}(\text{Ne.cl}(E'_1)) \subseteq E'_1$.

Definition 2.6. [9] $(X_N, \tau_N)$ be a N-T-S and $E'_1 = \{< x, \mu_{E'_1}(x), \sigma_{E'_1}(x), \gamma_{E'_1}(x) >= x \in X_N \}$ be a Neutrosophic set in $X_N$. Then $E'_1$ is named as

1. Neutrosophic closure of $E'_1$ is $\text{Ne.Cl}(E'_1) = \{H : H \in \text{Ne.CS in } X_N \text{ and } E'_1 \subseteq H\}$

2. Neutrosophic interior of $E'_1$ is $\text{Ne.Int}(E'_1) = \{M : M \in \text{Ne.OS in } X_N \text{ and } M \subseteq E'_1\}$

Definition 2.7. Let $(X_N, \mathcal{T}_N)$ be an NTS and be an NS in $X_N$.

The Neutrosophic closure $\beta$-closure $\&$-$\beta$-interior of $A$ are defined by

(i) $\mathcal{N}^\beta Cl(E'_1) = \{E'_1, E'_3 \in \beta CS \in X_N \text{ and } E'_3 \supseteq E'_1\}$. 

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(ii) $\mathcal{N}β\text{int}(E'_1) = \cup[E'_4; E'_4]$ is a $\mathcal{N}β\text{OS}$ in $X_Y$ and $E'_4 \subseteq E'_1$.

**Lemma 2.8.**

Let $E'_1$ be an NS in NTS $(X_Y, T_N)$. Then

(i) $\text{int}(E'_1) \subseteq \text{NP int}(E'_1) \subseteq E'_1 \subseteq \text{NPcl}(E'_1) \subseteq \text{Ncl}(E'_1)$

(ii) $\text{int}(E'_1) \subseteq \text{Nint}(E'_1) \subseteq E'_1 \subseteq \text{Nc}(E'_1) \subseteq \text{Ncl}(E'_1)$

(iii) $\text{int}(E'_1) \in \text{NSint}(E'_1) \subseteq E'_1 \subseteq \text{NScl}(E'_1) \subseteq \text{Ncl}(E'_1)$

(iv) $\text{int}(E'_1) \in \text{Nβ int}(E'_1) \subseteq E'_1 \subseteq \text{Nβc}(E'_1) \subseteq \text{Ncl}(E'_1)$

**Proof:** It is easy to prove.

3. **Neutrosophic Pre-α, Semi-α & Pre-β Irresolute Functions**

In this section Neutrosophic pre-α-irresolute, semi-α-irresolute, Neutrosophic pre-β-irresolute functions are defined. Also, the relationships of these functions with the other existing functions are studied.

**Definition 3.1.**

A function $\hat{\beta}: (X_Y, T_N) \rightarrow (Y_N, G_N)$ from an NTS $(X_Y, T_N)$ to another NTS $(Y_N, G_N)$ is named as Neutrosophic $\beta$-irresolute if $\hat{\beta}^{-1}(E'_2)$ is a $\mathcal{N}β\text{OS}$ in $(X_Y, T_N)$ for each $\mathcal{N}β\text{OS} E'_2$ in $(Y_N, G_N)$.

**Definition 3.2** A function $\hat{\beta}: (X_Y, T_N) \rightarrow (Y_N, G_N)$ from an NTS $(X_Y, T_N)$ to another NTS $(Y_N, G_N)$ is named as Neutrosophic pre-α-irresolute if $\hat{\beta}^{-1}(E'_2)$ is an NPOS in $(X_Y, T_N)$ for each NPOS $E'_2$ in $(Y_N, G_N)$.

**Definition 3.3** A function $\hat{\beta}: (X_Y, T_N) \rightarrow (Y_N, G_N)$ from an NTS $(X_Y, T_N)$ to another NTS $(Y_N, G_N)$ is named as Neutrosophic $\alpha$-irresolute if $\hat{\beta}^{-1}(E'_2)$ is a NaOS in $(X_Y, T_N)$ for each NaOS $E'_2$ in $(Y_N, G_N)$.

**Definition 3.4** A function $\hat{\beta}: (X_Y, T_N) \rightarrow (Y_N, G_N)$ from an NTS $(X_Y, T_N)$ to another NTS $(Y_N, G_N)$ is named as Neutrosophic semi-α-irresolute if $\hat{\beta}^{-1}(E'_2)$ is an NSOS in $(X_Y, T_N)$ for each NaOS $E'_2$ in $(Y_N, G_N)$.

**Definition 3.5** A function $\hat{\beta}: (X_Y, T_N) \rightarrow (Y_N, G_N)$ from an NTS $(X_Y, T_N)$ to another NTS $(Y_N, G_N)$ is named as Neutrosophic pre-$\beta$-irresolute if $\hat{\beta}^{-1}(E'_2)$ is a NPOS in $(X_Y, T_N)$ for each $\mathcal{N}β\text{OS} E'_2$ in $(Y_N, G_N)$.

**Proposition 3.6** Every Na-irresolute function is $\text{Npre-α (NSemi-α, resp.)-irresolute function.}$

**Proof:** Let $\hat{\beta}: (X_Y, T_N) \rightarrow (Y_N, G_N)$ be Na-irresolute function from NTS $(X_Y, T_N)$ to another NTS $(Y_N, G_N)$. Let $E'_2$ be NaOS in $Y_N$. Since $\hat{\beta}$ is Na-irresolute function, $\hat{\beta}^{-1}(E'_2)$ is NaOS in $X_Y$. Every NaOS is NPOS (NSOS, resp.). So $\hat{\beta}^{-1}(E'_2)$ is NPOS (NSOS, resp.) in $X_Y$. Hence $\hat{\beta}$ is Npre-α (NSemi-α, resp.)-irresolute function.

**Proposition 3.7** Every Npre-β-irresolute function is Npre-α (Npre, resp.) irresolute function.

**Proof:** Let $\hat{\beta}: (X_Y, T_N) \rightarrow (Y_N, G_N)$ be Npre-β irresolute function from NTS $(X_Y, T_N)$ to another NTS $(Y_N, G_N)$. Let $E'_2$ be NaOS (NPOS resp.) in $Y_N$. Every NaOS (NPOS, resp.) is $\mathcal{N}β\text{OS}$. Since $\hat{\beta}$ is Npre-β-irresolute function, $\hat{\beta}^{-1}(E'_2)$ is NPOS in $X_Y$. Hence $\hat{\beta}$ is Npre-α (Npre, resp.) irresolute function.

**Proposition 3.8** Every Npre-β-irresolute function is $\mathcal{N}β$-irresolute function.

**Proof:** Let $\hat{\beta}: (X_Y, T_N) \rightarrow (Y_N, G_N)$ be Npre-β irresolute function from NTS $(X_Y, T_N)$ to another NTS $(Y_N, G_N)$. Let $E'_2$ be $β\text{OS}$. Since $\hat{\beta}$ is Npre-β-irresolute function, $\hat{\beta}^{-1}(E'_2)$ is NPOS in $X_Y$. As every NPOS is $\mathcal{N}β\text{OS}$. $\hat{\beta}^{-1}(E'_2)$ is $\mathcal{N}β\text{OS}$ in $X_Y$. Hence $\hat{\beta}$ is $\mathcal{N}β$-irresolute function.

**Proposition 3.9** Every Nirresolute function is NS-α-irresolute function.
**Proof:** Let $\tilde{f} : (X_N, \mathcal{T}_N) \rightarrow (Y_N, \mathcal{G}_N)$ be Nirresolute function from NTS($X_N, \mathcal{T}_N$) to another NTS ($Y_N, \mathcal{G}_N$). Let $E_2'$ be NaOS in $Y_N$. Every NaOS is NSOS. Since $\tilde{f}$ is Nirresolute function, $\tilde{f}^{-1}(E_2')$ is NSOS in $X_N$. Hence $\tilde{f}$ is NS-$\alpha$-irresolute function.

**Example 3.10** Let $X_N=[a,b]$ $Y_N=[c,d]$ and $\mathcal{T}_N=\{0, E_1, 1\}$, $\mathcal{G}_N=\{0, E_2, 1\}$ are NTS on $X_N$ and $Y_N$ respectively where

\[
E_1^* = \{x\left(\frac{6}{10}, \frac{5}{10}, \frac{5}{10}\right), \left(\frac{5}{10}, \frac{5}{10}, \frac{5}{10}\right)\},
\]

\[
E_2^* = \{y\left(\frac{2}{10}, \frac{5}{10}, \frac{6}{10}\right), \left(\frac{4}{10}, \frac{5}{10}, \frac{5}{10}\right)\}.
\]

Define an Neutrosophic function $\tilde{f} : (X_N, \mathcal{T}_N) \rightarrow (Y_N, \mathcal{G}_N)$. By $\tilde{f} (a)=d$, $\tilde{f} (b)=c$ $E_2'$ is a NOS in $(Y_N, \mathcal{G}_N)$. So $E_2'$ is NaOS, NPOS, and $N^\beta OS$ in $Y_N$.

Since $\tilde{f}^{-1}(E_2') = \{x\left(\frac{6}{10}, \frac{5}{10}, \frac{5}{10}\right), \left(\frac{4}{10}, \frac{5}{10}, \frac{5}{10}\right)\}$ is an NPOS in $X_N$.

\[
\tilde{f}^{-1}(E_2') \subseteq Nint(Ncl(\tilde{f}^{-1}(E_2'))) = 1_N
\]

Also $\tilde{f}^{-1}(E_2') \subseteq Ncl(Nint(Ncl(\tilde{f}^{-1}(E_2')))) = 1_N$

So $\tilde{f}^{-1}(E_2')$ is a $N^\beta OS$ in $X_N$. Thus $\tilde{f}$ is Npre-$\beta$-Irresolute, Npre irresolute function, Npre-$\alpha$-irresolute function and $N^\beta$-irresolute function. Also $\tilde{f}$ is a N precontinuous and $N^\beta$-continuous. As $Nint(Ncl(Nint(\tilde{f}^{-1}(E_2')))) = 0_N$, $\tilde{f}^{-1}(E_2') \not\subseteq Ncl(Nint(Ncl(\tilde{f}^{-1}(E_2'))))$

$\tilde{f}^{-1}(E_2')$ is not NaOS in $X_N$. Also $\tilde{f}^{-1}(E_2') \not\subseteq Ncl(Nint(\tilde{f}^{-1}(E_2'))=0_N$. implies $\tilde{f}^{-1}(E_2')$ is not NSOS in $X_N$. Thus $\tilde{f}$ is not Na- irresolute function, not NSemi-$\alpha$-irresolute function, not Na-continuous, not NSemi continuous, and not Nirresolute function.

**Example 3.11**

Let $X_N=[a,b]$ $Y_N=[c,d]$ and $\mathcal{T}_N=\{0, E_1, 1\}$, $\mathcal{G}_N=\{0, E_2, 1\}$ are NTS on $X_N$ and $Y_N$ respectively is a NS in $Y_N$.

\[
E_1^* = \{x\left(\frac{2}{10}, \frac{5}{10}, \frac{3}{10}\right), \left(\frac{4}{10}, \frac{5}{10}, \frac{4}{10}\right)\},
\]

\[
E_2^* = \{y\left(\frac{4}{10}, \frac{5}{10}, \frac{4}{10}\right), \left(\frac{2}{10}, \frac{5}{10}, \frac{3}{10}\right)\},
\]

\[
E_3^* = \{y\left(\frac{4}{10}, \frac{5}{10}, \frac{4}{10}\right), \left(\frac{3}{10}, \frac{5}{10}, \frac{2}{10}\right)\},
\]

is a NS in $Y_N$.

Define a Neutrosophic function $\tilde{f} : (X_N, \mathcal{T}_N) \rightarrow (Y_N, \mathcal{G}_N)$ by $\tilde{f} (a)=d$, $\tilde{f} (b)=c$ $E_2'$ is a NOS in $(Y_N, \mathcal{G}_N)$. Also $E_2'$ is NaOS, NPOS and NSOS in $Y_N$.

\[
\tilde{f}^{-1}(E_2') = \{x\left(\frac{2}{10}, \frac{5}{10}, \frac{3}{10}\right), \left(\frac{4}{10}, \frac{5}{10}, \frac{4}{10}\right)\}
\]

and $\tilde{f}^{-1}(E_2') \subseteq Nint(Ncl(\tilde{f}^{-1}(E_2'))) = E_1^*$. So $\tilde{f}^{-1}(E_2') \subseteq Nint(Ncl(\tilde{f}^{-1}(E_2')))$ This implies $\tilde{f}^{-1}(E_2')$ is a NaOS in $X_N$. Also $\tilde{f}^{-1}(E_2')$ is NPOS and NSOS in $X_N$. Hence $\tilde{f}$ is a Na-irresolute.
function, NS-α-irresolute function, Npre-α-irresolute function, Nα-continuous, NSemicontinuous, and Nprecontinuous.E'_3 ⊆ Ncl(Nint(E'_3)) = E'_2. So E'_3 is a NOS in Y_N.

Also \( \hat{g}^{-1}(E'_2) = (x,(\frac{2}{10},\frac{5}{10},\frac{5}{10})) \). Then \( \hat{g}^{-1}(E'_2) \notin Ncl(Nint(\hat{g}^{-1}(E'_3))) = E'_1 \)

Hence \( \hat{g}^{-1}(E'_3) \) is not NaOS in X_N. Thus \( \hat{g} \) is not Nstrongly α-continuous.

**Example 3.12** Let \( X_N = \{a,b\} \) \( \gamma_N = \{c,d\} \) and \( T_N = \{0, E'_1, 1\}, \Gamma_N = \{0, E'_2, 1\} \) are NTS on \( X_N \) and \( \gamma_N \) respectively, where

\[
E'_1 = ([x,(\frac{6}{10},\frac{5}{10},\frac{4}{10}), (\frac{5}{10},\frac{5}{10},\frac{5}{10}))],
\]

\[
E'_2 = ([y,(\frac{2}{10},\frac{5}{10},\frac{6}{10}), (\frac{4}{10},\frac{5}{10},\frac{5}{10})])
\]

Define a Neutrosophic function \( \hat{g}: (X_N,\gamma_N) \rightarrow (\gamma_N,\gamma_N) \). By \( \hat{g} \) (a)=d, \( \hat{g} \) (b)=c. \( E'_2 \) is a NOS in \( \gamma_N \).

Hence \( \hat{g}^{-1}(E'_2) \) is NaOS, NPOS, NSOS and \( \mathcal{N}\beta OS \) in \( (\gamma_N,\gamma_N) \).

\( \hat{g}^{-1}(E'_2) \subseteq Ncl(Nint(\hat{g}^{-1}(E'_2))) = E'_1 \) implies \( \hat{g}^{-1}(E'_2) \) is a NSOS in \( X_N \). Also \( \hat{g}^{-1}(E'_2) \) is a \( \mathcal{N}\beta OS \) in \( X_N \), since \( \hat{g}^{-1}(E'_2) \subseteq Ncl(Nint(\hat{g}^{-1}(E'_2))) = E'_1 \). Hence \( \hat{g} \) is Nirresolute function, NS-α-irresolute function, NSemi continuous and \( \mathcal{N}\beta \) -continuous. Nint(Ncl(Nint(\hat{g}^{-1}(E'_2))) = E'_1. So \( \hat{g}^{-1}(E'_2) \) \( \notin \) Nint(Ncl(Nint(\hat{g}^{-1}(E'_2))). Hence \( \hat{g}^{-1}(E'_2) \) is not NaOS in \( X_N \). Also \( \hat{g}^{-1}(E'_2) \) \( \notin \) Nint(Ncl(\hat{g}^{-1}(E'_2))). Hence \( \hat{g}^{-1}(E'_2) \) is not NPOS in \( X_N \). Thus \( \hat{g} \) is not Na- irresolute function, not Npre-α-irresolute function, not Npre irresolute function, not Npre-β-irresolute function, not Nα-continuous and not Npre continuous.

**Example 3.13**

Let \( X_N = \{a,b,c\} = \gamma_N \) and \( T_N = \{0_n, 1_n, E'_1, E'_1 \cup E'_2, E'_1 \cap E'_2\}, \Gamma_N = \{0_n, 1_n, E'_3\} \) are NTS on \( X_N \) and \( \gamma_N \) where

\[
E'_1 = ([x,(\frac{5}{10},\frac{5}{10},\frac{7}{10}), (\frac{6}{10},\frac{5}{10},\frac{4}{10})]),
\]

\[
E'_2 = ([x,(\frac{2}{10},\frac{5}{10},\frac{7}{10}), (\frac{4}{10},\frac{5}{10},\frac{6}{10})]),
\]

\[
E'_3 = ([y,(\frac{5}{10},\frac{5}{10},\frac{6}{10}), (\frac{6}{10},\frac{5}{10},\frac{4}{10})])
\]

\[
E'_4 = ([y,(\frac{5}{10},\frac{5}{10},\frac{4}{10}), (\frac{6}{10},\frac{5}{10},\frac{4}{10})])
\]

is a NS in \( \gamma_N \). Define an identity Neutrosophic function \( \hat{g}: (X_N,\gamma_N) \rightarrow (\gamma_N,\gamma_N) \). \( E'_3 \) is a NOS in \( \gamma_N \) and \( \hat{g}^{-1}(E'_3) = ([x,(\frac{5}{10},\frac{5}{10},\frac{5}{10}), (\frac{6}{10},\frac{5}{10},\frac{4}{10})]) \).

Nint(Ncl(Nint(\hat{g}^{-1}(E'_3))) = E'_1 \cup E'_2. Thus \( \hat{g}^{-1}(E'_3) \) \( \subseteq \) Nint(Ncl(Nint(\hat{g}^{-1}(E'_3))) Hence \( \hat{g}^{-1}(E'_3) \) is a NaOS in \( (X_N,T_N) \). Also \( \hat{g}^{-1}(E'_3) \) is NPOS, NSOS and \( \mathcal{N}\beta OS \) in \( X_N \). Therefore \( \hat{g} \) is \( \mathcal{N}\alpha \)-continuous, Npre continuous, NSemicontinuous and \( \mathcal{N}\beta \) -continuous. \( E'_4 \) is a NS in \( \gamma_N \) and \( E'_4 \) \( \subseteq \) Nint(Ncl(Nint(E'_4))) = 1_n. Hence \( E'_4 \) is a NaOS in \( \gamma_N \). Also \( E'_4 \) is NPOS, NSOS and \( \mathcal{N}\beta OS \) in \( \gamma_N \).
\[
\hat{\mathcal{f}}^{-1}(E_1^* ) = \left( x \left( \frac{5}{10} \frac{5}{10} \frac{4}{10} \right), \left( \frac{4}{10} \frac{5}{10} \frac{4}{10} \right), \left( \frac{6}{10} \frac{5}{10} \frac{4}{10} \right) \right)
\]

And \( \hat{\mathcal{f}}^{-1}(E_1^* ) \subseteq \text{Ncl}(\text{Nt}(\hat{\mathcal{f}}^{-1}(E_1^* ))) = \overline{E_1^*} \). Hence \( \hat{\mathcal{f}}^{-1}(E_1^* ) \) is NSOS and also \( \mathcal{N} \beta OS \) in \( X_N \). So \( \hat{\mathcal{f}} \) is Nirresolve function, NS-\( \alpha \)- irresolute function and \( \mathcal{N} \beta \) - irresolute function. Since 
\[
\hat{\mathcal{f}}^{-1}(E_1^* ) \notin \text{Nnt}(\text{Ncl}(\hat{\mathcal{f}}^{-1}(E_1^* ))) = E_1^* \cup E_2^* , \quad \hat{\mathcal{f}}^{-1}(E_1^* ) \text{ is not NaOS in } X_N \text{ and } \hat{\mathcal{f}}^{-1}(E_1^* ) \notin \text{Nnt}(\text{Ncl}(\hat{\mathcal{f}}^{-1}(E_1^* )) = E_1^* \cup E_2^*, \quad \hat{\mathcal{f}}^{-1}(E_1^* ) \text{ is not NPOS in } X_N \text{. Thus } \hat{\mathcal{f}} \text{ is not } \text{Na- irresolute function and not Npre- } \alpha \text{- irresolute function and not Npre- } \beta \text{- irresolute function.}
\]

**Example 3.14**

Let \( X_N = [a,b] \) \( Y_N = [c,d] \) and \( I_N = [0, E_1^*, 1] \), \( \Gamma_N = [0, E_2^*, 1] \), are NTS on \( X_N \) and \( Y_N \) respectively where

\[
E_1^* = \left( x \left( \frac{3}{10} \frac{5}{10} \frac{6}{10} \right), \left( \frac{4}{10} \frac{5}{10} \frac{5}{10} \right) \right)
\]

\[
E_2^* = \left( y \left( \frac{4}{10} \frac{5}{10} \frac{5}{10} \right), \left( \frac{5}{10} \frac{5}{10} \frac{3}{10} \right) \right)
\]

And \( E_3^* = \left( y \left( \frac{4}{10} \frac{5}{10} \frac{5}{10} \right), \left( \frac{6}{10} \frac{5}{10} \frac{2}{10} \right) \right) \) is a NS in \( Y_N \). Define a Neutrosophic function \( \hat{\mathcal{f}} : (X_N, \mathcal{F}_N) \rightarrow (Y_N, \mathcal{G}_N) \). By \( \hat{\mathcal{f}} \) \((a)=d, \hat{\mathcal{f}} \) \((b)=c E_1^* \) is a NOS in \( (Y_N, \mathcal{G}_N) \). And \( \hat{\mathcal{f}}^{-1}(E_2^* ) = \left( y \left( \frac{5}{10} \frac{5}{10} \frac{3}{10} \right), \left( \frac{4}{10} \frac{5}{10} \frac{5}{10} \right) \right) \) and 

\[
\text{Ncl}(\text{Nnt}(\hat{\mathcal{f}}^{-1}(E_2^* ))) = \overline{E_1^*} \text{. Thus } \hat{\mathcal{f}}^{-1}(E_2^* ) \subseteq \text{Ncl}(\text{Nnt}(\hat{\mathcal{f}}^{-1}(E_2^* ))) \text{. Hence } \hat{\mathcal{f}}^{-1}(E_2^* ) \text{ is an NSOS in } X_N \text{, which implies } \hat{\mathcal{f}} \text{ is NSemi continuous and also } \hat{\mathcal{f}} \text{ is } \mathcal{N} \beta \text{-continuous. } E_3^* \text{ is a NS in } Y_N \text{. Also } E_3^* \subseteq \text{Nnt}(\text{Ncl}(\text{Nnt}(E_3^* ))= 1_N \text{which implies } E_3^* \text{ is a NaOS in } Y_N \text{. Hence } E_3^* \text{ is NPOS, NSOS and } \mathcal{N} \beta OS \text{ in } Y_N \text{.} \hat{\mathcal{f}}^{-1}(E_3^* ) = \left( y \left( \frac{5}{10} \frac{5}{10} \frac{3}{10} \right), \left( \frac{4}{10} \frac{5}{10} \frac{5}{10} \right) \right) \text{ So } \hat{\mathcal{f}}^{-1}(E_3^* ) \text{ is a NPOS and } \mathcal{N} \beta OS \text{ in } X_N \text{. Thus } \hat{\mathcal{f}} \text{ is Npre- } \alpha \text{- irresolute function, Npre- } \beta \text{- irresolute function and } \mathcal{N} \beta \text{- irresolute function. Since } \hat{\mathcal{f}}^{-1}(E_3^* ) \notin \text{Nnt}(\text{Ncl}(\hat{\mathcal{f}}^{-1}(E_3^* ))) = E_1^* , \quad \hat{\mathcal{f}}^{-1}(E_3^* ) \text{ is not NaOS in } X_N \text{. Also } \hat{\mathcal{f}}^{-1}(E_3^* ) \notin \text{Nnt}(\text{Ncl}(\hat{\mathcal{f}}^{-1}(E_3^* )) \neq \overline{E_1^*} \text{. So } \hat{\mathcal{f}}^{-1}(E_3^* ) \text{ is not NSOS in } X_N \text{. Hence } \hat{\mathcal{f}} \text{ is not } \text{Na- irresolute function, not } \text{Nirresolve function, and not NS- } \alpha \text{- irresolute function.}
\]

**Example 3.15**

Let \( X_N = [a,b] = Y_N \) and

\( \mathcal{J}_N = [0_n, 1_N, E_1^*, E_1^* \cup E_2^*, E_1^* \cap E_2^* ] \) 
\( \Gamma_N = [0_n, 1_N, E_3^* ] \) are NTS on \( X_N \) and \( Y_N \) where

\[
E_1^* = \left( x \left( \frac{2}{10} \frac{5}{10} \frac{5}{10} \right), \left( \frac{2}{10} \frac{5}{10} \frac{4}{10} \right) \right)
\]

\[
E_2^* = \left( x \left( \frac{4}{10} \frac{5}{10} \frac{6}{10} \right), \left( \frac{4}{10} \frac{5}{10} \frac{6}{10} \right) \right)
\]

\[
E_3^* = \left( y \left( \frac{2}{10} \frac{5}{10} \frac{6}{10} \right), \left( \frac{4}{10} \frac{5}{10} \frac{6}{10} \right) \right)
\]

\[
E_4^* = \left( y \left( \frac{3}{10} \frac{5}{10} \frac{3}{10} \right), \left( \frac{4}{10} \frac{5}{10} \frac{5}{10} \right) \right)
\]
is a NS in \( Y_N \). Define an identity Neutrosophic function \( \hat{f}: (X_N,T_N) \to (Y_N,G_N) \), \( \epsilon_3^+ \) is a NOS in \( Y_N \). \( \epsilon_3^+ \) is a NOS, NaOS, NPOS in \( (Y_N,G_N) \). \( \hat{f}^{-1}(\epsilon_3^+) = \{ \{ 2, 5, 3 \} , \{ 5, 10, 10 \} \} \)

So \( \hat{f}^{-1}(\epsilon_3^+) \subseteq \text{Nint}(\text{Ncl}(\hat{f}^{-1}(\epsilon_3^+))) = \epsilon_1 \cup \epsilon_2^+ \). Thus \( \hat{f}^{-1}(\epsilon_3^+) \) is a NaOS in \( X_N \). Hence \( \hat{f}^{-1}(\epsilon_3^+) \) is NPOS and NSOS in \( X_N \). Thus \( \hat{f} \) is \( \alpha \)-irresolute, \( \alpha \)-semi-irresolute and \( \alpha \)-pre-irresolute function, \( \alpha \)-continuous, \( \alpha \)-precontinuous and \( \alpha \)-semi-continuous. \( \epsilon_4 \) is a NS in \( Y_N \) and \( \text{Ncl}(\text{Nint}(\hat{f}^{-1}(\epsilon_4^+))) = \epsilon_3^+ \). Hence \( \epsilon_4^+ \subseteq \text{Ncl}(\text{Nint}(\epsilon_4^+)) \). Thus \( \epsilon_4^+ \) is a NSOS in \( Y_N \).

\[
\hat{f}^{-1}(\epsilon_3^+) = (x, \{ 3, 5, 3 \} , \{ 5, 10, 10 \} )
\]

and \( \text{Ncl}(\text{Nint}(\hat{f}^{-1}(\epsilon_4^+))) = \epsilon_3^+ \cup \epsilon_2^+ \).

So \( \hat{f}^{-1}(\epsilon_3^+) \nsubseteq \text{Ncl}(\text{Nint}(\hat{f}^{-1}(\epsilon_4^+))) \). Thus \( \hat{f}^{-1}(\epsilon_3^+) \) is not NSOS in \( X_N \). Hence \( \hat{f} \) is not \( \alpha \)-irresolute function.

**Example 3.16**

Let \( X_N = \{ a, b, c \} = Y_N \) and \( T_N = \{ 0, 1 \}, \epsilon_1 \), \( \Gamma_N = \{ 0, 1 \}, \epsilon_2^+ \) are NTS on \( X_N \) and \( Y_N \) where

\[
\epsilon_1^+ = (x, \{ 7, 5, 3 \} , \{ 5, 10, 10 \} )
\]

\[
\epsilon_2^+ = (x, \{ 5, 5, 2 \} , \{ 5, 10, 10 \} )
\]

Define a Neutrosophic function \( \hat{f}: (X_N,T_N) \to (Y_N,G_N) \) by \( \hat{f}(a) = b, \hat{f}(b) = c, \hat{f}(c) = a \). \( \epsilon_2^+ \) is a NOS in \( (Y_N,G_N) \). Also \( \epsilon_2^+ \) is NaOS, NPOS, NSOS and \( \mathcal{N}\beta OS \) in \( Y_N \) and

\[
\hat{f}^{-1}(\epsilon_2^+) = (x, \{ 3, 5, 4 \} , \{ 4, 5, 6 \} )
\]

\[
\text{Nint}(\text{Ncl}(\hat{f}^{-1}(\epsilon_2^+))) = 1_N. \text{Since } \hat{f}^{-1}(\epsilon_2^+) \subseteq \text{Nint}(\text{Ncl}(\hat{f}^{-1}(\epsilon_2^+))), \hat{f}^{-1}(\epsilon_2^+) \text{ is a NPOS in } (X_N,T_N) \text{ and also } \hat{f}^{-1}(\epsilon_2^+) \text{ is } \mathcal{N}\beta OS \text{ in } X_N. \text{Thus } \hat{f} \text{ is a Npre irresolute function, Npre-} \alpha \text{-irresolute function, Npre continuous and } \mathcal{N}\beta \text{ -continuous. Now } \hat{f}^{-1}(\epsilon_2^+) \nsubseteq \text{Ncl}(\text{Nint}(\hat{f}^{-1}(\epsilon_2^+))) = 0_N. \text{So } \hat{f}^{-1}(\epsilon_2^+) \text{ is not NSOS in } X_N. \text{Also } \hat{f}^{-1}(\epsilon_2^+) \nsubseteq \text{Nint}(\text{Ncl}(\hat{f}^{-1}(\epsilon_2^+))) = 0_N. \text{Hence } \hat{f}^{-1}(\epsilon_2^+) \text{ is not NaOS in } X_N. \text{Thus } \hat{f} \text{ is not NaOS function, not NS-} \alpha \text{-irresolute function and not Na-continuous and not NSsemi-continuous.}

**Example 3.17** Let \( X_N = \{ a, b \} \) \( Y_N = \{ c, d \} \) and \( T_N = \{ 0, 1 \}, \epsilon_1^+, \Gamma_N = \{ 0, 1 \}, \epsilon_2^+ \), \( \epsilon_2^+ \) are NTS on \( X_N \) and \( Y_N \) respectively where

\[
\epsilon_1^+ = (x, \{ 4, 5, 4 \} , \{ 5, 10, 10 \} )
\]

\[
\epsilon_2^+ = (y, \{ 4, 5, 5 \} , \{ 5, 10, 10 \} )
\]

\[
\epsilon_3^+ = (y, \{ 2, 5, 2 \} , \{ 4, 5, 5 \} )
\]

is a NS in \( Y_N \). Define an Neutrosophic function \( \hat{f}: (X_N,T_N) \to (Y_N,G_N) \) by \( \hat{f}(a) = c, \hat{f}(b) = d \). \( \epsilon_2^+ \) is a NOS in \( (Y_N,G_N) \). Also \( \epsilon_2^+ \) is NaOS, NPOS in \( Y_N \) and

\[
\hat{f}^{-1}(\epsilon_2^+) = (x, \{ 4, 5, 2 \} , \{ 3, 5, 5 \} )
\]

and \( \hat{f}^{-1}(\epsilon_2^+) \subseteq \text{Nint}(\text{Ncl}(\hat{f}^{-1}(\epsilon_2^+))) = 1_N. \)

Thus \( \hat{f}^{-1}(\epsilon_2^+) \subseteq \text{Nint}(\text{Ncl}(\hat{f}^{-1}(\epsilon_2^+))) \). Hence \( \hat{f}^{-1}(\epsilon_2^+) \) is a NPOS in \( X_N \).
Now $\mathcal{E}^*_3 \subseteq \text{Nint} (\text{Ncl}(\mathcal{E}^*_3)) = 1_N$. Therefore $\mathcal{E}^*_3$ is an NPOS in $\mathcal{Y}_N$. Also $\mathcal{E}^*_3$ is an NβOS in $\mathcal{Y}_N$.

$$\text{Nint}(\text{Ncl}(\tilde{\mathcal{E}}^{-1}(\mathcal{E}^*_3))) = 0_N.$$ Thus $\tilde{\mathcal{E}}^{-1}(\mathcal{E}^*_3) \subseteq \text{Nint}(\text{Ncl}(\tilde{\mathcal{E}}^{-1}(\mathcal{E}^*_3)))$.

Hence $\tilde{\mathcal{E}}^{-1}(\mathcal{E}^*_3)$ is not an NPOS in $\mathcal{X}_N$. So $\tilde{\mathcal{E}}$ is not Npre-β-irresolute function and $\tilde{\mathcal{E}}$ is not Npre irresolute function. Since $\tilde{\mathcal{E}}^{-1}(\mathcal{E}^*_3) \not\subseteq \text{Ncl}(\text{Nint}(\text{Ncl}(\mathcal{E}^*_3))) = 0_N$.

$\tilde{\mathcal{E}}^{-1}(\mathcal{E}^*_3)$ is not NβOS in $\mathcal{X}_N$. So $\tilde{\mathcal{E}}$ is not Nβ-irresolute function.

**Example 3.18** Let $\mathcal{X}_N = \{a, b\}$, $\mathcal{Y}_N = \{c, d\}$ and $\mathcal{T}_N = \{0, 1\}$, $\Gamma_N = \{0, 1\}$, are NTS on $\mathcal{X}_N$ and $\mathcal{Y}_N$ respectively where

$$\mathcal{E}^*_1 = \left( x, \left( \begin{array}{c} 4 \ 5 \\ 10 \ 10 \end{array} \right), \left( \begin{array}{c} 5 \ 0 \\ 10 \ 10 \end{array} \right) \right)$$

$$\mathcal{E}^*_2 = \left( y, \left( \begin{array}{c} 4 \ 5 \\ 10 \ 10 \end{array} \right), \left( \begin{array}{c} 5 \ 5 \\ 10 \ 10 \end{array} \right) \right)$$

$$\mathcal{E}^*_3 = \left( y, \left( \begin{array}{c} 2 \ 5 \\ 10 \ 10 \end{array} \right), \left( \begin{array}{c} 2 \ 5 \\ 10 \ 10 \end{array} \right) \right)$$

is a NS in $\mathcal{Y}_N$. Define an Neutrosophic function $\tilde{\mathcal{E}}: (\mathcal{X}_N, \mathcal{T}_N) \rightarrow (\mathcal{Y}_N, \mathcal{G}_N)$ by $\tilde{\mathcal{E}}(a) = c$, $\tilde{\mathcal{E}}(b) = d$. $\mathcal{E}^*_2$ is a NOS in $(\mathcal{Y}_N, \mathcal{G}_N)$. Also $\mathcal{E}^*_3$ is a NαOS, NPOS in $\mathcal{Y}_N$.

$$\tilde{\mathcal{E}}^{-1}(\mathcal{E}^*_2) = \left( x, \left( \begin{array}{c} 4 \ 5 \\ 10 \ 10 \end{array} \right), \left( \begin{array}{c} 5 \ 5 \\ 10 \ 10 \end{array} \right) \right)$$

and $\text{Nint} (\text{Ncl} (\tilde{\mathcal{E}}^{-1}(\mathcal{E}^*_2))) = 1_N$. Thus $\tilde{\mathcal{E}}^{-1}(\mathcal{E}^*_2) \subseteq \text{Nint} (\text{Ncl}(\tilde{\mathcal{E}}^{-1}(\mathcal{E}^*_2)))$. Hence $\tilde{\mathcal{E}}^{-1}(\mathcal{E}^*_2)$ is a NPOS in $\mathcal{X}_N$. Therefore $\tilde{\mathcal{E}}$ is a Npre irresolute, Npre-α-irresolute and Npre continuous.

$\mathcal{E}^*_3$ is a NS in $\mathcal{Y}_N$ and $\mathcal{E}^*_3 \subseteq \text{Ncl}(\text{Nint}(\tilde{\mathcal{E}}^{-1}(\mathcal{E}^*_2)) = \mathcal{E}^*_2$. Hence $\mathcal{E}^*_3$ is a NβOS in $\mathcal{Y}_N$.

$$\tilde{\mathcal{E}}^{-1}(\mathcal{E}^*_3) = \left( x, \left( \begin{array}{c} 2 \ 5 \\ 10 \ 10 \end{array} \right), \left( \begin{array}{c} 2 \ 5 \\ 10 \ 10 \end{array} \right) \right)$$

and $\text{Nint} (\text{Ncl}(\tilde{\mathcal{E}}^{-1}(\mathcal{E}^*_3))) = 0_N$. Thus $\tilde{\mathcal{E}}^{-1}(\mathcal{E}^*_3) \not\subseteq \text{Nint}(\text{Ncl}(\tilde{\mathcal{E}}^{-1}(\mathcal{E}^*_3)))$. So $\tilde{\mathcal{E}}^{-1}(\mathcal{E}^*_3)$ is not an NPOS in $\mathcal{X}_N$. Hence $\tilde{\mathcal{E}}$ is not Npre-β-irresolute function.

Diagram: I
4. PROPERTIES

**Theorem 4.1** If a function \( \tilde{f} : (X_N, T_N) \rightarrow (Y_N, G_N) \) is Npre-\( \alpha \)-irresolute (N\( \alpha \)-irresolute and NS\( \alpha \)-irresolute, resp.) then \( \tilde{f}^{-1}(E_1^\alpha) \) is NPCS (N\( \alpha \)-closed and NSemiclosed, resp.) in \( X_N \) for any N nowhere dense set \( E_1^\alpha \) of \( Y_N \).

**Proof:**
Let \( E_1^\alpha \) be an N nowhere dense set in \( Y_N \). Then \( \text{Nint}(\text{Ncl}(E_1^\alpha)) = \emptyset_N \). Now, \( \text{Nint}(\text{Ncl}(E_1^\alpha)) = 1_N \Rightarrow \text{Ncl}(\text{Nint}(E_1^\alpha)) = 1_N \). Hence \( E_1^\alpha \subseteq \text{Nint}(\text{Ncl}(E_1^\alpha)) \). Since \( \tilde{f} \) is Npre-\( \alpha \)-irresolute (N\( \alpha \)-irresolute and N semi-\( \alpha \)-irresolute, resp.), \( \tilde{f}^{-1}(E_1^\alpha) \) is a NPCS (N\( \alpha \)OS and NSOS, resp.) in \( X_N \). Hence \( \tilde{f}^{-1}(E_1^\alpha) \) is a NPCS (N\( \alpha \)CS and NSCS, resp.) in \( X_N \).

**Theorem 4.2** If a function \( \tilde{f} : (X_N, T_N) \rightarrow (Y_N, G_N) \) is Npre-\( \beta \)-irresolute, then \( \tilde{f}^{-1}(E_1^\beta) \) is NPCS in \( X_N \) for any Nnowheredense set \( E_1^\beta \) of \( Y_N \).

**Proof:**
Let \( E_1^\beta \) be an Nnowheredense set in \( Y_N \). Then \( \text{Nint}(\text{Ncl}(E_1^\beta)) = \emptyset_N \). Now, \( \text{Nint}(\text{Ncl}(E_1^\beta)) = 1_N \Rightarrow \text{Ncl}(\text{Nint}(E_1^\beta)) = 1_N \). Since \( \text{Nint}(E_1^\beta) = 1_N \) and \( \text{Ncl}(\text{Nint}(E_1^\beta)) \subseteq \text{Ncl}(\text{Nint}(E_1^\beta)) \), then \( E_1^\beta \subseteq 1_N \subseteq \text{Ncl}(\text{Nint}(E_1^\beta)) \). Hence \( E_1^\beta \) is a N\( \beta \)OS in \( Y_N \). Since \( \tilde{f} \) is Npre-\( \beta \)-irresolute, \( \tilde{f}^{-1}(E_1^\beta) \) is a NPO in \( X_N \). Hence \( \tilde{f}^{-1}(E_1^\beta) \) is a NPCS in \( X_N \).

**Theorem 4.3** A function \( \tilde{f} : (X_N, T_N) \rightarrow (Y_N, G_N) \) from an NTS \( X_N \) into an NTS \( Y_N \) is Npre-\( \alpha \)-irresolute if and only if for each NP \( p(\alpha, \beta) \) in \( X_N \) and NPOS \( E_1^\alpha \) in \( Y_N \), such that \( \tilde{f}(p(\alpha, \beta)) \subseteq E_2^\alpha \), there exists an NPO \( E_1^\alpha \) in \( X_N \) such that \( p(\alpha, \beta) \subseteq E_1^\alpha \) and \( \tilde{f}(E_1^\alpha) \subseteq E_2^\alpha \).

**Proof:**
Let \( \tilde{f} \) be any Npre-\( \alpha \)-irresolute function. \( p(\alpha, \beta) \) be an NP in \( X_N \) and \( E_2^\alpha \) be any NPO in \( Y_N \) such that \( \tilde{f}(p(\alpha, \beta)) \subseteq E_2^\alpha \). Then \( \tilde{f}^{-1}(E_2^\beta) \) is a NPO in \( X_N \) which containing NP \( p(\alpha, \beta) \) and \( \tilde{f}(E_1^\beta) \subseteq E_2^\beta \). Conversely, let \( E_2^\alpha \) be a NPO in \( Y_N \) and \( p(\alpha, \beta) \) be an NP in \( X_N \) such that \( p(\alpha, \beta) \subseteq \tilde{f}^{-1}(E_2^\beta) \). According to an assumption, there exists an NPO \( E_1^\alpha \) in \( X_N \) such that \( p(\alpha, \beta) \) be an NP in \( X_N \) and \( \tilde{f}(E_1^\alpha) \subseteq \tilde{f}^{-1}(E_2^\beta) \). Therefore, \( \tilde{f}^{-1}(E_2^\beta) \subseteq \text{Nint}(\text{Ncl}(\tilde{f}^{-1}(E_2^\beta))) \) is NPO in \( X_N \). Thus, \( \tilde{f} \) is a Npre-\( \alpha \)-irresolute function.

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Theorem 4.4. A function \( \tilde{f} : (X_N, J_N) \rightarrow (Y_N, S_N) \) from an NTS \( X_N \) into an NTS \( Y_N \) is \(-\alpha\)- irresolute if and only if for each NP \( p(a,\beta) \) in \( X_N \) and NαOS \( E_2 \) in \( Y_N \) such that \((p(a,\beta)) E_2\), there exists an NαOS \( E'_1 \) in \( X_N \) such that \( p(a,\beta) \) and \( \tilde{f} (E'_1) \subseteq E'_2\).

**Proof:** Let \( \tilde{f} \) be any Nα- irresolute function. \( p(a,\beta) \) be an NP in \( X_N \) and \( E'_2 \) be any NαOS in \( Y_N \) such that \((p(a,\beta)) E_2\). Then \( p(a,\beta) \in \tilde{f}^{-1}(E_2) = \text{NInt} \tilde{f}^{-1}(E_2) \). Let \( E'_1 = \text{NInt} \tilde{f}^{-1}(E_2) \). Then \( E'_1 \) is a NαOS in \( X_N \) containing NP \( p(a,\beta) \) and \( \tilde{f} (E'_1) = \text{NInt} \tilde{f}^{-1}(E_2) = f(\tilde{f}^{-1}(E'_2)) \subseteq E'_2\).

Conversely, let \( E'_2 \) be an NαOS in \( Y_N \) and \( p(a,\beta) \) be an NP in \( X_N \) such that \((p(a,\beta)) E_2\). According to an assumption, there exists an NαOS \( E'_1 \) in \( X_N \) such that \((p(a,\beta)) E'_1 \) and \( \tilde{f} (E'_1) \subseteq E'_2\). Hence \( p(a,\beta) \subseteq E'_1 \) and \( \tilde{f} (E'_1) \subseteq E'_2\). Also \( \tilde{f}^{-1}(E'_2) \subseteq \text{NInt}(\tilde{f}^{-1}(E'_2)) \). Since \( p(a,\beta) \) is an arbitrary NP \( p(a,\beta) \subseteq E'_1 \) and \( \tilde{f}^{-1}(E'_2) \) is union of all NPs containing in \( \tilde{f}^{-1}(E'_2) \), which gives that \( p(a,\beta) \subseteq \text{NInt} \tilde{f}^{-1}(E'_2) \) is NαOS in \( X_N \). Hence \( \tilde{f} \) is a Nα- irresolute function.

**Theorem 4.5** \( \tilde{f} \) function \((X_N,J_N) \rightarrow (Y_N,S_N)\) from an NTS \( X_N \) into an NTS \( Y_N \) is Nsemi-\(\alpha\)- irresolute if and only if for each NP \( p(a,\beta) \) in \( X_N \) and NαOS \( E_2 \) in \( Y_N \) such that \((p(a,\beta)) E_2\), there exists an NSOS \( E'_1 \) in \( X_N \) such that \( p(a,\beta) \subseteq E'_1 \) and \( \tilde{f} (E'_1) \subseteq E'_2\).

**Proof:** Let \( \tilde{f} \) be any NS-\(\alpha\)- irresolute function. \( p(a,\beta) \) be an NP in \( X_N \) and \( E'_2 \) be any NSOS in \( Y_N \) such that \((p(a,\beta)) E'_2\). Then \( p(a,\beta) \subseteq E'_2\). Let \( E'_1 = \tilde{f}^{-1}(E'_2) \). Then \( E'_1 \) is a NSOS in \( X_N \) containing NP \( p(a,\beta) \) and \( \tilde{f} (E'_1) = \tilde{f}^{-1}(E'_2) \subseteq E'_2\).

Conversely, let \( E'_2 \) be an NSOS in \( Y_N \) and \( p(a,\beta) \) be a NP in \( X_N \) such that \((p(a,\beta)) E'_2\). According to an assumption, there exists an NSOS \( E'_1 \) in \( X_N \) such that \( p(a,\beta) \subseteq E'_1 \) and \( \tilde{f} (E'_1) \subseteq E'_2\). Also \( p(a,\beta) \subseteq \text{NInt}(E'_1) \subseteq \text{NInt}(\tilde{f}^{-1}(E'_2)) \). Therefore, \( \tilde{f}^{-1}(E'_2) \subseteq \text{NInt}(\tilde{f}^{-1}(E'_2)) \). Since \( p(a,\beta) \) is an arbitrary NP \( \tilde{f}^{-1}(E'_2) \) is union of all NPs containing in \( \tilde{f}^{-1}(E'_2) \), which gives that \( \tilde{f}^{-1}(E'_2) \subseteq \text{NInt} \tilde{f}^{-1}(E'_2) \) is NSOS in \( X_N \). Hence \( \tilde{f} \) is a NS-\(\alpha\)- irresolute function.

**Theorem 4.6** A function \( \tilde{f} : (X_N, J_N) \rightarrow (Y_N, S_N) \) from an NTS \( X_N \) into an NTS \( Y_N \) is Npre-\(\beta\)- irresolute if and only if for each NP \( p(a,\beta) \) in \( X_N \) and \( N\beta OS \) \( E_2 \) in \( Y_N \) such that \((p(a,\beta)) E_2\), there exists an NPOS \( E'_1 \) in \( X_N \) such that \( p(a,\beta) \subseteq E'_1 \) and \( \tilde{f} (E'_1) \subseteq E'_2\).

**Proof:** Let \( \tilde{f} \) be any Npre-\(\beta\)- irresolute mapping. \( p(a,\beta) \) be an NP in \( X_N \) and \( E'_1 \) be any \( N\beta OS \) in \( Y_N \) such that \((p(a,\beta)) E'_1\). Then \( p(a,\beta) \subseteq \tilde{f}^{-1}(E'_2) \). Let \( E'_1 = \tilde{f}^{-1}(E'_2) \). Then \( E'_1 \) is a NPOS in \( X_N \) which containing NP \( p(a,\beta) \) and \( \tilde{f} (E'_1) = \tilde{f}^{-1}(E'_2) \subseteq E'_2\).

Conversely, let \( E'_2 \) be an \( N\beta OS \) in \( Y_N \) and \( p(a,\beta) \) be a NP in \( X_N \) such that \((p(a,\beta)) E'_2\). According to an assumption, there exists an NPOS \( E'_1 \) in \( X_N \) such that \( p(a,\beta) \subseteq E'_1 \) and \( \tilde{f} (E'_1) \subseteq E'_2\). Also \( p(a,\beta) \subseteq \text{NInt}(E'_1) \subseteq \text{NInt}(\tilde{f}^{-1}(E'_2)) \). Therefore, \( \tilde{f}^{-1}(E'_2) \subseteq \text{NInt}(\tilde{f}^{-1}(E'_2)) \). Since \( p(a,\beta) \) is an arbitrary NP \( \tilde{f}^{-1}(E'_2) \) is union of all NPs containing in \( \tilde{f}^{-1}(E'_2) \), which gives that \( \tilde{f}^{-1}(E'_2) \subseteq \text{NInt} \tilde{f}^{-1}(E'_2) \) is NPOS in \( X_N \). Hence \( \tilde{f} \) is a Npre-\(\beta\)- irresolute function.

**Theorem 4.7** A function \( \tilde{f} : (X_N, J_N) \rightarrow (Y_N, S_N) \) from an NTS \( X_N \) into an NTS \( Y_N \) is Npre-\(\alpha\)- irresolute if and only if for each NP \( p(a,\beta) \) in \( X_N \) and \( N\alpha OS \) \( E_2 \) in \( Y_N \) such that \((p(a,\beta)) E_2\), \( \tilde{f}^{-1}(E'_2) \) is a NN of NP \( p(a,\beta) \) in \( X_N \).

**Proof:** Let \( \tilde{f} \) be any Npre-\(\alpha\)- irresolute function. \( p(a,\beta) \) be an NP in \( X_N \) and \( E'_2 \) be any \( N\alpha OS \) in \( Y_N \) such that \((p(a,\beta)) E'_2\). Then \( p(a,\beta) \subseteq \tilde{f}^{-1}(E'_2) \). Then \( p(a,\beta) \subseteq \text{NInt}(\tilde{f}^{-1}(E'_2)) \). Hence \( \text{NInt}(\tilde{f}^{-1}(E'_2)) \) is IFN of \( p(a,\beta) \) in \( X_N \).

Conversely, let \( E'_2 \) be a \( N\alpha OS \) in \( Y_N \) and \( p(a,\beta) \) be an NP in \( X_N \) such that \((p(a,\beta)) E'_2\). Then \( p(a,\beta) \subseteq \tilde{f}^{-1}(E'_2) \). According to an assumption, \( \text{NInt}(\tilde{f}^{-1}(E'_2)) \) is NN of NP \( p(a,\beta) \) in \( X_N \). So \( p(a,\beta) \subseteq \text{NInt}(\tilde{f}^{-1}(E'_2)) \). Thus \( \tilde{f}^{-1}(E'_2) \subseteq \text{NInt}(\tilde{f}^{-1}(E'_2)) \). Hence \( \tilde{f}^{-1}(E'_2) \) is a NPOS in \( X_N \). Therefore \( \tilde{f} \) is a Npre-\(\alpha\)- irresolute function.

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Theorem 4.8: A function $\hat{f}: (X_N, T_N) \to (Y_N, S_N)$ from an NTS $X_N$ into an NTS $Y_N$ is N pre-\(\beta\)-irresolute if and only if for each NP $p(\alpha, \beta)$ in $X_N$ and $N\beta OS$ $E_2'$ in $Y_N$ such that $\hat{f}(p(\alpha, \beta)) \in E_2'$, $Ncl(\hat{f}^{-1}(E_2'))$, is a NN of NP $p(\alpha, \beta)$ in $X_N$.

Proof: Let $\hat{f}$ be any N pre-\(\beta\)-irresolute function. $p(\alpha, \beta)$ be an NP in $X_N$ and $E_2'$ be any $N\beta OS$ in $Y_N$ such that $\hat{f}(p(\alpha, \beta)) \in E_2'$. Then $p(\alpha, \beta) \in \hat{f}^{-1}(E_2') \subseteq Ncl(\hat{f}^{-1}(E_2')) \subseteq Ncl(Ncl(\hat{f}^{-1}(E_2'))).$

Hence $Ncl(Ncl(\hat{f}^{-1}(E_2'))) \subseteq Ncl(\hat{f}^{-1}(E_2'))$. Consequently, $E_2'$ is a $N\beta OS$ in $Y_N$. Conversely, let $E_2'$ be any $N\beta OS$ in $Y_N$ and $p(\alpha, \beta)$ be an NP in $X_N$ such that $\hat{f}(p(\alpha, \beta)) \in E_2'$. Then $p(\alpha, \beta) \in \hat{f}^{-1}(E_2')$. According to an assumption, $Ncl(Ncl(\hat{f}^{-1}(E_2'))) \subseteq Ncl(\hat{f}^{-1}(E_2'))$. Hence $\hat{f}^{-1}(E_2')$ is a NPOS in $N\beta OS$ in $X_N$. Therefore $\hat{f}$ is a N pre-\(\beta\)-irresolute function.

Theorem 4.9

The following hold for functions $\hat{f}: X_N \to Y_N$ and $\hat{g}: Y_N \to Z_N$:

i) If $\hat{f}$ is N pre irresolute and $g$ is N pre-\(\alpha\)-irresolute (N pre-\(\beta\)-irresolute, resp.), then $\hat{g} \circ \hat{f}$ is N pre-\(\alpha\)-irresolute (N pre-\(\beta\)-irresolute, resp.) function.

ii) If $\hat{f}$ is N pre-\(\alpha\)-irresolute (N pre-\(\beta\)-irresolute, resp.), and $g$ is N continuous (N\(\beta\) – continuous, resp.), then $\hat{g} \circ \hat{f}$ is N pre continuous.

iii) If $\hat{f}$ is N pre-\(\beta\)-irresolute (N pre-\(\alpha\)-irresolute, resp.) and $g$ is N irresolute (N\(\alpha\) – irresolute, resp.), then $\hat{g} \circ \hat{f}$ is N pre-\(\alpha\)-irresolute (N pre-\(\beta\)-irresolute, resp.).

iv) If $\hat{f}$ is NS-\(\alpha\)-irresolute (N irresolute, resp.) and $g$ is IF\(\alpha\)-continuous, then $\hat{g} \circ \hat{f}$ is N semi continuous (N irresolute, resp.).

v) If $\hat{f}$ is NS-\(\alpha\)-irresolute (N irresolute, resp.) and $g$ is IF\(\alpha\)-irresolute, then $\hat{g} \circ \hat{f}$ is NS-\(\alpha\)-irresolute (N irresolute, resp.).

vi) If $\hat{f}$ is N irresolute and $g$ is NS-\(\alpha\)-irresolute, then $\hat{g} \circ \hat{f}$ is N NS-\(\alpha\)-irresolute.

vii) If $\hat{f}$ is N irresolute and $g$ is N strongly \(\alpha\)-continuous, then $\hat{g} \circ \hat{f}$ is N strongly \(\alpha\)-continuous.

Proof:

(i) Let $E_2'$ be an N OS (N\(\beta\) OS, resp.) in $Z$. Since $g$ is N pre-\(\alpha\)-irresolute (N pre-\(\beta\)-irresolute, resp.) $g^{-1}(E_2')$ is a NPOS in $Y_N$. Now $(\hat{g} \circ \hat{f})^{-1}(E_2') = \hat{f}^{-1}(\hat{g}^{-1}(E_2'))$. Since $\hat{f}$ is N pre irresolute, $\hat{f}^{-1}(\hat{g}^{-1}(E_2'))$ is a NPOS in $X_N$. Hence $\hat{g} \circ \hat{f}$ is N pre-\(\alpha\)-irresolute (N pre-\(\beta\)-irresolute, resp.).

(ii) Let $E_2'$ be an N OS in $Z$. Since $g$ is N continuous (N\(\beta\) – continuous, resp.), $g^{-1}(E_2')$ is a N OS (N\(\beta\) OS, resp.) in $Y_N$. Now $(\hat{g} \circ \hat{f})^{-1}(E_2') = \hat{f}^{-1}(\hat{g}^{-1}(E_2'))$. Since $\hat{f}$ is N pre-\(\beta\)-irresolute (N pre-\(\alpha\)-irresolute, resp.), $\hat{f}^{-1}(\hat{g}^{-1}(E_2'))$ is a NPOS in $X_N$. Hence $\hat{g} \circ \hat{f}$ is N pre continuous.

(iii) Let $E_2'$ be an N OS (N\(\beta\) OS, resp.) in Z. Since $g$ is N irresolute (N\(\beta\) – irresolute, resp.), $\hat{g}^{-1}(E_2') = \hat{f}^{-1}(\hat{g}^{-1}(E_2'))$. Since $\hat{f}$ is N pre-\(\alpha\)-irresolute (N pre-\(\beta\)-irresolute, resp.), $\hat{f}^{-1}(\hat{g}^{-1}(E_2'))$ is a NPOS in $X_N$. Hence $\hat{g} \circ \hat{f}$ is N pre-\(\alpha\)-irresolute (N pre-\(\beta\)-irresolute, resp.).

(iv) Let $E_2'$ be an N OS in $Z$. Since $g$ is N continuous, $\hat{g}^{-1}(E_2')$ is an N OS in $Y_N$. Now $(\hat{g} \circ \hat{f})^{-1}(E_2') = \hat{f}^{-1}(\hat{g}^{-1}(E_2'))$. Since $\hat{f}$ is NS-\(\alpha\)-irresolute (N irresolute, resp.), $\hat{f}^{-1}(\hat{g}^{-1}(E_2'))$ is a NSOS (N OS, resp.) in $X_N$. Hence $\hat{g} \circ \hat{f}$ is NS semi continuous (N continuous, resp.).
(v) Let \( E^*_2 \) be an NαOS in \( Z \). Since \( g \) is Nα-irresolute, \( g^{-1}(E^*_2) \) is an NαOS in \( \mathcal{Y}_N \). Now Since \( \hat{f} \) is NS-\( \alpha \)-irresolute (Nα- irresolute, resp.), \( \hat{f}^{-1}(g^{-1}(E^*_2)) \) is a NSOS (NαOS, resp.) in \( X_N \). Hence \( \hat{g} \circ \hat{f} \) is NS-\( \alpha \)-irresolute (Nα-irresolute, resp.).

(vi) Let \( E^*_2 \) be an NαOS in \( Z \). Since \( g \) is NS-\( \alpha \)-irresolute, \( g^{-1}(E^*_2) \) is a NSOS in \( \mathcal{Y}_N \). Now \( (\hat{g} \circ \hat{f})^{-1}(E^*_2)=\hat{f}^{-1}(g^{-1}(E^*_2)) \). Since \( \hat{f} \) is Niirresolute, \( \hat{f}^{-1}(g^{-1}(E^*_2)) \) is a NSOS in \( X_N \). Hence \( \hat{g} \circ \hat{f} \) is NS-\( \alpha \)-irresolute.

(vii) Let \( E^*_2 \) be an NSOS in \( Z \). Since \( g \) is Nstrongly \( \alpha \)-continuous \( g^{-1}(E^*_2) \) is a NαOS in \( \mathcal{Y}_N \). Now \( (\hat{g} \circ \hat{f})^{-1}(E^*_2)=\hat{f}^{-1}(g^{-1}(E^*_2)) \). Since \( \hat{f} \) is Niirresolute, \( \hat{f}^{-1}(g^{-1}(E^*_2)) \) is a NSOS in \( X_N \). Hence \( \hat{g} \circ \hat{f} \) is Nstrongly \( \alpha \)-continuous.

5. CHARACTERIZATIONS

In this section, several characterizations of Neutrosophic pre-\( \alpha \)-irresolute functions, Neutrosophic \( \alpha \)-irresolute functions, Neutrosophic semi-\( \alpha \)-irresolute functions and Neutrosophic pre-\( \beta \)-irresolute functions are established.

**Theorem 5.1** If \( \hat{f} \) is a function from an NTS \( (X_N, \mathcal{T}_N) \) to another NTS \( (Y_N, \mathcal{G}_N) \), then the following are equivalent.

(a) \( \hat{f} \) is a Nprep-\( \alpha \)-irresolute.

(b) \( \hat{f}^{-1}(E^*_2) \subseteq \text{int}(\text{cl}(\hat{f}^{-1}(E^*_2))) \) for every NαOS \( E^*_2 \) in \( \mathcal{Y}_N \).

(c) \( \hat{f}^{-1}(E^*_2) \) is \( \mathcal{NPCS} \) in \( X_N \) for every NαCS \( E^*_2 \) in \( \mathcal{Y}_N \).

(d) \( \text{Ncl}(\text{Nint}(\hat{f}^{-1}(E^*_2)))=\text{Ncl}(\text{Ncl}(\hat{f}^{-1}(E^*_2))) \) for every NS \( E^*_2 \) of \( \mathcal{Y}_N \).

(e) \( \hat{f} \) (Ncl (Nint \( E^*_2 \))) = Ncl(\( f^{-1}(E^*_2) \)) for every NS \( E^*_2 \) of \( X_N \).

**Proof:**

(a) \( \Rightarrow \) (b): Let \( E^*_2 \) be an NαOS in \( \mathcal{Y}_N \). By (a), \( \hat{f}^{-1}(E^*_2) \) is \( \mathcal{NPCS} \) in \( X_N \). Hence (a) \( \Rightarrow \) (b) is proved.

(b) \( \Rightarrow \) (c): Let \( E^*_2 \) be any NαCS in \( \mathcal{Y}_N \). Then \( E^*_2 \) is NαOS in \( \mathcal{Y}_N \). By (b), \( \hat{f}^{-1}(E^*_2)=\text{Ncl}(\text{Ncl}(\hat{f}^{-1}(E^*_2))) \). But \( \hat{f}^{-1}(E^*_2) \subseteq \text{Ncl}(\text{Ncl}(\hat{f}^{-1}(E^*_2))) \) and \( \text{Ncl}(\text{Ncl}(\hat{f}^{-1}(E^*_2)))=\text{Ncl}(\text{Ncl}(\hat{f}^{-1}(E^*_2))) \).

(d) \( \Rightarrow \) (c): Let \( E^*_2 \) be an NS in \( X_N \). Then \( \text{Ncl}(\text{Nint}(\hat{f}^{-1}(E^*_2))) \subseteq \text{Ncl}(\text{Nint}(\hat{f}^{-1}(E^*_2))) \). Hence (c) \( \Rightarrow \) (d) is proved.

(e) \( \Rightarrow \) (a): Let \( E^*_2 \) be an NαOS in \( \mathcal{Y}_N \). Then \( \hat{f}^{-1}(E^*_2)=\text{Ncl}(\text{Ncl}(\hat{f}^{-1}(E^*_2))) \) is a NS in \( \mathcal{Y}_N \). Hence (e) \( \Rightarrow \) (a) is proved.

By (e),

\[
\hat{f}(\text{Ncl}(\text{Nint}(\hat{f}^{-1}(E^*_2)))) \subseteq \text{Ncl}(\hat{f}(\hat{f}^{-1}(E^*_2))) \subseteq \text{Ncl}(\text{Nint}(\hat{f}^{-1}(E^*_2))) = \text{Ncl}(\text{Ncl}(\hat{f}^{-1}(E^*_2))) \]

\[
\overline{\text{Nint}(\text{Ncl}(\hat{f}^{-1}(E^*_2)))}=\overline{\text{Ncl}(\text{Ncl}(\hat{f}^{-1}(E^*_2)))}=\overline{\text{Ncl}(\text{Ncl}(\hat{f}^{-1}(E^*_2)))}=\overline{\text{Ncl}(\text{Nint}(\hat{f}^{-1}(E^*_2)))} \]

Thus, \( \hat{f}(\text{Ncl}(\text{Nint}(\hat{f}^{-1}(E^*_2)))) \subseteq \text{Ncl}(\hat{f}(\hat{f}^{-1}(E^*_2))) \subseteq \text{Ncl}(\text{Nint}(\hat{f}^{-1}(E^*_2))) \), \( \text{Ncl}(\text{Nint}(\hat{f}^{-1}(E^*_2)))=\overline{\text{Ncl}(\text{Ncl}(\hat{f}^{-1}(E^*_2)))}=\overline{\text{Ncl}(\text{Ncl}(\hat{f}^{-1}(E^*_2)))}=\overline{\text{Ncl}(\text{Nint}(\hat{f}^{-1}(E^*_2)))} \)

Consider

\[
\text{Nint}(\text{Ncl}(\hat{f}^{-1}(E^*_2)))=\text{Ncl}(\text{Ncl}(\hat{f}^{-1}(E^*_2)))=\text{Ncl}(\text{Ncl}(\hat{f}^{-1}(E^*_2))) \]

---(2)
By (1) and (2), $\text{Nint}(Ncl(\tilde{f}^{-1}(E_2^2))) \subseteq \tilde{f}^{-1}(\text{Nint}(\tilde{f}^{-1}(E_2^2))) \subseteq \text{Nint}(Ncl(\tilde{f}^{-1}(E_2^2))) \Rightarrow \tilde{f}^{-1}(E_2^2) = \tilde{f}^{-1}(E_2^2) \Rightarrow \tilde{f}^{-1}(E_2) \subseteq \text{Nint}(Ncl(\tilde{f}^{-1}(E_2))) \Rightarrow \tilde{f}^{-1}(E_2)$ is NPOS in $X_N$. Thus $\tilde{f}$ is Npre-$\alpha$-irresolute. Hence (e) $\Rightarrow$ (a) is proved.

**Theorem 5.2** If $\tilde{f} : (X_N, T_N) \rightarrow (Y_N, G_N)$ be a mapping from NTS $X_N$ into NTS $Y_N$. Then the following are equivalent.

(a) $\tilde{f}$ is N-pre-irresolute.
(b) $\tilde{f}^{-1}(E_2^2)$ is NCS in $X_N$ for each NCS $E_2^2$ in $Y_N$.
(c) $\tilde{f}$ (Nacl($A$)) $\subseteq$ Nacl($\tilde{f}(A)$)) for each NS $E_1$ in $X_N$.
(d) Nac($\tilde{f}^{-1}(E_2^2)) \subseteq \tilde{f}^{-1}(\text{Nacl}(E_2^2))$ for each NS $E_2^2$ in $Y_N$.
(e) $\tilde{f}^{-1}(\text{Nint}(E_2^2)) \subseteq \text{Nint}(\tilde{f}^{-1}(E_2^2))$ for each NS $E_2^2$ in $Y_N$.

**Proof:**

(a)$\Rightarrow$(b): Let $E_2^2$ be NCS in $Y_N$. Then $\tilde{E}_2^2$ is NCS in $Y_N$. Since $\tilde{f}$ is N-$\alpha$-irresolute, $\tilde{f}^{-1}(E_2^2) = \tilde{f}^{-1}(E_2^2)$ is NCS in $X_N$. Hence $\tilde{f}^{-1}(E_2^2)$ is NCS in $X_N$. Thus (a)$\Rightarrow$(b) is proved.

(b)$\Rightarrow$(c): Let $E_1^1$ be NS in $X_N$. Then $E_1^1 \subseteq \tilde{f}^{-1}(\tilde{f}(E_1^1)) \subseteq \tilde{f}^{-1}(\text{Nacl}(\tilde{f}(E_1^1)))$. As Nacl($\tilde{f}(E_1^1)$) is NCS in $Y_N$, by (b), $\tilde{f}^{-1}(\text{Nacl}(\tilde{f}(E_1^1)))$ is a NCS in $X_N$. Nacl($E_1^1$) $\subseteq$ Nacl($\tilde{f}(E_1^1)$) $= \text{Nac}(\tilde{f}(E_1^1))$. Hence (b)$\Rightarrow$(c) is proved.

(c)$\Rightarrow$(d): For any NS $E_2^2$ in $Y_N$, let $\tilde{f}^{-1}(E_2^2) = E_1^1$. By (c), $\text{Nacl} (\tilde{f}^{-1}(E_2^2)) \subseteq \text{Nac} (\tilde{f}^{-1}(E_2^2)) \subseteq \text{Nac} (\tilde{f}(E_2^2))$. Thus $\text{Nac}(\tilde{f}^{-1}(E_2^2)) \subseteq \tilde{f}^{-1}(\text{Nac}(\tilde{f}(E_2^2)))$. Hence (c)$\Rightarrow$(d) is proved.

(d)$\Rightarrow$(e): For any NS $E_2^2$ in $Y_N$, Naint($E_2^2$) $= \text{Naint}(E_2^2)$. Now $\tilde{f}^{-1}(\text{Naint}(E_2^2)) = \tilde{f}^{-1}(\text{Naint}(E_2^2)) = \text{Naint}(\tilde{f}^{-1}(E_2^2)) = \text{Naint}(\text{Naint}(E_2^2))$. Hence (d)$\Rightarrow$(e) is proved.

(e)$\Rightarrow$(a): Let $E_2^2$ be NCS in $Y_N$. Then $E_2^2 = \text{Naint}(E_2^2)$ and $\tilde{f}^{-1}(E_2^2) = \tilde{f}^{-1}(E_2^2) \subseteq \text{Naint}(E_2^2)$. By definition $\tilde{f}^{-1}(E_2^2) \supseteq \text{Naint}(\tilde{f}^{-1}(E_2^2))$, so $\tilde{f}^{-1}(E_2^2) = \text{Naint}(\tilde{f}^{-1}(E_2^2))$. Thus $\tilde{f}^{-1}(E_2^2)$ is a NCS in $X_N$, which implies $\tilde{f}$ is N-$\alpha$-irresolute. Thus (e)$\Rightarrow$(a) is proved.

**Theorem 5.3** If $\tilde{f}$ is a function from an NTS $(X_N, T_N)$ to another NTS $(Y_N, G_N)$, then the following are equivalent.

(a) $\tilde{f}$ is a N-$\alpha$-irresolute.
(b) $\tilde{f}^{-1}(E_2^2) \subseteq \text{Ncl}(\text{Nint}(\tilde{f}^{-1}(E_2^2)))$ for every NCS $E_2^2$ in $Y_N$.
(c) $\tilde{f}^{-1}(E_2^2)$ is NSemisclosed in $X_N$ for every Nclosed set $E_2^2$ in $Y_N$.
(d) $\text{Nint}(\text{Ncl}(\tilde{f}^{-1}(E_2^2))) \subseteq \tilde{f}^{-1}(\text{Ncl}(\tilde{f}(E_2^2)))$ for every NS $E_2^2$ of $Y_N$.
(e) $\tilde{f}$ (Nint(\text{Ncl}($E_2^2$))) $\subseteq$ Ncl($E_2^2$) for every NS $E_2^2$ of $X_N$.

**Proof:**

(a)$\Rightarrow$(b): Let $E_2^2$ be an NCS in $Y_N$. By (a), $\tilde{f}^{-1}(E_2^2)$ is NS in $X_N$. $\Rightarrow$ $\tilde{f}^{-1}(E_2^2) \subseteq \text{Ncl}(\text{Nint}(\tilde{f}^{-1}(E_2^2)))$. Hence (a)$\Rightarrow$(b) is proved.

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(b)⇒ (c): Let $E_3$ be any NaCS in $\mathcal{Y}_N$. Then $\overline{E_3}$ is a NaOS in $\mathcal{Y}_N$. By (b), $\hat{f}^{-1}(\overline{E_3}) \subseteq \text{Ncl}(\text{Nint}(\hat{f}^{-1}(E_3)))$. Note that $\text{Ncl}(\text{Nint}(\hat{f}^{-1}(E_3))) \subseteq \text{Ncl}(\text{Ncl}(\hat{f}^{-1}(E_3))) = \text{Ncl}(\text{Ncl}(\hat{f}^{-1}(E_3))) \subseteq \hat{f}^{-1}(E_3)$.

(c)⇒ (d): Let $E_4$ be an NS in $\mathcal{Y}_N$. Then $\text{Na-cl}(E_4)$ is $\text{N} \beta$-closed in $\mathcal{Y}_N$.

(d)⇒ (e): Let $E_5$ be an NS in $\mathcal{Y}_N$. Then $\text{Ncl}(\text{Ncl}(E_5))$ is $\text{N} \beta$-closed in $\mathcal{Y}_N$.

(e)⇒ (a): Let $E_6$ be an NS in $\mathcal{Y}_N$. Then $\text{Ncl}(\text{Ncl}(E_6))$ is $\text{N} \beta$-closed in $\mathcal{Y}_N$.

Therefore $\text{Ncl}(\text{Ncl}(E_2)) \subseteq \hat{f}^{-1}(\text{Ncl}(\hat{f}^{-1}(E_2)))$.

Thus $\hat{f}$ is $\text{N} \alpha$-irresolute. Hence (e)⇒ (a) is proved.

**Theorem 5.4** If $\hat{f}$ is a function from an NT $(X_N,T_N)$ to another NT $(Y_N,G_N)$, then the following are equivalent.

(a) $\hat{f}$ is a Npre-$\beta$-irresolute.

(b) $\hat{f}^{-1}(E_2) \subseteq \int(\hat{f}^{-1}(E_2))$ for every $\mathcal{N} \beta$-OS $E_2$ in $\mathcal{Y}_N$.

(c) $\hat{f}^{-1}(E_3)$ is NPCS in $\mathcal{X}_N$ for every $\mathcal{N} \beta$-closed set $E_3$ in $\mathcal{Y}_N$.

(d) $\int(\hat{f}^{-1}(E_4)) \subseteq \hat{f}^{-1}(\text{Ncl}(\hat{f}^{-1}(E_4)))$ for every NS $E_4$ of $\mathcal{X}_N$.

(e) $\hat{f}^{-1}(\int(\hat{f}^{-1}(E_5))) \subseteq \hat{f}^{-1}(\text{Ncl}(\hat{f}^{-1}(E_5)))$ for every NS $E_5$ of $\mathcal{X}_N$.

**Proof:** (a)⇒ (b): Let $E_2$ be an $\mathcal{N} \beta$-OS in $\mathcal{Y}_N$. By (a), $\hat{f}^{-1}(E_2)$ is $\mathcal{N} \beta$-OS in $\mathcal{X}_N$. Hence (a)⇒ (b) is proved.

(b)⇒ (c): Let $E_3$ be any $\mathcal{N} \beta$-OS in $\mathcal{Y}_N$. Then $\overline{E_3}$ is $\mathcal{N} \beta$-OS in $\mathcal{Y}_N$. By (b), $\hat{f}^{-1}(\overline{E_3}) \subseteq \text{Ncl}(\text{Nint}(\hat{f}^{-1}(\overline{E_3})) = \hat{f}^{-1}(\text{Ncl}(\hat{f}^{-1}(\overline{E_3})))$.

(c)⇒ (d): Let $E_4$ be an NS in $\mathcal{Y}_N$. Then $\text{Ncl}(\text{Ncl}(E_4))$ is $\mathcal{N} \beta$-closed in $\mathcal{Y}_N$. By (c), $\hat{f}^{-1}(\text{Ncl}(\text{Ncl}(E_4))) \subseteq \hat{f}^{-1}(\text{Ncl}(\text{Ncl}(E_4)))$.

(d)⇒ (e): Let $E_5$ be an NS in $\mathcal{Y}_N$. Then $\text{Ncl}(\text{Ncl}(E_5)) \subseteq \text{Ncl}(\text{Ncl}(\hat{f}^{-1}(E_5))) \subseteq \text{Ncl}(\text{Ncl}(\hat{f}^{-1}(E_5)))$.

(e)⇒ (a): Let $E_2$ be an $\mathcal{N} \beta$-OS in $\mathcal{Y}_N$. Then $\hat{f}^{-1}(E_2)$ is $\mathcal{N} \beta$-OS in $\mathcal{X}_N$. Hence (e)⇒ (a) is proved.

This implies $\text{Ncl}(\text{Ncl}(\hat{f}^{-1}(E_5))) \subseteq \hat{f}^{-1}(\text{Ncl}(\hat{f}^{-1}(E_5)))$.

Hence (e)⇒ (a) is proved.

This implies $\text{Ncl}(\text{Ncl}(\hat{f}^{-1}(E_5))) \subseteq \hat{f}^{-1}(\text{Ncl}(\hat{f}^{-1}(E_5)))$.

Hence (e)⇒ (a) is proved.

This implies $\text{Ncl}(\text{Ncl}(\hat{f}^{-1}(E_5))) \subseteq \hat{f}^{-1}(\text{Ncl}(\hat{f}^{-1}(E_5)))$.

Hence (e)⇒ (a) is proved.
By (e), $\tilde{f}(Ncl(Nint(\tilde{f}^{-1}(E^2_2)))) \subseteq N\beta cl(\tilde{f}^{-1}((E^2_2))) \subseteq E^2_2$------(1) Consider $Nint(Ncl(\tilde{f}^{-1}(E^2_2)))$

$= Ncl(Ncl(\tilde{f}^{-1}(E^2_2))) = Nint(Ncl(\tilde{f}^{-1}(E^2_2))) \subseteq Ncl(Nint(\tilde{f}^{-1}(E^2_2))) \subseteq \tilde{f}^{-1}(\tilde{f}(Ncl(Nint(\tilde{f}^{-1}(E^2_2))))$------(2)

By (1) and (2), $Nint(Ncl(\tilde{f}^{-1}(E^2_2))) \subseteq \tilde{f}^{-1}(Ncl(Nint(\tilde{f}^{-1}(E^2_2)))) \subseteq \tilde{f}^{-1}(E^2_2)$ This implies $\tilde{f}^{-1}(E^2_2) \subseteq Nint(Ncl(\tilde{f}^{-1}(E^2_2)))$ which proves $\tilde{f}^{-1}(E^2_2)$ is NPOS in $X_N$. Thus $\tilde{f}$ is pre-$\beta$-irresolute. Hence (e)$\Rightarrow$(a) is proved.

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