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AH-Homomorphisms in Neutrosophic Rings and Refined Neutrosophic Rings

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Abstract: Algebraic relations between rings are determined by homomorphisms and isomorphisms. This paper introduces a new kind of algebraic functions between two neutrosophic rings to give more agility in the exploring of neutrosophic substructures properties, where it generalizes the concept of AHS-homomorphism in neutrosophic rings, and refined neutrosophic rings. Also, it determines the algebraic structure of neutrosophic AH-endomorphisms of the additive group of neutrosophic ring and refined neutrosophic ring.

Keywords: AH-homomorphism , AH-endomorphism , AH-ideal, AHS-ideal.

1. Introduction

Theory of neutrosophic algebra began with Smarandache and Kandasamy in [8]. They introduced interesting notions such as neutrosophic groups, neutrosophic rings, and neutrosophic loops. Recently, neutrosophic sets and their according concepts have many important applications in health care and Covid-19 identification, industry, optimization, and decision making algorithms [23,24,25,26,27].

More studies in neutrosophic algebra were carried out with a generalized view. The concepts of refined neutrosophic rings, and n-refined neutrosophic rings were defined and studied in [1,2,3,4,7]. Abobala and Smarandache presented AH-substructures in neutrosophic rings, refined neutrosophic rings, neutrosophic vector spaces, modules and n-refined neutrosophic rings. See [1,2,9,12,13,17,18,19,20,22]. Concepts such as AH-ideal, AHS-ideal, AHS-homomorphism, and AHS-isomorphism were defined and handled. These structures in general consists of many similar algebraic objects which help us to build a bridge between classical and neutrosophical algebra. For example, AH-ideal in a neutrosophic ring R(I) is a set with form $P = Q + SI$ where Q,S are ideals in the classical ring R. If Q=S we get an AHS-ideal. AHS-homomorphism is a well defined map $f: R(I) \to T(I); f(a + bI) = f_R(a) + f_R(b)I$, where f_R is a homomorphism between R and T. It can be understood as a ring homomorphism with two equal parts, each one is a classical homomorphism.

AHS-homomorphisms play an important role in the study of AHS-ideals (subsets with two equal classical ideals parts) in neutrosophic rings and refined neutrosophic rings. Thus we need weaker conditions and a generalized view to study the properties of such ideals with two different parts. For this goal, we will discuss the concept of AH-homomorphisms which are considered as a natural generalization of AHS-homomorphisms to deal with more complex substructures in neutrosophic rings and refined neutrosophic rings.

In this article we define AH-homomorphism to be a well defined map with different parts which are not supposed to be equal homomorphisms (weaker condition). These homomorphisms lead to better comprehension of some complex neutrosophic structures, especially AH-ideals. We study AH-homomorphisms in neutrosophic rings, and refined neutrosophic rings. Also, we determine the algebraic structure of neutrosophic AH-endomorphisms of the additive group of neutrosophic ring and refined neutrosophic ring.

Motivation

In the literature, AHS-homomorphism was a tool to investigate AHS-ideals, kernels and factors properties. From this point, the motivation of our work is to present and study a stronger tool (AH-homomorphism) to deal with complex neutrosophic substructures, which allows us to explore more interesting properties of AH-ideals, and kernels in neutrosophic rings and refined neutrosophic rings.

Also, these kinds of algebraic functions have an interesting structure itself. We will prove that endomorphisms of this kind has a structure of neutrosophic/refined neutrosophic ring respectively.

2. Preliminaries

Definition 2.1:[8]

Let (R,+,$\times$) be a ring, R(I)={a+bI ; a,b$\in R$} is called the neutrosophic ring, where I is a neutrosophic element with condition $I^2 = I$.

Definition 2.2:[4]

Let (R,+,$\times$) be a ring, (R(I_1,I_2) $,+,\times$) is called a refined neutrosophic ring generated by R , I_1,I_2.

Definition 2.3: [2]

Let (R(I_1,I_2) $,+,.$) be a refined neutrosophic ring and P_0,P_1,P_2 be ideals in the ring R then the set

$P = (P_0,P_1I_1,P_2I_2) = \{(a,bI_1,cI_2): a \in P_0, b \in P_1, c \in P_2\}$ is called a refined neutrosophic AH-ideal.

If $P_0 = P_1 = P_2$ then P is called a refined neutrosophic AHS-ideal.

Definition 2.4: [12]

Let (R,+,$\times$) be a ring and $I_k; 1 \leq k \leq n$ be n indeterminacies. We define

R_n(I)={$a_0 + a_1I + \cdots + a_nI_n$; $a_i \in R$} to be n-refined neutrosophic ring.

Addition and multiplication on R_n(I) are defined as:

$\sum_{i=0}^{n} x_i I_i + \sum_{i=0}^{n} y_i I_i = \sum_{i=0}^{n}(x_i + y_i) I_i$, $\sum_{i=0}^{n} x_i I_i \times \sum_{i=0}^{n} y_i I_i = \sum_{i,j=0}^{n}(x_i \times y_j) I_i I_j$.

Where × is the multiplication defined on the ring R.
Definition 2.5 :

Let (R,+,.), (T,+,.) be two rings and $f_R : R \to T$ is a homomorphism:

The map $f : \mathrm{R}(I_1, I_2) \to \mathrm{T}(I_1, I_2); f(x, yI_1, zI_2) = (f_R(x), f_R(y) I_1, f_R(z) I_2)$ is called an

AHS-homomorphism.

It is easy to see that $\forall\, x, y \in \mathrm{R}(I_1, I_2)$ $then$ $f(x + y) = f(x) + f(y), f(x.y) = f(x).f(y)$.

Definition 2.6: [2]

(a) Let $f : \mathrm{R}(I_1, I_2) \to \mathrm{T}(I_1, I_2)$ be an AHS-homomorphism we define AH-Kernel of f by :

$AH - Ker f = \{ (a, bI_1, cI_2); a, b, c \in Ker f_R \} = (Ker f_R, Ker f_R I_1, Ker f_R I_2)$

(b) let S=($S_0, S_1 I_1, S_2 I_2$) be a subset of $\mathrm{R}(I_1, I_2)$, then :

$f(S) = (f_R(S_0), f_R(S_1) I_1, f_R(S_2) I_2) = \{(f_R(a_0), f_R(a_1) I_1, f_R(a_2) I_2); a_i \in S_i\}$.

Definition 2.7 : [2]

Let (R$(I_1, I_2), +, .$) be a refined neutrosophic ring and P= $(P_0, P_1 I_1, P_2 I_2)$ be an AH-ideal:

(a) We say that P is a weak prime AH-ideal if $P_i ; i \in \{0,1,2\}$ are prime ideals in R.

(b) We say that P is a weak maximal AH-ideal if $P_i ; i \in \{0,1,2\}$ are maximal ideals in R.

(c) We say that P is a weak principal AH-ideal if $P_i ; i \in \{0,1,2\}$ are principal ideals in R.

3. Main concepts and results

Definition 3.1:

Let R , T be two rings and $f, g, h : R \to T$ be three homomorphisms:

(a) The map $[f, g] : R(I) \to T(I); [f, g](a + bI) = f(a) + g(b)I$ is called an AH-homomorphism.

(b) The map $[f, g, h] : R(I_1, I_2) \to T(I_1, I_2); [f, g, h](a, bI_1, cI_2) = (f(a), g(b) I_1, h(c) I_2)$ is called a

refined AH-homomorphism.

(c) If f, g, h are isomorphisms, then $[f, g]$, $[f, g, h]$ are called AH-isomorphism and refined AH-isomorphism respectively.

(d) $AH - Ker\ [f, g] = Ker\ (f) + (Ker\ (g))I$.

(e) $AH - ker\ [f, g, h] = (Ker(f), (Ker\ (g))I_1, (Ker\ (h))I_2)$.

Remark 3.2:

An AH-homomorphism is not supposed to be a neutrosophic homomorphism. See Example 3.11 in [1].

A refined AH-homomorphism is not supposed to be a refined neutrosophic homomorphism . See Example 3.3 in [2].

It is easy to see that if $f = g$, we get the concept of AHS-homomorphism in a neutrosophic ring R(I).

Also, we find that if $f = g = h$, we get the concept of AHS-homomorphism in a refined neutrosophic ring $R(I_1, I_2)$.

Theorem 3.3:

Let $R(I), T(I)$ be two neutrosophic rings and $[f, g]: R(I) \to T(I)$ be an AH-homomorphism:

(a) $[f, g]\big(R(I)\big) = f(R) + g(R)I$.

(b)$\forall\ x, y \in R(I), we\ have\ [f, g](x + y) = [f, g](x) + [f, g](y)$ and $[f, g](x.y) = [f, g](x).[f, g](y)$.

(c) If $P = Q + SI$ is an AH-ideal of R(I), $[f, g](P)$ is an AH-ideal of T(I).

(d) $AH - Ker\ [f, g]$ is an AH-ideal of R(I).

Proof :

(a) It is simple.

(b) Let $x = a + bI, y = c + dI$. We have:

$[f, g](x + y) = f(a + c) + g(b + d)I = f(a) + f(c) + [g(b) + g(d)]I = [f, g](x) + [f, g](y)$.

$[f, g](x.y) = f(a.c) + g(a.d + b.c + b.d)I =$

$f(a).f(c) + [g(a).g(d) + g(b).g(c) + g(b).g(d)]I = [f, g](x).[f, g](y)$.

(c) It is clear that $[f, g](P + QI) = f(P) + g(Q)I$, since $f(P), g(Q)$ are ideals in T, we get $f(P) + g(Q)I$ is an AH-ideal of T(J).

(d) Since $Ker(f), Ker(g)$ are ideals of R we find that $AH - Ker\ [f,g] = Ker\ f + (Ker\ g)I$ is an AH-ideal of R(I).

Theorem 3.4:

Let $R(I_1,I_2), T(I_1,I_2)$ be two refined neutrosophic rings and $[f,g,h]: R(I_1,I_2) \to T(I_1,I_2)$ be a refined AH-homomorphism, we have

(a) $[f,g,h]\big(R(I_1,I_2)\big) = (f(R), g(R)I_1, h(R)I_2)$.

(b)$\forall\ x, y \in R(I_1,I_2), we\ have\ [f,g,h](x+y) = [f,g,h](x) + [f,g,h](y)$ and

$[f,g,h](x.y) = [f,g,h](x).[f,g,h](y)$.

(c) If $P = (Q, S\,I_1, M\,I_2)$ is an AH-ideal of R(I_1,I_2) , then $[f,g,h](P)$ is an AH-ideal of T(I_1,I_2).

(d) $AH - Ker\ [f,g,h]$ is an AH-ideal of R(I_1,I_2).

Proof :

The proof is similar to Theorem 3.3 .

Definition 3.5:

(a) Let R(I) be a neutrosophic ring. The set of all AH-homomorphisms $[f,g]: R(I) \to R(I)$ is called AH-endomrphisms of R(I).We denote it by $AH - END(R(I))$.

(b) Let $R(I_1,I_2)$ be a refined neutrosophic ring. The set of all refined AH-homomorphisms $[f,g,h]: R(I_1,I_2) \to R(I_1,I_2)$ is called refined AH-endomorphisms of $R(I_1,I_2)$.We denote it by $AH - END\ (R(I_1,I_2))$.

Theorem 3.6:

Let $R(I), T(I)$ be two neutrosophic rings and $[f,g]: R(I) \to T(I)$ is an AH-homomorphism and $P = Q + SI$ is an AH-ideal of R(I) , where $AH - Ker\ [f,g] \leq P \neq R(I)$. We have

(a) P is a weak principal in R(I) if and only if $[f,g](P)$ is weak principal in T(I).

(b) P is a weak prime in R(I) if and only if $[f,g](P)$ is weak prime in T(I).

(c) P is a weak maximal in R(I) if and only if $[f,g](P)$ is weak maximal in T(I).

Proof :

(a) Since $[f,g](P) = f(Q) + g(S)I$, and $f(Q), g(S)$ are principal ideals in the ring T if and only if Q,S are principal in R, thus $[f,g](P)$ is weak principal in T(I) if and only if P is weak principal in R(I).

(b) Since $[f,g](P) = f(Q) + g(S)I$, and $f(Q), g(S)$ are prime ideals in the ring T if and only if Q,S are prime in R, thus $[f,g](P)$ is weak prime in T(I) if and only if P is weak prime in R(I).

(c) Since $[f,g](P) = f(Q) + g(S)I$, and $f(Q), g(S)$ are maximal ideals in the ring T if and only if Q,S are maximal in R, thus $[f,g](P)$ is weak maximal in T(I) if and only if P is weak maximal in R(I)..

Theorem 3.7:

Let $R(I_1,I_2), T(I_1,I_2)$ be two refined neutrosophic rings and $[f,g,h]: R(I) \to T(I)$ is a refined AH-homomorphism and P = (Q,S $I_1, M\, I_2$) is a refined AH-ideal of R(I) , where $AH - Ker\ [f,g,h] \leq P \neq R(I_1,I_2)$. Then :

(a) P is a weak principal in R(I_1,I_2) if and only if $[f,g,h](P)$ is weak principal in T(I_1,I_2).

(b) P is a weak prime in R(I_1,I_2) if and only if $[f,g,h](P)$ is weak prime in T(I_1,I_2).

(c) P is a weak maximal in R(I_1,I_2) if and only if $[f,g,h](P)$ is weak maximal in T(I_1,I_2).

Proof :

Since $[f,g,h](P) = (f(Q), g(S)I_1, h(M)I_2)$, we get the proof by similar argument to Theorem 3.6.

Example 3.8:

The following example clarifies the concept of AH-homomorphism between two neutrosophic rings.

Let $R = Z$ be the ring of integers, $T = Z_6$ be the ring of integers modulo 6, we have

(a) $f: R \to T; f(x) = x \bmod 6, g: R \to T; g(x) = 3x \bmod 6$ are two homomorphisms.

(b) $[f,g]: R(I) \to T(I); [f,g](x + yI) = f(x) + g(x)I = (x \bmod 6) + (3y \bmod 6)I$ is the corresponding AH-homomorphism.

(c) $[f,g](R(I)) = f(R) + g(R)I = Z_6 + \{0,3\}I = \{0,1,2,3,4,5,3I,1+3I,2+3I,3+3I,4+3I,5+3I\}$.

(d) $AH - Ker\,[f,g] = Ker(f) + Ker(g)I = 6Z + 2ZI = \{6x + 2yI; x, y \in Z\}$.

(e) We have $Q = <3>, S = <2>$ are two principal, maximal, and prime ideals in R,

$Ker(f) = 6Z \leq Q \text{ and } Ker(g) = 2Z \leq S$,

$P = Q + SI$ is weak principal, maximal, and prime ideal in R(I), $f(Q) = \{0,3\}, g(S) = \{0\}$,

$[f, g](P) = f(Q) + g(S)I = \{0,3\} + \{0\}I = \{0,3\}$ which is a weak principal, maximal, and prime AH-ideal of T(I).

Example 3.9:

This example clarifies the concept of AH-homomorphism between two refined neutrosophic rings.

Let $R = Z$ be the ring of integers, $T = Z_6$ be the ring of integers modulo 6, we have:

(a) $f: R \to T; f(x) = x \bmod 6, g: R \to T; g(x) = 3x \bmod 6, h: R \to T; h(x) = 4x \bmod 6$ are three homomorphisms.

(b) We have $Q = <2>, S = <3>$, are two principal, maximal, and prime ideals in R,

$P = (Q, QI_1, SI_2) = \{(2x, 2yI_1, 3zI_2); x, y, z \in Z\}$ is a weak principal, maximal, and prime AH-ideal of $R(I_1, I_2)$, $f(Q) = \{0,2,4\}, g(Q) = \{0\}, h(S) = \{0\}$.

(c) $[f, g, h](P) = (f(Q), g(Q)I_1, h(S)I_2) = \{(0,0,0), (2,0,0), (4,0,0)\}$ is a weak principal, maximal, and prime AH-ideal of $T(I_1, I_2)$.

(d) $AH - Ker[f, g, h] = (\ker(f), Ker(g)I_1, Ker(h)I_2) = (6Z, 2ZI_1, 3ZI_2) =$

$\{(6x, 2yI_1, 3zI_2); x, y, z \in Z\}$.

4. Algebraic structure of some endomorphisms

Remark 4.1:

A famous result in classical ring theory ensures that ring endomorphisms do not have a structure of a ring, since the addition of two endomorphisms does not preserve multiplication, but if we consider the additive group of any ring R, then group endomorphisms together form a non-commutative ring in general.

To study algebraic structures of AH-endomorphisms we introduce the following definition:

Definition 4.2:

Let R be any ring, consider $R(I)$, $R(I_1, I_2)$, the corresponding neutrosophic ring, and refined neutrosophic ring, respectively. We define:

(a) The set of all AH-homomorphisms on the additive group (R(I),+) is denoted by

$AAH - END(R(I))$.

(b) The set of all AH-homomorphisms on the additive group $(R(I_1, I_2), +)$ is denoted by

$RAAH - END\ (R(I_1, I_2))$.

Now we define algebraic binary operations on these sets.

Definition 4.3:

(a) Let R(I) be a neutrosophic ring. Addition and multiplication on AAH-END(R(I)) are defined as:

Addition : $[f, g] + [h, t] = [f + h\ , g + t]$.

Multiplication : $[f, g].[h, t] = [foh\ , fot + goh + got]$.

Where $f, g, h, t \in END(R, +)$.

(b) Let $R(I_1, I_2)$ be a refined neutrosophic ring. Addition and multiplication on

RAAH-END($R(I_1, I_2)$) are defined as:

Addition: $[f, g, m] + [h, t, n] = [f + h\ , g + t, m + n]$.

Multiplication:

$[f, g, m].[h, t, n] = [foh\ , fot + got + goh + gon + mot, mon + fon + moh]$.

Where $f, g, h, m, n, t \in END(R, +)$.

Theorem 4.4:

Let (R(I),+) be the additive abelian group of a neutrosophic ring R(I). Then $\mathrm{AAH} - \mathrm{END}(\mathrm{R}(\mathrm{I}))$ is a ring.

Proof :

Suppose that $[f, g]\ , [h, t]\ , [k, s]$ are three arbitrary elements in AAH-ENDR(I), we have :

$(AAH - END(R(I)), +)$ is an abelian group clearly.

Multiplication is distributive with respect to addition since:

$[f, g].([h, t] + [k, s]) = [fo(h + s)\ , fo(t + s) + go(h + k) + go(t + s)]$=

$[f, g].[h, t] + [f, g].[k, s]$.

Multiplication is associative:

$[f, g].([h, t].[k, s]) = [f, g].[hok, hos + tok + tos] =$

$[fohok, fo(hos+tok+tos)+gohok+go(hos+tok+tos)]=([f,g].[h,t]).[k,s].$

It is easy to check that multiplication and addition are well defined.

So, our proof is complete.

Theorem 4.5:

Let ($R(I_1,I_2)$,+) be the additive group of a refined neutrosophic ring. Then RAAH-END $R(I_1,I_2)$ is a ring.

Proof :

It is easy to see that (RAAH-END $R(I_1,I_2)$,+) is an abelian group.

Multiplication and addition are well defined clearly.

Multiplication is distributive with respect to addition, suppose that $[f,g,m],[h,t,n],[k,s,r]$ are three arbitrary elements in RAAH-END $R(I_1,I_2)$, we have:

$[f,g,m].([h,t,n]+[k,s,r])=$

$[fo(h+k), fo(t+s)+go(t+s)+go(n+r)+mo(t+s)+go(h+k), fo(n+r)+mo(h+k)+ mo(n+r)]$

$=[f,g,m].[h,t,n]+[f,g,m].[k,s,r].$

. Multiplication is associative:

$([f,g,m].[h,t,n]).[k,s,r]=$

$[foh, fot+got+goh+gon+mot, mon+fon+moh].[k,s,r]$, we put

$fot+got+goh+gon+mot=a,\ \ mon+fon+moh=b$, now we write:

$([f,g,m].[h,t,n]).[k,s,r]=$

$[foh,a,b].[k,s,r]=[fohok, aok+aos+aor+bos+fohos, fohor+bor+bok]=$

$[f,g,m].([h,t,n].[k,s,r]).$

Thus we get the desired proof.

Theorem 4.6:

Let R be a ring. Then $AAH-END(R(I))\cong END(R,+)(I)$,

$RAAH-END(R(I_1,I_2))\cong END(R,+)(I_1,I_2).$

Where $END(R,+)(I)$ is the neutrosophic ring generated by $END(R,+)$ and I, $END(R,+)(I_1,I_2)$ is the refined neutrosophic ring generated by $END(R,+)$ and I_1,I_2.

Proof:

We have $END(R,+)(I) = END(R,+) + END(R,+)I = \{f + gI; f,g \in END(R,+)\}$. Define the map

$f: AAH - END(R(I)) \to END(R,+)(I);\ f([g,h]) = g + hI$, f is a well defined map.

Let $x = [g,h], y = [m,n]$ be two arbitrary elements in $AAH - END(R(I))$,

$f(x + y) = (g + m) + (h + n)I = (g + hI) + (m + nI) = f(x) + f(y)$.

$f(x.y) = f([gom, gon + hom + hon]) = (gom) + (gon + hom + hon)I = f(x).f(y)$.

It is easy to see that f is bijective, thus f is an isomorphism.

Also, we have $END(R,+)(I_1,I_2) = \{(f, gI_1, hI_2); f,g,h \in END(R,+)\}$. Define the map

$f: RAAH - END(R(I_1,I_2)) \to END(R,+)(I_1,I_2); f[g,h,k] = (g, hI_1, kI_2)$.By a similar argument we find that f is an isomorphism.

Example 4.7:

This example clarifies operations on $AAH - END(R(I))$.

Let $R = Z_6$ the ring of integers modulo 6,

$f: R \to R; f(x) = 3x, g: R \to R; g(x) = 4x, h: R \to R; h(x) = x$ are three endomorphisms on (R,+).

$[f,g]: R(I) \to R(I); [f,g](x + yI) = 3x + 4yI$, $[h,g]: R(I) \to R(I); [h,g](x + yI) = x + 4yI$ are two AH-endomorphisms on (R(I),+).

We can clarify addition by:

$([f,g] + [h,g])(x + yI) = [f + h, g + g](x + yI) = (f + h)(x) + (2g)(y)I =$

$4x + 8yI = 4x + 2yI = [g,s](x + yI); s: R \to R; s(x) = 2x$ is an endomorphism on (R,+).

Multiplication can be clarified as:

$([f,g].[h,g])(x + yI) = foh(x) + (fog + goh + gog)(y)I = 3x + (12y + 4y + 16y)I =$

$3x + 2yI = [f,s](x + yI)$.

Example 4.8:

This example clarifies operations on RAAH-END R(I_1,I_2).

Let $R=Z_6$ the ring of integers modulo 6,

$f:R\to R;f(x)=3x,g:R\to R;g(x)=4x,h:R\to R;h(x)=x$ are three endomorphisms on (R,+).

$[f,h,g]:\mathrm{R}(I_1,I_2)\to\mathrm{R}(I_1,I_2),[h,g,h]:\mathrm{R}(I_1,I_2)\to\mathrm{R}(I_1,I_2)$ are two AH-endomorphisms on (R(I_1,I_2),+).

$([f,h,g]+[h,g,h])(x,yI_1,zI_2)=((f+h)(x),(h+g)(y)I_1,(g+h)(z)I_2)=$

$(4x,5yI_1,5zI_2)=[g,s,s](x,yI_1,zI_2)$; $s:R\to R;s(x)=5x$ is an endomorphism on (R,+).

$[f,h,g].[h,g,h](x,yI_1,zI_2)=$

$(foh(x),(fog+hoh+hog+hoh+gog)(y)I_1,(foh+goh+goh)(z)I_2)=$

$(3x,(12y+y+4y+y+16y)I_1,(3z+4z+4z)I_2)=(3x,4yI_1,5zI_2)=[f,g,s](x,yI_1,zI_2)$.

Conclusion

In this work, we have generalized the concept of AHS-homomorphism by introducing the concept of AH-homomorphism in neutrosophic rings, and refined neutrosophic rings. These functions play an important role in determining the structures and properties of AH-ideal, that is because they generalize AHS-homomorphisms. Also, AH-homomorphisms between any neutrosophic/refined neutrosophic ring and itself have an interesting structure, since they make a neutrosophic ring/refined neutrosophic ring with a suitable defined operation. Through this work, we have characterized the algebraic structure of AH-endomorphisms of the additive group in the case of neutrosophic ring and case of refined neutrosophic ring.

Some important open questions arise according to this work, we can summarize it as follows:

Define AH-homomorphisms on an n-refined neutrosophic ring $R_n(I)$.

Is the set of all endomorphisms over the additive group of the n-refined neutrosophic ring

$R_n(I)$form a ring?. If the answer is yes, then prove that it is isomorphic to the n-refined neutrosophic

ring generated by END(R,+).

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