Neutrosophic Sets and Systems

Volume 38 Article 29

11-25-2020

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NSS

Neutrosophic Sets and Systems,Vol. 38,2020

University of New Mexico

$\mathbf{N}_{\delta^*\mathbf{g}\alpha}$-Continuous and Irresolute Functions in Neutrosophic Topological Spaces

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Abstract. In this paper, the notions of $N_{\delta^*g\alpha}$-continuous and $N_{\delta^*g\alpha}$-irresolute functions in neutrosophic topological spaces are given. Furthermore, we analyze their characterizations and investigate their properties.

Keywords: $N_{\delta^*g\alpha}$-closed set; $N_{\delta^*g\alpha}$-continuous; $N_{\delta^*g\alpha}$-irresolute; $N_{\delta^*g\alpha}$ – homeomorphism; $N_{\delta^*g\alpha}$c-homeomorphism.

1. Introduction

The notion of fuzzy set (FS) and its logic are investigated and discussed by Zadeh [12]. Next, Chang [3] studied the conception of fuzzy topological space (FTS). After that, Atanassav [8] investigated the intuitionistic fuzzy set (IFS) in 1986. Neutrosophy has extend the grounds for a total family of new mathematical estimations. It is one of the non-classical sets, like fuzzy, nano, soft, permutation sets and so on, see ([17]-[39]). The neutrosophic set (NS) was presented by Smarandache[6]and expounded, (NS) is a popularization of (IFS) in intuitionistic fuzzy topological space $(IFTS)$ by coker [4]. In 2012 [1], the conception of neutrosophic topological space (NTS) is presented. Further the fundamental sets like semi/pre/α-open sets are presented in neutrosophic topological spaces $(NTSs)$, see ([13]-[16]). The neutrosophic closed sets $(NCSs)$ and neutrosophic continuous functions $(NCFs)$ were presented by Salama et al.[2] in 2014. Arokiarani et al.[7] presented the neutrosophic α-closed set (NαCS) in $(NTSs)$. The concepts of δ-closure are auxiliary tools in standard topology in the study of H-closed spaces. Damodharan et al.[9,10] presente the idea of N_δ -closure and N_δ -Interior in $(NTSs)$. Further, N_δ -continuous and Neutrosophic almost continuous in $(NTSs)$ were presented and established some of their related attributes. Recently Damodharan and Vigneshwaran [11] presented the conception of $N_{\delta^*g\alpha}$-closed sets in $(NTSs)$and studied some of its characteristics. In 2020, some applications of (NS) are applied by Abdel-Basset and others, see ([40]) In this work, we presented the $N_{\delta^*g\alpha}$-continuous functions and $N_{\delta^*g\alpha}$-irresolute functions in $(NTSs)$. Furthermore, the conceptions of $N_{\delta^*g\alpha}$ – homeomorphism and $N_{\delta^*g\alpha}$c-homeomorphism are presented and investigate their characteristics.

2. Preliminaries

In this section, we mention some pertinent basic preliminaries about neutrosophic sets (NSs)and its operations.

2.1. *Definition [1]*

Assume S is a non-empty fixed set. A neutrosophic set (NS) P is an object having the form:
$P = \{\langle s, \mu_m (P(s)), \sigma_i (P(s)), \nu_{nm} (P(s))\rangle \forall s \in S\}$, where $\mu_m (P(s))$ represents the degree of membership, $\sigma_i (P(s))$ represents the degree of indeterminacy and $\nu_{nm} (P(s))$ represents the degree of nonmembership $\forall s \in S$ to P.

2.2. *Remark [1]*

A (NS) $P = \{\langle s, \mu_m (P(s)), \sigma_i (P(s)), \nu_{nm} (P(s))\rangle \forall s \in S\}$ can be identified to an ordered triple $\langle \mu_m (P(s)), \sigma_i (P(s)), \nu_{nm} (P(s))\rangle$ in $]^-0, 1^+[$ on S.

2.3. *Definition [1]*

In (NTS) We have:

$$\begin{array}{ll}
0_N \text{ may is defined as } \forall s \in S & 1_N \text{ may be defined as } \forall s \in S \\
0_N = \langle s, 0, 0, 1\rangle & 1_N = \langle s, 1, 0, 0\rangle \\
0_N = \langle s, 0, 1, 1\rangle & 1_N = \langle s, 1, 0, 1\rangle \\
0_N = \langle s, 0, 1, 0\rangle & 1_N = \langle s, 1, 1, 0\rangle \\
0_N = \langle s, 0, 0, 0\rangle & 1_N = \langle s, 1, 1, 1\rangle
\end{array}$$

2.4. *Definition [1]*

Assume P is (NS) of the form:
$P = \{\langle s, \mu_m(P(s)), \sigma_i(P(s)), \nu_{nm}(P(s))\rangle \forall s \in S\}$, Then the complement of P $[P^c]$ may be defined as
$P^c = \{\langle s, \nu_{nm}(P(s)), \sigma_i(P(s)), \mu_m(P(s))\rangle \forall s \in S\}$

2.5. *Definition [1]*

Assume P and Q are two (NSs)of the form,
$P = \{\langle s, \mu_m(P(s)), \sigma_i(P(s)), \nu_{nm}(P(s))\rangle \forall s \in S\}$ and
$Q = \{\langle s, \mu_m(Q(s)), \sigma_i(Q(s)), \nu_{nm}(Q(s))\rangle \forall s \in S\}$. Then,

(1) Subsets $P \subseteq Q$ may be defined as follows
$P \subseteq Q \Leftrightarrow \mu_m(P(s)) \leq \mu_m(Q(s)), \sigma_i(P(s)) \geq \sigma_i(Q(s)), \nu_{nm}(P(s)) \geq \nu_{nm}(Q(s))$
(2) Subsets $P = Q \Leftrightarrow P \subseteq Q$ and $Q \subseteq P$
(3) Union of subsets $P \cup Q$ may be defined as follows
$$P \cup Q = \{s, max\{\mu_m(P(s), \mu_m(Q(s)))\}, min\{\sigma_i(P(s)), \sigma_i(Q(s))\},$$
$$min\{\nu_{nm}(P(s)), \nu_{nm}(Q(s))\} \forall s \in S\},$$
(4) Intersection of subsets $P \cap Q$ may be defined as follows
$$P \cap Q = \{s, min\{\mu_m(P(s), \mu_m(Q(s)))\}, max\{\sigma_i(P(s)), \sigma_i(Q(s))\},$$
$$max\{\nu_{nm}(P(s)), \nu_{nm}(Q(s))\} \forall s \in S\},$$

2.6. *Proposition [9]*

For any two (NSs) P and Q the following condition holds

i): $(P \cap Q)^c = P^c \cup Q^c$,
ii): $(P \cup Q)^c = P^c \cap Q^c$,

2.7. *Definition [1]*

A neutrosophic topology (NT) on a non-empty set S is a family τ of neutrosophic subsets in S satisfying the following axioms:

i): $0_N, 1_N \in \tau$,
ii): $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$,
iii): $\cup G_i \in \tau \forall \{G_i : i \in J\} \subseteq \tau$

Then the pair (S, τ) is named a neutroscopic topological space(NTS).

2.8. *Definition [1]*

Assume P is a (NS) in a (NTS) (S, τ). Then

i): $Nint\,(P) = \bigcup\{Q/Q$is a neutrosophic open set (NOS) in (s,τ) and$Q \subseteq P\}$ is named the neutrosophic interior of P;

ii): $Ncl\,(P) = \bigcap\{Q/Q$is a neutrosophic closed set (NCS)in (s,τ) and$Q \supseteq P\}$ is named the neutrosophic closure of P;

2.9. *Definition [7]*

A subset A of (S,τ) is named

i): neutrosophic semi-open set (NSOS)if $P \subseteq Ncl(Nint(P))$.

ii): neutrosophic pre-open set (NPOS)if $P \subseteq Nint(Ncl(P))$.

iii): neutrosophic semi-preopenset (NSPOS)if $P \subseteq Ncl(Nint(Ncl(P)))$.

iv): neutrosophic α-open set (NαOS)if $P \subseteq Nint(Ncl(Nint(P)))$.

v): neutrosophic regular open set (NROS)if $P = Nint(Ncl(P))$.

The complement of a (NSOS) (resp. (NPOS), (NSPOS), (NαOS), (NROS)) set is named (NSCS) (resp. (NPCS), (NSPCS), (NαCS, (NRCS)).

2.10. *Definition [9]*

Assume $\alpha,\beta,\lambda \in [0,1]$ and $\alpha+\beta+\lambda \leq 3$. A neutrosophic point $s_{(\alpha,\beta,\lambda)}$of S is a neutrosophic point (NP) of S which is clarified by

$$s_{(\alpha,\beta,\lambda)}(y) = \begin{cases} (\alpha,\beta,\lambda) & \text{when } y = s, \\ (0,0,1) & \text{when } y \neq s. \end{cases}$$

Here, S is named the support of $s_{(\alpha,\beta,\lambda)}$ and α,β and λ, respectively. A (NP) $s_{(\alpha,\beta,\lambda)}$ is named belong to a (NS)
$P = \langle \mu_m\,(P\,(s))\,, \sigma_i\,(P\,(s))\,, \nu_{nm}\,(P\,(s))\rangle$ in S, denoted by $s_{(\alpha,\beta,\lambda)} \in P$ if $\alpha \leq \mu_m\,(P\,(s))\,, \beta \geq \sigma_i\,(P\,(s))$ and $\lambda \geq \nu_{nm}\,(P\,(s))$ Clearly a (NP) can be represented by an ordered triple of (NP) as follows : $s_{(\alpha,\beta,\lambda)} = (s_\alpha, s_\beta, s_\lambda)$.

2.11. *Definition [9]*

Assume (S,τ) is a (NTS). Assume P is a (NS) and Assume $s_{(\alpha,\beta,\lambda)}$ is a (NP). $s_{(\alpha,\beta,\lambda)}$ is named neutrosophic quasi coincident with P [denoted by $s_{(\alpha,\beta,\lambda)}qP$] if $\alpha + \mu_m(P(s)) > 1; \beta + \sigma_i(P(s)) < 1$ and $\lambda + \nu_{nm}(P(s)) < 1$.

2.12. *Definition [9]*

Assume P and Q are two (NSs). P is named neutrosophic quasi coincident with Q [denoted by PqQ] if $\mu_m(P(s)) + \mu_m(Q(s)) > 1; \sigma_i(P(s)) + \sigma_i(Q(s)) < 1$ and $\nu_{nm}(P(s)) + \nu_{nm}(Q(s)) < 1$.

2.13. *Definition [9]*

Assume (S,τ) is an (NTS). An (NP) $s_{(\alpha,\beta,\lambda)}$ is named an neutroscopic δ-cluster point of an (NS) P if AqP for each neutrosophic regular open q-neighborhood A of $s_{(\alpha,\beta,\lambda)}$. The set of all neutroscopic δ-cluster points of P is named the neutrosophic δ-closure of Pand denoted by $Ncl_\delta\left(P\right)$. An (NS) P is named an N_δ-closed set (N_δ-CS) if $P = Ncl_\delta\left(P\right)$. The complement of an ($N_\delta$-CS) is named an N_δ-open set (N_δ-OS).

3. $\mathbf{N_{\delta^* g\alpha}}$-continuous functions

Here, some new conceptions are given by the authors.

3.1. *Definition*

A map $T : (S,\tau) \longrightarrow (Y,\sigma)$ is named a Neutrosophic delta star generalized alpha-continuous map(briefly $N_{\delta^* g\alpha}$-CM) if $T^{-1}(K)$ is $N_{\delta^* g\alpha}$-CS in (S,τ) for any (NCS) in (Y,σ).

3.2. *Theorem*

Any $N_{\delta^* g\alpha}$-CM is N_{gs}-CM.(resp $N_{\alpha g}$-CM, N_{gsp}-CM, N_{gp}-CM). Also converse part is not true as shown through the following examples.

Proof. Assume K is a (NCS) in (Y,σ). Since T is $N_{\delta^* g\alpha}$-CM. $T^{-1}(K)$ is $N_{\delta^* g\alpha}$-CS in (S,τ). Since any $N_{\delta^* g\alpha}$CS is N_{gs}-CS (resp $N_{\alpha g}$-CS, N_{gsp}-CS, N_{gp}-CS), therefore $T^{-1}(K)$ is N_{gs}-CS (resp $N_{\alpha g}$-CS, N_{gsp}-CS, N_{gp}-CS) in (S,τ). Hence T is N_{gs}-CM.(resp $N_{\alpha g}$-CM, N_{gsp}-CM, N_{gp}-CM).

3.3. *example*

Assume $S = \{p, q, r\}$. Define the (NSs)D_1, D_2, D_3, D_4 and G_1, G_2, G_3, G_4 as follows:

$D_1 = \left\langle \left(\frac{p}{0.3}, \frac{q}{0.3}, \frac{r}{0.2}\right), \left(\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.2}\right), \left(\frac{p}{0.4}, \frac{q}{0.6}, \frac{r}{0.6}\right) \right\rangle$

$D_2 = \left\langle \left(\frac{p}{0.4}, \frac{q}{0.6}, \frac{r}{0.6}\right), \left(\frac{p}{0.5}, \frac{q}{0.4}, \frac{r}{0.4}\right), \left(\frac{p}{0.3}, \frac{q}{0.3}, \frac{r}{0.2}\right) \right\rangle$

$D_3 = \left\langle \left(\frac{p}{0.4}, \frac{q}{0.6}, \frac{r}{0.6}\right), \left(\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.2}\right), \left(\frac{p}{0.3}, \frac{q}{0.3}, \frac{r}{0.2}\right) \right\rangle$

$D_4 = \left\langle \left(\frac{p}{0.3}, \frac{q}{0.3}, \frac{r}{0.2}\right), \left(\frac{p}{0.5}, \frac{q}{0.4}, \frac{r}{0.4}\right), \left(\frac{p}{0.4}, \frac{q}{0.6}, \frac{r}{0.6}\right) \right\rangle$

and $G_1 = \left\langle \left(\frac{p}{0.4}, \frac{q}{0.4}, \frac{r}{0.4}\right), \left(\frac{p}{0.5}, \frac{q}{0.4}, \frac{r}{0.4}\right), \left(\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.2}\right) \right\rangle$

$G_2 = \left\langle \left(\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.2}\right), \left(\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.2}\right), \left(\frac{p}{0.4}, \frac{q}{0.6}, \frac{r}{0.6}\right) \right\rangle$

$G_3 = \left\langle \left(\frac{p}{0.4}, \frac{q}{0.4}, \frac{r}{0.4}\right), \left(\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.2}\right), \left(\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.2}\right) \right\rangle$

$G_4 = \left\langle \left(\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.2}\right), \left(\frac{p}{0.5}, \frac{q}{0.4}, \frac{r}{0.4}\right), \left(\frac{p}{0.4}, \frac{q}{0.6}, \frac{r}{0.6}\right) \right\rangle$

Then the families $\tau = \{0_N, 1_N, D_1, D_2, D_3, D_4\}$ and $\xi = \{0_N, 1_N, G_1, G_2, G_3, G_4\}$ are neutrosophic topologies (NTs) on S. Thus, (S,τ) and (S,ξ) are (NTSs). Define $T : (S,\tau) \to (S,\xi)$ as $T\left(p\right) = p, T\left(q\right) = q, T\left(r\right) = r$. Then T is N_{gs}-CM but not $N_{\delta^* g\alpha}$-CM. Hence in (S,τ), $N_{\delta^* g\alpha}$-CS is $\left\langle \left(\frac{p}{0.4}, \frac{q}{0.5}, \frac{r}{0.5}\right), \left(\frac{p}{0.5}, \frac{q}{0.5}, \frac{r}{0.5}\right), \left(\frac{p}{0.4}, \frac{q}{0.4}, \frac{r}{0.3}\right) \right\rangle$ and

N_{gs}-CS is $\left\langle\left(\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.2}\right), \left(\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.2}\right), \left(\frac{p}{0.4}, \frac{q}{0.4}, \frac{r}{0.4}\right)\right\rangle$. Here $T^{-1}\left(G_3^c\right)$ is N_{gs}-CS but not $N_{\delta^* g\alpha}$-CS.

3.4. *example*

Assume $S = \{p, q, r\}$. Define the (NSs)D_1, D_2, D_3, D_4 and H_1, H_2, H_3, H_4 as follows:
$D_1 = \left\langle\left(\frac{p}{0.3}, \frac{q}{0.3}, \frac{r}{0.2}\right), \left(\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.2}\right), \left(\frac{p}{0.4}, \frac{q}{0.6}, \frac{r}{0.6}\right)\right\rangle$
$D_2 = \left\langle\left(\frac{p}{0.4}, \frac{q}{0.6}, \frac{r}{0.6}\right), \left(\frac{p}{0.5}, \frac{q}{0.4}, \frac{r}{0.4}\right), \left(\frac{p}{0.3}, \frac{q}{0.3}, \frac{r}{0.2}\right)\right\rangle$
$D_3 = \left\langle\left(\frac{p}{0.4}, \frac{q}{0.6}, \frac{r}{0.6}\right), \left(\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.2}\right), \left(\frac{p}{0.3}, \frac{q}{0.3}, \frac{r}{0.2}\right)\right\rangle$
$D_4 = \left\langle\left(\frac{p}{0.3}, \frac{q}{0.3}, \frac{r}{0.2}\right), \left(\frac{p}{0.5}, \frac{q}{0.4}, \frac{r}{0.4}\right), \left(\frac{p}{0.4}, \frac{q}{0.6}, \frac{r}{0.6}\right)\right\rangle$
and $H_1 = \left\langle\left(\frac{p}{0.3}, \frac{q}{0.3}, \frac{r}{0.2}\right), \left(\frac{p}{0.4}, \frac{q}{0.3}, \frac{r}{0.3}\right), \left(\frac{p}{0.3}, \frac{q}{0.4}, \frac{r}{0.4}\right)\right\rangle$
$H_2 = \left\langle\left(\frac{p}{0.4}, \frac{q}{0.5}, \frac{r}{0.6}\right), \left(\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.2}\right), \left(\frac{p}{0.3}, \frac{q}{0.5}, \frac{r}{0.5}\right)\right\rangle$
$H_3 = \left\langle\left(\frac{p}{0.3}, \frac{q}{0.3}, \frac{r}{0.2}\right), \left(\frac{p}{0.4}, \frac{q}{0.3}, \frac{r}{0.3}\right), \left(\frac{p}{0.3}, \frac{q}{0.5}, \frac{r}{0.5}\right)\right\rangle$
$H_4 = \left\langle\left(\frac{p}{0.4}, \frac{q}{0.5}, \frac{r}{0.6}\right), \left(\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.2}\right), \left(\frac{p}{0.3}, \frac{q}{0.4}, \frac{r}{0.4}\right)\right\rangle$
Then the families $\tau = \{0_N, 1_N, D_1, D_2, D_3, D_4\}$ and $\xi = \{0_N, 1_N, H_1, H_2, H_3, H_4\}$ are (NTs) on S. Thus, (S, τ) and (S, ψ) are (NTSs). Define $T : (S, \tau) \to (S, \psi)$ as $T(p) = p, T(q) = q, T(r) = r$. Then T is $N_{\alpha g}$-CM but not $N_{\delta^* g\alpha}$-CM. Hence in (S, τ),
$N_{\delta^* g\alpha}$-CS is $\left\langle\left(\frac{p}{0.4}, \frac{q}{0.5}, \frac{r}{0.5}\right), \left(\frac{p}{0.5}, \frac{q}{0.5}, \frac{r}{0.5}\right), \left(\frac{p}{0.4}, \frac{q}{0.4}, \frac{r}{0.3}\right)\right\rangle$ and
$N_{\alpha g}$-CS is $\left\langle\left(\frac{p}{0.3}, \frac{q}{0.5}, \frac{r}{0.5}\right), \left(\frac{p}{0.4}, \frac{q}{0.3}, \frac{r}{0.3}\right), \left(\frac{p}{0.3}, \frac{q}{0.3}, \frac{r}{0.2}\right)\right\rangle$. Here $T^{-1}\left(H_3^c\right)$ is $N_{\alpha g}$-CS but not $N_{\delta^* g\alpha}$-CS.

3.5. *example*

Assume $Y = \{u, v, w\}$. Define the (NSs)F_1, F_2, F_3, F_4 and I_1, I_2, I_3, I_4 as follows:
$F_1 = \left\langle\left(\frac{p}{0.3}, \frac{q}{0.3}, \frac{r}{0.2}\right), \left(\frac{p}{0.4}, \frac{q}{0.4}, \frac{r}{0.4}\right), \left(\frac{p}{0.4}, \frac{q}{0.6}, \frac{r}{0.6}\right)\right\rangle$
$F_2 = \left\langle\left(\frac{p}{0.4}, \frac{q}{0.6}, \frac{r}{0.6}\right), \left(\frac{p}{0.5}, \frac{q}{0.4}, \frac{r}{0.4}\right), \left(\frac{p}{0.4}, \frac{q}{0.4}, \frac{r}{0.3}\right)\right\rangle$
$F_3 = \left\langle\left(\frac{p}{0.4}, \frac{q}{0.6}, \frac{r}{0.6}\right), \left(\frac{p}{0.4}, \frac{q}{0.4}, \frac{r}{0.4}\right), \left(\frac{p}{0.4}, \frac{q}{0.4}, \frac{r}{0.3}\right)\right\rangle$
$F_4 = \left\langle\left(\frac{p}{0.3}, \frac{q}{0.3}, \frac{r}{0.2}\right), \left(\frac{p}{0.5}, \frac{q}{0.4}, \frac{r}{0.4}\right), \left(\frac{p}{0.4}, \frac{q}{0.6}, \frac{r}{0.6}\right)\right\rangle$
and $I_1 = \left\langle\left(\frac{p}{0.4}, \frac{q}{0.5}, \frac{r}{0.6}\right), \left(\frac{p}{0.5}, \frac{q}{0.6}, \frac{r}{0.6}\right), \left(\frac{p}{0.4}, \frac{q}{0.5}, \frac{r}{0.3}\right)\right\rangle$
$I_2 = \left\langle\left(\frac{p}{0.4}, \frac{q}{0.5}, \frac{r}{0.3}\right), \left(\frac{p}{0.5}, \frac{q}{0.4}, \frac{r}{0.4}\right), \left(\frac{p}{0.4}, \frac{q}{0.5}, \frac{r}{0.6}\right)\right\rangle$
$I_3 = \left\langle\left(\frac{p}{0.4}, \frac{q}{0.5}, \frac{r}{0.6}\right), \left(\frac{p}{0.5}, \frac{q}{0.4}, \frac{r}{0.4}\right), \left(\frac{p}{0.4}, \frac{q}{0.5}, \frac{r}{0.3}\right)\right\rangle$
$I_4 = \left\langle\left(\frac{p}{0.4}, \frac{q}{0.5}, \frac{r}{0.3}\right), \left(\frac{p}{0.5}, \frac{q}{0.6}, \frac{r}{0.6}\right), \left(\frac{p}{0.4}, \frac{q}{0.5}, \frac{r}{0.6}\right)\right\rangle$
Then the families $\vartheta = \{0_N, 1_N, F_1, F_2, F_3, F_4\}$ and $\zeta = \{0_N, 1_N, I_1, I_2, I_3, I_4\}$ are (NTs) on Y. Thus, (Y, ϑ) and (Y, ζ) are (NTSs). Define $g : (Y, \vartheta) \to (Y, \zeta)$ as $g(u) = u, g(v) = v, g(w) = w$. Then g is N_{gp}-CM but not $N_{\delta^* g\alpha}$-CM. Hence in (Y, ϑ),
$N_{\delta^* g\alpha}$-CS is $\left\langle\left(\frac{p}{0.4}, \frac{q}{0.5}, \frac{r}{0.5}\right), \left(\frac{p}{0.5}, \frac{q}{0.5}, \frac{r}{0.5}\right), \left(\frac{p}{0.4}, \frac{q}{0.4}, \frac{r}{0.3}\right)\right\rangle$ and
N_{gp}-CS is $\left\langle\left(\frac{p}{0.4}, \frac{q}{0.5}, \frac{r}{0.3}\right), \left(\frac{p}{0.5}, \frac{q}{0.4}, \frac{r}{0.4}\right), \left(\frac{p}{0.4}, \frac{q}{0.5}, \frac{r}{0.6}\right)\right\rangle$. Here $g^{-1}\left(I_3^c\right)$ is N_{gp}-CS but not $N_{\delta^* g\alpha}$-CS.

3.6. *example*

Assume $Y=\{u,v,w\}$. Define the (NSs)F_1,F_2,F_3,F_4 and J_1,J_2,J_3,J_4 as follows:
$F_1=\left\langle\left(\frac{p}{0.3},\frac{q}{0.3},\frac{r}{0.2}\right),\left(\frac{p}{0.4},\frac{q}{0.4},\frac{r}{0.4}\right),\left(\frac{p}{0.4},\frac{q}{0.6},\frac{r}{0.6}\right)\right\rangle$
$F_2=\left\langle\left(\frac{p}{0.4},\frac{q}{0.6},\frac{r}{0.6}\right),\left(\frac{p}{0.5},\frac{q}{0.4},\frac{r}{0.4}\right),\left(\frac{p}{0.4},\frac{q}{0.4},\frac{r}{0.3}\right)\right\rangle$
$F_3=\left\langle\left(\frac{p}{0.4},\frac{q}{0.6},\frac{r}{0.6}\right),\left(\frac{p}{0.4},\frac{q}{0.4},\frac{r}{0.4}\right),\left(\frac{p}{0.4},\frac{q}{0.4},\frac{r}{0.3}\right)\right\rangle$
$F_4=\left\langle\left(\frac{p}{0.3},\frac{q}{0.3},\frac{r}{0.2}\right),\left(\frac{p}{0.5},\frac{q}{0.4},\frac{r}{0.4}\right),\left(\frac{p}{0.4},\frac{q}{0.6},\frac{r}{0.6}\right)\right\rangle$
and $J_1=\left\langle\left(\frac{p}{0.4},\frac{q}{0.6},\frac{r}{0.5}\right),\left(\frac{p}{0.3},\frac{q}{0.4},\frac{r}{0.3}\right),\left(\frac{p}{0.4},\frac{q}{0.4},\frac{r}{0.5}\right)\right\rangle$
$J_2=\left\langle\left(\frac{p}{0.4},\frac{q}{0.4},\frac{r}{0.4}\right),\left(\frac{p}{0.3},\frac{q}{0.3},\frac{r}{0.2}\right),\left(\frac{p}{0.4},\frac{q}{0.6},\frac{r}{0.5}\right)\right\rangle$
$J_3=\left\langle\left(\frac{p}{0.4},\frac{q}{0.4},\frac{r}{0.4}\right),\left(\frac{p}{0.3},\frac{q}{0.4},\frac{r}{0.3}\right),\left(\frac{p}{0.4},\frac{q}{0.6},\frac{r}{0.5}\right)\right\rangle$
$J_4=\left\langle\left(\frac{p}{0.4},\frac{q}{0.6},\frac{r}{0.5}\right),\left(\frac{p}{0.3},\frac{q}{0.3},\frac{r}{0.2}\right),\left(\frac{p}{0.4},\frac{q}{0.4},\frac{r}{0.5}\right)\right\rangle$
Then the families $\vartheta=\{0_N,1_N,F_1,F_2,F_3,F_4\}$ and $\zeta=\{0_N,1_N,J_1,J_2,J_3,J_4\}$ are (NTs) on Y. Thus, (Y,ϑ) and (Y,φ) are (NTSs). Define $g:(Y,\vartheta)\to(Y,\varphi)$ as $g(u)=u, g(v)=w, g(w)=v$. Then g is N_{gsp}-C but not $N_{\delta^*g\alpha}$-C. Hence in (Y,ϑ),
$N_{\delta^*g\alpha}$-CS is $\left\langle\left(\frac{p}{0.4},\frac{q}{0.5},\frac{r}{0.5}\right),\left(\frac{p}{0.5},\frac{q}{0.5},\frac{r}{0.5}\right),\left(\frac{p}{0.4},\frac{q}{0.4},\frac{r}{0.3}\right)\right\rangle$ and
N_{gsp}-CS is $\left\langle\left(\frac{p}{0.4},\frac{q}{0.6},\frac{r}{0.5}\right),\left(\frac{p}{0.4},\frac{q}{0.4},\frac{r}{0.4}\right),\left(\frac{p}{0.3},\frac{q}{0.4},\frac{r}{0.3}\right)\right\rangle$. Here $g^{-1}(J_3^c)$ is N_{gsp}-CS but not $N_{\delta^*g\alpha}$-CS.

3.7. *Theorem*

The composition of two $N_{\delta^*g\alpha}$-CMs is also a $N_{\delta^*g\alpha}$-CM. Proof. Assume $T:(S,\tau)\longrightarrow(Y,\sigma)$ and $g:(Y,\sigma)\longrightarrow(Z,\eta)$ are two $N_{\delta^*g\alpha}$-CMs. Assume l is a NCS in (Z,η). Since g is a $N_{\delta^*g\alpha}$-CM, $g^{-1}(l)$ is $N_{\delta^*g\alpha}$-CS in (Y,σ). Since any $N_{\delta^*g\alpha}$-CS is NCS, $g^{-1}(l)$ is $NCSSin(Y,\sigma)$. Since T is a $N_{\delta^*g\alpha}$-CM, $T^{-1}\left(g^{-1}(l)\right)=goT(l)$ is $N_{\delta^*g\alpha}$-CS in (S,τ), therefore goT is also $N_{\delta^*g\alpha}$-CM.

4. $\mathbf{N_{\delta^*g\alpha}}$-Irresolute functions

Here, some new conceptions are given by the authors.

4.1. *Definition*

A map $T:(S,\tau)\longrightarrow(Y,\sigma)$ is named a Neutrosophic delta star generalized alpha-Irresolute map (briefly $N_{\delta^*g\alpha}$-IMM) if $T^{-1}(K)$ is $N_{\delta^*g\alpha}$-CS in (S,τ) for any $N_{\delta^*g\alpha}$-CS in (Y,σ).

4.2. *Theorem*

Assume $T:(S,\tau)\longrightarrow(Y,\sigma)$ and $g:(Y,\sigma)\longrightarrow(Z,\eta)$ are any two functions, then

(i) $goT:(S,\tau)\longrightarrow(Z,\eta)$ is $N_{\delta^*g\alpha}$-CM if g is N-CM and T is $N_{\delta^*g\alpha}$-CM.
(ii) $goT:(S,\tau)\longrightarrow(Z,\eta)$ is $N_{\delta^*g\alpha}$-IM if both g and T $N_{\delta^**g\alpha}$-IM.
(iii) $goT:(S,\tau)\longrightarrow(Z,\eta)$ is $N_{\delta^*g\alpha}$-CM if g is $N_{\delta^*g\alpha}$-CM and T is $N_{\delta^**g\alpha}$-IM.

Proof.

(i) Assume K is a(NCS) in (Z,η). Since g is N-CM, $g^{-1}(K)$ is NCS in (Y,σ). Since T is $N_{\delta^*g\alpha}$-CM, $T^{-1}\left(g^{-1}(K)\right)=(goT)^{-1}(K)$ is $N_{\delta^*g\alpha}$-CS in (S,τ), Therefore goT is $N_{\delta^*g\alpha}$-CM.

(ii) Assume K is a $N_{\delta^*g\alpha}$-CS in (Z,η). Since g is $N_{\delta^{*}*g\alpha}$-IM, $g^{-1}(K)$ is $N_{\delta^*g\alpha}$-CS in (Y,σ). Since T is $N_{\delta^{*}*g\alpha}$-IM, $T^{-1}\left(g^{-1}(K)\right)=(goT)^{-1}(K)$ is $N_{\delta^*g\alpha}$-CS in (S,τ), Therefore goT is $N_{\delta^{*}*g\alpha}$-IM.

(iii) Assume K is a(NCS) in (Z,η). Since g is $N_{\delta^*g\alpha}$-CM, $g^{-1}(K)$ is $N_{\delta^*g\alpha}$-CS in (Y,σ). Since T is $N_{\delta^{*}*g\alpha}$-IM, $T^{-1}\left(g^{-1}(K)\right)=(goT)^{-1}(K)$ is $N_{\delta^*g\alpha}$-CS in (S,τ), Therefore goT is $N_{\delta^*g\alpha}$-CM.

4.3. *Theorem*

Assume $T:(S,\tau)\longrightarrow(Y,\sigma)$ is $N_{\delta^*g\alpha}$-CM (N_{gs}-CM, $N_{\alpha g}$-CM, N_g-CM). If (S,τ) is an $N_{\alpha\delta T^{**}_{\frac{3}{4}}g\alpha}$-space ($N_{{}^{**}_{\delta\alpha}T_{\frac{1}{2}}}$-space, $N_{\delta\alpha T^{**}_c}$-space, $N_{\delta T^{**}_c}$-space) then T is contiuous.

Proof. Assume K is a(NCS) of (Y,σ). Since T is $N_{\delta^*g\alpha}$-CM (N_{gs}-CM, $N_{\alpha g}$-CM, N_g-CM), then $T^{-1}(K)$ is $N_{\delta^*g\alpha}$-CS (N_{gs}-CS, $N_{\alpha g}$-CS, N_g-CS) in (S,τ). Since (S,τ) is $N_{\alpha\delta T^{**}_{\frac{3}{4}}g\alpha}$-space ($N_{{}^{**}_{\delta\alpha}T_{\frac{1}{2}}}$-space, $N_{\delta\alpha T^{**}_c}$-space, $N_{\delta T^{**}_c}$-space), then $T^{-1}(K)$ is N_δ-CS in (S,τ). Any N_δ-CS is (NCS) in (S,τ). Therefore T is continuous.

4.4. *Theorem*

Assume $T:(S,\tau)\longrightarrow(Y,\sigma)$ is a surjective, $N_{*g\alpha}$-IM and N_δ-CM. Then $T(A)$ is $N_{\delta^*g\alpha}$-CS of (Y,σ) for any $N_{\delta^*g\alpha}$-CS A of (S,τ).

Proof. Assume A is a $N_{\delta^*g\alpha}$-CS of (S,τ). Assume U is a $N_{*g\alpha}$-OS of (Y,σ). such that $T(A)\subseteq U$. Since T is surjective and $N_{**g\alpha}$-IM, $T^{-1}(U)$ is $N_{*g\alpha}$-OS in (S,τ). Since $A\subseteq T^{-1}(U)$ and A is $N_{\delta^*g\alpha}$-CS of (S,τ), $Ncl_\delta(A)\subseteq T^{-1}(U)$. Then $T[Ncl_\delta(A)]\subseteq T\left[T^{-1}(U)\right]=U$, since T is N_δ-CS, $T[Ncl_\delta(A)]=Ncl_\delta[T[Ncl_\delta(A)]]$. This implies $Ncl_\delta[T(A)]\subseteq Ncl_\delta[T[Ncl_\delta(A)]]=T[Ncl_\delta(A)]\subseteq U$, Therefore $T(A)$ is a $N_{\delta^*g\alpha}$-CS of (Y,σ)

4.5. *Theorem*

Assume $T:(S,\tau)\longrightarrow(Y,\sigma)$ is a surjective, $N_{\delta^*g\alpha}$-IM and N_δ-CM. If (S,τ) is an $N_{\alpha\delta T^{**}_{\frac{3}{4}}g\alpha}$-space, then (Y,σ) is also an $N_{\alpha\delta T^{**}_{\frac{3}{4}}g\alpha}$-space.

Proof. Assume A is a $N_{\delta^*g\alpha}$-CS of (Y,σ). Since T is $N_{\delta^{*}*g\alpha}$-IM, $T^{-1}(A)$ is $N_{\delta^*g\alpha}$-CS in (S,τ). Since (S,τ) is $N_{\alpha\delta T^{**}_{\frac{3}{4}}g\alpha}$-space, $T^{-1}(A)$ is N_δ-CS of (S,τ). Since T is N_δ-CM and surjective, $T\left[T^{-1}(A)\right]=A$ is N_δ-CS in (Y,σ). Thus A is N_δ-CS in (Y,σ), Therefore (Y,σ) is an $N_{\alpha\delta T^{**}_{\frac{3}{4}}g\alpha}$-space.

5. $N_{\delta^* g\alpha}$-Homeomorphism

Here, some new conceptions are given by the authors.

5.1. *Definition*

A map $T:(S,\tau)\longrightarrow(Y,\sigma)$ is named a neutrosophic delta star generalized alpha-homeomorphism (briefly $N_{\delta^* g\alpha}$-H) if T is bijective, $N_{\delta^* g\alpha}$-CM and $N_{\delta^* g\alpha}$-OM.

5.2. *Theorem*

Any $N_{\delta^* g\alpha}$-H is N_{gs}-H.

Proof. Let $f:(X,\tau)\longrightarrow(Y,\sigma)$ be $N_{\delta^* g\alpha}$-H then f is bijective, $N_{\delta^* g\alpha}$-continuous and $N_{\delta^* g\alpha}$-OM. Let V be N-CS in (Y,σ), then $f^{-1}(V)$ is $N_{\delta^* g\alpha}$-CS in (X,τ). Since every $N_{\delta^* g\alpha}$-CS is N_{gs}-CS, then $f^{-1}(V)$ is N_{gs}-CS in (X,τ), Therefore f is N_{gs}-continuous. Let U be N-OS in (X,τ), then $f(U)$ is $N_{\delta^* g\alpha}$-OS in (Y,σ). Since every $N_{\delta^* g\alpha}$-OS is N_{gs}-OS, then $f(U)$ is N_{gs}-OS in (Y,σ), Therefore f is N_{gs}-OM. Hence f is N_{gs}-H.

5.3. *example*

Assume $S=\{p,q,r\}$. Define the (NSs)D_1,D_2,D_3,D_4 and G_1,G_2,G_3,G_4 as follows:

$D_1=\left\langle\left(\frac{p}{0.3},\frac{q}{0.3},\frac{r}{0.2}\right),\left(\frac{p}{0.3},\frac{q}{0.2},\frac{r}{0.2}\right),\left(\frac{p}{0.4},\frac{q}{0.6},\frac{r}{0.6}\right)\right\rangle$

$D_2=\left\langle\left(\frac{p}{0.4},\frac{q}{0.6},\frac{r}{0.6}\right),\left(\frac{p}{0.5},\frac{q}{0.4},\frac{r}{0.4}\right),\left(\frac{p}{0.3},\frac{q}{0.3},\frac{r}{0.2}\right)\right\rangle$

$D_3=\left\langle\left(\frac{p}{0.4},\frac{q}{0.6},\frac{r}{0.6}\right),\left(\frac{p}{0.3},\frac{q}{0.2},\frac{r}{0.2}\right),\left(\frac{p}{0.3},\frac{q}{0.3},\frac{r}{0.2}\right)\right\rangle$

$D_4=\left\langle\left(\frac{p}{0.3},\frac{q}{0.3},\frac{r}{0.2}\right),\left(\frac{p}{0.5},\frac{q}{0.4},\frac{r}{0.4}\right),\left(\frac{p}{0.4},\frac{q}{0.6},\frac{r}{0.6}\right)\right\rangle$

and $G_1=\left\langle\left(\frac{p}{0.4},\frac{q}{0.4},\frac{r}{0.4}\right),\left(\frac{p}{0.5},\frac{q}{0.4},\frac{r}{0.4}\right),\left(\frac{p}{0.3},\frac{q}{0.2},\frac{r}{0.2}\right)\right\rangle$

$G_2=\left\langle\left(\frac{p}{0.3},\frac{q}{0.2},\frac{r}{0.2}\right),\left(\frac{p}{0.3},\frac{q}{0.2},\frac{r}{0.2}\right),\left(\frac{p}{0.4},\frac{q}{0.6},\frac{r}{0.6}\right)\right\rangle$

$G_3=\left\langle\left(\frac{p}{0.4},\frac{q}{0.4},\frac{r}{0.4}\right),\left(\frac{p}{0.3},\frac{q}{0.2},\frac{r}{0.2}\right),\left(\frac{p}{0.3},\frac{q}{0.2},\frac{r}{0.2}\right)\right\rangle$

$G_4=\left\langle\left(\frac{p}{0.3},\frac{q}{0.2},\frac{r}{0.2}\right),\left(\frac{p}{0.5},\frac{q}{0.4},\frac{r}{0.4}\right),\left(\frac{p}{0.4},\frac{q}{0.6},\frac{r}{0.6}\right)\right\rangle$

Then the families $\tau=\{0_N,1_N,D_1,D_2,D_3,D_4\}$ and $\xi=\{0_N,1_N,G_1,G_2,G_3,G_4\}$ are (NTs) on S. Thus, (S,τ) and (S,ξ) are (NTSs). Define $T:(S,\tau)\rightarrow(S,\xi)$ as $T(p)=p,T(q)=q,T(r)=r$. Then T is N_{gs}-H but not $N_{\delta^* g\alpha}$-H. Hence in (S,τ),

$N_{\delta^* g\alpha}$-CS is $\left\langle\left(\frac{p}{0.4},\frac{q}{0.5},\frac{r}{0.5}\right),\left(\frac{p}{0.5},\frac{q}{0.5},\frac{r}{0.5}\right),\left(\frac{p}{0.4},\frac{q}{0.4},\frac{r}{0.3}\right)\right\rangle$ and

N_{gs}-CS is $\left\langle\left(\frac{p}{0.3},\frac{q}{0.2},\frac{r}{0.2}\right),\left(\frac{p}{0.3},\frac{q}{0.2},\frac{r}{0.2}\right),\left(\frac{p}{0.4},\frac{q}{0.4},\frac{r}{0.4}\right)\right\rangle$. Here $T^{-1}(G_3^c)$ is N_{gs}-CS but not $N_{\delta^* g\alpha}$-CS.

$N_{\delta^* g\alpha}$-OS is $\left\langle\left(\frac{p}{0.4},\frac{q}{0.4},\frac{r}{0.3}\right),\left(\frac{p}{0.5},\frac{q}{0.5},\frac{r}{0.5}\right),\left(\frac{p}{0.4},\frac{q}{0.5},\frac{r}{0.5}\right)\right\rangle$ and

N_{gs}-OS is $\left\langle\left(\frac{p}{0.4},\frac{q}{0.4},\frac{r}{0.4}\right),\left(\frac{p}{0.3},\frac{q}{0.2},\frac{r}{0.2}\right),\left(\frac{p}{0.3},\frac{q}{0.2},\frac{r}{0.2}\right)\right\rangle$ is N_{gs}-OS but not $N_{\delta^* g\alpha}$-OS.

5.4. *Theorem*

For any bijective map $T:(S,\tau)\longrightarrow(Y,\sigma)$ the following statement are equivalent.

(i) $T^{-1}:(Y,\tau)\longrightarrow(S,\sigma)$ is $N_{\delta^* g\alpha}$-CM.

(ii) T is an $N_{\delta^* g\alpha}$-OM.

(iii) T is an $N_{\delta^* g\alpha}$-CM.

Proof.

$(i) \Rightarrow (ii)$ Assume U is an (NOS)in (S,τ), then S-U is (NCS) in (S,τ) Since T^{-1} is $N_{\delta^*g\alpha}$-CM, then $\left(T^{-1}\right)^{-1}(U)$ is $N_{\delta^*g\alpha}$-CS in (Y,σ). that is $T(S-U)$ is $N_{\delta^*g\alpha}$-CS in (Y,σ), that is Y-$T(U)$ is $N_{\delta^*g\alpha}$-CS in (Y,σ). This implies that $T(U)$ is $N_{\delta^*g\alpha}$-OS in (Y,σ). Thus T is $N_{\delta^*g\alpha}$-OM.

$(ii) \Rightarrow (iii)$ Assume F is an (NCS) in (S,τ), then S-F is N-OS in (S,τ). Since T is $N_{\delta^*g\alpha}$-OM, then $T(S-F)$ is $N_{\delta^*g\alpha}$-OS in (Y,σ). That is Y-$T(F)$ is $N_{\delta^*g\alpha}$-OS in (Y,σ). This implies that $T(F)$ is $N_{\delta^*g\alpha}$-CS in (Y,σ). hence T is $N_{\delta^*g\alpha}$-CM.

$(iii) \Rightarrow (i)$ Assume K is an (NCS) in (S,τ), Since T is $N_{\delta^*g\alpha}$-CM, then $T(K)$ is $N_{\delta^*g\alpha}$-CS in (Y,σ). That is $[T^{-1}]^{-1}(K)$ is $N_{\delta^*g\alpha}$-CS in (Y,σ). hence T^{-1} is $N_{\delta^*g\alpha}$-CM.

5.5. *Theorem*

Assume $T:(S,\tau) \longrightarrow (Y,\sigma)$ is bijective and $N_{\delta^*g\alpha}$-CM. then the following statement are equivalent.

(i) T is an $N_{\delta^*g\alpha}$-OM.
(ii) T is an $N_{\delta^*g\alpha}$-H.
(iii) T is an $N_{\delta^*g\alpha}$-CM.

Proof.

$(i) \Rightarrow (ii)$ Assume T is an $N_{\delta^*g\alpha}$-OM. Since T is bijective and $N_{\delta^*g\alpha}$-CM, T is $N_{\delta^*g\alpha}$-H.

$(ii) \Rightarrow (iii)$ Assume T is an $N_{\delta^*g\alpha}$-H. Then T is $N_{\delta^*g\alpha}$-OM. If F is (NCS) in S, then $T(S-F)$ is $N_{\delta^*g\alpha}$-OS in (Y,σ). That is Y-$T(F)$ is $N_{\delta^*g\alpha}$-OS in (Y,σ). This implies that $T(F)$ is $N_{\delta^*g\alpha}$-CS in (Y,σ). hence T is $N_{\delta^*g\alpha}$-CM.

$(iii) \Rightarrow (i)$ Assume U is an (NOS) in (S,τ), Then S-U is (NCS) in (S,τ). Since T is $N_{\delta^*g\alpha}$-CS, then $T(S-U)$ is $N_{\delta^*g\alpha}$-CS in (Y,σ). That is $Y-T(U)$ is $N_{\delta^*g\alpha}$-CS in (Y,σ). Hence $T(U)$ is $N_{\delta^*g\alpha}$-OS in (Y,σ).

5.6. *Theorem*

The composition of two $N_{\delta^*g\alpha}$-Hs is also a $N_{\delta^*g\alpha}$-H.

Proof. Assume $T:(S,\tau) \longrightarrow (Y,\sigma)$ and $g:(Y,\sigma) \longrightarrow (Z,\eta)$ are two $N_{\delta^*g\alpha}$-CM. Assume U is a (NCS) in (Z,η). Since g is a $N_{\delta^*g\alpha}$-CM, $g^{-1}(U)$ is $N_{\delta^*g\alpha}$-CS in (Y,σ). Since any $N_{\delta^*g\alpha}$-CS is (NCS), $g^{-1}(U)$ is (NCS) in (Y,σ). Since T is a $N_{\delta^*g\alpha}$-CM, $T^{-1}\left(g^{-1}(U)\right) = goT(U)$ is $N_{\delta^*g\alpha}$-CS in (S,τ), therefore goT is also $N_{\delta^*g\alpha}$-CM.

Assume A is a (NCS) in (S,τ) then S-A is a (NOS)in (S,τ). Since T is $N_{\delta^*g\alpha}$-H, then $T(S-A)$ is a $N_{\delta^*g\alpha}$-OS in (Y,σ), implies $T(A)$ is $N_{\delta^*g\alpha}$-CS in (Y,σ). Since any $N_{\delta^*g\alpha}$-CS is (NCS), then $T(A)$ is (NCS) in (Y,σ), then $Y-T(A)$ is N-OS in (Y,σ). Since g is $N_{\delta^*g\alpha}$-H.

$g(Y-T(A))$ is $N_{\delta^*g\alpha}$-OS in (Z,η), implies $g(T(A)) = goT(A)$ is $N_{\delta^*g\alpha}$-CS in (Z,η) therefore goT is $N_{\delta^*g\alpha}$-CM and $N_{\delta^*g\alpha}$-OM, implies goT is $N_{\delta^*g\alpha}$-H.

5.7. *Definition*

A map $T:(S,\tau)\longrightarrow(Y,\sigma)$ is named $N_{\delta^*g\alpha}$c-H if T is bijective, T and T^{-1} are $N_{\delta^**g\alpha}$-IM.

5.8. *Theorem*

The composition of two $N_{\delta^*g\alpha}$c-Hs is also a $N_{\delta^*g\alpha}$c-H.

Proof. Assume $T:(S,\tau)\longrightarrow(Y,\sigma)$ and $g:(Y,\sigma)\longrightarrow(Z,\eta)$ are two $N_{\delta^*g\alpha}$c-Hs. Assume U is a $N_{\delta^*g\alpha}$-CS in (Z,η). Since g is a $N_{\delta^*g\alpha}$-IM, $g^{-1}(U)$ is $N_{\delta^*g\alpha}$-CS in (Y,σ). Since U is $N_{\delta^**g\alpha}$-IM, $T^{-1}\left(g^{-1}(U)\right)$ is $N_{\delta^*g\alpha}$-CS in (S,τ). that is $(goT)^{-1}(T)$ is $N_{\delta^*g\alpha}$-CS in (S,τ), therefore $goT:(Y,\sigma)\longrightarrow(Z,\eta)$ is $N_{\delta^*g\alpha}$-IM.
Assume G is a $N_{\delta^*g\alpha}$-CS in (S,τ), since T^{-1} is a $N_{\delta^*g\alpha}$-IM, $\left(T^{-1}\right)^{-1}(G)$ is $N_{\delta^*g\alpha}$-CS in (Y,σ), that is $T(G)$ is $N_{\delta^*g\alpha}$-CS in (Y,σ). Since g^{-1} is $N_{\delta^**g\alpha}$-IM, $\left(g^{-1}\right)^{-1}(T(G))$ is $N_{\delta^*g\alpha}$-CS in (Z,η), that is $g(T(G))$ is $N_{\delta^*g\alpha}$-CS in (Z,η), therefore $(goT)(G)$ is $N_{\delta^*g\alpha}$-CS in (Z,η). This implies that $\left((goT)^{-1}\right)^{-1}(G)$ is a $N_{\delta^*g\alpha}$-CS in (Z,η). This shows that $(goT)^{-1}:(Y,\sigma)\longrightarrow(Z,\eta)$ is $N_{\delta^**g\alpha}$-IM. Hence (goT) is a $N_{\delta^*g\alpha}$c-H.

5.9. *Theorem*

Any $N_{\delta^*g\alpha}$-H from a $N_{\alpha\delta T^{**}_{\frac{3}{4}}g\alpha}$-space into another $N_{\alpha\delta T^{**}_{\frac{3}{4}}g\alpha}$-space is a homeomorphism

Proof.

Assume $T:(S,\tau)\longrightarrow(Y,\sigma)$ is a $N_{\delta^*g\alpha}$-H. Then T is bijective, $N_{\delta^*g\alpha}$-OM and $N_{\delta^*g\alpha}$-CM. Assume U is an (NOS)in (S,τ). Since T is $N_{\delta^*g\alpha}$-OM and since (Y,σ) is $N_{\alpha\delta T^{**}_{\frac{3}{4}}g\alpha}$-space, $T(U)$ is (NOS)in (Y,σ). This implies that T is N-open map. Assume K is a (NCS) in (Y,σ), since T is $N_{\delta^*g\alpha}$-CM and since (S,τ) is $N_{\alpha\delta T^{**}_{\frac{3}{4}}g\alpha}$-space, $T^{-1}(K)$ is (NCS) in (S,τ). Therefore T is continuous. Hence T is a homeomorphism.

5.10. *Theorem*

Assume (Y,σ) is $N_{\alpha\delta T^{**}_{\frac{3}{4}}g\alpha}$-space. If $T:(S,\tau)\longrightarrow(Y,\sigma)$ and $g:(Y,\sigma)\longrightarrow(Z,\eta)$ are $N_{\delta^*g\alpha}$-H then (goT) is $N_{\delta^*g\alpha}$-H.

Proof. Assume $T:(S,\tau)\longrightarrow(Y,\sigma)$ and $g:(Y,\sigma)\longrightarrow(Z,\eta)$ are two $N_{\delta^*g\alpha}$-H. Assume U is an (NOS)in (S,τ). Since T is $N_{\delta^*g\alpha}$-OM, $T(U)$ is $N_{\delta^*g\alpha}$-OS in (Y,σ). Since (Y,σ) is $N_{\alpha\delta T^{**}_{\frac{3}{4}}g\alpha}$-space, $T(U)$ is N-OS in (Y,σ). Also since g is $N_{\delta^*g\alpha}$-OM, $g(T(U))$ is $N_{\delta^*g\alpha}$-OS in (Z,η). Hence goT is $N_{\delta^*g\alpha}$-OM. Assume v is a (NCS) in (Z,η). Since g is $N_{\delta^*g\alpha}$-CM and since (Y,σ) is $N_{\alpha\delta T^{**}_{\frac{3}{4}}g\alpha}$-space, $g^{-1}(V)$ is (NCS) in (Y,σ). Since T is $N_{\delta^*g\alpha}$-CM,

$T^{-1}\left(g^{-1}\left(V\right)\right) = \left(goT\right)^{-1}\left(V\right)$ is $N_{\delta^{*}g\alpha}$-CS in (S,τ). That is (goT) is $N_{\delta^{*}g\alpha}$-contnuous. Hence (goT) is $N_{\delta^{*}g\alpha}$-H.

5.11. *Theorem*

Any $N_{\delta^{*}g\alpha}$-H from $(N_{\alpha\delta T^{**}_{\frac{3}{4}}g\alpha}$-S)into another $(N_{\alpha\delta T^{**}_{\frac{3}{4}}g\alpha}$-S)is a $N_{\delta^{*}g\alpha}$c-H.

Proof. Assume $T:(S,\tau)\longrightarrow(Y,\sigma)$ is $N_{\delta^{*}g\alpha}$-H. Assume U be $N_{\delta^{*}g\alpha}$-CS in (Y,σ). Then U is (NCS) in (Y,σ). Since T is $N_{\delta^{*}g\alpha}$-CM, $T^{-1}\left(U\right)$ is $N_{\delta^{*}g\alpha}$-CS in (S,τ). Then T is a $N_{\delta^{*}g\alpha}$-IM. Let K be $N_{\delta^{*}g\alpha}$-OS in (S,τ). Then K is (NOS) in (S,τ). Since T is $N_{\delta^{*}g\alpha}$-OM, $T\left(K\right)$ is $N_{\delta^{*}g\alpha}$-OS in (Y,σ). That is $\left(T^{-1}\right)^{-1}\left(K\right)$ is $N_{\delta^{*}g\alpha}$-OS in (Y,σ) and hence T^{-1} is $N_{\delta^{*}*g\alpha}$-IM. Thus T is $N_{\delta^{*}g\alpha}$c-H.

6. **Conclusion**

The notions of $N_{\delta^{*}g\alpha}$-continuous and $N_{\delta^{*}g\alpha}$-irresolute functions in (NTS) are given in this work. Next, their characterizations and investigate their properties are analyzed. In future work, we will use the soft sets theory to investigate new classes of neutrosophic soft maps and then we can study these new classes of (NTS)in soft setting.

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Received: June 7, 2020. Accepted: Nov 25, 2020