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M. Vigneshwaran
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$N_{\delta^*g\alpha}$-Continuous and Irresolute Functions in Neutrosophic Topological Spaces

K. Damodharan$^1$, M. Vigneshwaran$^2$ and Shuker Khalil$^3$

$^1$Department of Mathematics, KPR Institute of Engineering and Technology (Autonomous), Coimbatore - 641407, India 1; catchmedamo@gmail.com
$^2$Department of Mathematics, Kongunadu Arts and Science College (Autonomous), Coimbatore - 641029, India 2; vignesh.mat@gmail.com
$^3$Department of Mathematics, College of Science, University of Basrah, Basrah 61004, Iraq
shuker.alsalem@gmail.com

*Correspondence: shuker.alsalem@gmail.com; Tel.: (+964 7713144239)

Abstract. In this paper, the notions of $N_{\delta^*g\alpha}$-continuous and $N_{\delta^*g\alpha}$-irresolute functions in neutrosophic topological spaces are given. Furthermore, we analyze their characterizations and investigate their properties.

Keywords: $N_{\delta^*g\alpha}$-closed set; $N_{\delta^*g\alpha}$-continuous; $N_{\delta^*g\alpha}$-irresolute; $N_{\delta^*g\alpha}$-homeomorphism; $N_{\delta^*g\alpha}$-c-homeomorphism.

1. Introduction

The notion of fuzzy set (FS) and its logic are investigated and discussed by Zadeh [12]. Next, Chang [3] studied the conception of fuzzy topological space (FTS). After that, Atanassav [8] investigated the intuitionistic fuzzy set (IFS) in 1986. Neutrosophy has extend the grounds for a total family of new mathematical estimations. It is one of the non-classical sets, like fuzzy, nano, soft, permutation sets and so on, see ([17]-[39]). The neutrosophic set (NS) was presented by Smarandache [6] and expounded, (NS) is a popularization of (IFS) in intuitionistic fuzzy topological space (IFTS) by coker [4]. In 2012 [1], the conception of neutrosophic topological space (NTS) is presented. Further the fundamental sets like semi/pre/α-open sets are presented in neutrosophic topological spaces (NTSs), see ([13]-[16]). The neutrosophic closed sets (NCSs) and neutrosophic continuous functions (NCFs) were presented by Salama et al. [2] in 2014. Arokiarani et al. [7] presented the neutrosophic α-closed set (NoCS) in (NTSs). The concepts of δ-closure are auxiliary tools in standard topology in
the study of H-closed spaces. Damodharan et al.[9,10] presented the idea of $N_\delta$-closure and $N_\delta$-Interior in (NTSs). Further, $N_\delta$-continuous and Neutrosophic almost continuous in (NTSs) were presented and established some of their related attributes. Recently Damodharan and Vigneshwaran [11] presented the conception of $N_\delta^*g\alpha$-closed sets in (NTSs) and studied some of its characteristics. In 2020, some applications of (NS) are applied by Abdel-Basset and others, see ([40]). In this work, we presented the $N_\delta^*g\alpha$-continuous functions and $N_\delta^*g\alpha$-irresolute functions in (NTSs). Furthermore, the conceptions of $N_\delta^*g\alpha$-homeomorphism and $N_\delta^*g\alpha$-c-homeomorphism are presented and investigate their characteristics.

2. Preliminaries

In this section, we mention some pertinent basic preliminaries about neutrosophic sets (NSs) and its operations.

2.1. Definition [1]

Assume S is a non-empty fixed set. A neutrosophic set (NS) P is an object having the form:

$P = \{s, \mu_m(P(s)), \sigma_i(P(s)), \nu_{nm}(P(s)) \forall s \in S\}$

where $\mu_m(P(s))$ represents the degree of membership, $\sigma_i(P(s))$ represents the degree of indeterminacy and $\nu_{nm}(P(s))$ represents the degree of nonmembership $\forall s \in S$ to P.

2.2. Remark [1]

A (NS) $P = \{s, \mu_m(P(s)), \sigma_i(P(s)), \nu_{nm}(P(s)) \forall s \in S\}$ can be identified to an ordered triple $\langle \mu_m(P(s)), \sigma_i(P(s)), \nu_{nm}(P(s)) \rangle$ in $]-1,1+$[ on S.

2.3. Definition [1]

In (NTS) We have:

$0_N$ may be defined as $\forall s \in S$

$0_N = \langle s, 0, 0, 0 \rangle$

$1_N$ may be defined as $\forall s \in S$

$1_N = \langle s, 1, 0, 0 \rangle$

$0_N = \langle s, 0, 1, 1 \rangle$

$0_N = \langle s, 0, 1, 0 \rangle$

$0_N = \langle s, 0, 0, 0 \rangle$

$0_N = \langle s, 0, 0, 1 \rangle$

$0_N = \langle s, 0, 0, 0 \rangle$

$0_N = \langle s, 0, 0, 1 \rangle$

$0_N = \langle s, 0, 0, 0 \rangle$

$0_N = \langle s, 0, 0, 0 \rangle$

$0_N = \langle s, 0, 0, 0 \rangle$

$0_N = \langle s, 0, 1, 0 \rangle$

$0_N = \langle s, 0, 0, 0 \rangle$

$0_N = \langle s, 0, 0, 0 \rangle$

$0_N = \langle s, 0, 0, 0 \rangle$
2.4. Definition [1]

Assume P is (NS) of the form:
\[ P = \{ \langle s, \mu_m(P(s)), \sigma_i(P(s)), \nu_{nm}(P(s)) \rangle \forall s \in S \}, \]
Then the complement of P \([P^c]\) may be defined as
\[ P^c = \{ \langle s, \nu_{nm}(P(s)), \sigma_i(P(s)), \mu_m(P(s)) \rangle \forall s \in S \}. \]

2.5. Definition [1]

Assume P and Q are two (NSs) of the form,
\[ P = \{ \langle s, \mu_m(P(s)), \sigma_i(P(s)), \nu_{nm}(P(s)) \rangle \forall s \in S \} \]
and
\[ Q = \{ \langle s, \mu_m(Q(s)), \sigma_i(Q(s)), \nu_{nm}(Q(s)) \rangle \forall s \in S \}. \]
Then,
1. Subsets \( P \subseteq Q \) may be defined as follows
   \[ P \subseteq Q \iff \mu_m(P(s)) \leq \mu_m(Q(s)), \sigma_i(P(s)) \geq \sigma_i(Q(s)), \nu_{nm}(P(s)) \geq \nu_{nm}(Q(s)) \]
2. Subsets \( P = Q \iff P \subseteq Q \) and \( Q \subseteq P \)
3. Union of subsets \( P \cup Q \) may be defined as follows
   \[ P \cup Q = \{ s, \max \{ \mu_m(P(s), \mu_m(Q(s))) \}, \min \{ \sigma_i(P(s), \sigma_i(Q(s))) \}, \min \{ \nu_{nm}(P(s), \nu_{nm}(Q(s)) \} \forall s \in S \}, \]
4. Intersection of subsets \( P \cap Q \) may be defined as follows
   \[ P \cap Q = \{ s, \min \{ \mu_m(P(s), \mu_m(Q(s))) \}, \max \{ \sigma_i(P(s), \sigma_i(Q(s))) \}, \max \{ \nu_{nm}(P(s), \nu_{nm}(Q(s)) \} \forall s \in S \}, \]

2.6. Proposition [9]

For any two (NSs) P and Q the following condition holds
1. \((P \cap Q)^c = P^c \cup Q^c,\)
2. \((P \cup Q)^c = P^c \cap Q^c,\)

2.7. Definition [1]

A neutrosophic topology (NT) on a non-empty set S is a family \( \tau \) of neutrosophic subsets in S satisfying the following axioms:
1. \( 0_N, 1_N \in \tau, \)
2. \( G_1 \cap G_2 \in \tau \) for any \( G_1, G_2 \in \tau, \)
3. \( \cup G_i \in \tau \forall \{ G_i : i \in J \} \subseteq \tau \)

Then the pair \((S, \tau)\) is named a neutrosopic topological space (NTS).

2.8. Definition [1]

Assume P is a (NS) in a (NTS) \((S, \tau)\). Then
Nint \( P = \bigcup \{ Q/Q \text{is a neutrosophic open set (NOS) in } (s, \tau) \text{ and } Q \subseteq P \} \) is named the neutrosophic interior of \( P \);

\( \text{ii): } Ncl \( P = \bigcap \{ Q/Q \text{is a neutrosophic closed set (NCS) in } (s, \tau) \text{ and } Q \supseteq P \} \) is named the neutrosophic closure of \( P \);

2.9. Definition [7]

A subset \( A \) of \( (S, \tau) \) is named

\( \text{i): } \) neutrosophic semi-open set (NSOS) if \( P \subseteq Ncl(Nint(P)) \).

\( \text{ii): } \) neutrosophic pre-open set (NPOS) if \( P \subseteq Nint(Ncl(P)) \).

\( \text{iii): } \) neutrosophic semi-preopen set (NSPOS) if \( P \subseteq Ncl(Nint(Ncl(P))) \).

\( \text{iv): } \) neutrosophic \( \alpha \)-open set (N\( \alpha \)OS) if \( P \subseteq Nint(Ncl(Nint(Ncl(P)))) \).

\( \text{v): } \) neutrosophic regular open set (NROS) if \( P = Nint(Ncl(P)) \).

The complement of a (NSOS) (resp. (NPOS), (NSPOS), (N\( \alpha \)OS), (NROS)) set is named (NSCS) (resp. (NPCS), (NSPCS), (N\( \alpha \)CS), (NRCS)).

2.10. Definition [9]

Assume \( \alpha, \beta, \lambda \in [0,1] \) and \( \alpha + \beta + \lambda \leq 3 \). A neutrosophic point \( s_{(\alpha, \beta, \lambda)} \) of \( S \) is a neutrosophic point (NP) of \( S \) which is clarified by

\[
\tag{1}
\begin{align*}
    s_{(\alpha, \beta, \lambda)}(y) = & \quad (\alpha, \beta, \lambda) \quad \text{when } y = s, \\
    & \quad (0,0,1) \quad \text{when } y \neq s.
\end{align*}
\]

Here, \( S \) is named the support of \( s_{(\alpha, \beta, \lambda)} \) and \( \alpha, \beta \) and \( \lambda \), respectively. A (NP) \( s_{(\alpha, \beta, \lambda)} \) is named belong to a (NS) \( P = \langle \mu_m(P(s)), \sigma_i(P(s)), \nu_{nm}(P(s)) \rangle \) in \( S \), denoted by \( s_{(\alpha, \beta, \lambda)} \in P \) if \( \alpha \leq \mu_m(P(s)), \beta \geq \sigma_i(P(s)) \) and \( \lambda \geq \nu_{nm}(P(s)) \) Clearly a (NP) can be represented by an ordered triple of (NP) as follows : \( s_{(\alpha, \beta, \lambda)} = (s_\alpha, s_\beta, s_\lambda) \).

2.11. Definition [9]

Assume \( (S, \tau) \) is a (NTS). Assume \( P \) is a (NS) and Assume \( s_{(\alpha, \beta, \lambda)} \) is a (NP). \( s_{(\alpha, \beta, \lambda)} \) is named neutrosophic quasi coincident with \( P \) [denoted by \( s_{(\alpha, \beta, \lambda)} qP \)] if \( \alpha + \mu_m(P(s)) > 1; \beta + \sigma_i(P(s)) < 1 \) and \( \lambda + \nu_{nm}(P(s)) < 1 \).

2.12. Definition [9]

Assume \( P \) and \( Q \) are two (NSs). \( P \) is named neutrosophic quasi coincident with \( Q \) [denoted by \( P qQ \)] if \( \mu_m(P(s)) + \mu_m(Q(s)) > 1; \sigma_i(P(s)) + \sigma_i(Q(s)) < 1 \) and \( \nu_{nm}(P(s)) + \nu_{nm}(Q(s)) < 1 \).
2.13. Definition [9]

Assume \( (S, \tau) \) is an \((NTS)\). An \((NP)\) \( s_{(\alpha, \beta, \lambda)} \) is named an neutroscopic \( \delta \)-cluster point of an \((NS)\) \( P \) if \( AqP \) for each neutroscopic regular open \( q \)-neighborhood \( A \) of \( s_{(\alpha, \beta, \lambda)} \). The set of all neutroscopic \( \delta \)-cluster points of \( P \) is named the neutroscopic \( \delta \)-closure of \( P \) denoted by \( Ncl_\delta (P) \). An \((NS)\) \( P \) is named an \( N_\delta \)-closed set \((N_\delta-CS)\) if \( P = Ncl_\delta (P) \). The complement of an \((N_\delta-CS)\) is named an \( N_\delta \)-open set \((N_\delta-OS)\).

3. \(N_\delta^{g_{\alpha}}\)-continuous functions

Here, some new conceptions are given by the authors.

3.1. Definition

A map \( T : (S, \tau) \rightarrow (Y, \sigma) \) is named a Neutrosophic delta star generalized alpha-continuous map(briefly \( N_\delta^{g_{\alpha}}-CM \)) if \( T^{-1}(K) \) is \( N_\delta^{g_{\alpha}}-CS \) in \((S, \tau)\) for any \((NCS)\) in \((Y, \sigma)\).

3.2. Theorem

Any \( N_\delta^{g_{\alpha}}-CM \) is \( N_{gs}-CM \) (resp \( N_{ag}-CM, \ N_{gsp}-CM, \ N_{gp}-CM \)). Also converse part is not true as shown through the following examples.

Proof. Assume \( K \) is a \((NCS)\) in \((Y, \sigma)\). Since \( T \) is \( N_\delta^{g_{\alpha}}-CM \), \( T^{-1}(K) \) is \( N_\delta^{g_{\alpha}}-CS \) in \((S, \tau)\). Since any \( N_\delta^{g_{\alpha}}-CS \) is \( N_{gs-CS} \) (resp \( N_{ag-CS}, \ N_{gsp-CS}, \ N_{gp-CS} \)), therefore \( T^{-1}(K) \) is \( N_{gs-CS} \) (resp \( N_{ag-CS}, \ N_{gsp-CS}, \ N_{gp-CS} \)) in \((S, \tau)\). Hence \( T \) is \( N_{gs-CS} \) (resp \( N_{ag-CS}, \ N_{gsp-CS}, \ N_{gp-CS} \)).

3.3. example

Assume \( S = \{ p, q, r \} \). Define the \((NSs)D_1, D_2, D_3, D_4\) and \( G_1, G_2, G_3, G_4 \) as follows:

\[
D_1 = \{ (\frac{p}{0.3}, q, r) , (\frac{p}{0.3}, q, r) , (\frac{p}{0.3}, q, r) \}
\]

\[
D_2 = \{ (\frac{p}{0.3}, q, r) , (\frac{p}{0.3}, q, r) , (\frac{p}{0.3}, q, r) \}
\]

\[
D_3 = \{ (\frac{p}{0.3}, q, r) , (\frac{p}{0.3}, q, r) , (\frac{p}{0.3}, q, r) \}
\]

\[
D_4 = \{ (\frac{p}{0.3}, q, r) , (\frac{p}{0.3}, q, r) , (\frac{p}{0.3}, q, r) \}
\]

and \( G_1 = \{ (\frac{p}{0.3}, q, r) , (\frac{p}{0.3}, q, r) , (\frac{p}{0.3}, q, r) \}
\]

\[
G_2 = \{ (\frac{p}{0.3}, q, r) , (\frac{p}{0.3}, q, r) , (\frac{p}{0.3}, q, r) \}
\]

\[
G_3 = \{ (\frac{p}{0.3}, q, r) , (\frac{p}{0.3}, q, r) , (\frac{p}{0.3}, q, r) \}
\]

\[
G_4 = \{ (\frac{p}{0.3}, q, r) , (\frac{p}{0.3}, q, r) , (\frac{p}{0.3}, q, r) \}
\]

Then the families \( \tau = \{ 0_N, 1_N, D_1, D_2, D_3, D_4 \} \) and \( \xi = \{ 0_N, 1_N, G_1, G_2, G_3, G_4 \} \) are neutroscopic topologies \((NTs)\) on \( S \). Thus, \((S, \tau)\) and \((S, \xi)\) are \((NTSs)\). Define \( T : (S, \tau) \rightarrow (S, \xi) \) as \( T(p) = p, T(q) = q, T(r) = r \). Then \( T \) is \( N_{gs-CS} \) but not \( N_\delta^{g_{\alpha}}-CM \). Hence in \((S, \tau)\),

\( N_\delta^{g_{\alpha}}-CS \) is \( \{ (\frac{p}{0.4}, q, r) , (\frac{p}{0.5}, q, r) , (\frac{p}{0.4}, q, r) \}
\] and

\( N_\delta^{g_{\alpha}}-Continuous \) and \( Irresolute \) \( Functions \) in \( Neutrosophic \) \( Topological \) \( Spacese \).
$N_{g\delta}\text{-CS}$ is $\langle (\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.2}), (\frac{p}{0.2}, \frac{q}{0.2}, \frac{r}{0.2}), (\frac{p}{0.4}, \frac{q}{0.4}, \frac{r}{0.4}) \rangle$. Here $T^{-1}(G_3^c)$ is $N_{g\delta}\text{-CS}$ but not $N_{\delta^*g\alpha}\text{-CS}$.

3.4. example

Assume $S = \{p, q, r\}$. Define the (NSs) $D_1, D_2, D_3, D_4$ and $H_1, H_2, H_3, H_4$ as follows:

$D_1 = \langle (\frac{p}{0.3}, \frac{q}{0.3}, \frac{r}{0.2}), (\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.2}), (\frac{p}{0.4}, \frac{q}{0.4}, \frac{r}{0.6}) \rangle$

$D_2 = \langle (\frac{p}{0.4}, \frac{q}{0.3}, \frac{r}{0.5}), (\frac{p}{0.3}, \frac{q}{0.3}, \frac{r}{0.1}), (\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.2}) \rangle$

$D_3 = \langle (\frac{p}{0.3}, \frac{q}{0.3}, \frac{r}{0.2}), (\frac{p}{0.3}, \frac{q}{0.3}, \frac{r}{0.2}), (\frac{p}{0.3}, \frac{q}{0.3}, \frac{r}{0.2}) \rangle$

$D_4 = \langle (\frac{p}{0.3}, \frac{q}{0.3}, \frac{r}{0.2}), (\frac{p}{0.3}, \frac{q}{0.3}, \frac{r}{0.2}), (\frac{p}{0.3}, \frac{q}{0.3}, \frac{r}{0.2}) \rangle$

and $H_1 = \langle (\frac{p}{0.3}, \frac{q}{0.3}, \frac{r}{0.2}), (\frac{p}{0.3}, \frac{q}{0.3}, \frac{r}{0.2}), (\frac{p}{0.3}, \frac{q}{0.3}, \frac{r}{0.4}) \rangle$

$H_2 = \langle (\frac{p}{0.3}, \frac{q}{0.3}, \frac{r}{0.2}), (\frac{p}{0.3}, \frac{q}{0.3}, \frac{r}{0.2}), (\frac{p}{0.3}, \frac{q}{0.3}, \frac{r}{0.4}) \rangle$

$H_3 = \langle (\frac{p}{0.3}, \frac{q}{0.3}, \frac{r}{0.2}), (\frac{p}{0.3}, \frac{q}{0.3}, \frac{r}{0.3}), (\frac{p}{0.3}, \frac{q}{0.3}, \frac{r}{0.4}) \rangle$

$H_4 = \langle (\frac{p}{0.3}, \frac{q}{0.3}, \frac{r}{0.2}), (\frac{p}{0.3}, \frac{q}{0.3}, \frac{r}{0.2}), (\frac{p}{0.3}, \frac{q}{0.3}, \frac{r}{0.4}) \rangle$

Then the families $\tau = \{0_N, 1_N, D_1, D_2, D_3, D_4\}$ and $\xi = \{0_N, 1_N, H_1, H_2, H_3, H_4\}$ are (NTs) on $S$. Thus, $(S, \tau)$ and $(S, \psi)$ are (NTs). Define $T : (S, \tau) \to (S, \psi)$ as $T(p) = p, T(q) = q, T(r) = r$. Then $T$ is $N_{g\delta}\text{-CM}$ but not $N_{\delta^*g\alpha}\text{-CM}$. Hence in $(S, \tau), N_{\delta^*g\alpha}\text{-CS}$ is $\langle (\frac{p}{0.4}, \frac{q}{0.5}, \frac{r}{0.5}), (\frac{p}{0.3}, \frac{q}{0.5}, \frac{r}{0.7}), (\frac{p}{0.1}, \frac{q}{0.4}, \frac{r}{0.6}) \rangle$ and

$N_{g\delta}\text{-CS}$ is $\langle (\frac{p}{0.3}, \frac{q}{0.5}, \frac{r}{0.5}), (\frac{p}{0.4}, \frac{q}{0.3}, \frac{r}{0.3}), (\frac{p}{0.3}, \frac{q}{0.3}, \frac{r}{0.2}) \rangle$. Here $T^{-1}(H_3^c)$ is $N_{g\delta}\text{-CS}$ but not $N_{\delta^*g\alpha}\text{-CS}$.

3.5. example

Assume $Y = \{u, v, w\}$. Define the (NSs) $F_1, F_2, F_3, F_4$ and $I_1, I_2, I_3, I_4$ as follows:

$F_1 = \langle (\frac{p}{0.3}, \frac{q}{0.3}, \frac{r}{0.2}), (\frac{p}{0.2}, \frac{q}{0.2}, \frac{r}{0.2}), (\frac{p}{0.4}, \frac{q}{0.4}, \frac{r}{0.6}) \rangle$

$F_2 = \langle (\frac{p}{0.4}, \frac{q}{0.3}, \frac{r}{0.6}), (\frac{p}{0.3}, \frac{q}{0.4}, \frac{r}{0.4}), (\frac{p}{0.3}, \frac{q}{0.3}, \frac{r}{0.3}) \rangle$

$F_3 = \langle (\frac{p}{0.4}, \frac{q}{0.6}, \frac{r}{0.6}), (\frac{p}{0.4}, \frac{q}{0.4}, \frac{r}{0.4}), (\frac{p}{0.4}, \frac{q}{0.4}, \frac{r}{0.3}) \rangle$

$F_4 = \langle (\frac{p}{0.3}, \frac{q}{0.3}, \frac{r}{0.2}), (\frac{p}{0.3}, \frac{q}{0.3}, \frac{r}{0.2}), (\frac{p}{0.3}, \frac{q}{0.3}, \frac{r}{0.2}) \rangle$

and $I_1 = \langle (\frac{p}{0.3}, \frac{q}{0.5}, \frac{r}{0.6}), (\frac{p}{0.5}, \frac{q}{0.6}, \frac{r}{0.6}), (\frac{p}{0.4}, \frac{q}{0.5}, \frac{r}{0.3}) \rangle$

$I_2 = \langle (\frac{p}{0.4}, \frac{q}{0.5}, \frac{r}{0.3}), (\frac{p}{0.3}, \frac{q}{0.4}, \frac{r}{0.4}), (\frac{p}{0.4}, \frac{q}{0.3}, \frac{r}{0.6}) \rangle$

$I_3 = \langle (\frac{p}{0.4}, \frac{q}{0.5}, \frac{r}{0.3}), (\frac{p}{0.4}, \frac{q}{0.4}, \frac{r}{0.4}), (\frac{p}{0.4}, \frac{q}{0.5}, \frac{r}{0.3}) \rangle$

$I_4 = \langle (\frac{p}{0.4}, \frac{q}{0.5}, \frac{r}{0.1}), (\frac{p}{0.5}, \frac{q}{0.6}, \frac{r}{0.6}), (\frac{p}{0.3}, \frac{q}{0.5}, \frac{r}{0.6}) \rangle$

Then the families $\theta = \{0_N, 1_N, F_1, F_2, F_3, F_4\}$ and $\zeta = \{0_N, 1_N, I_1, I_2, I_3, I_4\}$ are (NTs) on $Y$. Thus, $(Y, \theta)$ and $(Y, \zeta)$ are (NTs). Define $g : (Y, \theta) \to (Y, \zeta)$ as $g(u) = u, g(v) = v, g(w) = w$. Then $g$ is $N_{g\varpi}\text{-CM}$ but not $N_{\delta^*g\alpha}\text{-CM}$. Hence in $(Y, \theta), N_{\delta^*g\alpha}\text{-CS}$ is $\langle (\frac{p}{0.4}, \frac{q}{0.3}, \frac{r}{0.6}), (\frac{p}{0.3}, \frac{q}{0.5}, \frac{r}{0.5}), (\frac{p}{0.1}, \frac{q}{0.4}, \frac{r}{0.6}) \rangle$ and

$N_{g\varpi}\text{-CS}$ is $\langle (\frac{p}{0.4}, \frac{q}{0.6}, \frac{r}{0.3}), (\frac{p}{0.4}, \frac{q}{0.4}, \frac{r}{0.4}), (\frac{p}{0.3}, \frac{q}{0.3}, \frac{r}{0.2}) \rangle$. Here $g^{-1}(I_3^c)$ is $N_{g\varpi}\text{-CS}$ but not $N_{\delta^*g\alpha}\text{-CS}$.

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3.6. **example**

Assume \( Y = \{u, v, w\} \). Define the (NSs) \( F_1, F_2, F_3, F_4 \) and \( J_1, J_2, J_3, J_4 \) as follows:

\[
F_1 = (\left( \frac{p}{0.3}, \frac{q}{0.3}, \frac{r}{0.7} \right), (\frac{p}{0.4}, \frac{q}{0.4}, \frac{r}{0.4}), (\frac{p}{0.6}, \frac{q}{0.6}, \frac{r}{0.6}))\\
F_2 = (\left( \frac{p}{0.4}, \frac{q}{0.6}, \frac{r}{0.6} \right), (\frac{p}{0.5}, \frac{q}{0.4}, \frac{r}{0.4}), (\frac{p}{0.4}, \frac{q}{0.4}, \frac{r}{0.3}))\\
F_3 = (\left( \frac{p}{0.5}, \frac{q}{0.6}, \frac{r}{0.7} \right), (\frac{p}{0.7}, \frac{q}{0.4}, \frac{r}{0.4}), (\frac{p}{0.4}, \frac{q}{0.4}, \frac{r}{0.3}))\\
F_4 = (\left( \frac{p}{0.3}, \frac{q}{0.3}, \frac{r}{0.7} \right), (\frac{p}{0.5}, \frac{q}{0.4}, \frac{r}{0.4}), (\frac{p}{0.6}, \frac{q}{0.6}, \frac{r}{0.6}))\\
\]

and 

\[
J_1 = (\left( \frac{p}{0.3}, \frac{q}{0.6}, \frac{r}{0.3} \right), (\frac{p}{0.4}, \frac{q}{0.3}, \frac{r}{0.3}), (\frac{p}{0.6}, \frac{q}{0.4}, \frac{r}{0.3})),\\
J_2 = (\left( \frac{p}{0.4}, \frac{q}{0.4}, \frac{r}{0.7} \right), (\frac{p}{0.4}, \frac{q}{0.3}, \frac{r}{0.2}), (\frac{p}{0.4}, \frac{q}{0.4}, \frac{r}{0.3})),\\
J_3 = (\left( \frac{p}{0.5}, \frac{q}{0.3}, \frac{r}{0.4} \right), (\frac{p}{0.4}, \frac{q}{0.4}, \frac{r}{0.4}), (\frac{p}{0.6}, \frac{q}{0.4}, \frac{r}{0.4})),\\
J_4 = (\left( \frac{p}{0.3}, \frac{q}{0.6}, \frac{r}{0.5} \right), (\frac{p}{0.4}, \frac{q}{0.3}, \frac{r}{0.3}), (\frac{p}{0.4}, \frac{q}{0.4}, \frac{r}{0.3})),
\]

Then the families \( \vartheta = \{0_N, 1_N, F_1, F_2, F_3, F_4\} \) and \( \zeta = \{0_N, 1_N, J_1, J_2, J_3, J_4\} \) are (NTs) on \( Y \).

Thus, \( (Y, \vartheta) \) and \( (Y, \varphi) \) are (NTSs). Define \( g : (Y, \vartheta) \rightarrow (Y, \varphi) \) as \( g (u) = u, g (v) = w, g (w) = v \). Then \( g \) is \( N_{gsp} \)-C but not \( N_{\delta^* \varphi^* \alpha} \)-C. Hence in \( (Y, \vartheta) \),

\[
N_{\delta^* \varphi^* \alpha} \text{-CS is } (\left( \frac{p}{0.4}, \frac{q}{0.5}, \frac{r}{0.5} \right), (\frac{p}{0.6}, \frac{q}{0.5}, \frac{r}{0.5})), (\frac{p}{0.7}, \frac{q}{0.4}, \frac{r}{0.4})), \text{ and } N_{gsp} \text{-CS is } (\left( \frac{p}{0.4}, \frac{q}{0.6}, \frac{r}{0.5} \right), (\frac{p}{0.6}, \frac{q}{0.4}, \frac{r}{0.4})), (\frac{p}{0.4}, \frac{q}{0.4}, \frac{r}{0.3})). \text{ Here } g^{-1}(J_3) \text{ is } N_{gsp} \text{-CS but not } N_{\delta^* \varphi^* \alpha} \text{-CS}.
\]

3.7. **Theorem**

The composition of two \( N_{\delta^* \varphi^* \alpha} \)-CMs is also a \( N_{\delta^* \varphi^* \alpha} \)-CM. Proof. Assume \( T : (S, \tau) \rightarrow (Y, \sigma) \) and \( g : (Y, \sigma) \rightarrow (Z, \eta) \) are two \( N_{\delta^* \varphi^* \alpha} \)-CMs. Assume \( l \) is a NCS in \( (Z, \eta) \). Since \( g \) is a \( N_{\delta^* \varphi^* \alpha} \)-CM, \( g^{-1}(l) \) is \( N_{\delta^* \varphi^* \alpha} \)-CS in \( (Y, \sigma) \). Since any \( N_{\delta^* \varphi^* \alpha} \)-CS is NCS, \( g^{-1}(l) \) is NCS in \( (Y, \sigma) \). Since \( T \) is a \( N_{\delta^* \varphi^* \alpha} \)-CM, \( T^{-1}(g^{-1}(l)) = goT(l) \) is \( N_{\delta^* \varphi^* \alpha} \)-CS in \( (S, \tau) \), therefore \( goT \) is also \( N_{\delta^* \varphi^* \alpha} \)-CM.

4. **\( N_{\delta^* \varphi^* \alpha} \)-Irresolute functions**

Here, some new conceptions are given by the authors.

4.1. **Definition**

A map \( T : (S, \tau) \rightarrow (Y, \sigma) \) is named a Neutrosophic delta star generalized alpha-Irresolute map (briefly \( N_{\delta^* \varphi^* \alpha} \)-IMM) if \( T^{-1}(K) \) is \( N_{\delta^* \varphi^* \alpha} \)-CS in \( (S, \tau) \) for any \( N_{\delta^* \varphi^* \alpha} \)-CS in \( (Y, \sigma) \).

4.2. **Theorem**

Assume \( T : (S, \tau) \rightarrow (Y, \sigma) \) and \( g : (Y, \sigma) \rightarrow (Z, \eta) \) are any two functions, then

(i) \( goT : (S, \tau) \rightarrow (Z, \eta) \) is \( N_{\delta^* \varphi^* \alpha} \)-CM if \( g \) is \( N \)-CM and \( T \) is \( N_{\delta^* \varphi^* \alpha} \)-CM.

(ii) \( goT : (S, \tau) \rightarrow (Z, \eta) \) is \( N_{\delta^* \varphi^* \alpha} \)-IM if both \( g \) and \( T \) \( N_{\delta^* \varphi^* \alpha} \)-IM.

(iii) \( goT : (S, \tau) \rightarrow (Z, \eta) \) is \( N_{\delta^* \varphi^* \alpha} \)-CM if \( g \) is \( N_{\delta^* \varphi^* \alpha} \)-CM and \( T \) is \( N_{\delta^* \varphi^* \alpha} \)-IM.

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4.3. **Theorem**

Assume \( T : (S, \tau) \rightarrow (Y, \sigma) \) is \( N_{\delta^{*}} \)-CM (\( N_{g^{*}} \)-CM, \( N_{a^{*}} \)-CM, \( N_{g} \)-CM). If \((S, \tau)\) is an \( N_{a_{1}}T_{3/2}^{*} \)-space (\( N_{a_{1}T_{1}^{*}} \)-space, \( N_{a_{1}T_{c}^{*}} \)-space, \( N_{a_{1}T_{c}^{**}} \)-space) then \( T \) is continuous.

Proof. Assume \( K \) is a \( N_{\delta^{*}} \)-CM of \((Z, \eta)\). Since \( g \) is \( N_{g^{*}} \)-CM, \( g^{-1}(K) \) is \( N_{\delta^{*}} \)-CM in \((Y, \sigma)\). Since \( T \) is \( N_{\delta^{*}} \)-CM, \( T^{-1}(g^{-1}(K)) = (goT)^{-1}(K) \) is \( N_{\delta^{*}} \)-CS in \((S, \tau)\), Therefore \( goT \) is \( N_{\delta^{*}} \)-CM.

(ii) Assume \( K \) is a \( N_{\delta^{*}} \)-CS in \((Z, \eta)\). Since \( g \) is \( N_{\delta^{*}} \)-IM, \( g^{-1}(K) \) is \( N_{\delta^{*}} \)-CS in \((Y, \sigma)\). Since \( T \) is \( N_{\delta^{*}} \)-IM, \( T^{-1}(g^{-1}(K)) = (goT)^{-1}(K) \) is \( N_{\delta^{*}} \)-CS in \((S, \tau)\), Therefore \( goT \) is \( N_{\delta^{*}} \)-IM.

(iii) Assume \( K \) is a \( N_{\delta^{*}} \)-CS in \((Z, \eta)\). Since \( g \) is \( N_{\delta^{*}} \)-CM, \( g^{-1}(K) \) is \( N_{\delta^{*}} \)-CS in \((Y, \sigma)\). Since \( T \) is \( N_{\delta^{*}} \)-CS, \( T^{-1}(g^{-1}(K)) = (goT)^{-1}(K) \) is \( N_{\delta^{*}} \)-CS in \((S, \tau)\), Therefore \( goT \) is \( N_{\delta^{*}} \)-CM.

4.4. **Theorem**

Assume \( T : (S, \tau) \rightarrow (Y, \sigma) \) is a surjective, \( N_{g^{*}} \)-IM and \( N_{\delta^{*}} \)-CM. Then \( T(A) \) is \( N_{\delta^{*}} \)-CS of \((Y, \sigma)\) for any \( N_{\delta^{*}} \)-CS \( A \) of \((S, \tau)\).

Proof. Assume \( A \) is a \( N_{\delta^{*}} \)-CS of \((S, \tau)\). Assume \( U \) is a \( N_{g^{*}} \)-OS of \((Y, \sigma)\) such that \( T(A) \subseteq U \). Since \( T \) is surjective and \( N_{g^{*}} \)-IM, \( T^{-1}(U) \) is \( N_{g^{*}} \)-OS in \((S, \tau)\). Since \( A \subseteq T^{-1}(U) \) and \( A \) is \( N_{\delta^{*}} \)-CS of \((S, \tau)\), \( Ncl_{\delta}(A) \subseteq T^{-1}(U) \). Then \( T[Ncl_{\delta}(A)] \subseteq T[T^{-1}(U)] = U \), since \( T \) is \( N_{\delta^{*}} \)-CS, \( T[Ncl_{\delta}(A)] = Ncl_{\delta}[T[Ncl_{\delta}(A)]] \). This implies \( Ncl_{\delta}[T(A)] \subseteq Ncl_{\delta}[T[Ncl_{\delta}(A)]] = T[Ncl_{\delta}(A)] \subseteq U \), Therefore \( T(A) \) is a \( N_{\delta^{*}} \)-CS of \((Y, \sigma)\).

4.5. **Theorem**

Assume \( T : (S, \tau) \rightarrow (Y, \sigma) \) is a surjective, \( N_{\delta^{*}} \)-IM and \( N_{\delta^{*}} \)-CM. If \((S, \tau)\) is an \( N_{a_{1}}T_{3/4}^{*} \)-space, then \((Y, \sigma)\) is also an \( N_{a_{1}}T_{3/4}^{*} \)-space.

Proof. Assume \( A \) is a \( N_{\delta^{*}} \)-CS of \((Y, \sigma)\). Since \( T \) is \( N_{\delta^{*}} \)-IM, \( T^{-1}(A) \) is \( N_{\delta^{*}} \)-CS in \((S, \tau)\). Since \((S, \tau)\) is an \( N_{a_{1}}T_{3/4}^{*} \)-space, \( T^{-1}(A) \) is \( N_{\delta^{*}} \)-CS of \((S, \tau)\). Since \( T \) is \( N_{\delta^{*}} \)-CM and \( T^{-1}(A) \) is \( N_{\delta^{*}} \)-CS in \((S, \tau)\). Thus \( A \) is \( N_{\delta^{*}} \)-CS in \((Y, \sigma)\). Therefore \((Y, \sigma)\) is an \( N_{a_{1}}T_{3/4}^{*} \)-space.
5. $N_{\delta^* g\alpha}$-Homeomorphism

Here, some new conceptions are given by the authors.

5.1. Definition

A map $T : (S, \tau) \rightarrow (Y, \sigma)$ is named a neutrosophic delta star generalized alpha-homeomorphism (briefly $N_{\delta^* g\alpha}$-H) if $T$ is bijective, $N_{\delta^* g\alpha}$-CM and $N_{\delta^* g\alpha}$-OM.

5.2. Theorem

Any $N_{\delta^* g\alpha}$-H is $N_{gs}$-H.

Proof. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be $N_{\delta^* g\alpha}$-H then $f$ is bijective, $N_{\delta^* g\alpha}$-continuous and $N_{\delta^* g\alpha}$-OM. Let $V$ be N-CS in $(X, \sigma)$, then $f^{-1}(V)$ is $N_{\delta^* g\alpha}$-CS in $(X, \tau)$. Since every $N_{\delta^* g\alpha}$-CS is $N_{gs}$-CS, then $f^{-1}(V)$ is $N_{gs}$-CS in $(X, \tau)$, Therefore $f$ is $N_{gs}$-continuous. Let $U$ be N-OS in $(X, \tau)$, then $f(U)$ is $N_{\delta^* g\alpha}$-OS in $(Y, \sigma)$. Since every $N_{\delta^* g\alpha}$-OS is $N_{gs}$-OS, then $f(U)$ is $N_{gs}$-OS in $(Y, \sigma)$, Therefore $f$ is $N_{gs}$-OM. Hence $f$ is $N_{gs}$-H.

5.3. Example

Assume $S = \{p, q, r\}$. Define the (NSs)$D_1, D_2, D_3, D_4$ and $G_1, G_2, G_3, G_4$ as follows:

$D_1 = \langle (p_{0.3}, q_{0.5}, r_{0.2}), (p_{0.3}, q_{0.5}, r_{0.2}), (p_{0.3}, q_{0.5}, r_{0.2}) \rangle$

$D_2 = \langle (p_{0.4}, q_{0.6}, r_{0.4}), (p_{0.5}, q_{0.4}, r_{0.3}), (p_{0.5}, q_{0.4}, r_{0.3}) \rangle$

$D_3 = \langle (p_{0.4}, q_{0.6}, r_{0.4}), (p_{0.4}, q_{0.6}, r_{0.4}), (p_{0.4}, q_{0.6}, r_{0.4}) \rangle$

$D_4 = \langle (p_{0.4}, q_{0.6}, r_{0.4}), (p_{0.4}, q_{0.6}, r_{0.4}), (p_{0.4}, q_{0.6}, r_{0.4}) \rangle$

and $G_1 = \langle (p_{0.2}, q_{0.4}, r_{0.2}), (p_{0.2}, q_{0.4}, r_{0.2}), (p_{0.2}, q_{0.4}, r_{0.2}) \rangle$

$G_2 = \langle (p_{0.3}, q_{0.6}, r_{0.3}), (p_{0.3}, q_{0.6}, r_{0.3}), (p_{0.3}, q_{0.6}, r_{0.3}) \rangle$

$G_3 = \langle (p_{0.4}, q_{0.4}, r_{0.2}), (p_{0.4}, q_{0.4}, r_{0.2}), (p_{0.4}, q_{0.4}, r_{0.2}) \rangle$

$G_4 = \langle (p_{0.4}, q_{0.4}, r_{0.2}), (p_{0.4}, q_{0.4}, r_{0.2}), (p_{0.4}, q_{0.4}, r_{0.2}) \rangle$

Then the families $\tau = \{0_N, 1_N, D_1, D_2, D_3, D_4\}$ and $\xi = \{0_N, 1_N, G_1, G_2, G_3, G_4\}$ are (NTs) on $S$. Thus, $(S, \tau)$ and $(S, \xi)$ are (NTs). Define $T : (S, \tau) \rightarrow (S, \xi)$ as $T(p) = p, T(q) = q, T(r) = r$. Then $T$ is $N_{gs}$-H but not $N_{\delta^* g\alpha}$-H. Hence in $(S, \tau)$,

$N_{\delta^* g\alpha}$-CS is $\langle (p_{0.4}, q_{0.5}, r_{0.5}), (p_{0.5}, q_{0.5}, r_{0.5}), (p_{0.4}, q_{0.4}, r_{0.6}) \rangle$ and

$N_{gs}$-CS is $\langle (p_{0.3}, q_{0.2}, r_{0.2}), (p_{0.3}, q_{0.2}, r_{0.2}), (p_{0.3}, q_{0.2}, r_{0.2}) \rangle$. Here $T^{-1}(G_3)$ is $N_{gs}$-CS but not $N_{\delta^* g\alpha}$-CS.

$N_{\delta^* g\alpha}$-OS is $\langle (p_{0.4}, q_{0.4}, r_{0.4}), (p_{0.5}, q_{0.5}, r_{0.5}), (p_{0.6}, q_{0.6}, r_{0.6}) \rangle$ and

$N_{gs}$-OS is $\langle (p_{0.4}, q_{0.4}, r_{0.4}), (p_{0.4}, q_{0.4}, r_{0.4}), (p_{0.4}, q_{0.4}, r_{0.4}) \rangle$ is $N_{gs}$-OS but not $N_{\delta^* g\alpha}$-OS.

5.4. Theorem

For any bijective map $T : (S, \tau) \rightarrow (Y, \sigma)$ the following statement are equivalent.

(i) $T^{-1} : (Y, \tau) \rightarrow (S, \sigma)$ is $N_{\delta^* g\alpha}$-CM.

(ii) $T$ is an $N_{\delta^* g\alpha}$-OM.

(iii) $T$ is an $N_{\delta^* g\alpha}$-CM.

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5.5. Theorem

Assume \( T : (S, \tau) \rightarrow (Y, \sigma) \) is bijective and \( N_{\delta^*g\alpha} \)-CM. then the following statement are equivalent.

(i) \( T \) is an \( N_{\delta^*g\alpha} \)-OM.

(ii) \( T \) is an \( N_{\delta^*g\alpha} \)-H.

(iii) \( T \) is an \( N_{\delta^*g\alpha} \)-CM.

Proof.

(i) \( \Rightarrow \) (ii) Assume \( T \) is an \( N_{\delta^*g\alpha} \)-OM. Since \( T \) is bijective and \( N_{\delta^*g\alpha} \)-CM, \( T \) is \( N_{\delta^*g\alpha} \)-H.

(ii) \( \Rightarrow \) (iii) Assume \( T \) is an \( N_{\delta^*g\alpha} \)-H. Then \( T \) is an \( N_{\delta^*g\alpha} \)-OM. If \( F \) is \( (NCS) \) in \( S \), then \( T (S - F) \) is \( N_{\delta^*g\alpha} \)-OS in \( (Y, \sigma) \). That is \( Y - T (F) \) is \( N_{\delta^*g\alpha} \)-OS in \( (Y, \sigma) \). This implies that \( T (F) \) is \( N_{\delta^*g\alpha} \)-CS in \( (Y, \sigma) \). hence \( T \) is \( N_{\delta^*g\alpha} \)-CM.

(iii) \( \Rightarrow \) (i) Assume \( U \) is an \( (NOS) \) in \( (S, \tau) \), Then \( S - U \) is \( (NCS) \) in \( (S, \tau) \). Since \( T \) is \( N_{\delta^*g\alpha} \)-CS, then \( T (S - U) \) is \( N_{\delta^*g\alpha} \)-CS in \( (Y, \sigma) \). That is \( Y - T (U) \) is \( N_{\delta^*g\alpha} \)-CS in \( (Y, \sigma) \). Hence \( T (U) \) is \( N_{\delta^*g\alpha} \)-OS in \( (Y, \sigma) \).

5.6. Theorem

The composition of two \( N_{\delta^*g\alpha} \)-Hs is also a \( N_{\delta^*g\alpha} \)-H.

Proof. Assume \( T : (S, \tau) \rightarrow (Y, \sigma) \) and \( g : (Y, \sigma) \rightarrow (Z, \eta) \) are two \( N_{\delta^*g\alpha} \)-CM. Assume \( U \) is a \( (NCS) \) in \( (Z, \eta) \). Since \( g \) is a \( N_{\delta^*g\alpha} \)-CM, \( g^{-1} (U) \) is \( N_{\delta^*g\alpha} \)-CS in \( (Y, \sigma) \). Since any \( N_{\delta^*g\alpha} \)-CS is \( (NCS) \), \( g^{-1} (U) \) is \( (NCS) \) in \( (Y, \sigma) \). Since \( T \) is a \( N_{\delta^*g\alpha} \)-CM, \( T^{-1} (g^{-1} (U)) \) is \( gT (U) \) is \( N_{\delta^*g\alpha} \)-CS in \( (S, \tau) \), therefore \( gT \) is also \( N_{\delta^*g\alpha} \)-CM.

Assume \( A \) is a \( (NCS) \) in \( (S, \tau) \) then \( S - A \) is a \( (NOS) \) in \( (S, \tau) \). Since \( T \) is \( N_{\delta^*g\alpha} \)-H, then \( T (S - A) \) is \( N_{\delta^*g\alpha} \)-OS in \( (Y, \sigma) \), implies \( T (A) \) is \( N_{\delta^*g\alpha} \)-CS in \( (Y, \sigma) \). Since any \( N_{\delta^*g\alpha} \)-CS is \( (NCS) \), then \( T (A) \) is \( (NCS) \) in \( (Y, \sigma) \), then \( Y - T (A) \) is \( N \)-OS in \( (Y, \sigma) \). Since \( g \) is \( N_{\delta^*g\alpha} \)-H.
$g\left(Y - T\left(A\right)\right)$ is $N_{\delta^{*}ga}$-OS in $(Z, \eta)$, implies $g\left(T\left(A\right)\right) = goT\left(A\right)$ is $N_{\delta^{*}ga}$-CS in $(Z, \eta)$ therefore $goT$ is $N_{\delta^{*}ga}$-CM and $N_{\delta^{*}ga}$-OM, implies $goT$ is $N_{\delta^{*}ga}$-H.

5.7. Definition

A map $T : (S, \tau) \rightarrow (Y, \sigma)$ is named $N_{\delta^{*}ga}$-H if $T$ is bijective, $T$ and $T^{-1}$ are $N_{\delta^{*}ga}$-IM.

5.8. Theorem

The composition of two $N_{\delta^{*}ga}$-Hs is also a $N_{\delta^{*}ga}$-H.

Proof. Assume $T : (S, \tau) \rightarrow (Y, \sigma)$ and $g : (Y, \sigma) \rightarrow (Z, \eta)$ are two $N_{\delta^{*}ga}$-Hs. Assume $U$ is a $N_{\delta^{*}ga}$-CS in $(Z, \eta)$. Since $g$ is a $N_{\delta^{*}ga}$-IM, $g^{-1}\left(U\right)$ is $N_{\delta^{*}ga}$-CS in $(Y, \sigma)$. Since $U$ is $N_{\delta^{*}ga}$-IM, $T^{-1}\left(g^{-1}\left(U\right)\right)$ is $N_{\delta^{*}ga}$-CS in $(S, \tau)$. that is $(goT)^{-1}\left(T\right)$ is $N_{\delta^{*}ga}$-CS in $(S, \tau)$, therefore $goT : (Y, \sigma) \rightarrow (Z, \eta)$ is $N_{\delta^{*}ga}$-IM.

Assume $G$ is a $N_{\delta^{*}ga}$-CS in $(S, \tau)$, since $T^{-1}$ is a $N_{\delta^{*}ga}$-IM, $(T^{-1})^{-1}\left(G\right)$ is $N_{\delta^{*}ga}$-CS in $(Y, \sigma)$, that is $T\left(G\right)$ is $N_{\delta^{*}ga}$-CS in $(Y, \sigma)$. Since $g^{-1}$ is $N_{\delta^{*}ga}$-IM, $(g^{-1})^{-1}\left(T\left(G\right)\right)$ is $N_{\delta^{*}ga}$-CS in $(Z, \eta)$, that is $g\left(T\left(G\right)\right)$ is $N_{\delta^{*}ga}$-CS in $(Z, \eta)$, therefore $(goT)\left(G\right)$ is $N_{\delta^{*}ga}$-CS in $(Z, \eta)$. This implies that $(goT)^{-1}\left(G\right)$ is a $N_{\delta^{*}ga}$-CS in $(Z, \eta)$. This shows that $(goT)^{-1} : (Y, \sigma) \rightarrow (Z, \eta)$ is $N_{\delta^{*}ga}$-IM. Hence $(goT)$ is a $N_{\delta^{*}ga}$-c-H.

5.9. Theorem

Any $N_{\delta^{*}ga}$-H from a $N_{a\delta T_{a}^{*}ga}$-space into another $N_{a\delta T_{a}^{*}ga}$-space is a homeomorphism

Proof.

Assume $T : (S, \tau) \rightarrow (Y, \sigma)$ is a $N_{\delta^{*}ga}$-H. Then $T$ is bijective, $N_{\delta^{*}ga}$-OM and $N_{\delta^{*}ga}$-CM. Assume $U$ is an (NOS)in $(S, \tau)$. Since $T$ is $N_{\delta^{*}ga}$-OM and since $(Y, \sigma)$ is $N_{a\delta T_{a}^{*}ga}$-space, $T\left(U\right)$ is (NOS)in $(Y, \sigma)$. This implies that $T$ is N-open map. Assume $K$ is a (NCS) in $(Y, \sigma)$, since $T$ is $N_{\delta^{*}ga}$-CM and since $(S, \tau)$ is $N_{a\delta T_{a}^{*}ga}$-space, $T^{-1}\left(K\right)$ is (NCS) in $(S, \tau)$. Therefore $T$ is continuous. Hence $T$ is a homeomorphism.

5.10. Theorem

Assume $(Y, \sigma)$ is $N_{a\delta T_{a}^{*}ga}$-space. If $T : (S, \tau) \rightarrow (Y, \sigma)$ and $g : (Y, \sigma) \rightarrow (Z, \eta)$ are $N_{\delta^{*}ga}$-H then $(goT)$ is $N_{\delta^{*}ga}$-H.

Proof. Assume $T : (S, \tau) \rightarrow (Y, \sigma)$ and $g : (Y, \sigma) \rightarrow (Z, \eta)$ are two $N_{\delta^{*}ga}$-H. Assume $U$ is an (NOS)in $(S, \tau)$. Since $T$ is $N_{\delta^{*}ga}$-OM, $T\left(U\right)$ is $N_{\delta^{*}ga}$-OS in $(Y, \sigma)$. Since $(Y, \sigma)$ is $N_{a\delta T_{a}^{*}ga}$-space, $T\left(U\right)$ is N-OS in $(Y, \sigma)$. Also since $g$ is $N_{\delta^{*}ga}$-OM, $g\left(T\left(U\right)\right)$ is $N_{\delta^{*}ga}$-OS in $(Z, \eta)$. Hence $goT$ is $N_{\delta^{*}ga}$-OM. Assume $v$ is a (NCS) in $(Z, \eta)$. Since $g$ is $N_{\delta^{*}ga}$-CM and since $(Y, \sigma)$ is $N_{a\delta T_{a}^{*}ga}$-space, $g^{-1}\left(V\right)$ is (NCS) in $(Y, \sigma)$. Since $T$ is $N_{\delta^{*}ga}$-CM,
$T^{-1}(g^{-1}(V)) = (goT)^{-1}(V)$ is $N_{\delta^*g\alpha}$-CS in $(S, \tau)$. That is $(goT)$ is $N_{\delta^*g\alpha}$-continuous. Hence $(goT)$ is $N_{\delta^*g\alpha}$-H.

5.11. Theorem

Any $N_{\delta^*g\alpha}$-H from $(N_{a\delta T^*_g\alpha} S)$ into another $(N_{a\delta T^*_g\alpha} S)$ is a $N_{\delta^*g\alpha}$-H.

Proof. Assume $T : (S, \tau) \rightarrow (Y, \sigma)$ is $N_{\delta^*g\alpha}$-H. Assume $U$ be $N_{\delta^*g\alpha}$-CS in $(Y, \sigma)$. Then $U$ is $(NCS)$ in $(Y, \sigma)$. Since $T$ is $N_{\delta^*g\alpha}$-CM, $T^{-1}(U)$ is $N_{\delta^*g\alpha}$-CS in $(S, \tau)$. Then $T$ is a $N_{\delta^*g\alpha}$-IM. Let $K$ be $N_{\delta^*g\alpha}$-OS in $(S, \tau)$. Then $K$ is $(NOS)$ in $(S, \tau)$. Since $T$ is $N_{\delta^*g\alpha}$-OM, $T(K)$ is $N_{\delta^*g\alpha}$-OS in $(Y, \sigma)$. That is $T^{-1}(K)$ is $N_{\delta^*g\alpha}$-OS in $(Y, \sigma)$ and hence $T^{-1}$ is $N_{\delta^*g\alpha}$-IM. Thus $T$ is $N_{\delta^*g\alpha}$-c-H.

6. Conclusion

The notions of $N_{\delta^*g\alpha}$-continuous and $N_{\delta^*g\alpha}$-irresolute functions in (NTS) are given in this work. Next, their characterizations and investigate their properties are analyzed. In future work, we will use the soft sets theory to investigate new classes of neutrosophic soft maps and then we can study these new classes of (NTS) in soft setting.

References


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[19] S. M. Khalil and A. Rajah, Solving Class Equation $x^d = \beta$ in an Alternating Group for each $\beta \in H \cap C^\alpha$ and $n \notin \theta$, Journal of the Association of Arab Universities for Basic and Applied Sciences, 10,(2011), 42-50.

[20] S. M. Khalil and A. Rajah, Solving Class Equation $x^d = \beta$ in an Alternating Group for all $n \in \theta$ & $\beta \in H_n \cap C^\alpha$, journal of the Association of Arab Universities for Basic and Applied Sciences, 16 (2014), 38–45.


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