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T-MBJ NEUTROSOPHIC SET UNDER M-SUBALGEBRA

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Abstract. In this paper, the idea of T-MBJ neutrosophic set is introduced in which MBJ-neutrosophic set is used to present this new set called T-MBJ neutrosophic set. furthermore the notion of T-MBJ neutrosophic M-subalgebra on G -algebra is also introduced and provide the conditions for T-MBJ neutrosophic M-subalgebra. The word M in the term M-subalgebra, represents the initial of author's first name Mohsin. We study the T-MBJ neutrosophic set through different characteristics and also prove some results for better understanding of newly define T-MBJ neutrosophic set.

Keywords: G -algebra; T-MBJ neutrosophic set; T-MBJ neutrosophic M-subalgebra.

1. Introduction

Smarandacha [3,4] extended the intuitionistic fuzzy set to neutrosophic set. Barbhuiya [22] wrote in detailed about t-intuitionistic fuzzy using the concept of subalgebra and different other characteristics. Takallo et al. [5] extensively explained the BMBJ-neutrosophic subalgebra. Senapati et al. [27] used cubic set and applied it to subalgebras, ideals and closed ideals of B -algebra. Imai and Iseki [7, 28] defined the BCK -algebra and BCI -algebra. Bandaru et al. [21] first time introduced the G -algebra. Zadeh [8,9] introduced unique sets called fuzzy set and interval-valued fuzzy set. Saeid [1] studied interval-valued fuzzy subalgebra of B -algebra. Khalid et al. [10] defined T-neutrosophic cubic set and explained this set with important results. Senapati et al. [25] studied L -fuzzy G -subalgebra of G -algebra. Lots of work on BG -algebras [2] have been done by the researchers [26]. Khalid et al. [11] investigated neutrosophic soft cubic subalgebra. Khalid et al. [12] studied translation and multiplication of intuitionistic fuzzy set through some theorems. Khalid et al. [13] first time done the magnification of translation of set MBJ-neutrosophic and proved the results to explain the magnification. Khalid et al. [14]

studied MBJ-neutrosophic T-ideal on B-algebra. Khalid et al. [15] wrote the extensively important results for multiplicative interpretation of neutrosophic cubic set. Takallo et al. [16] discussed the application of MBJ-neutrosophic set. Neggers et al. [6] studied the fundamental theorem of B -homomorphism for B -algebra. Biswas [23] investigated the membership function of interval valued fuzzy set. Ahn [24] proved different results for fuzzy subalgebra. Jun et al. [29] deeply studied neutrosophic cubic set with different characteristics. Basset et. al [17] done the detailed study on hybrid neutrosophic multiple criteria group decision making approach for project selection. Hemavati et. al [20] investigated the β -subalgebra using the interval valed fuzzy set. Surya et. al [18] worked on MBJ neutrsophic β -subalgebra. Basset et. al [19] worked on novel group decision making model based on neutrsophic set for the heart disease.

This paper is presented to define the T-MBJ neutrosophic set and provide the condition for T-MBJ neutrosophic set [T-MBJ NS] to be a T-MBJ neutrosophic M-subalgebra on G -algebra. We also investigate some properties and proved some results for T-MBJ neutrosophic M-subalgebra [T-MBJ NMSU].

2. Preliminaries

Here, some basic definitions are written that are helpful to present this paper.

Definition 2.1. A nonempty set Y with a constant 0 and a binary operation $*$ is said to be G -algebra [21] if it fulfills these axioms:

$$G1: t_1 * t_1 = 0$$

$$G2: t_1 * (t_1 * t_2) = t_2, \text{ for all } t_1, t_2 \in Y.$$

A G -algebra is denoted by $(Y, *, 0)$.

Definition 2.2. A nonempty set Y with a constant 0 and a binary operation $*$ is said to be B -algebra [6] if it fulfills these axioms:

$$B1: t_1 * t_1 = 0$$

$$B2: t_1 * 0 = t_1 \quad B2: (t_1 * t_2) * t_3 = t_1 * (t_3(0 * t_2)), \text{ for all } t_1, t_2, t_3 \in Y.$$

Definition 2.3. Let S be a subset of G -algebra is called a subalgebra [21] of Y if $t_1 * t_2 \in S$ $\forall t_1, t_2 \in S$.

Definition 2.4. Function $f | Y \rightarrow X$ of B -algebra is called homomorphic [6] if $f(t_1 * t_2) = f(t_1) * f(t_2) \forall t_1, t_2 \in Y$. If $f | Y \rightarrow X$ is a B -homomorphic, then $f(0) = 0$.

Definition 2.5. Let C be a fuzzy set [8] in Y is defined as $C = \{ \langle t_1, \vartheta_C(t_1) \rangle \mid t_1 \in Y \}$, where $\vartheta_C(t_1)$ is called the existence ship value of t_1 in C and $\vartheta_C(t_1) \in [0, 1]$.

For a fuzzy set's family $C_i = \{ \langle t_1, \vartheta_{C_i}(t_1) \rangle \mid t_1 \in Y \}$ in Y , where $i \in H$ and H is index set, Join (\vee) and meet (\wedge) are defined as follow:

$$\vee_{i \in H} C_i = (\vee_{i \in H} \vartheta_{C_i})(t_1) = \sup\{\vartheta_{C_i} \mid i \in H\},$$

and

$$\wedge_{i \in H} C_i = (\wedge_{i \in H} \vartheta_{C_i})(t_1) = \inf\{\vartheta_{C_i} \mid i \in H\}$$

respectively, $\forall t_1 \in Y$.

Definition 2.6. [23] Let two elements $D_1, D_2 \in D[0, 1]$. If $D_1 = [(t_1)_1^-, (t_1)_1^+]$ and $D_2 = [(t_1)_2^-, (t_1)_2^+]$, then $rmax(D_1, D_2) = [max((t_1)_1^-, (t_1)_2^-), max((t_1)_1^+, (t_1)_2^+)]$ which is denoted by $D_1 \vee^r D_2$ and $rmin(D_1, D_2) = [min((t_1)_1^-, (t_1)_2^-), min((t_1)_1^+, (t_1)_2^+)]$ which is denoted by $D_1 \wedge^r D_2$. Thus, if $D_i = [((t_1)_1)_i^-, ((t_1)_2)_i^+] \in D[0, 1]$ for $i = 1, 2, 3, \dots$, then they defined $rsup_i(D_i) = [sup_i(((t_1)_1)_i^-), sup_i(((t_1)_1)_i^+)]$, i.e., $\vee_i^r D_i = [\vee_i(((t_1)_1)_i^-), \vee_i(((t_1)_1)_i^+)]$. In the same way they defined $rinf_i(D_i) = [inf_i(((t_1)_1)_i^-), inf_i(((t_1)_1)_i^+)]$, i.e., $\wedge_i^r D_i = [\wedge_i(((t_1)_1)_i^-), \wedge_i(((t_1)_1)_i^+)]$. Now they called $D_1 \geq D_2 \iff (t_1)_1^- \geq (t_1)_2^-$ and $(t_1)_1^+ \geq (t_1)_2^+$. Similarly they defined the relations $D_1 \leq D_2$ and $D_1 = D_2$.

Ahn et al. [24] defined fuzzy subalgebra, which is defined below.

Definition 2.7. A nonempty set $C = \{ \langle t_1, \vartheta_C(t_1) \rangle \mid t_1 \in Y \}$ is called a fuzzy subalgebra [24] of Y if $\vartheta_C(t_1 * t_2) \geq \min\{\vartheta_C(t_1), \vartheta_C(t_2)\} \forall t_1, t_2 \in Y$.

Definition 2.8. For any $C_i = (\rho_i, \lambda_i)$ [29] where $\rho_i = \{ \langle t_1; \rho_{iE}(t_1), \rho_{iI}(t_1), \rho_{iN}(t_1) \rangle \mid t_1 \in Y \}$, $\lambda_i = \{ \langle t_1; \lambda_{iE}(t_1), \lambda_{iI}(t_1), \lambda_{iN}(t_1) \rangle \mid t_1 \in Y \}$ for $i \in H$, P-union, P-inersection, R-union and R-intersection is defined respectively by **P-union** $\cup_P C_i = (\cup_{i \in H} \rho_i, \vee_{i \in H} \lambda_i)$, **P-intersection** $\cap_P C_i = (\cap_{i \in H} \rho_i, \wedge_{i \in H} \lambda_i)$, **R-union** $\cup_R C_i = (\cup_{i \in H} \rho_i, \wedge_{i \in H} \lambda_i)$, **R-intersection:** $\cap_R C_i = (\cap_{i \in H} \rho_i, \vee_{i \in H} \lambda_i)$, where

$$\begin{aligned} \cup_{i \in H} \rho_i &= \{ \langle t_1; (\cup_{i \in H} \rho_{iE})(t_1), (\cup_{i \in H} \rho_{iI})(t_1), (\cup_{i \in H} \rho_{iN})(t_1) \rangle \mid t_1 \in Y \}, \\ \vee_{i \in H} \lambda_i &= \{ \langle t_1; (\vee_{i \in H} \lambda_{iE})(t_1), (\vee_{i \in H} \lambda_{iI})(t_1), (\vee_{i \in H} \lambda_{iN})(t_1) \rangle \mid t_1 \in Y \}, \\ \cap_{i \in H} \rho_i &= \{ \langle t_1; (\cap_{i \in H} \rho_{iE})(t_1), (\cap_{i \in H} \rho_{iI})(t_1), (\cap_{i \in H} \rho_{iN})(t_1) \rangle \mid t_1 \in Y \}, \\ \wedge_{i \in H} \lambda_i &= \{ \langle t_1; (\wedge_{i \in H} \lambda_{iE})(t_1), (\wedge_{i \in H} \lambda_{iI})(t_1), (\wedge_{i \in H} \lambda_{iN})(t_1) \rangle \mid t_1 \in Y \}. \end{aligned}$$

Definition 2.9. Let $B = (\vartheta_B, \nu_B)$ be an IFS of BG-algebra Y and $t \in [0, 1]$, then the IFS B^t is said to be t-intuitionistic fuzzy subset [1] of Y w.r.t B and is defined as $B^t = \{ \langle t_1, \vartheta_{B^t}(t_1), \nu_{B^t}(t_1) \rangle \mid t_1 \in Y \} = \langle \vartheta_{B^t}, \nu_{B^t} \rangle$, where $\vartheta_{B^t}(t_1) = \min\{\vartheta_B(t_1), t\}$ and $\nu_{B^t}(t_1) = \max\{\nu_B(t_1), 1 - t\} \forall t_1 \in Y$.

Definition 2.10. Let $B^t = (\vartheta_{B^t}, \nu_{B^t})$ be a t-IFS of BG-algebra Y and $t \in [0, 1]$ then B^t is said to be t-IFSU [23] of Y if it fulfills these axioms.

- (i) $\vartheta_{B^t}(t_1 * t_2) \geq \min\{\vartheta_{B^t}(t_1), \vartheta_{B^t}(t_2)\}$ and
- (ii) $\nu_{B^t}(t_1 * t_2) \leq \max\{\nu_{B^t}(t_1), \nu_{B^t}(t_2)\} \forall t_1, t_2 \in Y$.

Definition 2.11. A MBJ-neutrosophic set [16] in Y is a structure of the form $C = \{\langle M_C t_1, \hat{B}_C t_1, J_C t_1 \rangle \mid t_1 \in Y\}$, where M_C and J_C are fuzzy sets in Y and M_C is a truth membership function, J_C is a false membership function and \hat{B} is an indeterminate interval valued membership function.

3. T-MBJ Neutrosophic M-Subalgebras

Definition 3.1. A nonempty set of the form $C^t = (M^t, \hat{B}^t, J^t)$ is called a T-MBJ neutrosophic set (**T-MBJ NS**) of Y , where $C^t = \{\langle t_1, M^t(t_1), \hat{B}^t(t_1), J^t(t_1) \rangle \mid t_1 \in Y\} = \langle M^t, \hat{B}^t, J^t \rangle$ with two independent components is defined as $M^t(t_1) = \{\min(M, t)(t_1)\}$, $\hat{B}^t = \{rmin(\hat{B}, t')(t_1)\}$ and $J^t(t_1) = \{\max(J, 2 - t - t')(t_1)\} \forall t, t', 2 - t - t' \in [0, 1]$, where M^t is truth membership function, \hat{B} is an indeterminate interval valued membership function and J^t is a false membership function.

Definition 3.2. Let $C^t = (M^t, \hat{B}, J^t)$ be a T-MBJ neutrosophic set. Then C^t is T-MBJ NMSU under binary operation $*$, where $t_1, t_2, t, t', 2 - t - t', \aleph, \Re \in [0, 1]$ if it satisfies the following three conditions: N1:

$$\begin{aligned} \min(M((t_1 * \aleph) * (t_2 * \Re)), t) &= M^t((t_1 * \aleph) * (t_2 * \Re)) \succeq \min\{M^t(t_1 * \aleph), M^t(t_2 * \Re)\} \\ \min(\hat{B}((t_1 * \aleph) * (t_2 * \Re)), t') &= \hat{B}^t((t_1 * \aleph) * (t_2 * \Re)) \succeq rmin\{\hat{B}^t(t_1 * \aleph), \hat{B}^t(t_2 * \Re)\} \\ \min(J((t_1 * \aleph) * (t_2 * \Re)), 2 - t - t') &= J^t((t_1 * \aleph) * (t_2 * \Re)) \preceq \max\{J^t(t_1 * \aleph), J^t(t_2 * \Re)\}. \end{aligned}$$

For our simplicity we replace the $2 - t - t'$ with \Im .

Example 3.3. Let $Y = \{0, t_1 * \aleph, t_2 * \Re\}$ be a G -algebra with the following Cayley table.

$*$	0	$t_1 * \aleph$	$t_2 * \Re$
0	0	$t_1 * \aleph$	$t_2 * \Re$
$t_1 * \aleph$	$t_1 * \aleph$	0	$t_2 * \Re$
$t_2 * \Re$	$t_2 * \Re$	$t_1 * \aleph$	0

A T-MBJ neutrosophic set $C^t = (M^t, \hat{B}, J^t)$ of X is defined by

M^t_T	0	$t_1 * \aleph$	$t_2 * \Re$	\hat{B}^t	[0.1,0.2]	[0.3,0.5]	[0.6,0.9]	J^t	0.2	0.4	0.8
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It is the routine work to check that set is T-MBJ neutrosophic M-subalgebra.

Proposition 3.4. Let $C^t = \{\langle t_1, M^t(t_1), \hat{B}^t(t_1), J^t(t_1) \rangle\}$ be a T-MBJ NMSU of Y , then $\forall t_1 \in Y, M^t(0 * \aleph) \succeq M^t(t_1 * \aleph), \hat{B}^t(0 * \aleph) \succeq \hat{B}^t(t_1 * \aleph)$ and $J^t(0 * \aleph) \preceq J^t(t_1 * \aleph)$. Thus, $M^t(0 * \aleph), \hat{B}^t(0 * \aleph)$ and $J^t(0 * \aleph)$ are the upper bounds and lower bounds of $M^t(t_1 * \aleph), \hat{B}^t(t_1 * \aleph)$ and $J^t(t_1 * \aleph)$ respectively.

Proof. $\forall t_1 \in Y$, we have $M^t((0 * \aleph)) = \min(M((0 * \aleph)), t) = \min(M((t_1 * \aleph) * (t_1 * \aleph)), t) \succeq \min\{\min(M((t_1 * \aleph)), t), \min(M(t_1 * \aleph), t)\} = \min(M(t_1 * \aleph), t) = M^t((t_1 * \aleph)) \Rightarrow M^t((0 * \aleph)) \succeq M^t((t_1 * \aleph))$, $\hat{B}^t(0 * \aleph) = r\min(\hat{B}(0 * \aleph), t) = r\min(\hat{B}((t_1 * \aleph) * (t_1 * \aleph)), t) \succeq r\min\{r\min(\hat{B}(t_1 * \aleph), t), r\min(\hat{B}(t_1 * \aleph), t)\} = r\min(\hat{B}(t_1 * \aleph), t) = \hat{B}^t(t_1 * \aleph) \Rightarrow \hat{B}^t(0 * \aleph) \succeq \hat{B}^t(t_1 * \aleph)$ and $\max(J(0 * \aleph), \Im) = \max(J((t_1 * \aleph) * (t_1 * \aleph)), \Im) \succeq \max\{\max(J(t_1 * \aleph), \Im), \max(J(t_1 * \aleph), \Im)\} = \max(J(t_1 * \aleph), t) = J^t(t_1 * \aleph) \Rightarrow J^t(0 * \aleph) \preceq J^t(t_1 * \aleph)$. \square

Theorem 3.5. Let $\mathcal{C}^t = \{\langle (t_1), M^t(t_1), \hat{B}^t(t_1), J^t(t_1) \rangle\}$ be a T-MBJ NMSU of Y . If there exists a sequence $\{(t_1 * \aleph)_n\}$ of Y such that $\lim_{n \rightarrow \infty} M^t((t_1 * \aleph)_n) = 0$, $\lim_{n \rightarrow \infty} \hat{B}^t((t_1 * \aleph)_n) = [1, 1]$ and $\lim_{n \rightarrow \infty} J^t((t_1 * \aleph)_n) = 0$. Then $M^t(0) = 0$, $\hat{B}^t(0) = [1, 1]$ and $J^t(0) = 0$.

Proof. Using Proposition 3.4, $M^t(0 * \aleph) \succeq M^t(t_1 * \aleph) \forall t_1 \in Y$, then $M^t(0 * \aleph) \succeq M^t((t_1 * \aleph)_n)$ for $n \in \mathbf{Z}^+$. Consider, $0 \succeq M^t(0 * \aleph) \succeq \lim_{n \rightarrow \infty} M^t((t_1 * \aleph)_n) = 0$. Hence, $M^t(0 * \aleph) = 0$. Using Proposition 3.4, $\hat{B}^t(0 * \aleph) \succeq \hat{B}^t(t_1 * \aleph) \forall t_1 \in Y$, so therefore $\hat{B}^t(0 * \aleph) \succeq \hat{B}^t((t_1 * \aleph)_n)$ for $n \in \mathbf{Z}^+$. Consider, $[1, 1] \succeq \hat{B}^t(0 * \aleph) \succeq \lim_{n \rightarrow \infty} \hat{B}^t((t_1 * \aleph)_n) = [1, 1]$. Hence, $\hat{B}^t(0 * \aleph) = [1, 1]$. Again, using Proposition 3.4, $J^t(0 * \aleph) \preceq J^t(t_1 * \aleph) \forall t_1 \in Y$, so therefore $J^t(0 * \aleph) \preceq J^t((t_1 * \aleph)_n)$ for $n \in \mathbf{Z}^+$. Consider, $0 \preceq J^t(0 * \aleph) \preceq \lim_{n \rightarrow \infty} J^t((t_1 * \aleph)_n) = 0$. Hence, $J^t(0 * \aleph) = 0$. \square

Theorem 3.6. The R-intersection of any set of T-MBJ NMSU of Y is also a T-MBJ NMSU of Y .

Proof. Let $\mathcal{C}_i^t = \{\langle t_1, M_i^t, \hat{B}_i^t, J_i^t \rangle \mid t_1 \in Y\}$ where $i \in k$, be a set of T-MBJ NMSU of Y and $t_1, t_2 \in Y$ and $t, \aleph, \mathfrak{R} \in [0, 1]$. Then

$$\begin{aligned} (\vee(M_i^t)_i)((t_1 * \aleph) * (t_2 * \mathfrak{R})) &= \vee(\min(M_i, t)((t_1 * \aleph) * (t_2 * \mathfrak{R}))) \\ &= \sup(\min(M_i, t)((t_1 * \aleph) * (t_2 * \mathfrak{R}))) \\ &\succeq \sup\{\min\{\min(M_i, t)(t_1 * \aleph), \min(M_i, t)(t_2 * \mathfrak{R})\}\} \\ &= \min\{\sup(\min(M_i, t)(t_1 * \aleph)), \sup(\min(M_i, t)(t_2 * \mathfrak{R}))\} \\ &= \min\{\sup(M_i^t)(t_1 * \aleph), \sup(M_i^t)(t_2 * \mathfrak{R})\} \\ &= \min\{\vee(M_i^t)(t_1 * \aleph), \vee(M_i^t)(t_2 * \mathfrak{R})\} \\ \Rightarrow \vee(M_i^t)_i((t_1 * \aleph) * (t_2 * \mathfrak{R})) &\succeq \min\{\vee(M_i^t)(t_1 * \aleph), \vee(M_i^t)(t_2 * \mathfrak{R})\} \end{aligned}$$

and

$$\begin{aligned}
 (\cap(\hat{B}^t)_i)((t_1 * \aleph) * (t_2 * \ale�)) &= \cap(\text{rmin}(\hat{B}_i, t')((t_1 * \aleph) * (t_2 * \ale�))) \\
 &= \text{rinf}(\text{rmin}(\hat{B}_i, t')((t_1 * \aleph) * (t_2 * \ale�))) \\
 &\succeq \text{rinf}\{\text{rmin}\{\text{rmin}(\hat{B}_i, t')(t_1 * \aleph), (\text{rmin}(\hat{B}_i, t')(t_1 * \ale�))\}\} \\
 &= \text{rmin}\{\text{rinf}(\text{rmin}(\hat{B}_i, t')(t_1 * \aleph)), \text{rinf}(\text{rmin}(\hat{B}_i, t')(t_1 * \ale�))\} \\
 &= \text{rmin}\{\text{rinf}(\hat{B}_i^t)(t_1 * \aleph), \text{rinf}(\hat{B}_i^t)(t_1 * \ale�)\} \\
 &= \text{rmin}\{\cap(\hat{B}_i^t)(t_1 * \aleph), \cap(\hat{B}_i^t)(t_1 * \ale�)\} \\
 \Rightarrow \cap(\hat{B}_i^t)((t_1 * \aleph) * (t_2 * \ale�)) &\succeq \text{rmin}\{\cap(\hat{B}_i^t)(t_1 * \aleph), \cap(\hat{B}_i^t)(t_1 * \ale�)\},
 \end{aligned}$$

and

$$\begin{aligned}
 (\vee(J^t)_i)((t_1 * \aleph) * (t_2 * \ale�)) &= \vee(\text{max}(J_i, \Im)((t_1 * \aleph) * (t_2 * \ale�))) \\
 &= \text{sup}(\text{max}(J_i, \Im)((t_1 * \aleph) * (t_2 * \ale�))) \\
 &\preceq \text{sup}\{\text{max}\{(\text{max}(J_i, \Im)(t_1 * \aleph)), (\text{max}(J_i, \Im)(t_1 * \ale�))\}\} \\
 &= \text{max}\{\text{sup}(\text{max}(J_i, \Im)(t_1 * \aleph)), \text{sup}(\text{max}(J_i, \Im)(t_1 * \ale�))\} \\
 &= \text{max}\{\text{sup}(J_i^t)(t_1 * \aleph), \text{sup}(J_i^t)(t_1 * \ale�)\} \\
 &= \text{max}\{\vee(J_i^t)(t_1 * \aleph), \vee(J_i^t)(t_1 * \ale�)\} \\
 \Rightarrow \vee(J_i^t)((t_1 * \aleph) * (t_2 * \ale�)) &\preceq \text{max}\{\vee(J_i^t)(t_1 * \aleph), \vee(J_i^t)(t_1 * \ale�)\},
 \end{aligned}$$

which show that R -intersection of \mathcal{C}_i^t is a T-MBJ NMSU of Y . \square

Theorem 3.7. Let $\mathcal{C}_i^t = \{\langle t_1, (M_i^t), (\hat{B}_i^t), (J_i^t) \rangle \mid t_1 \in Y\}$ be a set of T-MBJ NMSU of Y , where $i \in k$ and $t \in [0, 1]$. If $\text{inf}\{\min\{(M_i^t)(t_1 * \aleph), (M_i^t)(t_1 * \ale�)\}\} = \min\{\text{inf}(M_i^t)(t_1 * \aleph), \text{inf}(M_i^t)(t_1 * \ale�)\}$ and $\text{inf}\{\max\{(J_i^t)(t_1 * \aleph), (J_i^t)(t_1 * \ale�)\}\} = \max\{\text{inf}(J_i^t)(t_1 * \aleph), \text{inf}(J_i^t)(t_1 * \ale�)\} \forall t_1 \in Y$, then P -intersection of \mathcal{C}_i^t is also a T-MBJ NMSU of Y .

Proof. Suppose that $\mathcal{C}_i^t = \{\langle t_1, (M_i^t), (\hat{B}_i^t), (J_i^t) \rangle \mid t_1 \in Y\}$ where $i \in k$, is a family of sets of T-MBJ NMSU of Y such that $\text{inf}\{\min\{(M_i^t)(t_1 * \aleph), (M_i^t)(t_1 * \ale�)\}\} = \min\{\text{inf}(M_i^t)(t_1 * \aleph), \text{inf}(M_i^t)(t_1 * \ale�)\}$ and $\text{inf}\{\max\{(J_i^t)(t_1 * \aleph), (J_i^t)(t_1 * \ale�)\}\} = \max\{\text{inf}(J_i^t)(t_1 * \aleph), \text{inf}(J_i^t)(t_1 * \ale�)\}$

$\aleph\}} \forall t_1, t_2 \in Y$ and $t \in [0, 1]$. Then

$$\begin{aligned} (\wedge(M^t)_i)((t_1 * \aleph) * (t_2 * \aleph)) &= \wedge(\min(M_i, t)((t_1 * \aleph) * (t_2 * \aleph))) \\ &= \inf(\min(M_i, t)((t_1 * \aleph) * (t_2 * \aleph))) \\ &\succeq \inf\{\min\{(\min(M_i, t)(t_1 * \aleph)), (\min(M_i, t)(t_2 * \aleph))\}\} \\ &= \min\{\inf(\min(M_i, t)(t_1 * \aleph)), \inf(\min(M_i, t)(t_2 * \aleph))\} \\ &= \min\{\inf(M_i^t)(t_1 * \aleph), \inf(M_i^t)(t_2 * \aleph)\} \\ &= \min\{\wedge(M_i^t)(t_1 * \aleph), \wedge(M_i^t)(t_2 * \aleph)\} \\ \Rightarrow \wedge(M_i^t)((t_1 * \aleph) * (t_2 * \aleph)) &\succeq \min\{\wedge(M_i^t)(t_1 * \aleph), \wedge(M_i^t)(t_2 * \aleph)\} \end{aligned}$$

and

$$\begin{aligned} (\cap(\hat{B}^t)_i)((t_1 * \aleph) * (t_2 * \aleph)) &= \cap(\text{rmin}(\hat{B}_i, t')((t_1 * \aleph) * (t_2 * \aleph))) \\ &= \text{rinf}(\text{rmin}(\hat{B}_i, t')((t_1 * \aleph) * (t_2 * \aleph))) \\ &\succeq \text{rinf}\{\text{rmin}\{(\text{rmin}(\hat{B}_i, t')(t_1 * \aleph)), (\text{rmin}(\hat{B}_i, t')(t_2 * \aleph))\}\} \\ &= \text{rmin}\{\text{rinf}(\text{rmin}(\hat{B}_i, t')(t_1 * \aleph)), \text{rinf}(\text{rmin}(\hat{B}_i, t')(t_2 * \aleph))\} \\ &= \text{rmin}\{\text{rinf}(\hat{B}_i^t)(t_1 * \aleph), \text{rinf}(\hat{B}_i^t)(t_2 * \aleph)\} \\ &= \text{rmin}\{\cap(\hat{B}_i^t)(t_1 * \aleph), \cap(\hat{B}_i^t)(t_2 * \aleph)\} \\ \Rightarrow \cap(\hat{B}_i^t)((t_1 * \aleph) * (t_2 * \aleph)) &\succeq \text{rmin}\{\cap(\hat{B}_i^t)(t_1 * \aleph), \cap(\hat{B}_i^t)(t_2 * \aleph)\}, \end{aligned}$$

and

$$\begin{aligned} (\wedge(J^t)_i)((t_1 * \aleph) * (t_2 * \aleph)) &= \wedge(\max(J_i, \Im)((t_1 * \aleph) * (t_2 * \aleph))) \\ &= \inf(\max(J_i, \Im)((t_1 * \aleph) * (t_2 * \aleph))) \\ &\preceq \inf\{\max\{(\max(J_i, \Im)(t_1 * \aleph)), (\max(J_i, \Im)(t_2 * \aleph))\}\} \\ &= \max\{\inf(\max(J_i, \Im)(t_1 * \aleph)), \inf(\max(J_i, \Im)(t_2 * \aleph))\} \\ &= \max\{\inf(J_i^t)(t_1 * \aleph), \inf(J_i^t)(t_2 * \aleph)\} \\ &= \max\{\wedge(J_i^t)(t_1 * \aleph), \wedge(J_i^t)(t_2 * \aleph)\} \\ \Rightarrow \wedge(J_i^t)((t_1 * \aleph) * (t_2 * \aleph)) &\preceq \max\{\wedge(J_i^t)(t_1 * \aleph), \wedge(J_i^t)(t_2 * \aleph)\}, \end{aligned}$$

which show that P -intersection of C_i^t is a T-MBJ NMSU of Y . \square

Theorem 3.8. Let $C_i^t = \{(t_1, (M_i^t), (\hat{B}_i^t), (J_i^t)) \mid t_1 \in Y\}$ where $i \in k$, be a family of sets of T-MBJ NMSU of Y . If $\sup\{\min\{(M_i^t)(t_1 * \aleph), (M_i^t)(t_2 * \aleph)\}\} = \min\{\sup(M_i^t)(t_1 * \aleph), \inf(M_i^t)(t_1 * \aleph)\}$ and $\text{rsup}\{\text{rmin}\{(\hat{B}_i^t)(t_1 * \aleph), (\hat{B}_i^t)(t_2 * \aleph)\}\} = \text{rmin}\{\text{rsup}(\hat{B}_i^t)(t_1 * \aleph), \text{rsup}(\hat{B}_i^t)(t_2 * \aleph)\}$

and $\sup\{\max\{(J_i^t)(t_1 * \aleph), (J_i^t)(t_1 * \aleph)\}\} = \max\{\sup(J_i^t)(t_1 * \aleph), \sup(J_i^t)(t_1 * \aleph)\} \forall t_1, t_2 \in Y$, then P -union of \mathcal{C}_i^t is also a T -MBJ NMSU of Y .

Proof. Let $\mathcal{C}_i^t = \{\langle t_1, (\hat{B}_i^t), (J_i^t) \rangle \mid t_1 \in Y\}$ where $i \in k$, be a family of sets of T -MBJ NMSU of Y such that $\sup\{\min\{(M_i^t)(t_1 * \aleph), (M_i^t)(t_1 * \aleph)\}\} = \min\{\sup(M_i^t)(t_1 * \aleph), \sup(M_i^t)(t_1 * \aleph)\}$ and $\text{rsup}\{\text{rmin}\{(\hat{B}_i^t)(t_1 * \aleph), (\hat{B}_i^t)(t_1 * \aleph)\}\} = \text{rmin}\{\text{rsup}(\hat{B}_i^t)(t_1 * \aleph), \text{rsup}(\hat{B}_i^t)(t_1 * \aleph)\}$ and $\sup\{\max\{(J_i^t)(t_1 * \aleph), (J_i^t)(t_1 * \aleph)\}\} = \max\{\sup(J_i^t)(t_1 * \aleph), \sup(J_i^t)(t_1 * \aleph)\} \forall t_1, t_2 \in Y$. Then for $t_1, t_2 \in Y$, and $t \in [0, 1]$.

$$\begin{aligned} (\vee(M^t)_i)((t_1 * \aleph) * (t_2 * \aleph)) &= \vee(\min(M_i, t)((t_1 * \aleph) * (t_2 * \aleph))) \\ &= \sup(\min(M_i, t)((t_1 * \aleph) * (t_2 * \aleph))) \\ &\succeq \sup\{\min\{(\min(M_i, t)(t_1 * \aleph)), (\min(M_i, t)(t_1 * \aleph))\}\} \\ &= \min\{\sup(\min(M_i, t)(t_1 * \aleph)), \sup(\min(M_i, t)(t_1 * \aleph))\} \\ &= \min\{\sup(M_i^t)(t_1 * \aleph), \sup(M_i^t)(t_1 * \aleph)\} \\ &= \min\{\vee(M_i^t)(t_1 * \aleph), \vee(M_i^t)(t_1 * \aleph)\} \\ \Rightarrow \vee(M_i^t)((t_1 * \aleph) * (t_2 * \aleph)) &\succeq \min\{\vee(M_i^t)(t_1 * \aleph), \vee(M_i^t)(t_1 * \aleph)\} \end{aligned}$$

and

$$\begin{aligned} (\cup(\hat{B}^t)_i)((t_1 * \aleph) * (t_2 * \aleph)) &= \cup(\text{rmin}(\hat{B}_i, t')((t_1 * \aleph) * (t_2 * \aleph))) \\ &= \text{rsup}(\text{rmin}(\hat{B}_i, t')((t_1 * \aleph) * (t_2 * \aleph))) \\ &\succeq \text{rsup}\{\text{rmin}\{(\text{rmin}(\hat{B}_i, t')(t_1 * \aleph)), (\text{rmin}(\hat{B}_i, t')(t_1 * \aleph))\}\} \\ &= \text{rmin}\{\text{rsup}(\text{rmin}(\hat{B}_i, t')(t_1 * \aleph)), \text{rsup}(\text{rmin}(\hat{B}_i, t')(t_1 * \aleph))\} \\ &= \text{rmin}\{\text{rsup}(\hat{B}_i^t)(t_1 * \aleph), \text{rsup}(\hat{B}_i^t)(t_1 * \aleph)\} \\ &= \text{rmin}\{\cup(\hat{B}_i^t)(t_1 * \aleph), \cup(\hat{B}_i^t)(t_1 * \aleph)\} \\ \Rightarrow \cup(\hat{B}_i^t)((t_1 * \aleph) * (t_2 * \aleph)) &\succeq \text{rmin}\{\cup(\hat{B}_i^t)(t_1 * \aleph), \cup(\hat{B}_i^t)(t_1 * \aleph)\} \end{aligned}$$

and

$$\begin{aligned} (\vee(J^t)_i)((t_1 * \aleph) * (t_2 * \aleph)) &= \vee(\max(J_i, \Im)((t_1 * \aleph) * (t_2 * \aleph))) \\ &= \sup(\max(J_i, \Im)((t_1 * \aleph) * (t_2 * \aleph))) \\ &\preceq \sup\{\max\{(\max(J_i, \Im)(t_1 * \aleph)), (\max(J_i, \Im)(t_1 * \aleph))\}\} \\ &= \max\{\sup(\max(J_i, \Im)(t_1 * \aleph)), \sup(\max(J_i, \Im)(t_1 * \aleph))\} \\ &= \max\{\sup(J_i^t)(t_1 * \aleph), \sup(J_i^t)(t_1 * \aleph)\} \\ &= \max\{\vee(J_i^t)(t_1 * \aleph), \vee(J_i^t)(t_1 * \aleph)\} \\ \Rightarrow \vee(J_i^t)((t_1 * \aleph) * (t_2 * \aleph)) &\preceq \max\{\vee(J_i^t)(t_1 * \aleph), \vee(J_i^t)(t_1 * \aleph)\}, \end{aligned}$$

which show that P -union of \mathcal{C}_i^t is a T-MBJ NMSU of Y . \square

Theorem 3.9. Let $\mathcal{C}_i^t = \{ \langle t_1, (\hat{B}_i^t), (J_i^t) \rangle \mid t_1 \in Y \}$ where $i \in k$, be a family of sets of T-MBJ NMSU of Y . If $\inf\{\min\{(M_i^t)(t_1 * \aleph), (M_i^t)(t_1 * \mathfrak{R})\}\} = \min\{\inf(M_i^t)(t_1 * \aleph), \inf(M_i^t)(t_1 * \mathfrak{R})\}$ and $\inf\{\max\{(J_i^t)(t_1 * \aleph), (J_i^t)(t_1 * \mathfrak{R})\}\} = \max\{\inf(J_i^t)(t_1 * \aleph), \inf(J_i^t)(t_1 * \mathfrak{R})\}$ and $\text{rsup}\{\text{rmin}\{(\hat{B}_i^t)(t_1 * \aleph), (\hat{B}_i^t)(t_1 * \mathfrak{R})\}\} = \text{rmin}\{\text{rsup}(\hat{B}_i^t)(t_1 * \aleph), \text{rsup}(\hat{B}_i^t)(t_1 * \mathfrak{R})\} \forall t_1 \in Y$, and $t \in [0, 1]$ then R -union of \mathcal{C}_i^t is also a T-MBJ NMSU of Y .

Proof. Let $\mathcal{C}_i^t = \{ \langle t_1, (M_i^t), (\hat{B}_i^t), (J_i^t) \rangle \mid t_1 \in Y \}$ where $i \in k$, and $t \in [0, 1]$ be a family of sets of T-MBJ NMSU of Y such that $\inf\{\min\{(M_i^t)(t_1 * \aleph), (M_i^t)(t_1 * \mathfrak{R})\}\} = \min\{\inf(M_i^t)(t_1 * \aleph), \inf(M_i^t)(t_1 * \mathfrak{R})\}$ and $\inf\{\max\{(J_i^t)(t_1 * \aleph), (J_i^t)(t_1 * \mathfrak{R})\}\} = \max\{\inf(J_i^t)(t_1 * \aleph), \inf(J_i^t)(t_1 * \mathfrak{R})\}$ and $\text{rsup}\{\text{rmin}\{(\hat{B}_i^t)(t_1 * \aleph), (\hat{B}_i^t)(t_1 * \mathfrak{R})\}\} = \text{rmin}\{\text{rsup}(\hat{B}_i^t)(t_1 * \aleph), \text{rsup}(\hat{B}_i^t)(t_1 * \mathfrak{R})\} \forall t_1 \in Y$, and $t \in [0, 1]$. Then for $t_1, t_2 \in Y$ and $t \in [0, 1]$.

$$\begin{aligned} (\wedge(M_i^t))((t_1 * \aleph) * (t_2 * \mathfrak{R})) &= (\wedge(\min(M_i, t)((t_1 * \aleph) * (t_2 * \mathfrak{R}))) \\ &= \inf\{(\min(M_i, t)((t_1 * \aleph) * (t_2 * \mathfrak{R})))\} \\ &\succeq \inf\{\min\{\min(M_i, t)(t_1 * \aleph), \min(M_i, t)(t_1 * \mathfrak{R})\}\} \\ &= \min\{\inf(\min(M_i, t)(t_1 * \aleph)), \inf(\min(M_i, t)(t_1 * \mathfrak{R}))\} \\ &= \min\{\inf((M_i^t)(t_1 * \aleph)), \inf((M_i^t)(t_1 * \mathfrak{R}))\} \\ &= \min\{\wedge(M_i^t)(t_1 * \aleph), \wedge(M_i^t)(t_1 * \mathfrak{R})\} \\ \Rightarrow \wedge(M_i^t)((t_1 * \aleph) * (t_2 * \mathfrak{R})) &\succeq \min\{\wedge(M_i^t)(t_1 * \aleph), \wedge(M_i^t)(t_1 * \mathfrak{R})\}, \end{aligned}$$

and

$$\begin{aligned} (\cup(\hat{B}_i^t))((t_1 * \aleph) * (t_2 * \mathfrak{R})) &= (\cup(\text{rmin}(\hat{B}_i, t'))((t_1 * \aleph) * (t_2 * \mathfrak{R}))) \\ &= \text{rsup}\{\text{rmin}(\hat{B}_i, t')((t_1 * \aleph) * (t_2 * \mathfrak{R}))\} \\ &\succeq \text{rsup}\{\text{rmin}\{\text{rmin}(\hat{B}_i, t')(t_1 * \aleph), \text{rmin}(\hat{B}_i, t')(t_1 * \mathfrak{R})\}\} \\ &= \text{rmin}\{\text{rsup}(\text{rmin}(\hat{B}_i, t')(t_1 * \aleph)), \text{rsup}(\text{rmin}(\hat{B}_i, t')(t_1 * \mathfrak{R}))\} \\ &= \text{rmin}\{\text{rsup}((\hat{B}_i^t)(t_1 * \aleph)), \text{rsup}((\hat{B}_i^t)(t_1 * \mathfrak{R}))\} \\ &= \text{rmin}\{\cup(\hat{B}_i^t)(t_1 * \aleph), \cup(\hat{B}_i^t)(t_1 * \mathfrak{R})\} \\ \Rightarrow \cup(\hat{B}_i^t)((t_1 * \aleph) * (t_2 * \mathfrak{R})) &\succeq \text{rmin}\{\cup(\hat{B}_i^t)(t_1 * \aleph), \cup(\hat{B}_i^t)(t_1 * \mathfrak{R})\}, \end{aligned}$$

and

$$\begin{aligned}
 (\wedge(J_i^t))((t_1 * \aleph) * (t_2 * \aleph)) &= (\wedge(\max(J_i, \mathfrak{S}))((t_1 * \aleph) * (t_2 * \aleph))) \\
 &= \inf\{\max(J_i, \mathfrak{S})((t_1 * \aleph) * (t_2 * \aleph))\} \\
 &\preceq \inf\{\max\{\max(J_i, \mathfrak{S})(t_1 * \aleph), \max(J_i, \mathfrak{S})(t_2 * \aleph)\}\} \\
 &= \max\{\inf(\max(J_i, \mathfrak{S})(t_1 * \aleph)), \inf(\max(J_i, \mathfrak{S})(t_2 * \aleph))\} \\
 &= \max\{\inf((J_i^t)(t_1 * \aleph)), \inf((J_i^t)(t_2 * \aleph))\} \\
 &= \max\{\wedge(J_i^t)(t_1 * \aleph), \wedge(J_i^t)(t_2 * \aleph)\} \\
 \Rightarrow \wedge(J_i^t)((t_1 * \aleph) * (t_2 * \aleph)) &\preceq \max\{\wedge(J_i^t)(t_1 * \aleph), \wedge(J_i^t)(t_2 * \aleph)\},
 \end{aligned}$$

which show that R -union of \mathcal{C}_i^t is a T-MBJ NMSU of Y . \square

Proposition 3.10. *If a T-MBJ neutrosophic set $\mathcal{C}^t = (M^t, \hat{B}^t, J^t)$ of Y is a TMBJ-neutrosophic M -subalgebra, then $\forall t_1 \in Y, M^t(0 * t_1) \succeq M^t(t_1 * \aleph)$ and $\hat{B}^t(0 * t_1) \succeq \hat{B}^t(t_1 * \aleph)$ and $J^t(0 * t_1) \preceq J^t(t_1 * \aleph)$.*

Proof. $\forall t_1 \in Y, M^t(0 * t_1) = \min(M(0 * t_1), t) \succeq \min\{\min(M(0), t), \min(M(t_1 * \aleph), t)\} = \min\{\min(M(t_1 * \aleph), t), \min(M(t_1 * \aleph), t)\} \succeq \min\{\min\{\min(M(t_1 * \aleph), t), \min(M(t_1 * \aleph), t)\}, \min(M(t_1 * \aleph), t)\} = \min(M(t_1 * \aleph), t) = M^t(t_1 * \aleph)$ and $\hat{B}^t(0 * t_1) = \text{rmin}(\hat{B}(0 * t_1), t') \succeq \text{rmin}\{\text{rmin}(\hat{B}(0), t'), \text{rmin}(\hat{B}(t_1 * \aleph), t')\} = \text{rmin}\{\text{rmin}(\hat{B}(t_1 * \aleph), t'), \text{rmin}(\hat{B}(t_1 * \aleph), t')\} \succeq \text{rmin}\{\text{rmin}\{\text{rmin}(\hat{B}(t_1 * \aleph), t'), \text{rmin}(\hat{B}(t_1 * \aleph), t')\}, \text{rmin}(\hat{B}(t_1 * \aleph), t')\} = \text{rmin}(\hat{B}(t_1 * \aleph), t') = \hat{B}^t(t_1 * \aleph)$ and $J^t(0 * t_1) = \max(J(0 * t_1), \mathfrak{S}) \preceq \max\{\max(J(0), \mathfrak{S}), \max(J(t_1 * \aleph), \mathfrak{S})\} = \max\{\max(J(t_1 * \aleph), \mathfrak{S}), \max(J(t_1 * \aleph), \mathfrak{S})\} \preceq \max\{\max\{\max(J(t_1 * \aleph), \mathfrak{S}), \max(J(t_1 * \aleph), \mathfrak{S})\}, \max(J(t_1 * \aleph), \mathfrak{S})\} = \max(J(t_1 * \aleph), \mathfrak{S}) = J^t(t_1 * \aleph)$ \square

Lemma 3.11. *If a T-MBJ neutrosophic set $\mathcal{C}^t = (M^t, \hat{B}^t, J^t)$ of Y is a T-MBJ neutrosophic M -subalgebra, then $\mathcal{C}^t((t_1 * \aleph) * (t_2 * \aleph)) = \mathcal{C}^t((t_1 * \aleph) * (0 * (0 * (t_2 * \aleph)))) \forall t_1, t_2 \in Y$.*

Proof. Let Y be a G -algebra and $t_1, t_2 \in Y$. Then we know by lemma that $t_2 * \aleph = 0 * (0 * (t_2 * \aleph))$. Hence, $M^t((t_1 * \aleph) * (t_2 * \aleph)) = M^t((t_1 * \aleph) * (0 * (0 * (t_2 * \aleph))))$ and $\hat{B}^t((t_1 * \aleph) * (t_2 * \aleph)) = \hat{B}^t((t_1 * \aleph) * (0 * (0 * (t_2 * \aleph))))$ and $J^t((t_1 * \aleph) * (t_2 * \aleph)) = J^t((t_1 * \aleph) * (0 * (0 * (t_2 * \aleph))))$. Therefore, $\mathcal{C}^t((t_1 * \aleph) * (t_2 * \aleph)) = \mathcal{C}^t((t_1 * \aleph) * (0 * (0 * (t_2 * \aleph))))$ \square

Proposition 3.12. *If T-MBJ neutrosophic set $\mathcal{C}^t = (M^t, \hat{B}^t, J^t)$ of Y is a T-MBJ NMSU, then $\forall t_1, t_2 \in Y, M^t((t_1 * \aleph) * (0 * (t_2 * \aleph))) \succeq \min\{M^t(t_1 * \aleph), M^t(t_2 * \aleph)\}$ and $\hat{B}^t((t_1 * \aleph) * (0 * (t_2 * \aleph))) \succeq \text{rmin}\{\hat{B}^t(t_1 * \aleph), \hat{B}^t(t_2 * \aleph)\}$ and $J^t((t_1 * \aleph) * (0 * (t_2 * \aleph))) \preceq \max\{J^t(t_1 * \aleph), J^t(t_2 * \aleph)\}$.*

Proof. Let $t_1, t_2 \in Y$. Then we have $M^t((t_1 * \aleph) * (0 * (t_2 * \mathfrak{R}))) = \min(M((t_1 * \aleph) * (0 * (t_2 * \mathfrak{R}))), t) \succeq \min\{\min(M(t_1 * \aleph), t), \min(M(0 * (t_2 * \mathfrak{R})), t)\} \succeq \min\{\min(M(t_1 * \aleph), t), \min(M(t_2 * \mathfrak{R}), t)\} = \min\{M^t(t_1 * \aleph), M^t(t_2 * \mathfrak{R})\}$ and $\hat{B}^t((t_1 * \aleph) * (0 * (t_2 * \mathfrak{R}))) = r\min(\hat{B}((t_1 * \aleph) * (0 * (t_2 * \mathfrak{R}))), t') \succeq r\min\{r\min(\hat{B}(t_1 * \aleph), t'), r\min(\hat{B}(0 * (t_2 * \mathfrak{R})), t')\} \succeq r\min\{r\min(\hat{B}(t_1 * \aleph), t'), r\min(\hat{B}(t_2 * \mathfrak{R}), t')\} = r\min\{\hat{B}^t(t_1 * \aleph), \hat{B}^t(t_2 * \mathfrak{R})\}$ and $J^t((t_1 * \aleph) * (0 * (t_2 * \mathfrak{R}))) = \max(J((t_1 * \aleph) * (0 * (t_2 * \mathfrak{R}))), \mathfrak{S}) \preceq \max\{\max(J(t_1 * \aleph), \mathfrak{S}), \max(J(0 * (t_2 * \mathfrak{R})), \mathfrak{S})\} \preceq \max\{\max(J(t_1 * \aleph), t), \max(J(t_2 * \mathfrak{R}), \mathfrak{S})\} = \max\{J^t(t_1 * \aleph), J^t(t_2 * \mathfrak{R})\}$ by Definition and Proposition. \square

Proposition 3.13. *If T-MBJ neutrosophic set $\mathcal{C}^t = (M^t, \hat{B}^t, J^t)$ of Y fulfills the following statements, then \mathcal{C}^t refers to a T-MBJ NMSU of Y .*

- (1) $M^t(0 * t_1) \succeq M^t(t_1 * \aleph)$ and $\hat{B}^t(0 * t_1) \succeq \hat{B}^t(t_1 * \aleph)$ and $J^t(0 * t_1) \preceq J^t(t_1 * \aleph) \forall t_1 \in Y$.
- (2) $M^t(t_1 * (0 * t_2)) \succeq \min\{M^t(t_1 * \aleph), M^t(t_1 * \mathfrak{R})\}$ and $\hat{B}^t(t_1 * (0 * t_2)) \succeq r\min\{\hat{B}^t(t_1 * \aleph), \hat{B}^t(t_1 * \mathfrak{R})\}$ and $J^t(t_1 * (0 * t_2)) \preceq \max\{J^t(t_1 * \aleph), J^t(t_1 * \mathfrak{R})\} \forall t_1, t_2 \in Y$ and $t \in [0, 1]$.

Proof. Let T-MBJ neutrosophic set $\mathcal{C}^t = (M^t, \hat{B}^t, J^t)$ of Y fulfills the above statements (1 and 2). Then by Lemma 3.11, we have $M^t((t_1 * \aleph) * (t_2 * \mathfrak{R})) = \{\min(M((t_1 * \aleph) * (t_2 * \mathfrak{R})), t)\} = \{\min(M(t_1 * (0 * (0 * t_2))), t)\} \succeq \min\{\min(M(t_1 * \aleph), t), \min(M(0 * t_2), t)\} \succeq \min\{\min(M(t_1 * \aleph), t), \min(M(0 * t_2), t)\} = \min\{M^t(t_1 * \aleph), M^t(t_1 * \mathfrak{R})\}$ and $\hat{B}^t((t_1 * \aleph) * (t_2 * \mathfrak{R})) = \{r\min(\hat{B}((t_1 * \aleph) * (t_2 * \mathfrak{R}))), t'\} = \{r\min(\hat{B}(t_1 * (0 * (0 * t_2))), t')\} \succeq r\min\{r\min(\hat{B}(t_1 * \aleph), t), r\min(\hat{B}(0 * t_2), t')\} \succeq r\min\{r\min(\hat{B}(t_1 * \aleph), t), r\min(\hat{B}(0 * t_2), t')\} = r\min\{\hat{B}^t(t_1 * \aleph), \hat{B}^t(t_1 * \mathfrak{R})\}$ and $J^t((t_1 * \aleph) * (t_2 * \mathfrak{R})) = \{\max(J((t_1 * \aleph) * (t_2 * \mathfrak{R})), \mathfrak{S})\} = \{\max(J(t_1 * (0 * (0 * t_2))), \mathfrak{S})\} \preceq \max\{\max(J(t_1 * \aleph), t), \max(J(0 * t_2), \mathfrak{S})\} \preceq \max\{\max(J(t_1 * \aleph), \mathfrak{S}), \max(J(0 * t_2), \mathfrak{S})\} = \max\{J^t(t_1 * \aleph), J^t(t_1 * \mathfrak{R})\} \forall t_1, t_2 \in Y$. Hence, \mathcal{C}^t is T-MBJ NMSU of Y . \square

Theorem 3.14. *The T-MBJ neutrosophic set $\mathcal{C}^t = (M^t, \hat{B}^t, J^t)$ of Y is a T-MBJ NMSU of $Y \iff M^t$ and $\hat{B}^{t-}, \hat{B}^{t+}$ and J^t are fuzzy subalgebra of Y .*

Proof. Suppose $M^t, \hat{B}^{t-}, \hat{B}^{t+}$ and J^t are fuzzy subalgebra of Y and $t_1, t_2 \in Y$ and $t, t', \mathfrak{S} \in [0, 1]$. Then $M^t((t_1 * \aleph) * (t_2 * \mathfrak{R})) = \{\min(M((t_1 * \aleph) * (t_2 * \mathfrak{R})), t)\} \succeq \min\{\min(M(t_1 * \aleph), t), \min(M(t_2 * \mathfrak{R}), t)\} = \min\{M^t(t_1 * \aleph), M^t(t_2 * \mathfrak{R})\}$ and $\hat{B}^{t-}((t_1 * \aleph) * (t_2 * \mathfrak{R})) = \{r\min(\hat{B}^-((t_1 * \aleph) * (t_2 * \mathfrak{R}))), t'\} \succeq r\min\{r\min(\hat{B}^-(t_1 * \aleph), t'), r\min(\hat{B}^-(t_2 * \mathfrak{R}), t')\} = r\min\{\hat{B}^{t-}(t_1 * \aleph), \hat{B}^{t-}(t_2 * \mathfrak{R})\}$ and $\hat{B}^{t+}((t_1 * \aleph) * (t_2 * \mathfrak{R})) = \{r\min(\hat{B}^+((t_1 * \aleph) * (t_2 * \mathfrak{R}))), t'\} \succeq r\min\{r\min(\hat{B}^+(t_1 * \aleph), t'), r\min(\hat{B}^+(t_2 * \mathfrak{R}), t')\} = r\min\{\hat{B}^{t+}(t_1 * \aleph), \hat{B}^{t+}(t_2 * \mathfrak{R})\}$ and $J^t((t_1 * \aleph) * (t_2 * \mathfrak{R})) = \{\max(J((t_1 * \aleph) * (t_2 * \mathfrak{R})), \mathfrak{S})\} \preceq \max\{\max(J(t_1 * \aleph), \mathfrak{S}), \max(J(t_2 * \mathfrak{R}), \mathfrak{S})\} = \max\{J^t(t_1 * \aleph), J^t(t_2 * \mathfrak{R})\}$. Now, $\hat{B}^t((t_1 * \aleph) * (t_2 * \mathfrak{R})) = [\hat{B}^{t-}((t_1 * \aleph) * (t_2 * \mathfrak{R})), \hat{B}^{t+}((t_1 * \aleph) * (t_2 * \mathfrak{R}))] = [r\min(\hat{B}^-((t_1 * \aleph) * (t_2 * \mathfrak{R}))), t'), r\min(\hat{B}^+((t_1 * \aleph) * (t_2 * \mathfrak{R}))), t')] \succeq$

$[rmin\{rmin(\hat{B}^-(t_1 * \aleph), t'), rmin(\hat{B}^-(t_1 * \aleph), t')\}, rmin\{rmin(\hat{B}^+(t_1 * \aleph), t'), rmin(\hat{B}^+(t_1 * \aleph), t')\}] = [rmin\{\hat{B}^{t-}(t_1 * \aleph), \hat{B}^{t-}(t_1 * \aleph)\}, rmin\{\hat{B}^{t+}(t_1 * \aleph), \hat{B}^{t+}(t_1 * \aleph)\}] \succeq rmin\{[\hat{B}^{t-}(t_1 * \aleph), \hat{B}^{t+}(t_1 * \aleph)], [\hat{B}^{t-}(t_1 * \aleph), \hat{B}^{t+}(t_1 * \aleph)]\} = rmin\{\hat{B}^t(t_1 * \aleph), \hat{B}^t(t_1 * \aleph)\}$. Therefore, \mathcal{C}^t is T-MBJ NMSU of Y .

Conversely, assume that \mathcal{C}^t is a T-MBJ NMSU of Y . For any $t_1, t_2 \in Y$, $M^t((t_1 * \aleph) * (t_2 * \aleph)) = \{min(M((t_1 * \aleph) * (t_2 * \aleph)), t)\} \succeq min\{min(M(t_1 * \aleph), t), min(M(t_2 * \aleph), t)\} = min\{M^t(t_1 * \aleph), M^t(t_2 * \aleph)\}$ and $[\hat{B}^{t-}((t_1 * \aleph) * (t_2 * \aleph)), \hat{B}^{t+}((t_1 * \aleph) * (t_2 * \aleph))] = \hat{B}^t((t_1 * \aleph) * (t_2 * \aleph)) = \{rmin(\hat{B}^t((t_1 * \aleph) * (t_2 * \aleph)), t')\} \succeq rmin\{rmin(\hat{B}^t(t_1 * \aleph), t'), rmin(\hat{B}^t(t_2 * \aleph), t')\} = rmin\{[rmin(\hat{B}^-(t_1 * \aleph), t'), rmin(\hat{B}^+(t_1 * \aleph), t'), rmin(\hat{B}^-(t_2 * \aleph), t'), rmin(\hat{B}^+(t_2 * \aleph), t')]\} = [rmin\{rmin(\hat{B}^-(t_1 * \aleph), t'), rmin(\hat{B}^-(t_2 * \aleph), t')\}, rmin\{rmin(\hat{B}^+(t_1 * \aleph), t'), rmin(\hat{B}^+(t_2 * \aleph), t')\}] = [rmin\{\hat{B}^{t-}(t_1 * \aleph), \hat{B}^{t-}(t_2 * \aleph)\}, rmin\{\hat{B}^{t+}(t_1 * \aleph), \hat{B}^{t+}(t_2 * \aleph)\}]. Thus, $\hat{B}^{t-}((t_1 * \aleph) * (t_2 * \aleph)) \succeq min\{\hat{B}^{t-}(t_1 * \aleph), \hat{B}^{t-}(t_2 * \aleph)\}$, $\hat{B}^{t+}((t_1 * \aleph) * (t_2 * \aleph)) \succeq min\{\hat{B}^{t+}(t_1 * \aleph), \hat{B}^{t+}(t_2 * \aleph)\}$ and $J^t((t_1 * \aleph) * (t_2 * \aleph)) = \{max(J((t_1 * \aleph) * (t_2 * \aleph)), \mathfrak{S})\} \preceq max\{max(J(t_1 * \aleph), \mathfrak{S}), max(J(t_2 * \aleph), \mathfrak{S})\} = max\{J^t(t_1 * \aleph), J^t(t_2 * \aleph)\}$. Hence M^t and $\hat{B}^{t+}, \hat{B}^{t-}$ and J^t are fuzzy subalgebra of Y . $\square$$

Theorem 3.15. *Let $\mathcal{C}^t = (M^t, \hat{B}^t, J^t)$ be a T-MBJ NMSU of Y . Then the sets I_{M^t} , $I_{\hat{B}^t}$ and I_{J^t} which are defined as $I_{M^t} = \{t_1 \in Y \mid M^t(t_1 * \aleph) = M^t(0)\}$, $I_{\hat{B}^t} = \{t_1 \in Y \mid \hat{B}^t(t_1 * \aleph) = \hat{B}^t(0)\}$ and $I_{J^t} = \{t_1 \in Y \mid J^t(t_1 * \aleph) = J^t(0)\}$ are T-MBJ neutrosophic M-subalgebra of Y .*

Proof. Let $t_1, t_2 \in I_{M^t}$. Then $M^t(t_1 * \aleph) = M^t(0) = M^t(t_2 * \aleph)$ and so, $M^t((t_1 * \aleph) * (t_2 * \aleph)) = \{min(M((t_1 * \aleph) * (t_2 * \aleph)), t)\} \succeq min\{(min(M(t_1 * \aleph), t), (min(M(t_2 * \aleph), t))\} = M^t(0)$. By using Proposition 3.4, as we know that $M^t((t_1 * \aleph) * (t_2 * \aleph)) = M^t(0)$ or equivalently $(t_1 * \aleph) * (t_2 * \aleph) \in I_{M^t}$.

Now we let $t_1, t_2 \in I_{\hat{B}^t}$. Then $\hat{B}^t(t_1 * \aleph) = \hat{B}^t(0) = \hat{B}^t(t_2 * \aleph)$ and so, $\hat{B}^t((t_1 * \aleph) * (t_2 * \aleph)) = \{rmin(\hat{B}^t((t_1 * \aleph) * (t_2 * \aleph)), t')\} \succeq rmin\{(rmin(\hat{B}^t(t_1 * \aleph), t'), (rmin(\hat{B}^t(t_2 * \aleph), t'))\} = rmin\{\hat{B}^t(t_1 * \aleph), \hat{B}^t(t_2 * \aleph)\} = \hat{B}^t(0)$. By using Proposition 3.4, as we know that $\hat{B}^t((t_1 * \aleph) * (t_2 * \aleph)) = \hat{B}^t(0)$ or equivalently $(t_1 * \aleph) * (t_2 * \aleph) \in I_{\hat{B}^t}$.

Again we let $t_1, t_2 \in I_{J^t}$. Then $J^t(t_1 * \aleph) = J^t(0) = J^t(t_2 * \aleph)$ and so, $J^t((t_1 * \aleph) * (t_2 * \aleph)) = \{max(J((t_1 * \aleph) * (t_2 * \aleph)), t)\} \preceq max\{(max(J(t_1 * \aleph), t), (max(J(t_2 * \aleph), t))\} = J^t(0)$. Again by using Proposition 3.4, as we know that $J^t((t_1 * \aleph) * (t_2 * \aleph)) = J^t(0)$ or equivalently $(t_1 * \aleph) * (t_2 * \aleph) \in I_{J^t}$. Hence the sets I_{M^t} , $I_{\hat{B}^t}$ and I_{J^t} are subalgebra of Y . \square

Definition 3.16. Let $\mathcal{C}^t = \{M^t, \hat{B}^t, J^t\}$ be a T-MBJ neutrosophic set of Y . For $[s_1, s_2] \in D[0, 1]$ and $t_1, t_2 \in [0, 1]$, the set $U(M^t \mid \acute{t}) = \{t_1 \in Y \mid M^t(t_1 * \aleph) \succeq \acute{t}\}$ is called upper t_1 -level of \mathcal{C}^t and the set $U(\hat{B}^t \mid [s_1, s_2]) = \{s_1, s_2 \in Y \mid \hat{B}^t(t_1 * \aleph) \succeq [s_1, s_2]\}$ is called upper $[s_1, s_2]$ -level

of \mathcal{C}^t and $L(J^t | \acute{t}) = \{t_1 \in Y \mid J^t(t_1 * \aleph) \preceq \acute{t} \text{ is called lower } (t_1 * \aleph)\text{-level of } \mathcal{C}^t.$

Theorem 3.17. *If $\mathcal{C}^t = (M^t, \hat{B}^t, J^t)$ is T-MBJ NMSU of Y , then the upper \acute{t} -level, upper $[s_1, s_2]$ -level and lower \acute{t} -level of \mathcal{C}^t are subalgebra of Y .*

Proof. Let $t_1, t_2 \in U(M^t | \acute{t})$. Then $M^t(t_1 * \aleph) \succeq \acute{t}$ and $M^t(t_2 * \aleph) \succeq \acute{t}$. It follows that $M^t((t_1 * \aleph) * (t_2 * \aleph)) = \{min(M((t_1 * \aleph) * (t_2 * \aleph)), t)\} \succeq min\{min(M(t_1 * \aleph), M(t_2 * \aleph)), t\} = min\{min(M(t_1 * \aleph), t), min(M(t_2 * \aleph), t)\} = min\{M^t(t_1 * \aleph), M^t(t_2 * \aleph)\} \succeq \acute{t} \Rightarrow (t_1 * \aleph) * (t_2 * \aleph) \in U(M^t | \acute{t})$. Hence $U(M^t | \acute{t})$ is a subalgebra of Y . Let $t_1, t_2 \in U(\hat{B}^t | [s_1, s_2])$. Then $\hat{B}^t(t_1 * \aleph) \succeq [s_1, s_2]$ and $\hat{B}^t(t_2 * \aleph) \succeq [s_1, s_2]$. It follows that $\hat{B}^t((t_1 * \aleph) * (t_2 * \aleph)) = \{rmin(\hat{B}((t_1 * \aleph) * (t_2 * \aleph)), t')\} \succeq rmin\{rmin(\hat{B}(t_1 * \aleph), \hat{B}(t_2 * \aleph)), t'\} = rmin\{rmin(\hat{B}(t_1 * \aleph), t'), rmin(\hat{B}(t_2 * \aleph), t')\} = rmin\{\hat{B}^t(t_1 * \aleph), \hat{B}^t(t_2 * \aleph)\} \succeq [s_1, s_2] \Rightarrow (t_1 * \aleph) * (t_2 * \aleph) \in U(\hat{B}^t | [s_1, s_2])$. Hence, $U(\hat{B}^t | [s_1, s_2])$ is a subalgebra of Y . Let $t_1, t_2 \in L(J^t | \acute{t})$. Then $J^t(t_1 * \aleph) \preceq \acute{t}$ and $J^t(t_2 * \aleph) \preceq \acute{t}$. It follows that $J^t((t_1 * \aleph) * (t_2 * \aleph)) = \{max(J((t_1 * \aleph) * (t_2 * \aleph)), \Im)\} \preceq max\{max(J(t_1 * \aleph), J(t_2 * \aleph)), \Im\} = max\{max(J(t_1 * \aleph), t), max(J(t_2 * \aleph), t)\} = max\{J^t(t_1 * \aleph), J^t(t_2 * \aleph)\} \preceq \acute{t} \Rightarrow (t_1 * \aleph) * (t_2 * \aleph) \in L(J^t | \acute{t})$. Hence $L(J^t | \acute{t})$ is a subalgebra of Y . \square

Theorem 3.18. *Any subalgebra of Y can be considered as upper \acute{t} -level, upper $[s_1, s_2]$ -level and lower \acute{t} -level of some T-MBJ NMSU of Y .*

Proof. Let \mathcal{D}^t be a T-MBJ NMSU of Y , and \mathcal{C}^t be a T-MBJ neutrosophic set on Y defined by

$$M^t = \begin{cases} [\nu] & \text{if } t_1 \in \mathcal{D}^t \\ 1, & \text{otherwise.} \end{cases} \quad \hat{B}^t = \begin{cases} [\mu_1, \mu_2] & \text{if } t_1 \in \mathcal{D}^t \\ [0, 0] & \text{otherwise.} \end{cases}, \quad J^t = \begin{cases} [\nu] & \text{if } t_1 \in \mathcal{D}^t \\ 0, & \text{otherwise.} \end{cases}$$

$\forall [\mu_1, \mu_2] \in D[0, 1]$ and $\nu \in [0, 1]$. Now we discuss the following cases.

Case 1. If $\forall t_1, t_2 \in \mathcal{D}^t$ then $M^t(t_1 * \aleph) = \nu$, $\hat{B}^t(t_1 * \aleph) = [\mu_1, \mu_2]$, $J^t(t_1 * \aleph) = \nu$ and $M^t(t_2 * \aleph) = \nu$, $\hat{B}^t(t_2 * \aleph) = [\mu_1, \mu_2]$, $J^t(t_2 * \aleph) = \nu$. Thus $M^t((t_1 * \aleph) * (t_2 * \aleph)) = \nu = min\{\nu, \nu\} = min\{M^t(t_1 * \aleph), M^t(t_2 * \aleph)\}$ and $\hat{B}^t((t_1 * \aleph) * (t_2 * \aleph)) = [\mu_1, \mu_2] = rmin\{[\mu_1, \mu_2], [\mu_1, \mu_2]\} = rmin\{\hat{B}^t(t_1 * \aleph), \hat{B}^t(t_2 * \aleph)\}$ and $J^t((t_1 * \aleph) * (t_2 * \aleph)) = \nu = max\{\nu, \nu\} = max\{J^t(t_1 * \aleph), J^t(t_2 * \aleph)\}$.

Case 2. If $t_1 \in \mathcal{R}^t$ and $t_2 \notin \mathcal{R}^t$, then $M^t(t_1 * \aleph) = \nu$, $\hat{B}^t(t_1 * \aleph) = [\mu_1, \mu_2]$, $J^t(t_1 * \aleph) = \nu$ and $M^t(t_2 * \aleph) = 0$, $\hat{B}^t(t_2 * \aleph) = [0, 0]$, $J^t(t_2 * \aleph) = 1$. Thus $M^t((t_1 * \aleph) * (t_2 * \aleph)) \succeq 0 = min\{\nu, 0\} = min\{M^t(t_1 * \aleph), M^t(t_2 * \aleph)\}$, $\hat{B}^t((t_1 * \aleph) * (t_2 * \aleph)) \succeq [0, 0] = rmin\{[\mu_1, \mu_2], [0, 0]\} = rmin\{\hat{B}^t(t_1 * \aleph), \hat{B}^t(t_2 * \aleph)\}$ and $J^t((t_1 * \aleph) * (t_2 * \aleph)) \preceq 1 = max\{\nu, 1\} = max\{J^t(t_1 * \aleph), J^t(t_2 * \aleph)\}$.

Case 3. If $t_1 \notin \mathcal{R}^t$ and $t_2 \in \mathcal{R}^t$, then $M^t(t_1 * \aleph) = 0$, $\hat{B}^t(t_1 * \aleph) = [0, 0]$, $J^t(t_1 * \aleph) = 1$ and $M^t(t_2 * \aleph) = \nu$, $\hat{B}^t(t_2 * \aleph) = [\mu_1, \mu_2]$, $J^t(t_2 * \aleph) = \nu$. Thus $M^t((t_1 * \aleph) * (t_2 * \aleph)) \succeq 0 =$

$min\{0, \nu\} = min\{M^t(t_1 * \aleph), M^t(t_2 * \aleph)\}, \hat{B}^t((t_1 * \aleph) * (t_2 * \aleph)) \succeq [0, 0] = rmin\{[0, 0], [\mu_1, \mu_2]\}$
 $= rmin\{\hat{B}^t(t_1 * \aleph), \hat{B}^t(t_2 * \aleph)\}$ and $J^t((t_1 * \aleph) * (t_2 * \aleph)) \preceq 1 = max\{1, \nu\} = max\{J^t(t_1 * \aleph), J^t(t_2 * \aleph)\}$.

Case 4. If $t_1 \notin \aleph^t$ and $t_2 \notin \aleph^t$, then $M^t(t_1 * \aleph) = 0, \hat{B}^t(t_1 * \aleph) = [0, 0], J^t(t_1 * \aleph) = 1$ and $M^t(t_2 * \aleph) = 0, \hat{B}^t(t_2 * \aleph) = [0, 0], J^t(t_2 * \aleph) = 1$. Thus $M^t((t_1 * \aleph) * (t_2 * \aleph)) \succeq 1 = min\{0, 0\} = min\{M^t(t_1 * \aleph), M^t(t_2 * \aleph)\}, \hat{B}^t((t_1 * \aleph) * (t_2 * \aleph)) \succeq [0, 0] = rmin\{[0, 0], [0, 0]\} = rmin\{\hat{B}^t(t_1 * \aleph), \hat{B}^t(t_2 * \aleph)\}$ and $J^t((t_1 * \aleph) * (t_2 * \aleph)) \preceq 1 = max\{1, 1\} = max\{J^t(t_1 * \aleph), J^t(t_2 * \aleph)\}$. Therefore, \mathcal{C}^t is a T-MBJ NMSU of Y . \square

Theorem 3.19. *Let \mathcal{C}^t be a subset of Y and \mathcal{C}^t be a T-MBJ neutrosophic set on Y which is given in the proof of above Theorem. If \mathcal{C}^t is considered as lower level subalgebra and upper level subalgebra of some T-MBJ NMSU of Y , then \mathcal{C}^t is a T-MBJ neutrosophic cubic one of Y .*

Proof. Let \mathcal{C}^t be a T-MBJ NMSU of Y , and $t_1, t_2 \in \mathcal{C}^t$. Then $M^t(t_1 * \aleph) = M^t(t_2 * \aleph) = \gamma, \hat{B}^t(t_1 * \aleph) = \hat{B}^t(t_2 * \aleph) = [\alpha_1, \alpha_2]$ and $J^t(t_1 * \aleph) = J^t(t_2 * \aleph) = \beta$. Thus $M^t((t_1 * \aleph) * (t_2 * \aleph)) = \{min(M^t((t_1 * \aleph) * (t_2 * \aleph)), t)\} \succeq min\{min(M^t(t_1 * \aleph), t), min(M^t(t_2 * \aleph), t)\} = min\{M^t(t_1 * \aleph), M^t(t_2 * \aleph)\} = min\{\gamma, \gamma\} = \gamma, \Rightarrow (t_1 * \aleph) * (t_2 * \aleph) \in \mathcal{C}^t, \hat{B}^t((t_1 * \aleph) * (t_2 * \aleph)) = \{rmin(\hat{B}^t((t_1 * \aleph) * (t_2 * \aleph)), t')\} \succeq rmin\{rmin(\hat{B}^t(t_1 * \aleph), \hat{B}^t(t_2 * \aleph)), t'\} = rmin\{rmin(\hat{B}^t(t_1 * \aleph), t'), rmin(\hat{B}^t(t_2 * \aleph), t')\} = rmin\{\hat{B}^t(t_1 * \aleph), \hat{B}^t(t_2 * \aleph)\} = rmin\{[\alpha_1, \alpha_2], [\alpha_1, \alpha_2]\} = [\alpha_1, \alpha_2]$ and $J^t((t_1 * \aleph) * (t_2 * \aleph)) = \{max(M^t((t_1 * \aleph) * (t_2 * \aleph)), \aleph)\} \preceq max\{max(M^t(t_1 * \aleph), \aleph), max(M^t(t_2 * \aleph), \aleph)\} = max\{M^t(t_1 * \aleph), M^t(t_2 * \aleph)\} = max\{\beta, \beta\} = \beta, \Rightarrow (t_1 * \aleph) * (t_2 * \aleph) \in \mathcal{C}^t$. Hence, proof is completed. \square

4. Conclusions

In this paper, T-MBJ neutrosophic set is defined and notion of T-MBJ neutrosophic M-subalgebra is also introduced by set of conditions on G-algebra. T-MBJ neutrosophic M-subalgebra of G-algebra has investigated by p-union, P-intersection, R-union, R-intersection and some results. For future work this study will be use to discuss the normal ideals, multiplication, translation and magnification of T-MBJ neutrosophic set.

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