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Neutrosophic Φ-open sets and neutrosophic Φ-continuous functions

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Abstract: We introduce the notion of neutrosophic Φ-open set and neutrosophic Φ-continuous mapping via neutrosophic topological spaces and investigate several properties of it. By defining neutrosophic Φ-open set, neutrosophic Φ-continuous mapping, and neutrosophic Φ-open mapping, we prove some remarks, theorems on neutrosophic topological spaces.

Keywords: Neutrosophic set; Neutrosophic topology; Neutrosophic supra topology; Neutrosophic α-open set; Neutrosophic Φ-open set.

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1. Introduction

Smarandache [53] defined the Neutrosophic Set (NS) in 1998 by extending fuzzy set [58], and intuitionistic fuzzy set [2] to deal with uncertain, inconsistent and indeterminate information. An NS Θ defined over the universe Ω, $\alpha = \alpha(\xi,\psi,\zeta) \in \Theta$ with ξ,ψ and ζ being the real standard or non-standard subsets of $]0^-, 1^+[$. ξ,ψ and ζ are the degrees of true membership function, indeterminate membership function and falsity membership function respectively in the set Θ. Wang, Smarandache, Zhang, and Sunderraman [56] defined Interval NS (INS) as an instance and a subclass of NS by considering the subunitary interval [0, 1]. An INS τ defined on universe Ω, $\alpha = \alpha(\xi,\psi,\zeta) \in \tau$ with ξ,ψ and ζ being the subinterval of [0, 1]. Wang, Smarandache, Zhang, and Sunderraman [57] defined Single Valued NS (SVNS) as an instance of NS. In SVNS, the degrees of truth-membership function, indeterminacy-membership function and falsity-membership function lie in the interval [0,1]. NS has drawn many researchers' much attention for theoretical as well as practical applications [3-18, 24, 26-34, 36-46, 54-55].

Salama and Alblowi [49] grounded the concept of Neutrosophic Topological Space (NTS). Salama and Alblowi [50] also studied the generalized NS and generalized NTS. Salama, Smarandache and Alblowi [51] presented a new concept on neutrosophic crisp topology. Iswaraya and Bageerathi [23] presented the neutrosophic semi-closed set and neutrosophic semi-open set. Arokiarani, Dhavaseelan, Jafari, and Parimala [1] present the neutrosophic semi-open functions and established some relations between them. Rao and Srinivasa [48] presented neutrosophic pre-open set and pre-closed set. Dhavaseelan, Parimala, Jafari, and Smarandache [20] presented the neutrosophic semi-supra open set and neutrosophic semi-supra continuous functions.

Dhavaseelan, Ganster, Jafari, and Parimala [21] presented the neutrosophic α-supra open set and neutrosophic α-supra continuous functions. Parimala, Karthika, Dhavaseelan, & Jafari [35] presented the neutrosophic supra pre- continuous functions, the neutrosophic supra pre-open maps, and the neutrosophic supra pre-closed maps in terms of neutrosophic supra pre-open sets and neutrosophic supra pre-closed sets. Dhavaseelan, and Jafari [21] studied Generalized Neutrosophic Closed Set (GNCS). Pushpalatha and Nandhini [47] defined the GNCS in NTSs. Ebenanjar, Immaculate, and Wilfred [22] studied neutrosophic *b*-open sets in NTSs. Maheswari, Sathyabama, and Chandrasekar [25] studied the neutrosophic generalized b- closed sets in NTSs. Das and Pramanik [17] presented the generalized neutrosophic b-open sets in NTSs.

Research gap: No study on neutrosophic Φ-open sets and neutrosophic Φ-continuous functions neutrosophic generalized *b*-open set has been reported in the recent literature.

Motivation: To fill the research gap, we introduce the neutrosophic Φ-open set.

In this paper, we develop the notion of neutrosophic Φ-open set and neutrosophic Φ-continuous mapping, neutrosophic Φ-open mapping, and neutrosophic Φ-closed mapping via NTSs.

The rest of the paper is designed as follows:

Section 2 recalls the definitions neutrosophic set, neutrosophic topological space, neutrosophic supra topological space, neutrosophic α-open sets, and neutrosophic α-closed sets. Section 3 introduces neutrosophic Φ-open set, neutrosophic Φ-continuous mapping, and neutrosophic Φ-open mapping and proofs of some remarks, and theorems on neutrosophic Φ-open sets and neutrosophic Φ-continuous mapping. Section 4 presents concluding remarks.

2. Preliminaries and some properties

In this section, we discuss some existing definitions and theorems which are already defined by many researchers.

Definition 2.1. Assume that W be a universal set. Then D, an NS [53] over W is denoted as follows: D= $\{(m, T_D(m), I_D(m), F_D(m)): m \in W$ and $T_D(m), I_D(m), F_D(m) \in]^-0,1^+[\}$ where T_D, I_D and F_D are the functions from D to $]^-0,1^+[$ and for each $y \in W$, $^-0 \leq T_D(m)+I_D(m)+F_D(m) \leq 3^+$.

Definition 2.2. Assume that $D = \{(m, T_D(m), I_D(m), F_D(m)): m \in W\}$ and $K = \{(m, T_K(m), I_K(m), F_K(m)): m \in W\}$ are any two NS over W, then $D\cup K$ and $D\cap K$ [53] are defined by

i. $D\cup K = \{(m, T_D(m)\vee T_K(m), I_D(m)\wedge I_K(m), F_D(m)\wedge F_K(m)): m \in W\}$;

ii. $D\cap K = \{(m, T_D(m)\wedge T_K(m), I_D(m)\vee I_K(m), F_D(m)\vee F_K(m)): m \in W\}$.

Definition 2.3. Assume that $D = \{(m, T_D(m), I_D(m), F_D(m)): m \in W\}$ is an NS over W. Then the complement [53] of D is defined by $D^c = \{(m, 1-T_D(m), 1-I_D(m), 1-F_D(m)): m \in W\}$.

Definition 2.4. Assume that $D = \{(m, T_D(m), I_D(m), F_D(m)): m \in W\}$ and $K = \{(m, T_K(m), I_K(m), F_K(m)): m \in W\}$ are any two NSs over W. Then D is contained in K [53] if and only if $T_D(m) \leq T_K(m)$, $I_D(m) \geq I_K(m)$, $F_D(m) \geq F_K(m)$, for all $m \in W$.

Now we may consider two NSs 0_N and 1_N over W as follows:

1) $0_N = \{(m, 0, 1, 1): m \in W\}$;
2) $1_N = \{(m, 1, 0, 0): m \in W\}$.

Clearly, $0_N \subseteq 1_N$.

Definition 2.5. Assume that W is a universe of discourse and τ is the collection of some NSs over W. Then the collection τ is said to be a Neutrosophic Topology (NT) [49] on W if the following axioms hold:

1. $0_N, 1_N \in \tau$
2. $C_1, C_2 \in \tau \Rightarrow C_1 \cap C_2 \in \tau$
3. $\cup C_i \in \tau$, for every $\{C_i: i \in \Delta\} \subseteq \tau$.

The pair (W,τ) is said to be an NTS. If $H \in \tau$, then H is called a Neutrosophic Open Set (NOS) and the complement of H i.e. H^c is called a Neutrosophic Closed Set (NCS).

Example 2.1. Assume that $W = \{s_1, s_2, s_3\}$ is a set with three NSs over W as follows:
$M_1 = \{(s_1, 0.9, 0.5, 0.7), (s_2, 0.7, 0.6, 0.8), (s_3, 0.7, 0.4, 0.7): s_1, s_2, s_3 \in W\}$;
$M_2 = \{(s_1, 1.0, 0.3, 0.4), (s_2, 0.9, 0.5, 0.5), (s_3, 1.0, 0.1, 0.5): s_1, s_2, s_3 \in W\}$;
$M_3 = \{(s_1, 0.9, 0.3, 0.5), (s_2, 0.8, 0.5, 0.8), (s_3, 0.9, 0.3, 0.5): s_1, s_2, s_3 \in W\}$;
Then (W,τ) is an NTS, where $\tau = \{0_N, 1_N, M_1, M_2, M_3\}$ is an NT on W.

Remark 2.1. The collection of all NOSs and NCSs in (W, τ) may be denoted as NOS(W) and NCS(W) respectively. The neutrosophic interior and neutrosophic closure [49] of a neutrosophic subset H of W is denoted by $N_{int}(H)$ and $N_{cl}(H)$ respectively and defined as follows:
$N_{int}(H) = \cup\{D: D$ is an NOS in W and $D \subseteq H\}$,
$N_{cl}(H) = \cap\{L: L$ is an NCS in W and $H \subseteq L\}$.
Clearly $N_{int}(H) \subseteq H \subseteq N_{cl}(H)$.

Definition 2.6. Assume that (W,τ) is an NTS and H be an NS over W. Then H is

1) Neutrosophic Pre-Open (NPO) set [48] iff $H \subseteq N_{int}N_{cl}(H)$;
2) Neutrosophic Semi-Open (NSO) set [23] iff $H \subseteq N_{cl}N_{int}(H)$;
3) Neutrosophic α-Open (Nα-O) set [1] iff $H \subseteq N_{int}N_{cl}N_{int}(H)$.

Definition 2.7. Assume that W is a universal set and Ω be the collection of some NSs over W. Then Ω is said to be a Neutrosophic Supra Topology (NST) [19] on W if the following axioms hold:

1) $0_N, 1_N \in \Omega$
2) $\cup C_i \in \Omega$, for every $\{C_i: i \in \Delta\} \subseteq \Omega$.

The pair (W, Ω) is said to be a Neutrosophic Supra Topological Space (NSTS). If $H \in \Omega$, then H is called a Neutrosophic-Supra Open (N-SO) set and its complement H^c is called a Neutrosophic-Supra Closed (N-SC) set in (W, Ω). The neutrosophic-supra interior and neutrosophic-supra closure of an NS H is denoted by $N_{int}^{\Omega}(H)$ and $N_{cl}^{\Omega}(H)$ respectively and are defined as follows:
$N_{int}^{\Omega}(H)=\cup\{D: D$ is an N-SO set in W and $D \subseteq H\}$,
$N_{cl}^{\Omega}(H)=\cap\{L: L$ is an N-SC set in W and $H \subseteq L\}$.

Definition 2.8. Assume that (W, Ω) be an NSTS and H is an NS over W. Then H is

1) Neutrosophic-Pre Supra Open (N-PSO) set [35] iff $H \subseteq N_{int}^{\Omega}(N_{cl}^{\Omega}(H))$;
2) Neutrosophic-Semi Supra Open (N-SSO) set [20] if and only if $H \subseteq N_{cl}^{\Omega}(N_{int}^{\Omega}(H))$;
3) Neutrosophic-α-Supra Open (N-αSO) set [19] if and only if $H \subseteq N_{int}^{\Omega}(N_{cl}^{\Omega}(N_{int}^{\Omega}(H)))$.

The complement of N-PSO set, N-SSO set and N-αSO set are called Neutrosophic Pre Supra-Closed (N-PSC) set, Neutrosophic Semi Supra-Closed (N-SSC) set and Neutrosophic α-Supra-Closed (N-αSC) set respectively.

Theorem 2.1. Assume that (W, Ω) be an NSTS.

Then

i. Every N-SO set is an N-αSO set.
ii. Every N-αSO set is an N-PSO set (N-SSO set).

For proof, see Parimala, Karthika, Dhavaseelan, and Jafari (2018).

Theorem 2.2. Assume that (W, Ω) be an NSTS.

Then

i. Union of two N-αSO sets is an N-αSO set.
ii. Intersection of two N-αSO sets may not be an N-αSO set in general.

For proof, see [19].

Definition 2.9. Let (W, Ω) and (M,Π) be any two NTSs. Then a function ξ:(W, Ω)→(Y, M) is called a neutrosophic continuous function [52] if the inverse image of each NOS G in M is an NOS in W.

Definition 2.10. Let (W, Ω) and (M,Π) be any two NSTSs. Then a function ξ:(W, Ω)→(Y, M) is called a neutrosophic supra continuous [19} if and only if the inverse image of each N-SO set G in M is an N-SO set in W.

Definition 2.11. A function ξ:(W, Ω)→(M,Π), where (W, Ω) and (M,Π) are two NSTSs is said to be a neutrosophic α-supra [19] continuous iff $\xi^{-1}(G)$ is an N-αSO set in W whenever G is an N-SO set in M.

Theorem 2.3. Assume that ξ be a function from an NSTS (W, Ω) to another NSTS (M,Π). Then the following statements [19] are equivalent:

i. ξ is a neutrosophic α-supra continuous mapping.

ii. $\xi^{-1}(G)$ is an N-αSC set in W whenever G is an N-SC set in M.

3. Neutrosophic Φ-open set and neutrosophic Φ-continuous mapping

Definition 3.1. Assume that (W, τ) is an NTS and H s an NS over W. Then H is called a Neutrosophic Φ-Open (N-Φ-O) set iff there exist an Nα-O set K such that $K \subseteq H \subseteq N_{cl}(K)$, where $N_{cl}(K)$ denotes the neutrosophic closure of K with respect to the NT τ on W.

Theorem 3.1. In an NTS (W, τ),

1) Every NOS is a neutrosophic Φ-open set;
2) Every Nα-O set is a neutrosophic Φ-open set.

Proof.

1) Assume that Q is an NOS in an NTS (W, τ). Since every NOS is an Nα-O set, so Q is an Nα-O set in (W, τ). Clearly $Q \subseteq Q \subseteq N_{cl}(Q)$. Therefore, Q is a neutrosophic Φ-open set. Hence every NOS in (W, τ) is a neutrosophic Φ-open set.
2) Assume that R is an Nα-O set in an NTS (W, τ). For any neutrosophic set R, $R \subseteq R \subseteq N_{cl}(R)$. Therefore, there exists an Nα-O set R in (W, τ) such that $R \subseteq R \subseteq N_{cl}(R)$.
   Hence R is a neutrosophic Φ-open set.
   Thus, every Nα-O set in (W, τ) is a neutrosophic Φ-open set.

Theorem 3.2. Assume that (W,τ) is an NTS and θ is a neutrosophic supra topology such that $\tau \subseteq \theta$. Then

1) Every neutrosophic Φ-open set in (W,τ) is a neutrosophic Φ-supra open set in (W, θ);
2) Every NOS in (W,τ) is a neutrosophic Φ-supra open set in (W, θ).

Proof.

1) Assume that (W,τ) is an NTS and θ is an NST such that $\tau \subseteq \theta$.
   Assume that Q is an arbitrary neutrosophic Φ-open set in (W,τ).
   Then there exists an Nα-O set K such that $K \subseteq Q \subseteq N_{cl}(K)$, where $N_{cl}(K)$ denotes the neutrosophic closure of K with respect to the topology τ.
   Since $\tau \subseteq \theta$ and θ is an NST on W, so $N_{cl}(K) \subseteq N_{cl}^{\theta}(K)$, where $N_{cl}^{\theta}(K)$ denotes the neutrosophic supra-closure of K with respect to the NST θ.
   Therefore $K \subseteq Q \subseteq N_{cl}^{\theta}(K)$.

Hence Q is a neutrosophic Φ-supra open set in (W, θ).

2) Assume that (W,τ) is an NTS and θ be an NST on W such that $\subseteq \theta$.
Assume that Q be an arbitrary NOS in (W,τ). From Theorem 3.1, it is clear that every NOS in an NTS (W,τ) is a neutrosophic Φ-open set. So, Q is a neutrosophic Φ-open set in (W,τ). From the first part of the theotem 3.2, it is clear that Q is a neutrosophic Φ-open set in (W, θ). Hence every NOS in an NTS (W, τ) is a neutrosophic Φ-supra open set in the NSTS (W, θ).

Lemma 3.1. In an NTS (W,τ), the union of two neutrosophic Φ-open sets is a neutrosophic Φ-open set.

Proof.
Assume that K and L are any two neutrosophic Φ-open sets in an NTS (W,τ). Then there exist two Nα-O sets Q_1 and Q_2 in (W,τ) such that $Q_1 \subseteq K \subseteq N_{cl}(Q_1)$, $Q_2 \subseteq L \subseteq N_{cl}(Q_2)$.
Now, $Q_1 \cup Q_2 \subseteq K \cup L \subseteq N_{cl}(Q_1) \cup N_{cl}(Q_2) = N_{cl}(Q_1 \cup Q_2)$ and $Q_1 \cup Q_2$ is an Nα-O set in (W,τ). Therefore $K \cup L$ is a neutrosophic Φ-open set in (W,τ). Hence the union of two neutrosophic Φ-open sets in an NTS (W,τ) is a neutrosophic Φ-open set.

Theorem 3.3. Assume that (W,τ) is an NTS. Then

1) Union of an NOS and a neutrosophic Φ-open set is a neutrosophic Φ-open set.
2) Union of an Nα-O set and a neutrosophic Φ-open set is a neutrosophic Φ-open set.

Proof. Let Q be an NOS and R be a neutrosophic Φ-open set in an NTS (W,τ). From Theorem 3.1, Q is a neutrosophic Φ-open set. Again, from Lemma 3.1, it is clear that $Q \cup R$ is a neutrosophic Φ-open set in (W,τ).

1) Assume that H is an Nα-O set and G is a neutrosophic Φ-open set in an NTS (W,τ). From Theorem 3.1, it is clear that H is a neutrosophic Φ-open set. Again, from Remark 3.1, it is clear that $H \cup G$ is a neutrosophic Φ-open set in (W,τ).

Definition 3.2. Assume that (W,τ) and (M,δ) are two NTSs. Then a function ξ: (W, τ)→ (M,δ) is called a neutrosophic Φ-continuous function iff the inverse image of every NOS G in M is a neutrosophic Φ-open set in W.

Definition 3.3. Assume that (W,τ), and (M,δ) are two NTSs and θ is an NST on W such that $\tau \subseteq \theta$. Then a function ξ: (W, τ)→ (M,δ) is called a neutrosophic Φ-supra continuous function iff the inverse image of every NOS G in M is a neutrosophic Φ-supra open set in W with respect to the NST θ on W.

Theorem 3.3. Every neutrosophic continuous function from an NTS (W,τ) to another NTS (M,δ) is a neutrosophic Φ-continuous function.

Proof. Assume that $\xi:(W, \tau)\rightarrow(M,\delta)$ is a neutrosophic continuous function and K be an arbitrary NOS in M. Then by hypothesis, $\xi^{-1}(K)$ is an NOS in W. Since each NOS is a neutrosophic Φ-open set, so $\xi^{-1}(K)$ is a neutrosophic Φ-open set in W. Therefore, for each NOS K in M, $\xi^{-1}(K)$ is a neutrosophic Φ-open set in W. Hence ξ is a neutrosophic Φ-continuous function. Therefore, every neutrosophic continuous function is a neutrosophic Φ-continuous function.

Theorem 3.4. Assume that (W,τ) and (M,δ) are two NTSs and $\tau \subseteq \theta$, where θ is an NST on W. Then every neutrosophic Φ-continuous function from (W,τ) to (M,δ) is a neutrosophic Φ-supra continuous function from (W,θ) to (M,δ).

Proof. Assume that $\xi:(W, \tau)\rightarrow(M,\delta)$ is a neutrosophic Φ-continuous mapping. Let θ be an NST such that $\tau \subseteq \theta$. Let T be an NOS in M. Then by hypothesis $\xi^{-1}(T)$ is a neutrosophic Φ-open set in W. Since each neutrosophic Φ-open set (W,τ) is a neutrosophic Φ-supra open set in (W,θ), so $\xi^{-1}(T)$ is a neutrosophic Φ-supra open set in (W,θ). Therefore ξ is a neutrosophic Φ-supra continuous mapping from (W, θ) to (M, δ).

Definition 3.4. Let (W,τ) and (M,δ) be two NTSs. A function $\xi:(W, \tau)\rightarrow (M, \delta)$ is called a neutrosophic Φ-open function if $\xi(Q)$ is a neutrosophic Φ-open set in M for each NOS Q in W.

Definition 3.5. Let (W,τ) and (M,δ) be two NTSs. A function $\xi:(W,\tau)\rightarrow(M,\delta)$ is called a neutrosophic Φ-closed function if $\xi(Q)$ is a neutrosophic Φ-closed set in M for each NCS Q in W.

Theorem 3.5. Assume that (W,τ) and (M,δ) are any two NTSs. Then $\xi:(W,\tau)\rightarrow(M,\delta)$ is a neutrosophic Φ-open function iff $\xi(N_{int}(K))\subseteq N_{int}(\xi(K))$, for each neutrosophic subset K of W.

Proof. Let $\xi:(W,\tau)\rightarrow(M,\delta)$ be a neutrosophic Φ-open function and K be a neutrosophic subset of W. Clearly $N_{int}(K)$ is an NOS in W and $N_{int}(K) \subseteq K$. Since ξ is a neutrosophic Φ-open function, so $\xi(N_{int}(K))$ is a neutrosophic Φ-open set in M and $\xi(N_{int}(K))\subseteq \xi(K)$. Since each NOS is a neutrosophic Φ-open set and $N_{int}(\xi(K))$ is the largest NOS contained in $\xi(K)$, so $N_{int}(\xi(K))$ is the largest neutrosophic Φ-open set contained in $\xi(K)$. Therefore $\xi(N_{int}(K)) \subseteq N_{int}(\xi(K)) \subseteq \xi(K)$ i.e. $\xi(N_{int}(K)) \subseteq N_{int}(\xi(K))$. Hence for each neutrosophic subset K of W, $\xi(N_{int}(K))\subseteq N_{int}(\xi(K))$.

Conversely, let L be an NOS in (W, τ). Therefore, $N_{int}(L) = L$. Now by hypothesis $\xi(N_{int}(L)) \subseteq N_{int}(\xi(L))$. This implies $\xi(L) \subseteq N_{int}(\xi(L))$. We know that $N_{int}(\xi(L)) \subseteq \xi(L)$. Therefore $\xi(L) = N_{int}(\xi(L))$. This means that $\xi(L)$ is an NOS in (M,δ). Since each NOS is a neutrosophic Φ-open set, so $\xi(L)$ is a neutrosophic Φ-open set in (M,δ). Hence for each NOS L in (W,τ), $\xi(L)$ is a neutrosophic Φ-open set in (M,δ). Therefore ξ is a neutrosophic Φ-open function.

Theorem 3.6. Assume that ξ is a bijective function from an NTS (W,τ) to another NTS (M,δ). Then the following mathematical statements are equivalent:

1) ξ is a neutrosophic Φ-continuous function;
2) ξ is a neutrosophic Φ-closed function;
3) ξ is a neutrosophic Φ-open function.

Proof.

(1)⇒(2) Assume that $\xi:(W,\tau)\rightarrow(M,\delta)$ is a neutrosophic Φ-continuous function. Let Q be any arbitrary NCS in (W,τ). Then Q^c is an NOS in (W,τ). Since each NOS is a neutrosophic Φ-open set, so Q^c is a neutrosophic Φ-open set in (W,τ). Since ξ is a bijective function, so $\xi(Q^c)=(\xi(Q))^c$ is an NOS in (M,δ). Hence $\xi(Q)$ is an NCS in (M,δ). Therefore, for each NCS Q in (W,τ), $\xi(Q)$ is a neutrosophic Φ-closed set in (M,δ). Hence ξ is a neutrosophic Φ-closed function.

(2)⇒(3) Assume that $\xi:(W,\tau)\rightarrow(M,\delta)$ be a neutrosophic Φ-closed function. Let L be any arbitrary NOS in (W,τ). Then L^c is an NCS in (W,τ). Since ξ is a neutrosophic Φ-closed function, so $\xi(L^c)=(\xi(L))^c$ is a neutrosophic Φ-closed set in (M,δ). Then $\xi(L)$ is a neutrosophic Φ-open set in (M,δ). Therefore, for each NOS L in (W,τ), $\xi(L)$ is a neutrosophic Φ-open set in (M,δ). Hence ξ is a neutrosophic Φ-open function.

(3)⇒(1) Assume that $\xi:(W,\tau)\rightarrow(M,\delta)$ is a neutrosophic Φ-open function. Let P be any arbitrary NOS in (M,δ). Then P is a neutrosophic Φ-open set in (M,δ). Since ξ is a bijective function, so $\xi^{-1}(P)$ is an NOS in (W,τ). Again, since each NOS is a neutrosophic Φ-open set, so $\xi^{-1}(P)$ is a neutrosophic Φ-open set in (W,τ). Therefore, for each NOS P in (M,δ), $\xi^{-1}(P)$ is a neutrosophic Φ-open set in (W,τ). Hence ξ is a neutrosophic Φ-continuous function.

4. Conclusion

In this study we have introduced neutrosophic Φ-open set, neutrosophic Φ-continuous mapping via NTSs and investigated their several properties. By defining neutrosophic Φ-open set, neutrosophic Φ-continuous mapping, we have proved some remarks, and theorems on NTSs. In the future, we hope that based on Φ-open set, neutrosophic Φ-continuous mapping via NTSs, many new investigations can be carried out.

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