
Chinnadurai V
Sindhu M. P

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Chinnadurai V1, Sindhu M P2

1 Department of Mathematics, Annamalai University, Annamalainagar, Tamilnadu, India; kv.chinnaduraimaths@gmail.com
2 Department of Mathematics, Karpagam College of Engineering, Coimbatore, Tamilnadu, India; sindhu.mp@kce.ac.in

Correspondence: kv.chinnaduraimaths@gmail.com

Abstract: Neutrosophic soft set is a parametric set of uncertainty, whereas the neutrosophic soft point is an exceptional type of it which used highly to explore the separation axioms. In this study, the impression of neutrosophic soft topological space is stretched to a new topology which contains neutrosophic soft points as its elements and named as neutro-spot topological space. One more topology is defined on the complement of neutrosophic soft points which satisfies the condition of supra topological space and named as neutro-supra spot topological space. Also, defined the notion of interior and closure, and are approached in a different way, along with the concept of subspace topology of such topological spaces. Some related properties have been proved and disproved with counterexamples. Moreover, the approach to separation axioms in such spaces has been presented with descriptive examples. The current epidemic situation discussed as a real life application in decision making problem to detect the major impact of COVID-19 and recover them quickly. The affected people investigated by the doctors according to their symptoms and other medical issues. The process of solving specified in the algorithm and the estimation formula stated for calculation. The appropriate treatment is provided for affected people as per the estimated value.

Keywords: Neutro-spot topological space; neutro-spot absolute interior; neutro-supra spot topological space; neutro-supra spot absolute closure; neutro-spot subspace topological space; neutro-supra spot subspace topological space; neutro-spot $T_{i=0,1,2}$-spaces and neutro-supra spot $T_{i=0,1,2}$-spaces; decision making problem on COVID-19.

1. Introduction

The values of three independent membership degrees such as truth, falsity, and indeterminacy, consigned to each element of a set which characterized to neutrosophic set (NS) as originated by (1998) Smarandache [16, 17], which is a simplification of a fuzzy set (FS) defined by (1965) Zadeh [36], and intuitionistic fuzzy set (IFS) created by (1986) Atanassov [34]. It turns out to be a valuable mathematical utensil to examine formless, faulty, unclear data. In recent years many researchers have further expanded and developed the theory and application of NSs [1, 10, 12-15]. Also, (2017) Smarandache [18] originated a new trend set called plithogenic set (PS) and others developed [4, 8, 11].
A soft set (SS) is a study of parameterization of vagueness introduced by (1999) Molodtsov [32]. Later (2013) Maji [30] pooled two-hybrid sets NS and SS, and urbanized its construction as neutrosophic soft sets (NSSs). This type of set is extended by different researchers [2, 7, 9] and its application in decision making (DM) problems [3]. Topology plays a vital role among all these types of sets such as fuzzy topology (FT), soft topology (ST), fuzzy soft topology (FST), neutrosophic topology (NT) [31], etc. Likewise, (2017) Bera & Mahapatra [26, 6] created the conception of neutrosophic soft topological spaces (NSTSs) and others [19, 20, 24]. Neutrosophic soft point (NSP) is a special type of NSS, which gives rise to the concept of separation axioms on NSTS [23, 25], and their application to DM was considered in [21, 22, 28]. Mashhour et al. [35] (1983) diminished the conditions of general topology, termed as supra topological spaces (SpTSs). The real-life application of SpTS is applied and defined on various sets such as FS [33], SS [29, 27], NS [5], and so on.

The major contribution of this work is to initiate a topology on NSPs, whose open sets defines the concept of the interior on it. The complement of NSP is defined and named as neutrosophic soft whole set. Such type of sets obeys the condition of supra topology, and so generated new types of supra topology, whose open sets defines the concept of closure on it. Some essential definitions and remarkable properties are studied with appropriate examples. On the other hand, the impact of separation axioms is examined in both the topologies with suitable examples. The recent wide-ranging problem extended as a major application to provide appropriate treatment for COVID-19 patients. Those people are under investigation as stated by its symptoms and other medical issues. Also, provided the solving procedure and formulae for calculation. The main aim of this DM problem to recover them speedily via proper treatment.

This study is prearranged as follows. Some important definitions related to the study are presented in part 2. Part 3 introduces the definition of neutro-spot topology, its interior, its subspace topology with fundamental properties, and related examples. Part 4 introduces the definition of neutro-supra spot topology, its closure, its subspace topology with fundamental properties, and related examples. Part 5 extends this study to separation axioms on both the topologies with explanatory examples. Part 6 solves the DM problem to detect the impact of COVID-19. The algorithm and formulae are presented to find the final result and provided proper treatment for them. At last, concluded with few ideas for upcoming work in part 7.

2. Preliminaries

In this part, some essential definitions connected to this work are pointed.

Definition 2.1 [26] Let \( V \) be an initial universe set, \( E \) be a set of parameters, and \( P, Q \) is any two NSSs over \((V, E)\). Then a NSS \( P \) over \( V \) is a set defined by a set-valued function \( f^{(E)P} \) representing a mapping \( f^{(E)P}: E \rightarrow NS(V) \) where \( f^{(E)P}(e) \) is called the approximate function of the NSS \( P \) and \( NS(V) \) is a family of NS over \( V \).

\[
P = f^{(E)P} = \{ e \in V: T_{f^{(E)P}(e)}, I_{f^{(E)P}(e)}, F_{f^{(E)P}(e)} \supseteq v \in V \}; e \in E \}.
\]

Definition 2.2 [24] A NSS \( P \) over \((V, E)\) is said to be null NSS if \( T_{f^{(E)P}(e)}(v) = 0, I_{f^{(E)P}(e)}(v) = 0, F_{f^{(E)P}(e)}(v) = 1 \), \( \forall e \in E \), \( \forall v \in V \). It is denoted by \( \phi \).

A NSS \( P \) over \((V, E)\) is said to be absolute NSS if \( T_{f^{(E)P}(e)}(v) = 1, I_{f^{(E)P}(e)}(v) = 1, F_{f^{(E)P}(e)}(v) = 0 \), \( \forall e \in E \), \( \forall v \in V \). It is denoted by \( 1 \).

Clearly, \( (\phi)^c = 1 \) and \( (1)^c = \phi \).

Definition 2.3 [25] Let \( V \) be a universe and \( E \) be a set of parameters. Let \( K \) be a NSS over \((V, E)\). Let \( e \) be an element of \( E \) and let
Definition 3.1 Let $V$ be a universe and $E$ be a set of parameters. Let $\text{NSP}(V, E)$ be the family of all NSPs over $V$. Then $\tau \subset \text{NSP}(V, E)$ is said to be neutro-spot topology (NSPT) over $(V, E)$ if it satisfies the following conditions

(i) $\phi, 1, \in \tau$.

(ii) The finite intersection of NSPs in $\tau$ belongs to $\tau$.

The trio $(V, \tau, E)$ is said to be neutro-spot topological space (NSPTS) over $(V, E)$. Elements in $\tau$ are called neutro-spot open sets (NSPOSs).

Example 3.2 A survey is taken based on the characteristics of houses owned by the people living in a slum area. Let $V = \{p_1, p_2, p_3\}$ be the set of sample people living in different areas in the slum and $E = \{e_1, e_2, e_3, e_4\}$ be the set of parameters of houses, where $e_1 = \text{neat}$, $e_2 = \text{beautiful}$, $e_3 = \text{compact}$, and $e_4 = \text{large}$. Let $\tau = \{\phi, 1, K_1, K_2, K_3\}$. According to the survey, the results have proceeded in the form of NSPs $K_1, K_2, K_3$ over $V$ as follows:

\[
K_1 = \begin{cases} 
 f^{(e_1)}K_1 = \langle p_1, (0,0,1), p_2, (0,0,1), p_3, (0,0,1) \rangle \\
 f^{(e_2)}K_1 = \langle p_1, (0,0,1), p_2, (0,0,1), p_3, (0,0,1) \rangle \\
 f^{(e_3)}K_1 = \langle p_1, (0,0,1), p_2, (3,4,2), p_3, (0,0,1) \rangle \\
 f^{(e_4)}K_1 = \langle p_1, (0,0,1), p_2, (0,0,1), p_3, (0,0,1) \rangle 
\end{cases}
\]

\[
K_2 = \begin{cases} 
 f^{(e_1)}K_2 = \langle p_1, (0,0,1), p_2, (0,0,1), p_3, (0,0,1) \rangle \\
 f^{(e_2)}K_2 = \langle p_1, (0,0,1), p_2, (0,0,1), p_3, (0,0,1) \rangle \\
 f^{(e_3)}K_2 = \langle p_1, (5,2,6), p_2, (0,0,1), p_3, (0,0,1) \rangle \\
 f^{(e_4)}K_2 = \langle p_1, (0,0,1), p_2, (0,0,1), p_3, (0,0,1) \rangle 
\end{cases}
\]

\[
K_3 = \begin{cases} 
 f^{(e_1)}K_3 = \langle p_1, (0,0,1), p_2, (0,0,1), p_3, (0,0,1) \rangle \\
 f^{(e_2)}K_3 = \langle p_1, (0,0,1), p_2, (0,0,1), p_3, (0,0,1) \rangle \\
 f^{(e_3)}K_3 = \langle p_1, (0,0,1), p_2, (0,0,1), p_3, (1,2,3) \rangle \\
 f^{(e_4)}K_3 = \langle p_1, (0,0,1), p_2, (0,0,1), p_3, (0,0,1) \rangle 
\end{cases}
\]

Here $\phi \cap K_1 = \phi$, $\phi \cap K_2 = \phi$, $\phi \cap K_3 = \phi$, $1 \cap K_1 = K_1$, $1 \cap K_2 = K_2$, $1 \cap K_3 = K_3$, $K_1 \cap K_2 = \phi$, $K_1 \cap K_3 = \phi$, $K_2 \cap K_3 = \phi$.

Then $K_1, K_2$ and $K_3$ are NSPOSs.

Thus $(V, \tau, E)$ is a NSPTS over $(V, E)$.

Proposition 3.3 Let $(V, \tau_{11}, E)$ and $(V, \tau_{12}, E)$ be two NSPTSs over $(V, E)$. Then $(V, \tau_{11} \cap \tau_{12}, E)$ is also a NSPTS over $(V, E)$.

Proof. Let $(V, \tau_{11}, E)$ and $(V, \tau_{12}, E)$ be two NSPTSs over $(V, E)$.
(i) Obviously, $\phi, 1_i \in \tau_1 \cap \tau_2$.

(ii) Let $K_1, K_2 \in \tau_1 \cap \tau_2$.

Then $K_1, K_2 \in \tau_1$ and $K_1, K_2 \in \tau_2$.

$\Rightarrow K_1 \cap K_2 \in \tau_1$ and $K_1 \cap K_2 \in \tau_2$.

$\Rightarrow K_1 \cap K_2 \in \tau_1 \cap \tau_2$.

Thus $(V, \tau_1 \cap \tau_2, E)$ is a NSPTS over $(V, E)$.

**Remark 3.4** Let $(V, \tau_1, E)$ and $(V, \tau_2, E)$ be two NSPTSs over $(V, E)$. Then $(V, \tau_1 \cup \tau_2, E)$ is not NSPTS over $(V, E)$.

**Example 3.5** Let $V = \{g_1, g_2, g_3\}$, $E = \{e_1, e_2, e_3, e_4\}$. Let $\tau_1 = \{\phi, 1_i, K\}$ and $\tau_2 = \{\phi, 1_i, L\}$ where the NSPs $K$ and $L$ over $V$ are defined as

$K = \begin{cases} f^{(1)}(K) = \{<g_1,(0,0,1)>, <g_2,(0,0,1), <g_3,(0,0,1)>\} \\ f^{(2)}(K) = \{<g_1,(0,0,1)>, <g_2,(0,0,1), <g_3,(0,0,1)>\} \\ f^{(3)}(K) = \{<g_1,(0,0,1)>, <g_2,(0,0,1), <g_3,(0,0,1)>\} \\ f^{(4)}(K) = \{<g_1,(0,0,1)>, <g_2,(0,0,1), <g_3,(0,0,1)>\} \end{cases}$

$L = \begin{cases} f^{(1)}(L) = \{<g_1,(0,0,1)>, <g_2,(0,0,1), <g_3,(0,0,1)>\} \\ f^{(2)}(L) = \{<g_1,(0,0,1)>, <g_2,(0,0,1), <g_3,(0,0,1)>\} \\ f^{(3)}(L) = \{<g_1,(0,0,1)>, <g_2,(0,3,9,5), <g_3,(0,0,1)>\} \\ f^{(4)}(L) = \{<g_1,(0,0,1)>, <g_2,(0,0,1), <g_3,(0,0,1)>\} \end{cases}$

Then $\tau_1 \cup \tau_2 = \{\phi, 1_i, K, L\}$ is not an NSPTS over $(V, E)$, since $K \cup L \not\in \tau_1 \cup \tau_2$.

Thus $(V, \tau_1 \cup \tau_2, E)$ is not NSPTS over $(V, E)$.

**Proposition 3.6** Let $K$ and $L$ be any two NSPs on NSPTS $(V, \tau, E)$ over $(V, E)$. Then

(i) $(K \cup L)^c = K^c \cup L^c$.

(ii) $(K \cap L)^c = K^c \cup L^c$.

Proof. Straight forward.

**Proposition 3.7** Let $(V, \tau, E)$ be a NSPTS over $(V, E)$ and

$\tau = \{K_i : K_i \in \text{NSP}(V, E)\} = \{e, f^{(E)}(K) : K_i \in \text{NSP}(V, E)\}$

where $f^{(E)}(K) = \{v, T^{(E)}(V, f^{(E)}(K))(v), I^{(E)}(K)(v), F^{(E)}(K)(v)) : v \in V, e \in E\}$.

Define

$\tau_1 = \{T^{(E)}(V, f^{(E)}(K))(v) \}_{v \in V}$,

$\tau_2 = \{I^{(E)}(K)(v) \}_{v \in V}$ and

$\tau_3 = \{F^{(E)}(K)(v) \}_{v \in V}$.

Then $\tau_1, \tau_2$ and $\tau_3$ are FSTs on $(V, E)$.

Proof. Let $(V, \tau, E)$ be a NSPTS over $(V, E)$.

(i) Since $\phi, 1_i \in \tau$,

$\Rightarrow 0, 1 \in \tau_1, 1, 0 \in \tau_2, 1, 0 \in \tau_3$.

(ii) Let $K_1, K_2 \in \tau_1$. Then $K_1 \cap K_2 \in \tau_1$.

That is,
(iii) Let $K_i \in \tau_i$, where $i \in I$. Then $\bigcup_{i \in I} K_i \in \tau_i$.

That is,

$$\bigcup_{i \in I} K_i = \left\{ \min_{x \in E} [T_{f^{(i)}(K_i)}(V)], \max_{x \in E} [I_{f^{(i)}(K_i)}(V)], \min_{x \in E} [F_{f^{(i)}(K_i)}(V)], \max_{x \in E} [F_{f^{(i)}(K_i)}(V)] \right\} \in \tau_i.$$ 

Thus $\min_{x \in E} [T_{f^{(i)}(K_i)}(V)], \max_{x \in E} [I_{f^{(i)}(K_i)}(V)], \min_{x \in E} [F_{f^{(i)}(K_i)}(V)], \max_{x \in E} [F_{f^{(i)}(K_i)}(V)] \in \tau_i$, and $\tau_i \neq \emptyset$.

Hence $\tau_i, \tau_2, \tau_3$ are FSTs on $(V, E)$.

**Remark 3.8** The following example illustrates that the converse of Proposition 3.7 is not true.

**Example 3.9** Let $V = \{g_1, g_2, g_3\}, E = \{e_1, e_2, e_3, e_4\}$. Let $\tau_i = [\phi, 1, K_i, K_2]$, where NSPs $K_1$ and $K_2$ over $V$ are defined as

$$K_1 = \left\{ f^{(i)}(K_1) = \begin{cases} f^{(i)}(K_1) = \emptyset, \\
 f^{(i)}(K_i) = \emptyset, \\
 f^{(i)}(K_i) = \emptyset, \\
 f^{(i)}(K_i) = \emptyset.
\end{cases}\right.$$

and

$$K_2 = \left\{ f^{(i)}(K_2) = \begin{cases} f^{(i)}(K_2) = \emptyset, \\
 f^{(i)}(K_i) = \emptyset, \\
 f^{(i)}(K_i) = \emptyset, \\
 f^{(i)}(K_i) = \emptyset.
\end{cases}\right.$$

Then

$$\tau_i = \left\{ T_{f^{(i)}(K_1)}(V), T_{f^{(i)}(K_1)}(V), T_{f^{(i)}(K_1)}(V), T_{f^{(i)}(K_1)}(V) \right\} \in \tau_i,$$

$$\tau_i = \left\{ I_{f^{(i)}(K_2)}(V), I_{f^{(i)}(K_2)}(V), I_{f^{(i)}(K_2)}(V), I_{f^{(i)}(K_2)}(V) \right\} \in \tau_i,$$

$$\tau_i = \left\{ F_{f^{(i)}(K_1)}(V), F_{f^{(i)}(K_1)}(V), F_{f^{(i)}(K_1)}(V), F_{f^{(i)}(K_1)}(V) \right\} \in \tau_i.$$ are FSTs on $(V, E)$, where

$$\tau_i = \left\{ e_1, e_2, e_3, e_4 \right\} \in \tau_i $$

Thus $\tau_i \neq \emptyset$.

**Proposition 3.10** Let $(V, \tau_i, E)$ be a NSPTS over $(V, E)$ and

$$\tau_i = [K_i : K_i \in NSP(V, E)] = \left\{ e, f^{(i)}(K) : K_i \in NSP(V, E) \right\}\left\{ e, f^{(i)}(K) : K_i \in NSP(V, E) \right\}.$$
where \( f^{(E \cup V)} = \{ v, T^{(\omega_K)}(v), I^{(\omega_K)}(v), F^{(\omega_K)}(v) : v \in V, e \in E \} \).

Define
\[
\tau_1 = \{ T^{(\omega_K)}(V) \}_{e \in E},
\tau_2 = \{ I^{(\omega_K)}(V) \}_{e \in E},
\tau_3 = \{ F^{(\omega_K)}(V) \}_{e \in E}
\]
as FSTs on \((V, E)\).

Then \( \tau_1 \cup \tau_3 \) and \( \tau_2 \cup \tau_3 \) are not FSTs on \((V, E)\).

Proposition 3.10 is illustrated by the following example.

**Example 3.11** Let \( V = \{g_1, g_2, g_3\} \), \( E = \{e_1, e_2, e_3, e_4\} \). Let \( \tau = \{ \phi, 1, K_1, K_2, K_3 \} \) where the NSPs \( K_1, K_2, K_3 \) over \( V \) are defined as

\[
K_1 = \{ f^{(e_i)}(K_1) = \{ < g_1, (0,0,1) >, < g_2, (0,0,1) >, < g_3, (0,0,1) > \}
\]

\[
K_2 = \{ f^{(e_i)}(K_2) = \{ < g_1, (0,0,1) >, < g_2, (0,0,1) >, < g_3, (0,0,1) > \}
\]

\[
K_3 = \{ f^{(e_i)}(K_3) = \{ < g_1, (0,0,1) >, < g_2, (0,0,1) >, < g_3, (0,0,1) > \}
\]

Here \( \phi \cap K_1 = \phi \), \( \phi \cap K_2 = \phi \), \( \phi \cap K_3 = \phi \), \( 1 \cap K_1 = K_1 \), \( 1 \cap K_2 = K_2 \), \( 1 \cap K_3 = K_3 \), \( K_1 \cap K_2 = K_3 \), \( K_1 \cap K_3 = K_2 \), \( K_2 \cap K_3 = K_1 \).

Then \( K_1, K_2 \) and \( K_3 \) are NSPOSs.

Thus \((V, \tau, E)\) is a NSPTS over \((V, E)\).

Then
\[
\tau_1 = \{ e_1, (0,0,0), (1,1,1), (0,0,0), (0,0,0), (0,0,0), (0,0,0), (0,0,7), (0,0,8), (0,0,7) \}
\]

and so on, are FSTs on \((V, E)\).

But, \( \tau_1 \cup \tau_3 \) and \( \tau_2 \cup \tau_3 \) are not FSTs on \((V, E)\).

**Proposition 3.12** Let \((V, \tau, E)\) be a NSPTS over \((V, E)\). Then
\[
\epsilon \tau_1 = \{ f^{(e, \omega_K)}(V) : K \in \tau \},
\epsilon \tau_2 = \{ f^{(e, \omega_K)}(V) : K \in \tau \},
\epsilon \tau_3 = \{ f^{(e, \omega_K)}(V) : K \in \tau \}
\]
for each \( e \in E \), define FTs on \((V, E)\).
Proof. Follows from Proposition 3.7.

Remark 3.13 The following example illustrates that the converse of Proposition 3.12 is not true.

Example 3.14 Consider Example 3.9.
Then
$$\alpha_{\tau_1} = \{ f^{(\alpha)}(V), T_f^{(\alpha)}(V), T_f^{(\alpha)K_1}(V), T_f^{(\alpha)K_2}(V) \}$$
$$\alpha_{\tau_2} = \{ f^{(\alpha)}(V), I_f^{(\alpha)}(V), I_f^{(\alpha)K_1}(V), I_f^{(\alpha)K_2}(V) \}$$
$$\alpha_{\tau_3} = \{ f^{(\alpha)}(V), F_f^{(\alpha)}(V), F_f^{(\alpha)K_1}(V), F_f^{(\alpha)K_2}(V) \}$$
are FTs on $V$, where
$$\alpha_{\tau_1} = \{0,0,0,1,1,1,0,0,0,0,0\}$$
and so on.

Consequently, \{\alpha_{\tau_1}, \alpha_{\tau_2}, \alpha_{\tau_3}\} are fuzzy tritopologies on $V$.
Thus \(\tau_i = \{\phi, 1, K_1, K_2\}\) is not NSPT over $\langle V, E \rangle$, since $K_1 \cap K_2 \not\in \tau_i$.

Definition 3.15 Let $\langle V, \tau_i, E \rangle$ be a NSPTS over $\langle V, E \rangle$ and $L \in \text{NSP}(V,E)$ be any NSP. Then the neutro-spot absolute interior of $L$ is denoted by $\bar{L}$ and defined as
(i) $\bar{L} = \{ K : K \in \text{NSPOS} \& K \subseteq L \}$ i.e., the union of all neutro-spot open subsets of $L$.
(ii) $\bar{L} = \left\{ \left. \bigcap_{f(e)K} \bigcap_{f(e)K_1} \bigcap_{f(e)K_2} \bigcap_{f(e)K_3} \right\} : K_i \in \tau_i \& f(K_i) \subseteq f(L) \right\}$

Example 3.16 Let $V = \{g_1, g_2\}$, $E = \{e_1, e_2, e_3\}$. Let $\tau_i = \{\phi, 1, K_1, K_2, K_3\}$ where the NSPs $K_1, K_2, K_3$ over $V$ are defined as
$$K_1 = \left\{ \left. f^{(e)}K_1 \right| \langle g_1, (0,0,1) >, g_2, (0,0,1) \rangle \right\}$$
$$K_2 = \left\{ \left. f^{(e)}K_2 \right| \langle g_1, (0,0,1) >, g_2, (0,0,1) \rangle \right\}$$
$$K_3 = \left\{ \left. f^{(e)}K_3 \right| \langle g_1, (0,0,1) >, g_2, (0,0,1) \rangle \right\}$$
Here $\phi \cap K_1 = \phi$, $\phi \cap K_2 = \phi$, $\phi \cap K_3 = \phi$, $1 \cap K_1 = K_1$, $1 \cap K_2 = K_2$, $1 \cap K_3 = K_3$, $K_1 \cap K_2 = \phi$, $K_1 \cap K_3 = K_3$, $K_2 \cap K_3 = \phi$.
Then $K_1, K_2$ and $K_3$ are NSPOSs.
Thus $\langle V, \tau_i, E \rangle$ is a NSPTS over $\langle V, E \rangle$.
Let $L \in \text{NSP}(V,E)$ be any NSP defined as
$$L = \left\{ \left. f^{(e)}L \right| \langle g_1, (0,0,1) >, g_2, (0,0,1) \rangle \right\}$$
$$f^{(e)}L = \langle g_1, (0,0,1) >, g_2, (0,0,1) \rangle$$
Then $\phi, K_1, K_3 \subseteq L$. 
Thus $\bar{L}_e = \phi \cup K_1 \cup K_3 = K_1$.

Also, 
\[
\begin{align*}
&f^{(e_1)}(K_1) \subseteq f^{(e_2)}(K_2) \subseteq f^{(e_3)}(K_3) \\
&f^{(e_2)}(K_2) \subseteq f^{(e_3)}(K_3) \subseteq f^{(e_1)}(L) \\
&f^{(e_3)}(K_3) \subseteq f^{(e_2)}(K_2) \subseteq f^{(e_1)}(L) \\
&f^{(e_1)}(K_1) \subseteq f^{(e_2)}(K_2) \subseteq f^{(e_3)}(K_3) \\
&f^{(e_2)}(K_2) \subseteq f^{(e_3)}(K_3) \subseteq f^{(e_1)}(L).
\end{align*}
\]

Then 
\[
\begin{align*}
&\bigg\{ (\bar{f}_{f^{(e_1)}(L)}, \bar{f}_{f^{(e_2)}(L)}, \bar{f}_{f^{(e_3)}(L)}) = (g_1, (0,0,1), g_2, (0,0,1)), \\
&\bigg\{ (\bar{f}_{f^{(e_2)}(L)}, \bar{f}_{f^{(e_3)}(L)}, \bar{f}_{f^{(e_3)}(L)}) = (g_1, (0,0,1), g_2, (0,0,1)), \\
&\bigg\{ (\bar{f}_{f^{(e_3)}(L)}, \bar{f}_{f^{(e_3)}(L)}, \bar{f}_{f^{(e_3)}(L)}) = (g_1, (0,0,1), g_2, (0,0,1)).
\end{align*}
\]

Thus 
\[
\begin{align*}
&\bigg\{ (\bar{f}_{f^{(e_1)}(L)}, \bar{f}_{f^{(e_2)}(L)}, \bar{f}_{f^{(e_3)}(L)}) \subseteq K_1.
\end{align*}
\]

**Theorem 3.17** Let $(V, \tau, E)$ be a NSPTS over $(V, E)$ and $L, S \in \text{NSP}(V, E)$ be any two NSPs. Then,
\[
\begin{align*}
&\text{(i)} \quad \bar{L}_e \subseteq L \text{ and } \bar{L}_e \text{ is the largest NSPOS}. \\
&\text{(ii)} \quad L \subseteq S \Rightarrow \bar{L}_e \subseteq \bar{S}_e. \\
&\text{(iii)} \quad \bar{L}_e \text{ is a NSPOS i.e., } \bar{L}_e \in \tau_e. \\
&\text{(iv)} \quad L \text{ is a NSPOS iff } \bar{L}_e = L. \\
&\text{(v)} \quad \bar{L}_e = L. \\
&\text{(vi)} \quad \bar{L}_e = \bar{L}_e \text{ and } \bar{L}_e = \bar{L}_e. \\
&\text{(vii)} \quad \bar{L}_e \cap \bar{S}_e = \bar{L}_e \cap \bar{S}_e. \\
&\text{(viii)} \quad \bar{L}_e \cup \bar{S}_e = \bar{L}_e \cup \bar{S}_e.
\end{align*}
\]

Proof. Let $(V, \tau, E)$ be a NSPTS over $(V, E)$ and $L, S \in \text{NSP}(V, E)$ be any two NSPs.

(i) Follows from Definition 3.15.

(ii) Let $L \subseteq S$. Then $\bar{L}_e \subseteq L \subseteq S \Rightarrow \bar{L}_e \subseteq S$ and $\bar{S}_e \subseteq S$.

Since $\bar{S}_e$ is the largest NSPOS contained in $S$, hence $\bar{L}_e \subseteq \bar{S}_e$.

(iii) Follows from Definition 3.15.

(iv) Let $L$ be a NSPOS. Then $\bar{L}_e$ is the largest NSPOS which contained in $L$ is equal to $L$.

Hence $\bar{L}_e = L$.

Conversely, assume that $\bar{L}_e = L$.

By (iii), $\bar{L}_e \in \tau_e$.

Then $L$ is a NSPOS.

(v) Let $\bar{L}_e = K$. Then $K \in \tau_e$ iff $\bar{K}_e = K$.

Thus $\bar{L}_e = \bar{L}_e$.

(vi) Since $\phi, \bar{L}_e \in \tau_e$ and by (iv), hence $\bar{L}_e = \bar{L}_e$ and $\bar{L}_e = \bar{L}_e$.

(vii) $L \cap S \subseteq L$ and $L \cap S \subseteq S$.

\[ \Rightarrow \bar{L}_e \cap \bar{S}_e \subseteq \bar{L}_e \text{ and } \bar{L}_e \cap \bar{S}_e \subseteq \bar{L}_e. \]

Also, $\bar{L}_e \subseteq L$ and $\bar{P}_e \subseteq P$.

Then $\bar{L}_e \cap \bar{S}_e \subseteq L \cap S$.

Since $\bar{L}_e \cap \bar{S}_e \subseteq L \cap S$ and it is the largest NSPOS, then $\bar{L}_e \cap \bar{S}_e = \bar{L}_e \cap \bar{S}_e$.
Define Thus \( \left( \overline{L} \cap \overline{S} \right) = \overline{L} \cap \overline{S} \).

(vii) \( L \subseteq L \cup S \) and \( S \subseteq L \cup S \).
\( \Rightarrow \overline{L} \subseteq \left( \overline{L} \cup \overline{S} \right) \) and \( \overline{S} \subseteq \left( \overline{L} \cup \overline{S} \right) \).

**Definition 3.18** Let \( (V, \tau, E) \) be a NSPTS over \( (V, E) \) where \( \tau \) is a NSPT over \( (V, E) \) and \( L \in \text{NSP}(V, E) \) be any NSP. Then the subspace topology on NSPTS is denoted by \( \tau_L^E \) and defined as \( \tau_L^E = \{ L \cap K_i : K_i \in \tau \} \) and \( \phi \cap \tau_L^E \). Thus \( (V, \tau_L^E, E) \) is a neutro-spot subspace topological space (NSPSTS) of \( (V, \tau, E) \), where \( \tau_L^E \) is also a NSPT over \( (V, E) \).

**Example 3.19** Let \( V = \{ g_1, g_2, g_3 \}, \ E = \{ e_1, e_2 \} \). Let \( \tau = \{ \phi, L_1, K_1, K_2, K_3 \} \) where the NSPs \( K_1, K_2, K_3 \) over \( V \) are defined as
\[
K_1 = \begin{cases}
\langle f(e_1), K_1 \rangle = \langle g_1, (0,0,1), g_2, (0,0,1), < g_3, (0,0,1) > \\
\langle f(e_2), K_1 \rangle = \langle g_1, (0,0,1), g_2, (1,4,5), < g_3, (0,0,1) >
\end{cases}
\] ;
\[
K_2 = \begin{cases}
\langle f(e_1), K_2 \rangle = \langle g_1, (0,0,1), g_2, (0,0,1), < g_3, (0,0,1) > \\
\langle f(e_2), K_2 \rangle = \langle g_1, (0,0,1), g_2, (4,7,8), < g_3, (0,0,1) >
\end{cases} \quad \text{and}
\]
\[
K_3 = \begin{cases}
\langle f(e_1), K_3 \rangle = \langle g_1, (0,0,1), g_2, (0,0,1), < g_3, (0,0,1) > \\
\langle f(e_2), K_3 \rangle = \langle g_1, (0,0,1), g_2, (1,4,8), < g_3, (0,0,1) >
\end{cases} .
\]

Here \( \phi \cap K_1 = \phi, \ \phi \cap K_2 = \phi, \ \phi \cap K_3 = \phi, \ \phi_1 \cap K_1 = K_1, \ \phi_1 \cap K_2 = K_2, \ \phi_1 \cap K_3 = K_3, \ K_1 \cap K_2 = K_3, \ K_1 \cap K_3 = K_2, \ K_2 \cap K_3 = K_3, \)

Then \( K_1, K_2, \) and \( K_3 \) are NSPOSs.
Thus \( (V, \tau, E) \) is a NSPTS over \( (V, E) \).

Define \( L_4 \in \text{NSP}(V, E) \) as
\[
L_4 = \begin{cases}
\langle f(e_1), L_4 \rangle = \langle g_1, (0,0,1), g_2, (0,0,1), < g_3, (0,0,1) > \\
\langle f(e_2), L_4 \rangle = \langle g_1, (0,0,1), g_2, (7,3,6), < g_3, (0,0,1) >
\end{cases} .
\]

Thus we denote \( L_2 \cap \phi = \phi, \ L_1 \cap L_1 = L_4, \ L_1 \cap K_1 = L_2, \ L_1 \cap K_2 = L_3, \ L_1 \cap K_3 = L_4 \).

Then the NSPs \( L_2, L_3, L_4 \) over \( V \) are defined as
\[
L_2 = \begin{cases}
\langle f(e_1), L_2 \rangle = \langle g_1, (0,0,1), g_2, (0,0,1), < g_3, (0,0,1) > \\
\langle f(e_2), L_2 \rangle = \langle g_1, (0,0,1), g_2, (1,3,6), < g_3, (0,0,1) >
\end{cases} ;
\]
\[
L_3 = \begin{cases}
\langle f(e_1), L_3 \rangle = \langle g_1, (0,0,1), g_2, (0,0,1), < g_3, (0,0,1) > \\
\langle f(e_2), L_3 \rangle = \langle g_1, (0,0,1), g_2, (4,3,8), < g_3, (0,0,1) >
\end{cases} \quad \text{and}
\]
\[
L_4 = \begin{cases}
\langle f(e_1), L_4 \rangle = \langle g_1, (0,0,1), g_2, (0,0,1), < g_3, (0,0,1) > \\
\langle f(e_2), L_4 \rangle = \langle g_1, (0,0,1), g_2, (1,3,8), < g_3, (0,0,1) >
\end{cases} .
\]

Then \( \tau_L^E = \{ \phi, L_1, L_2, L_3, L_4 \} \) is a NSPST over \( (V, E) \).
Thus \( (V, \tau_L^E, E) \) is a NSPSTS of \( (V, \tau, E) \).
4. Neutro-Supra Spot Topology

This part defines the neutro-supra spot topology, neutro-supra spot absolute closure, and its subspace topology with some properties and examples.

**Definition 4.1** Let \( V \) be a universe and \( E \) be a set of parameters. Then the complement of NSP is said to be neutrosophic soft whole set (NSWS) over \((V, E)\).

**Definition 4.2** Let \( V \) be a universe and \( E \) be a set of parameters. Let \( NSWS(V, E) \) be the family of all neutrosophic soft whole sets (NSWSs) over \( V \). Then \((\tau, \tau') \subset NSWS(V, E)\) is said to be neutro-supra spot topology (NSSPT) on \( V \) if it satisfies the following conditions

(i) \( \emptyset, 1, \in (\tau') \).

(ii) the arbitrary union of NSWSs in \((\tau')\) belongs to \((\tau, \tau')\). Then \((\tau, \tau') \) is said to be a neutro-supra spot topology (NSSPT) over \((V, E)\), and the trio \((V, (\tau, \tau'), E)\) is said to be a neutro-supra spot topological spaces (NSSPTSs) over \((V, E)\). Elements in \((\tau, \tau')\) are called neutro-supra spot open sets (NSSPOSs).

**Example 4.3** Consider Example 3.2. Here \((\tau, \tau') = \{ \emptyset, 1, (K_1)^\tau, (K_2)^\tau, (K_3)^\tau \}\). Then NSWS \((K_1)^\tau, (K_2)^\tau, (K_3)^\tau\) over \( V \) are defined as

\[
(K_1)^\tau = \begin{cases} f^{(e_1)}(K_1)^\tau = \{ p_1, (1,1,0), p_2, (1,1,0), p_3, (1,1,0) \} \\
\end{cases}
\]

and

\[
(K_2)^\tau = \begin{cases} f^{(e_1)}(K_2)^\tau = \{ p_1, (1,1,0), p_2, (1,1,0), p_3, (1,1,0) \} \\
\end{cases}
\]

Here \( \emptyset \cup (K_1)^\tau = (K_1)^\tau \), \( \emptyset \cup (K_2)^\tau = (K_2)^\tau \), \( \emptyset \cup (K_3)^\tau = (K_3)^\tau \), \( I_1 \cup (K_1)^\tau = I_1 \), \( I_1 \cup (K_2)^\tau = I_1 \), \( I_1 \cup (K_3)^\tau = I_1 \), \( (K_1)^\tau \cup (K_2)^\tau = 1 \), \( (K_1)^\tau \cup (K_3)^\tau = 1 \), \( (K_2)^\tau \cup (K_3)^\tau = 1 \).

Thus \((V, (\tau, \tau'), E)\) is a NSSPTS over \((V, E)\).

**Proposition 4.4** Let \((V, (\tau_1, \tau_2)', E)\) and \((V, (\tau_2, \tau_3)', E)\) be two NSSPTSs over \((V, E)\). Then \((V, (\tau_1, \tau_2) \cap (\tau_2, \tau_3)', E)\) is also a NSSPTS over \((V, E)\).

Proof. Let \((V, (\tau_1, \tau_2)', E)\) and \((V, (\tau_2, \tau_3)', E)\) be two NSSPTSs over \((V, E)\).

(i) Obviously, \( \emptyset, 1, \in (\tau_2, \tau_3)' \cap (\tau_2, \tau_3)' \).
(ii) Let \( \{ K_i \in \text{NSWS}(V,E) : i \in I \} \subseteq (\tau_1)' \cap (\tau_2)' \). Then \( \{ K_i \} \subseteq (\tau_1)' \) and \( \{ K_i \} \subseteq (\tau_2)' \). 
\[ \bigcup_{i \in I} K_i \subseteq (\tau_1)' \text{ and } \bigcup_{i \in I} K_i \subseteq (\tau_2)' \] 
Thus, \( (\tau_1)' \cap (\tau_2)' \) is a NSSPTS over \((V,E)\).

**Remark 4.5**

Let \( (\tau_1)' \) and \( (\tau_2)' \) be two NSWSs on NSSPTS \((V,E)\). Then \( (\tau_1)' \cup (\tau_2)' \) is not NSSPTS over \((V,E)\).

**Example 4.6** Consider Example 3.5. Here \( (\tau_1)' = \{ \phi_1, 1, K^c \} \) and \( \tau_2 = \{ \phi_1, 1, L^c \} \) where the NSWSs \( K^c \) and \( L^c \) over \( V \) are defined as

\[
K^c = \begin{cases}
\begin{align*}
\{ & \left< g_1, (1,1,0) \right>, \left< g_2, (1,1,0) \right>, \left< g_3, (1,1,0) \right> \\
& \left< g_1, (1,1,0), g_2, (1,1,0), g_3, (1,1,0) \right>
\end{align*}
\end{cases}
\]

and

\[
L^c = \begin{cases}
\begin{align*}
\{ & \left< g_1, (1,1,0) \right>, \left< g_2, (1,1,0) \right>, \left< g_3, (1,1,0) \right> \\
& \left< g_1, (1,1,0), g_2, (1,1,0), g_3, (1,1,0) \right>
\end{align*}
\end{cases}
\]

Thus, \( (\tau_1)' \cup (\tau_2)' = \{ \phi_1, 1, K^c, L^c \} \) is not an NSSPTS over \((V,E)\), since \( K^c \cup L^c \notin (\tau_1)' \cup (\tau_2)' \).

**Proposition 4.7** Let \( K \) and \( L \) be any two NSWSs on NSSPTS \((V, (\tau,)' , E)\) over \((V,E)\). Then

(i) \( (K \cup L)' = K^c \cap L^c \).

(ii) \( (K \cap L)' = K^c \cup L^c \).

Proof. Straight forward.

**Proposition 4.8** Let \((V, (\tau,)' , E)\) be a NSSPTS over \((V,E)\) and 
\[ (\tau,)' = \{ K_i : K_i \in \text{NSWS}(V,E) = \{ \phi, \underbrace{\{ (E)K \}}_{E \in E}, K_i \in \text{NSWS}(V,E) \} \} \]
where \( f^{(E)K}(\cdot) = \{ (v,T_f^{(\nu)_K}(v)), I_f^{(\nu)_K}(v), F_f^{(\nu)_K}(v) : v \in V, \nu \in E, \} \).

Define
\[
(\tau_1)' = \{ T_f^{(\nu)_K}(V) \}_{\nu \in E}, \\
(\tau_2)' = \{ I_f^{(\nu)_K}(V) \}_{\nu \in E} \text{ and} \\
(\tau_3)' = \{ F_f^{(\nu)_K}(V) \}_{\nu \in E}.
\]

Then \( (\tau_1)', (\tau_2)' \) and \( (\tau_3)' \) are FSTs on \((V,E)\).

Proof. Let \((V, (\tau,)' , E)\) be a NSSPTS over \((V,E)\).

(i) Since \( \phi_1, 1 \in (\tau,)' \),
\[
0,1 \in (\tau_1)', 0,1 \in (\tau_2)', 1,0 \in (\tau_3)'.
\]

(ii) Let \( \{ K_i \in \text{NSWS}(V,E) : i \in I \} \subseteq (\tau_1)' \).

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Then \( \bigcup_{i=1}^{n} K_i \in (\tau_i)' \).

That is,
\[
\bigcup_{i=1}^{n} K_i = \left\{ \left\{ \sup_{\tau \in E} [f_{i}(\tau, K_i)(V)]_{\tau \in E}, \sup_{\tau \in E} [f_{j}(\tau, K_j)(V)]_{\tau \in E}, \inf_{\tau \in E} [F_{i}(\tau, K_i)(V)]_{\tau \in E}, \inf_{\tau \in E} [F_{j}(\tau, K_j)(V)]_{\tau \in E} \right\} \right\} \in (\tau_i)'.
\]

Thus
\[
\left\{ \sup_{\tau \in E} [f_{i}(\tau, K_i)(V)]_{\tau \in E} \right\} \in (\tau_1)',
\]
\[
\left\{ \sup_{\tau \in E} [f_{j}(\tau, K_j)(V)]_{\tau \in E} \right\} \in (\tau_2)',
\]
and
\[
\left\{ \sup_{\tau \in E} [F_{i}(\tau, K_i)(V)]_{\tau \in E} \right\} \in (\tau_3)'.
\]

(iii) Let \( K_1, K_2 \in (\tau_i)' \).

Then \( K_1 \cap K_2 \in (\tau_i)' \).

That is,
\[
K_1 \cap K_2 = \left\{ \left\{ \min_{\tau \in E} [f_{i}(\tau, K_i)(V), T_{j}(\tau, K_j)(V)]_{\tau \in E}, \min_{\tau \in E} [f_{j}(\tau, K_j)(V), T_{j}(\tau, K_j)(V)]_{\tau \in E}, \max_{\tau \in E} [F_{i}(\tau, K_i)(V), F_{j}(\tau, K_j)(V)]_{\tau \in E} \right\} \right\} \in (\tau_i)'.
\]

Thus
\[
\left\{ \min_{\tau \in E} [f_{i}(\tau, K_i)(V), T_{j}(\tau, K_j)(V)]_{\tau \in E} \right\} \in (\tau_1)'
\]
\[
\left\{ \min_{\tau \in E} [f_{j}(\tau, K_j)(V), T_{j}(\tau, K_j)(V)]_{\tau \in E} \right\} \in (\tau_2)'
\]
and
\[
\left\{ \min_{\tau \in E} [F_{i}(\tau, K_i)(V), F_{j}(\tau, K_j)(V)]_{\tau \in E} \right\} \in (\tau_3)'.
\]

Hence \((\tau_1)',(\tau_2)'\) and \((\tau_3)'\) are FSTs on \((V,E)\).

**Remark 4.9** The following example illustrates that the converse of Proposition 4.8 is not true.

**Example 4.10** Consider Example 3.9. Here \((\tau)_i = \left\{ \phi, 1, (K_i)^c, (K_2)^c \right\} \) where the NSWSs \((K_1)^c\) and \((K_2)^c\) over \(V\) are defined as

\[
(K_1)^c = \left\{ f^{(e)}(K_1)^c = \{< g_1, (1,1,0), < g_2, (1,1,0), < g_3, (1,1,0) > \}, \right\}
\]
\[
(K_2)^c = \left\{ f^{(e)}(K_2)^c = \{< g_1, (1,1,0), < g_2, (1,1,0), < g_3, (1,1,0) > \}, \right\}
\]

Then
\[
(\tau_1)' = \left\{ \sup_{\tau \in E} [f_{i}(\tau, K_i)(V), T_{j}(\tau, K_j)(V), T_{j}(\tau, K_j)(V)]_{\tau \in E}, \inf_{\tau \in E} [F_{i}(\tau, K_i)(V), F_{j}(\tau, K_j)(V)]_{\tau \in E} \right\}
\]
\[
(\tau_2)' = \left\{ \sup_{\tau \in E} [f_{i}(\tau, K_i)(V), I_{j}(\tau, K_j)(V), I_{j}(\tau, K_j)(V)]_{\tau \in E}, \inf_{\tau \in E} [F_{i}(\tau, K_i)(V), F_{j}(\tau, K_j)(V)]_{\tau \in E} \right\}
\]

and
(τ₁)’ = \\left\{ F^{(i)}_{F(V)}, F^{(i)}_{I(V)}, F^{(i)}_{\tau_{K}}, F^{(i)}_{\tau_{K,2}}, F^{(i)}_{\tau_{K,3}} \right\}_{i \in E} \] are FSTs on \((V, E)\),

where

\((τ₁)’ = \{ e_1, (0,0,0), (1,1,1), (1,1,1) \}, \{ e_2, (0,0,0), (1,1,1), (1,1,1) \}, \{ e_3, (0,0,0), (1,1,1), (1,1,1) \}, \{ e_4, (0,0,0), (1,1,1), (1,1,1) \} \) and so on.

Thus \((τ₁)’ = \{ φ, 1, (K₁)’, (K₂)’ \} \) is not NSSPT over \((V, E)\), since \((K₁)’ \cap (K₂)’ \neq (τ₁)’\).

**Proposition 4.11** Let \((V, (τ₁), E)\) be a NSSPTS over \((V, E)\) and

\((τ₁)’ = \{ K_i : K_i \in NSWS(V, E) \} = \{ \langle v, f^{(E)}K \rangle : K_i \in NSWS(V, E) \} \)

Where

\[ f^{(E)K} = \left\{ v, T^{(v)K}(v), I^{(v)K}(v), F^{(v)K}(v) \right\} : v \in V, e \in E. \]

Define

\((τ₁)’ = \{ F^{(v)K} \} \)

and

\[(τ₁)’ = \{ F^{(v)K} \} \]

are FSTs on \((V, E)\).

Then \((τ₁)’ \cup (τ₁)’\) and \((τ₁)’ \cup (τ₁)’\) are not FSTs on \((V, E)\).

**Proposition 4.11** is illustrated by the following example.

**Example 4.12** Consider Example 3.11. Here \((τ₁)’ = \{ φ, 1, (K₁)’, (K₂)’ \} \) where the NSWSs \((K₁)’, (K₂)’ \) over \(V\) are defined as

\((K₁)’ = \left\{ f^{(i)}_{K₁} \right\} = \left\{ < g₁, (1,1,0) >, < g₂, (1,1,0) >, < g₃, (1,1,0) > \right\} \)

\((K₂)’ = \left\{ f^{(i)}_{K₂} \right\} = \left\{ < g₁, (1,1,0) >, < g₂, (1,1,0) >, < g₃, (1,1,0) > \right\} \)

\((K₃)’ = \left\{ f^{(i)}_{K₃} \right\} = \left\{ < g₁, (1,1,0) >, < g₂, (1,1,0) >, < g₃, (1,1,0) > \right\} \)

\[ (φ) \cup (K₁)’ = (K₁)’ \]

\[ φ \cup (K₂)’ = (K₂)’ \]

\[ (K₁)’ \cup (K₂)’ = (K₂)’ \]

\[ (K₁)’ \cup (K₃)’ = (K₃)’ \]

Then \((K₁)’, (K₂)’ \) and \((K₃)’ \) are NSSPOSs.

Thus \((V, (τ₁), E)\) is a NSSPTS over \((V, E)\).

Then

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\[(\tau_{1})^{*} = \{e_{1}, (0,0,0),(1,1,1),(1,1,1),(1,1,1),(1,1,1)\}, \{e_{2}, (0,0,0),(1,1,1),(1,1,2),(1,1,5),(1,1,5)\},
\{e_{3}, (0,0,0),(1,1,1),(1,1,1),(1,1,1)\}, \{e_{4}, (0,0,0),(1,1,1),(1,1,1),(1,1,1)\}\]

and so on, are FSTs on \((V, E)\).

But, \((\tau_{1})^{*} \cup (\tau_{3})^{*}\) and \((\tau_{2})^{*} \cup (\tau_{3})^{*}\) are not FSTs on \((V, E)\).

**Proposition 4.13** Let \((V, (\tau_{i})^{*}, E)\) be a NSSPTS over \((V, E)\). Then

\[
(\tau_{1})^{*} = \left\{ T_{f}(\alpha_{K}) : K \in (\tau_{i})^{*} \right\},
\]

\[
(\tau_{2})^{*} = \left\{ I_{f}(\alpha_{K}) : K \in (\tau_{i})^{*} \right\},
\]

\[
(\tau_{3})^{*} = \left\{ F_{f}(\alpha_{K}) : K \in (\tau_{i})^{*} \right\}
\]

for each \(e \in E\), define FTs on \((V, E)\).

**Proof.** Follows from Proposition 4.8.

**Remark 4.14** The following example illustrates that the converse of Proposition 4.13 is not true.

**Example 4.15** Consider Example 4.10. Then

\[
(\tau_{1})^{*} = \left\{ T_{f}(\alpha_{K}) : K \in (\tau_{i})^{*} \right\},
\]

\[
(\tau_{2})^{*} = \left\{ I_{f}(\alpha_{K}) : K \in (\tau_{i})^{*} \right\},
\]

\[
(\tau_{3})^{*} = \left\{ F_{f}(\alpha_{K}) : K \in (\tau_{i})^{*} \right\}
\]

are FTs on \(V\), where

\[
(\tau_{1})^{*} = \{(0,0,0),(1,1,1),(1,1,1),(1,1,1)\}
\]

and so on.

Consequently, \(\{(\tau_{1})^{*}, (\tau_{2})^{*}, (\tau_{3})^{*}\}\) and \(\{(\tau_{1})^{*}, (\tau_{2})^{*}, (\tau_{3})^{*}\}\) are fuzzy tritopologies on \(V\).

Thus \((\tau_{1})^{*} = \phi, 1, (K_{1})^{*}, (K_{2})^{*}\) is not NSSPT over \((V, E)\), since \((K_{1})^{*} \cap \tau_{2}^{*} \neq (\tau_{1})^{*}\).

**Definition 4.16** Let \((V, (\tau_{i})^{*}, E)\) be a NSSPTS over \((V, E)\) and \(L \in \text{NSWS}(V, E)\) be any NSWS. Then the neutro- supr spot absolute closure of \(L\) is denoted by \(\overline{L}_{-}\) and defined as

\[
(i) \quad \overline{L}_{-} = \{K : K \in \text{NSSPOS} & K \subseteq L\} \quad \text{i.e., the intersection of all neutro-supra spot open subsets of } L.
\]

\[
(ii) \quad \overline{L}_{-} = \left\{ \min_{e \in E} T_{f}(\alpha_{K}) \min_{e \in E} I_{f}(\alpha_{K}) \max_{e \in E} F_{f}(\alpha_{K}) \right\} E : K \in (\tau_{i})^{*} \text{ & } f \left( \frac{\alpha_{L}}{f} \right) \subseteq f \left( \frac{\alpha_{L}}{f} \right)
\]

**Example 4.17** Consider Example 3.16. Here \((\tau_{i})^{*} = \phi, 1, (K_{1})^{*}, (K_{2})^{*}, (K_{3})^{*}\) where the NSWSs

\[(K_{1})^{*}, (K_{2})^{*}, (K_{3})^{*}\] over \(V\) are defined as

\[
\begin{cases}
\begin{align*}
(K_{1})^{*} &= \left\{ g_{1}, (1,1,0) > < g_{2}, (1,1,0) \right\} \\
(K_{2})^{*} &= \left\{ g_{1}, (2,6,9) > < g_{2}, (1,1,0) \right\} \\
(K_{3})^{*} &= \left\{ g_{1}, (1,1,0) > < g_{2}, (1,1,0) \right\}
\end{align*}
\end{cases}
\]

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Let \( (V, (\tau, \tau')', E) \) be a NSSPTS over \( (V, E) \) and \( L \in NSWS(V, E) \) be any NSWS defined as

\[
\begin{align*}
L &= \begin{cases} f^{(\alpha)}L = \prec g_1, (1,1,0), >, g_2, (1,1,0) \}, & \text{and} \\
& f^{(\alpha)}L = \prec g_1, (1,1,0), >, g_2, (1,1,0) \}, & f^{(\alpha)}L = \prec g_1, (1,1,0), >, g_2, (1,1,0) \}, \end{cases}
\]

Then \( 1, (K_1)^c \), \( (K_3)^c \) \( \supseteq \) \( L \).

Thus \( L = 1, (K_1)^c \), \( (K_3)^c \) \( = \) \( (K_1)^c \).

Also,

\[
\begin{align*}
& f^{(c_1)}(K_1)^c, f^{(c_3)}(K_2)^c, f^{(c_2)}(K_3)^c \supseteq f^{(c_1)}L, \\
& f^{(c_2)}(K_1)^c, f^{(c_3)}(K_2)^c, f^{(c_2)}(K_3)^c \supseteq f^{(c_2)}L \text{ and} \\
& f^{(c_1)}(K_1)^c, f^{(c_3)}(K_2)^c \supseteq f^{(c_3)}L.
\end{align*}
\]

Then

\[
\begin{align*}
& \bar{T}_{f^{(\alpha)}L} \bar{T}_{f^{(\alpha)}L} \subseteq \bar{T}_{f^{(\alpha)}L} \supseteq \bar{T}_{f^{(\alpha)}L} = \prec g_1, (1,1,0), >, g_2, (1,1,0) >, \\
& \bar{T}_{f^{(\alpha)}L} \bar{T}_{f^{(\alpha)}L} \subseteq \bar{T}_{f^{(\alpha)}L} \supseteq \bar{T}_{f^{(\alpha)}L} = \prec g_1, (2,6,9), >, g_2, (1,1,0) >, \\
& \bar{T}_{f^{(\alpha)}L} \bar{T}_{f^{(\alpha)}L} \subseteq \bar{T}_{f^{(\alpha)}L} \supseteq \bar{T}_{f^{(\alpha)}L} = \prec g_1, (1,1,0), >, g_2, (1,1,0) >.
\end{align*}
\]

Thus \( \bar{T}_{f^{(\alpha)}L} \bar{T}_{f^{(\alpha)}L} \subseteq \bar{T}_{f^{(\alpha)}L} \subseteq \bar{T}_{f^{(\alpha)}L} = (K_1)^c \).

**Theorem 4.18** Let \( (V, (\tau, \tau'), E) \) be a NSSPTS over \( (V, E) \) and \( L, S \in NSWS(V, E) \) be any two NSWSs. Then,

(i) \( L \subseteq \bar{L} \) and \( \bar{L} \) is the smallest NSSPOS.

(ii) \( L \subseteq S \Rightarrow \bar{L} \subseteq \bar{S} \).

(iii) \( \bar{L} \) is a NSSPOS i.e., \( \bar{L} \in (\tau')^c \).

(iv) \( L \) is a NSSPOS iff \( \bar{L} = L \).

(v) \( \bar{L} = L \).
(vi) \((\overline{\phi}_-) = \overline{\phi}_-\) and \((\overline{1}_-) = \overline{1}_-\).

(vii) \((\overline{L \cup S}) = \overline{L} \cup \overline{S}\).

(ix) \((\overline{L \cap S}) \subseteq \overline{L} \cap \overline{S}\).

Proof. Let \((V, (\tau_u^\prime), E)\) be a NSSPTS over \((V, E)\) and \(L, S \in \text{NSWS}(V, E)\) be any two NSWSs.

(i) Follows from Definition 4.16.

(ii) Let \(L \subseteq S\). Then \(L \subseteq \overline{L}\) and \(P \subseteq \overline{S}_- \Rightarrow L \subseteq S \subseteq \overline{S}_- \Rightarrow L \subseteq \overline{S}_-\).

Since \(\overline{L}_-\) is the smallest NSSPOS containing \(L\), hence \(\overline{L}_- \subseteq \overline{S}_-\).

(iii) Follows from Definition 4.16.

(iv) Let \(L\) be a NSSPOS. Then \(\overline{L}_-\) is the smallest NSSPOS which containing \(L\) is equal to \(L\).

Hence \(\overline{L}_- = L\).

Conversely, assume that \(\overline{L}_- = L\).

By (iii), \(\overline{L}_- \in (\tau_u^\prime)^c\).

Then \(L\) is a NSSPOS.

(v) Let \(\overline{L}_- = K\). Then \(K \in (\tau_u^\prime)^c\) iff \(\overline{K}_- = K\).

Thus \((\overline{L}_-) = \overline{L}_-\).

(vi) Since \(\overline{\phi}_- \in (\tau_u^\prime)^c\) and by (iv), hence \((\overline{\phi}_-) = \overline{\phi}_-\) and \((\overline{1}_-) = \overline{1}_-\).

(vii) \(L \subseteq \overline{L \cup S}\) and \(S \subseteq \overline{L \cup S}\).

\[\Rightarrow \overline{L} \subseteq \overline{L \cup S}\text{ and } \overline{S}_- \subseteq \overline{L \cup S}\.
\]

\[\Rightarrow \overline{L \cup S} \subseteq \overline{L \cup S}_-\text{.}
\]

Also, \(L \subseteq \overline{L}_-\) and \(S \subseteq \overline{S}_-\).

Then \(\overline{L \cup S} \subseteq \overline{L}_- \cup \overline{S}_-\).

Since \(\overline{L \cup S} \subseteq \overline{L \cup S}_-\), and it is the smallest NSSPOS, then \((\overline{L \cup S}) \subseteq \overline{L} \cup \overline{S}_-\).

Hence \((\overline{L \cup S}) = \overline{L} \cup \overline{S}_-\).

(vii) \(L \cap S \subseteq L\) and \(L \cap S \subseteq S\).

\[\Rightarrow \overline{L \cap S} \subseteq \overline{L}_-\text{ and } \overline{L \cap S} \subseteq \overline{S}_-\text{.}
\]

\[\Rightarrow \overline{L \cap S} \subseteq \overline{L} \cap \overline{S}_-\text{.}
\]

\[\text{Definition 4.19} \text{ Let } (V, (\tau_n^\prime), E) \text{ be a NSSPTS over } (V, E) \text{ where } (\tau_n^\prime) \text{ is a NSSPT over } (V, E) \text{ and } L \in \text{NSWS}(V, E) \text{ be any NSWS. Then the subspace topology on NSSPTS is denoted by } (\tau_n^\prime)^c \text{ and defined as } (\tau_n^\prime)^c = \{L \cap K: K \in (\tau_n^\prime)\} \text{ and } \phi_1 \in (\tau_n^\prime)^c\text{. Thus } (V, (\tau_n^\prime)^c), E) \text{ is a Neutro-supra spot subspace topological space (NSSPSTS) of } (V, (\tau_n^\prime), E) \text{, where } (\tau_n^\prime)^c \text{ is also a NSSPT over } (V, E)\text{.}
\]

\[\text{Example 4.20} \text{ Consider Example 3.19. Here } (\tau_n^\prime)^c = \{\phi_1, (K_1)^c, (K_2)^c, (K_3)^c\} \text{ where the NSWSs } (K_1)^c, (K_2)^c, (K_3)^c \text{ over } V \text{ are defined as}
\]

\[(K_1)^c = \begin{cases} f^{(\phi_1)}(K_1)^c = \{< g_1, \{1,1,0\}, < g_2, \{1,1,0\}, < g_3, \{1,1,0\}> \} \\
 f^{(\phi_1)}(K_1)^c = \{< g_1, \{1,1,0\}, < g_2, \{1,1,0\}, < g_3, \{1,1,0\}> \}
\end{cases};
\]

\[(K_2)^c = \begin{cases} f^{(\phi_1)}(K_2)^c = \{< g_1, \{1,1,0\}, < g_2, \{1,1,0\}, < g_3, \{1,1,0\}> \} \\
 f^{(\phi_1)}(K_2)^c = \{< g_1, \{1,1,0\}, < g_2, \{1,3,4\}, < g_3, \{1,1,0\}> \}
\end{cases}\text{ and}
\]
(K_3)^c = \begin{cases} f^{(c_1)}(K_3)^c = \langle g_1, (1,1,0), < g_2, (1,1,0), < g_3, (1,1,0) > \\ f^{(c_2)}(K_3)^c = \langle g_1, (1,1,0), < g_2, (8,6,1), < g_3, (1,1,0) > \end{cases}.

Here \( \phi \cup (K_3)^c = (K_3)^c \), \( \phi \cup (K_3)^c = (K_3)^c \), \( 1 \cup (K_1)^c = 1 \), \( 1 \cup (K_2)^c = 1 \), \( 1 \cup (K_3)^c = 1 \), \( (K_1)^c \cup (K_2)^c = (K_2)^c \), \( (K_2)^c \cup (K_3)^c = (K_3)^c \), \( (K_2)^c \cup (K_3)^c = (K_3)^c \).

Then \((K_1)^c, (K_2)^c\) and \((K_3)^c\) are NSSPOSs.

Thus \((V, (\tau_r)^c, E)\) is a NSSPTS over \((V, E)\).

Define \(L_4 \in \text{NSWS}(V, E)\) as

\[
L_4 = \begin{cases} f^{(c_1)}(L_4) = \langle g_1, (1,1,0), < g_2, (1,1,0), < g_3, (1,1,0) > \\ f^{(c_2)}(L_4) = \langle g_1, (1,1,0), < g_2, (6,7,7), < g_3, (1,1,0) > \end{cases}.
\]

Thus we denote \(L_4 \cap \phi = L_4\), \(L_4 \cap 1 = L_4\), \(L_4 \cap (K_1)^c = L_2\), \(L_4 \cap (K_2)^c = L_3\), \(L_4 \cap (K_3)^c = L_4\).

Then the NSWSs \(L_2, L_3, L_4\) over \(V\) is defined as

\[
L_2 = \begin{cases} f^{(c_1)}(L_2) = \langle g_1, (1,1,0), < g_2, (1,1,0), < g_3, (1,1,0) > \\ f^{(c_2)}(L_2) = \langle g_1, (1,1,0), < g_2, (5,6,7), < g_3, (1,1,0) > \end{cases}.
\]

\[
L_3 = \begin{cases} f^{(c_1)}(L_3) = \langle g_1, (1,1,0), < g_2, (1,1,0), < g_3, (1,1,0) > \\ f^{(c_2)}(L_3) = \langle g_1, (1,1,0), < g_2, (6,3,7), < g_3, (1,1,0) > \end{cases}.
\]

\[
L_4 = \begin{cases} f^{(c_1)}(L_4) = \langle g_1, (1,1,0), < g_2, (1,1,0), < g_3, (1,1,0) > \\ f^{(c_2)}(L_4) = \langle g_1, (1,1,0), < g_2, (6,6,7), < g_3, (1,1,0) > \end{cases}.
\]

Then \((\tau_r)^c, (K_1)^c, L_2, L_3, L_4\) is a NSSPST over \((V, E)\).

Thus \((V, (\tau_r)^c, E)\) is a NSSPSTS of \((V, (\tau_r)^c, E)\).

5. Separation Axioms

This part is split into two parts as separation axioms on NSPTS and NSSPTS are defined with examples.

5.1. Separation Axioms on NSPTS

**Definition 5.1.1** Let \((V, \tau_r, E)\) be a NSPTS over \((V, E)\) where \(\tau_r\) is a NSPT over \((V, E)\). Let \(p\) and \(q\) be any distinct NSPs. If there exists NSPOSs \(R\) and \(S\) such that

\[ p \in R \quad \text{and} \quad p \cap S = \phi \quad \text{or} \]

\[ q \in S \quad \text{and} \quad q \cap R = \phi, \]

Then \((V, \tau_r, E)\) is said to be a neutro-spot \(T_0\)-space.

**Definition 5.1.2** Let \((V, \tau_r, E)\) be a NSPTS over \((V, E)\) where \(\tau_r\) is a NSPT over \((V, E)\). Let \(p\) and \(q\) be any distinct NSPs. If there exists NSPOSs \(R\) and \(S\) such that

\[ p \in R \quad \text{and} \quad p \cap S = \phi \quad \text{and} \]

\[ q \in S \quad \text{and} \quad q \cap R = \phi, \]

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Then $(V, \tau, E)$ is said to be a neutro-spot $T_1$-space.

**Definition 5.1.3** Let $(V, \tau, E)$ be a NSPTS over $(V, E)$ where $\tau$ is a NSPT over $(V, E)$. Let $p$ and $q$ be any distinct NSPs. If there exists NSPOSs $R$ and $S$ such that

$$p \in R, \quad q \in S \quad \text{and} \quad R \cap S = \phi,$$

Then $(V, \tau, E)$ is said to be a neutro-spot $T_2$-space.

**Theorem 5.1.4** Let $(V, \tau, E)$ be a NSPTS over $(V, E)$. Every neutro-spot $T_2$-space is also a neutro-spot $T_1$-space and every neutro-spot $T_1$-space is also a neutro-spot $T_0$-space.

**Proof.** Follows from Definitions 5.1.1, 5.1.2, and 5.1.3.

**Example 5.1.5** Let $V = \{v_1, v_2\}$, $E = \{e_1, e_2\}$ where the NSPs $R_1, R_2, R_3, R_4$ over $V$ are defined as

$$R_1 = \begin{cases} f^{(e_1)}R_1 = \{ <v_1,(0,0,1)>, <v_2,(0,0,1) > \} \\ f^{(e_2)}R_1 = \{ <v_1,(7,4,2)>, <v_2,(0,0,1) > \} \end{cases};$$

$$R_2 = \begin{cases} f^{(e_1)}R_2 = \{ <v_1,(0,0,1)>, <v_2,(0,0,1) > \} \\ f^{(e_2)}R_2 = \{ <v_1,(3,5,6)>, <v_2,(0,0,1) > \} \end{cases};$$

$$R_3 = \begin{cases} f^{(e_1)}R_3 = \{ <v_1,(0,0,1)>, <v_2,(0,0,1) > \} \\ f^{(e_2)}R_3 = \{ <v_1,(3,4,6)>, <v_2,(0,0,1) > \} \end{cases} \quad \text{and}$$

$$R_4 = \begin{cases} f^{(e_1)}R_4 = \{ <v_1,(0,0,1)>, <v_2,(0,0,1) > \} \\ f^{(e_2)}R_4 = \{ <v_1,(0,0,1)>, <v_2,(8,4,5) > \} \end{cases}.$$

Here $\phi_1 \cap R_1 = \phi_1$, $\phi_1 \cap R_2 = \phi_1$, $\phi_1 \cap R_3 = \phi_1$, $\phi_1 \cap R_4 = \phi_1$, $\phi_2 \cap R_1 = \phi_2$, $\phi_2 \cap R_2 = \phi_2$, $\phi_2 \cap R_3 = \phi_2$, $\phi_2 \cap R_4 = \phi_2$, $\phi_3 \cap R_1 = \phi_3$, $\phi_3 \cap R_2 = \phi_3$, $\phi_3 \cap R_3 = \phi_3$, $\phi_3 \cap R_4 = \phi_3$.

Then $R_1, R_2, R_3$ and $R_4$ are NSPOSs.

Thus $(V, \tau, E)$ is a NSPTS over $(V, E)$.

Let $p$ and $q$ be any distinct NSPs which are defined as

$$p = \begin{cases} f^{(e_1)}p = \{ <v_1,(0,0,1)>, <v_2,(0,0,1) > \} \\ f^{(e_2)}p = \{ <v_1,(1,4,7)>, <v_2,(0,0,1) > \} \end{cases} \quad \text{and}$$

$$q = \begin{cases} f^{(e_1)}q = \{ <v_1,(0,0,1)>, <v_2,(0,0,1) > \} \\ f^{(e_2)}q = \{ <v_1,(0,0,1)>, <v_2,(2,1,5) > \} \end{cases}.$$

Hence $(V, \tau, E)$ is a neutro-spot $T_2$-space, also a neutro-spot $T_1$-space and a neutro-spot $T_0$-space.

### 5.2. Separation Axioms on NSSPTS

**Definition 5.2.1** Let $(V, (\tau)_o, E)$ be a NSSPTS over $(V, E)$ where $(\tau)_o$ is a NSSPT over $(V, E)$. Let $p$ and $q$ be any distinct NSPs. If there exists NSSPOSs $R$ and $S$ such that

$$p \in R \quad \text{and} \quad p \cap \phi \neq \phi_1 \quad \text{or}$$

$$q \in S \quad \text{and} \quad q \cap \phi \neq \phi_2 \quad \text{or}$$

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Thus (V, (τ), E) is said to be a neutro-supra spot T₀-space.

**Definition 5.2.2** Let (V, (τ), E) be a NSSPTS over (V, E) where (τ) is a NSSPT over (V, E). Let p and q be any distinct NSPs. If there exists NSSPOSs R and S such that

\[ p \in R \quad \text{and} \quad q \cap R \equiv \phi, \]

Then (V, (τ), E) is said to be a neutro-supra spot T₁-space.

**Definition 5.2.3** Let (V, (τ), E) be a NSSPTS over (V, E) where (τ) is a NSSPT over (V, E). Let p and q be any distinct NSPs. If there exists NSSPOSs R and S such that

\[ p \in R \quad \text{and} \quad q \cap S \equiv \phi, \]

Then (V, (τ), E) is said to be a neutro-supra spot T₂-space.

**Theorem 5.2.4** Let (V, (τ), E) be a NSSPTS over (V, E). Every neutro-supra spot T₂-space is also a neutro-supra spot T₁-space and every neutro-supra spot T₁-space is also a neutro-supra spot T₀-space.

Proof. Follows from Definitions 5.2.1, 5.2.2, and 5.2.3.

**Example 5.2.5** Consider Example 5.1.5. Here (τ) = \{φ₁, 1, (R₁) c, (R₂), (R₃), (R₄) c\} where the NSWSs (R₁)c, (R₂)c, (R₃)c, (R₄)c over V is defined as

\[
(R₁)c = \begin{cases} 
    f(φ₁)(R₁)c = \{< v₁, (1,1,0) >, < v₂, (1,1,0) > \} \\
    f(φ₂)(R₁)c = \{< v₁, (2,2,7) >, < v₂, (1,1,0) > \}
\end{cases}
\]

\[
(R₂)c = \begin{cases} 
    f(φ₁)(R₂)c = \{< v₁, (1,1,0) >, < v₂, (1,1,0) > \} \\
    f(φ₂)(R₂)c = \{< v₁, (6,6,3) >, < v₂, (1,1,0) > \}
\end{cases}
\]

\[
(R₃)c = \begin{cases} 
    f(φ₁)(R₃)c = \{< v₁, (1,1,0) >, < v₂, (1,1,0) > \} \\
    f(φ₂)(R₃)c = \{< v₁, (6,6,3) >, < v₂, (1,1,0) > \}
\end{cases}
\]

\[
(R₄)c = \begin{cases} 
    f(φ₁)(R₄)c = \{< v₁, (1,1,0) >, < v₂, (1,1,0) > \} \\
    f(φ₂)(R₄)c = \{< v₁, (1,1,0) >, < v₂, (5,6,8) > \}
\end{cases}
\]

Here \( φ₁ \cup (R₁)c = (R₁)c \), \( φ₂ \cup (R₂)c = (R₂)c \), \( φ₁ \cup (R₃)c = (R₃)c \), \( φ₂ \cup (R₄)c = (R₄)c \), \( 1, \cup (R₁)c = 1, \), \( 1, \cup (R₂)c = 1, \), \( 1, \cup (R₃)c = 1, \), \( (R₁)c \cup (R₂)c = (R₂)c \), \( (R₁)c \cup (R₃)c = (R₃)c \), \( (R₁)c \cup (R₄)c = (R₄)c \), \( (R₂)c \cup (R₃)c = (R₃)c \), \( (R₂)c \cup (R₄)c = (R₄)c \), \( (R₃)c \cup (R₄)c = (R₄)c \), \( (R₁)c \cup (R₂)c \cup (R₃)c = (R₃)c \), \( (R₁)c \cup (R₂)c \cup (R₄)c = (R₄)c \), \( (R₂)c \cup (R₃)c \cup (R₄)c = (R₄)c \), \( (R₁)c \cup (R₂)c \cup (R₃)c \cup (R₄)c = 1, \)

Then (R₁)c, (R₂)c, (R₃)c and (R₄)c are NSSPOSs.

Thus (V, (τ), E) is a NSSPTS over (V, E).

Let p and q be any distinct NSPs which are defined as

\[
p = \begin{cases} 
    f(φ₁)p = \{< v₁, (0,0,1) >, < v₂, (0,0,1) > \} \\
    f(φ₂)p = \{< v₁, (0.001, 0.002, 0.987) >, < v₂, (0,0,1) > \}
\end{cases}
\]

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\[ q = \left\{ \begin{array}{ll} f^{(e_1)}(q) = \{v_1, (0,0,1), <v_2, (0,0,1)\} \\ f^{(e_2)}(q) = \{v_1, (0,0,1), <v_2, (0.002, 0.001, 0.998)\} \end{array} \right. \]

Hence \((V, (\tau_1', E))\) is a neutro-supra spot \(T_2\)-space, also a neutro-supra spot \(T_1\)-space and a neutro-supra spot \(T_0\)-space.

6. DM Problem to Detect the Impact on COVID-19

In this part, the DM problem explains the COVID-19 situation and detected its impact on corona virus patients to undergoing exact treatment for them according to their medical report. The process of evaluation is pointed out in the algorithm and formula defined for computing the result.

**Definition 6.1** Let \((V, \tau_{11}, E_1)\) and \((V, \tau_{12}, E_2)\) be two NSPTSs over \((V, E_1)\) and \((V, E_2)\), respectively where \(V\) is the set of risk factors and \(E_1, E_2\) are two different parametric sets of medical issues. Let \(M\) be a corona virus patient, were \(E_1 \in \tau_{11}\) and \(E_2 \in \tau_{12}\) are two NSPs.

Then for each \(v \in V\), the Risk State Value (RSV) of \(M(v)\) is given as:

\[
\text{RSV}[M(v)^{(E_1, E_2)}] = \left[ \frac{A-C}{2} \right] \times \left[ 1 - \frac{B}{2} \right],
\]

where

\[
A = \sum_{i=1}^{k} T\left[ f^{(v)}(M)\right]^{(v)} + \sum_{j=1}^{k} T\left[ f^{(v)}(M)\right]^{(v)},
\]

\[
B = \sum_{i=1}^{k} I\left[ f^{(v)}(M)\right]^{(v)} + \sum_{j=1}^{k} I\left[ f^{(v)}(M)\right]^{(v)},
\]

\[
C = \sum_{i=1}^{k} F\left[ f^{(v)}(M)\right]^{(v)} + \sum_{j=1}^{k} F\left[ f^{(v)}(M)\right]^{(v)}
\]

and for all \(s_i \in E_1\), \(r_j \in E_2\).

Then \(\forall v \in V\), the Total Risk State Value (TRSV) of \(M\) is given as:

\[
\text{TRSV}(M) = \sum_{n} \text{RSV}[M(v_n)^{(E_1, E_2)}],
\]

**Algorithm**

**Step 1:** List the set of risk factors \(v \in V\).

**Step 2:** List two different parametric sets, say \(E_1\) and \(E_2\), where \(E_1\) represents the symptoms of COVID-19 and \(E_2\) represents the pre-medical issues.

**Step 3:** Pick out the people affected by COVID-19, say \(M\).

**Step 4:** Go through the medical status of each patients.

**Step 5:** Test their corona virus symptoms \((E_1)\) and categories its risk factors \((V)\).

**Step 6:** Collect those data in the form of NSPs.

**Step 7:** Define a NSPT \(\tau_{11}\) and so \((V, \tau_{11}, E_1)\) is a NSPTS over \((V, E_1)\).

**Step 8:** Check the pre-medical issues of each patient \((E_2)\) and categories its risk factors \((V)\).

**Step 9:** Collect those data in the form of NSPs, which satisfies the condition of NSPT \(\tau_{12}\).

**Step 10:** Define a NSPTS \((V, \tau_{12}, E_2)\) over \((V, E_2)\).

**Step 11:** Use the formula 6.1.1 to calculate the RSV of \(M(v)\), for each \(v \in V\), and tabulate it.

**Step 12:** Use the formula 6.1.2 to calculate the TRSV of \(M\), for all \(v \in V\), and tabulate it.
Step 13: (i) If TRSV(M) is greater than 2.5, then M undergoes ventilation treatment.
(ii) If TRSV(M) lies between 1.5 and 2.5, then M needs to be hospitalized.
(iii) If TRSV(M) is less than 1.5, then M should be self-isolated at home.

Step 14: Step 13 is also concluded according to the risk factor V.

Step 15: If two or more patients have the same TRSV, then each patient required the same treatment.

Problem 6.2 The survey on COVID-19 patients tested in a particular area. Its low-level risk scenarios are talking to someone face to face, walking, jogging, cycling, etc. The medium level risk scenarios are grocery shopping. The high-level risk scenarios are restaurants, public bathrooms, indoor spaces, and common areas. The very high-level risk scenarios are schools, colleges, parties, weddings, cinemas, and workplaces. The people who are affected by COVID-19 are tested by the doctors according to their symptoms and are under investigation. Also, considered other medical issues. They are categorized with some risk factors. Our problem is to recover them quickly by giving appropriate treatment for those affected people.

1. Let $V = \{v_1, v_2, v_3, v_4\}$ be the set of risk factors, where $v_1$ – low risk, $v_2$ – medium risk, $v_3$ – high risk and $v_4$ – very high risk.

2. Let $E_1 = \{s_1, s_2, s_3, s_4\}$ and $E_2 = \{\eta_1, \eta_2, \eta_3, \eta_4\}$ be two different parametric sets. The set $E_1$ represents the symptoms of COVID-19 such as $s_1$ – fever, $s_2$ – dry cough, $s_3$ – chest pain, and $s_4$ – shortness of breath. The set $E_2$ represents pre-medical issues such as $\eta_1$ – diabetes, $\eta_2$ – blood pressure, $\eta_3$ – cardiac diseases, and $\eta_4$ – respiratory diseases.

3. Let $M_1, M_2, M_3$ and $M_4$ be the people affected by COVID-19.

4. Go through the medical status of each patient.

5. First test their corona virus symptoms ($E_1$) and categories its risk factors ($V$).

6. Those data are collected in the form of NSPs, as are follows:

   \[ E_1 M_1 = \{ f^{(1)} M_1 = \{ (v_1, 0, 0, 1) >, (v_2, 0, 0, 1), (v_3, 0, 0, 1), (v_4, 0, 0, 1) \} \} \]

   \[ E_2 M_2 = \{ f^{(2)} M_2 = \{ (v_1, 0, 0, 1) >, (v_2, 0, 0, 1), (v_3, 0, 0, 1), (v_4, 0, 0, 1) \} \} \]

   \[ E_3 M_3 = \{ f^{(3)} M_3 = \{ (v_1, 0, 0, 1) >, (v_2, 0, 0, 1), (v_3, 0, 0, 1), (v_4, 0, 0, 1) \} \} \]

   \[ E_4 M_4 = \{ f^{(4)} M_4 = \{ (v_1, 0, 0, 1) >, (v_2, 0, 0, 1), (v_3, 0, 0, 1), (v_4, 0, 0, 1) \} \} \]

7. Then $\tau_1 = \{ E_1 M_1, E_2 M_2, E_3 M_3, E_4 M_4 \}$ is a NSPT.

Thus ($V, \tau_1, E_1$) is a NSPTS over ($V, E_1$).
8. Then, check the pre-medical issues of each patient (E2) and categories its risk factors (V).
9. Those data are collected in the form of NSPs, as follows:

\[ E^2 M_1 = \begin{cases} f^{(1)} M_1 = \langle v_1, (0, 0, 1), v_2, (0, 0, 1), v_3, (0, 0, 1), v_4, (0, 0, 1) \rangle \\ f^{(2)} M_1 = \langle v_1, (0, 0, 1), v_2, (0, 0, 1), v_3, (0, 0, 1), v_4, (0, 0, 1) \rangle \\ f^{(3)} M_1 = \langle v_1, (0, 0, 1), v_2, (0, 0, 1), v_3, (0, 0, 1), v_4, (0, 0, 1) \rangle \\ f^{(4)} M_1 = \langle v_1, (0, 0, 1), v_2, (0, 0, 1), v_3, (0, 0, 1), v_4, (0, 0, 1) \rangle \end{cases} \]

\[ E^2 M_2 = \begin{cases} f^{(1)} M_2 = \langle v_1, (0, 0, 1), v_2, (0, 0, 1), v_3, (0, 0, 1), v_4, (0, 0, 1) \rangle \\ f^{(2)} M_2 = \langle v_1, (0, 0, 1), v_2, (0, 0, 1), v_3, (0, 0, 1), v_4, (0, 0, 1) \rangle \\ f^{(3)} M_2 = \langle v_1, (0, 0, 1), v_2, (0, 0, 1), v_3, (0, 0, 1), v_4, (0, 0, 1) \rangle \\ f^{(4)} M_2 = \langle v_1, (0, 0, 1), v_2, (0, 0, 1), v_3, (0, 0, 1), v_4, (0, 0, 1) \rangle \end{cases} \]

\[ E^2 M_3 = \begin{cases} f^{(1)} M_3 = \langle v_1, (0, 0, 1), v_2, (0, 0, 1), v_3, (0, 0, 1), v_4, (0, 0, 1) \rangle \\ f^{(2)} M_3 = \langle v_1, (0, 0, 1), v_2, (0, 0, 1), v_3, (0, 0, 1), v_4, (0, 0, 1) \rangle \\ f^{(3)} M_3 = \langle v_1, (0, 0, 1), v_2, (0, 0, 1), v_3, (0, 0, 1), v_4, (0, 0, 1) \rangle \\ f^{(4)} M_3 = \langle v_1, (0, 0, 1), v_2, (0, 0, 1), v_3, (0, 0, 1), v_4, (0, 0, 1) \rangle \end{cases} \]

\[ E^2 M_4 = \begin{cases} f^{(1)} M_4 = \langle v_1, (0, 0, 1), v_2, (0, 0, 1), v_3, (0, 0, 1), v_4, (0, 0, 1) \rangle \\ f^{(2)} M_4 = \langle v_1, (0, 0, 1), v_2, (0, 0, 1), v_3, (0, 0, 1), v_4, (0, 0, 1) \rangle \\ f^{(3)} M_4 = \langle v_1, (0, 0, 1), v_2, (0, 0, 1), v_3, (0, 0, 1), v_4, (0, 0, 1) \rangle \\ f^{(4)} M_4 = \langle v_1, (0, 0, 1), v_2, (0, 0, 1), v_3, (0, 0, 1), v_4, (0, 0, 1) \rangle \end{cases} \]

10. Then \( \tau_{E2} = \{ E^2 M_1, E^2 M_2, E^2 M_3, E^2 M_4 \} \) is a NSPT.
Thus \( (V, \tau_{E2}) \) is a NSPTS over \( (V, E2) \).
11. By using the formula 6.1.1, the RSV of \( M(v) \) are calculated, for each \( v \in V \).
These values are tabulated in the following table.

<table>
<thead>
<tr>
<th>( v_1 )</th>
<th>( v_2 )</th>
<th>( v_3 )</th>
<th>( v_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_1 )</td>
<td>1.755</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( M_2 )</td>
<td>0</td>
<td>1.75</td>
<td>0</td>
</tr>
<tr>
<td>( M_3 )</td>
<td>1.625</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( M_4 )</td>
<td>0</td>
<td>0</td>
<td>1.59</td>
</tr>
</tbody>
</table>

12. By using the formula 6.1.2, the TRSV of \( M \) are calculated, for all \( v \in V \).
These values are tabulated in the following table.

<table>
<thead>
<tr>
<th>( v_1 )</th>
<th>( v_2 )</th>
<th>( v_3 )</th>
<th>( v_4 )</th>
<th>TRSV</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_1 )</td>
<td>1.755</td>
<td>0</td>
<td>0</td>
<td>1.43</td>
</tr>
<tr>
<td>( M_2 )</td>
<td>0</td>
<td>1.75</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( M_3 )</td>
<td>1.625</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( M_4 )</td>
<td>0</td>
<td>0</td>
<td>1.59</td>
<td>1.855</td>
</tr>
</tbody>
</table>

13. Here \( \text{TRSV}(M_1) = 3.185 \) and \( \text{TRSV}(M_4) = 3.445 \), which are greater than 2.5.
Thus they both are at a very high-risk stage. So, they each should be treated in ventilation for quick recovery.

Next, $\text{TRSV}(M_2) = 1.75$ and $\text{TRSV}(M_3) = 1.625$, which are greater than 1.5. Here $M_2$ is under medium risk stage, and so need to be hospitalized. Even though the TRSV of $M_3$ lies between 1.5 and 2.5, $M_3$ is under the low-risk stage. So, $M_3$ should be self-isolated at home itself.

### 7. Conclusions

In this paper, NSPTS and NSSPTS are introduced and defined a subspace topology on them. Along with its absolute interior of NSPT and absolute closure of NSSPT are also defined. Few properties are examined with illustrative examples. This study extended to introduce a concept of separation axioms of NSPTS and NSSPTS are defined as neutro-spot $T_{i=0,1,2}$-$\text{-spaces}$ and neutro-supra spot $T_{i=0,1,2}$-$\text{-spaces}$ respectively with related examples. Additionally, the DM problem explains the COVID-19 situation and detected its impact on corona virus patients to undergoing exact treatment for them according to their medical report. The process of evaluation is pointed out in the algorithm and formula defined for computing the result. The appropriate treatment is provided for affected people as per the estimated value. Some more practical applications of such types of topologies can be explored for future work. Many more sets like open sets, closed sets, rough sets, crisp sets, etc., can be developed on NSPTS and NSSPTS. Later these concepts will step ahead on multi-criteria DM problems by upcoming researchers.

### References
