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On New Types of Weakly Neutrosophic Crisp Continuity

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Abstract: The article processes the conceptualizations of neutrosophic crisp α-open and neutrosophic crisp semi-α-open sets to define some new types of weakly “neutrosophic crisp continuity” essentially, neutrosophic crisp α*-continuous, neutrosophic crisp α**-continuous, neutrosophic crisp semi-α-continuous, neutrosophic crisp semi-α*-continuous and neutrosophic crisp semi-α**-continuous functions. Also, we shall explain the relationships between these types of weakly neutrosophic crisp continuity and the concepts of neutrosophic crisp continuity.

Keywords: Neutrosophic crisp α*-continuous, neutrosophic crisp α**-continuous, neutrosophic crisp semi-α-continuous, neutrosophic crisp semi-α*-continuous, and neutrosophic crisp semi-α**-continuous functions.

1. Introduction


2. Preliminaries

For the whole of the disquisition, (X, 𝒓1), (Y, 𝒓2), and (Z, 𝒓3) (merely X, Y, and Z) habitually intend NCTSs. Let C be a neutrosophic crisp set (shortly, NCS) in NCTS (X, 𝒓1) and denote its complement by Cc. Indicate the neutrosophic crisp open set as NC-OS, and the neutrosophic crisp closed set (its complement) as NC-CS in NCTS (X, 𝒓1). Additionally, we refer to the neutrosophic crisp closure and neutrosophic crisp interior of C via NCcl(C) and NCint(C), correspondingly.
Definition 2.1 [1]: Assume that nonempty particular under study space $X$ has mutually disjoint subsets $C_1, C_2$ and $C_3$. A $NCS:\mathcal{C}$ with form $\mathcal{C} = \langle C_1, C_2 \rangle$ is called an object.

Definition 2.2: For any $NCS:\mathcal{C}$ in $NCTS(X, I_1)$, we have
(i) if $\mathcal{C} \subseteq NCint(NCcl(NCint(\mathcal{C})))$, then it is called a neutrosophic crisp $\alpha$-open set and symbolize by $NCS\alpha$-OS. Furthermore, its complement is named neutrosophic crisp $\alpha$-closed set and signified by $NCS\alpha$-CS. Likewise, we reveal the collection consisting of all $NCS\alpha$-OSs in $X$ with $NCS\alphaO(X)$. [12]
(ii) if $\mathcal{C} \subseteq NCcl(NCint(NCcl(NCint(\mathcal{C}))))$, then it is said to be a neutrosophic crisp semi-$\alpha$-open set and indicated via $NCS\alpha$-OS. Moreover, its complement is known as a neutrosophic crisp semi-$\alpha$-closed set and referred with $NCS\alpha$-CS. Besides, we mentioned the collection of all $NCS\alpha$-OSs in $X$ through $NCS\alphaO(X)$. [2]

Proposition 2.3 [12]: For any $NCS:\mathcal{C}$ in $NCTS(X, I_1)$, then $\mathcal{C} \in NCS\alphaO(X)$ iff we have at least a $NC$-OS $D$ satisfying $D \subseteq \mathcal{C} \subseteq NCint(NCcl(D))$.

Proposition 2.4 [14]: Every $NC$-OS is a $NCS\alpha$-OS, but the opposite is not valid in general.

Proposition 2.5 [2]: In a $NCTS(X, I_1)$, the next assertions stand, but not vice versa:
(i) All $NC$-OSs are $NCS\alpha$-OSs.
(ii) All $NCS\alpha$-OSs are $NCS\alpha$-OSs.

Definition 2.6 [1]: Let $\eta: (X, I_1) \rightarrow (Y, I_2)$ be a function, we called it a neutrosophic crisp continuous and denoted by $NC$-continuous iff for all $NC$-OSs $D$ from $Y$, then its inverse image $\eta^{-1}(D)$ is a $NC$-OS from $X$.

Theorem 2.7 [1]: A function $\eta: (X, I_1) \rightarrow (Y, I_2)$ is $NC$-continuous iff $\eta^{-1}(NCint(D)) \subseteq NCint(\eta^{-1}(D))$ for every $D \subseteq Y$.

Definition 2.8 [1]: Let $\eta: (X, I_1) \rightarrow (Y, I_2)$ be a function, we named it a neutrosophic crisp open and indicated via $NC$-open iff for all $NC$-OSs $C$ from $X$, then its image $\eta(C)$ is a $NC$-OS from $Y$.

Definition 2.9 [13]: Let $\eta: (X, I_1) \rightarrow (Y, I_2)$ be a function, we said it a neutrosophic crisp $\alpha$-continuous and referred through $NC\alpha$-continuous iff for all $NC$-OSs $D$ from $Y$, then its inverse image $\eta^{-1}(D)$ is a $NC\alpha$-OS from $X$.

Proposition 2.10 [14]: Every $NC$-continuous function is a $NC\alpha$-continuous, but the opposite is not valid in general.

3. Weakly Neutrosophic Crisp Continuity Functions

Definition 3.1: Let $\eta: (X, I_1) \rightarrow (Y, I_2)$ be a function, we call it as
(i) a neutrosophic crisp $\alpha^*$-continuous and denoted by $NC\alpha^*$-continuous iff for all $NC\alpha$-OSs $\mathcal{D}$ from $\mathcal{Y}$, then its inverse image $\eta^{-1}(\mathcal{D})$ is a $NC\alpha$-OS from $\mathcal{X}$.
(ii) a neutrosophic crisp $\alpha^{**}$-continuous and indicated via $NC\alpha^{**}$-continuous iff for all $NC\alpha$-OS $\mathcal{D}$ from $\mathcal{Y}$, then its inverse image $\eta^{-1}(\mathcal{D})$ is a $NC\alpha$-OS from $\mathcal{X}$.

**Definition 3.2:** Let $\eta: (\mathcal{X}, \mathcal{I}_1) \rightarrow (\mathcal{Y}, \mathcal{I}_2)$ be a function, we named it as
(i) a neutrosophic crisp semi-$\alpha$-continuous and referred through $NCS\alpha$-continuous iff for all $NC\alpha$-OSs $\mathcal{D}$ from $\mathcal{Y}$, then its inverse image $\eta^{-1}(\mathcal{D})$ is a $NCS\alpha$-OS from $\mathcal{X}$.
(ii) a neutrosophic crisp semi-$\alpha^*$-continuous and symbolize by $NCS\alpha^*$-continuous iff for all $NCS\alpha$-OSs $\mathcal{D}$ from $\mathcal{Y}$, then its inverse image $\eta^{-1}(\mathcal{D})$ is a $NCS\alpha$-OS from $\mathcal{X}$.
(iii) a neutrosophic crisp semi-$\alpha^{**}$-continuous and signified via $NCS\alpha^{**}$-continuous iff for all $NCS\alpha$-OSs $\mathcal{D}$ from $\mathcal{Y}$, then its inverse image $\eta^{-1}(\mathcal{D})$ is a $NCS\alpha$-OS from $\mathcal{X}$.

**Theorem 3.3:** Let $\eta: (\mathcal{X}, \mathcal{I}_1) \rightarrow (\mathcal{Y}, \mathcal{I}_2)$ be a function, then the next declarations are same:

(i) $\eta$ is a $NCS\alpha$-continuous.
(ii) Its inverse image of each $NC-CS$ from $\mathcal{Y}$ is $NCS\alpha$-CS from $\mathcal{X}$.
(iii) $\eta(NCint(NCcl(NCint(C))) \subseteq NCcl(\eta(C)))$, for each $C \in \mathcal{X}$.
(iv) $NCint(NCcl(NCint(\eta^{-1}(\mathcal{D})))) \subseteq \eta^{-1}(NCcl(\mathcal{D}))$, for each $\mathcal{D} \in \mathcal{Y}$.

**Proof:**

(i) $\Rightarrow$ (ii) Suppose $\mathcal{D}$ is a $NC-CS$ from $\mathcal{Y}$. This implies that $\mathcal{D}^c$ stands a $NC$-OS. Hence $\eta^{-1}(\mathcal{D}^c)$ is a $NCS\alpha$-OS from $\mathcal{X}$. In other words, $(\eta^{-1}(\mathcal{D}))^c$ stands a $NCS\alpha$-OS from $\mathcal{X}$. Thus $\eta^{-1}(\mathcal{D})$ is a $NCS\alpha$-CS in $\mathcal{X}$.

(ii) $\Rightarrow$ (iii) Let $C \in \mathcal{X}$, then $NCcl(\eta(C))$ stays a $NC-CS$ from $\mathcal{Y}$. Hence $\eta^{-1}(NCcl(\eta(C)))$ is $NCS\alpha$-CS in $\mathcal{X}$. Thus we have $\eta^{-1}(NCcl(\eta(C))) \supseteq NCint(NCcl(\eta^{-1}(NCcl(C)))) \supseteq NCint(NCcl(\eta(C)))$.

(iii) $\Rightarrow$ (iv) Since $\mathcal{D} \in \mathcal{Y}$, $\eta^{-1}(\mathcal{D}) \in \mathcal{X}$, so, we have by our hypothesis the corresponding notation $NCint(NCcl(\eta(C))) \subseteq NCcl(\eta^{-1}(\mathcal{D})) \subseteq NCcl(\mathcal{D})$, and that leads us to this fact $NCint(NCcl(\eta(C))) \subseteq \eta^{-1}(NCcl(\mathcal{D}))$.

(iv) $\Rightarrow$ (i) Let $\mathcal{D}$ be a $NC$-OS of $\mathcal{Y}$. Let $C = \mathcal{D}^c$ and $\mathcal{D} = \eta^{-1}(C)$ by (iii) we have $NCint(NCcl(\eta(C))) \subseteq \eta^{-1}(NCcl(\mathcal{D}))$. That is $NCint(NCcl(\eta(C))) \subseteq \eta^{-1}(\mathcal{D})$. Or $NCint(NCcl(\eta(C))) \supseteq \eta^{-1}(\mathcal{D})$. Hence $\eta^{-1}(\mathcal{D})$ is a $NCS\alpha$-OS in $\mathcal{X}$ and thus $\eta$ be there a $NCS\alpha$-continuous.

**Proposition 3.4:**

(i) all $NC$-continuous functions are $NCS\alpha$-continuous, but the opposite is not valid in general.

(ii) all $NC\alpha$-continuous functions are $NCS\alpha$-continuous, but the opposite is not exact in general.

**Proof:**

(i) Suppose $\eta: (\mathcal{X}, \mathcal{I}_1) \rightarrow (\mathcal{Y}, \mathcal{I}_2)$ is a $NC$-continuous function, and $\mathcal{D}$ be a $NC$-OS from $\mathcal{Y}$. Next $\eta^{-1}(\mathcal{D})$ remains a $NC$-OS from $\mathcal{X}$. Since any $NC$-OS is a $NCS\alpha$-OS, $\eta^{-1}(\mathcal{D})$ stays a $NCS\alpha$-OS from $\mathcal{X}$. Thus $\eta$ exists a $NCS\alpha$-continuous function.
(ii) Let \( \eta : (X, \Gamma_1) \rightarrow (Y, \Gamma_2) \) be a \( NCA \)-continuous function and \( D \) be a \( NCA \)-OS from \( Y \). Subsequently \( \eta^{-1}(D) \) happens a \( NCA \)-OS from \( X \). Since any \( NCA \)-OS is \( NCA \)-OS, \( \eta^{-1}(D) \) stays a \( NCA \)-OS from \( X \). Thus \( \eta \) is a \( NCA \)-continuous function.

**Example 3.5:** Suppose \( X = \{p, q, r, s\} \) and \( Y = \{u, v, w\} \). Then \( \Gamma_1 = \{\phi_X, X_0\} \cup \{(p), (\phi, \phi)\} \) and \( \Gamma_2 = \{\phi_Y, Y_0\} \cup \{(u), (\phi, \phi)\} \) be neurotropic crisp topologies (shortly, NCTs) on \( X \) and \( Y \), correspondingly. Define the function \( \eta : (X, \Gamma_1) \rightarrow (Y, \Gamma_2) \) via \( \eta((p), (\phi, \phi)) = \eta((q), (\phi, \phi)) = \eta((r), (\phi, \phi)) = \eta((s), (\phi, \phi)) = \eta((w), (\phi, \phi)) \). Then \( \eta \) is a \( NCA \)-continuous function but not \( NC \)-continuous since \( \eta((u), (\phi, \phi)) \) is \( NCA \)-OS but \( \eta^{-1}((u), (\phi, \phi)) = \{(p, q), (\phi, \phi)\} \) which is not \( NCA \)-OS in \( X \). Also, \( \eta \) is a \( NCA \)-continuous function but not \( NCA \)-continuous, since \( \eta((u), (\phi, \phi)) = \{(p, q), (\phi, \phi)\} \) is \( NCA \)-OS in \( Y \) but \( \eta^{-1}((u), (\phi, \phi)) = \{(p, q), (\phi, \phi)\} \) is not \( NCA \)-OS from \( X \).

**Example 3.6:** Suppose \( X = \{p, q, r\} \). Then \( \Gamma = \{\phi_X, X_0\} \cup \{(p), (\phi, \phi), (q), (\phi, \phi), (p, q), (\phi, \phi)\} \) be a \( NCT \) on \( X \).

Define the function \( \eta : (X, \Gamma) \rightarrow (X, \Gamma) \) via \( \eta((p), (\phi, \phi)) = \{(p), (\phi, \phi), (q), (\phi, \phi), (p, q), (\phi, \phi)\} \). It is easily seen that \( \eta \) is a \( NCA \)-continuous function but not \( NC \)-continuous, since \( \eta((q), (\phi, \phi)) = \{(p, q), (\phi, \phi)\} \) is \( NCA \)-OS in \( X \) but \( \eta^{-1}((q), (\phi, \phi)) = \{(p, q), (\phi, \phi)\} \) is not \( NCA \)-OS in \( X \).

**Remark 3.7:** The concepts of \( NCA \)-continuity and \( NCA \)-continuity are independent, for examples.

**Example 3.8:** Suppose \( X = \{p, q, r, s\} \) and \( Y = \{u, v, w\} \). Then \( \Gamma_1 = \{\phi_X, X_0\} \cup \{(p), (\phi, \phi), (q), (r), (\phi, \phi), (p, q, r), (\phi, \phi)\} \) and \( \Gamma_2 = \{\phi_Y, Y_0\} \cup \{(u), (\phi, \phi)\} \) be NCTs on \( X \) and \( Y \), correspondingly. Define the function \( \eta : (X, \Gamma_1) \rightarrow (Y, \Gamma_2) \) via \( \eta((p), (\phi, \phi)) = \{(u), (\phi, \phi)\} \). Then \( \eta \) is a \( NCA \)-continuous function but not \( NC \)-continuous, since \( \{(p, q), (\phi, \phi)\} \) is \( NCA \)-OS in \( Y \) but \( \eta^{-1}((p, q), (\phi, \phi)) = \{(p, q), (\phi, \phi)\} \) is not \( NCA \)-OS in \( X \).

**Example 3.9:** Assume \( X = \{p, q, r, s\} \) and \( Y = \{u, v, w\} \). Then \( \Gamma_1 = \{\phi_X, X_0\} \cup \{(p), (\phi, \phi)\} \) and \( \Gamma_2 = \{\phi_Y, Y_0\} \cup \{(u), (\phi, \phi)\} \) be NCTs on \( X \) and \( Y \), correspondingly. Define the function \( \eta : (X, \Gamma_1) \rightarrow (Y, \Gamma_2) \) via \( \eta((p), (\phi, \phi)) = \{(u), (\phi, \phi)\} \). Then \( \eta \) is a \( NCA \)-continuous function but not \( NC \)-continuous, since \( \{(p, q), (\phi, \phi)\} \) is \( NC \)-OS in \( Y \) but \( \eta^{-1}((p, q), (\phi, \phi)) = \{(p, q), (\phi, \phi)\} \) is not \( NC \)-OS in \( X \).

**Theorem 3.10:**

(i) If a function \( \eta : (X, \Gamma_1) \rightarrow (Y, \Gamma_2) \) is \( NC \)-open, \( NC \)-continuous, and bijective, then \( \eta \) is a \( NCA \)-continuous.

(ii) A function \( \eta : (X, \Gamma_1) \rightarrow (Y, \Gamma_2) \) is \( NCA \)-continuous iff \( \eta : (X, NC\alpha O(X)) \rightarrow (Y, NC\alpha O(Y)) \) is a \( NC \)-continuous.

**Proof:**

(i) Let \( D \in NC\alpha O(Y) \), to prove that \( \eta^{-1}(D) \in NC\alpha O(X) \), i.e., \( \eta^{-1}(D) \subseteq NC\alpha int(NC\alpha CL(\eta^{-1}(D))) \). Let \( r \in \eta^{-1}(D) \Rightarrow \eta(r) \in \subseteq \). Hence \( \eta(r) \in NC\alpha int(NC\alpha CL(\eta^{-1}(D))) \) (since \( D \in NC\alpha O(Y) \)). Therefore, at least \( NC \)-OS \( \eta \) from \( Y \) where \( \eta(r) \in \subseteq \). Then \( r \in \eta^{-1}(\subseteq) \subseteq \)

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\[ \eta^{-1}(NCcl(NCint(D))) \text{, but } \eta^{-1}(NCcl(NCint(D))) \subseteq NCcl(\eta^{-1}(NCint(D))) \text{ (since } \eta^{-1} \text{ is a NC -continuous, which is equivalent to } \eta \text{ is a NC-open and bijective). Then } r \in \eta^{-1}(H) \subseteq NCcl(\eta^{-1}(NCint(D))). \text{ Hence } r \in \eta^{-1}(H) \subseteq NCcl(\eta^{-1}(NCint(D))) \subseteq NCcl(\eta^{-1}(NCint(D))) \text{ (since } \eta \text{ is a NC-continuous). Hence } r \in \eta^{-1}(H) \subseteq NCcl(\eta^{-1}(NCint(D))), \text{ but } \eta^{-1}(H) \text{ remains a NC-OS from } X \text{ (because } \eta \text{ be present a NC-continuous). Therefore, } r \in NCint(NCcl(\eta^{-1}(NCint(D)))). \text{ Hence } \eta^{-1}(D) \subseteq NCint(NCcl(\eta^{-1}(NCint(D)))) \Rightarrow \eta^{-1}(D) \in NCaO(X) \Rightarrow \eta \text{ is a NC}\alpha^*\text{-continuous.}

(ii) The proof of (ii) is easily. □

**Theorem 3.11:** A function \( \eta : (X, \Gamma_1) \rightarrow (Y, \Gamma_2) \) is a NCS\( \alpha^*\)-continuous iff \( \eta : (X, NCaO(X)) \rightarrow (Y, NCaO(Y)) \) is a NC-continuous.

**Proof:** Obvious. □

**Remark 3.12:** The concepts of NC-continuity and NC\( \alpha^*\)-continuity are independent, for examples.

**Example 3.13:** Suppose \( X = \{p, q, r, s\} \) and \( Y = \{u, v, w\} \). Then \( \Gamma_1 = \{p_n, X_n\} \cup \{(p), (q), (r), (s), (p, q, r, \phi, \lambda)\} \) and \( \Gamma_2 = \{p_n, Y_n\} \cup \{(u), (v), (w)\} \) be NCT's on \( X \) and \( Y \), correspondingly. Define the function \( \eta : (X, \Gamma_1) \rightarrow (Y, \Gamma_2) \) via \( \eta((p), (q), (r), (s), (p, q, r, \phi, \lambda)) = ((u), (v), (w)) \). Then \( \eta \) is a NC-continuous function but not NC\( \alpha^*\)-continuous, since \( (u, v, w) \) is not NC\( \alpha^*\)-OS in \( Y \).

**Example 3.14:** Assume \( X = \{p, q, r, s\} \) and \( Y = \{u, v, w\} \). Then \( \Gamma_1 = \{p_n, X_n\} \cup \{(p), (q), (r), (s), (p, q, r, \phi, \lambda)\} \) and \( \Gamma_2 = \{p_n, Y_n\} \cup \{(u), (v), (w)\} \) be NCT's on \( X \) and \( Y \), correspondingly. Define the function \( \eta : (X, \Gamma_1) \rightarrow (Y, \Gamma_2) \) via \( \eta((p), (q), (r), (s), (p, q, r, \phi, \lambda)) = ((u), (v), (w)) \). Then \( \eta \) is a NC\( \alpha^*\)-continuous function but not NC-continuous, since \( (u, v, w) \) is NC-OS in \( Y \), but \( \eta^{-1}((u, v, w)) = ((p, q), (r, \phi, \lambda)) \) is NC-OS in \( X \).

**Proposition 3.15:** Every NC\( \alpha^*\)-continuous function is a NC\( \alpha \)-continuous and NC\( \alpha^* \)-continuous; however, the reverse generally is not valid.

**Proof:** Assume \( \eta : (X, \Gamma_1) \rightarrow (Y, \Gamma_2) \) is a NC\( \alpha^*\)-continuous function and let \( D \) be any NC-OS from \( Y \). Then we have \( D \) as a NC\( \alpha \)-OS from \( Y \) [from proposition 2.4]. Since \( \eta \) is a NCS\( \alpha^*\)-continuous, then \( \eta^{-1}(D) \) considers a NC\( \alpha \)-OS from \( X \). Thus, \( \eta \) stands a NC\( \alpha \)-continuous. Also, \( \eta \) is a NCS\( \alpha \)-continuous. □

**Example 3.16:** Let \( X = \{p, q, r, s\} \).

Then \( \Gamma = \{p_n, X_n\} \cup \{(p), (q), (r), (s), (p, q, r, \phi, \lambda)\} \) be a NCT on \( X \). Define the function \( \eta : (X, \Gamma) \rightarrow (X, \Gamma) \) by \( \eta((p), (q), (r), (s), (p, q, r, \phi, \lambda)) = ((p), (q), (r), (s), (p, q, r, \phi, \lambda)) \). It is easily seen that \( \eta \) is a NC\( \alpha \)-continuous function but not NC\( \alpha^*\)-continuous, since \( ((p, q, r), (p, q, r)) \) is NC\( \alpha \)-OS in \( X \), but \( \eta^{-1}((p, q, r)) = ((p, s), (p, s)) \) is not NC\( \alpha \)-OS in \( X \).

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Example 3.17: Let $X = \{p, q, r\}$. Then $\Gamma = \{\phi_N, \phi_S, \phi_L\} \cup \{\{(p, \phi, \phi), (q, \phi, \phi), (p, q, \phi, \phi)\}\}$ be a NCT on $X$. Define a function $\eta: (X, \Gamma) \rightarrow (X, \Gamma)$ by $\eta(((p, \phi, \phi), (q, \phi, \phi), (p, q, \phi, \phi))) = ((q, \phi, \phi))$. It is easily seen that $\eta$ is a NCS$\alpha$-continuous function but not NCS$\alpha$-continuous, since $\eta^{-1}(((q, \phi, \phi))) = ((q, \phi, \phi))$ is not NCS$\alpha$-OS in $X$. Thus, $\eta$ is a NCS$\alpha$-continuous but not NCS$\alpha$-continuous function.

Definition 3.18: A function $\eta: (X, \Gamma) \rightarrow (Y, \Gamma)$ is called $\mathcal{M}$-function iff $\eta^{-1}(NCint(NCcl(D))) \subseteq NCint(NCcl(\eta^{-1}(D)))$, for every NCS$\alpha$-OS $D$ from $Y$.

Theorem 3.19: If $\eta: (X, \Gamma) \rightarrow (Y, \Gamma)$ is a NCS$\alpha$-continuous function and $\mathcal{M}$-function, then $\eta$ is a NCS$\alpha$-continuous function.

Proof: Let $C$ be any NCS$\alpha$-OS of $Y$, then we have at least a NCS$\alpha$-OS of $Y$ where $D \subseteq C \subseteq NCint(NCcl(D))$. Since $\eta$ is a $\mathcal{M}$-function, we have $\eta^{-1}(D) \subseteq \eta^{-1}(C) \subseteq \eta^{-1}(NCint(NCcl(D))) \subseteq NCint(NCcl(\eta^{-1}(D)))$. By proposition 2.3, we have $\eta^{-1}(\mathcal{C})$ is a NCS$\alpha$-OS. Hence, $\eta$ is a NCS$\alpha$-continuous function.

Remark 3.20: The concepts of NCS$\alpha$-continuity and NCS$\alpha$-continuity are independent as the following examples show.

Example 3.21: Assume $X = \{p, q, r, s\}$ and $Y = \{u, v, w\}$. Then $\Gamma_1 = \{\phi_N, \phi_S, \phi_L\} \cup \{(p, \phi, \phi), (q, \phi, \phi), (p, q, \phi, \phi)\}$ and $\Gamma_2 = \{\phi_N, \phi_S, \phi_L\} \cup \{(u, \phi, \phi), (v, \phi, \phi), (u, v, \phi, \phi)\}$ be NCT’s on $X$ and $Y$, correspondingly. Define the function $\eta: (X, \Gamma_1) \rightarrow (Y, \Gamma_2)$ via $\eta(((p, \phi, \phi), (q, \phi, \phi))) = (\{(v, \phi, \phi), \{(q, \phi, \phi), (w, \phi, \phi)\})$ and $\eta(((q, \phi, \phi))) = (\{(u, \phi, \phi), (v, \phi, \phi), (u, v, \phi, \phi)\})$. It is easily seen that $\eta$ is a NCS$\alpha$-continuous function but not NCS$\alpha$-continuous, since $\{(v, \phi, \phi), \{(q, \phi, \phi), (w, \phi, \phi)\}$ is NCS$\alpha$-OS in $Y$ but $\eta^{-1}(((q, \phi, \phi))) = (\{(p, \phi, \phi), \{(p, q, \phi, \phi)\})$ is not NCS$\alpha$-OS in $X$.

Example 3.22: Suppose $X = \{p, q, r, s\}$. Then $\Gamma = \{\phi_N, \phi_L, \phi_S\} \cup \{(p, \phi, \phi), (q, \phi, \phi), (p, q, \phi, \phi)\}$ be a NCT on $X$. Define the function $\eta: (X, \Gamma) \rightarrow (X, \Gamma)$ via $\eta(((p, \phi, \phi))) = \eta(((q, \phi, \phi))) = ((s, \phi, \phi)), \eta(((p, q, \phi, \phi))) = ((r, \phi, \phi))$. It is easily seen that $\eta$ is a NCS$\alpha$-continuous function but not NCS$\alpha$-continuous, since $\{(p, q, \phi, \phi)\}$ is NCS$\alpha$-OS in $X$, but $\eta^{-1}(((p, q, \phi, \phi))) = (\{(p, q, \phi, \phi)\})$ is not NCS$\alpha$-OS in $X$.

Theorem 3.23: If a function $\eta: (X, \Gamma_1) \rightarrow (Y, \Gamma_2)$ is NCS$\alpha$-continuous, NC-open and bijective, then it is NCS$\alpha$-continuous.

Proof: Let $\eta: (X, \Gamma_1) \rightarrow (Y, \Gamma_2)$ be a NCS$\alpha$-continuous, NC-open and bijective. Let $D$ be a NCS$\alpha$-OS in $Y$. Then we have at least a NCS$\alpha$-OS say $P$ where $P \subseteq D \subseteq NCcl(P)$. Therefore $\eta^{-1}(P) \subseteq \eta^{-1}(NCcl(P)) \subseteq NCcl(\eta^{-1}(P))$ since $\eta$ is a NC-open, but $\eta^{-1}(P) \in NCS\alpha\alpha(X)$ since $\eta$ is a NC$\alpha$-continuous. Hence $\eta^{-1}(P) \subseteq \eta^{-1}(D) \subseteq NCcl(\eta^{-1}(P))$. Thus, $\eta^{-1}(D) \in NCS\alpha\alpha(X)$. Therefore, $\eta$ is a NCS$\alpha$-continuous function.

Remark 3.24: Let $\eta_1: (X, \Gamma_1) \rightarrow (Y, \Gamma_2)$ and $\eta_2: (Y, \Gamma_2) \rightarrow (Z, \Gamma_3)$ be two functions, then:
(i) If \( \eta_1 \) and \( \eta_2 \) are NC \( \alpha \)-continuous, then \( \eta_2 \circ \eta_1 \colon (X, \Gamma_1) \to (Z, \Gamma_3) \) need not to be a NC\( \alpha \)-continuous.

(ii) If \( \eta_1 \) and \( \eta_2 \) are NCS \( \alpha \)-continuous, then \( \eta_2 \circ \eta_1 \colon (X, \Gamma_1) \to (Z, \Gamma_3) \) need not to be a NCS\( \alpha \)-continuous.

**Theorem 3.25:** Let \( \eta_1 \colon (X, \Gamma_1) \to (Y, \Gamma_2) \) and \( \eta_2 \colon (Y, \Gamma_2) \to (Z, \Gamma_3) \) be two functions, then:

(i) If \( \eta_1 \) is NC \( \alpha \)-continuous and \( \eta_2 \) is NC -continuous, then \( \eta_2 \circ \eta_1 \colon (X, \Gamma_1) \to (Z, \Gamma_3) \) is a NC\( \alpha \)-continuous.

(ii) If \( \eta_1 \) is NC \( \alpha^* \)-continuous and \( \eta_2 \) is NC \( \alpha \)-continuous, then \( \eta_2 \circ \eta_1 \colon (X, \Gamma_1) \to (Z, \Gamma_3) \) is a NC\( \alpha \)-continuous.

(iii) If \( \eta_1 \) and \( \eta_2 \) are NC\( \alpha^* \)-continuous, then \( \eta_2 \circ \eta_1 \colon (X, \Gamma_1) \to (Z, \Gamma_3) \) is a NC\( \alpha^* \)-continuous.

(iv) If \( \eta_1 \) and \( \eta_2 \) are NCS\( \alpha^* \)-continuous, then \( \eta_2 \circ \eta_1 \colon (X, \Gamma_1) \to (Z, \Gamma_3) \) is a NCS\( \alpha^* \)-continuous.

(v) If \( \eta_1 \) and \( \eta_2 \) are NC\( \alpha^{**} \)-continuous, then \( \eta_2 \circ \eta_1 \colon (X, \Gamma_1) \to (Z, \Gamma_3) \) is a NC\( \alpha^{**} \)-continuous.

(vi) If \( \eta_1 \) and \( \eta_2 \) are NCS\( \alpha^{**} \)-continuous, then \( \eta_2 \circ \eta_1 \colon (X, \Gamma_1) \to (Z, \Gamma_3) \) is a NCS\( \alpha^{**} \)-continuous.

(vii) If \( \eta_1 \) is NC \( \alpha^{**} \)-continuous and \( \eta_2 \) is NC \( \alpha^* \)-continuous, then \( \eta_2 \circ \eta_1 \colon (X, \Gamma_1) \to (Z, \Gamma_3) \) is a NC\( \alpha^* \)-continuous.

(viii) If \( \eta_1 \) is NC \( \alpha^{**} \)-continuous and \( \eta_2 \) is NC \( \alpha \)-continuous, then \( \eta_2 \circ \eta_1 \colon (X, \Gamma_1) \to (Z, \Gamma_3) \) is a NC\( \alpha \)-continuous.

(ix) If \( \eta_1 \) is NC \( \alpha \)-continuous and \( \eta_2 \) is NC \( \alpha^* \)-continuous, then \( \eta_2 \circ \eta_1 \colon (X, \Gamma_1) \to (Z, \Gamma_3) \) is a NC\( \alpha^* \)-continuous.

(x) If \( \eta_1 \) is NC -continuous and \( \eta_2 \) is NC \( \alpha^* \)-continuous, then \( \eta_2 \circ \eta_1 \colon (X, \Gamma_1) \to (Z, \Gamma_3) \) is a NC\( \alpha^* \)-continuous.

**Proof:**

(i) Assume \( \mathcal{F} \) considers a NC-OS from \( Z \). Since \( \eta_2 \) is a NC-continuous, \( \eta_2^{-1}(F) \) is a NC\( \alpha \)-OS in \( Y \). Since \( \eta_1 \) is a NC \( \alpha \)-continuous, \( \eta_1^{-1}(\eta_2^{-1}(F)) = (\eta_2 \circ \eta_1)^{-1}(F) \) is a NC \( \alpha \)-OS in \( X \). Thus, \( \eta_2 \circ \eta_1 \colon (X, \Gamma_1) \to (Z, \Gamma_3) \) exists a NC\( \alpha \)-continuous.

(ii) Let \( \mathcal{F} \) be a NC-OS in \( Z \). Subsequently \( \eta_2 \) stands a NC \( \alpha \)-continuous, and \( \eta_2^{-1}(F) \) stays a NC\( \alpha \)-OS from \( Y \). Since \( \eta_1 \) is a NC\( \alpha^* \)-continuous, \( \eta_1^{-1}(\eta_2^{-1}(F)) = (\eta_2 \circ \eta_1)^{-1}(F) \) is a NC\( \alpha \)-OS in \( X \). Thus, \( \eta_2 \circ \eta_1 \colon (X, \Gamma_1) \to (Z, \Gamma_3) \) is a NC\( \alpha \)-continuous.

(iii) Let \( \mathcal{F} \) be a NC\( \alpha \)-OS in \( Z \). Since \( \eta_2 \) is a NC\( \alpha^* \)-continuous, \( \eta_2^{-1}(F) \) is a NC\( \alpha \)-OS in \( Y \). Since \( \eta_1 \) is a NC\( \alpha^* \)-continuous, \( \eta_1^{-1}(\eta_2^{-1}(F)) = (\eta_2 \circ \eta_1)^{-1}(F) \) is a NC\( \alpha \)-OS in \( X \). Thus, \( \eta_2 \circ \eta_1 \colon (X, \Gamma_1) \to (Z, \Gamma_3) \) is a NC\( \alpha \)-continuous.

(iv) Let \( \mathcal{F} \) be a NCS\( \alpha \)-OS in \( Z \). Since \( \eta_2 \) is a NCS\( \alpha^* \)-continuous, \( \eta_2^{-1}(F) \) is a NCS\( \alpha \)-OS in \( Y \). Since \( \eta_1 \) is a NCS\( \alpha^* \)-continuous, \( \eta_1^{-1}(\eta_2^{-1}(F)) = (\eta_2 \circ \eta_1)^{-1}(F) \) is a NCS\( \alpha \)-OS in \( X \). Thus, \( \eta_2 \circ \eta_1 \colon (X, \Gamma_1) \to (Z, \Gamma_3) \) is a NCS\( \alpha \)-continuous.

(v) Let \( \mathcal{F} \) be a NC\( \alpha \)-OS in \( Z \). Since \( \eta_2 \) is a NC\( \alpha^{**} \)-continuous, \( \eta_2^{-1}(F) \) is a NC-OS in \( Y \). Since any NC-OS is a NC\( \alpha \)-OS, \( \eta_2^{-1}(F) \) is a NC-OS in \( Y \). Since \( \eta_1 \) is a NC\( \alpha^{**} \)-continuous, \( \eta_1^{-1}(\eta_2^{-1}(F)) = (\eta_2 \circ \eta_1)^{-1}(F) \) is a NC-OS in \( X \). Thus, \( \eta_2 \circ \eta_1 \colon (X, \Gamma_1) \to (Z, \Gamma_3) \) is a NC\( \alpha^{**} \)-continuous. The proof is obvious for others.

**Remark 3.26:** The next figure describes the relationship between various classes of weakly NC-continuous functions:
4. Conclusion

We shall use the concepts of $\text{NC}_\alpha$-OS and $\text{NCS}_\alpha$-CS to define several new types of weakly $\text{NC}$-continuity such as; $\text{NC}_\alpha^*$-continuous, $\text{NC}_\alpha^{**}$-continuous, $\text{NCS}_\alpha$-continuous, $\text{NCS}_\alpha^*$-continuous and $\text{NCS}_\alpha^{**}$-continuous functions. The neutrosophic crisp $\alpha$-open and neutrosophic crisp semi-$\alpha$-open sets can be used to derive some new types of weakly $\text{NC}$-open ($\text{NC}$-closed) functions.

References


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