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Pairwise Neutrosophic- b -Open Set in Neutrosophic Bitopological Spaces

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Abstract: In this paper we introduce the notion of neutrosophic- b -open set, pairwise neutrosophic- b -open set in neutrosophic bitopological spaces. We have investigated some of their basic properties and established relation between the other existing notions.

Keywords: *Neutrosophic set; Neutrosophic topology; Neutrosophic bitopology; Neutrosophic- b -open set.*

1. Introduction

Smarandache (1998) introduced the notion of neutrosophic set as a generalization of intuitionistic fuzzy set. The concept of neutrosophic topological space was introduced by Salama and Alblowi (2012a). Salama and Alblowi (2012b) introduced the concept of generalized *neutrosophic set* and generalized neutrosophic topological space. Thereafter Ozturk and Ozkan (2019) introduce the concept of neutrosophic bitopological space. The concept of b -open sets in topological space was introduced by Andrijevic (1996). Ebenanjar, Immaculate, and Wilfred (2018) introduced neutrosophic b -open sets in neutrosophic topological spaces. Thangavelu and Thamizharsi (2011) introduce the concept of b_t -open sets in bitopological spaces. In this paper, we introduce the notion of pairwise neutrosophic b -open set in neutrosophic bitopological spaces.

2. Preliminaries and some properties

Definition 2.1. [Smarandache, 2005] Let X be a non-empty set. Then H , a neutrosophic set (NS in short) over X is denoted as follows:

$H = \{(y, T_H(y), I_H(y), F_H(y)) : y \in X \text{ and } T_H(y), I_H(y), F_H(y) \in]0, 1+[, \text{ where } T_H(y), I_H(y) \text{ and } F_H(y) \text{ are the degree of truthness, indeterminacy and falseness respectively.}$

There is no restriction on the sum of $T_C(y), F_C(y)$ and $I_C(y)$, so

$$-0 \leq T_H(y) + I_H(y) + F_H(y) \leq 3^+.$$

Definition 2.2. [Smarandache, 2005] Let $H = \{(y, T_H(y), I_H(y), F_H(y)): y \in X\}$ be a neutrosophic set over X . Then the complement of H is defined by $H^c = \{(y, 1-T_H(y), 1-I_H(y), 1-F_H(y)): y \in X\}$.

Definition 2.3. [Smarandache, 2005] A neutrosophic set $H = \{(y, T_H(y), I_H(y), F_H(y)): y \in X\}$ is contained in the other neutrosophic set $K = \{(y, T_K(y), I_K(y), F_K(y)): y \in X\}$ (i.e. $H \subseteq K$) if and only if $T_H(y) \leq T_K(y)$, $I_H(y) \geq I_K(y)$, $F_H(y) \geq F_K(y)$, for each $y \in X$.

Definition 2.4. [Smarandache, 2005] If $H = \{(y, T_H(y), I_H(y), F_H(y)): y \in X\}$ and $K = \{(y, T_K(y), I_K(y), F_K(y)): y \in X\}$ are any two neutrosophic sets over X , then $H \cup K$ and $H \cap K$ is defined by
 $H \cup K = \{(y, T_H(y) \vee T_K(y), I_H(y) \wedge I_K(y), F_H(y) \wedge F_K(y)): y \in X\};$
 $H \cap K = \{(y, T_H(y) \wedge T_K(y), I_H(y) \vee I_K(y), F_H(y) \vee F_K(y)): y \in X\}.$

Here we can construct two neutrosophic set 0_N and 1_N over X as follows:

- 1) $0_N = \{(y, 0, 0, 1): y \in X\};$
- 2) $1_N = \{(y, 1, 0, 0): y \in X\}.$

The neutrosophic set 0_N is known as neutrosophic null set and neutrosophic set 1_N is known as neutrosophic whole set over X . Also, 0_N and 1_N over X have three other types of representation too. Clearly, $0_N \subseteq 1_N$.

The neutrosophic topological space is defined as follows:

Definition 2.5. [Salama & Alblowi, 2012a] Let X be a non-empty fixed set and τ be the family of some NSs over X . Then τ is said to be a neutrosophic topology (NT in short) on X if the following properties holds:

1. $0_N, 1_N \in \tau,$
2. $T_1, T_2 \in \tau \Rightarrow T_1 \cap T_2 \in \tau,$
3. $\cup_{i \in \Delta} T_i \in \tau,$ for every $\{T_i: i \in \Delta\} \subseteq \tau.$

Then the pair (X, τ) is called a neutrosophic topological space (NTS in short). The members of τ are called neutrosophic-open set (NOS in short). A NS D is called a neutrosophic-closed set (NCS in short) in (X, τ) if and only if D^c is a neutrosophic-open set.

Example 2.1. Let $X = \{z_1, z_2\}$ and let

$$G = \{(z_1, 0.6, 0.5, 0.3), (z_2, 0.6, 0.7, 0.3): z_1, z_2 \in X\}$$

$$H = \{(z_1, 0.5, 0.6, 0.8), (z_2, 0.4, 0.9, 0.8): z_1, z_2 \in X\}$$

$$K = \{(z_1, 0.6, 0.6, 0.3), (z_2, 0.4, 0.8, 0.6): z_1, z_2 \in X\}$$
 be three NSs over X .

Then clearly the family $\tau = \{0_N, 1_N, G, H, K\}$ is a NT on X .

Example 2.2. Let $X = \{z_1, z_2, z_3\}$ and let

$$L = \{(z_1, 0.6, 0.7, 0.4), (z_2, 0.5, 0.6, 0.8), (z_3, 0.5, 0.5, 0.4): z_1, z_2, z_3 \in X\}$$

$$K = \{(z_1, 0.4, 0.9, 0.8), (z_2, 0.3, 0.7, 0.8), (z_3, 0.4, 0.6, 0.8) : z_1, z_2, z_3 \in X \}$$

$$J = \{(z_1, 0.4, 0.9, 0.9), (z_2, 0.2, 0.8, 0.9), (z_3, 0.3, 0.5, 0.8) : z_1, z_2, z_3 \in X \}$$

be three NSs over X .

Here the collection $\tau = \{0_N, 1_N, L, K, J\}$ is not a neutrosophic topology on X because $K \cap J \notin \tau$.

Definition 2.5. [Salama & Alblowi, 2012a] Let (X, τ) be a NTS and H be a NS over X . The neutrosophic-interior (in short N_{int}) and neutrosophic-closure (in short N_{cl}) of H are defined by

$$N_{int}(H) = \cup \{P : P \text{ is an NOS in } X \text{ and } P \subseteq H\};$$

$$N_{cl}(H) = \cap \{Q : Q \text{ is an NCS in } X \text{ and } H \subseteq Q\}.$$

Proposition 2.1. [Salama & Alblowi, 2012a] Let C, D are two neutrosophic subsets of (X, τ) . Then the following properties hold:

- 1) $C \subseteq N_{cl}(C)$;
- 2) $N_{int}(C) \subseteq C$;
- 3) $N_{int}(C) \subseteq N_{cl}(C)$;
- 4) $C \subseteq D \Rightarrow N_{int}(C) \subseteq N_{int}(D)$;
- 5) $C \subseteq D \Rightarrow N_{cl}(C) \subseteq N_{cl}(D)$;
- 6) $N_{cl}(0_N) = 0_N$;
- 7) $N_{int}(1_N) = 1_N$;
- 8) $N_{cl}(C \cup D) = N_{cl}(C) \cup N_{cl}(D)$;
- 9) $N_{int}(C \cup D) \supseteq N_{int}(C) \cup N_{int}(D)$;
- 10) $N_{int}(C \cap D) = N_{int}(C) \cap N_{int}(D)$;
- 11) $N_{cl}(C \cap D) \subseteq N_{cl}(C) \cap N_{cl}(D)$;
- 12) C is neutrosophic closed if and only if $N_{cl}(C) = C$;
- 13) C is neutrosophic open if and only if $N_{int}(C) = C$.

The neutrosophic bitopological space is defined as follows:

Definition 2.6. [Ozturk & Ozkan, 2019] Assume that (X, τ_1) and (X, τ_2) be two different NTSs. Then the triplet (X, τ_1, τ_2) is called a neutrosophic bitopological space (NBTS in short).

Example 2.3. Let $X = \{z_1, z_2\}$ and let

$$U_1 = \{(z_1, 0.6, 0.5, 0.4), (z_2, 0.8, 0.7, 0.6) : z_1, z_2 \in X \},$$

$$U_2 = \{(z_1, 0.4, 0.6, 0.5), (z_2, 0.7, 0.8, 0.8) : z_1, z_2 \in X \},$$

$$U_3 = \{(z_1, 0.4, 0.6, 0.8), (z_2, 0.6, 0.9, 0.8) : z_1, z_2 \in X \},$$

$$U_4 = \{(z_1, 0.6, 0.8, 0.7), (z_2, 0.4, 0.6, 0.7) : z_1, z_2 \in X \},$$

$$U_5 = \{(z_1, 0.8, 0.4, 0.5), (z_2, 0.6, 0.4, 0.5) : z_1, z_2 \in X \},$$

$$U_6 = \{(z_1, 0.7, 0.5, 0.6), (z_2, 0.6, 0.5, 0.5) : z_1, z_2 \in X \}$$
 are six NSs over X .

Then clearly $\tau_1 = \{0_N, 1_N, U_1, U_2, U_3\}$ and $\tau_2 = \{0_N, 1_N, U_4, U_5, U_6\}$ are two different NTs on X . So the triplet (X, τ_1, τ_2) is a neutrosophic bitopological space.

Definition 2.7. [Ozturk & Ozkan, 2019] Let (X, τ_1, τ_2) be a neutrosophic bitopological space. Then H , a neutrosophic set over X is called a pairwise open set in (X, τ_1, τ_2) if there exist an open set G_1 in τ_1 and an open set G_2 in τ_2 such that $H = G_1 \cup G_2$.

Remark 2.2. Let G be a neutrosophic subset of a neutrosophic bitopological space (X, τ_1, τ_2) . Then we shall use the following notations:

- 1) $N_{cl}^i(G) = \tau_i$ -neutrosophic-closure of G ($i=1, 2$);
- 2) $N_{int}^i(G) = \tau_i$ -neutrosophic-interior of G ($i=1, 2$).

3. τ_i -neutrosophic- b -open set:

Definition 3.1. Let (X, τ_1, τ_2) be a neutrosophic bitopological space. Then P , a NS over X is called

- 1) τ_i -neutrosophic-semi-open if and only if $P \subseteq N_{cl}^i N_{int}^i(P)$;
- 2) τ_i -neutrosophic-pre-open if and only if $P \subseteq N_{int}^i N_{cl}^i(P)$;
- 3) τ_i -neutrosophic- b -open if and only if $P \subseteq N_{cl}^i N_{int}^i(P) \cup N_{int}^i N_{cl}^i(P)$.

Remark 3.1. In a neutrosophic bitopological space (X, τ_1, τ_2) , a NS P over X is called a τ_i -neutrosophic- b -closed set if and only if its complement is τ_i -neutrosophic- b -open set.

We formulate the following results based on the above definitions.

Proposition 3.1. In a neutrosophic bitopological space (X, τ_1, τ_2) , if P is τ_i -neutrosophic-semi-open (τ_i -neutrosophic-pre-open), then P is τ_i -neutrosophic- b -open.

Proposition 3.2. In a neutrosophic bitopological space (X, τ_1, τ_2) , the union of two τ_i -neutrosophic- b -open set is a τ_i -neutrosophic- b -open set.

4. τ_{ij} -neutrosophic- b -open set:

Definition 4.1. Assume that (X, τ_1, τ_2) be a neutrosophic bitopological space. Then P , a NS over X is called

- 1) τ_{ij} -neutrosophic-semi-open if and only if $P \subseteq N_{cl}^i N_{int}^j(P)$;
- 2) τ_{ij} -neutrosophic-pre-open if and only if $P \subseteq N_{int}^j N_{cl}^i(P)$;
- 3) τ_{ij} -neutrosophic- b -open if and only if $P \subseteq N_{cl}^i N_{int}^j(P) \cup N_{int}^j N_{cl}^i(P)$.

Remark 4.1. A neutrosophic set P over X is called a τ_{ij} -neutrosophic- b -closed set if and only if P^c (complement of P) is τ_{ij} -neutrosophic- b -open set in (X, τ_1, τ_2) .

Definition 4.2. Assume that (X, τ_1, τ_2) be a neutrosophic bitopological space. Then a neutrosophic set G over X is said to be a

- 1) τ_{ij} -neutrosophic- p -set if and only if $N_{cl}^i N_{int}^j(G) \subseteq N_{int}^i N_{cl}^j(G)$;

- 2) Contra τ_{ij} -neutrosophic- p -set if and only if $N_{cl}^j N_{int}^i(G) \subseteq N_{int}^i N_{cl}^j(G)$;
- 3) τ_{ij} -neutrosophic- q -set if and only if $N_{int}^j N_{cl}^i(G) \subseteq N_{cl}^i N_{int}^j(G)$.
- 4) Contra τ_{ij} -neutrosophic- q -set if and only if $N_{int}^i N_{cl}^j(G) \subseteq N_{cl}^i N_{int}^j(G)$.

Theorem 4.1. In a neutrosophic bitopological space (X, τ_1, τ_2) ,

- 1) if G is τ_i -neutrosophic-closed and τ_{ij} -neutrosophic-pre-open then G is τ_{ij} -neutrosophic-semi-open.
- 2) if G is τ_j -neutrosophic-open and τ_{ij} -neutrosophic-semi-open then G is τ_{ij} -neutrosophic-pre-open.

Proof:

- 1) Let (X, τ_1, τ_2) be a neutrosophic bitopological space and G be a neutrosophic set over X , which is both τ_i -neutrosophic-closed and τ_{ij} -neutrosophic-pre-open. So, we have

$$G = N_{cl}^i(G) \dots\dots\dots(1)$$

$$\text{and } G \subseteq N_{int}^j N_{cl}^i(G) \dots\dots\dots(2)$$

From eq (2) we have $G \subseteq N_{int}^j N_{cl}^i(G)$

$$\begin{aligned} &= N_{int}^j(G) \quad [\text{by eq (1)}] \\ \Rightarrow G &\subseteq N_{int}^j(G) \subseteq N_{cl}^i N_{int}^j(G) \\ \Rightarrow G &\subseteq N_{cl}^i N_{int}^j(G) \end{aligned}$$

Hence, G is a τ_{ij} -neutrosophic-semi-open set in (X, τ_1, τ_2) .

- 2) Let (X, τ_1, τ_2) be a neutrosophic bitopological space and G be a NS over X , which is both τ_j -neutrosophic-open and τ_{ij} -neutrosophic-semi-open. So, we have

$$G = N_{int}^j(G) \dots\dots\dots(3)$$

$$\text{and } G \subseteq N_{cl}^i N_{int}^j(G) \dots\dots\dots(4)$$

From eq (4) we have

$$\begin{aligned} G &\subseteq N_{cl}^i N_{int}^j(G) \\ &= N_{cl}^i(G) \quad [\text{by eq (3)}] \\ \Rightarrow G &\subseteq N_{cl}^i(G) \\ \Rightarrow N_{int}^j(G) &\subseteq N_{int}^j N_{cl}^i(G) \\ \Rightarrow G = N_{int}^j(G) &\subseteq N_{int}^j N_{cl}^i(G) \quad [\text{since } G = N_{int}^j(G)] \\ \Rightarrow G &\subseteq N_{int}^j N_{cl}^i(G) \end{aligned}$$

Hence, G is a τ_{ij} -neutrosophic-pre-open set in (X, τ_1, τ_2) .

Theorem 4.2. Let (X, τ_1, τ_2) be a neutrosophic bitopological space. If A is τ_{ij} -neutrosophic-semi-open (τ_{ij} -neutrosophic-pre-open), then A is τ_{ij} -neutrosophic- b -open.

Proof: Let us assume that A is τ_{ij} -neutrosophic-semi-open set in a neutrosophic bitopological space (X, τ_1, τ_2) . Then $A \subseteq N_{cl}^i N_{int}^j(A)$.

Now, $A \subseteq N_{cl}^i N_{int}^j(A)$

$\Rightarrow A \subseteq N_{cl}^i N_{int}^j(A) \cup N_{int}^j N_{cl}^i(A)$.

Therefore, A is τ_{ij} -neutrosophic- b -open in (X, τ_1, τ_2) .

Similarly, we can show that if A is τ_{ij} -neutrosophic-pre-open set in (X, τ_1, τ_2) then it is τ_{ij} -neutrosophic- b -open.

Theorem 4.3. Let (X, τ_1, τ_2) be a neutrosophic bitopological space.

- 1) If A is τ_{ij} -neutrosophic- b -open, contra τ_{ji} -neutrosophic- p -set then A is τ_{ij} -neutrosophic-pre-open set;
- 2) If A is τ_{ij} -neutrosophic- b -open, contra τ_{ij} -neutrosophic- q -set then A is τ_{ij} -neutrosophic-semi-open set;
- 3) If A is τ_{ij} -neutrosophic- b -open, τ_{ij} -neutrosophic- p -set and contra τ_{ji} -neutrosophic- q -set then A is τ_{ji} -neutrosophic- b -open set;
- 4) If A is τ_{ij} -neutrosophic- q -set (τ_{ij} -neutrosophic- p -set) then A^c is contra τ_{ji} -neutrosophic- p -set (contra τ_{ij} -neutrosophic- q -set).

Proof:

- 1) Let A be both τ_{ij} -neutrosophic- b -open and contra τ_{ji} -neutrosophic- p -set in a neutrosophic bitopological space (X, τ_1, τ_2) .

Then, we have $A \subseteq N_{cl}^i N_{int}^j(A) \cup N_{int}^j N_{cl}^i(A)$ (5)

and $N_{cl}^i N_{int}^j(A) \subseteq N_{int}^j N_{cl}^i(A)$ (6)

From eqs (5) & (6) we get

$$\begin{aligned} A &\subseteq N_{cl}^i N_{int}^j(A) \cup N_{int}^j N_{cl}^i(A) \\ &\subseteq N_{int}^j N_{cl}^i(A) \cup N_{int}^j N_{cl}^i(A) \\ &= N_{int}^j N_{cl}^i(A) \\ \Rightarrow A &\subseteq N_{int}^j N_{cl}^i(A) \end{aligned}$$

Therefore, A is τ_{ij} -neutrosophic-pre-open set in (X, τ_1, τ_2) .

- 2) The proof is analogous to the proof of part (1), so omitted.
- 3) Let A be τ_{ij} -neutrosophic- b -open, τ_{ij} -neutrosophic- p -set and contra τ_{ji} -neutrosophic- q -set in a neutrosophic bitopological space (X, τ_1, τ_2) . Then we have

$$A \subseteq N_{cl}^i N_{int}^j(A) \cup N_{int}^j N_{cl}^i(A), \quad \dots\dots\dots(7)$$

$$N_{cl}^i N_{int}^j(A) \subseteq N_{int}^j N_{cl}^i(A) \quad \dots\dots\dots(8)$$

$$\text{and } N_{int}^j N_{cl}^i(A) \subseteq N_{cl}^i N_{int}^j(A) \quad \dots\dots\dots(9)$$

$$\begin{aligned} \text{From eq (7) we get } A &\subseteq N_{cl}^i N_{int}^j(A) \cup N_{int}^j N_{cl}^i(A) \\ &\subseteq N_{int}^j N_{cl}^i(A) \cup N_{cl}^i N_{int}^j(A) \quad [\text{ by eqs (8) \& (9)}] \\ \Rightarrow A &\subseteq N_{int}^j N_{cl}^i(A) \cup N_{cl}^i N_{int}^j(A) \end{aligned}$$

Therefore, A is τ_{ji} -neutrosophic- b -open set in (X, τ_1, τ_2) .

Theorem 4.4. In a neutrosophic bitopological space (X, τ_1, τ_2)

- 1) if A is τ_{ij} -neutrosophic-semi-open and τ_{ij} -neutrosophic- p -set then A is τ_{ji} -neutrosophic-pre-open;
- 2) If A is τ_{ji} -neutrosophic-semi-open and contra τ_{ji} -neutrosophic- p -set then A is τ_{ji} -neutrosophic-pre-open.

Proof:

- 1) Let (X, τ_1, τ_2) be a neutrosophic bitopological space and A is both τ_{ij} -neutrosophic-semi-open and τ_{ji} -neutrosophic- p -set.

Since, A is τ_{ij} -neutrosophic-semi-open, so we have

$$A \subseteq N_{cl}^i N_{int}^j(A) \dots\dots\dots(10)$$

Since, A is τ_{ji} -neutrosophic- p -set, so

$$N_{cl}^i N_{int}^j(A) \subseteq N_{int}^i N_{cl}^j(A) \dots\dots\dots(11)$$

From eqs (10) & (11), we've got

$$A \subseteq N_{int}^i N_{cl}^j(A).$$

Hence, A is τ_{ji} -neutrosophic-pre-open in (X, τ_1, τ_2) .

- 2) The proof is analogous to the proof of the first part, so omitted.

Theorem 4.5. Let (X, τ_1, τ_2) be an neutrosophic bitopological space.

- 1) If A is τ_{ij} -neutrosophic- p -set and τ_{ji} -neutrosophic- q -set then $N_{cl}^i N_{int}^j(A) \subseteq N_{cl}^j N_{int}^i(A)$;
- 2) If A is contra τ_{ij} -neutrosophic- p -set and contra τ_{ji} -neutrosophic- q -set then $N_{cl}^j N_{int}^i(A) \subseteq N_{cl}^i N_{int}^j(A)$.

Proof:

- 1) Let (X, τ_1, τ_2) be a neutrosophic bitopological space and A be both τ_{ij} -neutrosophic- p -set and τ_{ji} -neutrosophic- q -set. Then, we have

$$N_{cl}^i N_{int}^j(A) \subseteq N_{int}^i N_{cl}^j(A) \dots\dots\dots(12)$$

$$\text{and } N_{int}^i N_{cl}^j(A) \subseteq N_{cl}^j N_{int}^i(A) \dots\dots\dots(13)$$

From eqs (12) & (13), we get

$$N_{cl}^i N_{int}^j(A) \subseteq N_{cl}^j N_{int}^i(A).$$

- 2) Let (X, τ_1, τ_2) be a neutrosophic bitopological space and A be both contra τ_{ij} -neutrosophic- p -set and contra τ_{ji} -neutrosophic- q -set. Then, we have

$$N_{cl}^j N_{int}^i(A) \subseteq N_{int}^i N_{cl}^j(A) \dots\dots\dots(14)$$

$$N_{int}^i N_{cl}^j(A) \subseteq N_{cl}^j N_{int}^i(A) \dots\dots\dots(15)$$

From eqs (14) & (15), we get

$$N_{cl}^j N_{int}^i(A) \subseteq N_{cl}^i N_{int}^j(A).$$

5. Pairwise τ_{ij} -*b*-open:

Definition 5.1. A neutrosophic set H is said to be pairwise τ_{ij} -neutrosophic-semi-open set (pairwise τ_{ij} -neutrosophic-pre-open set) in a neutrosophic bitopological space (X, τ_1, τ_2) if $H=K \cup L$, where K is a τ_{ij} -neutrosophic-semi-open set (τ_{ij} -neutrosophic-pre-open set) and L is a τ_{ji} -neutrosophic-semi-open set (τ_{ji} -neutrosophic-pre-open set) in (X, τ_1, τ_2) .

Definition 5.2. A neutrosophic set H is said to be pairwise τ_{ij} -neutrosophic-*b*-open set in a neutrosophic bitopological space (X, τ_1, τ_2) if $H=K \cup L$, where K is a τ_{ij} -neutrosophic-*b*-open set and L is a τ_{ji} -neutrosophic-*b*-open set in (X, τ_1, τ_2) .

Theorem 5.1. The union of two pairwise τ_{ij} -neutrosophic-*b*-open set in a neutrosophic bitopological space (X, τ_1, τ_2) is again a pairwise τ_{ij} -neutrosophic-*b*-open set.

Proof: Let A, B be two pairwise τ_{ij} -neutrosophic-*b*-open set in a neutrosophic bitopological space (X, τ_1, τ_2) . Then there exists two τ_{ij} -neutrosophic-*b*-open set G_1, G_2 and two τ_{ji} -neutrosophic-*b*-open set H_1, H_2 such that $A= G_1 \cup H_1$ and $B= G_2 \cup H_2$.

Since, G_1, G_2 are τ_{ij} -neutrosophic-*b*-open set so

$$G_1 \subseteq N_{cl}^i N_{int}^j(G_1) \cup N_{int}^j N_{cl}^i(G_1) \dots\dots\dots(16)$$

$$\text{and } G_2 \subseteq N_{cl}^i N_{int}^j(G_2) \cup N_{int}^j N_{cl}^i(G_2) \dots\dots\dots(17)$$

Since, H_1, H_2 are τ_{ji} -neutrosophic-*b*-open set so

$$H_1 \subseteq N_{cl}^j N_{int}^i(H_1) \cup N_{int}^i N_{cl}^j(H_1) \dots\dots\dots(18)$$

$$\text{and } H_2 \subseteq N_{cl}^j N_{int}^i(H_2) \cup N_{int}^i N_{cl}^j(H_2) \dots\dots\dots(19)$$

Now, we have

$$\begin{aligned} G_1 \cup G_2 &\subseteq N_{cl}^i N_{int}^j(G_1) \cup N_{int}^j N_{cl}^i(G_1) \cup N_{cl}^i N_{int}^j(G_2) \cup N_{int}^j N_{cl}^i(G_2) \quad [\text{using eqs (16) \& (17)}] \\ &= N_{cl}^i N_{int}^j(G_1) \cup N_{cl}^i N_{int}^j(G_2) \cup N_{int}^j N_{cl}^i(G_1) \cup N_{int}^j N_{cl}^i(G_2) \\ &\subseteq N_{cl}^i(N_{int}^j(G_1) \cup N_{int}^j(G_2)) \cup N_{int}^j(N_{cl}^i(G_1) \cup N_{cl}^i(G_2)) \\ &\subseteq N_{cl}^i(N_{int}^j(G_1 \cup G_2)) \cup N_{int}^j(N_{cl}^i(G_1 \cup G_2)) \end{aligned}$$

$\Rightarrow G_1 \cup G_2$ is a τ_{ij} -neutrosophic-*b*-open set.

Further, we have

$$\begin{aligned} H_1 \cup H_2 &\subseteq N_{cl}^j N_{int}^i(H_1) \cup N_{int}^i N_{cl}^j(H_1) \cup N_{cl}^j N_{int}^i(H_2) \cup N_{int}^i N_{cl}^j(H_2) \quad [\text{using eqs (18) \& (19)}] \\ &= N_{cl}^j N_{int}^i(H_1) \cup N_{cl}^j N_{int}^i(H_2) \cup N_{int}^i N_{cl}^j(H_1) \cup N_{int}^i N_{cl}^j(H_2) \\ &\subseteq N_{cl}^j(N_{int}^i(H_1) \cup N_{int}^i(H_2)) \cup N_{int}^i(N_{cl}^j(H_1) \cup N_{cl}^j(H_2)) \\ &\subseteq N_{cl}^j(N_{int}^i(H_1 \cup H_2)) \cup N_{int}^i(N_{cl}^j(H_1 \cup H_2)) \end{aligned}$$

$\Rightarrow H_1 \cup H_2$ is a τ_{ji} -neutrosophic-*b*-open set.

Hence, $A \cup B = (G_1 \cup H_1) \cup (G_2 \cup H_2) = (G_1 \cup G_2) \cup (H_1 \cup H_2) = G \cup H$.

Therefore there exists a τ_{ij} -neutrosophic-*b*-open set $G=(G_1 \cup G_2)$ and a τ_{ji} -neutrosophic-*b*-open set $H=(H_1 \cup H_2)$ such that $A \cup B= G \cup H$. Hence $A \cup B$ is a pairwise τ_{ij} -neutrosophic-*b*-open set. Thus the

union of two pairwise τ_{ij} -neutrosophic- b -open set in a neutrosophic bitopological space (X, τ_1, τ_2) is again a pairwise τ_{ij} -neutrosophic- b -open set.

Theorem 5.2. In a neutrosophic bitopological space (X, τ_1, τ_2) , every pairwise τ_{ij} -neutrosophic-semi open set (pairwise τ_{ij} -neutrosophic-pre-open set) is a pairwise τ_{ij} -neutrosophic- b -open set.

Proof: Let G be a pairwise τ_{ij} -neutrosophic-semi-open set (pairwise τ_{ij} -neutrosophic-pre-open set). Then there exist a τ_{ij} -neutrosophic-semi-open set A (τ_{ij} -neutrosophic-pre-open set A) and a τ_{ij} -neutrosophic-semi-open set B (τ_{ij} -neutrosophic-pre-open set B) such that $G=A \cup B$.

In theorem 4.2, it is clearly shown that every τ_{ij} -neutrosophic-semi-open set (τ_{ij} -neutrosophic-pre-open set) is a τ_{ij} -neutrosophic- b -open set and every τ_{ij} -neutrosophic-semi-open set (τ_{ij} -neutrosophic-pre-open set) is a τ_{ij} -neutrosophic- b -open set. So A be τ_{ij} -neutrosophic- b -open and B be τ_{ij} -neutrosophic- b -open set. Therefore, there exist a τ_{ij} -neutrosophic- b -open set A and a τ_{ij} -neutrosophic- b -open set B such that $G=A \cup B$. Hence, G is a pairwise τ_{ij} -neutrosophic- b -open set. Thus every pairwise τ_{ij} -neutrosophic-semi-open set (pairwise τ_{ij} -neutrosophic-pre-open set) is a pairwise τ_{ij} -neutrosophic- b -open set.

6. Conclusion

In this article, we studied neutrosophic- b -open set, pairwise neutrosophic- b -open set in neutrosophic bitopological spaces and investigate their basic properties. By defining neutrosophic- b -open set, pairwise neutrosophic- b -open set, we prove some theorems on neutrosophic bitopological spaces and some examples are given.

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