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**Abstract**. In this paper some elementary types of neutrosophic functions and their inverse functions are defined based on Smarandache's definition. Also composition of two neutrosophic functions is introduced and some elementary theorems on them are developed.

**Keywords:** Neutro function; Neutro inverse function; Neutro composition mapping

## 1. **Introduction**

The importance of neutrosophic set invented by Smarandache [1] is increasing rapidly due to its characteristic inherent in its definition itself. It has acquired in latest years in extensive applicational areas [2–9]. Recently it is applied in professional selection [10], supply chain problem [11], evaluation of manufacturer industry [12], smart product-service system [13] (Mohammed Abdel Basset et al., 2020). Even neutrosophic set is included in the research-arena of algebra [14–16], calculas [17], topology [18–22] etc.

All laws in the world are not deterministic. So, the axioms need to be more flexible to cope up with our dynamic world. Neutrosophic algebraic and N algebraic structure, notion of groups, N-groups, semigroups, N-semigroups, N-loops etc. were discussed in [23] by Kandasamy et al. The author (Kandasamy et al.) also studied about neutrosohic rings [24]. Popular research papers are on neutrosohic groups, subgroups [25], neutrosophic rings [27,28], hyper groups [26] and hyper rings [28]. In the context of neutrosohic theory N-bi-ideal [29] was discussed in semigroup (Porselvi et al.). In BCI/BCK-algebras BMBJ-neutrosophic subalgebras as well as their related properties were studied in [30] by H. Bordbar et al. G.R. Rezaei et al. [31] investigated about neutrosophic quadruple a-ideal. The idea of neutrosophic lattice ideals and LI-ideals [32] was introduced by Rajab Ali Borzooei et al.

In this way in algebra on various topics there are different types of researches based on neutrosophic theory. But on mapping, which has an significant role, a few research are there. In classical algebra the axioms characterized on a set are well defined. Yet there are various circumstances in science and in space of information with a maxim characterized on a set where algebraic axioms are partially followed. Herein lies the importance of neutro axioms. Smarandache introduced neutroalgebraic, antialgebraic structure in [33] and discussed the importance of neutro-axiom. Also the author [34] extended neutro algebra as a generalization of partial algebra and defined neutrosophic function. In this article first time different types of netro functions- one-one, onto, bijective neutro mapping, their composition and inverse neutro mapping are defined. Based on these definitions, some elementary theorems are also developed.

## 2. **Preliminaries**

### 2.1. *Neutrosophic set [1]*

Let $U$ be an universe of discourse, then the neutrosophic set $A$ is defined as $A = \{\langle x : T_A(x), I_A(x), F_A(x)\rangle, x \in U\}$, where the functions $T$, $I$, $F$: $U \to ]^-0, 1^+[$ define respectively the degree of membership (or Truth), the degree of indeterminacy and the degree of non-membership (or falsehood) of the element $x \in U$ to the set $A$ with the condition $^-0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+$.

### 2.2. *Neutro Function [34]*

A function$f : X \to Y$ is called a Neutro function if it has elements in $X$ for which the function is well-defined, elements in $X$ for which the function is indeterminate, and elements in $X$ for which the function is outer-defined.

## 3. **Physical implication of the research work**

In this section let us consider some practical examples which indicate the implication of the proposed discussion.

Example 1. In case of radio active decaying element, among the disintegrating atoms of radio active element, some may have only a short existence, while others may remain unchanged for a long time- why we don't know. The disintegration of a particular atom is a chance incident, however, only half life period can be conveniently represented. Now if we want to make a relation between atoms of an radioactive element and its decay within half life period, mapping can not be defined rather a neutro function can be established.

Example 2. Suppose $(ct, c/t)$ denotes the position of a moving particle at any time t. Then the mapping can not be defined from $[0, T]$ (representing time interval) to its corresponding position. Because at $t = 0$, it is undefined; though it is a neutro mapping.

Example 3. Suppose a coin is tossed. Let us think a rule of correspondence $f(Head) = 1$, $f(Tail) = 0$. But it may happen that the coin is stucked at a split and it is neither head nor tail. Here also mapping is not defined., however neutro function is accepted.

In this way there are so many practical events where functions are not defined to give a rule for making relations, whereas neutro functions can be defined. Now based on the concepts of neutro function's definition and their variation, the idea of different types of functions and their elementary properties are introduced.

## 4. **some proposed basic definitions**

### 4.1. *Neutro one-one function*

A function $f : X \to Y$ is called a neutro one-one function if for each well defined pair of distinct elements of X, their $f$ images are distinct or it has elements in X for which the function is undefined.

Consider $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $X = \{1, 2, 3, 4, 5, 6\}$, $Y = \{7, 8, 9, 10, 11\}$, $f(1) = 7, f(2) = 8, f(3) = 9, f(4) = undefined, f(5) = 10, f(6) = 11$. Then f is not a function but its a neutro one to one mapping.

Example 2., example 3. discussed in section 3 are the practical example of neutro one-one mapping.

### 4.2. *Neutro onto function*

A neutro mapping $f : X \to Y$ is said to be a neutro onto if for any element $y \in Y$ it is confirmed that every $y \in Y$ has its pre-image but exactly which one that may not be determined.

#### 4.2.1. *Example*

Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $X = \{1, 2, 3, 4, 5, 6\}$, $Y = \{7, 8, 9\}$, $f(1) = 7$, $f(2) = 8$, $f(3) = 8$, $f(\textit{some numbers greater than } 4)$=9.

Example 1. in sec. 3 is a neutro onto mapping. Because only confirmation is within half life period half of the radioactive quantity will decay, but it is not predictable particular which atom will decay.

4.3. *Neutro bijection mapping*

A neutro mapping $f : A \to B$ is said to be neutro bijective if $f$ is both neutro one to one and neutro onto.

4.3.1. *Example*

Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $X = \{1, 2, 3, 4, 5, 6\}$, $Y = \{7, 8, 9, 10, 11\}$, $f(1) = 7, f(2) = 8, f(3) = 9, f(4) = undefined, f(5) = 10, f(6) = 11$. Clearly $f$ is a neutro bijective mapping.

Consider example 3. (sec. 3). If we define the correspondence from position of the coin after toss to the set $\{0, 1\}$. Then it,s a neutro bijective mapping.

4.3.2. *Neither one-one nor onto*

Let $U = \{1, 2, 3, ..., 9\}$, $X = \{1, 2, 3, 4, 5, 6\}$, $Y = \{7, 8, 9, 10, 11\}$, $f(1) = 7$, $f(2) = 7$,$f(3) = 9$ *or* 10 or 11. $f$ is not one-one as images of $1, 2$ are same $(7)$. It is not onto also because it is not confirmed that 9, 10 and 11 have a pre-image.

**Theorem 4.1.** *Let $f : A \to B$ and $g : B \to C$ be both neutro one-one mapping, then the composite mapping $gof : A \to C$ is neutro one-one.*

*Proof.* Clearly, the undefined points does not affect the theorem. Consider two elements $x_1$, $x_2$ in A where $f$ is well defined. Then clearly for $x_1 \neq x_2$, $gof(x_1) \neq gof(x_2)$. □

**Theorem 4.2.** *Let $f : A \to B$ and $g : B \to C$ be two neutro mapping such that $gof : A \to C$ is neutro one-one, then f is neutro one-one.*

*Proof.* If possible let $f$ be not neutro one-one. Then there exists two elements $x$ and $y$ in A such that for $x_1 \neq x_2$, $f(x_1) \neq f(x_2)$. So,$gof(x_1) \neq gof(x_2)$, which is a contradiction. So, $f$ is neutro one to one. □

**Theorem 4.3.** *Let $f : A \to B$ and $g : B \to C$ be both neutro onto mapping, then the composite mapping $gof : A \to C$ is neutro onto.*

*Proof.* Since g is onto, for any c in C, $g($*some element b in B*$)$=$C$ and in the same way $f($*some a in A*$)$=$b$. So, $gof(a) = c$. □

**Theorem 4.4.** *Let $f : A \to B$ and $g : B \to C$ be two neutro mapping then $gof : A \to C$ is neutro onto then $g$ is also neutro onto.*

*Proof.* Let c be an element in C, then there exist some element a in A such that $gof(a) = c$, i.e., $g(b) = c$, for some b in B. Hence g is neutro onto. □

4.4. *Neutrosophic identity mapping*

A mapping $f : A \to A$ is said to be the neutrosophic identity mapping on A if $f(x) = x$ at each elements where the function is well defined.

According to the definition of neutrosophic identity mapping the concept of inverse neutro mapping is developed:

4.5. *Inverse Neutrosophic mapping*

Let $f : A \to B$ be a neutrosophic mapping. If there exists a neutrosophic mapping $g : B \to A$, such that $gof = I_A$, then $g$ is said to be left neutro inverse of $f$.

If there exists a mapping $h : B \to A$ such that $foh = I_B$, $h$ is said to be right neutrosophic inverse of $f$.

Example 1: Take two neutrosophic mapping such that $f : R \to [-1, 1]$ by $f(x) = \sin x$ and $g : [-1, 1] \to R$ by $g(x) = \sin^{-1} x$, then$fog(x) = x$, but $gof(x)$ does not always give the value x. For example, for $x = \pi/2$, $gof(x) = (4n+1)\pi/2$, $n = 0, 1, 2...$ So, g is the right neutrosophic inverse of f but not left neutrosophic inverse.

4.6. *Neutrosophic inverse*

A function $g$ is called neurosophic inverse of $f$ if $gof = fog = I$.

Consider the mappings f and g from $R \to R$ by $f(x) = 1/x$ and$g(x) = 1/x$, then $fog = gof = I$. Here $g$ is the neutrosophic inverse of $f$.

**Theorem 4.5.** *A neutro mapping $f : A \to B$ is invertible if and only if $f$ is neutro one-one.*

*Proof.* Let $f : A \to B$ be a neutrosohic invertible, then there exists $g : B \to A$ such that $gof = I_A$. Clearly, $I_A$ is neutro one to one mapping. So, $f$ is neutro one to one.

Conversely, let $f$ is neutro one-one mapping. Let $b \in f(A)$, since $f$ is one-one, there exist $a \in A$ such that $f(a) = b$. Define a neutro mappping $g : B \to A$ such that $g(b) = a$. Then $gof(a) = g(b) = a$ and $fog(b) = f(a) = b$. Hence the proof. □

**Theorem 4.6.** *Let $f$ is a neutrosophic mapping and $f(P) \subseteq f(Q)$. Then $p \subseteq Q$.*

*Proof.*
Let $f(P) = \{(T_{f_P}(x), I_{f_P}(x), F_{f_P}(x)) : x \in P\}$ and $f(Q) = \{(T_{f_Q}(x), I_{f_Q}(x), F_{f_Q}(x)) : x \in Q\}$.
Since $f(P) \subseteq f(Q)$, $T_{f_P}(x) \leq T_{f_Q}(x)$, $I_{f_P}(x) \geq I_{f_Q}(x)$, $F_{f_P}(x) \geq F_{f_Q}(x)$.

Evidently, the existence of x in P is less than Q, otherwise it would violate $f(P) \subseteq f(Q)$. □

**Theorem 4.7.** *Let $f$ is a onto neutrosophic mapping and $f^{-1}(P) \subseteq f^{-1}(Q)$, then $P \subseteq Q$.*

*Proof.* For an element y, $T_{f_P^{-1}}(y) \leq T_{f_Q^{-1}}(y)$, $I_{f_P^{-1}}(y) \geq I_{f_Q^{-1}}(y)$, $F_{f_P^{-1}}(y) \geq F_{f_Q^{-1}}(y)$, where, $T_{f_P^{-1}}(y)$, $F_{f_P^{-1}}(y)$ and $I_{f_P^{-1}}(y)$ denotes the truth, falsity and indeterminacy degree of belongingness of $f^{-1}(y)$ in $f_P^{-1}$ respectively.

It assures that the belongingness of y in P less than Q, otherwise it would violate the above relations. □

Remark: To develop any theorem on neutro algebra, the necessary definitions should be initially defined. All the theories are true only on the basis of the proposed definition.

## 5. Conclusion

In this paper several types of neutro mappings similar to classical algebra are defined. Also some elementary theorems are established following the proposed definitions. Since in science and technology the laws that describe them can hardly be rigorously defined, the neutro mapping based on neutro axioms will keep its incredible utilities.

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