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On NeutroNilpotentGroups

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Abstract. In this paper, we introduce the notion of commutator of two elements in a specific NeutroGroup. Then we define the notion of a NeutroNilpotentGroup and we study some of their properties. Moreover, we show that the intersection of two NeutroNilpotentGroups is a NeutroNilpotentGroup. Also, we show that the quotient of a NeutroNilpotentGroup is a NeutroNilpotentGroup. Specially, using NeutroHomomorphism we prove the NeutroNilpotentcy is closed with respect to homomorphic image.

Keywords: NeutroGroup; NeutroSubgroup; NeutroNilpotentGroup; NeutroQuotientGroup; NeutroGroup Homomorphism.

1. Introduction

One of the most important concepts in the study of groups is the notion of nilpotency [6]. Nilpotent groups arose in Galois theory, as well as in the classification of groups. By Galois theory, certain problems in field theory reduced to group theory. In [10][11], Smarandache introduced the notions of NeutroDefined, AntiDefined laws, NeutroAxiom and AntiAxiom. Then in [9], he studied NeutroAlgebras and AntiAlgebras. Rezaei et al. in [5], proved that there are $(2^n - 1)$ NeutroAlgebras and $(3^n - 2^n)$ AntiAlgebras in a classical algebra $S$ with $n$ operations and axioms all together, where $n \geq 1$. Agboola et al. in [1], studied NeutroGroups $(NG, \ast)$ where the law of composition and axioms defined on $NG$ may either be only partially defined (partially true), or partially undefined (partially false), or totally undefined (totally false) with respect to $\ast$. Moreover, they considered three NeutroAxioms (NeutroAssociativity, existence of NeutroNeutral element and existence of NeutroInverse element) to show the difference between groups and NeutroGroups. Also, in [3], Agboola studied NeutroRings by considering three NeutroAxioms (NeutroAbelianGroup (additive), NeutroSemigroup (multiplicative) and
NeutroDistributivity (multiplication over addition)). Scholars applied the notion of NeutroAxioms and NeutroLaw on Rings, Subrings, Ideals, QuotientRings and Ring Homomorphism to present some new notions and several results are obtained (see [3], [7]). In this paper, we consider a class of NeutroGroups was introduced in [1], and define the notion of NeutroNilpotentGroups. Moreover, we investigate elementary properties of NeutroNilpotentGroups. Specially, we show that the intersection of two NeutroNilpotentGroups is a NeutroNilpotentGroup. Also, we prove the NeutroNilpotency is closed with respect to homomorphic image.

2. Preliminaries

We recall some basic definitions and results which are proposed by the pioneers of this subject.

**Definition 2.1 ([8]).**

(i) A classical operation is well defined for all the set’s elements.

(ii) A NeutroOperation is an operation partially well defined, partially indeterminate, and partially outer defined on the given set.

(iii) A classical law/axiom defined on a nonempty set is totally true (i.e. true for all set’s elements).

(iv) A NeutroLaw/NeutroAxiom (or NeutrosophicLaw/NeutrosophicAxiom) defined on a nonempty set is a law/axiom that is true for some set’s elements (degree of truth (T)), indeterminate for other set’s elements (degree of indeterminacy (I)), or false for the other set’s elements (degree of falsehood (F)), where $T, I, F \in [0, 1]$, with $(T, I, F) \neq (1, 0, 0)$ that represents the classical axiom.

(v) A NeutroAlgebra is an algebra that has at least one NeutroOperation or one NeutroAxiom (axiom that is true for some elements, indeterminate for other elements and false for other elements).

**Definition 2.2 ([4]).** For a nonempty set $G$ and a binary operation $*$ on $G$ the couple $(G, *)$ is called a classical group if the following conditions hold:

(G1) $x * y \in G$ for all $x, y \in G$.

(G2) $x * (y * z) = (x * y) * z$ for all $x, y, z \in G$.

(G3) There exists $e \in G$ such that $x * e = e * x = x$ for all $x \in G$.

(G4) There exists $y \in G$ such that $x * y = y * x = e$ for all $x \in G$, where $e$ is the neutral element of $G$.

If for all $x, y \in G$, (G5) $x * y = y * x$, then $(G, *)$ is called an abelian group.

Note that $x * y$ will be written as $xy$ for all $x, y \in G$. 

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**Definition 2.3** ([6]). A group \((G,\ast)\) is called nilpotent if it has a central series, that is, a normal series \(e = G_0 \leq G_1 \leq \cdots \leq G_n = G\) such that \(G_{i+1}/G_i\) is contained in the center of \(G/G_i\) for all \(i\). The length of a shortest centreal series of \(G\) is the nilpotent class of \(n\).

**Definition 2.4** ([6]). Let \((G,\ast)\) be a group and \(x_1, \ldots, x_n\) be elements of \(G\). Commutator of \(x_1\) and \(x_2\) is \([x_1, x_2] = x_1^{-1}x_2^{-1}x_1x_2\). A commutator of weight \(n \geq 2\) is defined by \([x_1, \ldots, x_n] = [x_1, \ldots, x_{n-1}], x_n\] where by convention \([x_1] = x_1\).

A NeutroGroup is an alternative of a group that has either one NeutroOperation (partially well-defined, partially indeterminate and partially outer-defined), or at least one NeutroAxiom (NeutroAssociativity, NeutroNeutralElement or NeutroInverseElement) with no AntiOperation (is an operation outer-defined for all the set’s elements (totally falsehood)) or AntiAxion (is an axiom that is false for all set’s elements). It is possible to define NeutroGroup in another way by considering only one NeutroAxiom or by considering two NeutroAxioms or etc.

**Definition 2.5.** Let \(NG\) be a nonempty set and \(\ast\) be a binary operation on \(NG\). The couple \((NG,\ast)\) is called a NeutroGroup if the following conditions are satisfied:

\((NG1)\) There exists some triplet \((x, y, z) \in NG\) such that 
\[
x \ast (y \ast z) = (x \ast y) \ast z \text{ and } u \ast (v \ast w) \neq (u \ast v) \ast w
\]
for some \((u, v, w) \in NG\) or there exists some \((r, s, t) \in NG\) such that \(\ast\)\text{-indeterminate} (NeutroAssociativity).

\((NG2)\) There exists at least an element \(a \in NG\) that has a single neutral element i.e., we have 
\[
e \in NG \text{ such that } a \ast e = e \ast a = a \text{ and for } b \in NG \text{ there does not exist } e \in NG \text{ such that } b \ast e = e \ast b \neq b \text{ or there exists } e_1, e_2 \in NG \text{ such that } b \ast e_1 = e_1 \ast b = b \text{ or } b \ast e_2 = e_2 \ast b = b \text{ with } e_1 \neq e_2 \text{ or there exists at least an element } c \in NG \text{ that there is } d \in NG \text{ such that } c \ast d = d \ast c \text{ =indeterminate (NeutroNeutralElement).}
\]

\((NG3)\) There exists an element \(a \in NG\) that has an inverse \(b \in NG\) w.r.t. a unit element \(e \in NG\) i.e., \(a \ast b = b \ast a = e\), or there exists at least one element \(b \in NG\) that has two or more inverses \(c, d \in NG\) w.r.t. some unit element \(u \in NG\) i.e., \(b \ast c = c \ast b = u\), \(b \ast d = d \ast b = u\) or there exists at least one element \(r \in NG\) that has one element \(s \in NG\) such that \(\ast\)-indeterminate (NeutroInverseElement).

\((NG4)\) There exists some duplet \((a, b) \in NG\) such that \(a \ast b = b \ast a\) and there exists some duplet \((c, d) \in NG\) such that \(c \ast d \neq d \ast c\), or there exists some \((r, s) \in NG\), \(\ast\text{-indeterminate} or s\ast r \text{-indeterminate, then } (NG,\ast) \text{ is called a NeutroAbelianGroup (NeutroAbelianGroup).}

**Example 2.6.** Let \(U = \{a, b, c, d, e, f\}\) be a universe of discourse and \(NG = \{a, b, c, d\}\) be a subset of \(U\). Define the operation \(*_1\) on \(NG\) in table [1]. Then \(*_1\) is a NeutroLow since \(c *_1 d = \text{indeterminate. Also,}
\[
a *_1 (b *_1 c) = (a *_1 b) *_1 c \text{ and } c *_1 (a *_1 c) = c *_1 d = \text{indeterminate}
\]

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Table 1. The table of NeutroGroup \((NG, *_1)\)

<table>
<thead>
<tr>
<th>*_1</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>b</td>
<td>c</td>
<td>d</td>
<td>a</td>
</tr>
<tr>
<td>b</td>
<td>c</td>
<td>d</td>
<td>a</td>
<td>c</td>
</tr>
<tr>
<td>c</td>
<td>d</td>
<td>a</td>
<td>b</td>
<td>?</td>
</tr>
<tr>
<td>d</td>
<td>a</td>
<td>b</td>
<td>?</td>
<td>a</td>
</tr>
</tbody>
</table>

Thus, \((NG, *_1)\) is a NeutroGroup.

Note that \(x * y\) will be written as \(xy\) for all \(x, y \in NG\).

**Theorem 2.7** ([1]). Let \((NH, *)\) be a NeutroSubgroup of the NeutroGroup \((NG, *)\). The sets \((NG/NH)_l = \{xNH : x \in NG\}\) and \((NG/NH)_r = \{NHx : x \in NG\}\) are two NeutroGroups with operations \(\circ_l, \circ_r\) where for any \(xNH, yNH \in (NG/NH)_l, NHx, NHy \in (NG/NH)_r, x, y \in NG\) we have

\[xNH \circ_l yNH = xyNH, \quad NHx \circ_r NHy = NHxy.\]

**Definition 2.8** ([1]). Let \((NG, *)\) and \((NK, \circ)\) be two NeutroGroups. The mapping \(\varphi : NG \rightarrow NK\) is called a NeutroGroup Homomorphism if for every duplet \((x, y) \in G\), we have \(\varphi(x * y) = \varphi(x) \circ \varphi(y)\).

In addition, if \(\varphi\) is a NeutroBijection, then \(\varphi\) is called a NeutroGroup Isomorphism. NeutroGroup Epimorphism, NeutroGroup Monomorphism, NeutroGroup Endomorphism are defined similarly.

**Theorem 2.9** ([1]). Let \((NG, *)\) and \((NK, \circ)\) be NeutroGroups and let \(e_{NG}\) and \(e_{NH}\) be NeutroNeutralElements in \(NG\) and \(NK\) respectively. Suppose that \(\varphi : NG \rightarrow NK\) is a NeutroGroup Homomorphism. Then \(\varphi(e_{NG}) = e_{NK}\).

From now on, \(NG\) is a NeutroGroup with tree NeutroAxioms (NeutroAssociativity, NeutroNeutralElement and NeutroInverseElement). Also, for all \(x \in NG\), \(N_x\) and \(I_x\) represent the NeutroNeutralElement and the NeutroInverseElement respectively.

3. **Some Results On NeutroNilpotentGroups**

In this section, we introduce the notion of commutator of two elements in a NeutroGroup and study a new concept as NeutroNilpotentGroups and their properties are given.

Let \(x, y\) be elements of a NeutroGroup \(NG\). The commutator of \(x, y\), denoted by \([x, y]\), is the element \(I_xI_yxy\), i.e., \([x, y] = I_xI_yxy\). If \(I_x\) or \(I_y\) does not exist, then put \(I_x = x\) and \(I_y = y\).

Also, for any \(x, y_1, \ldots, y_n \in NG\), define the commutator \([x, y_1, \ldots, y_n]\) by \([x, y_1, \ldots, y_n] = [[x, y_1, \ldots, y_{n-1}], y_n]\).
Table 2. The table of NeutroNilpotentGroup (NG, \(*_2\))

<table>
<thead>
<tr>
<th>(*_2)</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>b</td>
<td>c</td>
<td>d</td>
<td>a</td>
</tr>
<tr>
<td>b</td>
<td>c</td>
<td>d</td>
<td>a</td>
<td>c</td>
</tr>
<tr>
<td>c</td>
<td>d</td>
<td>a</td>
<td>b</td>
<td>d</td>
</tr>
<tr>
<td>d</td>
<td>a</td>
<td>b</td>
<td>c</td>
<td>a</td>
</tr>
</tbody>
</table>

Table 3. The table of NeutroAbelianGroup (NG, \(*_3\))

<table>
<thead>
<tr>
<th>(*_3)</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>e</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>b</td>
<td>a</td>
<td>b</td>
<td>a</td>
</tr>
<tr>
<td>b</td>
<td>c</td>
<td>f</td>
<td>c</td>
<td>b</td>
</tr>
<tr>
<td>c</td>
<td>d</td>
<td>c</td>
<td>e</td>
<td>c</td>
</tr>
<tr>
<td>e</td>
<td>a</td>
<td>b</td>
<td>c</td>
<td>e</td>
</tr>
</tbody>
</table>

Definition 3.1. A NeutroGroup (NG, *) is called NeutroNilpotentGroup if $Z_n(NG) = NG$ for some $n \in \mathbb{N}$, where

$Z_n(NG) = \{x \in NG : [x, g_1, g_2, \ldots, g_n] = N_z \text{ for at least one } g_1, \ldots, g_n, z \in NG\}.$

The smallest such $n$ is called the NeutroNilpotency of NG.

Note that, if NG is a NeutroNilpotentGroup, then for any $x \in NG$ there exists at least one $g_1, \ldots, g_n, z \in NG$ such that $[x, g_1, g_2, \ldots, g_n] = N_z$.

Example 3.2. Let $U = \{a, b, c, d, e, f\}$ be a universe of discourse and $NG = \{a, b, c, d\}$ be a subset of $U$. Define the operation \(*_2\) on $NG$ in table 2. Since $[a, b] = d, [d, b] = d, [c, c] = d$ and $[b, b] = a$, we have $[c, d, b] = [b, b, b] = [a, b] = d = N_a, [d, b, b] = [d, b] = N_a$ and $[a, b, b] = [d, b] = N_a$. Therefore, NG is a NeutroNilpotentGroup of class 2.

Example 3.3. Let $U = \{a, b, c, d, e, f\}$ be a universe of discourse and let $NG = \{e, a, b, c\}$ be a subset of $U$. Define the operation \(*_3\) on $NG$ in table 3. Since $[a, b, a] = e = N_c, [b, a, e] = [e, e] = e, [c, e, c] = [c, c] = e, [e, a, a] = [b, a] = e$, we have NG is NeutroAbelianGroup and a NeutroNilpotentGroup of class 2.

In what follows we have a non Abelian NeutroNilpotentGroup.

Example 3.4. Let $U = \{a, b, c, d\}$ be a universe and $NG = \{a, b, c\}$ be a NeutroGroup by the Cayley table 4. Then $H = \{a, b\}$, by the operation \(*_4\), is a NeutroSubgroup of $NG$ (see 1). Since $N_a = a, I_a = a, N_b, I_b$ does not exist, we have $[a, a] = aaaa = a = N_a$ and $[b, b] = a = N_a$. Therefore, $H$ is a NeutroNilpotentSubgroup that is not an AbelianNeutroGroup.

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Table 4. The table of non Abelian NeutoAbelianGroup \((NG, \ast_4)\)

<table>
<thead>
<tr>
<th>(\ast_4)</th>
<th>a</th>
<th>b</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>a</td>
<td>c</td>
<td>b</td>
</tr>
<tr>
<td>b</td>
<td>c</td>
<td>a</td>
<td>c</td>
</tr>
<tr>
<td>c</td>
<td>a</td>
<td>c</td>
<td>d</td>
</tr>
</tbody>
</table>

Table 5. The table of NeutoSubgroup \((H, \ast_4)\)

<table>
<thead>
<tr>
<th>(\ast_4)</th>
<th>a</th>
<th>b</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>a</td>
<td>c</td>
</tr>
<tr>
<td>b</td>
<td>c</td>
<td>a</td>
</tr>
</tbody>
</table>

Table 6. The table of NeutoSubgroup \((NH, \ast_5)\) of \((NG, \ast_5)\)

<table>
<thead>
<tr>
<th>(\ast_5)</th>
<th>a</th>
<th>c</th>
<th>d</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>b</td>
<td>d</td>
<td>a</td>
</tr>
<tr>
<td>c</td>
<td>d</td>
<td>b</td>
<td>d</td>
</tr>
<tr>
<td>d</td>
<td>a</td>
<td>c</td>
<td>a</td>
</tr>
</tbody>
</table>

Theorem 3.5. Let \(NG\) and \(NK\) be two NeutoGroups. Then \(Z_n(NG \times NK) = Z_n(NG) \times Z_n(NK)\). Moreover, \(NG \times NK\) is a NeutoNilpotentGroup of class \(n\) if and only if \(NG\) and \(NK\) are NeutoNilpotentGroups of class \(n\).

Proof. Assume \((x, y) \in Z_n(NG \times NK)\), \(z \in NG\) and \(t \in NK\). Then for some \((x_1, y_1), \ldots, (x_n, y_n) \in NG \times NK\), we have

\[
(N_z, N_t) = [(x, y), (x_1, y_1), \ldots, (x_n, y_n)] = ([x, x_1, \ldots, x_n], [y, y_1, \ldots, y_n])
\]

\[
\iff [x, x_1, \ldots, x_n] = N_z, [y, y_1, \ldots, y_n] = N_t
\]

\[
\iff x \in Z_n(NG), y \in Z_n(NK)
\]

\[
\iff (x, y) \in Z_n(NG) \times Z_n(NK).
\]

Therefore, \(Z_n(NG \times NK) = Z_n(NG) \times Z_n(NK)\).

Moreover, \(NG \times NK\) is NeutoNilpotentGroup if and only if \(Z_n(NG \times NK) = NG \times NK = Z_n(NG) \times Z_n(NK)\) if and only if \(NG\) and \(NK\) are NeutoNilpotentGroups. \(\square\)

In what follows we have a NeutoSubgroup that is not NeutoNilpotentGroup.

Example 3.6. Consider the NeutoGroup \(NG\) from Example 3.2. Define the operation \(\ast_5\) on \(NG\) in table 6. Then \(NH = \{a, c, d\}\) is a NeutoSubgroup of \(NG\) (see [1]). Since \([a, d] = a\), \([a, a]\) and \([a, c]\) does not exist, we get \([a, g_1, \ldots, g_n]\) does not exist for any \(g_1, \ldots, g_n \in NH\), and so \(a \notin Z_n(NH)\) i.e., \(NH\) is not NeutoNilpotentGroup.
Theorem 3.7. Let $NH$ be a NeutroSubgroup of the NeutroNilpotentGroup $NG$. Then NeutroQuotientGroups $(NG/NH)_l$ and $(NG/NH)_r$ are NeutroNilpotentGroups.

Proof. Assume $NH$ be a NeutroSubgroup of $NG$ and $gH \in (NG/NH)_l$. Since $NG$ is a NeutroNilpotentGroup, we have $[g, g_1, \ldots, g_n] = N_z$ for some $g_1, \ldots, g_n, z \in NG$, and so $[gNH, g_1NH, \ldots, g_nNH] = [g, g_1, \ldots, g_n]NH = N_zNH$. Since $(zNH) \circ_l (N_zNH) = (z \ast N_z)NH = zNH = (N_z)NH \circ_l zNH$, we get $(N_z)NH$ is a NeutroNaturalElement of $(NG/NH)_l$. Therefore, $(NG/NH)_l$ is a NeutroNilpotentGroup. Similarly, $(NG/NH)_r$ is a NeutroNilpotentGroup. \(\square\)

We recall that the intersection of two NeutroGroups is a NeutroGroup (see \(\square\)). Now we have the following:

Theorem 3.8. Let $NG$ and $NK$ be two NeutroNilpotentGroups. Then $NG \cap NK$ is a NeutroNilpotentGroup.

Proof. Straightforward. \(\square\)

Theorem 3.9. Let $NH$ be a NeutroNilpotentSubgroup of a NeutroGroup $NG$ and for all $x, t \in NG$ we have $xNH = NH \Rightarrow x \in NH$, $(N_t)NH = NH$.

If $(NG/NH)_l$ is a NeutroNilpotentQuotientGroup, then $NG$ is a NeutroNilpotentGroup.

Proof. Assume $(NG/NH)_l$ is NeutroNilpotentGroup of class $n$ and $NH$ is NeutroNilpotentGroup of class $m$. Then for any $xNH \in (NG/NH)_l$, there exist $g_1NH, \ldots, g_nNH \in (NG/NH)_l$ such that $[xNH, g_1NH, \ldots, g_nNH] = (N_z)NH$, where $z \in NG$. Then $[x, g_1, \ldots, g_n]NH = (N_z)NH = NH$, and so $[x, g_1, \ldots, g_n] \in NH$. Since $NH$ is NeutroNilpotentGroup, we get there exist $k_1, \ldots, k_m \in NH$ such that $[[x, g_1, \ldots, g_n], k_1, \ldots, k_m] = N_t$, for some $t \in NG$. Consequently, $NG$ is NeutroNilpotentGroup of class $n + m$. \(\square\)

Theorem 3.10. Let $NH$ be a NeutroNilpotentSubgroup of a NeutroGroup $NG$ and for all $x, t \in NG$ we have $NHx = NH \Rightarrow x \in NH$, $NH(N_t) = NH$.

If $(NG/NH)_r$ is a NeutroNilpotentQuotientGroup, then $NG$ is a NeutroNilpotentGroup.

Proof. Similar to the proof of Theorem 3.9. \(\square\)

Theorem 3.11. Every homomorphic image of a NeutroNilpotentGroup is NeutroNilpotentGroup.

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Proof. Assume \( NH \) be a NeutroSubgroup of a NeutroNilpotentGroup \( NG \) and \( e_1, e_2 \) be NeutroNeutralElements in \( NG \) and \( NH \), respectively. Suppose that \( \psi : NG \to NH \) is a NeutroGroup Epimorphism. Then for any \( h \in NH \), there exists \( x \in NG \) such that \( h = \psi(x) \). Since \( NG \) is NeutroNilpotentGroup, for \( x \in NG \), there exist \( g_1, \ldots, g_n \in NG \) such that \( [x, g_1, \ldots, g_n] = e_1 \). Take \( k_1 = \psi(g_1), \ldots, k_n = \psi(g_n) \). Therefore, \([h, k_1, \ldots, k_n] = \psi([x, g_1, \ldots, g_n]) = \psi(e_1) = e_2\), and so \( NH \) is a NeutroNilpotentGroup. \( \Box \)

4. Conclusion

In this paper, we defined a class of NeutroGroups, named NeutroNilpotentGroups, and their elementary properties were presented. The intersection of two NeutroSubgroups is not necessarily a NeutroSubgroup while their union is a NeutroSubgroup. We hope to study NeutroSolvabelGroups, NeutroEngelGroups in our future works.

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