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## Introduction to NeutroHyperGroups

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◊ In commemoration of the 60th birthday of the second author

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**Abstract.** Neutrosophication and AntiSophication are processes through which NeutroAlgebraic and AntiAlgebraic structures can be generated from any classical structures. Given any classical structure with  $m$  operations (laws and axioms) where  $m \geq 1$  we can generate  $(2^m - 1)$  NeutroStructures and  $(3^m - 2^m)$  AntiStructures. In this paper, we introduce for the first time the concept of NeutroHyperGroups. Specifically, we study a particular class of NeutroHyperGroups called  $[2, 3]$ -NeutroHyperGroups and present their basic properties and several examples. It is shown that the intersection of two  $[2, 3]$ -NeutroSubHyperGroups is not necessarily a  $[2, 3]$ -NeutroSubHypergroup but their union may produce a  $[2, 3]$ -NeutroSubhypergroup. Also, the quotient of a  $[2, 3]$ -NeutroHyperGroup factored by a  $[2, 3]$ -NeutroSubHyperGroup is shown to be a  $[2, 3]$ -NeutroHyperGroup. Examples are provided to show that in the neutrosophic environment,  $[2, 3]$ -NeutroHyperGroups are associated with dismutation reactions in some chemical reactions and biological processes.

**Keywords:** NeutroHyperGroup, NeutroSubHyperGroup, NeutroHyperGroupHomomorphism, NeutroHyperGroupIsomorphism.

### 1. Introduction

In 2013, Agboola and Davvaz established the connections between neutrosophic set and algebraic hyperstructures. In [1–3] they studied neutrosophic hypergroup, neutrosophic canonical hypergroup and neutrosophic hyperrings. Since then several neutrosophic algebraic structures have been studied and many results have been obtained and published. Recently, Ibrahim and Agboola in [13] studied Neutrosophic Hypernearings and presented some of their properties. In 2019, Florentin Smarandache in [20] presented the concept of NeutroAlgebraicStructures and AntiAlgebraicStructures which can be generated from classical algebraic structures through processes called Neutrosophication and AntiSophication respectively. He recalled, improved and extended several definitions and properties of these new structures in [19]. These new concepts have provided new methodologies for handling indeterminate,

incomplete and imprecise information and processes. The work of Smarandache in [20] was studied viz-a-viz the classical number systems  $\mathbb{N}$ ,  $\mathbb{Z}$ ,  $\mathbb{Q}$ ,  $\mathbb{R}$  and  $\mathbb{C}$  by Agboola et al. in [4]. In [5,6], Agboola formally presented the notions of NeutroGroups, NeutroSubgroups, NeutroRings, NeutroSubrings, NeutroIdeal, NeutroQuotientRings, and he established several properties of these structures and their substructures for the classes he considered. Recently, Rezaei and Smarandache in [17] introduced the concepts of Neutro-BE-algebras and Anti-BE-algebras and in [7,12] Agboola and Ibrahim introduced the concept of NeutroVectorSpaces and AntiRings. The present paper will be concerned with the introduction of the concept of NeutroHyperGroups and presentations of their basic properties and examples. For more details on Neutrosophy and applications, the readers should see [8,9,14–16,18,22].

## 2. Preliminaries

In this section, we will give some definitions, examples and results that will be used in the sequel.

**Definition 2.1.** Let  $H$  be a non-empty set and  $\circ : H \times H \rightarrow P^*(H)$  be a hyperoperation. The couple  $(H, \circ)$  is called a hypergroupoid. For any two non-empty subsets  $A$  and  $B$  of  $H$  and  $x \in H$ , we define

$$A \circ B = \bigcup_{a \in A, b \in B} a \circ b, \quad A \circ x = A \circ \{x\} \quad \text{and} \quad x \circ B = \{x\} \circ B.$$

**Definition 2.2.** A hypergroupoid  $(H, \circ)$  is called a semihypergroup if for all  $a, b, c$  of  $H$  we have  $(a \circ b) \circ c = a \circ (b \circ c)$ , which means that

$$\bigcup_{u \in a \circ b} u \circ c = \bigcup_{v \in b \circ c} a \circ v.$$

A hypergroupoid  $(H, \circ)$  is called a quasihypergroup if for all  $a$  of  $H$  we have  $a \circ H = H \circ a = H$ . This condition is also called the reproduction axiom.

**Definition 2.3.** A hypergroupoid  $(H, \circ)$  which is both a semihypergroup and a quasi-hypergroup is called a hypergroup.

**Definition 2.4.** Let  $(H, \circ)$  and  $(H', \circ')$  be two hypergroupoids. A map  $\phi : H \rightarrow H'$ , is called

- (1) an inclusion homomorphism if for all  $x, y$  of  $H$ , we have  $\phi(x \circ y) \subseteq \phi(x) \circ' \phi(y)$ ;
- (2) a good homomorphism if for all  $x, y$  of  $H$ , we have  $\phi(x \circ y) = \phi(x) \circ' \phi(y)$ .

**Definition 2.5.** Let  $H$  be a non-empty set and let  $+$  be a hyperoperation on  $H$ . The couple  $(H, +)$  is called a canonical hypergroup if the following conditions hold:

- (1)  $x + y = y + x$ , for all  $x, y \in H$ ,
- (2)  $x + (y + z) = (x + y) + z$ , for all  $x, y, z \in H$ ,
- (3) there exists a neutral element  $0 \in H$  such that  $x + 0 = \{x\} = 0 + x$ , for all  $x \in H$ ,
- (4) for every  $x \in H$ , there exists a unique element  $-x \in H$  such that  $0 \in x + (-x) \cap (-x) + x$ ,
- (5)  $z \in x + y$  implies  $y \in -x + z$  and  $x \in z - y$ , for all  $x, y, z \in H$ .

**Definition 2.6.** [21]

- (i) A classical operation is an operation well-defined for all the set's elements.
- (ii) A classical hyper-operation is a hyper-operation well-defined for all the set's elements.
- (iii) A neutro operation is an operation partially well-defined or partially indeterminate or partially outer defined on a given set.
- (iv) (Anti) Operation is an operation that is outer defined for all set's elements.
- (v) A classical law/axiom defined on a nonempty set is a law/axiom that is totally true (i.e., true for all set's elements).
- (vi) A NeutroLaw/NeutroAxiom defined on a nonempty set is a law/axiom that is true for some set's element [degree of truth ( $T$ )], indeterminate for other set's elements [degree of indeterminacy ( $I$ )], or false for the other set's elements [degree of falsehood ( $F$ )], where  $T, I, F \in [0, 1]$ , with  $(T, I, F) \neq (1, 0, 0)$  that represents the classical axiom, and  $(T, I, F) \neq (0, 0, 1)$  that represents the AntiAxiom.
- (vii) An AntiLaw/AntiAxiom defined on a nonempty set is a law/axiom that is false for all set's elements.
- (viii) NeutroHyperOperation is a hyper-operation partially well-defined, partially indeterminate, and partially outer-defined on a given set.
- (ix) AntiHyperOperation is a hyper-operation outer-defined for all set's elements.
- (x) A NeutroAlgebra is an algebra that has at least one NeutroOperation or one NeutroAxiom (axiom that is true for some elements, indeterminate for other elements, and false for other elements).
- (xi) An AntiAlgebra is an algebra endowed with at least one AntiOperation or at least one Anti-Axiom.

**Theorem 2.7.** [17] *Let  $\mathbb{U}$  be a nonempty finite or infinite universe of discourse and let  $S$  be a finite or infinite subset of  $\mathbb{U}$ . If  $n$  classical operations (laws and axioms) are defined on  $S$  where  $n \geq 1$ , then there will be  $(2^n - 1)$  NeutroAlgebras and  $(3^n - 2^n)$  AntiAlgebras.*

**Definition 2.8.** [Classical group] Let  $G$  be a nonempty set and let  $*$  :  $G \times G \rightarrow G$  be a binary operation on  $G$ . The couple  $(G, *)$  is called a classical group if the following conditions hold:

- (G1)  $x * y \in G \forall x, y \in G$  [closure law].
- (G2)  $x * (y * z) = (x * y) * z \forall x, y, z \in G$  [axiom of associativity].
- (G3) There exists  $e \in G$  such that  $x * e = e * x = x \forall x \in G$  [axiom of existence of neutral element].
- (G4) There exists  $y \in G$  such that  $x * y = y * x = e \forall x \in G$  [axiom of existence of inverse element] where  $e$  is the neutral element of  $G$ .

If in addition  $\forall x, y \in G$ , we have

- (G5)  $x * y = y * x$ , then  $(G, *)$  is called an abelian group.

**Definition 2.9.** [Neutrosophication of the law and axioms of the classical group] [5]

- (NG1) There exist some duplets  $(x, y), (u, v), (p, q), \in G$  such that  $x * y \in G$  (inner-defined with degree of truth T) and  $[u * v = \text{indeterminate (with degree of indeterminacy I) or } p * q \notin G \text{ (outer-defined/falsehood with degree of falsehood F)}]$  [NeutroClosureLaw].
- (NG2) There exist some triplets  $(x, y, z), (p, q, r), (u, v, w) \in G$  such that  $x * (y * z) = (x * y) * z$  (inner-defined with degree of truth T) and  $[[p * (q * r)] \text{ or } [(p * q) * r] = \text{indeterminate (with degree of indeterminacy I) or } u * (v * w) \neq (u * v) * w \text{ (outer-defined/falsehood with degree of falsehood F)}]$  [NeutroAxiom of associativity (NeutroAssociativity)].
- (NG3) There exists an element  $e \in G$  such that  $x * e = e * x = x$  (inner-defined with degree of truth T) and  $[[x * e] \text{ or } [e * x] = \text{indeterminate (with degree of indeterminacy I) or } x * e \neq x \neq e * x \text{ (outer-defined/falsehood with degree of falsehood F)}]$  for at least one  $x \in G$  [NeutroAxiom of existence of neutral element (NeutroNeutralElement)].
- (NG4) There exists an element  $u \in G$  such that  $x * u = u * x = e$  (inner-defined with degree of truth T) and  $[[x * u] \text{ or } [u * x] = \text{indeterminate (with degree of indeterminacy I) or } x * u \neq e \neq u * x \text{ (outer-defined/falsehood with degree of falsehood F)}]$  for at least one  $x \in G$  [NeutroAxiom of existence of inverse element (NeutroInverseElement)] where  $e$  is a NeutroNeutralElement in  $G$ .
- (NG5) There exist some duplets  $(x, y), (u, v), (p, q) \in G$  such that  $x * y = y * x$  (inner-defined with degree of truth T) and  $[[u * v] \text{ or } [v * u] = \text{indeterminate (with degree of indeterminacy I) or } p * q \neq q * p \text{ (outer-defined/falsehood with degree of falsehood F)}]$  [NeutroAxiom of commutativity (NeutroCommutativity)].

**Definition 2.10.** [AntiSophication of the law and axioms of the classical group] [5]

- (AG1) For all the duplets  $(x, y) \in G, x * y \notin G$  [AntiClosureLaw].
- (AG2) For all the triplets  $(x, y, z) \in G, x * (y * z) \neq (x * y) * z$  [AntiAxiom of associativity (AntiAssociativity)].
- (AG3) There does not exist an element  $e \in G$  such that  $x * e = e * x = x \forall x \in G$  [AntiAxiom of existence of neutral element (AntiNeutralElement)].
- (AG4) There does not exist  $u \in G$  such that  $x * u = u * x = e \forall x \in G$  [AntiAxiom of existence of inverse element (AntiInverseElement)] where  $e$  is an AntiNeutralElement in  $G$ .
- (AG5) For all the duplets  $(x, y) \in G, x * y \neq y * x$  [AntiAxiom of commutativity (AntiCommutativity)].

**Definition 2.11.** [5] A NeutroGroup  $NG$  is an alternative to the classical group  $G$  that has at least one NeutroLaw or at least one of  $\{NG1, NG2, NG3, NG4\}$  with no AntiLaw or AntiAxiom.

**Definition 2.12.** [5] An AntiGroup  $AG$  is an alternative to the classical group  $G$  that has at least one AntiLaw or at least one of  $\{AG1, AG2, AG3, AG4\}$ .

**Definition 2.13.** [5] A NeutroAbelianGroup  $NG$  is an alternative to the classical abelian group  $G$  that has at least one NeutroLaw or at least one of  $\{NG1, NG2, NG3, NG4\}$  and  $NG5$  with no AntiLaw or AntiAxiom.

**Definition 2.14.** [5] An AntiAbelianGroup  $AG$  is an alternative to the classical abelian group  $G$  that has at least one AntiLaw or at least one of  $\{AG1, AG2, AG3, AG4\}$  and  $AG5$ .

**Proposition 2.15.** [5] Let  $(G, *)$  be a finite or infinite classical non abelian group. Then:

- (i) there are 15 types of NeutroNonAbelianGroups,
- (ii) there are 65 types of AntiNonAbelianGroups.

**Proposition 2.16.** [5] Let  $(G, *)$  be a finite or infinite classical abelian group. Then:

- (i) there are 31 types of NeutroAbelianGroups,
- (ii) there are 211 types of AntiAbelianGroups.

**Definition 2.17.** [5] Let  $(NG, *)$  be a NeutroGroup. A nonempty subset  $NH$  of  $NG$  is called a NeutroSubgroup of  $NG$  if  $(NH, *)$  is also a NeutroGroup of the same type as  $NG$ . If  $(NH, *)$  is a NeutroGroup of a type different from that of  $NG$ , then  $NH$  will be called a QuasiNeutroSubgroup of  $NG$ .

**Example 2.18.** [5]

- (i) Let  $NG = \mathbb{N} = \{1, 2, 3, 4 \dots\}$ . Then  $(NG, \cdot)$  is a finite NeutroGroup where  $\cdot$  is the binary operation of ordinary multiplication.
- (ii) Let  $AG = \mathbb{Q}_+^*$  be the set of all irrational positive numbers. Then  $(AG, *)$  is an infinite AntiGroup.
- (iii) Let  $\mathbb{U} = \{a, b, c, d, e, f\}$  be a universe of discourse and let  $AG = \{a, b, c\}$  be a subset of  $\mathbb{U}$ . Let  $*$  be a binary operation defined on  $AG$  as shown in the Cayley table below:

*	a	b	c
a	d	c	b
b	c	e	a
c	b	a	f

Then  $(AG, *)$  is a finite AntiGroup.

### 3. Formulation of a NeutroHyperGroup

**Definition 3.1.** [Classical Hypergroup]

Let  $H$  be a non-empty set and  $\circ : H \times H \rightarrow P^*(H)$  be a hyperoperation. Then  $(H, \circ)$  is a hypergroup if the following conditions hold:

- (H1) for all  $x, y \in H$ ,  $x \circ y \subseteq H$  (closure law),
- (H2) for all  $x, y, z \in H$ ,  $(x \circ y) \circ z = x \circ (y \circ z)$  (associative axiom),

(H3) for all  $x \in H$ ,  $x \circ H = H \circ x = H$  (reproductive axiom).

**Definition 3.2.** [Neutrosophication of the law and axioms of the classical hypergroup]

(NH1) There exist some duplets  $(u, v), (x, y), (p, q) \in H$  such that  $u \circ v \subseteq H$  (inner-defined with the degree of truth T) and  $[x \circ y = \text{indeterminate (with the degree of indeterminacy I) or } p \circ q \not\subseteq H$  (outer-defined/falsehood with degree of falsehood F)].

(NH2) There exist some triplets  $(u, v, w), (x, y, z), (p, q, r) \in H$  such that  $u \circ (v \circ w) = (u \circ v) \circ w$  (inner-defined with the degree of truth T) and  $[x \circ (y \circ z) \text{ or } (x \circ y) \circ z = \text{indeterminate (with the degree of indeterminacy I) or } (p \circ q) \circ r \neq p \circ (q \circ r)$  (outer-defined/falsehood with degree of falsehood F)].

(NH3) There exists at least a triplet  $(u, v, x) \in H$  such that  $u \circ H = H \circ u = H$  (inner-defined with the degree of truth T) and  $[v \circ H \text{ or } H \circ v = \text{indeterminate (with the degree of indeterminacy I) or } x \circ H \neq H \neq H \circ x$  (outer-defined/falsehood with degree of falsehood F)].

**Definition 3.3.** [AntiSophication of the law and axioms of the classical hypergroup ]

(AH1)  $u \circ v \not\subseteq H \forall u, v \in H$  (anti closure law).

(AH2)  $u \circ (v \circ w) \neq (u \circ v) \circ w \forall u, v, w \in H$  (anti associative axiom)

(AH3)  $x \circ H \neq H$  and  $H \circ x \neq H \forall x \in H$  (anti reproductive axiom).

**Definition 3.4.** A NeutroHyperGroup  $(NH, \circ)$  is an alternative to the classical hypergroup  $(H, \circ)$  that has a NeutroLaw or at least one of NH2 and NH3 with no Antilaw or AntiAxiom.

**Definition 3.5.** An AntiHyperGroup  $(AH, \circ)$  is an alternative to the classical hypergroup  $(H, \circ)$  that has an AntiLaw or at least one of AH2 and AH3.

**Theorem 3.6.** Let  $(H, \circ)$  be a classical hypergroup. Then,

- (1) there are 7 classes of NeutroHyperGroup.
- (2) there are 19 classes of AntiHyperGroup.

*Proof.* The proof follows easily from Theorem 2.7 .  $\square$

Theorem 3.6 shows that there are 7 classes of NeutroHypergroups. The classes where NH1 – NH3 hold are called the trivial NeutroHyperGroups. Examples of NeutroHyperGroups in this class are presented below.

**Example 3.7.** Let  $V = \{u, v, w, s, t, z\}$  be a universe of discourse and let  $NH = \{v, w, s, z\}$  be a subset of  $V$ . Define on  $NH$  the binary Operation  $\circ$  as shown in the table below.

It can easily be deduced from the table that  $(NH, \circ)$  is a trivial NeutroGroup. The subset  $NK = \{v, s\}$  of  $NH$  is also a trivial NeutroGroup and hence a NeutroSubgroup of  $NH$ .

$\circ$	$v$	$w$	$s$	$z$
$v$	$s$	$t$	$z$	$v$
$w$	$z$	$u$	$v$	$s$
$s$	$v$	$w$	$s$	$z$
$z$	$z$	$u$	$v$	$s$

Now, consider the NeutroSubgroup  $NK = \{v, s\}$ . Defined on  $NK$  a hyperoperation  $\star_{NH}$  as follows

:

$$x \star_{NK} y = \begin{cases} x \circ NK \circ y = \{x \circ z \circ y : z \in NK\} \text{ if } x = y, \\ \{x, x \circ y\} \text{ if } x \neq y. \end{cases}$$

From this definition we construct the table below:

TABLE 1. Cayley table for the hyperoperation " $\star_{NK}$ "

$\star_{NK}$	$v$	$w$	$s$	$z$
$v$	$\{v, z\}$	$\{t, v\}$	$\{v, z\}$	$\{v\}$
$w$	$\{w, z\}$	$\{t, u\}$	$\{v, w\}$	$\{s, w\}$
$s$	$\{s, v\}$	$\{s, w\}$	$\{s, z\}$	$\{s, z\}$
$z$	$\{z\}$	$\{u, z\}$	$\{v, z\}$	$\{s, v\}$

It can be seen from Table 1 that  $\star_{NK}$  satisfies :

- (1) NeutroClosureLaw (NH1) : Except for the composition

$v \star_{NK} w = \{t, v\}, w \star_{NK} w = \{t, u\}$  and  $z \star_{NK} w = \{u, z\}$  which are false with 18.75% degree of falsehood, all other composition are true with 81.25% degree of truth.

- (2) NeutroAssociative (NH2) :

$$s \star_{NK} (v \star_{NK} v) = (s \star_{NK} v) \star_{NK} v = \{s, v, z\}.$$

$$s \star_{NK} (w \star_{NK} v) = \{s, w, z\} \text{ but } (s \star_{NK} w) \star_{NK} v = \{s, v, w, z\} \neq \{s, w, z\}.$$

- (3) NeutroReproductionAxiom (NH3) :

$$s \star_{NK} NH = NH \star_{NK} s = \{v, w, s, z\} = NH.$$

$$w \star_{NK} NH = NH \star_{NK} w = \{u, v, w, s, t, z\} \neq NH.$$

Hence,  $(NH, \star_{NK})$  is a trivial NeutroHyperGroup.

**Example 3.8.** Let  $V = \{u, v, w, s, t, z\}$  be a universe of discourse and let  $NH = \{u, v, w, z\}$  be a subset of  $V$ . Define on  $NH$  the binary operation  $\circ$  as shown in the table below.

It can be shown from the table that  $(NH, \circ)$  is a NeutroGroup and the subset  $NK = \{u, v\}$  of  $NH$  is a classical group with respect to  $\circ$ .



$\circ$	$u$	$v$	$s$	$z$
$u$	$u$	$v$	$s$	$z$
$v$	$v$	$u$	$t$	$w$
$s$	$s$	$w$	$u$	$t$
$z$	$z$	$t$	$w$	$u$

Now, defined on  $NH$  a hyperoperation  $\star_{NK}$  as follows :

$$x \star_{NK} y = \begin{cases} x \circ NK \circ y = \{x \circ z \circ y : z \in NK\} & \text{if } x \neq u, y \neq u \text{ and } x \neq y, \\ x \circ y & \text{if } x = y, \\ \{x, y\} & \text{otherwise.} \end{cases}$$

Note, if  $x \circ y$  is indeterminate, we write  $x \circ y = I$ .

From this definition we construct the table below:

TABLE 2. Cayley table for the hyperoperation " $\star_{NK}$ "

$\star_{NK}$	$u$	$v$	$s$	$z$
$u$	$u$	$\{u, v\}$	$\{u, s\}$	$\{u, z\}$
$v$	$\{u, v\}$	$u$	$\{s, t\}$	$\{w, z\}$
$s$	$\{u, s\}$	$\{w, s\}$	$u$	$\{v, t\}$
$z$	$\{u, z\}$	$\{t, I\}$	$\{w, I\}$	$u$

It can be seen from the table that  $\star_{NK}$  satisfies :

- (1) NeutroClosureLaw ( $NH1$ ) : Except for the compositions

$v \star_{NK} s = \{s, t\}, v \star_{NH} z = \{w, z\}, s \star_{NK} v = \{w, s\}$  and  $s \star_{NH} z = \{v, t\}$  which are false with 25.0% degree of falsehood, and the compositions  $z \star_{NK} v = \{t, I\}$  and  $z \star_{NK} s = \{w, I\}$  which are indeterminate with 12.5% degree of indeterminacy all other compositions are true with 62.5% degree of truth.

- (2) NeutroAssociative ( $NH2$ ) :

$$u \star_{NK} (v \star_{NK} u) = (u \star_{NK} v) \star_{NK} u = \{u, v\}.$$

$$s \star_{NK} (u \star_{NK} v) = \{u, w, s\} \text{ but } (s \star_{NK} u) \star_{NK} v = \{u, v, w, s\} \neq \{u, w, s\}.$$

- (3) NeutroReproductionAxiom ( $NH3$ ) :

$$u \star_{NK} NH = NH \star_{NK} s = \{u, v, s, z\} = NH.$$

$$z \star_{NK} NH = \{u, w, t, z, I\} \neq NH \text{ and } NH \star_{NK} z = \{u, v, w, t, z\} \neq NH.$$

Hence,  $(NH, \star_{NK})$  is a trivial NeutroHyperGroup.

#### 4. Study of a Class of NeutroHyperGroup

In this section, we are going to consider a particular class of NeutroHyperGroups  $(NH, \star)$  where

- (i)  $(NH, \star)$  is a classical hypergroupoid,
- (ii) the hypergroupoid  $(NH, \star)$  is a NeutroSemiHyperGroup and
- (iii) the hypergroupoid  $(NH, \star)$  is a NeutroQuasiHyperGroup.

We will refer to this class of NeutroHyperGroups as [2,3]-NeutroHyperGroup (i.e., H2 and H3 of Definition 3.1 are NeutroAxioms).

**Example 4.1.** Let  $NH = \{u, v, s, t\}$  be a non empty set and let  $\cdot$  be a binary operations defined on  $NH$  as shown in the table below.

$\cdot$	$u$	$v$	$s$	$t$
$u$	$v$	$t$	$s$	$u$
$v$	$v$	$u$	$t$	$u$
$s$	$s$	$t$	$v$	$u$
$t$	$u$	$u$	$u$	$u$

Now consider the subset  $NK = \{u, v\}$ . Defined on  $NH$  a hyperoperation  $\star_{NK}$  as follows :

$$x \star_{NK} y = \begin{cases} x \cdot NK \cdot y = \{x \cdot z \cdot y : z \in NK\} & \text{if } x \neq y \text{ and } x, y \neq u, \\ x \cdot y & \text{otherwise.} \end{cases}$$

From this definition we construct the table below.

TABLE 3. Cayley table for the hyperoperation  $\star_{NK}$

$\star_{NK}$	$u$	$v$	$s$	$t$
$u$	$v$	$t$	$s$	$u$
$v$	$v$	$\{u, t\}$	$\{s, t\}$	$u$
$s$	$s$	$\{u, t\}$	$\{u, v\}$	$u$
$t$	$u$	$t$	$s$	$u$

It can be seen from Table 3 that :

- (1)  $(NH, \star_{NK})$  is a hypergroupoid.
- (2)  $\star_{NK}$  is NeutroAssociative, since

$$(t \star_{NK} v) \star_{NK} t = t \star_{NK} (v \star_{NK} t) = \{u\}.$$

$$(v \star_{NK} s) \star_{NK} t = \{u\} \text{ but } v \star_{NK} (s \star_{NK} t) = \{v\} \neq \{u\}.$$

Hence, the hypergroupoid  $(NH, \star_{NK})$  is NeutroSemiHyperGroup.

(3)  $\star_{NK}$  satisfies NeutroReproductiveAxiom, since

$$s \star_{NK} NH = NH \star_{NK} s = \{u, v, s, t\} = NG.$$

$$v \star_{NK} NH = \{u, v, s, t\} \neq \{u, t\} = NH \star_{NK} v.$$

Hence, the hypergroupoid  $(NH, \star_{NK})$  is a NeutroQuasiHyperGroup.

Accordingly,  $(NH, \star_{NK})$  is a  $[2, 3]$ -NeutroHyperGroup .

**Example 4.2.** Let  $NH = \{\alpha, \beta, \gamma, \phi, \psi\}$  and let  $\star$  be a hyperoperation defined on  $NH$  as shown in the table below ;

TABLE 4. Cayley table for the hyperoperation " $\star$ "

$\star$	$\alpha$	$\beta$	$\gamma$	$\phi$	$\psi$
$\alpha$	$\alpha$	$\alpha$	$\alpha$	$\alpha$	$\alpha$
$\beta$	$\alpha$	$\{\gamma, \phi\}$	$\{\phi, \psi\}$	$\{\beta, \gamma\}$	$\{\alpha, \psi\}$
$\gamma$	$\alpha$	$\{\alpha, \gamma\}$	$\{\alpha, \gamma\}$	$\gamma$	$\{\alpha, \gamma\}$
$\phi$	$\alpha$	$\{\alpha, \phi\}$	$\phi$	$\{\alpha, \phi\}$	$\{\alpha, \phi\}$
$\psi$	$\alpha$	$\{\gamma, \phi\}$	$\{\beta, \phi\}$	$\{\gamma, \psi\}$	$\{\alpha, \beta\}$

Then,  $(NH, \star)$  is a  $[2, 3]$ -NeutroHyperGroup.

It can be seen from Table 4 that :

- (1)  $(NH, \star)$  is a hypergroupoid.
- (2)  $\star$  is NeutroAssociative, since

$$(\phi \star \beta) \star \psi = \phi \star (\beta \star \psi) = \{\alpha, \phi\}.$$

$$(\beta \star \phi) \star \gamma = \{\alpha, \gamma, \phi, \psi\} \text{ but } \beta \star (\phi \star \gamma) = \{\beta, \gamma\} \neq \{\alpha, \gamma, \phi, \psi\}.$$

Hence, the hypergroupoid  $(NH, \star)$  is NeutroSemiHyperGroup.

- (3)  $\star$  satisfies NeutroReproductiveAxiom, since

$$\psi \star NH = NH \star \psi = \{\alpha, \beta, \gamma, \phi, \psi\} = NH.$$

$$\phi \star NH = \{\alpha, \phi\} \neq \{\alpha, \beta, \gamma, \phi, \psi\} = NH \star \phi.$$

Hence, the hypergroupoid  $(NH, \star)$  is a NeutroQuasiHyperGroup.

Accordingly,  $(NH, \star)$  is a  $[2, 3]$ -NeutroHyperGroup.

**Example 4.3.** Let  $NH = \{m, n, p, q\}$  and let  $\circ$  be a hyperoperation defined on  $NH$  as shown in table 5 below.

Then,  $(NH, \circ)$  is a  $[2, 3]$ -NeutroHyperGroup. It can be seen from the Table 5 that :

- (1)  $(NH, \circ)$  is a Hypergroupoid.

TABLE 5. Cayley table for the hyperoperation "o"

o	m	n	p	q
m	m	{m, n}	{m, p}	{m, q}
n	{m, n}	p	n	{n, p}
p	p	{p, q}	p	{p, q}
q	q	{m, q}	q	{m, q}

(2) o is NeutroAssociative, since

$$(m \circ m) \circ n = m \circ (m \circ n) = \{m, n\}.$$

$$(m \circ n) \circ q = \{m, n, p, q\} \text{ but } m \circ (n \circ q) = \{m, n, p\} \neq \{m, n, p, q\}.$$

Hence, the hypergroupoid (NH, o) is NeutroSemiHyperGroup.

(3) o satisfies NeutroReproductiveAxiom, since

$$m \circ NH = NH \circ m = \{m, n, p, q\} = NH.$$

$$p \circ NH = \{p, q\} \neq \{m, n, p, q\} = NH \circ p.$$

Hence, the hypergroupoid (NH, o) is a NeutroQuasiHyperGroup.

Accordingly, (NH, o) is a [2, 3]-NeutroHyperGroup.

**Example 4.4.** Let  $NH = \{1, 2, 3, 4, 5, 6\}$  and let  $\star$  be a hyperoperation defined on  $NH$  as shown in the table below ;

TABLE 6. Cayley table for the hyperoperation "star"

star	1	2	3	4	5	6
1	1	1	1	1	1	1
2	1	{1, 2}	2	{2, 4}	{1, 2}	{2, 4}
3	1	3	{1, 3}	3	{1, 3}	{1, 3}
4	1	{1, 4}	4	{2, 4}	{1, 4}	{2, 4}
5	1	{3, 5}	{2, 5}	{5, 6}	{1, 2, 3, 5}	{2, 4, 5, 6}
6	1	{3, 6}	{4, 6}	{5, 6}	{1, 3, 4, 6}	{2, 4, 5, 6}

It can be shown from Table 6 that, (NH, star) is a [2, 3]-NeutroHyperGroup.

**Proposition 4.5.** Let  $(NH_1, \star_1)$  and  $(NH_2, \star_2)$  be any two [2, 3]-NeutroHyperGroups. Let

$$NH_1 \times NH_2 = \{(v, k) : v \in NH_1 \text{ and } k \in NH_2\},$$

for  $x = (v_1, k_1), y = (v_2, k_2) \in NH_1 \times NH_2$  define :

$$x \star y = ((v_1 \star_1 v_2), (k_1 \star_2 k_2)).$$

Then  $(NH_1 \times NH_2, \star)$  is a [2, 3]-NeutroHyperGroup.

*Proof.* (1) Let  $x = (v_1, k_1), y = (v_2, k_2) \in NH_1 \times NH_2$ , then

$$x \star y = (v_1, k_1) \star (v_2, k_2) = (v_1 \star_1 v_2, k_1 \star_2 k_2) \subseteq NH_1 \times NH_2. \quad \because v_1 \star_1 v_2 \subseteq NH_1 \text{ and } k_1 \star_2 k_2 \subseteq NH_2.$$

Hence,  $(NH_1 \times NH_2, \star)$  is a hypergroupoid.

(2) There exists at least a triplet  $((v_1, k_1), (v_2, k_2), (v_3, k_3)) \in NH_1 \times NH_2$  such that

$$\begin{aligned} ((v_1, k_1) \star (v_2, k_2)) \star (v_3, k_3) &= ((v_1 \star_1 v_2), (k_1 \star_2 k_2)) \star (v_3, k_3) \\ &= ((v_1 \star_1 v_2) \star_1 v_3, (k_1 \star_2 k_2) \star_2 k_3) \\ &= (v_1 \star_1 (v_2 \star_1 v_3), k_1 \star_2 (k_2 \star_2 k_3)) \quad \because NH2 \text{ holds in } NH_1 \text{ and } NH_2. \\ &= (v_1, k_1) \star ((v_2 \star_1 v_3), (k_2 \star_2 k_3)) \\ &= (v_1, k_1) \star ((v_2, k_2) \star (v_3, k_3)). \end{aligned}$$

Also, there exists at least a triplet  $((a_1, b_1), (a_2, b_2), (a_3, b_3)) \in NH_1 \times NH_2$  such that

$$\begin{aligned} ((a_1, b_1) \star (a_2, b_2)) \star (a_3, b_3) &= ((a_1 \star_1 a_2), (b_1 \star_2 b_2)) \star (a_3, b_3) \\ &= ((a_1 \star_1 a_2) \star_1 a_3, (b_1 \star_2 b_2) \star_2 b_3) \\ &\neq (a_1 \star_1 (a_2 \star_1 a_3), b_1 \star_2 (b_2 \star_2 b_3)) \quad \because NH2 \text{ holds in } NH_1 \text{ and } NH_2. \\ &= (a_1, b_1) \star ((a_2 \star_1 a_3), (b_2 \star_2 b_3)) \\ &= (a_1, b_1) \star ((a_2, b_2) \star (a_3, b_3)). \end{aligned}$$

Hence,  $NH2$  holds in  $NH_1 \times NH_2$ .

(3) There exists at least a  $(v, k) \in NH_1 \times NH_2$  with  $v \in NH_1$  and  $k \in NH_2$  such that

$$\begin{aligned} (v, k) \star NH_1 \times NH_2 &= (v, k) \star \{(v_1, k_1) : v_1 \in NH_1, k_1 \in NH_2\} \\ &= \{(v, k) \star (v_1, k_1) : v_1 \in NH_1, k_1 \in NH_2\} \\ &= \{((v \star_1 v_1), (k \star_2 k_1)) : v_1 \in NH_1, k_1 \in NH_2\} \\ &= (v \star_1 NH_1, k \star_2 NH_2) \\ &= (NH_1 \star_1 v, NH_2 \star_2 k) \quad (\because NH3 \text{ holds in } NH_1 \text{ and } NH_2) \\ &= NH_1 \times NH_2 \star (v, k) \\ &= NH_1 \times NH_2. \end{aligned}$$

Also, there exists at least a  $(u, q) \in NH_1 \times NH_2$  with  $u \in NH_1$  and  $q \in NH_2$  such that

$$\begin{aligned} (u, q) \star NH_1 \times NH_2 &= (u, q) \star \{(v_1, k_1) : v_1 \in NH_1, k_1 \in NH_2\} \\ &= \{(u, q) \star (v_1, k_1) : v_1 \in NH_1, k_1 \in NH_2\} \\ &= \{((u \star_1 v_1), (q \star_2 k_1)) : v_1 \in NH_1, k_1 \in NH_2\} \\ &= (u \star_1 NH_1, q \star_2 NH_2) \\ &\neq (NH_1 \star_1 u, NH_2 \star_2 q) \quad (\because NH3 \text{ holds in } NH_1 \text{ and } NH_2) \\ &= NH_1 \times NH_2 \star (u, q). \end{aligned}$$

Accordingly,  $(NH_1 \times NH_2, \star)$  is a  $[2, 3]$ -NeutroHyperGroup.  $\square$

**Proposition 4.6.** Let  $(NV, \star_1)$  be a  $[2, 3]$ -NeutroHyperGroup and  $(H, \star_2)$  be any hypergroup. Let

$$NV \times H = \{(v, h) : v \in NV \text{ and } h \in H\},$$

for  $x = (v_1, h_1), y = (v_2, h_2) \in NV \times H$  define :

$$x \star y = ((v_1 \star_1 v_2), (h_1 \star_2 h_2)).$$

Then  $(NV \times H, \star)$  is a  $[2, 3]$ -NeutroHyperGroup.

*Proof.* The proof follows similar approach as the proof of 4.5 .  $\square$

**Definition 4.7.** Let  $NH$  be a  $[2, 3]$ –NeuroHyperGroup, a non-empty subset  $NK$  of  $NH$  is called a  $[2, 3]$ –NeuroSubHyperGroup of  $NH$  if  $NK$  is itself a  $[2, 3]$ –NeuroHyperGroup.

**Example 4.8.** Let  $(NH, \circ)$  be the  $[2, 3]$ – NeuroHyperGroup defined in Example 4.3 and let  $NK = \{m, p, q\}$  be a subset of  $NH$ . Let  $\circ$  be defined as shown in the table below.

TABLE 7. Cayley table for the hyperoperation " $\circ$ "

$\circ$	$m$	$p$	$q$
$m$	$m$	$\{m, p\}$	$\{m, q\}$
$p$	$p$	$p$	$\{p, q\}$
$q$	$q$	$q$	$\{m, q\}$

It can be shown from Table 7 that  $(NK, \circ)$  is a  $[2, 3]$ –NeuroHyperGroup. Then,  $(NK, \circ)$  is a  $[2, 3]$ –NeuroSubHyperGroup of  $NH$ .

**Example 4.9.** Let  $(NH, \circ)$  be the  $[2, 3]$ – NeuroHyperGroup defined in Example 4.4 and let  $NK = \{1, 2, 3, 4\}$  and  $NW = \{1, 2, 3, 5\}$  be subsets of  $NH$ .

Let  $\star$  be defined as shown in the tables below :

TABLE 8. Cayley table for the hyperoperation " $\star$ "

$\star$	1	2	3	4
1	1	1	1	1
2	1	$\{1, 2\}$	2	$\{2, 4\}$
3	1	3	$\{1, 3\}$	3
4	1	$\{1, 4\}$	4	$\{2, 4\}$

TABLE 9. Cayley table for the hyperoperation " $\star$ "

$\star$	1	2	3	5
1	1	1	1	1
2	1	$\{1, 2\}$	2	$\{1, 2\}$
3	1	3	$\{1, 3\}$	$\{1, 3\}$
5	1	$\{3, 5\}$	$\{2, 5\}$	$\{1, 2, 3, 5\}$

We can see from Tables 8 and 9 that  $(NK, \star)$  and  $(NW, \star)$  are  $[2, 3]$ –NeuroHyperGroups. Then,  $(NK, \star)$  and  $(NW, \star)$  are  $[2, 3]$ – NeuroSubHyperGroups of  $NH$ .

Now, consider the following:

- (1)  $NK \cup NW = \{1, 2, 3, 4, 5\}$ .
- (2)  $NK \cap NW = \{1, 2, 3\}$ .

It can be shown from Table 6 that  $NK \cup NW$  is a  $[2, 3]$ – NeuroSubHyperGroup of  $NR$  but  $NK \cap NW$  is a non-trivial NeuroSemiHyperGroup of  $NH$ .

These observations are recorded in Remark 4.10

**Remark 4.10.** Let  $NK$  and  $NW$  be two  $[2, 3]$ –NeuroSubHyperGroups of a  $[2, 3]$ – NeuroHyperGroup  $NH$ . Then

- (1)  $NK \cup NW$  can be a  $[2, 3]$ - NeutroSubHyperGroup of  $NH$ .
- (2)  $NK \cap NW$  is not necessarily a  $[2, 3]$ - NeutroSubHyperGroup of  $NH$ .

**Proposition 4.11.** *Let  $NH$  be a  $[2, 3]$ - NeutroHyperGroup and let  $NW$  be a  $[2, 3]$ - NeutroSubHyperGroup of  $NH$ . For  $aNW, bNW \in NH/NW$  with  $a, b \in NH$ , let  $\star$  be a hyperoperation defined on  $NH/NW$  by*

$$aNW \star bNW = \{cd \mid c \in aNW, d \in bNW\}.$$

*Then,  $(NH/NW, \star)$  is a  $[2, 3]$ - NeutroHyperGroup which is known as a  $[2, 3]$ - NeutroQuotientHyperGroup.*

*Proof.* The proof of this Proposition will be by a constructed example as given in Example 4.12.  $\square$

**Example 4.12.** Let  $(NH, \circ)$  be the  $[2, 3]$ - NeutroHyperGroup defined in Example 4.4 and let  $NW$  be the  $[2, 3]$ -NeutroSubHyperGroup of Example 4.9 . Then we have

$$NH/NW = \{NW, p \circ NW, n \circ NW, q \circ NW\}.$$

Define on  $NH/NW$  a hyperoperation  $\star$  as shown in the table below

TABLE 10. Cayley table for the hyperoperation " $\star$ "

$\star$	$NW$	$pNW$	$nNW$	$qNW$
$NW$	$\{NW, pNW, qNW\}$	$\{NW, pNW, qNW\}$	$\{NW, pNW, nNW\}$	$\{NW, pNW, qNW\}$
$pNW$	$\{pNW, qNW\}$	$\{pNW, qNW\}$	$\{pNW, qNW\}$	$\{pNW, qNW\}$
$nNW$	$\{NW, pNW, nNW\}$	$\{pNW, nNW\}$	$\{NW, nNW\}$	$\{pNW, qNW, nNW\}$
$qNW$	$\{NW, qNW\}$	$\{NW, qNW\}$	$\{nNW, qNW\}$	$qNW$

Then, it can be seen from the table that :

- (1)  $(NH/NW, \star)$  is a hypergroupoid.
- (2) there exists at least a triplet  $(pNW, nNW, qNW) \in NH/NW$  such that

$$pNW \star (nNW \star qNW) = (pNW \star nNW) \star qNW = \{pNW, qNW\}.$$

And, there exists at least a triplet  $(qNW, nNW, qNW) \in NH/NW$  such that

$$(qNW \star nNW) \star qNW = \{pNW, nNW, qNW\} \neq \{NW, nNW, qNW\} = qNW \star (nNW \star qNW).$$

- (3) there exists  $nNW \in NH/NW$  such that

$$nNW \star NH/NW = NH/NW \star nNW = NH/NW.$$

And, there exists  $pNW \in NH/NW$  such that

$$pNW \star NH/NW = \{pNW, qNW\} \neq \{NW, pNW, nNW, qNW\} = NH/NW \star pNW.$$

Accordingly,  $(NH/NW, \star)$  is a  $[2, 3]$ - NeutroHyperGroup.

**Definition 4.13.** Let  $(NH, \star)$  and  $(NW, \circ)$  be any two  $[2, 3]$ - NeutroHyperGroups . The mapping

$$\phi : NH \longrightarrow NW$$

- (1) is called a NeutroHyperGroupHomomorphism if  $\phi(a \star b) \subseteq \phi(a) \circ \phi(b)$  for at least a duplet  $(x, y) \in NH$ .
- (2) is called a good NeutroHyperGroupHomomorphism if  $\phi(a \star b) = \phi(a) \circ \phi(b)$  for at least a duplet  $(x, y) \in NH$ .
- (3) is called NeutroHyperGroupIsomorphism if  $\phi$  is a NeutroHyperGroupHomomorphism and  $\phi^{-1}$  is also a NeutroHyperGroupHomomorphism.

**Definition 4.14.** Let  $(NH, \star)$  and  $(NW, \star)$  be any two  $[2, 3]$ - NeutroHyperGroups with NeutroNeutralElements  $e_{NH}$  and  $e_{NW}$  respectively.

Let  $\phi : NH \longrightarrow NW$  be a good NeutroHyperGroupHomomorphism.

The kernel of  $\phi$  denoted by  $NHKer\phi$  is defined as

$$NHKer\phi = \{x : \phi(x) = e_{NW}\}.$$

The image of  $\phi$  denoted by  $NHIm\phi$  is defined as

$$NHIm\phi = \{y \in NW : y = \phi(x) \text{ for at least one } x \in NH\}.$$

**Example 4.15.** Let  $(NK, \circ)$  be the  $[2, 3]$ - NeutroHyperGroup of Example 4.8 and let

$$\phi : NK \times NK \longrightarrow P^*(NK)$$

be given by  $\phi(k_1, k_2) = k_1 \circ k_2$  for all  $k_1, k_2 \in NK$ .

Then  $\phi$  is a good NeutroHyperGroupHomomorphism.

We have  $NHKer\phi = \{(m, m), (p, m), (p, p), (q, m), (q, p)\}$  and

$NHIm\phi = \{m, p, q, \{m, p\}, \{m, q\}, \{p, q\}\}$ .

It can be shown that  $NHKer\phi$  is a  $[2, 3]$ - NeutroSubHyperGroup of  $(NK \times NK, \circ)$  and  $NHIm\phi$  is a  $[2, 3]$ - NeutroSubHyperGroup of  $(P^*(NK), \circ)$ .

**Proposition 4.16.** Let  $(NH, \star)$  and  $(NW, \star)$  be any two  $[2, 3]$ -NeutroHypergroups.

Let  $\phi : NH \longrightarrow NW$  be a good NeutroHyperGroupHomomorphism. Then :

- (1)  $NHKer\phi$  is a NeutroSubHyperGroup of  $NH$ .
- (2)  $NHIm\phi$  is a NeutroSubHyperGroup of  $NW$ .

*Proof.* The proof follows from Example 4.15 .  $\square$



5. Applications of [2, 3]– NeutroHyperGroups in Biological and Chemical Sciences

In [10], Davvaz et al. provided some examples of hyperstructures and weak hyperstructures associated with dismutation reactions. In what follows, we will provide examples to show that when dismutation reactions take place in the neutrosophic environment, they are associated with [2, 3]– NeutroHyperGroups.

**Example 5.1.** Let  $NX = \{x_0 = Sn, x_2 = Sn^{2+}, x_4 = Sn^{4+}\}$  be a set of Tin in different oxidation state. Define on  $NX$ , a hyperoperation  $\star$  as shown in the table below, where  $\star$  is a comproportionation reaction (without energy). Then, it can be seen from Table 11 that :

TABLE 11. Cayley table for the hyperoperation " $\star$ "

$\star$	$x_0$	$x_2$	$x_4$
$x_0$	$x_0$	$\{x_0, x_2\}$	$x_2$
$x_2$	$\{x_0, x_2\}$	$x_2$	$\{x_2, x_4\}$
$x_4$	$x_2$	$\{x_2, x_4\}$	$x_4$

- (1)  $(NX, \star)$  is a hypergroupoid.
- (2) For the triplet  $(x_2, x_4, x_2) \in NX$ , we have

$$x_2 \star (x_4 \star x_2) = (x_2 \star x_4) \star x_2 = \{x_2, x_4\}$$

and for the triplet  $(x_0, x_2, x_4) \in NX$ , we have have

$$(x_0 \star x_2) \star x_4 = \{x_2, x_4\} \neq \{x_0, x_2\} = x_0 \star (x_2 \star x_4).$$

- (3) For  $x_2 \in NX$ , we have

$$x_2 \star NX = NX \star x_2 = \{x_0, x_2, x_4\}$$

and for  $x_4 \in NX$ , we have

$$x_4 \star NX = NX \star x_4 = \{x_2, x_4\} \neq NX.$$

Accordingly,  $(NX, \star)$  is a [2, 3]– NeutroHyperGroup.

**Example 5.2.** Let  $BG = \{a_0 = AA, a_1 = AS, a_3 = SS\}$  be a set of blood group. Define on  $BG$ , a hyperoperation  $\star$  as shown in the table below, where  $\star$  denote mating.

Then, it can be seen from table 12 that :

- (1)  $(BG, \star)$  is a hypergroupoid.

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A Comproportionation is a chemical reaction where two reactants each containing the same element but with a different oxidation number, will give a product with oxidation number intermediate of the two reactant.

TABLE 12. Cayley table for the hyperoperation " $\star$ "

$\star$	$a_0$	$a_1$	$a_3$
$a_0$	$a_0$	$\{a_0, a_1\}$	$a_1$
$a_1$	$\{a_0, a_1\}$	$\{a_0, a_1, a_3\}$	$\{a_1, a_3\}$
$a_3$	$a_1$	$\{a_1, a_3\}$	$a_3$

(2) For the triplet  $(a_1, a_3, a_1) \in BG$ , we obtain

$$a_1 \star (a_3 \star a_1) = (a_1 \star a_3) \star a_1 = \{a_0, a_1, a_3\}.$$

and for the triplet  $(a_0, a_1, a_3) \in BG$ , we obtain

$$(a_0 \star a_1) \star a_3 = \{a_1, a_3\} \neq \{a_0, a_1\} = a_0 \star (a_1 \star a_3).$$

(3) For an element  $a_1 \in BG$ , we obtain

$$a_1 \star BG = BG \star a_1 = \{a_0, a_1, a_3\} = BG$$

and for an an element  $a_3 \in BG$ , we obtain

$$a_3 \star BG = BG \star a_3 = \{a_1, a_3\} \neq BG.$$

Accordingly,  $(BG, \star)$  is a  $[2, 3]$ -NeutroHyperGroup.

**Remark 5.3.** It is evident from Examples 5.1 and 5.2 that in the neutrosophic environment,  $[2, 3]$ -NeutroHyperGroups are associated with dismutation reactions in some chemical reactions and biological processes.

## 6. Conclusions

In this paper, we have for the first time introduced the concept of NeutroHyperGroups. Specifically, a class of NeutroHyperGroups called  $[2, 3]$ -NeutroHyperGroup was investigated and some of their elementary properties and several examples were presented. It was shown that the intersection of two  $[2, 3]$ -NeutroSubHyperGroups is not necessarily a  $[2, 3]$ -NeutroSubHyperGroup but their union may produce a  $[2, 3]$ -NeutroSubHyperGroup. Also, it was shown that the quotient of a  $[2, 3]$ -NeutroHyperGroup factored by a  $[2, 3]$ -NeutroSubHyperGroup is a  $[2, 3]$ -NeutroHyperGroup. Examples were provided to show that in the neutrosophic environment,  $[2, 3]$ -NeutroHyperGroups are associated with dismutation reactions in some chemical reactions and biological processes. In our future work, we hope to use the algebraic properties of NeutroHyperGroups to analyze some chemical reactions and biological processes.

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