A short history of fuzzy, intuitionistic fuzzy, neutrosophic and plithogenic sets

A. Rezaei

T. Oner

T. Katikan

Florentin Smarandache

N. Gandotra

A short history of fuzzy, intuitionistic fuzzy, neutrosophic and plithogenic sets

Article · January 2022

Akbar Rezaei
Payame Noor University

Tahsin Oner
Ege University

Tugce Katican
Izmir University of Economics

Florentin Smarandache
University of New Mexico Gallup

A short history of fuzzy, intuitionistic fuzzy, neutrosophic and plithogenic sets

[1]A. Rezaei *, [2]T. Oner, [3]T. Katican, [4]F. Smarandache and [5]N. Gandotra
[1]Department of Mathematics, Payame Noor University, P.O.Box. 19395-4697, Tehran, Iran.
[2,3]Department of Mathematics, Ege University, 35100 Izmir, Turkey.
[4]Department of Mathematics and Science, University of New Mexico, Gallup, UNM 87301, USA.
[5]Yogananda School of AI, Comput. Data Sci., Shoolini University, Solan 173229, Himachal Pradesh, India.
*Corresponding author: rezaei@pnu.ac.ir[1], tahsin.oner@ege.edu.tr[2], tugcektcn@gmail.com[3], smarand@unm.edu[4], neerajgandotra@shooliniuniversity.com[5]

Abstract

Recently, research on uncertainty modeling is progressing rapidly and many essential and breakthrough studies have already been done. There are various ways such as fuzzy, intuitionistic and neutrosophic sets to handle these uncertainties. Although these concepts can handle incomplete information in various real-world issues, they cannot address all types of uncertainty such as indeterminate and inconsistent information. Also, plithogenic sets as a generalization of crisp, fuzzy, intuitionistic fuzzy, and neutrosophic sets, which is a set whose elements are characterized by many attributes' values. In this paper, our aim is to demonstrate and review the history of fuzzy, intuitionistic and neutrosophic sets. For this purpose, we divided the paper as: section 1. History of Fuzzy Sets, section 2. History of Intuitionistic Fuzzy Sets and section 3. History of Neutrosophic Theories and Applications, section 4. History of Plithogenic Sets.

1 History of Fuzzy Sets

In the twentieth century, the problem of representing vagueness in logic, in physics, in linguistics and the questioning of the notion of set caused many first suggestions for the fuzzy set theory. The fuzzy set theory is not a unusual, arbitrary object that abruptly come out of nowhere, yet it clarified the intuitions of some spearheading scientists in the century. The philosophers Charles Peirce[109] and Bertrand Russell[113] thought that vagueness is neglected in mathematical information. There were discussions on the relationships between logic and vagueness in the philosophical literature ([49] [76]). It is stated that notions in natural language do not have a apparent class of their properties but they have enlargeable boundaries.[145] To determine vague symbols, the American philosopher Max Black propounded consistency profiles which are the ancestors of fuzzy membership functions.[39] In 1940, H. Weyl gave a generalization of the usual characteristic function by substituting it with a continuous characteristic function.[144] Then Kaplan and Schott proposed a similar generalization of characteristic functions of vague predicates and the fundamental fuzzy set connectives in these works.[82] In 1951, Karl Menger used the term "ensemble flou" which is the French counterpart of fuzzy set for the first-time.[99]

Although the term "fuzzy set" is often worst defined and misunderstood, it is fashinable in scientific areas and daily life. The doctrine of fuzzyiness has been properly defined by M. M. Gupta as "a body of concepts and techniques aimed at providing a systematic framework for dealing with the vagueness and imprecision inherent in human thought processes".[73] Thus, the three basic keywords, the three pillards of this doctrine which form its philosophical basis, are following: thinking, vagueness, and imprecision.[58] Since the activity of thinking is not basically correlated with linguistic expressions,[63] it designates to operations or constructions in the mind, which are in charge of analysis and logical tests. On the other hand, the whole doctrine of fuzziness is founded on vagueness and imprecision, and so, it is handled as properties of language and knowledge. For centuries, scientist have started different enterprises in order to explain these phenomena. Various logical calculus were constructed to the aim of dealing with different aspects of language and information. The fuzzy theory is also one of them. Two different kinds of theories of perception and thought exist.[115] Some suppose

Doi: https://doi.org/10.54216/IJNS.180109
Received Aug. 30, 2021 Accepted: Jan 16, 2022

that the content of perception is never identical with the perceived entity. However, others assert that this content is identical with the perceived object, and provided that perception is veridical. Identity theories were strongly criticized by B. Russell,[113] who clearly distinguished the properties of words from the properties of things. In this famous paper, he writes that vagueness and precision are characteristics which can only belong to a language and that "apart from representation there can be no such thing as vagueness or precision". He means that vagueness is a characteristic of words, not of things.[58] The aim of fuzzy set theory is to built a formal framework for unfinished and gradual knowledge in natural language. The concepts of ambiguity and imprecision of information can be variously understood by researchers as vagueness, uncertainty, etc. When these concepts are regarded as related for representational systems, everyone agree that language is a typical example.

In the twentieth century, Zadeh introduced the degree of membership/truth (T) in 1965 and defined the fuzzy set.[148] In contrast to classical binary logic, this aims to express thoughts and notions which do not mathematically explain in natural language such as uncertainty and randomness. Thus, Zadeh preferred classes of objects consisting of relative notions in natural language terms such as age, size, height, temperature. However, classical binary logic is not appropriate to analyze such classes where it appears that membership is a gradual notion rather than an all-or-nothing matter. The fuzzy theory summarizes significantly perceptive phenomenas that expound the complexity of the world. Analytical representations of physical phenomena can model natural language but they are sometimes difficult to understand since they have nebulous explanations and is not be obvious for the non-specialist people. Mental representations has uncertainty and vagueness, the lack of specificity of linguistic terms and the lack of well-defined boundaries of the class of preferred objects. In natural language, objects are represented by proper names, properties are expressed by adjectives and nouns, whereas prepositions and verbs tend to express relations between two or more things.[114] In artificial languages, for example, in a language based on first order logic, objects are represented by singular names, whereas properties and relations are represented by predicates.[58]

For years, producing of the devices simulated human thoughts and behaviors is many scientists' dream. Since we have entered the era of information management as soon as the emergence of computers, this dream have started to realize. The developments of science and technology needing detailed knowledge to produce of the devices human thoughts and behaviors and easy attainableness to computers helps to realize the dream. An important problem is to stock and make use of the information in various areas. Because the fuzzy theory attend to this tendency,[56] it has close connection with artificial intelligence.

When Zadeh propounded fuzzy sets, his concerns concentrated on their potential contribution in the fields of model classification,[38] processing and transportation of knowledge, abstraction and summarization.[149] Despite the claims that fuzzy sets were related in these fields emerged in the early sixties, the developments of sciences, technologies and engineering showed that these intuitions were true.

The terms "depressed" and "old" are fuzzy concepts since they cannot be sharply defined. However, people use it to decide. The fuzzy concepts is absolutely contrast to the terms as married, under 25 years old or over 2.5 feet tall. Pure mathematics is interested in classes of objects such as the subset of rational numbers in the set of real numbers. However, when the subset of young people in a given set of people, it may be impossible to decide whether a person is in that subset or not. It is possible to answer it but there can be losing of information when it does. Even though this case has existed from time immemorial, the dominant context is the situation in which statements are exact.

In this day and age, quickly developing technology and the dream of producing devices that mimic human reasoning human thoughts and behaviors which is based on uncertain and imprecise knowledge has drawn many scientists' attention. The theory and applications of fuzzy concepts is largely in the fieds of engineering and applied sciences. The widely usage of fuzzy concepts in technology and science are witnessed by means of the developments of automatic control and expert systems. Although the mathematical elements that form the basis of fuzzy notions have existed for a long time, the rising of its applications has satisfied a motivation for mathematics. Until the rising of fuzzy set theory as a crucial tool in practical applications, there did not exist a force majeure to analyse its mathematics. By virtue of the practical significance of such improvements, studying the mathematical basis of this theory has become important.

In the literature of fuzzy sets, the word "fuzzy" means the word "vague". According to the Oxford English Dictionary, the word "fuzzy" has some various meanings. Unlike any other meaning, "fuzzy" can be used to form a predicate of the form "something is fuzzy". For instance, it can sound normal to say that "a fox is fuzzy" but sound weirdo to say that "old is fuzzy". In this case, the adjective "old" is vague (but not fuzzy in the material sense) since its sense is not set by certain boundaries. However, the word "fuzzy" is applied to predicates and needs to state the gradual nature of some of these words that appear as vague. Additionally, "vagueness" specifies mostly a much larger kind of tabulation for words including ambiguity. Vagueness is interested the meaning of signs in a language and is treated as a certain type of ambiguity. If any notion

Doi: https://doi.org/10.54216/IJNS.180109
Received Aug. 30, 2021 Accepted: Jan 16, 2022

is not sharply defined, then it is called vague. Since there exists a graceful degradation between situations to which a vague concept fully applies and situations to which it does not never apply, there exist a whole range of situations to which a vague concept fractionally apply.[58] This is called "membership gradience" by Lakoff.[90] It is keynoted that the fuzziness is an main property of the notion of vagueness which is different from uncertainty. Many researchers think that full applicability of a vague notion to a particular situation may be dubious.[112] If a vague concept applies or not to a particular situation, then such doubt can only emerge in the case which is forced to decide. If a proposition involves gradual predicates, then it is called vague. A characteristic property of these propositions is that they can be neither true nor false when they applied to describe a given situation.[49] In a word, they can be stated as a degree. Symbolic forms of these degrees of truth exist in natural languages, for example, "very", "rather", "almost no", "less", etc. If a person of the class of "depressive people" characterizes "less depressive", then it is said that the truth level of the statement "this person is depressive" is "less" and this person has a low degree of membership to the class of "depressive people". These linguistic restrictions apply only to gradual predicates. For example, "married" is not a gradual predicate since the term "less married" does not exist.

There exist two reasons that there are fuzziness and ambiguity in all languages. A first reason is that any language is discrete and the external world seems to be continuous. According to Aristotle, this represents the extensity of ambiguity in languages. Specially, a certain number of words import ostensible continuous numerical scales. For example, let the word "old" apply to humans. In order to fix an age threshold below which old fully applies and it does not at all is difficult. There exists a conflict between the linguistic representation of the age scale (for example, {old, teen, young, mature}) and its numerical representation (for instance, the real interval [0, 120] years for humans). The range of functions which Zadeh[148] calls membership functions, can involve as many elements as in the age scale and continuous itself. Terms such as old, young, teen, etc. is called fuzzy predicates. A second reason is that natural language overlooks the existence of exceptions. For instance, some of snakes are venomous but other is not since some don't have venom. Thus, the collection of snakes is not just a set, and also it is a set partially ordered by a relation of compared typicality. Then the membership function is only a partially arbitrary encoding of the partial ordering relation.

Imprecision is also characteristic of language and relates to measurable concepts. These concepts are represented by numbers. This imprecision originates from the fact that any measurement results bounded truth. Then it is possible to mention many correspondences between the results of measurement and real numbers. Chwistek suggested the usage of intervals instead of single numbers[47] and Mellor improved in various direction by Mellor ([97 98]). Imprecision seems in the form of disjunctions in logic. For instance, a proposition "p or q is true" contains imprecision since it is not known whether p is true, q is true, or both. Hence, imprecision is represented by sets as a disjunction (not conjunction) of elements. R. Young attempts to improve a formal theory allowing manipulations with indefinite quantities.[147] This leading work is called interval mathematics. R. Young substituted a variable which supposes severally considered values by a notion of many-valuedness. This notion was stated as a set of values but it considered collectively. S. Lesniewski studied widely on these sets ([91 92]).

Despite imprecision and vagueness mention the contents of a part of knowledge in a language, uncertainty states the ability of an agent to claim whether a proposition holds or not. Uncertainty of Boolean propositions can be gradual as in probability theory where partial doctrine ranges on the unit interval while uncertainty is three-valued in propositional logic, i.e., a proposition is known, its negation is known or both is not known. Even though stating and reasoning these three cases explicitly requires the features of modal logic,[78] the modal possibility has the lack of certainty and certainty reflects logical consistency. However, uncertainty modeled by probability is different from uncertainty modeled by propositional logic. Probability theory often models uncertainty irritating from conflicting, exactly observed, part of knowledge. This situation seems in statistics where a random experiment runs several times an it does not produce the same outcomes. Probability logic is a tool for making inferences from data. Also, Probability logic is used in cases where propositions are either true or false, but the knowledge is imprecise. If the knowledge is fuzzy, then a model of gradual uncertainty is produced since this uncertainty stems from a lack of knowledge. Hence, fuzzy sets causes gradual uncertainty which is different from the one of frequentest probability. This is possibility theory ([58 59]).

In complex real world cases, many types of uncertainty exist. For example, "morality" or "beauty" of each population of humans chosen at random can be researched and so on. These examples are fuzzy concepts that is accurately formulated by fuzzy sets. Each type of uncertainty has its mathematical model. Different mathematical theories are like tools which are advantageous to use than another in a given situation. Some of these theories may be used individually or in conjunction. However, the right combination of mathematical theories should be chosen.

This discussion points out that uncertainty differs from imprecision and vagueness and only result from them. In order to see better the differences between the three notions discussed here, let us consider the

Doi: https://doi.org/10.54216/IJNS.180109

Received Aug. 30, 2021 Accepted: Jan 16, 2022

following assertions about some car:

- This car is between 10 and 15 years old (pure imprecision).
- This car is very big (imprecision and vagueness).
- This car was probably made in Germany (uncertainty).

In the first case there is a lack of knowledge, due to a lack of ability to measure or to evaluate numerical features. In the second case there is a lack of precise definition of the notion "big" and the modifier "very" indicates a rough degree of "bigness", and the third case expresses uncertainty about a well-defined proposition, perhaps based on statistics.[58]

The originality of fuzzy sets is to catch the idea of partial membership. The membership function of a fuzzy set is a function whose range is an ordered membership set containing more that two values. Thus, a fuzzy set can be thought as a function. Also, the fuzzy set theory are to handle functions as if they were subsets of their domains because such functions are used to state gradual classes. Therefore, the concepts such as intersection, union, complement, inclusion are enlarged in order to combine functions ranging on an ordered membership set. In fuzzy set theory, the union, intersection, inclusion and complementation of functions are committed by taking their point wise maximum, minimum, inequality between functions, and by means of order-reversing automorphism of the membership scale, respectively. Because degrees of membership may be thought as degrees of truth, intersection as conjunction, union as disjunction, complementation as negation and set-inclusion as implication, it seems easily that the fuzzy set theory is closely related to many-valued logics. Moreover, many mathematical notions are extended to fuzzy sets. It is defined special types of fuzzy sets such as convex fuzzy sets, fuzzy numbers, fuzzy intervals and fuzzy relations, and some types of non-classical sets that are different from fuzzy sets.

Gottwald[71] and Ostasiewicz ([106 107]) defined the early developments of fuzzy sets in detailed. Also, Ostasiewicz gave the basic philosophical parts of fuzzy set theory.[105] The first published book on fuzzy sets wrote by Kaufmann in French (1973) and translated into English in 1975.[83] Up to now. Then a mathematical dissertation is followed by Negoita and Ralescu.[100] Dubois and Prade ([60 61]), Kandel,[80] Novak,[102] Yager et al.[146] and Klir and Bo Yuan ([84 85]) are fundamental books on fuzzy sets. Kruse et al. (1994) focused on probabilistic foundations.[89] Gottwald,[72] Lowen[96] and Nguyen and Walker[101] studied on a mathematical structure of fuzzy sets. Zimmerman,[150] Terano, Asai and Sugeno,[141] Klir and Folger,[86] Klir et al[87] and Pedrycz and Gomide[108] investigate methodological problems and applications. Also, the anthology edited by Dubois, Prade and Yager is seen for some basic fuzzy papers of the first twenty five years and other books.[57]

2 History of Intuitionistic Fuzzy Sets

The set theory based on the concept of element membership to sets has proved itself to be one of the most powerful tools of Modern Mathematics and it has allowed to model and to improve other sciences. However, the concept of element membership to a set is a bivalent concept, useded by the values 0 (there is no membership) and 1 (there is membership) and it does not approve other set possibilities in logic.

In today's word, science, technology, biology, social sciences, linguistic, psychology and economics consists of complicated operations and situations for which complete information does not always exist. For these situations, mathematical models are improved to treat various types of systems containing elements of uncertainty. Some of these models are based fuzzy sets. The fuzzy set (FS, briefly) theory introduced by Zadeh regards membership degree and the non-membership degree is the complement of the membership degree. However, this linguistic complement does not provide the logical negation. Since a membership function can be Gaussian, triangular, exponential or any other membership function, and so, there exist a hesitation when a membership function is defined, non-membership degree is less than or equal to the complement of the membership degree. Because different results are obtained with different membership functions.

In 1985, Atanassov introduced the degree of non-membership (T) and falsehood (F) (or non-membership) and defined the intuitionistic fuzzy set (IFS, for short). An intuitionistic fuzzy set may be thought as an alternative approach to describe a fuzzy sets on is not sufficient for the definition of an incomplete notion by means of an ordinary fuzzy set. In the intuitionistic fuzzy set theory, the non-membership degree is not equal to the complement of the membership degree owing to the fact that a lack of information exists when the membership function is defined. In contrast to fuzzy sets, a intuitionistic fuzzy set has two uncertainties called membership and non-membership degrees. Actually, the idea of intuitionistic fuzziness was a coincidence as a mathematical game. Atanassov obtained extension of the ordinary fuzzy set by adding a second degree (degree of non-membership). Since he did not immediately notice properties of intuitionistic fuzzy sets that are different from properties of fuzzy sets, his first studies in this area followed step by step the existing results in fuzzy sets. Some notions of fuzzy sets are not very difficult to extend formally notions of intuitionistic fuzzy

Doi: https://doi.org/10.54216/IJNS.180109
Received Aug. 30, 2021 Accepted: Jan 16, 2022

sets. However, it is interesting to show that the relevant extension has certain properties which do not exist in the basic concept. Since the method of fuzzification has the idea of intuitionism, the name "intuitionistic fuzzy set" (IFS) is suggested by George Gargov who is Atanassov's former lecturer in the Mathematical Faculty of Sofia University ([6 77]). The intuitionistic fuzzy set is a successful generalization of the fuzzy set. The fuzzy set has already achieved a great success in theoretical researches and practical applications.[94] Therefore, it is expected that the intuitionistic fuzzy set could be used to simulate human decision-making processes and any activities requiring human expertise, experience, and knowledge, which are inevitably imprecise or not totally reliable. Thus, it is completely believed that the intuitionistic fuzzy set has a wide prospect of applications to the fields such as management, economics, business and environment as well as military.[95]

The first example for intuitionistic fuzzy sets is given as follow:[6]

This is a story with Johnny and Mary, characters in many Bulgarian anecdotes. They bought a box of chocolates with 10 pieces inside. Being more nimble, Johnny ate seven of them, while Mary– only two.

One of the candies fell into the floor.

In this moment, a girl friend of Mary came and Mary said, *"We can't treat you with chocolates, because Johnny ate them all"*.

Let us estimate the truth value of this statement at the moment of speaking, i.e., before we have any knowledge of subsequent events.

From classical logic point of view, which uses for estimations the members of the set $\{0, 1\}$, the statement has truth value of 0, since Mary has also taken part in eating the candies and Johnny was not the only one who has eaten them. On the other hand, we are intuitively convinced that the statement is more true than false. Mary greets her guest – statement's truth value is obviously 0.7. However, at the next moment Johnny can take the fallen candy and place it back into the box of treat the guest, preserving the truth value of Mary's statement at 0.7, and falsity 0.3. But he can always take advantage of the distraction and eat the last candy which would result in truth value of 0.8 and falsity of 0.2. In this sense the statement depends to a great extent on Johnny's actions. Therefore, the apparatus of IFSs gives us the most accurate answer to the question: $[0.7, 0.2]$. The degree of uncertainty now is 0.1 and it corresponds to our ignorance of the boundaries of Johnny's gluttony.

In the beginning of the last century, L. Brouwer introduced the notion of the intuitionism.[41] He proposed to the mathematicians to remove Aristoteles' law of excluded middle. He said

An immediate consequence was that for a mathematical assertion the two cases of truth and falsehood, formerly exclusively admitted, were replaced by the following three:

(1) has been proved to be true;

(2) has been proved to be absurd;

(3) has neither been proved to be true nor to be absurd, nor do we know a finite algorithm leading to the statement either that is true or that is absurd".[143]

Thus, if a proposition A exists, then it can be said that either A is true, or A is false, or that it is not known whether A is true or false. Also, the proposition $A \vee \neg A$ is always valid in the first order logic. It has truth value "true" (or 1) in Boolean algebras bu this proposition can take value smaller than 1 in the ordinary fuzzy logic of Zadeh and in many-valued logics starting with Lukasiewicz. The same is true for intuitionistic fuzzy sets but the situation happen on semantical and prediction' level. the prediction in Brouwer's sense is fuzzified by clarifying the three possibilities. Thus, Gargov proposed the name "intuitionistic fuzzy set".

By now, the relations between the intuitionistic fuzzy set theory and Brouwer's intuitionism don't have been researched in detail.

The intuitionistic fuzzy sets allow the definition of operators which are, in a way, similar to the modal operators. Since these operators reduce to identity, they are meaningless in ordinary fuzzy sets.[7] G. Takeuti and S. Titani just attribute a very different meaning in the notion of an "intuitionistic fuzzy sets".[140] Atanassov's first two communications in Bulgarian[7] and English.[8]

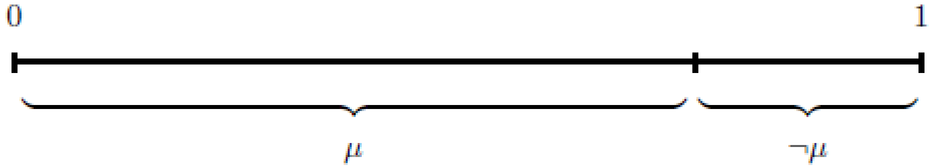
Atanassaov started a discussion on the accuracy of the name of the intuitionistic fuzzy sets.[62] Besides, T. Trifonov and Atanassov built a similar of Takeuti and Titani's research (,[910]). H. Bustince and P. Burillo showed to coincide vague sets with intuitionistic fuzzy sets.[42]

The notion of "interval-valued fuzzy set" (IVFS, shortly) are defined in.[70] Atanassov and Gargov discussed that this notion equals[88] to the intuitionistic fuzzy sets ([11 33]). They showed that each intuitionistic fuzzy set can be stated by an interval-valued fuzzy set and vice versa. Since the intuitionistic fuzzy sets are extensions of the ordinary fuzzy sets, every studies on fuzzy sets may be described in terms of intuitionistic fuzzy sets.

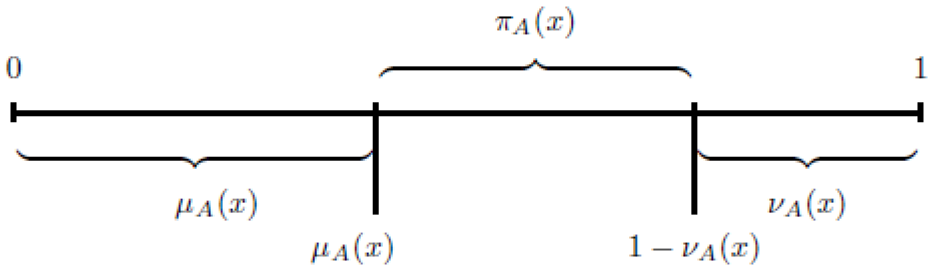
The relations between ordinary fuzzy sets and intuitionistic fuzzy sets are examined from geometrical and probabilistical aspects. Also, some scientists discuss the statement $\mu \vee \neg\mu \leq 1$ in ordinary fuzzy sets as a declaration of the idea of intuitionism. This inequality does not provide the Law of Excluded Middle. However,

Doi: https://doi.org/10.54216/IJNS.180109
Received Aug. 30, 2021 Accepted: Jan 16, 2022

this is not the situation in a geometrical interpretation. Having in mind that in fuzzy set theory $\neg\mu = 1 - \mu$, it is obtained that the geometrical interpretation as follows:



The situation in the intuitionistic fuzzy sets case is different and as below:



Now, the geometrical sums of both degrees can really be smaller than 1, i.e., the Law of Excluded Middle is not valid. From probabilistic point of view, for case of the ordinary fuzzy sets, if $\mu\&\neg\mu = 0$, then the probability $p(\mu \vee \neg\mu) = p(\mu) + p(\neg\mu) = 1$ as in the geometrical case, while in the case of intuitionistic fuzzy sets, the inequality $p(\mu \vee \neg\mu) \leq 1$ follows that for proper elements of intuitionistic fuzzy sets will be strong. All of these constructions are only on the level of the definition of the set (fuzzy set or intuitionistic fuzzy sets), i.e., not related to the possible operations that can be defined over these sets.[6]

The notion of vague sets is defined.[64] It is proved that the notion of vague sets is equivalent to the notion of intuitionistic fuzzy sets.[42]

A non-probabilistic type of entropy measure is offered for intuitionistic fuzzy sets.[134] Li and Cheng examined similarity measures of intuitionistic fuzzy sets and their application to pattern recognitions.[54]

De et al. studied on Sanchez's approach for medical diagnosis, generalized this concept with the notion of intuitionistic fuzzy set theory and described some operations of intuitionistic fuzzy sets ([50 51]).

Some types of fuzzy connectedness in intuitionistic fuzzy topological spaces are defined.[142] Szmidt and Kacprzyk contemplated the usage of intuitionistic fuzzy sets for constructing soft decision-making models with incomplete information and they offered two solutions about the intuitionistic fuzzy core and the concurrence winner for group decision-making by using intuitionistic fuzzy sets ([135 136 137 138]). Besides, they analyzed distances between intuitionistic fuzzy sets are analyzed.[139] A novel and effective approach to deal with decision-making in medical diagnosis using the composition of intuitionistic fuzzy relations was suggested.[55]

Different theorems for building intuitionistic fuzzy relations on a set with preestablished features are given.[46]

Different forms of fuzzy propositional representations and their relationships are investigated.[48]

However, there has been barely existed researches on multicriteria or multiattribute in discrete decision situations and/or group decision-making using intuitionistic fuzzy sets, which is in fact one of the most crucial fields in decision analysis as in real world decision problems contain multiple criteria and a group of decision makers (,[5294]). Multiattribute decision-making using intuitionistic fuzzy sets is researched, a lot of suitable linear programming models are built to produce optimal weights of attributes and the suitable decision-making methods are suggested.[93]

Moreover, various types of intuitionistic fuzzy sets are introduced ([30 31 32]). Two versions of intuitionistic fuzzy propositional calculus, intuitionistic fuzzy predicate calculus, two versions of intuitionistic fuzzy modal logic and temporal intuitionistic fuzzy logic (TIFL, briefly) were defined and investigated in a series of communications ([33 34 35 65]).

Versions of FORTRAN, C, and PASCAL software packages implementing operations, relations and operators over the intuitionistic fuzzy sets are introduced.[29] $V-$fuzzy Petri nets, reduced $V-$fuzzy generalized nets,[128] intuitionistic fuzzy generalized nets of type I[19] and II ([20 21]) and intuitionistic fuzzy programs[22] have

Doi: https://doi.org/10.54216/IJNS.180109
Received Aug. 30, 2021 Accepted: Jan 16, 2022

been investigated. A gravity field of many bodies is examined [428]. Intuitionistic fuzzy models of neural networks is improved on the basis of intuitionistic fuzzy sets. Intuitionistic fuzzy expert systems ([2324 25 110 111]), intuitionistic fuzzy systems,[26] intuitionistic fuzzy PROLOG ([27 28 67]) and intuitionistic fuzzy constraint logic programming ([16 17]) are studied. Some new results on the intuitionistic fuzzy sets theory and its applications were reported to the "Mathematical Foundation of Artificial Intelligence" Seminars held in Sofia in October, 1989 ([18 66 67 74]), March, 1990 ([14 15 30 68 129 133]), June, 1990 ([130 131]), November, 1990 ([13 132]) and October, 1994 ([36 37 43 44 45]). A collection of open questions and problems in the theory of intuitionistic fuzzy sets is presented[13] and discussed.[29]

3 History of Neutrosophic Theories and Applications

The neutrosophic theory that was founded by F. Smarandache in 1998 constitutes a further generalization of fuzzy set, intuitionistic fuzzy set, picture fuzzy set, Pythagorean fuzzy set, spherical fuzzy set, etc. Since then, this logic has been applied in various domains of science and engineering. F. Smarandache introduced the degree of indeterminacy/neutrality (I) as independent component in 1995 (published in 1998) and he defined the neutrosophic set on three components: (T, I, F) = (Truth, Indeterminacy, Falsehood), where in general T, I, F are subsets of the interval $[0, 1]$; in particular T, I, F may be intervals, hesitant sets, or single-values; see
F. Smarandache, Neutrosophy / Neutrosophic probability, set, and logic", Proquest, Michigan, USA, 1998,
https://arxiv.org/ftp/math/papers/0101/0101228.pdf
http://fs.unm.edu/eBook-Neutrosophics6.pdf;
reviewed in Zentralblatt fuer Mathematik (Berlin, Germany):
https://zbmath.org/?q=an:01273000 and cited by Denis Howe in The Free Online Dictionary of Computing, England, 1999.

Neutrosophic Set and Logic are generalizations of classical, fuzzy, and intuitionistic fuzzy set and logic:
https://arxiv.org/ftp/math/papers/0404/0404520.pdf
https://arxiv.org/ftp/math/papers/0303/0303009.pdf

Nonstandard Neutrosophic Logic, Set, Probability (1998, 2019)
https://arxiv.org/ftp/arxiv/papers/1903/1903.04558.pdf

While Neutrosophic Probability and Statistics are generalizations of classical and imprecise probability and statistics.

Etymology.
The words "neutrosophy" and "neutrosophic" were coined/invented by F. Smarandache in his 1998 book.

Neutrosophy: A branch of philosophy, introduced by F. Smarandache in 1980, which studies the origin, nature, and scope of neutralities, as well as their interactions with different idealization spectra. Neutrosophy considers a proposition, theory, event, concept, or entity <A> in relation to its opposite <antiA>, and with their neutral <neutA>. Neutrosophy (as dynamic of opposites and their neutrals) is an extension of the Dialectics (which is the dynamic of opposites only).

Neutrosophy is the basis of neutrosophic logic, neutrosophic probability, neutrosophic set, and neutrosophic statistics.
https://arxiv.org/ftp/math/papers/0010/0010099.pdf

Neutrosophic Logic is a general framework for unification of many existing logics, such as fuzzy logic (especially intuitionistic fuzzy logic), paraconsistent logic, intuitionistic logic, etc. The main idea of NL is to characterize each logical statement in a 3D-Neutrosophic Space, where each dimension of the space represents respectively the truth (T), the falsehood (F), and the indeterminacy (I) of the statement under consideration, where T, I, F are standard or non-standard real subsets of $]^{-}0, 1^{+}[$ with not necessarily any connection between them. For software engineering proposals the classical unit interval $[0, 1]$ may be used.

Degrees of Dependence and Independence between Neutrosophic Components T, I, F are independent components, leaving room for incomplete information (when their superior sum < 1), paraconsistent and contradictory information (when the superior sum > 1), or complete information (sum of components = 1).

For software engineering proposals the classical unit interval $[0, 1]$ is used.
For single valued neutrosophic logic, the sum of the components is:
$0 \leq t + i + f \leq 3$ when all three components are independent;
$0 \leq t + i + f \leq 2$ when two components are dependent, while the third one is independent from them;
$0 \leq t + i + f \leq 1$ when all three components are dependent.

Doi: https://doi.org/10.54216/IJNS.180109

Received Aug. 30, 2021 Accepted: Jan 16, 2022

When three or two of the components T, I, F are independent, one leaves room for incomplete information (sum < 1), paraconsistent and contradictory information (sum > 1), or complete information (sum $= 1$).

If all three components T, I, F are dependent, then similarly one leaves room for incomplete information (sum < 1), or complete information (sum $= 1$).

In general, the sum of two components x and y that vary in the unitary interval $[0, 1]$ is: $0 \leq x + y \leq 2 - d^0(x, y)$, where $d^0(x, y)$ is the degree of dependence between x and y, while $d^0(x, y)$ is the degree of independence between x and y.
https://doi.org/10.5281/zenodo.571359
http://fs.unm.edu/NSS/DegreeOfDependenceAndIndependence.pdf

In 2013 Smarandache refined the neutrosophic set to n components:
$(T_1, T_2, \cdots ; I_1, I_2, \cdots ; F_1, F_2, \cdots)$;
https://arxiv.org/ftp/arxiv/papers/1407/1407.1041.pdf
http://fs.unm.edu/n-ValuedNeutrosophicLogic-PiP.pdf

The Most Important Books and Papers in the Advancement of Neutrosophics 1995-1998 – Smarandache generalized the dialectics to neutrosophy; introduced the neutrosophic set/logic/probability/statistics; introduces the single-valued neutrosophic set (pp. 7-8);
https://arxiv.org/ftp/math/papers/0101/0101228.pdf (fourth edition)
http://fs.unm.edu/eBook-Neutrosophics6.pdf (online edition)

2002 – Introduction of corner cases of sets / probabilities / statistics / logics, such as:

-Neutrosophic intuitionistic set (different from intuitionistic fuzzy set), neutrosophic paraconsistent set, neutrosophic faillibilist set, neutrosophic paradoxist set, neutrosophic pseudo-paradoxist set, neutrosophic tautological set, neutrosophic nihilist set, neutrosophic dialetheist set, neutrosophic trivialist set;

-Neutrosophic intuitionistic probability and statistics, neutrosophic paraconsistent probability and statistics, neutrosophic faillibilist probability and statistics, neutrosophic paradoxist probability and statistics, neutrosophic pseudo-paradoxist probability and statistics, neutrosophic tautological probability and statistics, neutrosophic nihilist probability and statistics, neutrosophic dialetheist probability and statistics,neutrosophic trivialist probability and statistics;

-Neutrosophic paradoxist logic (or paradoxism), neutrosophic pseudo-paradoxist logic (or neutrosophic pseudo-paradoxism), neutrosophic tautological logic (or neutrosophic tautologism):
https://arxiv.org/ftp/math/papers/0301/0301340.pdf
http://fs.unm.edu/DefinitionsDerivedFromNeutrosophics.pdf

2003 – Introduction by Kandasamy and Smarandache of Neutrosophic Numbers ($a + bI$, where I = indeterminacy, $I^2 = I$), I-Neutrosophic Algebraic Structures and Neutrosophic Cognitive Maps
https://arxiv.org/ftp/math/papers/0311/0311063.pdf
http://fs.unm.edu/NCMs.pdf

2005 - Introduction of Interval Neutrosophic Set/Logic
https://arxiv.org/pdf/cs/0505014.pdf
http://fs.unm.edu/INSL.pdf

2006 – Introduction of Degree of Dependence and Degree of Independence between the Neutrosophic Components T, I, F
http://fs.unm.edu/eBook-Neutrosophics6.pdf (p. 92)
http://fs.unm.edu/NSS/DegreeOfDependenceAndIndependence.pdf

2007 – The Neutrosophic Set was extended [Smarandache, 2007] to Neutrosophic Overset (when some neutrosophic component is > 1), since he observed that, for example, an employee working overtime deserves a degree of membership > 1, with respect to an employee that only works regular full-time and whose degree of membership $= 1$; and to Neutrosophic Underset (when some neutrosophic component is < 0), since, for example, an employee making more damage than benefit to his company deserves a degree of membership < 0, with respect to an employee that produces benefit to the company and has the degree of membership > 0; and to and to Neutrosophic Offset (when some neutrosophic components are off the interval $[0, 1]$, i.e. some neutrosophic component > 1 and some neutrosophic component < 0).

Then, similarly, the Neutrosophic Logic/Measure/Probability/Statistics etc. were extended to respectively Neutrosophic Over-/Under-/Off- Logic, Measure, Probability, Statistics etc.
https://arxiv.org/ftp/arxiv/papers/1607/1607.00234.pdf
http://fs.unm.edu/NeutrosophicOversetUndersetOffset.pdf
http://fs.unm.edu/SVNeutrosophicOverset-JMI.pdf
http://fs.unm.edu/IV-Neutrosophic-Overset-Underset-Offset.pdf

2007 – Smarandache introduced the Neutrosophic Tripolar Set and Neutrosophic Multipolar Set and consequently the Neutrosophic Tripolar Graph and Neutrosophic Multipolar Graph

Doi: https://doi.org/10.54216/IJNS.180109
Received Aug. 30, 2021 Accepted: Jan 16, 2022

http://fs.unm.edu/eBook-Neutrosophics6.pdf (p. 93)
http://fs.unm.edu/IFS-generalized.pdf

2009 – Introduction of N-norm and N-conorm
https://arxiv.org/ftp/arxiv/papers/0901/0901.1289.pdf
http://fs.unm.edu/N-normN-conorm.pdf

2013 - Development of Neutrosophic Measure and Neutrosophic Probability (chance that an event occurs, indeterminate chance of occurrence, chance that the event does not occur)
https://arxiv.org/ftp/arxiv/papers/1311/1311.7139.pdf
http://fs.unm.edu/NeutrosophicMeasureIntegralProbability.pdf

2013 – Smarandache Refined the Neutrosophic Components (T, I, F) as
$(T_1, T_2, \cdots; I_1, I_2, \cdots; F_1, F_2, \cdots)$
http://fs.unm.edu/n-ValuedNeutrosophicLogic-PiP.pdf

2014 – Introduction of the Law of Included Multiple Middle
$(<A>; <neut1A>, <neut2A>, \cdots, <antiA>)$
http://fs.unm.edu/LawIncludedMultiple-Middle.pdf

2014 - Development of Neutrosophic Statistics (indeterminacy is introduced into classical statistics with respect to the sample/population, or with respect to the individuals that only partially belong to a sample/population)
https://arxiv.org/ftp/arxiv/papers/1406/1406.2000.pdf
http://fs.unm.edu/NeutrosophicStatistics.pdf

2015 - Introduction of Neutrosophic Precalculus and Neutrosophic Calculus
https://arxiv.org/ftp/arxiv/papers/1509/1509.07723.pdf
http://fs.unm.edu/NeutrosophicPrecalculusCalculus.pdf

2015 – Refined Neutrosophic Numbers $(a + b_1I_1 + b_2I_2 + \cdots + b_nI_n)$, where $I_1, I_2, \cdots, I_n$ are subindeterminacies of indeterminacy I;

2015 – (t, i, f)-neutrosophic graphs;

2015 - Thesis-Antithesis-Neutrothesis, and Neutrosynthesis, Neutrosophic Axiomatic System, neutrosophic dynamic systems, symbolic neutrosophic logic, (t, i, f)-Neutrosophic Structures, I-Neutrosophic Structures, Refined Literal Indeterminacy, Quadruple Neutrosophic Algebraic Structures, Multiplication Law of Subindeterminacies:
https://arxiv.org/ftp/arxiv/papers/1512/1512.00047.pdf
http://fs.unm.edu/SymbolicNeutrosophicTheory.pdf

2015 – Introduction of the Subindeterminacies of the form $(I_0)^n = \frac{k}{0}$, for $k \in \{0, 1, 2, \cdots, n-1\}$, into the ring of modulo integers $\mathbb{Z}_n$ - called natural neutrosophic indeterminacies (Vasantha-Smarandache)
http://fs.unm.edu/MODNeutrosophicNumbers.pdf

2015 – Introduction of Neutrosophic Crisp Set and Topology (Salama & Smarandache)
http://fs.unm.edu/NeutrosophicCrispSetTheory.pdf

2016 – Introduction of Neutrosophic Multisets (as generalization of classical multisets)
http://fs.unm.edu/NeutrosophicMultisets.htm

2016 – Introduction of Neutrosophic Triplet Structures and m-valued refined neutrosophic triplet structures [Smarandache - Ali]
http://fs.unm.edu/NeutrosophicTriplets.htm

2016 – Introduction of Neutrosophic Duplet Structures
http://fs.unm.edu/NeutrosophicDuplets.htm

2017 – 2020 - Neutrosophic Score, Accuracy, and Certainty Functions form a total order relationship on the set of (single-valued, interval-valued, and in general subset-valued) neutrosophic triplets (T, I, F); and these functions are used in MCDM (Multi-Criteria Decision Making):
http://fs.unm.edu/NSS/TheScoreAccuracyAndCertainty1.pdf

2017 - In biology Smarandache introduced the Theory of Neutrosophic Evolution: Degrees of Evolution, Indeterminacy or Neutrality, and Involution
http://fs.unm.edu/neutrosophic-evolution-PP-49-13.pdf

2017 - Introduction by F. Smarandache of Plithogeny (as generalization of Dialectics and Neutrosophy), and Plithogenic Set/Logic/Probability/Statistics (as generalization of fuzzy, intuitionistic fuzzy, neutrosophic set/logic/probability/statistics)
https://arxiv.org/ftp/arxiv/papers/1808/1808.03948.pdf
http://fs.unm.edu/Plithogeny.pdf

2018 – Introduction to Neutrosophic Psychology (Neutropsyche, Refined Neutrosophic Memory: conscious, aconscious, unconscious, Neutropsychic Personality, Eros/ Aoristos/ Thanatos, Neutropsychic Crisp

Doi: https://doi.org/10.54216/IJNS.180109
Received Aug. 30, 2021 Accepted: Jan 16, 2022

Personality)
http://fs.unm.edu/NeutropsychicPersonality-ed3.pdf

2019 - Introduction to Neutrosophic Sociology (Neutrosociology) [neutrosophic concept, or (T, I, F)-concept, is a concept that is T
http://fs.unm.edu/Neutrosociology.pdf

2019 & 2020 – Generalization of the classical Algebraic Structures to NeutroAlgebraic Structures (or NeutroAlgebras) [whose operations and axioms are partially true, partially indeterminate, and partially false] and AntiAlgebraic Structures (or AntiAlgebras) [with operations and axioms totally false]:
http://fs.unm.edu/NeutroAlgebra.htm

Applications in: Artificial Intelligence, Information Systems, Computer Science, Cybernetics, Theory Methods, Mathematical Algebraic Structures, Applied Mathematics, Automation, Control Systems, Big Data, Engineering, Electrical, Electronic, Philosophy, Social Science, Psychology, Biology, Biomedical, Engineering, Medical Informatics, Operational Research, Management Science, Imaging Science, Photographic Technology, Instruments, Instrumentation, Physics, Optics, Economics, Mechanics, Neurosciences, Radiology Nuclear, Medicine, Medical Imaging, Interdisciplinary Applications, Multidisciplinary Sciences etc.

[Xindong Peng and Jingguo Dai, A bibliometric analysis of neutrosophic set: two decades review from 1998 to 2017, Artificial Intelligence Review, Springer, 18 August 2018;
http://fs.unm.edu/BibliometricNeutrosophy.pdf]

Neutrosophic Sets and Systems (NSS) international journal started in 2013 and it is indexed by Scopus, Web of Science (ESCI), DOAJ, Index Copernicus, Redalyc - Universidad Autonoma del Estado de Mexico (IberoAmerica), Publons, CNKI, Google Scholar, Chinese Baidu Scholar etc.
http://fs.unm.edu/NSS/
Submit papers on neutrosophic set/logic/probability/statistics and their applications to the editor-in-chief: smarand@unm.edu.

Encyclopedia of Neutrosophic Researchers

The authors who have published or presented papers on neutrosophics and are not included in the Encyclopedia of Neutrosophic Researchers (ENR), vols. 1, 2, 3, and 4,
http://fs.unm.edu/EncyclopediaNeutrosophicResearchers.pdf
http://fs.unm.edu/EncyclopediaNeutrosophicResearchers2.pdf
http://fs.unm.edu/EncyclopediaNeutrosophicResearchers3.pdf
http://fs.unm.edu/EncyclopediaNeutrosophicResearchers4.pdf
are pleased to send their CV, photo, and List of Neutrosophic Publications to smarand@unm.edu in order to be included into the fifth volume of ENR.

4 History of Plithogenic Sets

In 2018, F. Smarandache introduced the plithogenic set (as generalization of crisp, fuzzy, intuitionistic fuzzy, and neutrosophic sets), which is a set whose elements are characterized by many attributes' values. An attribute value v has a corresponding (fuzzy, intuitionistic fuzzy, or neutrosophic) degree of appurtenance d(x,v) of the element x, to the set P, with respect to some given criteria. In order to obtain a better accuracy for the plithogenic aggregation operators in the plithogenic set, and for a more exact inclusion (partial order), a (fuzzy, intuitionistic fuzzy, or neutrosophic) contradiction (dissimilarity) degree is defined between each attribute value and the dominant (most important) attribute value. The plithogenic intersection and union are linear combinations of the fuzzy operators tnorm and tconorm, while the plithogenic complement, inclusion (inequality), equality are influenced by the attribute values contradiction (dissimilarity) degrees.[123]

Plithogeny is the genesis or origination, creation, formation, development, and evolution of new entities from dynamics and organic fusions of contradictory and/or neutrals and/or non-contradictory multiple old entities. While plithogenic means what is pertaining to plithogeny.

A plithogenic set P is a set whose elements are characterized by one or more attributes, and each attribute may have many values. Each attribute's value v has a corresponding degree of appurtenance d(x,v) of the element x, to the set P, with respect to some given criteria.

In order to obtain a better accuracy for the plithogenic aggregation operators, a contradiction (dissimilarity) degree is defined between each attribute value and the dominant (most important) attribute value.

{However, there are cases when such dominant attribute value may not be taking into consideration or may

Doi: https://doi.org/10.54216/IJNS.180109
Received Aug. 30, 2021 Accepted: Jan 16, 2022

not exist [therefore it is considered zero by default], or there may be many dominant attribute values. In such cases, either the contradiction degree function is suppressed, or another relationship function between attribute values should be established.}

The plithogenic aggregation operators (intersection, union, complement, inclusion, equality) are based on contradiction degrees between attributes' values, and the first two are linear combinations of the fuzzy operators' tnorm and tconorm.

Plithogenic set is a generalization of the crisp set, fuzzy set, intuitionistic fuzzy set, and neutrosophic set, since these four types of sets are characterized by a single attribute value (appurtenance): which has one value (membership) – for the crisp set and fuzzy set, two values (membership, and nonmembership) – for intuitionistic fuzzy set, or three values (membership, nonmembership, and indeterminacy) – for neutrosophic set (see[123]).
http://fs.unm.edu/NSS/PlithogenicSetAnExtensionOfCrisp.pdf

Acknowledgments: The authors wish to sincerely thank the referees for several useful comments.

Conflicts of Interest: "The authors declare no conflict of interest."

References

[1] E. Abo-El-Hamd, H. M. Shamma, M. Saleh, I. El-Khodary, Neutrosophic Logic Theory and Applications, Neutrosophic Sets and Systems, vol. 41, pp. 30–51, 2021.

[2] A.A.A. Agboola, Introduction to NeutroGroups, International Journal of Neutrosophic Science, vol. 6, no. 1, pp. 41–47, 2020.

[3] M. Al-Tahan, B. Davvaz, F. Smarandache and O. Anis, On Some NeutroHyperstructures, Symmetry, 13, 355. https://doi.org/10.3390/sym13040535

[4] M. Al-Tahan, F. Smarandache, B. Davvaz, NeutroOrderedAlgebra: Applications to Semigroups, Neutrosophic Sets and Systems, vol. 39, pp. 133–147, 2021.

[5] K. T. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems, vol.20, pp. 87–96, 1986.

[6] K. T. Atanassov, On intuitionistic fuzzy sets theory, Springer, vol. 283, 2012.

[7] K. T. Atanassov, Intuitionistic fuzzy sets, VII ITKR's Session, Sofia (Deposed in Central Sci. - Techn. Library of Bulg. Acad. of Sci., 1697/84), June 1983, (in Bulg.).

[8] K. T. Atanassov and S. Stoeva, Intuitionistic fuzzy sets, Proc. of Polish Symp. on Interval & Fuzzy Mathematics, Poznan, pp. 23–26, August 1983.

[9] K. T. Atanassov, Answer to Dubois, D., Gottwald, S., Hajek, P., Kacprzyk, J., Prade's, H., paper : Terminological difficulties in fuzzy set theory- the case of "intuitionistic fuzzy sets", Fuzzy Sets and Systems 156(3), 496–499 (2005)

[10] K. T. Atanassov and T. Trifonov, Towards combining two kinds of intuitionistic fuzzy sets, Notes on Intuitionistic Fuzzy Sets, vol. 11, no. 2, pp. 1–11, 2005. http://ifigenia.org/wiki/issue:nifs/11/2/01-11

[11] K. T. Atanassov and G. Gargov, Interval valued intuitionistic fuzzy sets, Fuzzy Sets and Systems, vol. 31, no. 3, pp. 343–349, 1989.

[12] K. T. Atanassov, Review and new results on intuitionistic fuzzy sets, Preprint, IM-MFAIS-1-88, Sofia, 1988.

[13] K. T. Atanassov, On intuitionistic fuzzy sets research in Institute for Microsystems and some open problems in the intuitionistic fuzzy set theory, Ninetieth Session of the Nat. Seminar of Informatics of the Union of Bulg. Mathematicians and Fourth Scientific Session of the "Mathematical Foundation Artificial Intelligence" Seminar, Sofia, Nov. 5, Preprint, IM-MFAIS-5-90, Sofia, pp. 7–14, 1990.

Doi: https://doi.org/10.54216/IJNS.180109
Received Aug. 30, 2021 Accepted: Jan 16, 2022

[14] K. T. Atanassov, Remark on a temporal intuitionistic fuzzy logic, Second Sci. Session of the "Mathematical Foundation of Artificial Intelligence" Seminar, Sofia, March 30, Preprint, IM-MFAIS-I-90, pp. 1–5, 1990.

[15] K. T. Atanassov, G. Gargov and C. Georgiev, Remark on intuitionistic fuzzy Modus Ponens, Second Sci. Session of the "Mathematical Foundation of Artificial Intelligence" Seminar, Sofia, March 30, Preprint, IM-MFAIS-I-90, pp. 10–13, 1990.

[16] K. T. Atanassov, Intuitionistic fuzzy constraint logic programming, Preprint, MRL-I-92, Sofia, 1992.

[17] K. T. Atanassov, Constraint logic programming and intuitionistic fuzzy logics, BUSEFAL, vol. 56, pp. 98–107, 1993.

[18] K. T. Atanassov, One variant of the intuitionistic fuzzy relations, First Sci. Session of the " Mathematical Foundation of Artificial Intelligence" Seminar, Sofia, October 10, Preprint IM-MFAIS-7-89, pp. 1–3, 1989.

[19] K. T. Atanassov, Generalized nets and their fuzzings, AMSE Review, vol. 2, no. 3, pp. 39–49, 1985.

[20] K. T. Atanassov, Second type intuitionistic fuzzy generalized nets, AMSE Review, vol. 17, no.1, 5-8, 1991.

[21] K. T. Atanassov, Algorithms of the functioning of the second type of intuitionistic fuzzy generalized nets, Advances in Modelling & Analysis, A, vol. 32, no. 1-2, pp. 57–61, 1997.

[22] K. T. Atanassov and S. Stoeva, Intuitionistic fuzzy programs, Proc. of the Second Polish Symp. on Interval & Fuzzy Mathematics, Poznan, Sept. pp. 13–16, 1986.

[23] K. T. Atanassov, Intuitionistic fuzzy sets and expert estimations, BUSEFAL, vol. 55, pp. 67–71, 1993.

[24] K. T. Atanassov, Remark on intuitionistic fuzzy expert systems, BUSEFAL, vol. 59, pp. 71–76, 1994.

[25] K. T. Atanassov, Intuitionistic fuzzy sets and expert estimations, II, BUSEFAL, vol. 59, pp. 64–69, 1994.

[26] K. T. Atanassov, Intuitionistic fuzzy systems, BUSEFAL, vol. 58, pp. 92–96, 1994.

[27] K. T. Atanassov and C. Georgiev, Intuitionistic fuzzy Prolog, Fuzzy sets and Systems, vol. 53, no.1, pp. 121–128, 1993.

[28] K. T. Atanassov, H. Georgiev, A. Drumev and I. Kazalarski, Intuitionistic fuzzy PROLOG, Preprint, IM-MFAIS-5-89, Sofia, 1989.

[29] K. T. Atanasov, Intuitionistic fuzzy sets: theory and applications, Physica-Verlag, 1999.

[30] K. T. Atanasov, Intuitionistic fuzzy sets over different universes, Second Sci. Session of the "Mathematical Foundation of Artificial Intelligence" Seminar, Sofia, March 30 1990, Prepr. IM-MFAIS-I-90, pp. 6–9.

[31] K. T. Atanasov, Temporal intuitionistic fuzzy sets, Comptes Rendus de l' Academie Bulgare des Sciences, Tome 44, No. 7, pp. 5–7, 1991.

[32] K. T. Atanasov, A second type of intuitionistic fuzzy sets, BUSEFAL, Vol. 56, pp. 66–70, 1993.

[33] K. T. Atanasov, Two variants of intuitonistic fuzzy propositional calculus, Preprint IM-MFAIS-5-88, Sofia, 1988.

[34] K. T. Atanasov, Two variants of intuitionistic fuzzy modal logic, Preprint IMMFAIS-3-89, Sofia, 1989.

[35] K. T. Atanassov and G. Gargov, Intuitionistic fuzzy logic, Compt. rend. Acad. bulg. Sci., Tome 43, no. 3, pp. 9–12, 1990.

[36] K. T. Atanassov, Index matrix representation of the intuitionistic fuzzy graphs, 5th Sci. Session of the "Mathematical Foundation of Artificial Intelligence" Seminar, Sofia, Oct. 5, Preprint MRL-MFAIS-10-94, Sofia, pp. 36–41, 1994.

Doi: https://doi.org/10.54216/IJNS.180109
Received Aug. 30, 2021 Accepted: Jan 16, 2022

[37] K. T. Atanassov, Remark on the concept intuitionistic fuzzy relation, Fifth Sci. Session of the ”Mathematical Foundation of Artificial Intelligence” Seminar, Sofia, October 5, Preprint, MRL-MFAIS-10-94, Sofia, pp. 42–46, 1994.

[38] R. E. Bellman, R. Kalaba and L. A. Zadeh, Abstraction and pattern classification, J. Math. Anal. Appl., vol. 13, pp. 1–7, 1966.

[39] M. Black, “Vagueness”, [*Phil. of Science*], vol. 4, pp. 427–455. Reprinted in Language and Philosophy: Studies in Method, Cornell University Press, Ithaca and London, pp. 23–58, 1949. Also in Int. J. of General Systems, vol. 17, pp. 107–128, 1990.

[40] A. Borumand Saeid, Y. B. Jun, Neutrosophic sub-algebras of BCK/BCI-algebras based on neutrosophic points, *Annals of Fuzzy Mathematics and Informatics*, vol.14, pp. 87–97, 2017.

[41] L.E.J. Brouwer, Collected Works, vol. 1, North Holland, Amsterdam, 1975.

[42] P. Burillo and H. Bustince, Vague sets are intuitionistic fuzzy sets, Fuzzy Sets and Systems, vol. 79, pp. 403-–405, 1996.

[43] P. Burillo and H. Bustince, Algebraic structures for intuitionistic fuzzy sets, Fifth Sci. Session of the ”Mathematical Foundation of Artificial Intelligence” Seminar, Sofia, October 5, 1994, Preprint, MRL-MFAIS-10-94, Sofia, pp. 1–13, 1994.

[44] P. Burillo and H. Bustince, Numerical measurements of information on intuitionistic fuzzy sets and interval-valued fuzzy sets ($\Phi-$fuzzy), Fifth Sci. Session of the ”Mathematical Foundation of Artificial Intelligence” Seminar, Sofia, October 5, 1994, Preprint MRL-MFAIS-10-94, Sofia, pp. 14–26, 1994.

[45] H. Bustince, Correlation of interval-valued intuitionistic fuzzy sets, Fifth Sci. Session of the ”Mathematical Foundation of Artificial Intelligence” Seminar, Sofia, October 5, 1994, Preprint MRL-MFAIS-I0-94, Sofia, pp. 27–35, 1994.

[46] H. Bustince, Construction of intuitionistic fuzzy relations with predetermined properties, Fuzzy Sets and Systems, vol. 109, pp. 379–403, 2000.

[47] L. Chwistek, The limits of science, Routledge and Kegan Paul, London, 1948.

[48] T. Ciftcibasi and D. Altunay, Two-sided (intuitionistic) fuzzy reasoning, IEEE Trans. Systems Man Cybernet, vol. A-28, pp. 662–677, 1998.

[49] I. M. Copilowish, Border-line cases, vagueness, and ambiguity, Phil. of Science, vol. 6, pp. 181–195, 1939.

[50] S. K. De, R. Biswas and A. R. Roy, Some operations on intuitionistic fuzzy sets, Fuzzy Sets and Systems, vol. 114, pp. 477–484, 2000.

[51] S. K. De, R. Biswas and A. R. Roy, An application of intuitionistic fuzzy sets in medical diagnosis, Fuzzy Sets and Systems, vol. 117, pp. 209–213, 2001.

[52] L. I. Dengfeng, Fuzzy multiattribute decision making models and methods with incomplete information, Fuzzy Sets and Systems, vol. 106, no. 2, pp. 113–119, 1999.

[53] L. I. Dengfeng, Fuzzy multi objective many-person decision-makings and games, National Defense Industry Press, Beijing, 2003.

[54] L. I. Dengfeng and C. Cheng, New similarity measures of intuitionistic fuzzy sets and application to pattern recognitions, Pattern Recognition Lett., vol. 23, no. 1–3, pp. 221–225, 2002.

[55] G. Deschrijver and E. E. Kerre, On the composition of intuitionistic fuzzy relations, Fuzzy sets and Systems, vol. 136, no. 3, pp. 333–361, 2003.

[56] D. Dubois D., H. Prade and R. R. Yager (Eds.), Fuzzy information engineering: a guided tour of applications, John Wiley & Sons, New York, 1997.

[57] D. Dubois D., H. Prade and R. R. Yager (Eds.), Readings in Fuzzy Sets and Intelligent Systems, Morgan Kaufman, San Mateo, CA, 1993.

Doi: https://doi.org/10.54216/IJNS.180109
Received Aug. 30, 2021 Accepted: Jan 16, 2022

[58] D. Dubois, W. Ostasiewicz and H. Prade, Fuzzy sets: history and basic notions, Fundamentals of fuzzy sets, Springer, Boston, MA, pp. 21–124, 2000.

[59] D. Dubois and H. Prade, Possibility theory, 1988.

[60] D. Dubois and H. Prade, Fuzzy sets and systems- theory and applications, Academic Press, New York, 1980.

[61] D. Dubois and H. Prade, New results about properties and semantics of fuzzy-set-theoretic operators, fuzzy sets: theory and applications to policy analysis and information systems, (Wang P.P. and Chang S.K., eds.), Plenum Publ., pp. 59–75, 1980.

[62] D. Dubois, S. Gottwald, P. Hajek, and J. Kacprzyk, H. Prade, Terminological difficulties in fuzzy set theory - the case of "intuitionistic fuzzy sets", Fuzzy Sets and Systems, vol. 156, no. 3, pp. 485–491, 2005.

[63] A. A. Fraenkel, Y. A. Bar-Hillel and A. Levy, Foundations of set theory, North-Holland, 1973.

[64] W. L. Gau and D. J. Buehrer, Vague sets, IEEE Trans. Systems Man Cybernet, vol. 23, pp. 610–614, 1993.

[65] G. Gargov and K. Atanassov, Two results in intuitionistic fuzzy logic, Compt. rend. Acad. bulg. Sci., Tome 45, no. 12, pp. 29–3l, 1992.

[66] G. Gargov, Notes on the intuitionistic fuzzy predicate logic, First Sci. Session of the "Mathematical Foundation of Artificial Intelligence" Seminar, Sofia, October 10, Preprint, IM-MFAIS-7-89, pp. 19–21, 1989.

[67] C. Georgiev and K. T. Atanassov, A possibility for incorporating constraints into intuitionistic fuzzy PROLOG, First Sci. Session of the "Mathematical Foundation of Artificial Intelligence" Seminar, Sofia, October 10, Preprint, IM-MFAIS-7-89, pp. 22–23, 1989.

[68] C. Georgiev, Variant of the combination of evidence in the framework of intuitionistic fuzziness, Second Sci. Session of the "Mathematical Foundation of Artificial Intelligence" Seminar, Sofia, March 30, Prepr. IM-MFAIS-1-90, pp. 22–24, 1990.

[69] K. Georgiev, A simplification of the neutrosophic sets, neutrosophic logic and intuitionistic fuzzy sets, Notes on Intuitionistic Fuzzy Sets, vol. 11, no. 2, pp. 28–31, 2005.

[70] M. Gorzalczany, A method of inference in approximate reasoning based on interval-valued fuzzy sets, Fuzzy Sets and Systems, vol. 21, no. 1, pp. 1–17, 1987.

[71] S. Gottwald, Fuzzy sets theory: some aspects of the early development, aspects of vagueness, (Skala H.J., Termini S. and Trillas E., eds.), D. Reidel, pp. 13–29. 1984.

[72] S. Gottwald, Fuzzy sets and fuzzy logic- foundations of applications from a mathematical point of view, Vieweg, Wiesbaden, Germany, 1993.

[73] M. M. Gupta, Fuzzy-ism, the first decade, Fuzzy Automate and Decision Processes (Gupta M.M., Saridis G.N. and Gaines B.R., eds.), North-Holland, Amsterdam, pp. 5–10, 1977.

[74] L. Hadjyiski and N. Kamburov, Program package for calculations of intuitionistic fuzzy sets, First Sci. Session of the "Mathematical Fundation of Artificial Intelligence" Seminar, Sofia, October 10, Preprint, IM-MFAIS-7-89, pp. 24–25, 1989.

[75] P. Hájek, Metamathematics of fuzzy logic, Springer Science & Business Media, 4, 2013.

[76] C. G. Hempel, Vagueness and logic, Phil. of Science, vol. 6, pp. 163–180, 1939.

[77] A. Heyting, Intuitionism. An Introduction, North-Holland, Amsterdam, 1956.

[78] G. E. Hughes and M. J. Cresswell, An Introduction to modal logic, Methuen, London, 1968.

[79] Y. B. Jun, F. Smarandache, Florentin and H. Bordbar, Neutrosophic $\mathcal{N}-$structures applied to BCK/BCI-algebras, Information, vol. 8, no. 128, pp. 1–12, 2017.

[80] A. Kandel, Fuzzy mathematical techniques with applications, Addison Wesley Publ. Corp., Reading, 1986.

[81] M. Khan, S. Anis, F. Smarandache and Y.B. Jun, Neutrosophic N-structures and their applications in semigroups, 2017, Infinite Study.

[82] A. Kaplan and H. F. Schott, A calculus for empirical classes, Methods, vol. III, pp. 165–188, 1951.

[83] A. Kaufmann, Introduction to the Theory of Fuzzy Subsets, Fundamental Theoretical Elements, Academic Press, 1, New York, 1975.

[84] G. J. Klir and B. Yuan, Fuzzy sets and fuzzy logic- theory and applications, Prentice Hall, Upper Saddle River, NJ, 1995.

[85] G. J. Klir and B. Yuan, Fuzzy sets, Fuzzy logic, and fuzzy systems-selected papers by Lotfi A. Zadeh, World Scientific, Singapore, 1996.

[86] G. J. Klir and T. A. Folger, Fuzzy sets, uncertainty, and information, Prentice Hall, Englewook Cliffs, NJ, 1988.

[87] G. J. Klir, U. H. St Clair and B. Yuan, Fuzzy set theory- foundations and applications, Prentice Hall, Englewook Cliffs, NJ, 1997.

[88] N. Kondakov, Logical Dictionary, Moskow, Nauka, 1971, (in Russian).

[89] R. Kruse, J. Gebhardt and F. Klawonn, Foundations of fuzzy systems, Wiley, New York, 1994.

[90] G. Lakoff, Women, fire, and dangerous things - what categories reveal about the mind, University of Chicago Press, Chicago, 1987.

[91] S. Lesniewski, Podstawy ogolnej teorii mnogosci, Prace Polskiego Kola Naukowego w Moskwie, 2, Moscow, 1916.

[92] S. Lesniewski, Collected works, Polish Scientific Publishers, Warszawa, 1992.

[93] D. F. Li, Multiattribute decision making models and methods using intuitionistic fuzzy sets, Journal of computer and System Sciences, vol. 70, no. 1, pp. 73–85.

[94] D. F. Li, Fuzzy Multi objective Many Person Decision Makings and Games, National Defense Industry Press, Beijing, 2003 (in Chinese).

[95] D. F. Li, Decision and game theory in management with intuitionistic fuzzy sets, vol. 308, pp. 1–441, Berlin, Springer, 2014.

[96] R. Lowen, Fuzzy set theory- basic concepts, techniques and bibliography, Kluwer Academic Publ., Dordrecht, 1996.

[97] D. H. Mellor, Experimental error and deductibility, Phil. of Science, vol. 32, pp. 102–122, 1965.

[98] D. H. Mellor, Inexactness and explanation, Phil. of Science, vol. 33, pp. 345–359, 1966.

[99] K. Menger, Ensembles flous et fonctions aleatoires, Comptes Rendus de l'Académie des Sciences de Paris, vol. 232, pp. 2001–2003, 1951.

[100] C. V. Negoita and D. A. Ralescu, Applications of fuzzy sets to systems analysis, Interdisciplinary Systems Research Series, Birkhaeuser, Basel & Stuttgart and Halsted Press, 11, New York, 1975.

[101] H. T. Nguyen and E. A. Walker, A first course in fuzzy logic, CRC Press, New York, 1996.

[102] V. Novak, Fuzzy sets and their applications, Adam Hilger, Bristol, UK, 1986.

[103] T. Oner, T, Katican and A. Borumand Saeid, Neutrosophic $\mathcal{N}-$structures on Sheffer stroke Hilbert algebras, Neutrosophic Sets and Systems, in press, 2021.

[104] T. Oner, T, Katican and A. Rezaei, Neutrosophic $\mathcal{N}-$structures on strong Sheffer stroke non-associative MV-algebras, Neutrosophic Sets and Systems, vol. 40, pp. 235–252, 2021.

Doi: https://doi.org/10.54216/IJNS.180109
Received Aug. 30, 2021 Accepted: Jan 16, 2022

[105] W. Ostasiewicz, Uncertainty and vagueness, Advances in fuzzy sets and applications, (Tofan I., Gil Aluja J., Costinescu O. and Teodorescu H.N., Eds.), Iasi, 1992.

[106] W. Ostasiewicz, Half a century of fuzzy sets, Supplement to Kybernetika, vol. 28, pp. 17–20, 1992. (Proc. of the Inter. Symp. on Fuzzy Approach to Reasoning and Decision Making, Bechyne, Czechoslovakia, June 25-29, 1990).

[107] W. Ostasiewicz, Pioneers of fuzziness, Busefal, vol. 46, pp. 4–15, 1991.

[108] W. Pedrycz and F. Gomide, An introduction to fuzzy sets- analysis and design, MIT Press, Cambridge, Ma, 1998.

[109] C. S. Peirce, Collected Papers of Charles Sanders Peirce, C. Hartshorne and P. Weiss, eds., Harvard University Press, Cambridge, MA, 1931.

[110] S. Petkov and K. T. Atanassov, Intuitionistic fuzzy reasoning and expert systems on the example of the CONTEXT tool, Preprint IM-MFAIS-6-88, Sofia, 1988.

[111] S. Petkov and K. T. Atanassov, Generalized net model of the fuzzy reasoning in the context expert system tool, Proc. of the XX Spring Conf. of the Union of Bulg. Math., Varna, Drujba, April 1991, pp. 336–340.

[112] N. Rescher, Introduction to logic, St. Martins Press, New York, 1964.

[113] B. Russell, Vagueness, Austr. J. of Philosophy, vol. 1, pp. 84–92, 1923.

[114] B. Russell, The problem of philosophy, Williams & Norgate, London, 1934.

[115] B. Rolf, Topics on vagueness, Ph. D. Thesis, University of Lund, Sweden, 1981.

[116] M. Şahin, A. Kargın and M. A. Çoban, Fixed point theorem for neutrosophic triplet partial metric space, Symmetry, vol. 10, no. 7, pp. 240, 2018.

[117] A. Rezaei, F. Smarandache, On Neutro-BE-algebras and Anti-BE-algebras, International Journal of Neutrosophic Science, vol. 4, no. 1, pp. 8–15, 2020.

[118] A. Rezaei, F. Smarandache and S. Mirvakili, Applications of (Neutro / Anti) Sophications to Semihypergroups, Journal of Mathematics, Vol. 2021, Article ID 6649349, 7 pages. https://doi.org/10.1155/2021/66493492021.

[119] F. Smarandache, NeutroAlgebra is a Generalization of Partial Algebra, International Journal of Neutrosophic Science, vol. 2, no. 1, pp. 08–17, 2020.

[120] F. Smarandache, Introduction to NeutroAlgebraic Structures and AntiAlgebraic Structures (revisited), Neutrosophic Sets and Systems, vol. 31, pp. 1–16, 2020.

[121] F. Smarandache, Extension of HyperGraph to n-SuperHyperGraph and to Plithogenic n-SuperHyperGraph, and Extension of HyperAlgebra to n-ary (Classical- / Neutro- / Anti-) HyperAlgebra, Neutrosophic Sets and Systems, vol. 33, pp. 290–296, 2020.

[122] F. Smarandache, Introduction to NeutroAlgebraic Structures and AntiAlgebraic Structures, in Advances of Standard and Nonstandard Neutrosophic Theories, Pons Publishing House Brussels, Belgium, Ch. 6, pp. 240–265, 2019.

[123] F. Smarandache, Plithogenic Set, an Extension of Crisp, Fuzzy, Intuitionistic Fuzzy, and Neutrosophic Sets - Revisited, t Neutrosophic Sets and Systems, vol. 21, pp. 153–166, 2018.

[124] F. Smarandache, Symbolic Neutrosophic Theory, Europa Nova asbl Clos du Parnasse, 3E 1000, Bruxelles Belgium, 2015.

[125] F. Smarandache, A unifying field in logics. neutrosophy: Neutrosophic probability, set and logic, American Research Press, Rehoboth, 1999.

[126] F. Smarandache, Neutrosophy / Neutrosophic Probability, Set, and Logic, American Research Press, 1998.

Doi: https://doi.org/10.54216/IJNS.180109
Received Aug. 30, 2021 Accepted: Jan 16, 2022

[127] F. Smarandache, Neutrosophic set a generalization of the intuitionistic fuzzy set, International journal of pure and applied mathematics, vol. 24, no. 3, pp. 287–297, 2005.

[128] S. Stoeva and K. T. Atanassov, V-fizzy Petri nets and reduced V-fuzzy Generalized nets, AMSE Review, vol. 3, no. 3, pp. 1–5, 1986.

[129] D. Stoyanova, A variant of a cartesian product over intuitionistic fuzzy sets, Second Sci. Session of the" Mathematical Foundation of Artificial Intelligence" Seminar, Sofia, March 30, Preprint, IM-MFAIS-1-90, pp. 43–45, 1990.

[130] D. Stoyanova, Algebraic structures of intuitionistic fuzzy sets, Third Sci. Session of the "Mathematical Foundation of Artificial Intelligence" Seminar, Sofia, June 12, Preprint, IM-MFAIS-2-90, Part 1, pp. 19–21, 1990.

[131] D. Stoyanova, Algebraic structures of fuzzy sets, Third Sci. Session of the "Mathematical Foundation of Artificial Intelligence" Seminar, Sofia, June 12, Preprint IM-MFAIS-2-90, Part 1, pp. 15–18, 1990.

[132] D. Stoyanova, Sets from (α, β)-level generated by an intuitionistic fuzzy sets, Ninetieth Session of the Nat. Seminar of Informatics of the Union of Bulg. Mathematicians and Fourth Scientific Session of the "Mathematical Foundation Artificial Intelligence" Seminar, Sofia, Nov. 5, Preprint, IM-MFAIS-5-90, Sofia, pp. 40–42, 1990.

[133] D. Stoyanova and K. T. Atanassov, Relations between operators, defined over intuitionistic fuzzy sets, Second Sci. Session of the "Mathematical Foundation of Artificial Intelligence" Seminar, Sofia, March 30, Preprint, IM-MFAIS-I-90, pp. 46–49, 1990.

[134] E. Szmidt and J. Kacprzyk, Entropy for intuitionistic fuzzy sets, Fuzzy Sets and Systems, vol. 118, pp. 467–477, 2001.

[135] E. Szmidt and J. Kacprzyk, Intuitionistic fuzzy sets in group decision making, NIFS, vol. 2, no. 1, pp. 15–32, 1996.

[136] E. Szmidt and J. Kacprzyk, Remarks on some applications of intuitionistic fuzzy sets in decision making, NIFS, vol. 2, no. 3, pp. 22–31, 1996.

[137] E. Szmidt and J. Kacprzyk, Group decision making via intuitionistic fuzzy sets, FUBEST'96, Sofia, Bulgaria, October 9–11, pp. 107–112, 1996.

[138] E. Szmidt and J. Kacprzyk, Intuitionistic fuzzy sets for more realistic group decision making, International Conference on Transition to Advanced Market Institutions and Economies, Warsaw, June 18–21, pp. 430–433, 1997.

[139] E. Szmidt and J. Kacprzyk, Distances between intuitionistic fuzzy sets, Fuzzy Sets and Systems, vol. 114, pp. 505–518, 2001.

[140] G. Takeuti and S. Titani, Intuitionistic fuzzy logic and intuitionistic fuzzy set theory, The Journal of Symbolic Logic, vol. 49, no. 3, pp. 851–866, 1984.

[141] T. Terano, K. Asai and M. Sugeno, Fuzzy systems theory and its applications, Academic Press, 1987.

[142] N. Turanli and D. Coker, Fuzzy connectedness in intuitionistic fuzzy topological spaces, Fuzzy Sets and Systems, vol. 116, pp. 369–375, 2000.

[143] D. Van Dalen (ed.), Brouwer's Cambridge Lectures on Intuitionism, Cambridge Univ. Press, Cambridge, 1981.

[144] H. Weyl, The ghost of modality, Philosophical essays in memory of Edmund Husserl, Cambridge, MA, pp. 278–303, 1940.

[145] L. Wittgenstein, Philosophical investigations, Macmillan, New York, 1953.

[146] R. R. Yager, S. Ovchinnikov, R. M. Tong and H. T. Nguyen (Eds.), Fuzzy sets and applications: selected papers by L. A. Zadeh, Wiley, New York, 1987.

[147] C. Young, The algebra of many-valued quantities, Mathematische Annalen, vol. 104, pp. 260–290, 1931.

Doi: https://doi.org/10.54216/IJNS.180109

Received Aug. 30, 2021 Accepted: Jan 16, 2022

[148] L. A. Zadeh, Fuzzy sets, Information and Control, vol. 8, no. 3, pp. 338–353, 1965.

[149] L. A. Zadeh, , Outline of a new approach to the analysis of complex systems and decision processes, IEEE Trans. on Systems, Man and Cybernetics, vol. 3, pp. 28–44, 1973.

[150] H. J. Zimmermann, Fuzzy sets theory- and its applications, Kluwer/Nijhoff Publ., Boston, 1985. Second revised edition, Kluwer Academic Publ., Boston, 1991. Third revised edition, 1996.

Doi: https://doi.org/10.54216/IJNS.180109
Received Aug. 30, 2021 Accepted: Jan 16, 2022