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New Types of Neutrosophic Crisp Closed Sets

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Abstract: The neutrosophic sets were known since 1999, and because of their wide applications and their great flexibility to solve the problems, we used these the concepts to define a new types of neutrosophic crisp closed sets and limit points in neutrosophic crisp topological space, namely [neutrosophic crisp Gem sets and neutrosophic crisp Turig points] respectvely, we stady their properties in details and join it with topological concepts. Finally we used [neutrosophic crisp Gem sets and neutrosophic crisp Turig points] to introduce of topological concepts as : neutrosophic crisp closed (open) sets , neutrosophic crisp closure, neutrosophic crisp interior, neutrosophic crisp extrior and neutrosophic crisp boundary which are fundamental for further reserch on neutrosophic crisp topology and will setrenghen the foundations of theory of neutrosophic topological spaces.

Keywords: Neutrosophic crisp set, Neutrosophic crisp topology, Neutrosophic crisp closed set,

1. Introduction

In 1999, Smarandache firstly proposed the theory of neutrosophic set [1] which is the generalization of the class sets, conventional fuzz set [2] and intuitionistic set fuzzy [3]. After Smarandache, neutrosophic sets have been successfully applied to many fields such as; topology, control theory, databases, medical diagnosis problem, decision making problem and so on, [4-37] .

A.A. Salama, et, al.[38] proposed a new mathematical model called " Neutrosophic crisp sets and Neutrosophic crisp topological spaces " .

The idea of "Gem-Set", which is a characterization of the concept of closure is introduced by AL-Nafee ,Al-Swidi [39] . After AL-Nafee, the idea of "Gem-Set has been successfully using to many topological concepts such as; interior, exterior, boundary ,separation axioms, continuous functions , bitopological spaces, compactness, soft topological spaces, and so on, [40,41,42,43,44,45,46,47,48].

The idea of "controlling soft Gem-Set" and join it with topological concepts in soft topological space is introduced by [49]. The concept of the soft Turing point and used it with separation axioms in soft topological space is introduced by [50,51].

The goal of this research is to combine the concept of "Gem-Set" and Turing point with neutrosophic crisp set to define a new types of neutrosophic crisp closed sets and limit points in neutrosophic crisp topological space, namely [neutrosophic crisp Gem sets and neutrosophic crisp Turig points] respectvely, we stady their properties in details and we also use it to introduce the some of topological concepts as : neutrosophic crisp closed (open) sets , neutrosophic crisp closure, neutrosophic crisp interior, neutrosophic crisp extrior and neutrosophic crisp boundary which are

fundamental for further research on neutrosophic crisp topology and will strengthen the foundations of theory of neutrosophic topological spaces.

The paper is structured as follows; In section 2, we first recall the necessary background on neutrosophic and neutrosophic crisp points [NCP_N for short]. In section 3, a neutrosophic crisp Turing points properties are introduced with their properties. In section 4, the concept of neutrosophic crisp Gem sets are introduced and studied their properties.

Throughout this paper, NCTS means a neutrosophic crisp topological space, also we write (H) by H (for short), the collection of all neutrosophic crisp sets on H will be denoted by N(H).

2. Preliminaries

2.1. Definition [52]

Let H be a non-empty fixed set, a neutrosophic crisp set (for short NCS) D is an object having the form $D = \langle D_1, D_2, D_3 \rangle$ where D_1, D_2 and D_3 are subsets of H.

We will exhibit the basic neutrosophic operations definitions (union, intersection and complement). Since there are different definitions of neutrosophic operations, we will organize the existing definitions into two types in each type these operations will be consistent and functional. In this work we will use one Type of neutrosophic crisp sets operations.

2.2. Definition [52]

A neutrosophic crisp topology (NCTS) on a non-empty set H is a family T of neutrosophic crisp subsets in H satisfying the following conditions;

$$\emptyset_N, H_N \in T.$$

$$C \cap D \in T, \text{ for } C, D \in T$$

The union of any number of set in T belongs to T.

The pair (H, T) is said to be a neutrosophic crisp topological space (NCTS) in H. Moreover the elements in T are said to be neutrosophic crisp open sets. A neutrosophic crisp set F is closed iff its complement (F^c) is an open neutrosophic crisp set.

2.3. Definition [52]

Let NI be a non-null collection of neutrosophic crisp sets over a universe H. Then NI is called neutrosophic crisp ideal on H if;

- $C \in NI$ and $D \in NI$ then $C \cup D \in NI$.
- $C \in NI$ and $D \subseteq C$ then $D \in NI$.

2.4. Definition [52]

Let (H, T) be NCTS, A be a neutrosophic crisp set then: The intersection of any neutrosophic crisp closed sets contained A is called neutrosophic crisp cluster of A (for short NC-CL(A)).

2.5. Definition [52]

((neutrosophic crisp sets operations of Type.I))

Let H be a non-empty set and $C = \langle C_1, C_2, C_3 \rangle$, $D = \langle D_1, D_2, D_3 \rangle$ be two neutrosophic crisp sets, where D_1, C_1, D_2, C_2 and D_3, C_3 are subsets of H, such that $(D_1 \cap D_2) = \emptyset$, $(D_1 \cap D_3) = \emptyset$, $(D_2 \cap D_3) = \emptyset$, $(C_1 \cap C_2) = \emptyset$, $(C_1 \cap C_3) = \emptyset$, $(C_2 \cap C_3) = \emptyset$ then:

- $\emptyset_N = \langle \emptyset, \emptyset, H \rangle$ (Neutrosophic empty set).
- $H_N = \langle H, \emptyset, \emptyset \rangle$ (Neutrosophic universal set).
- $C \cap D = [C_1 \cap D_1], [C_2 \cap D_2]$ and $[C_3 \cup D_3]$.
- $C \cup D = [C_1 \cup D_1], [C_2 \cup D_2]$ and $[C_3 \cap D_3]$.
- $C \subseteq D \Leftrightarrow C_1 \subseteq D_1, C_2 \subseteq D_2$ and $D_3 \subseteq C_3$.

- The complement of a NCS (D) may be defined as: $D^c = \langle D_3, D_2, D_1 \rangle$.
- $C = D \Leftrightarrow C \subseteq D, D \subseteq C$.

2.6. Definition [53]

((neutrosophic crisp sets operations of Type.2))

Let H be a non-empty set and $C = \langle C_1, C_2, C_3 \rangle$, $D = \langle D_1, D_2, D_3 \rangle$ be two neutrosophic crisp sets, where D_1, C_1, D_2, C_2 and D_3, C_3 are subsets of H then:

- $\emptyset_N = \langle \emptyset, \emptyset, \emptyset \rangle$ (Neutrosophic empty set) .
- $H_N = \langle H, H, H \rangle$ (Neutrosophic universal set) .
- $C \cap D = [C_1 \cap D_1], [C_2 \cap D_2]$ and $[C_3 \cap D_3]$.
- $C \cup D = [C_1 \cup D_1], [C_2 \cup D_2]$ and $[C_3 \cup D_3]$.
- $C \subseteq D \Leftrightarrow C_1 \subseteq D_1, C_2 \subseteq D_2$ and $C_3 \subseteq D_3$.
- The complement of a NCS (D) may be defined as: $D^c = \langle D_1^c, D_2^c, D_3^c \rangle$.
- $C = D \Leftrightarrow C \subseteq D, D \subseteq C$.

2.7. Definition [53]

For all $a, b, c \in H$.Then the neutrosophic crisp points related to a, b, c are defined as follows ;

- $a_{N_1} = \langle \{a\}, \emptyset, \emptyset \rangle$ on H .
- $b_{N_2} = \langle \emptyset, \{b\}, \emptyset \rangle$ on H .
- $c_{N_3} = \langle \emptyset, \emptyset, \{c\} \rangle$ on H .

(The set of all neutrosophic crisp points $(a_{N_1}, b_{N_2}, c_{N_3})$ is denoted by NCPN) .

3. Neutrosophic crisp turing point

In this work, we will use Type.2 of neutrosophic crisp sets operations , this was necessary to homogeneous suitable results for the upgrade of this research .

3.1. Definition

Let (H, T) be NCTS , $P \in$ NCPN in H, we define a neutrosophic crisp ideal NI with respect to a neutrosophic crisp point P, as follows :

$$NI(P) = \{D \in T : P \in (D)^c\}$$

3.2. Definition

Let (H, T) be NCTS, $P \in$ NCPN in (H, T) , $Y \subseteq H$, we define a neutrosophic crisp ideal ${}^YNI(P)$ respect to subspace (Y, T_Y) , as follows:

$${}^YNI(P) = \{D \in T_Y : P \in (H \setminus D)\} .$$

3.3. Remark

Let (H, T) be NCTS, $Y \subseteq H$, for each $D \neq \emptyset_N$ and $P \in$ NCPN in Y , then ;

$${}^YNI(P) = \{D \in T_Y : P \in (H \setminus D)\} = \{D \in T_Y : P \in (Y \setminus D)\} .$$

Proof

$${}^YNI(P) = \{D \in T_Y : P \in (H \setminus D)\} = \{D \in T_Y : P \notin D, \text{ for each } P \in Y\} = \{D \in T_Y : P \in (Y \setminus D), \text{ for each } P \in Y\} .$$

3.4. Remark

Let (H, T) be NCTS , $Y \subseteq H$, for each $D \neq \emptyset_N$ and $P \in$ NCPN in H, then ;

$${}^YNI(P) = \{D \in T_Y : P \in (H \setminus D)\} = \{D \cap Y : \text{for each } D \neq \emptyset_N \in NI(P)\} .$$

3.5. Example

Let (H, T) be NCTS , such that $H = \{1\}$,

$T = \{ \emptyset_N, H_N, A, B, C, D, E, F, G \}$, $P_1 = \langle \emptyset, \{1\}, \emptyset \rangle$, such that;

$A = \langle \{1\}, \emptyset, \emptyset \rangle$, $B = \langle \emptyset, \{1\}, \emptyset \rangle$, $C = \langle \{1\}, \{1\}, \emptyset \rangle$, $D = \langle \{1\}, \emptyset, \{1\} \rangle$,

$E = \langle \emptyset, \{1\}, \{1\} \rangle$, $F = \langle \emptyset, \emptyset, \{1\} \rangle$, $G = \langle \{1\}, \{1\}, \{1\} \rangle$.

Then, $NI(P_1) = \{\emptyset_N, A, D, F\}$.

3.6. Definition

Let (H, T) be NCTS, $P \in$ NCPN in H and NI be a neutrosophic crisp ideal on (H, T) , we say that p is a neutrosophic crisp turing point of NI if $D^c \in NI$ for each $D \in T_P$, T_P is collection of all neutrosophic crisp open set of neutrosophic crisp point p .

3.7. Remark

Let (H, T) be NCTS, $P \in$ NCPN in H and $NI(P) = \{D \in T: P \in (D)^c\}$ be a neutrosophic crisp ideal on (H, T) .

Then, p is a neutrosophic crisp turing point of $NI(P)$.

3.8. Example

Let (H, T) be NCTS, such that $H = \{1\}$,
 $T = \{\emptyset_N, H_N, A, B, C, D, E, F, G\}$, $P_1 = \langle \emptyset, \{1\}, \emptyset \rangle$, $P_2 = \langle \{1\}, \emptyset, \emptyset \rangle$, such that;
 $A = \langle \{1\}, \emptyset, \emptyset \rangle$, $B = \langle \emptyset, \{1\}, \emptyset \rangle$, $C = \langle \{1\}, \{1\}, \emptyset \rangle$, $D = \langle \{1\}, \emptyset, \{1\} \rangle$,
 $E = \langle \emptyset, \{1\}, \{1\} \rangle$, $F = \langle \emptyset, \emptyset, \{1\} \rangle$, $G = \langle \{1\}, \{1\}, \{1\} \rangle$.

Then, P_1 is a neutrosophic crisp turing point of neutrosophic crisp ideal $NI(P_1)$, but not P_2 .

3.9. Theorem

Let (H, T) be NCTS, $a_{N_1} \neq b_{N_1} \in$ NCPN in H , then, $\langle \{b\}, \emptyset, \emptyset \rangle$ is a neutrosophic crisp closed set if and only if a_{N_1} is not a neutrosophic crisp turing point of $NI(b_{N_1})$.

Proof

Let $a_{N_1} \neq b_{N_1} \in$ NCPN in H . Assume that $\langle \{b\}, \emptyset, \emptyset \rangle$ is a neutrosophic crisp closed set, so that $\langle \{b\}, \emptyset, \emptyset \rangle = \text{cl}(\langle \{b\}, \emptyset, \emptyset \rangle)$. But $a_{N_1} \neq b_{N_1}$ get that $a_{N_1} \notin \text{cl}(\langle \{b\}, \emptyset, \emptyset \rangle)$. Therefore, there exists a neutrosophic crisp open set U such that, $a_{N_1} \in U$, $U \cap \langle \{b\}, \emptyset, \emptyset \rangle = \emptyset_N$. So that $a_{N_1} \in U$, $U^c \notin NI(b_{N_1})$, because if $U^c \in NI(b_{N_1})$, then $\langle \{b\}, \emptyset, \emptyset \rangle \in U$, that means $U \cap \langle \{b\}, \emptyset, \emptyset \rangle \neq \emptyset_N$, this a contradiction!. Hence a_{N_1} is not a neutrosophic crisp turing point of $NI(b_{N_1})$.

Conversely,

Let $a_{N_1} \neq b_{N_1} \in$ NCPN in H . Since a_{N_1} is not a neutrosophic crisp turing point of $NI(b_{N_1})$, then there exists a neutrosophic crisp open set U such that, $a_{N_1} \in U$, $U^c \notin NI(b_{N_1})$, so $\langle \{b\}, \emptyset, \emptyset \rangle \notin U$. Thus $a_{N_1} \in U$, $U \cap \langle \{b\}, \emptyset, \emptyset \rangle = \emptyset_N$ implies $a_{N_1} \notin \text{cl}(\langle \{b\}, \emptyset, \emptyset \rangle)$.

Hence $\langle \{b\}, \emptyset, \emptyset \rangle = \text{cl}(\langle \{b\}, \emptyset, \emptyset \rangle)$, thus $\langle \{b\}, \emptyset, \emptyset \rangle$ is a neutrosophic crisp closed set in H .

Proof by the same proof of 2.10. Theorem.

4. Neutrosophic crisp Gem set

4.1. Definition

Let (H, T) be NCTS, $P \in$ NCPN in H , $NI(P)$ be a neutrosophic crisp ideal on (H, T) and $D \subseteq (H, T)$, we defined the neutrosophic crisp set ND^{*P} with respect to space (H, T) as follows:

$ND^{*P} = \{P_1 \in$ NCPN in $H; F \cap D \notin NI(P), \text{ for each } F \in T_{P_1}, T_{P_1} \text{ is collection of all neutrosophic crisp open set of neutrosophic crisp point } P_1. \text{ The neutrosophic crisp set } ND^{*P} \text{ is called neutrosophic crisp Gem-Set.}$

4.2. Example

Let (H, T) be NCTS, such that $H = \{1, 2, 3\}$,
 $T = \{\emptyset_N, H_N, A, B, C, D, E, F, G\}$, $P = \langle \emptyset, \{1\}, \emptyset \rangle$, $D = \langle \emptyset, \{1, 3\}, \emptyset \rangle$, such that;
 $A = \langle \emptyset, \{1\}, \emptyset \rangle$, $B = \langle \emptyset, \{2\}, \emptyset \rangle$, $C = \langle \emptyset, \{3\}, \emptyset \rangle$, $D = \langle \emptyset, \{1, 2\}, \emptyset \rangle$.

$E = \langle \emptyset, \{1,3\}, \emptyset, \emptyset \rangle, F = \langle \emptyset, \{2,3\}, \emptyset, \emptyset \rangle, G = \langle \emptyset, \{1,2,3\}, \emptyset, \emptyset \rangle.$

Then, $NI(P) = \{\emptyset, N, B, C, F\}$ and $ND^{*P} = \langle \{1\}, \emptyset, \emptyset \rangle.$

4.3. Theorem

Let (H, T) be NCTS, $P \in$ NCPN in H , and let D, C be subsets of (H, T) . Then

1. $\emptyset_N^{*P} = \emptyset_N$
2. $H_N^{*P} = H_N$, whenever $NI(P) = \emptyset_N$.
3. $C \subseteq D \rightarrow NC^{*P} \subseteq ND^{*P}.$
4. For any points $P_1, P_2 \in$ NCPN in H , with $NI(P_2) \supseteq NI(P_1)$ then $ND^{*P_2} \subseteq ND^{*P_1}.$
5. $P \in D$ if and only if $P \in ND^{*P}.$
6. If $P \in D$, then $(ND^{*P})^{*P} = ND^{*P}.$
7. If $P_1 \in D, P_2 \in C$ with $P_1 \neq P_2, D \cap C = \emptyset_N$, then $ND^{*P_1} \cap NC^{*P_2} = \emptyset_N.$
8. If $a_{N_1}, b_{N_1} \in$ NCPN in H with $a_{N_1} \neq b_{N_1}$, then $b_{N_1} \in (a_{N_1})^c$ implies $a_{N_1} \notin (a_{N_1})^* b_{N_1}$ and $b_{N_1} \notin (b_{N_1})^* a_{N_1}.$

4.4 . Remark

The equality of theorem part (3),(4) does not necessarily hold as shown :

Let (H, T) be NCTS, such that $H = \{1,2\}, D = \langle \emptyset, \{2\}, \emptyset, \emptyset \rangle, C = \langle \emptyset, \{1\}, \emptyset, \emptyset \rangle,$

$T = \{ \emptyset_N, H_N, A, B, G \}, P_1 = \langle \emptyset, \{2\}, \emptyset, \emptyset \rangle, P_2 = \langle \emptyset, \{1\}, \emptyset, \emptyset \rangle,$

$A = \langle \emptyset, \{1\}, \emptyset, \emptyset \rangle, B = \langle \emptyset, \{2\}, \emptyset, \emptyset \rangle, G = \langle \emptyset, \{1,2\}, \emptyset, \emptyset \rangle,$

Then, $NI(P_1) = \{\emptyset_N, A\}, NI(P_2) = \{\emptyset_N, B\}$ and $ND^{*P_1} = \langle \emptyset, \{2\}, \emptyset, \emptyset \rangle, ND^{*P_2} = \emptyset_N, NC^{*P_1} = \emptyset_N$

Note that,

- 1) $ND^{*P_2} \subseteq ND^{*P_1}$, but $NI(P_2) \not\subseteq NI(P_1).$
- 2) $NC^{*P_1} \subseteq ND^{*P_1}$, but $C \not\subseteq D.$

4.5 .Theorem

Let (H, T) be NCTS, $P_1 \in$ NCPN in H and D, C be subsets of (H, T) . Then $ND^{*P_1} \cup NC^{*P_1} = N(D \cup C)^{*P_1}.$

Proof

It is obviously known that $D \subset (D \cup C)$ and $C \subset (D \cup C)$, then from theorem 3.3 part(3) we get, $ND^{*P_1} \subset N(D \cup C)^{*P_1}$ and $NC^{*P_1} \subset N(A \cup C)^{*P_1}$, for any $P_1 \in$ NCPN in H . Hence

$$ND^{*P_1} \cup NC^{*P_1} \subset N(D \cup C)^{*P_1} \text{ ----(1)}$$

For reverse inclusion, let $P_2 \notin ND^{*P_1}$. Then there exists neutrosophic crisp open set U containing p , with $D \cap U \in NI(P_1)$. Similarly, if $P_2 \notin NC^{*P_1}$, then there exists neutrosophic crisp open set V containing p , with $C \cap V \in NI(P_1)$. Then by hereditary property of neutrosophic crisp ideal, we get, $D \cap U \cap V \in NI(P_1)$ and $C \cap U \cap V \in NI(P_1)$. Again by the finite additivity condition of neutrosophic crisp ideal, we get $(D \cup C) \cap U \cap V \in NI(P_1)$. Hence $P_2 \notin N(D \cup C)^{*P_1}$. So,

$$N(D \cup C)^{*P_1} \subset ND^{*P_1} \cup NC^{*P_1} \text{ ----(2)}$$

From (1) and (2) we get, $ND^{*P_1} \cup NC^{*P_1} = N(D \cup C)^{*P_1}.$

4.6 .Theorem

Let (H, T) be NCTS, $P_1 \in$ NCPN in H and D, C be subsets of (H, T) . Then $N(D \cap C)^{*P_1} \subset ND^{*P_1} \cap NC^{*P_1}$.

Proof

It is known that $D \cap C \subset D$ and $D \cap C \subset C$, then from theorem part (3), $N(D \cap C)^{*P_1} \subset ND^{*P_1}$ and $N(D \cap C)^{*P_1} \subset NC^{*P_1}$. Hence $N(D \cap C)^{*P_1} \subset ND^{*P_1} \cap NC^{*P_1}$, for any $P_1 \in$ NCPN in H .

4.7. Theorem

Let (H, T) be NCTS, $a_{N_1} \in$ NCPN in H , for each neutrosophic crisp open set U containing a_{N_1} , then $(a_{N_1})^* a_{N_1} \subseteq U$.

proof

Let $b_{N_1} \notin U$, so $a_{N_1} \neq b_{N_1}$, then we get that $U \cap (b_{N_1}) = \emptyset_N \in NI((a_{N_1}))$. That means $(b_{N_1}) \notin (a_{N_1})^* a_{N_1}$. Thus $(a_{N_1})^* a_{N_1} \subseteq U$.

4.8. Theorem

Let (H, T) be NCTS, $P_1 \in$ NCPN in H and D be subsets of (H, T) . Then

$$D^{*P_1} = \begin{cases} \emptyset_N & \text{if } P_1 \notin D \\ \text{cl}(P_1) & \text{if } P_1 \in D \end{cases}$$

Proof

Case(1)

If $P_1 \notin D$, To prove $D^{*P_1} = \emptyset_N$. Let $D^{*P_1} \neq \emptyset_N$, then there exists least one element. say $P_2 \in D^{*P_1}$ (by definition of D^{*P_1}), we have $C_{P_2} \cap D \notin NI(P_1)$. Hence $P_1 \in D \cap C_{P_2}$. So $P_1 \in D$ which contradiction!, then $D^{*P_1} = \emptyset_N$.

Case (2)

If $P_1 \in D$, to prove $D^{*P_1} = \text{cl}(P_1)$. Let $P_2 \in D^{*P_1}$ implies $P_1 \in D \cap V_{P_2}$ for each $V_{P_2} \in T_{P_2}$ implies that $P_1 \in V_{P_2}$ for each $V_{P_2} \in T_{P_2}$ it follows $P_2 \in \text{cl}(P_1)$ then $D^{*P_1} \subseteq \text{cl}(P_1)$ for each D be subsets of (H, T) . Let $P_2 \in \text{cl}(P_1)$ and $P_2 \notin D^{*P_1}$ then there exists neutrosophic crisp open set V_{P_2} containing P_2 such that $D \cap V_{P_2} \in NI(P_1)$, which implies that $P_1 \notin D \cap V_{P_2}$ then $P_1 \notin D$ or $P_1 \notin V_{P_2}$ which means that $P_1 \notin D$ or $P_2 \notin \text{cl}(P_1)$ which contradiction! in two case. Hence $P_2 \in D^{*P_1}$ implies that $\text{cl}(P_1) \subseteq D^{*P_1}$. Therefore, $D^{*P_1} = \text{cl}(P_1)$. if $P_1 \in D$.

4.9. Definition

Let $(H, T), (Y, \delta)$ be NCTS. Then, the mapping $f: (H, T) \rightarrow (Y, \delta)$ is called NI^* -map, if and only if, for every subset D of (H, T) , $P_1 \in$ NCPN in H , $f(D^{*P_1}) = (f(D))^{*f(P_1)}$.

4.10. Example

Let $(H, T), (Y, \delta)$ be NCTS, such that $H = \{1, 2, 3\}$, $Y = \{a, b, c\}$,

$T = \{ \emptyset_N, H_N, A, B \}$, $\delta = \{ \emptyset_N, Y_N, G \}$, such that .

$A = \{1\}, \emptyset, \emptyset, B = \{2, 3\}, \emptyset, \emptyset, G = \{a\}, \emptyset, \emptyset$.

Define $f(2) = f(1) = c$ and $f(3) = a$, Put $D = \{3\}$ subset of (H, T) .

Then $D^{*3} = B = \{2, 3\}, \emptyset, \emptyset$, so $f(D^{*3}) = (f(D))^{*f(3)=a} = (\{a, c\}, \emptyset, \emptyset)^* a = \{a, b, c\}, \emptyset, \emptyset$.

4.11. Definition

Let $(H, T), (Y, \delta)$ be NCTS. Then, the mapping $f: (H, T) \rightarrow (Y, \delta)$ is called NI^{**} -map if and only if, for every subset D of (Y, δ) , $p \in$ NCPN in Y , $f^{-1}(D^{*p}) = (f^{-1}(D))^{*f^{-1}(p)}$.

4.12. Example

Let $(H, T), (Y, \delta)$ be NCTS, such that $H = \{a, b, c\}$, $Y = \{1, 2, 3\}$

$T = \{ \emptyset_N, H_N, A, B \}$, $\delta = \{ \emptyset_N, Y_N, G \}$, such that .

$A = \langle \{a\}, \emptyset, \emptyset \rangle$, $B = \langle \{b, c\}, \emptyset, \emptyset \rangle$, $G = \langle \{1\}, \emptyset, \emptyset \rangle$.

Define $f(b)=f(a)=3$ and $f(c)=1$. Put $D=\{3\}$ subset of (Y, δ) .

Then $D^{*1} = B = \langle \{2,3\}, \emptyset, \emptyset \rangle$, so $f^{-1}(D^{*1}) = (f^{-1}(D))^{*c} = (\langle \{b, a\}, \emptyset, \emptyset \rangle)^{*c} = \langle \{b, c\}, \emptyset, \emptyset \rangle$.

Conclusion

We defined a new types of neutrosophic crisp closed sets and limit points in neutrosophic crisp topological space namely [neutrosophic crisp Gem sets and neutrosophic crisp Turig points] respectively , we study their properties in details and we also use it to introduce the some of topological concepts as : neutrosophic crisp closed (open) sets , neutrosophic crisp closure , neutrosophic crisp interior , neutrosophic crisp extrior and neutrosophic crisp boundary which are fundamental for further reserch on neutrosophic crisp topology and will setrengthen the foundations of theory of neutrosophic topological spaces .

We expect, this paper will promote the future study on neutrosophic crisp topological spaces and many other general frameworks .

References

- [1] F. Smarandache. A Unifying Field in Logics: Neutrosophic Logic. Neutrosophy, Neutrosophic Set, Neutrosophic Probability, American Re- search Press, Rehoboth, NM, (1999).
- [2] Zadeh, L.A., *Fuzzy sets*, Information and Control,(1965),8 ,338-353. Accepted: 22.04.2014.
- [3] Atanassov, K., *Intuitionistic Fuzzy Sets*, Fuzzy Sets and Systems, 1986, 20 ,87-96.
- [4] F. Smarandache. An introduction to the Neutrosophy probability applied in Quantum Physics, International Conference on introduction Neutrosophic Physics, Neutrosophic Logic, Set, Probability, and Statistics, University of New Mexico, Gallup, NM 87301, USA2-4 December (2011).
- [5] Aggarwal, S., Biswas, R., Ansari, A.Q., *Neutrosophic Modeling and Control*, Computer and Communication Technology, 2010, 718-723.
- [6] Arora, M., Biswas, R., Pandey, U.S., *Neutrosophic Relational Database Decomposition*, International Journal of Advanced Computer Science and Applications, 2011,2(8), 121-125.
- [7] Arora, M., Biswas, R., *Deployment of Neutrosophic Technology to Re-trieve Answers for Queries Posed in Natural Language*, in 3rd International Conference on Computer Science and Information Technology, 2010,435-439.
- [8] Ansari, Biswas, Aggarwal, *Proposal for Applicability of Neutrosophic Set Theory in Medical AI*, International Journal of Computer Applications, 2011, 27/5 ,5-11.
- [9] Kharal, A., *A Neutrosophic Multicriteria Decision Making Method*, New Mathematics and Natural Computation, Creighton University, USA,2013.
- [10] Lupianez, F.G., *On neutrosophic topology*, Kybernetes, 2008 ,37(6) ,797-800.
- [11] A. A. Salama. Neutrosophic Crisp Points and Neutrosophic Crisp Ideals. Neutrosophic Sets and Systems, 2013, Vol.1, Issue 1, 50-54.
- [12] Salama, A., Florentin Smarandache, and Valeri Kromov. "Neutrosophic Closed Set and Neutrosophic Continuous Functions." Neutrosophic Sets and Systems 4.1 (2014): 2.
- [13] Salama, A. A., Alblowi, S. A., Smarandache, F. "Neutrosophic crisp open set and neutrosophic crisp continuity via neutrosophic crisp ideals". I.J. Information Engineering and Electronic Business vol. 3, pp.1 – 8, 2014.
- [14] Salama, A. A., Elghawalby, H. " *- Neutrosophic Crisp Set & Relations", Neutrosophic Sets and Systems, Vol. 6, pp. 12-17, 2014.
- [15] Salama, A. A., Alblowi, S. A., and Smarandache, F. "The characteristic function of a neutrosophic set", Neutrosophic Sets and Systems vol. 3, pp.14 – 18, 2014.
- [16] Salama, A. A., Smarandache, F. "Neutrosophic Crisp Set Theory", Educational Publisher, Columbus, Ohio,USA., 2015.
- [17] Salama, A. A., Smarandache, F, "Filters via Neutrosophic Crisp Sets", Neutrosophic Sets and Systems, Vol. 1, No. 1, pp. 34-38, 2013 .
- [18] .Salama, A. A., Alblowi, S. "Neutrosophic set and neutrosophic topological spaces", ISOR J.

- Mathematics vol. 3, no. 3, pp.31 – 35, 2012..
- [19] H. E. Khalid, An Original Notion to Find Maximal Solution in the Fuzzy Neutrosophic Relation Equations (FNRE) with Geometric Programming(GP),Neutrosophic Sets and Systems,2015vol.7 ,pp.3-7.
- [20] M. Abdel-Basset, M. Saleh, A. Gamal, F.Smarandache An approach of TOPSIS technique for developing supplier selection with group decision making under type-2 neutrosophic number, Applied Soft Computing, April 2019,Volume 77, Pages 438-452.
- [21] M. Abdel-Basset, G. Manogaran, A. Gamal, F. Smarandach, A Group Decision Making Framework Based on Neutrosophic TOPSIS Approach for Smart Medical Device Selection. J. Medical Systems, 2019, 43(2): 38:1-38:13 .
- [22] M. Abdel Basset, V. Chang, A. Gamal, F. Smarandache, Anintegrated neutrosophic ANP and VIKOR method for achieving sustainable supplier selection: A case study in importing field. Computers in Industry, (2019),106: 94-110 .
- [23] Abdel-Basset, Mohamed, et al. "An integrated plithogenic MCDM approach for financial performance evaluation of manufacturing industries." Risk Management (2020): 1-27.
- [24] Abdel-Basset, M., Mohamed, R., Sallam, K., & Elhoseny, M. (2020). A novel decision-making model for sustainable supply chain finance under uncertainty environment. Journal of Cleaner Production, 122324.
- [25] Abdel-Basst, M., Mohamed, R., & Elhoseny, M. (2020). A novel framework to evaluate innovation value proposition for smart product-service systems. Environmental Technology & Innovation, 101036.
- [26] M. Parimala, M. Karthika, Jafari, F. Smarandache, R.Udhayakumar. Neutrosophic Nano ideal topological structure, Neutrosophic Sets and Systems, 2019,Vol. 24,70-76.
- [27] R.K. Al-Hamido, "Neutrosophic Crisp Bi-Topological Spaces", Neutrosophic Sets and Systems, 2018. vol. 21, 66-73.
- [28] Giri, B. C., Molla, M. U., and Biswas, P. TOPSIS Method for MADM based on Interval Trapezoidal Neutrosophic Number. *Neutrosophic Sets and Systems*, (2018), 22, 151-167.
- [29] Elhassouny, A., Idbrahim, S., and Smarandache, F. Machine learning in Neutrosophic Environment: A Survey *Neutrosophic Sets and Systems*, (2019) 28, 58-68.
- [30] Gonzalez-Ortega, R., Leyva-Vazquez, M., Sganderla-Figueiredo, J. A., and Guijarro-Rodriguez, A. Sinos river basin social-environmental prospective assessment of water quality management using fuzzy cognitive maps and neutrosophic AHP-TOPSIS. *Neutrosophic Sets and Systems*, (2018),23, 160-171.
- [31] Wadei F.AL-omeri and SaeidJafari ,Neutrosophic pre continuous multifunction and almost pre continuous multifunctions,Neutrosophic sets and system, 2019,vol 27,53-69 .
- [32] T. Nandhini and M. Vigneshwaran, $N_{\alpha\beta}\psi$ -open map, $N_{\alpha\beta}\psi$ -closed map and $N_{\alpha\beta}\psi$ -homomorphism in neutrosophic topological spaces. *Neutrosophic Sets and Systems*, (2019), 29,186-196.
- [33] V. Banupriya and S. Chandrasekar, Neutrosophic $\alpha\beta$ s Continuity and Neutrosophic $\alpha\beta$ s Irresolute Maps. *Neutrosophic Sets and Systems*,(2019), 28,162-170.11.
- [34] Jayaparthasarathy, G.; Little Flower, V.F.; Arockia Dasan, M. Neutrosophic Supra Topological Applications in Data Mining Process, *Neutrosophic Sets and System*, 2019, Volume 27, pp. 80 – 97.
- [35] Aal, S. I. A.; Ellatif, A.M.A.A.; Hassan, M.M. Two Ranking Methods of Single Valued Triangular Neutrosophic Numbers to Rank and Evaluate Information Systems Quality, *Neutrosophic Sets and Systems*, 2018, Volume 19, pp. 132-141.
- [36] R. Dhavaseelan, R. Narmada Devi, S. Jafari and Qays Hatem Imran, Neutrosophic α_m -continuity. *Neutrosophic Sets and Systems*, (2019) 27, 171-179.10.
- [37] Karaaslan, F. Gaussian single-valued neutrosophic numbers and its application in multi-attribute decision making, *Neutrosophic Sets and Systems*, 2018, Volume 22, pp.101–117.
- [38] Salama, A. A., Florentin Smarandache, and Valeri Kroumov. "Neutrosophic Crisp Sets & Neutrosophic Crisp Topological Spaces." *Neutrosophic Sets and Systems* 2.1 (2014)
- [39] L. AL-Swidi and A. AL-Nafee, "New Separation Axioms Using the idea of "Gem-Set", in Topological Space", *Mathematical Theory and modeling*, 2013,vol.3, no.3, pp. 60 - 66.

- [40] L. Al-Swidi and M. A. AL-Ethary " Compactness with "Gem-Set ", *Int. Journal of Math. Analysis*, 2014,vol. 8, no.23, pp. 1105 - 1117.
- [41] L. AL-Swidi. and M. AL-Rubaye, " On Separation Axioms with Respect to Gem set" *International Journal of Engineering Research and Technology*, 2014,vol. 3, pp 2789 – 2792.
- [42] L. AL-Swidi. and M. AL-Rubaye, " New Classes of Separation Axiom via Special Case of Local Function" , *International Journal of Mathematical Analysis*. 2014,vol.8, no.23,pp1119-1131.
- [43] L. Al-Swidi and M. A. AL-Ethary " New Function With "Gem-Set" in Topological Space" , *International Journal of Engineering Research & Technology (IJERT)*, 2014,vol3, no.2, pp.2324 –2327.
- [44] R. Rathinam and C. Elango, " Gem-Separation Axioms in Topological Space" *Annals of Pure and Applied Mathematics*, 2014,vol. 15, pp 133 – 144.
- [45] I. Diaa . " δ - Continuous in Bitopological Space By Gem-set ",*JUBPAS*, 2016,vol.24, no.3, pp. 622 - 629.
- [46] AL-Swidi,L.A. and Al-Nafee,A.B." Separation Axioms Via Turing point of an Ideal in Topological Space" *Archive Des sciences* , 2013,Vol.66, No3,pp510-516.
- [47] L. A. AL-Swidi and Dheargham A. Al-Sada "Turing Point of Proper Ideal" *Archive Des Science* , 2012,Volume 65 No. 7 Jul, 219-220.
- [48] AL-Nafee, A. B.; Hamed, A. H. The Concept of (Gem-Set) in Topology, A Review . *JUBPAS* **2019**, *27*, 213-217.
- [49] AL-Nafee, A. B.; R. D. Ali. On Idea of Controlling Soft Gem-Set in Soft Topological Space, *Jour of Adv Research in Dynamical & Control Systems*, Vol. 10, 13-Special Issue, 2018 , 606-615
<http://jardcs.org/backissues/abstract.php?archiveid=6022>.
- [50] AL-Nafee, A. B. On *Soft Turing Point With Separation Axioms. *JUBPAS* **2018**, *26*, 200 - 209.
- [51] AL-Nafee, A. B.; L.A. Al-Swidi, L.A. "Separation Axioms Using Soft Turing Point of a Soft Ideal in Soft Topological Space". *Journal of New Theory*, 2018, 29-37.
- [52] A. A. Salama, F. Smarandache and Valeri Kroumov. Neutrosophic Crisp Sets and Neutrosophic Crisp Topological Spaces, *Neutrosophic Sets and Systems*, 2014, (2),25-30.
- [53] A. AL-Nafee, R. Al-Hamido and F. Smarandache, " Separation Axioms in Neutrosophic Crisp Topological Spaces", *Neutrosophic Sets and Systems*, 2019, vol. 25 , pp. 25 – 33.

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