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Mookambika. A.P

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# Neutrosophic Nano Semi-Frontier

R. Vijayalakshmi <sup>1\*</sup>, Mookambika.A.P<sup>2</sup>

<sup>1</sup> Assistant Professor, PG & Research Department of Mathematics, Arignar Anna Government Arts College,; Namakkal(DT), Tamil Nadu, India.; E-mail: vijji\_lakshmi80@rediffmail.com

<sup>2</sup> Assistant Professor, Department of Mathematics, Mahatma Gandhi Government Arts College,; New Mahe (PO), Mahe, Puducherry, India.; E-mail: mookiratna@gmail.com

\*Correspondence: Author (vijji\_lakshmi80@rediffmail.com)

**Abstract:** Smarandache presented and built up the new idea of Neutrosophic set from the Intuitionistic fuzzy sets. A.A. Salama presented Neutrosophic topological spaces by utilizing the Neutrosophic sets. M.L.Thivagar et al., created Nano topological spaces and Neutrosophic nano topological spaces. Point of this paper is we present and study the properties of Neutrosophic Nano semi frontier in Neutrosophic nano topological spaces and its portrayal are talked about subtleties.

**Keywords:** Neutrosophic Nano semi open set, Neutrosophic Nano semi closed set, Neutrosophic Nano frontier, Neutrosophic Nano semi frontier, Neutrosophic nano topology.

## 1. Introduction

Nano topology explored by M.L.Thivagar [15] et.al can be communicated as an assortment of nano approximations, Neutrosophic sets set up by F.Smarandache[14]. Neutrosophic set is illustrate by three functions: a membership, indeterminacy and nonmembership functions that are independently related. Neutrosophic set have wide scope of uses, all things considered. M.L.Thivagar et al., created Neutrosophic nano topological spaces .Neutrosophic nano semi closed, neutrosophic nano  $\alpha$ closed, neutrosophic nano pre closed, neutrosophic nano semi pre closed and neutrosophic nano regular closed are presented by M.Parimala[17] et al. Point of the current paper is we learned about properties of Neutrosophic Nano frontier, Neutrosophic Nano semi frontier in Neutrosophic nano topological spaces

## 2. PRELIMINARIES

In this section, we recall needed basic definition and operation of Neutrosophic sets

### Definition 2.1 : [15]

Let  $U$  be a non-empty set and  $R$  be an equivalence relation on  $U$ . Let  $F$  be a neutrosophic set in  $U$  with the membership function  $\mu_F$ , the indeterminacy function  $\sigma_F$  and the non-membership function  $\nu_F$ . The neutrosophic nano lower, neutrosophic nano upper approximation and neutrosophic nano boundary of  $F$  in the approximation  $(U, R)$  denoted by  $\underline{N}(F)$ ,  $\overline{N}(F)$  and  $B_N(F)$  are respectively defined as follows:

- (i)  $\underline{N}(F) = \{ \langle u, \mu_{\underline{R}}(M_1^*)(u), \sigma_{\underline{R}}(M_1^*)(u), \nu_{\underline{R}}(M_1^*)(u) \rangle / y \in [u]_R, u \in U \}$ .
- (ii)  $\overline{N}(F) = \{ \langle u, \mu_{\overline{R}}(M_1^*)(u), \sigma_{\overline{R}}(M_1^*)(u), \nu_{\overline{R}}(M_1^*)(u) \rangle / y \in [u]_R, u \in U \}$ .
- (iii)  $B_N(F) = \overline{N}(F) - \underline{N}(F)$

### Definition 2.2 : [15]

Let  $U$  be an universe,  $R$  be an equivalence relation on  $U$  and  $F$  be a neutrosophic set in  $U$  and if the collection  $N_N(\tau) = \{0_{N_N}, 1_{N_N}, \underline{N}(F), \overline{N}(F), B_N(F)\}$

forms a topology then it is said to be a neutrosophic nano topology. We call  $(U, N_N(\tau))$  as the neutrosophic nano topological space. The elements of  $N_N(\tau)$  are called neutrosophic nano open sets.

**Definition 2.3 : [15]**

Let  $U$  be a nonempty set and the Neutrosophic sets  $M_1^*$  and  $M_2^*$  in the form  $M_1^* = \{ \langle u: \mu_{M_1^*}(u), \sigma_{M_1^*}(u), \nu_{M_1^*}(u) \rangle, u \in U \}$ ,

$$M_2^* = \{ \langle u: \mu_{M_2^*}(u), \sigma_{M_2^*}(u), \nu_{M_2^*}(u) \rangle, u \in U \}.$$

Then the following statements hold:

- (i)  $0_{N_N} = \{ \langle u, 0, 0, 1 \rangle : u \in U \}$  and  $1_{N_N} = \{ \langle u, 1, 1, 0 \rangle : u \in U \}$ .
- (ii)  $M_1^* \subseteq M_2^*$  iff  $\{ \mu_{M_1^*}(u) \leq \mu_{M_2^*}(u), \sigma_{M_1^*}(u) \leq \sigma_{M_2^*}(u), \nu_{M_1^*}(u) \geq \nu_{M_2^*}(u), \forall u \in U \}$ .
- (iii)  $M_1^* = M_2^*$  iff  $M_1^* \subseteq M_2^*$  and  $M_2^* \subseteq M_1^*$ .
- (iv)  $M_1^{*c} = \{ \langle u, \nu_{M_1^*}(u), 1 - \sigma_{M_1^*}(u), \mu_{M_1^*}(u) \rangle : u \in U \}$ .
- (v)  $M_1^* \cap M_2^* = \{ \langle u, \mu_{M_1^*}(u) \wedge \mu_{M_2^*}(u), \sigma_{M_1^*}(u) \wedge \sigma_{M_2^*}(u), \nu_{M_1^*}(u) \vee \nu_{M_2^*}(u) \rangle, \forall u \in U \}$ .
- (vi)  $M_1^* \cup M_2^* = \{ \langle u, \mu_{M_1^*}(u) \vee \mu_{M_2^*}(u), \sigma_{M_1^*}(u) \vee \sigma_{M_2^*}(u), \nu_{M_1^*}(u) \wedge \nu_{M_2^*}(u) \rangle, \forall u \in U \}$ .
- (vii)  $\cup M_j^* = \langle u, \vee, \vee, \wedge \rangle$
- (viii)  $\cap M_j^* = \langle u, \wedge, \wedge, \vee \rangle$
- (ix)  $M_1^* - M_2^* = M_1^* \cap \odot M_2^{*c}$

**Proposition 2.4 [15]**

For any Neutrosophic Nano set  $M_1^*$  in  $(U, N_N(\tau))$  we have

- (1)  $N^N Cl ((M_1^*)^c) = (N^N Int (M_1^*))^c$ ,
- (2)  $N^N Int ((M_1^*)^c) = (N^N Cl (M_1^*))^c$ .
- (3)  $M_1^* \subseteq M_2^* \Rightarrow N^N Int (M_1^*) \subseteq N^N Int (M_2^*)$ ,
- (4)  $M_1^* \subseteq M_2^* \Rightarrow N^N Cl (M_1^*) \subseteq N^N Cl (M_2^*)$ ,
- (5)  $N^N Int (N^N Int (M_1^*)) = N^N Int (M_1^*)$ ,
- (6)  $N^N Cl (N^N Cl (M_1^*)) = N^N Cl (M_1^*)$ ,
- (7)  $N^N Int (M_1^* \cap M_2^*) = N^N Int (M_1^*) \cap N^N Int (M_2^*)$ ,
- (8)  $N^N Cl (M_1^* \cup M_2^*) = N^N Cl (M_1^*) \cup N^N Cl (M_2^*)$ ,
- (9)  $N^N Int (0_{N_N}) = 0_{N_N}$ ,
- (10)  $N^N Int (1_{N_N}) = 1_{N_N}$ ,
- (11)  $N^N Cl (0_{N_N}) = 0_{N_N}$ ,
- (12)  $N^N Cl (1_{N_N}) = 1_{N_N}$ ,
- (13)  $M_1^* \subseteq M_2^* \Rightarrow (M_2^{*c} \subseteq M_1^{*c})$ ,
- (14)  $N^N Cl (M_1^* \cap M_2^*) \subseteq N^N Cl (M_1^*) \cap N^N Cl (M_2^*)$ ,
- (15)  $N^N Int (M_1^* \cup M_2^*) \supseteq N^N Int (M_1^*) \cup N^N Int (M_2^*)$ .

**3. NEUTROSOPHIC NANO FRONTIER**

In this section, the concepts of the Neutrosophic Nano frontier in Neutrosophic Nano topological space are introduced and also discussed their characterizations with some related examples.

**Definition 3.1.**

Let  $U$  be a  $N-N-T-S$  and let  $M_1^* \in NNS(U)$ . Neutrosophic Nano frontier of  $M_1^*$  and is denoted by  $NFr(M_1^*)$ . i.e.,  $N^N Fr (M_1^*) = N^N Cl(M_1^*) \cap \odot N^N Cl((M_1^*)^c)$ .

**Proposition 3.2.** For each  $M_1^* \in NNS(U)$ ,  $M_1^* \cup \odot N^N Fr (M_1^*) \subseteq N^N Cl (M_1^*)$ .

**Proof :** Let  $M_1^*$  be the  $NNS$  in the  $N-N-T-S$   $U$ . Using Definition 3.1.,

$$M_1^* \cup \odot N^N Fr (M_1^*) = M_1^* \cup ( N^N Cl (M_1^*) \cap \odot N^N Cl ((M_1^*)^c) )$$

$$= (M_1^* \cup \odot N^N Cl (M_1^*) ) \cap \odot ( M_1^* \cup \odot N^N Cl ((M_1^*)^c) )$$

$$\subseteq N^N Cl (M_1^*) \cap \odot N^N Cl (M_1^*)$$

$$\subseteq N^N Cl (M_1^*)$$

Hence  $M_1^* \cup \odot N^N Fr (M_1^*) \subseteq N^N Cl (M_1^*)$ .

**Example 3.3.**

Let  $U$  and  $\mathcal{A}$  be two non-empty finite sets,

where U is the universe and  $\mathcal{A}$  the set of attributes  
 The members of  $U = \{P_1, P_2, P_3, P_4\}$  are pressure patients  
 Let  $U/R = \{\{P_1, P_2, P_3\}, \{P_4\}\}$  be an equivalence relation  
 $\mathcal{A} = \{\text{Salt food, colostreal food}\}$  are two attributes

$$P_1 = \langle x, \left(\frac{5}{10}, \frac{10}{10}, \frac{1}{10}\right), \left(\frac{9}{10}, \frac{2}{10}, \frac{5}{10}\right) \rangle$$

$$P_2 = \langle x, \left(\frac{2}{10}, \frac{5}{10}, \frac{9}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{10}{10}\right) \rangle$$

$$P_3 = \langle x, \left(\frac{5}{10}, \frac{10}{10}, \frac{1}{10}\right), \left(\frac{9}{10}, \frac{5}{10}, \frac{5}{10}\right) \rangle$$

$$P_4 = \langle x, \left(\frac{2}{10}, \frac{5}{10}, \frac{9}{10}\right), \left(\frac{0}{10}, \frac{2}{10}, \frac{10}{10}\right) \rangle$$

$$N_N(\tau) = \{0_{N_N}, 1_{N_N}, \underline{N(F)}, \overline{N(F)}, B_N(F)\}$$

$$\underline{N(F)} = \langle x, \left(\frac{2}{10}, \frac{5}{10}, \frac{9}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{10}{10}\right) \rangle$$

$$\overline{N(F)} = \langle x, \left(\frac{5}{10}, \frac{10}{10}, \frac{1}{10}\right), \left(\frac{9}{10}, \frac{5}{10}, \frac{5}{10}\right) \rangle$$

$$B_N(F) = \langle \left(\frac{5}{10}, \frac{5}{10}, \frac{2}{10}\right), \left(\frac{9}{10}, \frac{5}{10}, \frac{5}{10}\right) \rangle$$

$$N_N(\tau) = \{0_{N_N}, 1_{N_N}, \langle x, \left(\frac{2}{10}, \frac{5}{10}, \frac{9}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{10}{10}\right) \rangle, \langle \left(\frac{5}{10}, \frac{10}{10}, \frac{1}{10}\right), \left(\frac{9}{10}, \frac{5}{10}, \frac{5}{10}\right) \rangle, \langle \left(\frac{5}{10}, \frac{5}{10}, \frac{2}{10}\right), \left(\frac{9}{10}, \frac{5}{10}, \frac{5}{10}\right) \rangle, \langle \left(\frac{2}{10}, \frac{5}{10}, \frac{9}{10}\right), \left(\frac{0}{10}, \frac{2}{10}, \frac{10}{10}\right) \rangle\}$$

$$\text{Here } N^N Cl(P_3) = 1_{N_N} \text{ and } N^N Cl(P_3^c) = \langle \left(\frac{2}{10}, \frac{5}{10}, \frac{5}{10}\right), \left(\frac{5}{10}, \frac{5}{10}, \frac{9}{10}\right) \rangle.$$

$$\text{Using Definition 2.1, } N^N Fr(M_1^*) = \langle \left(\frac{2}{10}, \frac{5}{10}, \frac{5}{10}\right), \left(\frac{5}{10}, \frac{5}{10}, \frac{9}{10}\right) \rangle.$$

$$\text{Also } M_1^* \cup \circledast N^N Fr(M_1^*) = \langle \left(\frac{5}{10}, \frac{10}{10}, \frac{1}{10}\right), \left(\frac{9}{10}, \frac{5}{10}, \frac{5}{10}\right) \rangle \subseteq 1_{N_N}.$$

$$\text{Therefore } N^N Cl(M_1^*) = 1_{N_N} \not\subseteq \langle \left(\frac{5}{10}, \frac{10}{10}, \frac{1}{10}\right), \left(\frac{9}{10}, \frac{5}{10}, \frac{5}{10}\right) \rangle.$$

**Theorem 3.5.**

For a NNS  $M_1^*$  in the N-N-T-S U,  $N^N Fr(M_1^*) = N^N Fr(M_1^{*c})$ .

**Proof :** Let  $M_1^*$  be the NNS in the N-N-T-S U. Using Definition 3.1.,

$$N^N Fr(M_1^*) = N^N Cl(M_1^*) \cap \circledast N^N Cl(M_1^{*c})$$

$$= N^N Cl(M_1^{*c}) \cap \circledast N^N Cl(M_1^*)$$

$$= N^N Cl(M_1^{*c}) \cap \circledast N^N Cl(M_1^{*c})^c$$

Again by Definition 3.1.,  $= N^N Fr(M_1^{*c})$

Hence  $N^N Fr(M_1^*) = N^N Fr(M_1^{*c})$ .

**Theorem 3.6.**

If a  $N^N S M_1^*$  is a NCS, then  $N^N Fr(M_1^*) \subseteq \circledast M_1^*$ .

**Proof :**

Let  $M_1^*$  be the  $N^N S$  in the Neutrosophic Nano topological space U. Using Definition 3.1.,

$$N^N Fr(M_1^*) = N^N Cl(M_1^*) \cap N^N Cl(M_1^{*c}) \subseteq N^N Cl(M_1^*)$$

By Proposition (2.4),  $= M_1^*$

Hence  $N^N Fr(M_1^*) \subseteq \circledast M_1^*$ , if  $M_1^*$  is  $N^N CS$  in U.

The converse of the above theorem needs not be true as shown by the following example.

**Theorem 3.7.**

If a NNS  $M_1^*$  is  $N^N OS$ , then  $N^N Fr(M_1^*) \subseteq \circledast M_1^{*c}$ .

**Proof :**

Let  $M_1^*$  be the NNS in the N-N-T-S U. Using Definition 3.1 ,

$M_1^*$  is  $N^N OS$  implies  $M_1^{*c}$  is  $N^N CS$  in U. By Theorem 3.6,  $N^N Fr(M_1^{*c}) \subseteq M_1^{*c}$  and by Theorem 3.5,

we get  $N^N Fr(M_1^*) \subseteq M_1^{*c}$

**Theorem 3.8.**

For a NNS  $M_1^*$  in the  $N^N TS$  U,  $(N^N Fr(M_1^*))^c = N^N Int(M_1^*) \cup \circledast N^N Int(M_1^{*c})$ .

**Proof :**

Let  $M_1^*$  be the  $N^N S$  in the N-N-T-S U. Using Definition 3.1.,

$$(N^N Fr(M_1^*))^c = (N^N Cl(M_1^*))^c \cap \circledast (N^N Cl(M_1^{*c}))^c$$

By Propositon (2.4)  $= (N^N Cl (M_1^*))^C \cup \otimes (N^N Cl (M_1^{*C}))^C$

By Propositon (2.4),  $= N^N Int (M_1^{*C}) \cup \otimes N^N Int (M_1^*)$

Hence  $(N^N Fr (M_1^*))^C = N^N Int (M_1^*) \cup \otimes N^N Int (M_1^{*C})$ .

**Theorem 3.9**

Let  $M_1^* \subseteq \otimes M_2^*$  and  $M_2^* \otimes N^N C (U)$  ( resp.,  $M_2^* \otimes N^N O (U)$  ). Then  $N^N Fr (M_1^*) \subseteq \otimes M_2^*$  ( resp.,  $N^N Fr (M_1^*) \subseteq (N^N Cl (M_2^*))^C$  ), where  $N^N C (U)$  ( resp.,  $N^N O (U)$  ) denotes the class of Neutrosophic Nano closed ( resp., Neutrosophic Nano open) sets in U.

**Proof :** Use Prop.,2.4 ,  $M_1^* \subseteq \otimes M_2^*$  ,

$$N^N Cl (M_1^*) \subseteq \otimes N^N Cl (M_2^*) \text{ ----- (1)}$$

By Definition 3.1.,

$$N^N Fr (M_1^*) = N^N Cl (M_1^*) \cap \otimes N^N Cl (M_1^{*C})$$

$$\subseteq \otimes N^N Cl (M_2^*) \cap \otimes N^N Cl (M_1^{*C}) \text{ by (1)}$$

$$\subseteq \otimes N^N Cl (M_2^*) = M_2^*$$

Hence  $N^N Fr (M_1^*) \subseteq \otimes M_2^*$ .

**Theorem 3.10**

Let  $M_1^*$  be the NNS in the N-N-T-S U. Then

$$N^N Fr (M_1^*) = N^N Cl (M_1^*) - N^N Int (M_1^*).$$

**Proof :** Let  $M_1^*$  be the NNS in the N-N-T-S U. By Propositon (2.4),

$$((N^N Cl (M_1^{*C}))^C = N^N Int (M_1^*) \text{ and by Definition 3.1.,}$$

$$N^N Fr (M_1^*) = N^N Cl (M_1^*) \cap \otimes N^N Cl (M_1^{*C})$$

$$= N^N Cl (M_1^*) - (N^N Cl (M_1^{*C}))^C$$

by using  $M_1^* - M_2^* = M_1^* \cap \otimes M_2^{*C}$

By Propositon (2.4),

$$= N^N Cl (M_1^*) - N^N Int (M_1^*)$$

Hence  $N^N Fr (M_1^*) = N^N Cl (M_1^*) - N^N Int (M_1^*)$ .

**Theorem 3.11.**

For a NNS  $M_1^*$  in the  $N^N TS$  U,  $N^N Fr (N^N Int (M_1^*)) \subseteq \otimes N^N Fr (M_1^*)$ .

**Proof :**

Let  $M_1^*$  be the NNS in the N-N-T-S U. Using Definition 3.1.,

$$N^N Fr (N^N Int (M_1^*)) = N^N Cl (N^N Int (M_1^*)) \cap N^N Cl (N^N Int (M_1^*))^C$$

By Propositon (2.4),

$$= N^N Cl (N^N Int (M_1^*)) \cap N^N Cl (N^N Cl (M_1^{*C}))$$

By Propositon (2.4),

$$= N^N Cl (N^N Int (M_1^*)) \cap N^N Cl (M_1^{*C})$$

. By Propositon (2.4), ,

$$\subseteq \otimes N^N Cl (M_1^*) \cap N^N Cl (M_1^{*C})$$

Again by Definition 3.1.,

$$= N^N Fr (M_1^*)$$

Hence  $N^N Fr (N^N Int (M_1^*)) \subseteq \otimes N^N Fr (M_1^*)$ .

**Example 3.12.**

Let U and  $\mathcal{A}$  be two non-empty finite sets,

where U is the universe and  $\mathcal{A}$  the set of attributes

The members of  $U = \{P_1, P_2, P_3, P_4\}$  are pressure patients

Let  $U/R = \{\{P_1, P_2, P_3\}, \{P_4\}\}$  be an equivalence relation

$\mathcal{A} = \{\text{Head ache, Temperature}\}$  are two attributes

$$P_1 = \langle x, (\frac{5}{10}, \frac{6}{10}, \frac{7}{10}), (\frac{10}{10}, \frac{9}{10}, \frac{4}{10}) \rangle$$

$$P_2 = \langle x, (\frac{3}{10}, \frac{9}{10}, \frac{2}{10}), (\frac{4}{10}, \frac{1}{10}, \frac{6}{10}) \rangle$$

$$P_3 = \langle x, \left(\frac{5}{10}, \frac{9}{10}, \frac{2}{10}\right), \left(\frac{4}{10}, \frac{9}{10}, \frac{4}{10}\right) \rangle$$

$$P_4 = \langle x, \left(\frac{3}{10}, \frac{6}{10}, \frac{7}{10}\right), \left(\frac{1}{10}, \frac{1}{10}, \frac{6}{10}\right) \rangle$$

$$N_N(\tau) = \{0_{N_N}, 1_{N_N}, \underline{N(F)}, \overline{N(F)}, B_N(F)\}$$

$$\underline{N(F)} = \langle x, \left(\frac{3}{10}, \frac{6}{10}, \frac{7}{10}\right), \left(\frac{4}{10}, \frac{1}{10}, \frac{6}{10}\right) \rangle$$

$$\overline{N(F)} = \langle x, \left(\frac{5}{10}, \frac{9}{10}, \frac{2}{10}\right), \left(\frac{10}{10}, \frac{9}{10}, \frac{4}{10}\right) \rangle$$

$$B_N(F) = \langle x, \left(\frac{5}{10}, \frac{4}{10}, \frac{3}{10}\right), \left(\frac{6}{10}, \frac{9}{10}, \frac{4}{10}\right) \rangle$$

$$N_N(\tau) = \{0_{N_N}, 1_{N_N}, \langle x, \left(\frac{3}{10}, \frac{6}{10}, \frac{7}{10}\right), \left(\frac{4}{10}, \frac{1}{10}, \frac{6}{10}\right) \rangle, \langle \left(\frac{5}{10}, \frac{9}{10}, \frac{2}{10}\right), \left(\frac{10}{10}, \frac{9}{10}, \frac{4}{10}\right) \rangle, \langle \left(\frac{5}{10}, \frac{4}{10}, \frac{3}{10}\right), \left(\frac{6}{10}, \frac{9}{10}, \frac{4}{10}\right) \rangle, \left(\frac{3}{10}, \frac{6}{10}, \frac{7}{10}\right), \left(\frac{1}{10}, \frac{1}{10}, \frac{6}{10}\right)\}$$

$$M_3^* = \langle \left(\frac{5}{10}, \frac{4}{10}, \frac{3}{10}\right), \left(\frac{6}{10}, \frac{9}{10}, \frac{4}{10}\right) \rangle$$

Therefore by Definition 3.1.,  $N^N Fr (M_3^*) = \not\subseteq N^N Fr (N^N Int (M_3^*))$ .

**Theorem 3.13.**

For a NNS  $M_1^*$  in the N-N-T-S U,  $N^N Fr (N^N Cl (M_1^*)) \subseteq N^N Fr (M_1^*)$ .

**Proof :** Let  $M_1^*$  be the NNS in the N-N-T-S U. Using Definition 3.1.,  $N^N Fr (N^N Cl (M_1^*)) = N^N Cl (N^N Cl (M_1^*)) \cap N^N Cl ((N^N Cl (M_1^*)))^C$  By Propositon (2.4),  $= N^N Cl (M_1^*) \cap N^N Cl (N^N Int (M_1^{*C}))$  By Propositon (2.4),  $\subseteq N^N Cl (M_1^*) \cap N^N Cl (M_1^{*C})$  Again by Definition 3.1.,  $= N^N Fr (M_1^*)$

Hence  $N^N Fr (N^N Cl (M_1^*)) \subseteq N^N Fr (M_1^*)$ .

**Theorem 3.14.**

Let  $M_1^*$  be the NNS in the N-N-T-S U. Then  $N^N Int (M_1^*) \subseteq \ominus M_1^* - N^N Fr (M_1^*)$ .

**Proof :** Let  $M_1^*$  be the NNS in the N-N-T-S U. Now by Definition 3.1.,  $M_1^* - N^N Fr (M_1^*) = M_1^* - (N^N Cl (M_1^*) \cap \ominus N^N Cl (M_1^{*C}))$   
 $= (M_1^* - N^N Cl (M_1^*)) \cup (M_1^* - \ominus N^N Cl (M_1^{*C}))$   
 $= M_1^* - N^N Cl (M_1^{*C})$   
 $N^N Int (M_1^*)$ .

Hence  $N^N Int (M_1^*) \subseteq M_1^* - N^N Fr (M_1^*)$ .

**Remark 3.15.**

In general topology, the following conditions are hold :

$$N^N Fr (M_1^*) \cap \ominus N^N Int (M_1^*) = 0_{N_N}$$

$$N^N Int (M_1^*) \cup \ominus N^N Fr (M_1^*) = N^N Cl (M_1^*),$$

$$N^N Int (M_1^*) \cup \ominus N^N Int (M_1^{*C}) \cup \ominus N^N Fr (M_1^*) = 1_{N_N}.$$

**Theorem 3.16.**

Let  $M_1^*$  and  $M_2^*$  be the two NNSs in the N-N-T-S.

Then  $N^N Fr (M_1^* \cup \ominus M_2^*) \subseteq \ominus N^N Fr (M_1^*) \cup \ominus N^N Fr (M_2^*)$ .

**Proof :** Let  $M_1^*$  and  $M_2^*$  be the two NNSs in the N-N-T-S U.

Using Definition 3.1.,

$$N^N Fr (M_1^* \cup \ominus M_2^*) = N^N Cl (M_1^* \cup \ominus M_2^*) \cap \ominus N^N Cl ((M_1^* \cup M_2^*)^C)$$

. By Propositon (2.4),

$$= N^N Cl (M_1^* \cup \ominus M_2^*) \cap \ominus N^N Cl (M_1^{*C} \cap \ominus M_2^{*C})$$

$$\subseteq \ominus (N^N Cl (M_1^*) \cup \ominus N^N Cl (M_2^*)) \cap \ominus (N^N Cl (M_1^{*C}) \cap \ominus N^N Cl (M_2^{*C}))$$

$$= [(N^N Cl (M_1^*) \cup \ominus N^N Cl (M_2^*)) \cap \ominus N^N Cl (M_1^{*C})] \cap \ominus [(N^N Cl (M_1^*) \cup \ominus N^N Cl (M_2^*)) \cap \ominus N^N Cl (M_2^{*C})]$$

$$= [(N^N Cl (M_1^*) \cap \ominus N^N Cl (M_1^{*C})) \cup (N^N Cl (M_2^*) \cap \ominus N^N Cl (M_1^{*C}))] \cap \ominus [(N^N Cl (M_1^*) \cap \ominus N^N Cl (M_2^{*C})) \cup (N^N Cl (M_2^*) \cap \ominus N^N Cl (M_2^{*C}))]$$

Again by Definition 3.1.,

$$= [N^N Fr (M_1^*) \cup \ominus (N^N Cl (M_2^*) \cap \ominus N^N Cl (M_1^{*C}))] \cap \ominus [(N^N Cl (M_1^*) \cap \ominus N^N Cl (M_2^{*C})) \cup \ominus N^N Fr (M_2^*)]$$

$$= (N^N Fr (M_1^*) \cup \ominus N^N Fr (M_2^*)) \cap \ominus [(N^N Cl (M_2^*) \cap \ominus N^N Cl (M_1^{*C}))]$$

$$\cup \circ ( N^N Cl ( M_1^* ) \cap \circ N^N Cl ( M_2^{*C} ) ) ]$$

$$\subseteq \circ N^N Fr ( M_1^* ) \cup \circ N^N Fr ( M_2^* ).$$

Hence  $N^N Fr ( M_1^* \cup \circ M_2^* ) \subseteq \circ N^N Fr ( M_1^* ) \cup \circ N^N Fr ( M_2^* )$ .

**Note 3.17.**

$N^N Fr ( M_1^* \cap \circ M_2^* ) \not\subseteq N^N Fr ( M_1^* ) \cap \circ N^N Fr ( M_2^* )$  and

$N^N Fr ( M_1^* ) \cap \circ N^N Fr ( M_2^* ) \not\subseteq N^N Fr ( M_1^* \cap \circ M_2^* )$ .

**Theorem 3.18.**

For any NNSs  $M_1^*$  and  $M_2^*$  in the  $N-N-T-S U$ ,

$$N^N Fr ( M_1^* \cap \circ M_2^* ) \subseteq \circ ( N^N Fr ( M_1^* ) \cap \circ N^N Cl ( M_2^* ) ) \cup \circ ( N^N Fr ( M_2^* ) \cap N^N Cl ( M_1^* ) ).$$

**Proof :** Let  $M_1^*$  and  $M_2^*$  be the two NNSs in the  $N-N-T-S U$ .

Using Definition 3.1.,

$$N^N Fr ( M_1^* \cap \circ M_2^* ) = N^N Cl ( M_1^* \cap \circ M_2^* ) \cap \circ N^N Cl ( ( M_1^* \cap M_2^* )^C )$$

Use Prop., 3.2 (1) [18] ,

$$= N^N Cl ( M_1^* \cap \circ M_2^* ) \cap \circ N^N Cl ( M_1^{*C} \cup \circ M_2^{*C} )$$

. By Propositon (2.4),

$$\subseteq \circ ( N^N Cl ( M_1^* ) \cap \circ N^N Cl ( M_2^* ) ) \cap \circ ( N^N Cl ( M_1^{*C} ) \cup \circ N^N Cl ( M_2^{*C} ) )$$

$$= [ ( N^N Cl ( M_1^* ) \cap \circ N^N Cl ( M_2^* ) ) \cap \circ N^N Cl ( M_1^{*C} ) ] \cup \circ [ ( N^N Cl ( M_1^* ) \cap \circ N^N Cl ( M_2^* ) ) \cap \circ N^N Cl ( M_2^{*C} ) ]$$

Again by Definition 3.1.,

$$= ( N^N Fr ( M_1^* ) \cap \circ N^N Cl ( M_2^* ) ) \cup \circ ( N^N Fr ( M_2^* ) \cap \circ N^N Cl ( M_1^* ) )$$

Hence  $N^N Fr ( M_1^* \cap \circ M_2^* ) \subseteq \circ ( N^N Fr ( M_1^* ) \cap \circ N^N Cl ( M_2^* ) ) \cup$

$$( N^N Fr ( M_2^* ) \cap \circ N^N Cl ( M_1^* ) ).$$

**Corollary 3.19.**

For any NNSs  $M_1^*$  and  $M_2^*$  in the  $N-N-T-S U$ ,

$$N^N Fr ( M_1^* \cap \circ M_2^* ) \subseteq \circ N^N Fr ( M_1^* ) \cup \circ N^N Fr ( M_2^* ).$$

**Proof :**

Let  $M_1^*$  and  $M_2^*$  be the two NNSs in the  $N-N-T-S U$ . Using Definition 3.1.,

$$N^N Fr ( M_1^* \cap \circ M_2^* ) = N^N Cl ( M_1^* \cap \circ M_2^* ) \cap \circ N^N Cl ( ( M_1^* \cap M_2^* )^C )$$

. By Propositon (2.4),,

$$= N^N Cl ( M_1^* \cap \circ M_2^* ) \cap \circ N^N Cl ( M_1^{*C} \cup \circ M_2^{*C} )$$

. By Propositon (2.4),,

$$\subseteq \circ ( N^N Cl ( M_1^* ) \cap \circ N^N Cl ( M_2^* ) ) \cap \circ ( N^N Cl ( M_1^{*C} ) \cup \circ N^N Cl ( M_2^{*C} ) )$$

$$= ( N^N Cl ( M_1^* ) \cap \circ N^N Cl ( M_2^* ) \cap \circ N^N Cl ( M_1^{*C} ) ) \cup \circ ( N^N Cl ( M_1^* ) \cap \circ N^N Cl ( M_2^* ) \cap \circ N^N Cl ( M_2^{*C} ) )$$

Again by Definition 3.1.,

$$= ( N^N Fr ( M_1^* ) \cap \circ N^N Cl ( M_2^* ) ) \cup \circ ( N^N Cl ( M_1^* ) \cap \circ N^N Fr ( M_2^* ) )$$

$$\subseteq \circ N^N Fr ( M_1^* ) \cup \circ N^N Fr ( M_2^* )$$

Hence  $N^N Fr ( M_1^* \cap \circ M_2^* ) \subseteq \circ N^N Fr ( M_1^* ) \cup \circ N^N Fr ( M_2^* )$ .

**Theorem 3.20**

For any NNS  $M_1^*$  in the  $N-N-T-S U$ ,

$$(1) N^N Fr ( N^N Fr ( M_1^* ) ) \subseteq \circ N^N Fr ( M_1^* ),$$

$$(2) N^N Fr ( N^N Fr ( N^N Fr ( M_1^* ) ) ) \subseteq \circ N^N Fr ( N^N Fr ( M_1^* ) ).$$

**Proof :** (1) Let  $M_1^*$  be the NNS in the Neutrosophic Nano topological space  $U$ . Using Definition 3.1.,

$$N^N Fr ( N^N Fr ( M_1^* ) ) = N^N Cl ( N^N Fr ( M_1^* ) ) \cap \circ N^N Cl ( ( N^N Fr ( M_1^* ) )^C )$$

$$\text{Again by Definition 3.1.,}$$

$$= N^N Cl ( N^N Cl ( M_1^* ) \cap \circ N^N Cl ( M_1^{*C} ) ) \cap N^N Cl ( ( ( N^N Cl ( M_1^* ) \cap N^N Cl ( M_1^{*C} ) )^C )$$

By Propositon (2.4), and by Propositon (2.4),

$$\subseteq \circ ( N^N Cl ( N^N Cl ( M_1^* ) ) \cap \circ N^N Cl ( N^N Cl ( M_1^{*C} ) ) ) \cap \circ N^N Cl ( N^N Int ( M_1^{*C} ) \cup \circ N^N Int ( M_1^* ) )$$

Use Prop., 1.18 (f) [18] ,

$$= ( N^N Cl ( M_1^* ) \cap \circ N^N Cl ( M_1^{*C} ) ) \cap \circ ( N^N Cl ( N^N Int ( M_1^{*C} ) ) \cup \circ N^N Cl ( N^N Int ( M_1^* ) ) )$$

$$\subseteq \circ N^N Cl ( M_1^* ) \cap \circ N^N Cl ( M_1^{*C} )$$

By Definition 3.1.,

$$= N^N Fr (M_1^*)$$

Therefore  $N^N Fr (N^N Fr (M_1^*)) \subseteq \odot N^N Fr (M_1^*)$ .

(2) By Definition 3.1.,

$$N^N Fr (N^N Fr (N^N Fr (M_1^*))) = N^N Cl (N^N Fr (N^N Fr (M_1^*))) \cap N^N Cl ((N^N Fr (N^N Fr (M_1^*)))^c)$$

Use Prop., 1.18 (f) [18],

$$\subseteq \odot (N^N Fr (N^N Fr (M_1^*))) \cap \odot N^N Cl ((N^N Fr (N^N Fr (M_1^*)))^c) \subseteq \odot N^N Fr (N^N Fr (M_1^*)).$$

Hence  $N^N Fr (N^N Fr (N^N Fr (M_1^*))) \subseteq \odot N^N Fr (N^N Fr (M_1^*))$ .

**Example 3.21.**

Let U and  $\mathcal{A}$  be two non-empty finite sets,

where U is the universe and  $\mathcal{A}$  the set of attributes

The members of  $U = \{P_1, P_2, P_3, P_4\}$  are patients

Let  $U/R = \{\{P_1, P_2, P_3\}, \{P_4\}\}$  be an equivalence relation

$\mathcal{A} = \{\text{Head ache, Temperature}\}$  are two attributes

$$P_1 = \langle x, \left(\frac{8}{10}, \frac{4}{10}, \frac{5}{10}\right), \left(\frac{4}{10}, \frac{6}{10}, \frac{7}{10}\right) \rangle$$

$$P_2 = \langle x, \left(\frac{4}{10}, \frac{2}{10}, \frac{9}{10}\right), \left(\frac{1}{10}, \frac{4}{10}, \frac{9}{10}\right) \rangle$$

$$P_3 = \langle x, \left(\frac{5}{10}, \frac{6}{10}, \frac{8}{10}\right), \left(\frac{7}{10}, \frac{4}{10}, \frac{4}{10}\right) \rangle$$

$$P_4 = \langle x, \left(\frac{9}{10}, \frac{8}{10}, \frac{4}{10}\right), \left(\frac{9}{10}, \frac{6}{10}, \frac{1}{10}\right) \rangle$$

$$N_N(\tau) = \{0_{N_N}, 1_{N_N}, \underline{N(F)}, \overline{N(F)}, B_N(F)\}$$

$$\underline{N(F)} = \langle x, \left(\frac{4}{10}, \frac{2}{10}, \frac{9}{10}\right), \left(\frac{1}{10}, \frac{4}{10}, \frac{9}{10}\right) \rangle$$

$$\overline{N(F)} = \langle x, \left(\frac{8}{10}, \frac{6}{10}, \frac{5}{10}\right), \left(\frac{7}{10}, \frac{6}{10}, \frac{4}{10}\right) \rangle$$

$$B_N(F) = \langle x, \left(\frac{9}{10}, \frac{6}{10}, \frac{4}{10}\right), \left(\frac{7}{10}, \frac{6}{10}, \frac{4}{10}\right) \rangle$$

$$N_N(\tau) = \{0_{N_N}, 1_{N_N}, \langle x, \left(\frac{4}{10}, \frac{2}{10}, \frac{9}{10}\right), \left(\frac{1}{10}, \frac{4}{10}, \frac{9}{10}\right) \rangle, \langle \left(\frac{8}{10}, \frac{6}{10}, \frac{5}{10}\right), \left(\frac{7}{10}, \frac{6}{10}, \frac{4}{10}\right) \rangle, \langle \left(\frac{9}{10}, \frac{6}{10}, \frac{4}{10}\right), \left(\frac{7}{10}, \frac{6}{10}, \frac{4}{10}\right) \rangle, \langle \left(\frac{9}{10}, \frac{8}{10}, \frac{4}{10}\right), \left(\frac{9}{10}, \frac{6}{10}, \frac{1}{10}\right) \rangle\}$$

$$M_1^* = \langle x, \left(\frac{6}{10}, \frac{7}{10}, \frac{8}{10}\right), \left(\frac{5}{10}, \frac{4}{10}, \frac{5}{10}\right) \rangle$$

$$\text{Then } N^N Fr (M_1^*) = \langle x, \left(\frac{9}{10}, \frac{8}{10}, \frac{4}{10}\right), \left(\frac{9}{10}, \frac{6}{10}, \frac{1}{10}\right) \rangle$$

$$N^N Fr (N^N Fr (M_1^*)) = \langle x, \left(\frac{4}{10}, \frac{2}{10}, \frac{9}{10}\right), \left(\frac{1}{10}, \frac{4}{10}, \frac{9}{10}\right) \rangle.$$

$$N^N Fr (M_1^*) \not\subseteq N^N Fr (N^N Fr (M_1^*))$$

**III. NEUTROSOPHIC NANO SEMI-FRONTIER**

In this section, we introduce the Neutrosophic Nano semi-frontier and their properties in N-N-T-S.

**Definition 4.1.**

Let  $M_1^*$  be a NNS in the N-N-T-S U. Then the Neutrosophic Nano semi-frontier of  $M_1^*$  is defined as

$$NN(S)Fr (M_1^*) = N^N(S)Cl (M_1^*) \cap N^N(S)Cl(M_1^{*c}).$$

Obviously  $N^N(S)Fr (M_1^*)$  is a NN(S)C set in U.

**Theorem 4.2.**

Let  $M_1^*$  be a NNS in the N-N-T-S U. Then the following conditions are holds :

(i)  $N^N(S)Cl (M_1^*) = M_1^* \cup \odot N^N Int (N^N Cl (M_1^*))$ ,

(ii)  $N^N(S)Int (M_1^*) = M_1^* \cap \odot N^N Cl (N^N Int (M_1^*))$ .

**Proof :** (i) Let  $M_1^*$  be a NNS in U. Consider

$$\begin{aligned} N^N Int (N^N Cl (M_1^* \cup \odot N^N Int (N^N Cl (M_1^*)))) & \\ = N^N Int (N^N Cl (M_1^*) \cup \odot N^N Cl (N^N Int (N^N Cl (M_1^*)))) & \\ = N^N Int (N^N Cl (M_1^*)) & \\ \subseteq \odot M_1^* \cup \odot N^N Int (N^N Cl (M_1^*)) & \end{aligned}$$

It follows that  $M_1^* \cup \odot N^N Int (N^N Cl (M_1^*))$  is a NN(S)C set in U.

$$\text{Hence } N^N(S)Cl (M_1^*) \subseteq \odot M_1^* \cup \odot N^N Int (N^N Cl (M_1^*)) \dots (1)$$

Use Prop  $N^N(S)Cl (M_1^*)$  is  $N^N(S)C$  set in



U. We have  $N^N Int (N^N Cl (M_1^*)) \subseteq \otimes N^N Int (N^N Cl (N^N(S)Cl (M_1^*))) \subseteq \otimes N^N(S)Cl (M_1^*)$ .

Thus  $M_1^* \cup \otimes N^N Int (N^N Cl (M_1^*)) \subseteq \otimes N^N(S)Cl (M_1^*) \dots (2)$ .

From (1) and (2),  $N^N(S)Cl (M_1^*) = M_1^* \cup \otimes N^N Int (N^N Cl (M_1^*))$ .

(ii) This can be proved in a similar manner as (i).

**Theorem 4.3.**

For a NNS  $M_1^*$  in the N-N-T-S U,  $N^N(S)Fr (M_1^*) = N^N(S)Fr (M_1^{*C})$ .

**Proof :** Let  $M_1^*$  be the NNS in the N-N-T-S U. Using Definition 4.1,

$$\begin{aligned} N^N(S)Fr (M_1^*) &= N^N(S)Cl (M_1^*) \cap \otimes N^N(S)Cl (M_1^{*C}) \\ &= N^N(S)Cl (M_1^{*C}) \cap \otimes N^N(S)Cl (M_1^*) \\ &= N^N(S)Cl (M_1^{*C}) \cap \otimes N^N(S)Cl (M_1^{*C})^C \end{aligned}$$

Again by Definition 4.1,

$$= N^N(S)Fr (M_1^{*C})$$

Hence  $N^N(S)Fr (M_1^*) = N^N(S)Fr (M_1^{*C})$ .

**Theorem 4.4.**

If  $M_1^*$  is NN(S)C set in U, then  $N^N(S)Fr (M_1^*) \subseteq \otimes M_1^*$ .

**Proof :** Let  $M_1^*$  be the NNS in the N-N-T-S U. Using Definition 4.1,

$$\begin{aligned} N^N(S)Fr (M_1^*) &= N^N(S)Cl (M_1^*) \cap \otimes N^N(S)Cl (M_1^{*C}) \\ &\subseteq \otimes N^N(S)Cl (M_1^*) = M_1^* \end{aligned}$$

Hence  $N^N(S)Fr (M_1^*) \subseteq \otimes M_1^*$ , if  $M_1^*$  is NN(S)C in U.

The converse of the above theorem is not true as shown by the following example.

**Example 4.5.**

Let U and  $\mathcal{A}$  be two non-empty finite sets,

where U is the universe and  $\mathcal{A}$  the set of attributes

$$U = \{F_1, F_2, F_3, F_4\} \text{ are Fruits}$$

Let  $U/R = \{\{ F_1, F_2, F_3\}, \{ F_4\}\}$  be an equivalence relation

$\mathcal{A} = \{\text{Proteins, minerals, vitamins}\}$  are three attributes ,its Neutrosophic values are given below

$$F_1 = \left\langle \left( \frac{4}{10}, \frac{5}{10}, \frac{3}{10} \right), \left( \frac{3}{10}, \frac{2}{10}, \frac{2}{10} \right), \left( \frac{9}{10}, \frac{5}{10}, \frac{8}{10} \right) \right\rangle$$

$$F_2 = \left\langle \left( \frac{2}{10}, \frac{4}{10}, \frac{5}{10} \right), \left( \frac{1}{10}, \frac{1}{10}, \frac{2}{10} \right), \left( \frac{6}{10}, \frac{5}{10}, \frac{8}{10} \right) \right\rangle$$

$$F_3 = \left\langle \left( \frac{5}{10}, \frac{5}{10}, \frac{3}{10} \right), \left( \frac{4}{10}, \frac{2}{10}, \frac{2}{10} \right), \left( \frac{9}{10}, \frac{5}{10}, \frac{8}{10} \right) \right\rangle$$

$$F_4 = \left\langle \left( \frac{4}{10}, \frac{4}{10}, \frac{5}{10} \right), \left( \frac{2}{10}, \frac{2}{10}, \frac{2}{10} \right), \left( \frac{6}{10}, \frac{5}{10}, \frac{8}{10} \right) \right\rangle$$

$$N_N(\tau) = \{0_{N_N}, 1_{N_N}, \overline{N(M)}, \overline{N(M)}, B_N(M)\}$$

$$\overline{N(F)} = \left\langle \left( \frac{2}{10}, \frac{4}{10}, \frac{5}{10} \right), \left( \frac{1}{10}, \frac{1}{10}, \frac{2}{10} \right), \left( \frac{6}{10}, \frac{5}{10}, \frac{8}{10} \right) \right\rangle$$

$$\overline{N(F)} = \left\langle \left( \frac{5}{10}, \frac{5}{10}, \frac{3}{10} \right), \left( \frac{4}{10}, \frac{2}{10}, \frac{2}{10} \right), \left( \frac{9}{10}, \frac{5}{10}, \frac{8}{10} \right) \right\rangle$$

$$B_N(F) = \left\langle \left( \frac{5}{10}, \frac{5}{10}, \frac{3}{10} \right), \left( \frac{2}{10}, \frac{2}{10}, \frac{2}{10} \right), \left( \frac{8}{10}, \frac{5}{10}, \frac{8}{10} \right) \right\rangle$$

$$\begin{aligned} N_N(\tau) = \{0_{N_N}, 1_{N_N}, &\left\langle \left( \frac{2}{10}, \frac{4}{10}, \frac{5}{10} \right), \left( \frac{1}{10}, \frac{1}{10}, \frac{2}{10} \right), \left( \frac{6}{10}, \frac{5}{10}, \frac{8}{10} \right) \right\rangle, \\ &\left\langle \left( \frac{5}{10}, \frac{5}{10}, \frac{3}{10} \right), \left( \frac{4}{10}, \frac{2}{10}, \frac{2}{10} \right), \left( \frac{9}{10}, \frac{5}{10}, \frac{8}{10} \right) \right\rangle, \\ &\left\langle \left( \frac{5}{10}, \frac{5}{10}, \frac{3}{10} \right), \left( \frac{4}{10}, \frac{2}{10}, \frac{2}{10} \right), \left( \frac{9}{10}, \frac{5}{10}, \frac{8}{10} \right) \right\rangle, \\ &\left\langle \left( \frac{5}{10}, \frac{5}{10}, \frac{3}{10} \right), \left( \frac{2}{10}, \frac{2}{10}, \frac{2}{10} \right), \left( \frac{8}{10}, \frac{5}{10}, \frac{8}{10} \right) \right\rangle, \\ &\left\langle \left( \frac{4}{10}, \frac{4}{10}, \frac{5}{10} \right), \left( \frac{2}{10}, \frac{2}{10}, \frac{2}{10} \right), \left( \frac{6}{10}, \frac{5}{10}, \frac{8}{10} \right) \right\rangle\} \end{aligned}$$

$M_1^* \left\langle \left( \frac{2}{10}, \frac{5}{10}, \frac{5}{10} \right), \left( \frac{2}{10}, \frac{7}{10}, \frac{2}{10} \right), \left( \frac{7}{10}, \frac{5}{10}, \frac{5}{10} \right) \right\rangle$ , is Neutrosophic Nano semi-closed set

Then  $N^N(S)Fr (M_1^*) \subseteq M_1^*$

**Theorem 4.6.**

If  $M_1^*$  is NNSO set in U, then  $N^N(S)Fr (M_1^*) \subseteq M_1^{*C}$

**Proof :** Let  $M_1^*$  be the NNS in the N-N-T-S U. Using Proposition 4.3 [18],

$M_1^*$  is NNSO set implies  $M_1^{*C}$  is NN(S)C set in U. By Theorem 3.4,  $N^N(S)Fr (M_1^{*C}) \subseteq M_1^{*C}$  and by Theorem 3.3, we get  $N^N(S)Fr (M_1^*) \subseteq M_1^{*C}$

**Theorem 4.7.**

Let  $M_1^* \subseteq M_2^*$  and  $M_2^* \in N^N(S)C (U)$  ( resp.,  $M_2^* \in N^N(S)SO (U)$  ). Then  $N^N(S)Fr (M_1^*) \subseteq M_2^*$  ( resp.,  $N^N(S)Fr (M_1^*) \subseteq M_2^{*C}$ , where  $N^N(S)C (U)$  ( resp.,  $N^N(S)SO (U)$  ) denotes the class of Neutrosophic Nano semi-closed ( resp., Neutrosophic Nano semi-open) sets in U.

**Proof :** Use Prop., 6.3 (iv) [18],  $M_1^* \subseteq M_2^*$ ,

$$N^N(S)Cl (M_1^*) \subseteq N^N(S)Cl (M_2^*) \text{ ----- (1)}$$

By Definition 4.1,

$$\begin{aligned} N^N(S)Fr (M_1^*) &= N^N(S)Cl(M_1^*) \cap N^N(S)Cl (M_1^{*C}) \\ &\subseteq N^N(S)Cl(M_2^*) \cap N^N(S)Cl (M_1^{*C}) \text{ by (1)} \\ &\subseteq N^N(S)Cl (M_2^*) \text{ Use Prop., 6.3 (ii) [18],} = M_2^* \end{aligned}$$

Hence  $N^N(S)Fr (M_1^*) \subseteq M_2^*$ .

**Theorem 4.8.**

Let  $M_1^*$  be the NNS in the N-N-T-S U. Then  $(N^N(S)Fr (M_1^*))^C = N^N(S)Int (M_1^*) \cup N^N(S)Int (M_1^{*C})$ .

**Proof :** Let  $M_1^*$  be the NNS in the N-N-T-S U. Using Definition 4.1,

$$\begin{aligned} (N^N(S)Fr (M_1^*))^C &= (N^N(S)Cl (M_1^*) \cap N^N(S)Cl (M_1^{*C}))^C \text{ Use Prop., 3.2 (1) [18],} \\ &= (N^N(S)Cl (M_1^*))^C \cup (N^N(S)Cl (M_1^{*C}))^C \text{ Use Prop., 6.2 (ii) [18],} \\ &= N^N(S)Int (M_1^{*C}) \cup N^N(S)Int (M_1^*) \end{aligned}$$

Hence  $(N^N(S)Fr (M_1^*))^C = N^N(S)Int (M_1^*) \cup N^N(S)Int (M_1^{*C})$ .

**Theorem 4.9.**

For a NNS  $M_1^*$  in the N-N-T-S U, then  $N^N(S)Fr (M_1^*) \subseteq N^N(S)Fr (M_1^*)$ .

**Proof :** Let  $M_1^*$  be the NNS in the N-N-T-S U. Using Proposition 6.4 [18],

$$\begin{aligned} N^N(S)Cl (M_1^*) &\subseteq N^N(S)Cl (M_1^*) \text{ and } N^N(S)Cl (M_1^{*C}) \subseteq N^N(S)Cl (M_1^{*C}). \text{ Now by Definition 4.1,} \\ N^N(S)Fr (M_1^*) &= N^N(S)Cl (M_1^*) \cap N^N(S)Cl (M_1^{*C}) \subseteq N^N(S)Cl (M_1^*) \cap N^N(S)Cl (M_1^{*C}) \text{ By Definition 3.1,} \\ &= N^N(S)Fr (M_1^*) \end{aligned}$$

Hence  $N^N(S)Fr (M_1^*) \subseteq N^N(S)Fr (M_1^*)$ .

**Theorem 4.10.**

For a NNS  $M_1^*$  in the N-N-T-S U, then  $N^N(S)Cl (N^N(S)Fr (M_1^*)) \subseteq N^N(S)Fr (M_1^*)$ .

**Proof :** Let  $M_1^*$  be the NNS in the N-N-T-S U. Using Definition 4.1,

$$\begin{aligned} N^N(S)Cl (N^N(S)Fr (M_1^*)) &= N^N(S)Cl (N^N(S)Cl (M_1^*) \cap N^N(S)Cl ((M_1^{*C}))) \\ &\subseteq N^N(S)Cl (N^N(S)Cl (M_1^*)) \cap N^N(S)Cl (N^N(S)Cl ((M_1^{*C}))) \text{ Use Prop., 6.3 (iii) [18],} \\ &= N^N(S)Cl (M_1^*) \cap N^N(S)Cl ((M_1^{*C})) \text{ By Definition 4.1,} \\ &= N^N(S)Fr (M_1^*) \text{ By Theorem 3.10,} \\ &\subseteq N^N(S)Fr (M_1^*) \end{aligned}$$

Hence  $N^N(S)Cl (N^N(S)Fr (M_1^*)) \subseteq N^N(S)Fr (M_1^*)$ .

**Theorem 4.11**

Let  $M_1^*$  be a  $N^N(S)$  in the N-N-T-S U. Then  $N^N(S)Fr (M_1^*) = N^N(S)Cl (M_1^*) - N^N(S)Int (M_1^*)$ .

**Proof :**

Let  $M_1^*$  be the NNS in the N-N-T-S U. Use Prop., 6.2 (ii) [18],

$$\begin{aligned} (N^N(S)Cl (M_1^{*C}))^C &= N^N(S)Int (M_1^*) \text{ and by Definition 4.1,} \\ N^N(S)Fr (M_1^*) &= N^N(S)Cl (M_1^*) \cap N^N(S)Cl ((M_1^{*C})) \\ &= N^N(S)Cl (M_1^*) - (N^N(S)Cl ((M_1^{*C})))^C \text{ by using } M_1^* - M_2^* = M_1^* \cap (M_2^{*C}) \text{ Use Prop., 6.2 (ii) [18],} \\ &= N^N(S)Cl (M_1^*) - N^N(S)Int (M_1^*) \end{aligned}$$

Hence  $N^N(S)Fr (M_1^*) = N^N(S)Cl (M_1^*) - N^N(S)Int (M_1^*)$ .

**Theorem 4.12.**

For a NNS  $M_1^*$  in the N-N-T-S U, then  $N^N(S)Fr (N^N(S)Int (M_1^*)) \subseteq N^N(S)Fr (M_1^*)$ .

**Proof :** Let  $M_1^*$  be the NNS in the N-N-T-S U. Using Definition 4.1,

$$N^N(S)Fr (N^N(S)Int (M_1^*)) = N^N(S)Cl (N^N(S)Int (M_1^*)) \cap N^N(S)Cl ((N^N(S)Int (M_1^*))^C) \text{ Use Prop., 6.2 (i) [18],}$$

$$\begin{aligned}
 &= N^N(S)Cl(N^N(S)Int(M_1^*)) \cap N^N(S)Cl(N^N(S)Cl((M_1^{*C}))) \text{ Use Prop., 6.3 (iii) [18],} \\
 &= N^N(S)Cl(N^N(S)Int(M_1^*)) \cap N^N(S)Cl(M_1^{*C}) \text{ Use Prop., 5.2 (ii) [18],} \\
 &\subseteq N^N(S)Cl(M_1^*) \cap N^N(S)Cl(M_1^{*C}) \text{ By Definition 4.1,} \\
 &= N^N(S)Fr(M_1^*)
 \end{aligned}$$

Hence  $N^N(S)Fr(N^N(S)Int(M_1^*)) \subseteq N^N(S)Fr(M_1^*)$ .

**Example 4.13.**

Let  $U$  and  $\mathcal{A}$  be two non-empty finite sets, where  $U$  is the universe and  $\mathcal{A}$  the set of attributes

$U = \{P_1, P_2, P_3, P_4\}$  are Patients

Let  $U/R = \{\{P_1, P_2, P_3, P_4\}, \{P_5\}\}$  be an equivalence relation

$\mathcal{A} = \{\text{Head ache, Temperature, Cold}\}$  are three attributes

its Neutrosophic values are given below

$$P_1 = \left\langle \left(\frac{3}{10}, \frac{4}{10}, \frac{2}{10}\right), \left(\frac{5}{10}, \frac{6}{10}, \frac{7}{10}\right), \left(\frac{9}{10}, \frac{5}{10}, \frac{2}{10}\right) \right\rangle$$

$$P_2 = \left\langle \left(\frac{3}{10}, \frac{5}{10}, \frac{1}{10}\right), \left(\frac{4}{10}, \frac{6}{10}, \frac{2}{10}\right), \left(\frac{8}{10}, \frac{4}{10}, \frac{6}{10}\right) \right\rangle$$

$$P_3 = \left\langle \left(\frac{3}{10}, \frac{5}{10}, \frac{1}{10}\right), \left(\frac{4}{10}, \frac{6}{10}, \frac{2}{10}\right), \left(\frac{8}{10}, \frac{4}{10}, \frac{6}{10}\right) \right\rangle$$

$$P_4 = \left\langle \left(\frac{3}{10}, \frac{4}{10}, \frac{2}{10}\right), \left(\frac{4}{10}, \frac{6}{10}, \frac{7}{10}\right), \left(\frac{8}{10}, \frac{4}{10}, \frac{6}{10}\right) \right\rangle$$

$$P_5 = \left\langle \left(\frac{5}{10}, \frac{6}{10}, \frac{1}{10}\right), \left(\frac{6}{10}, \frac{7}{10}, \frac{1}{10}\right), \left(\frac{9}{10}, \frac{5}{10}, \frac{2}{10}\right) \right\rangle$$

$$N_N(\tau) = \{0_{N_N}, 1_{N_N}, N(M), \overline{N(M)}, B_N(M)\}$$

$$\frac{N(F)}{N(F)} = \left\langle \left(\frac{3}{10}, \frac{4}{10}, \frac{2}{10}\right), \left(\frac{4}{10}, \frac{6}{10}, \frac{7}{10}\right), \left(\frac{8}{10}, \frac{4}{10}, \frac{6}{10}\right) \right\rangle$$

$$\frac{N(F)}{N(F)} = \left\langle \left(\frac{3}{10}, \frac{5}{10}, \frac{1}{10}\right), \left(\frac{5}{10}, \frac{6}{10}, \frac{2}{10}\right), \left(\frac{9}{10}, \frac{5}{10}, \frac{2}{10}\right) \right\rangle$$

$$B_N(F) = \left\langle \left(\frac{2}{10}, \frac{5}{10}, \frac{3}{10}\right), \left(\frac{5}{10}, \frac{6}{10}, \frac{4}{10}\right), \left(\frac{6}{10}, \frac{5}{10}, \frac{6}{10}\right) \right\rangle$$

$$\begin{aligned}
 N_N(\tau) = \{ &0_{N_N}, 1_{N_N}, \left\langle \left(\frac{3}{10}, \frac{4}{10}, \frac{2}{10}\right), \left(\frac{4}{10}, \frac{6}{10}, \frac{7}{10}\right), \left(\frac{8}{10}, \frac{4}{10}, \frac{6}{10}\right) \right\rangle, \left\langle \left(\frac{3}{10}, \frac{5}{10}, \frac{1}{10}\right), \left(\frac{5}{10}, \frac{6}{10}, \frac{2}{10}\right), \left(\frac{9}{10}, \frac{5}{10}, \frac{2}{10}\right) \right\rangle \right. \\
 &\left. \left\langle \left(\frac{2}{10}, \frac{5}{10}, \frac{3}{10}\right), \left(\frac{5}{10}, \frac{6}{10}, \frac{4}{10}\right), \left(\frac{6}{10}, \frac{5}{10}, \frac{6}{10}\right) \right\rangle, \left\langle \left(\frac{5}{10}, \frac{6}{10}, \frac{1}{10}\right), \left(\frac{6}{10}, \frac{7}{10}, \frac{1}{10}\right), \left(\frac{9}{10}, \frac{5}{10}, \frac{2}{10}\right) \right\rangle \right\}
 \end{aligned}$$

$$M_1^* = \left\langle \left(\frac{2}{10}, \frac{6}{10}, \frac{5}{10}\right), \left(\frac{5}{10}, \frac{4}{10}, \frac{5}{10}\right), \left(\frac{6}{10}, \frac{5}{10}, \frac{7}{10}\right) \right\rangle \text{ is a NNSO}$$

Therefore  $N^N(S)Fr(M_1^*) \not\subseteq N^N(S)Fr(N^N(S)Int(M_1^*))$ .

**Theorem 4.14.**

For a NNS  $M_1^*$  in the  $N-N-T-S$   $U$ , then  $N^N(S)Fr(N^N(S)Cl(M_1^*)) \subseteq N^N(S)Fr(M_1^*)$ .

**Proof :**

Let  $M_1^*$  be the  $N^N S$  in the  $N-N-T-S$   $U$ . Using Definition 4.1,

$$N^N(S)Fr(N^N(S)Cl(M_1^*)) = N^N(S)Cl(N^N(S)Cl(M_1^*)) \cap N^N(S)Cl((N^N(S)Cl(M_1^*)))^C$$

Use Prop., 6.3 (iii) and Proposition 6.2 (ii) [18],

$$= N^N(S)Cl(M_1^*) \cap N^N(S)Cl(N^N(S)Int(M_1^{*C})) \text{ Use Prop., 5.2 (i) [18],}$$

$$\subseteq N^N(S)Cl(M_1^*) \cap N^N(S)Cl(M_1^{*C}) \text{ By Definition 4.1,}$$

$$= N^N(S)Fr(M_1^*)$$

Hence  $N^N(S)Fr(N^N(S)Cl(M_1^*)) \subseteq N^N(S)Fr(M_1^*)$ .

**Remark 4.15.**

In general topology, the following conditions conditions are hold :

$$N^N(S)Fr(M_1^*) \cap \otimes N^N(S)Int(M_1^*) = 0_N,$$

$$N^N(S)Int(M_1^*) \cup \otimes N^N(S)Fr(M_1^*) = N^N(S)Cl(M_1^*),$$

$$N^N(S)Int(M_1^*) \cup \otimes N^N(S)Int(M_1^{*C}) \cup \otimes N^N(S)Fr(M_1^*) = 1_{N_N}.$$

**Theorem 4.16.**

Let  $M_1^*$  and  $M_2^*$  be NNSs in the  $N-N-T-S$   $U$ .

Then  $N^N(S)Fr(M_1^* \cup \otimes M_2^*) \subseteq \otimes N^N(S)Fr(M_1^*) \cup \otimes N^N(S)Fr(M_2^*)$ .

**Proof :** Let  $M_1^*$  and  $M_2^*$  be NNSs in the  $N-N-T-S$   $U$ . Using Definition 4.1,

$$\begin{aligned}
 N^N(S)Fr(M_1^* \cup M_2^*) &= N^N(S)Cl(M_1^* \cup M_2^*) \cap N^N(S)Cl(M_1^* \cup M_2^*)^c. \text{ By Propositon (2.4),} \\
 &= N^N(S)Cl(M_1^* \cup M_2^*) \cap N^N(S)Cl((M_1^{*c}) \cap (M_2^{*c})) \text{ Use Prop., 6.5 (i) and (ii) [18],} \\
 &\subseteq \ominus(N^N(S)Cl(M_1^*) \cup N^N(S)Cl(M_2^*)) \cap (N^N(S)Cl(M_1^{*c}) \cap N^N(S)Cl(M_2^{*c})) \\
 &= [(N^N(S)Cl(M_1^*) \cup N^N(S)Cl(M_2^*)) \cap N^N(S)Cl((M_1^{*c}) \cap (N^N(S)Cl(M_1^*) \cup N^N(S)Cl(M_2^*))) \\
 &\cap N^N(S)Cl(M_2^{*c})] \\
 &= [(N^N(S)Cl(M_1^*) \cap N^N(S)Cl(M_1^{*c})) \cup (N^N(S)Cl(M_2^*) \cap N^N(S)Cl(M_1^{*c}))] \\
 &\cap [(N^N(S)Cl(M_1^*) \cap N^N(S)Cl(M_2^{*c})) \cup (N^N(S)Cl(M_2^*) \cap N^N(S)Cl(M_2^{*c}))] \text{ By Definition 4.1,} \\
 &= [N^N(S)Fr(M_1^*) \cup (N^N(S)Cl(M_2^*) \cap N^N(S)Cl(M_1^{*c}))] \cap [(N^N(S)Cl(M_1^*) \cap N^N(S)Cl(M_2^{*c})) \\
 &\cup N^N(S)Fr(M_2^*)] \\
 &= (N^N(S)Fr(M_1^*) \cup N^N(S)Fr(M_2^*)) \cap [(N^N(S)Cl(M_2^*) \cap N^N(S)Cl(M_1^{*c})) \cup (N^N(S)Cl(M_1^*) \cap N^N(S)Cl \\
 &(M_2^{*c}))] \\
 &\subseteq \ominus N^N(S)Fr(M_1^*) \cup \ominus N^N(S)Fr(M_2^*).
 \end{aligned}$$

Hence  $N^N(S)Fr(M_1^* \cup M_2^*) \subseteq \ominus N^N(S)Fr(M_1^*) \cup \ominus N^N(S)Fr(M_2^*)$ .

**Example 4.17.**

Let U and A be two non-empty finite sets,

where U is the universe and A the set of attributes

$$U = \{P_1, P_2, P_3, P_4\} \text{ are Patients}$$

Let  $U/R = \{\{P_1, P_2, P_3\}, \{P_4\}\}$  be an equivalence relation

$A = \{\text{Temperature}\}$  are one attributes

$$U/R = \{P_1\} \{P_2, P_3, P_4\}$$

$$P_1 = \left\langle \left( \frac{6}{10}, \frac{5}{10}, \frac{4}{10} \right) \right\rangle$$

$$P_2 = \left\langle \left( \frac{4}{10}, \frac{5}{10}, \frac{7}{10} \right) \right\rangle$$

$$P_3 = \left\langle \left( \frac{6}{10}, \frac{5}{10}, \frac{4}{10} \right) \right\rangle$$

$$P_4 = \left\langle \left( \frac{3}{10}, \frac{5}{10}, \frac{6}{10} \right) \right\rangle.$$

Then  $N_N(\tau) = \{0_{N_N}, 1_{N_N}, \underline{N(M)}, \overline{N(M)}, B_N(M)\}$

$$\underline{N(F)} = \left\langle \left( \frac{4}{10}, \frac{5}{10}, \frac{7}{10} \right) \right\rangle$$

$$\overline{N(F)} = \left\langle \left( \frac{6}{10}, \frac{5}{10}, \frac{4}{10} \right) \right\rangle$$

$$B_N(F) = \left\langle \left( \frac{6}{10}, \frac{5}{10}, \frac{4}{10} \right) \right\rangle$$

$$N_N(\tau) = \{0_{N_N}, 1_{N_N}, \left\langle \left( \frac{4}{10}, \frac{5}{10}, \frac{7}{10} \right) \right\rangle, \left\langle \left( \frac{6}{10}, \frac{5}{10}, \frac{4}{10} \right) \right\rangle, \left\langle \left( \frac{6}{10}, \frac{5}{10}, \frac{4}{10} \right) \right\rangle, \left\langle \left( \frac{3}{10}, \frac{8}{10}, \frac{6}{10} \right) \right\rangle\}$$

$$N_N(\tau) = \{0_{N_N}, 1_{N_N}, \left\langle \left( \frac{4}{10}, \frac{5}{10}, \frac{7}{10} \right) \right\rangle, \left\langle \left( \frac{6}{10}, \frac{5}{10}, \frac{4}{10} \right) \right\rangle, \left\langle \left( \frac{3}{10}, \frac{5}{10}, \frac{6}{10} \right) \right\rangle\}$$

$$M_1^* = \left\langle \left( \frac{4}{10}, \frac{5}{10}, \frac{6}{10} \right) \right\rangle, M_2^* = \left\langle \left( \frac{6}{10}, \frac{5}{10}, \frac{4}{10} \right) \right\rangle \text{ are NN(S)C}$$

$$M_1^* \cup M_2^* = \left\langle \left( \frac{6}{10}, \frac{5}{10}, \frac{4}{10} \right) \right\rangle$$

$$N^N(S)Fr(M_1^* \cup M_2^*) \subseteq \ominus N^N(S)Fr(M_1^*) \cup \ominus N^N(S)Fr(M_2^*).$$

**Theorem 4.18.**

For any NNSs  $M_1^*$  and  $M_2^*$  in the N-N-T-S U,

$$N^N(S)Fr(M_1^* \cap M_2^*) \subseteq \ominus (N^N(S)Fr(M_1^*) \cap \ominus N^N(S)Cl(M_2^*)) \cup (N^N(S)Fr(M_2^*) \cap \ominus N^N(S)Cl(M_1^*)).$$

**Proof :** Let  $M_1^*$  and  $M_2^*$  be  $N^N$ Ss in the  $N^N$ TS U. Using Definition 4.1,

$$N^N(S)Fr(M_1^* \cap M_2^*) = N^N(S)Cl(M_1^* \cap M_2^*) \cap \ominus N^N(S)Cl(M_1^* \cap M_2^*)^c \text{ Use Prop., 3.2 (1) [18],}$$

$$= N^N(S)Cl(M_1^* \cap M_2^*) \cap \ominus N^N(S)Cl((M_1^{*c}) \cup (M_2^{*c})) \text{ Use Prop., 6.5 (ii) and (i) [18],}$$

$$\subseteq \ominus (N^N(S)Cl(M_1^*) \cap N^N(S)Cl(M_2^*)) \cap (N^N(S)Cl((M_1^{*c}) \cup N^N(S)Cl((M_2^{*c})))$$

$$= [(N^N(S)Cl(M_1^*) \cap \ominus N^N(S)Cl(M_2^*)) \cap \ominus N^N(S)Cl((M_1^{*c}) \cup [(N^N(S)Cl(M_1^*) \cap \ominus N^N(S)Cl(M_2^*)) \cap \ominus N^N(S)Cl(M_2^{*c})])] \text{ By Definition 4.1,}$$

$$= (N^N(S)Fr(M_1^*) \cap \ominus N^N(S)Cl(M_2^*)) \cup \ominus (N^N(S)Fr(M_2^*) \cap N^N(S)Cl(M_1^*)$$

$$\text{Hence } N^N(S)Fr(M_1^* \cap M_2^*) \subseteq \ominus (N^N(S)Fr(M_1^*) \cap \ominus N^N(S)Cl(M_2^*)) \cup (N^N(S)Fr(M_2^*) \cap \ominus N^N(S)Cl(M_1^*)).$$

**Corollary 4.19**

For any NNSs  $M_1^*$  and  $M_2^*$  in the N-N-T-S U,

$$N^N(S)Fr (M_1^* \cap M_2^*) \subseteq \ominus N^N(S)Fr (M_1^*) \cup \ominus N^N(S)Fr (M_2^*).$$

**Proof :** Let  $M_1^*$  and  $M_2^*$  be NNSs in the N-N-T-S U. Using Definition 4.1,

$$N^N(S)Fr (M_1^* \cap M_2^*) = N^N(S)Cl (M_1^* \cap M_2^*) \cap \ominus N^N(S)Cl (M_1^* \cap M_2^*)^c \text{ Use Prop., 3.2 (1) [18] ,}$$

$$= N^N(S)Cl (M_1^* \cap M_2^*) \cap \ominus N^N(S)Cl ((M_1^{*c}) \cup (M_2^{*c})),$$

$$\subseteq \ominus (N^N(S)Cl (M_1^*) \cap N^N(S)Cl (M_2^*)) \cap (N^N(S)Cl ((M_1^{*c}) \cup (M_2^{*c})))$$

$$= (N^N(S)Cl (M_1^*) \cap \ominus N^N(S)Cl (M_2^*)) \cap \ominus N^N(S)Cl ((M_1^{*c}) \cup (N^N(S)Cl (M_1^*) \cap \ominus N^N(S)Cl (M_2^*) \cap \ominus N^N(S)Cl ((M_2^{*c})))$$

By Definition 4.1,

$$= (N^N(S)Fr (M_1^*) \cap \ominus N^N(S)Cl (M_2^*)) \cup \ominus (N^N(S)Cl (M_1^*) \cap \ominus N^N(S)Fr (M_2^*)) \subseteq \ominus N^N(S)Fr (M_1^*) \cup \ominus N^N(S)Fr (M_2^*).$$

Hence  $N^N(S)Fr (M_1^* \cap M_2^*) \subseteq \ominus N^N(S)Fr (M_1^*) \cup \ominus N^N(S)Fr (M_2^*)$ .

#### Theorem 4.20

For any NNS  $M_1^*$  in the N-N-T-S U,

$$(1) N^N(S)Fr (N^N(S)Fr (M_1^*)) \subseteq \ominus N^N(S)Fr (M_1^*),$$

$$(2) N^N(S)Fr (N^N(S)Fr (N^N(S)Fr (M_1^*))) \subseteq \ominus N^N(S)Fr (N^N(S)Fr (M_1^*)).$$

**Proof :** (1) Let  $M_1^*$  be the NNS in the N-N-T-S U. Using Definition 4.1,

$$N^N(S)Fr (N^N(S)Fr (M_1^*)) = N^N(S)Cl (N^N(S)Fr (M_1^*)) \cap \ominus N^N(S)Cl ((N^N(S)Fr (M_1^*))^c (N^N(S)Fr (M_1^*)))$$

By Definition 4.1,

$$= N^N(S)Cl (N^N(S)Cl (M_1^*) \cap \ominus N^N(S)Cl (M_1^{*c})) \cap N^N(S)Cl (((N^N(S)Cl (M_1^*) \cap N^N(S)Cl ((M_1^{*c})))^c))$$

Use Prop., 6.3 (iii) and 6.2 (ii) [18] ,

$$\subseteq \ominus (N^N(S)Cl (N^N(S)Cl (M_1^*)) \cap \ominus N^N(S)Cl (N^N(S)Cl ((M_1^{*c}))) \cap N^N(S)Cl (N^N(S)Int ((M_1^{*c})))$$

$$\cup \ominus N^N(S)Int (M_1^*)) \text{ Use Prop., 6.3 (iii) [18] ,}$$

$$= (N^N(S)Cl (M_1^*) \cap \ominus N^N(S)Cl (M_1^{*c})) \cap \ominus (N^N(S)Cl (N^N(S)Int (M_1^{*c})) \cup \ominus N^N(S)Cl (N^N(S)Int (M_1^*)))$$

$$\subseteq \ominus N^N(S)Cl (M_1^*) \cap \ominus N^N(S)Cl (M_1^{*c}) \text{ By Definition 4.1,}$$

$$= N^N(S)Fr (M_1^*)$$

Therefore  $N^N(S)Fr (N^N(S)Fr (M_1^*)) \subseteq \ominus N^N(S)Fr (M_1^*)$ . (2) By Definition 4.1,

$$N^N(S)Fr (N^N(S)Fr (N^N(S)Fr (M_1^*))) = N^N(S)Cl (N^N(S)Fr (N^N(S)Fr (M_1^*))) \cap \ominus N^N(S)Cl ((N^N(S)Fr (N^N(S)Fr (M_1^*)))^c)$$

Use Prop., 6.3 (iii) [18] ,

$$\subseteq \ominus (N^N(S)Fr (N^N(S)Fr (M_1^*))) \cap \ominus N^N(S)Cl ((N^N(S)Fr (N^N(S)Fr (M_1^*)))^c (N^N(S)Fr (N^N(S)Fr (M_1^*))))$$

$$\subseteq \ominus N^N(S)Fr (N^N(S)Fr (M_1^*)).$$

Hence  $N^N(S)Fr (N^N(S)Fr (N^N(S)Fr (M_1^*))) \subseteq \ominus N^N(S)Fr (N^N(S)Fr (M_1^*))$ .

#### Conclusion

This research article shared some fundamental properties of introduce the Neutrosophic Nano semi-frontier .This concepts for further research will be on elaborating the structure of Neutrosophic Nano topology to more new classes of weak and strong forms of nano-open sets, new classes of generalized sets and new classes of continuous functions. There is further scope of launching into wider applications of Neutrosophic nano topology in different branches of Sciences and Humanities.

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