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Neutrosophic g^* -Closed Sets and its maps

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Abstract: Topology is one of the classical subjects in Mathematics. A lot of researchers have published their ideas.As a generalization of topological concepts many new kind of closed and open sets are published continuously. Salama presented Neutrosophic topological spaces by using Smarandache 's Neutrosophic sets. Many Researchers introduced so many closed sets in Neutrosophic topological spaces. Purpose of this research paper is we introduce Neutrosophic g^* -Closed sets and Neutrosophic g^* -open sets in Neutrosophic topological spaces. Also we study about study about mappings of Neutrosophic g^* -Closed sets

Keywords: Neutrosophic g -Closed sets Neutrosophic g^* -Closed sets, Neutrosophic g^* -open sets, Neutrosophic g^* -continuous.

1. Introduction

Smarandache [10,11] characterized the Neutrosophic set on three segment Neutrosophic sets(T Truth, I-Indeterminacy, F-Falsehood). Neutrosophic topological spaces(NS-T-S) presented by Salama [19,20]et al. Neutrosophic have wide scope of constant applications for the fields of Electrical & Electronic, Artificial Intelligence, Mechanics, Computer Science, Information Systems, Applied Mathematics , basic leadership. Prescription and Management Science and so on.

Neutrosophic semi closed, α - closed, pre closed and regular closed sets are introduced by I. Arokiarani[6] et al.,R.Dhavaseelan[8] et al. introduced Neutrosophic g closed sets and $g\alpha$ closed sets .Point of this paper is R .Dhavaseelan[9] and S.Jafari, are introduced Generalized Neutrosophic Closed sets . D.Jayanthi [13]presented αG Closed Sets in Neutrosophic Topological Spaces, V.K.Shanthi [22] developed Neutrosophic gs and sg closed set. C.Mahesawri[14,15] et al introduced Neutrosophic gb closed sets.

Aim of this present paper is, we introduce and study the concepts of Neutrosophic g^* -Closed sets and Neutrosophic g^* -open sets in Neutrosophic topological spaces. Also we study about mappings of Neutrosophic g^* -Closed sets

2. Preliminaries

In this section, we recall required and necessary definition and results of Neutrosophic sets

Definition 2.1 [16,17] Let Nu_X^* be a non-empty fixed set. A Neutrosophic set W_1^* is a object having the form $W_1^* = \{ \langle w, \mu_{W_1^*}(w), \sigma_{W_1^*}(w), \gamma_{W_1^*}(w) \rangle : w \in Nu_X^* \}$,

$\mu_{W_1^*}(w)$ - membership function

$\sigma_{W_1^*}(w)$ - Indeterminacy function

$\gamma_{W_1^*}(w)$ - Non-Membership function

Definition 2.2 [16,17]. Neutrosophic set $W_1^* = \{ \langle w, \mu_{W_1^*}(w), \sigma_{W_1^*}(w), \gamma_{W_1^*}(w) \rangle : w \in Nu_X^* \}$, on Nu_X^* and $\forall w \in Nu_X^*$ then complement of W_1^* is

$$W_1^{*C} = \{ \langle w, \gamma_{W_1^*}(w), 1 - \sigma_{W_1^*}(w), \mu_{W_1^*}(w) \rangle : w \in Nu_X^* \}$$

Definition 2.3 [16,17]. Let W_1^* and W_2^* are two Neutrosophic sets, $\forall w \in Nu_X^*$

$$W_1^* = \{ \langle w, \mu_{W_1^*}(w), \sigma_{W_1^*}(w), \gamma_{W_1^*}(w) \rangle : w \in Nu_X^* \}$$

$$W_2^* = \{ \langle w, \mu_{W_2^*}(w), \sigma_{W_2^*}(w), \gamma_{W_2^*}(w) \rangle : w \in Nu_X^* \}$$

Then $W_1^* \subseteq W_2^* \Leftrightarrow \mu_{W_1^*}(w) \leq \mu_{W_2^*}(w), \sigma_{W_1^*}(w) \leq \sigma_{W_2^*}(w) \ \& \ \gamma_{W_1^*}(w) \geq \gamma_{W_2^*}(w)$

Definition 2.4[16,17]. Let Nu_X^* be a non-empty set, and Let W_1^* and W_2^* be two Neutrosophic sets are

$$W_1^* = \{ \langle w, \mu_{W_1^*}(w), \sigma_{W_1^*}(w), \gamma_{W_1^*}(w) \rangle : w \in Nu_X^* \}, W_2^* = \{ \langle w, \mu_{W_2^*}(w), \sigma_{W_2^*}(w), \gamma_{W_2^*}(w) \rangle : w \in Nu_X^* \}$$

Then $W_1^* \cap W_2^* = \{ \langle w, \mu_{W_1^*}(w) \cap \mu_{W_2^*}(w), \sigma_{W_1^*}(w) \cup \sigma_{W_2^*}(w), \gamma_{W_1^*}(w) \cup \gamma_{W_2^*}(w) \rangle : w \in Nu_X^* \}$

$$W_1^* \cup W_2^* = \{ \langle w, \mu_{W_1^*}(w) \cup \mu_{W_2^*}(w), \sigma_{W_1^*}(w) \cup \sigma_{W_2^*}(w), \gamma_{W_1^*}(w) \cap \gamma_{W_2^*}(w) \rangle : w \in Nu_X^* \}$$

Definition 2.5 [19,20]. Let Nu_X^* be non-empty set and Nu_τ be the collection of Neutrosophic subsets of Nu_X^* satisfying the accompanying properties:

1. $0_{Nu}, 1_{Nu} \in Nu_\tau$
2. $Nu_{T_1} \cap Nu_{T_2} \in Nu_\tau$ for any $Nu_{T_1}, Nu_{T_2} \in Nu_\tau$
3. $\cup Nu_{T_i} \in Nu_\tau$ for every $\{Nu_{T_i} : i \in j\} \subseteq Nu_\tau$

Then the space (Nu_X^*, Nu_τ) , is called a Neutrosophic topological space(NS-T-S) The component of Nu_τ are called Nu-OS (Neutrosophic open set)and its complement is Nu-CS(Neutrosophic closed set)

Example 2.6. Let $Nu_X^* = \{w\}$ and $\forall w \in Nu_X^*, W_1^* = \langle w, \frac{6}{10}, \frac{6}{10}, \frac{5}{10} \rangle, W_2^* = \langle w, \frac{5}{10}, \frac{7}{10}, \frac{9}{10} \rangle$

$W_3^* = \langle w, \frac{6}{10}, \frac{7}{10}, \frac{5}{10} \rangle, W_4^* = \langle w, \frac{5}{10}, \frac{6}{10}, \frac{9}{10} \rangle$ Then the collection $Nu_\tau = \{0_{Nu}, W_1^*, W_2^*, W_3^*, W_4^*, 1_{Nu}\}$ is called a NS-T-S on Nu_X^* .

Definition 2.7. Let (Nu_X^*, Nu_τ) , be a NS-T-S

and $W_1^* = \{ \langle w, \mu_{W_1^*}(w), \sigma_{W_1^*}(w), \gamma_{W_1^*}(w) \rangle : w \in Nu_X^* \}$ be a Neutrosophic set in Nu_X^* . Then W_1^* is said to be

- [1] Neutrosophic α -closed set [6] (Nu – α CS in short) $Nu-cl(Nu-in(Nu-cl(W_1^*))) \subseteq W_1^*$,
- [2] Neutrosophic pre-closed set [22] (Nu-PCS in short) $Nu-cl(Nu-in(W_1^*)) \subseteq W_1^*$,
- [3] Neutrosophic regular closed set [6] (Nu -RCS in short) $Nu-cl(Nu-in(W_1^*)) = W_1^*$,
- [4] Neutrosophic semi closed set [7] (Nu-SCS in short) $Nu-in(Nu-cl(W_1^*)) \subseteq W_1^*$,
- [5] Neutrosophic generalized closed set [4] (Nu -GCS in short) $Nu-cl(W_1^* \subseteq \mathcal{H}$ whenever $W_1^* \subseteq \mathcal{H}$ and \mathcal{H} is a Nu -OS, in Nu_X^*
- [6] Neutrosophic α generalized closed set [13] (Nu - (α G) CS in short) $Nu\alpha cl(W_1^*) \subseteq \mathcal{H}$ whenever $W_1^* \subseteq \mathcal{H}$ And \mathcal{H} is a Nu-OS, in Nu_X^*
- [7] Neutrosophic generalized semi closed set [21](Nu-GSCS in short) $Nu-Scl(W_1^*) \subseteq \mathcal{H}$ whenever $W_1^* \subseteq \mathcal{H}$ and \mathcal{H} is a Nu-OS in Nu_X^*
- [8] Neutrosophic semi generalized closed set[21](Nu-SGCS in short) if $Nu scl(W_1^*) \subseteq \mathcal{H}$ whenever $W_1^* \subseteq \mathcal{H}$ and \mathcal{H} is a Nu-SOS in Nu_X^* ,
- [9] Neutrosophic generalized alpha closed set[9]. (Nu-G α CS in short) if $Nu-\alpha cl(W_1^*) \subseteq \mathcal{H}$ whenever $W_1^* \subseteq \mathcal{H}$ and \mathcal{H} is a Nu- α OS in Nu_X^*
- [10] Neutrosophic generalized b closed set[14](Nu-GbCS in short) if $Nu-bcl(W_1^*) \subseteq \mathcal{H}$ whenever $W_1^* \subseteq \mathcal{H}$ and \mathcal{H} is a Nu-OS in Nu_X^*

Definition 2.8.[13] An (NS)S W_1^* in an (NS)TS (Nu_X^*, Nu_τ) , is said to be a Neutrosophic weakly generalized closed set ((Nu-WG)CS) $Nu-cl(Nu-in(W_1^*)) \subseteq \mathcal{K}$ whenever $W_1^* \subseteq \mathcal{K}$, \mathcal{K} is (Nu)OS in Nu_X^* .

Definition 2.9. (Nu_X^*, Nu_τ) , be a NS-T-S and $W_1^* = \{ \langle w, \mu_{W_1^*}(w), \sigma_{W_1^*}(w), \gamma_{W_1^*}(w) \rangle : w \in Nu_X^* \}$ Nu_X^* .

Then Neutrosophic closure of W_1^* is $Nu-Cl(W_1^*) = \cap \{ \mathcal{H} : \mathcal{H} \text{ is a Nu-CS in } Nu_X^* \text{ and } W_1^* \subseteq \mathcal{H} \}$
 Neutrosophic interior of W_1^* is $Nu-Int(W_1^*) = \cup \{ M : M \text{ is a Nu-OS in } Nu_X^* \text{ and } M \subseteq W_1^* \}$.

Definition 2.10.[2] Let (Nu_X^*, Nu_τ) , be a NS-T-S and

- $W_1^* = \{ \langle w, \mu_{W_1^*}(w), \sigma_{W_1^*}(w), \gamma_{W_1^*}(w) \rangle : w \in Nu_X^* \}$
- $Nu-Sint(W_1^*) = \cup \{ \mathcal{H} / \mathcal{H} \text{ is a Nu-SOS in } Nu_X^* \text{ and } \mathcal{H} \subseteq W_1^* \}$,
- $Nu-Scl(W_1^*) = \cap \{ \mathcal{K} / \mathcal{K} \text{ is a Nu-SCS in } Nu_X^* \text{ and } W_1^* \subseteq \mathcal{K} \}$.
- $Nu-\alpha int(W_1^*) = \cup \{ \mathcal{H} / \mathcal{H} \text{ is a Nu-}\alpha\text{OS in } Nu_X^* \text{ and } \mathcal{H} \subseteq W_1^* \}$,
- $Nu-\alpha cl(W_1^*) = \cap \{ \mathcal{K} / \mathcal{K} \text{ is a Nu-}\alpha\text{CS in } Nu_X^* \text{ and } W_1^* \subseteq \mathcal{K} \}$.

3. NEUTROSOPHIC G* CLOSED SETS

In this section we introduce Neutrosophic G*-Closed sets and studied some of its basic properties.

Definition 3.1: An NS W_1^* in (Nu_X^*, Nu_τ) is said to be a Neutrosophic G*-Closed set (Nu-G*CS in short) if $Nu-cl(W_1^*) \subseteq \mathcal{K}$ whenever $W_1^* \subseteq \mathcal{K}$ and \mathcal{K} is Nu-GOS in (Nu_X^*, Nu_τ) .

The family of all Nu-G*CS's of A NTS (Nu_X^*, Nu_τ) is denoted by $Nu-G^*C(Nu_X^*)$.

Example 3.2: Let $Nu_X^* = \{ w_1, w_2 \}$ and let $Nu_\tau = \{ 0_{Nu}, \mathcal{K}, 1_{Nu} \}$ is NT on Nu_X^* , where $\mathcal{K} = \langle w, (\frac{3}{10}, \frac{5}{10}, \frac{7}{10}), (\frac{4}{10}, \frac{5}{10}, \frac{6}{10}) \rangle$. Then the NS $W_1^* = \langle w, (\frac{7}{10}, \frac{5}{10}, \frac{1}{10}), (\frac{6}{10}, \frac{5}{10}, \frac{0}{10}) \rangle$ is Nu-G*CS in (Nu_X^*, Nu_τ)

Theorem 3.3: Every Nu-CS is Nu-G*CS .

Proof: Let W_1^* be a Nu-CS in (Nu_X^*, Nu_τ) . Then $Nu-cl(W_1^*) = W_1^*$. Let $W_1^* \subseteq \mathcal{K}$ and \mathcal{K} is Nu-GOS in (Nu_X^*, Nu_τ) . Therefore $Nu-cl(W_1^*) = W_1^* \subseteq \mathcal{K}$. Thus W_1^* is Nu-G*CS in Nu_X^* .

Example 3.4: Let $Nu_X^* = \{ w_1, w_2 \}$ and let $Nu_\tau = \{ 0_{Nu}, \mathcal{K}, 1_{Nu} \}$ is NT on Nu_X^* , where

$\mathcal{K} = \langle w, (\frac{4}{10}, \frac{5}{10}, \frac{6}{10}), (\frac{2}{10}, \frac{5}{10}, \frac{7}{10}) \rangle$ Then the NS $W_1^* = \langle w, (\frac{6}{10}, \frac{5}{10}, \frac{1}{10}), (\frac{7}{10}, \frac{5}{10}, \frac{1}{10}) \rangle$ is Nu-G*CS but not an

Nu-CS in Nu_X^* .

Theorem 3.5: Every Nu-G*CS is Nu-GCS.

Proof: Let W_1^* be a Nu-G*CS in (Nu_X^*, Nu_τ) . Let $W_1^* \subseteq \mathcal{K}$ and \mathcal{K} is Nu-OS in (Nu_X^*, Nu_τ) . Since every Nu-OS is Nu-GOS and since W_1^* is Nu-G*CS in Nu_X^* . Therefore $Nu-cl(W_1^*) \subseteq \mathcal{K}$ whenever $W_1^* \subseteq \mathcal{K}$, \mathcal{K} is Nu-OS in Nu_X^* . Thus W_1^* is Nu-GCS in Nu_X^* .

Example 3.6: Let $Nu_X^* = \{w_1, w_2, w_3\}$ and

let $Nu_\tau = \{0_{Nu}, \mathcal{K}, 1_{Nu}\}$ is NT on Nu_X^* , where $\mathcal{K} = \langle w, (\frac{2}{10}, \frac{5}{10}, \frac{7}{10}), (\frac{5}{10}, \frac{5}{10}, \frac{4}{10}), (\frac{3}{10}, \frac{5}{10}, \frac{5}{10}) \rangle$.

Then the NS $W_1^* = \langle w, (\frac{5}{10}, \frac{5}{10}, \frac{5}{10}), (\frac{2}{10}, \frac{5}{10}, \frac{7}{10}), (\frac{3}{10}, \frac{5}{10}, \frac{6}{10}) \rangle$ is Nu-GCS but not an Nu-G*CS in Nu_X^* .

Theorem 3.7: Every Nu-G*CS is Nu- α GCS.

Proof: Let W_1^* be a Nu-G*CS in (Nu_X^*, Nu_τ) . By Theorem 3.6 W_1^* is Nu-GCS in Nu_X^* . Since $Nu\alpha-cl(W_1^*) \subseteq Nu-cl(W_1^*)$ and W_1^* is a Nu-GCS in Nu_X^* . Therefore $Nu\alpha-cl(W_1^*) \subseteq Nu-cl(W_1^*) \subseteq \mathcal{K}$ whenever $W_1^* \subseteq \mathcal{K}$, \mathcal{K} is Nu-OS in Nu_X^* . Thus W_1^* is Nu- α GCS in Nu_X^* .

Example 3.8: Let $Nu_X^* = \{w_1, w_2\}$ and let $Nu_\tau = \{0_{Nu}, \mathcal{K}, 1_{Nu}\}$ is NT on Nu_X^* ,

where $\mathcal{K} = \langle w, (\frac{1}{10}, \frac{5}{10}, \frac{6}{10}), (\frac{4}{10}, \frac{5}{10}, \frac{5}{10}) \rangle$. Then the NS $W_1^* = \langle w, (\frac{3}{10}, \frac{5}{10}, \frac{4}{10}), (\frac{5}{10}, \frac{5}{10}, \frac{5}{10}) \rangle$

is Nu- α GCS but not an Nu-G*CS in Nu_X^* .

Theorem 3.9: Every Nu-RCS is Nu-G*CS.

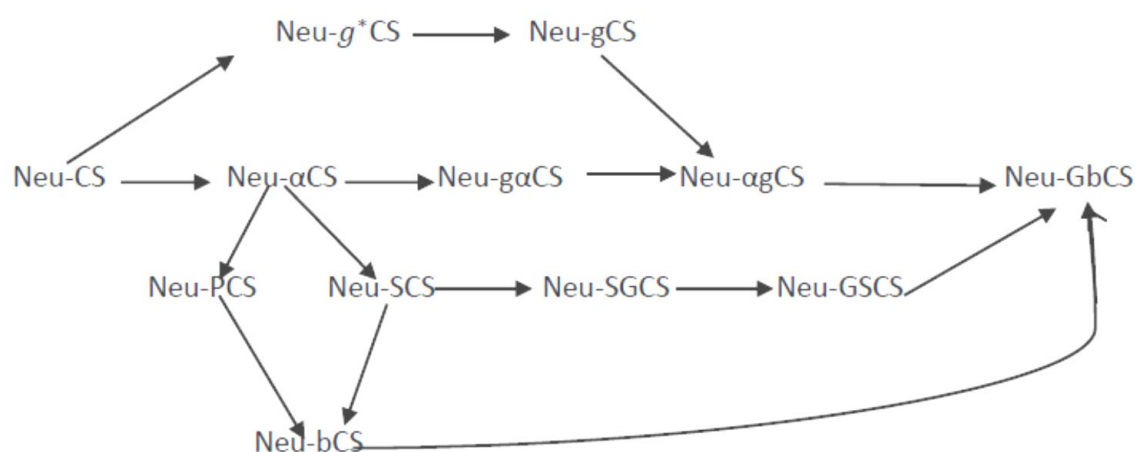
Proof: Let W_1^* be a Nu-RCS in (Nu_X^*, Nu_τ) . Then $W_1^* = Nu-cl(Nu-int(W_1^*))$. Let $W_1^* \subseteq \mathcal{K}$ and \mathcal{K} is Nu-GOS in (Nu_X^*, Nu_τ) . Therefore $Nu-cl(W_1^*) \subseteq Nu-cl(Nu-int(W_1^*))$. This implies $Nu-cl(W_1^*) \subseteq \mathcal{K}$. Thus W_1^* is Nu-G*CS in Nu_X^* .

Example 3.10: Let $Nu_X^* = \{w_1, w_2\}$ and let $Nu_\tau = \{0_{Nu}, \mathcal{K}, 1_{Nu}\}$ is NT on Nu_X^* , Where

$\mathcal{K} = \langle w, (\frac{3}{10}, \frac{5}{10}, \frac{6}{10}), (\frac{7}{10}, \frac{5}{10}, \frac{3}{10}) \rangle$ Then NS $W_1^* = \langle w, (\frac{7}{10}, \frac{5}{10}, \frac{3}{10}), (\frac{6}{10}, \frac{5}{10}, \frac{4}{10}) \rangle$ is Nu-G*CS but not an

Nu-RCS in Nu_X^* .

Diagram:I



Remark 3.11:

Nu-G*CS is independent from Nu- α CS, Nu-SCS, Nu-PCS, and Nu-bCS as seen from the following example.

Example 3.12: Let $Nu_X^* = \{w_1, w_2\}$ and let $Nu_\tau = \{0_{Nu}, \mathcal{K}, 1_{Nu}\}$ is NT on Nu_X^* , where

$$\mathcal{K} = \langle w \left(\frac{2}{10}, \frac{5}{10}, \frac{6}{10} \right), \left(\frac{4}{10}, \frac{5}{10}, \frac{5}{10} \right) \rangle. \text{ Then } NS W_1^* = \langle w, \left(\frac{3}{10}, \frac{5}{10}, \frac{5}{10} \right), \left(\frac{4}{10}, \frac{5}{10}, \frac{5}{10} \right) \rangle$$

is NuSCS, Nu-bCS, but not an Nu-G*CS in Nu_X^* .

Example 3.13: Let $Nu_X^* = \{w_1, w_2\}$ and let $Nu_\tau = \{0_{Nu}, \mathcal{K}, 1_{Nu}\}$ is NT on Nu_X^* , where

$$\mathcal{K} = \langle w \left(\frac{6}{10}, \frac{5}{10}, \frac{2}{10} \right), \left(\frac{5}{10}, \frac{5}{10}, \frac{4}{10} \right) \rangle. \text{ Then } NS W_1^* = \langle w, \left(\frac{1}{10}, \frac{5}{10}, \frac{7}{10} \right), \left(\frac{4}{10}, \frac{5}{10}, \frac{5}{10} \right) \rangle$$

is Nu-PCS, Nu- α CS, but not an Nu-G*CS in Nu_X^* .

Example 3.14: Let $Nu_X^* = \{w_1, w_2\}$ and let $Nu_\tau = \{0_{Nu}, \mathcal{K}, 1_{Nu}\}$ is NT on Nu_X^* , where

$$\mathcal{K} = \langle w, \left(\frac{6}{10}, \frac{5}{10}, \frac{2}{10} \right), \left(\frac{5}{10}, \frac{5}{10}, \frac{2}{10} \right) \rangle \text{ Then the } NS W_1^* = \langle w, \left(\frac{1}{10}, \frac{5}{10}, \frac{3}{10} \right), \left(\frac{4}{10}, \frac{5}{10}, \frac{2}{10} \right) \rangle$$

is Nu-G*CS but not NuSCS, Nu-bCS Nu_X^* .

Example 3.15: Let $Nu_X^* = \{w_1, w_2\}$ and let $Nu_\tau = \{0_{Nu}, \mathcal{K}, 1_{Nu}\}$ is NT on Nu_X^* , where

$$\mathcal{K} = \langle w, \left(\frac{2}{10}, \frac{5}{10}, \frac{6}{10} \right), \left(\frac{3}{10}, \frac{5}{10}, \frac{5}{10} \right) \rangle \text{ Then the } NS W_1^* = \langle w, \left(\frac{3}{10}, \frac{5}{10}, \frac{1}{10} \right), \left(\frac{3}{10}, \frac{5}{10}, \frac{3}{10} \right) \rangle$$

is Nu-G*CS but not Nu α CS, Nu-PCS Nu_X^* .

Theorem 3.16: The union of two Nu-G*CS's is Nu-G*CS

Proof: Let W_1^* and W_2^* be the two Nu-G*CS's in Nu_X^* and let $W_1^* \cup W_2^* \subseteq \mathcal{K}$, where \mathcal{K} is a Nu-GOS in Nu_X^* . Therefore $W_1^* \subseteq \mathcal{K}$ or $W_2^* \subseteq \mathcal{K}$ or both contained \mathcal{K} . Since W_1^* and W_2^* are Nu-G*CS, Nu-cl(W_1^*) $\subseteq \mathcal{K}$ and Nu-cl(W_2^*) $\subseteq \mathcal{K}$. Therefore Nu-cl($W_1^* \cup W_2^*$) $\subseteq \mathcal{K}$. Thus $W_1^* \cup W_2^*$ is Nu-G*CS.

Remark 3.17: The intersection of any two Nu-G*CSs is not an Nu-G*CS in general as seen in the following example.

Example 3.18: Let $Nu_X^* = \{w_1, w_2\}$ and let $Nu_\tau = \{0_{Nu}, \mathcal{K}, 1_{Nu}\}$ is NT on Nu_X^* ,

$$\text{where } \mathcal{K} = \langle w, \left(\frac{5}{10}, \frac{5}{10}, \frac{1}{10} \right), \left(\frac{1}{10}, \frac{5}{10}, \frac{8}{10} \right) \rangle.$$

$$\text{Then NS's } W_1^* = \langle w, \left(\frac{2}{10}, \frac{5}{10}, \frac{5}{10} \right), \left(\frac{7}{10}, \frac{5}{10}, \frac{0}{10} \right) \rangle \quad W_2^* = \langle w, \left(\frac{6}{10}, \frac{5}{10}, \frac{0}{10} \right), \left(\frac{3}{10}, \frac{5}{10}, \frac{7}{10} \right) \rangle$$

are Nu-G*CS's in Nu_X^* but $W_1^* \cap W_2^*$ is not a Nu-G*CS in Nu_X^* .

Theorem 3.19: If W_1^* is Nu-G*CS in (Nu_X^*, Nu_τ) , such that $W_1^* \subseteq W_2^* \subseteq Nu-cl(W_1^*)$. Then W_2^* is also a Nu-G*CS of (Nu_X^*, Nu_τ)

Proof: Let \mathcal{K} be a Nu-GOS in (Nu_X^*, Nu_τ) such that $W_2^* \subseteq \mathcal{K}$. Since $W_1^* \subseteq W_2^*$, $W_1^* \subseteq \mathcal{K}$ and \mathcal{K} be a Nu-GOS. Also since W_1^* is Nu-G*CS, Nu-cl(W_1^*) $\subseteq \mathcal{K}$. By hypothesis $W_2^* \subseteq Nu-cl(W_1^*)$. This implies Nu-cl(W_2^*) $\subseteq Nu-cl(Nu-cl(W_1^*)) \subseteq \mathcal{K}$. Therefore Nu-cl(W_2^*) $\subseteq \mathcal{K}$. Hence W_2^* is Nu-G*CS of Nu_X^* .

Theorem 3.20: If W_1^* is both Nu-GOS and Nu-G*CS of (Nu_X^*, Nu_τ) , then W_1^* is Nu-CS in Nu_X^* .

Proof: Let W_1^* is Nu-GOS in Nu_X^* . Since $W_1^* \subseteq W_1^*$, by hypothesis Nu-cl(W_1^*) $\subseteq W_1^*$. But from the Definition, $W_1^* \subseteq Nu-cl(W_1^*)$. Therefore Nu-cl(W_1^*) $= W_1^*$. Hence W_1^* is Nu-CS of Nu_X^* .

Theorem 3.21: Let (Nu_X^*, Nu_τ) be a NTS. Then Nu-GO(Nu_X^*) $= Nu-GC(Nu_X^*)$ iff every NS in (Nu_X^*, Nu_τ) is Nu-G*CS in Nu_X^* .

Proof:

Necessity: Suppose that Nu-GO(Nu_X^*) $= Nu-GC(Nu_X^*)$. Let $W_1^* \subseteq \mathcal{K}$ and \mathcal{K} is Nu-GOS in Nu_X^* . This implies Nu-cl(W_1^*) $\subseteq Nu-cl(\mathcal{K})$. Since \mathcal{K} is Nu-GOS in Nu_X^* . Since by hypothesis \mathcal{K} is Nu-GCS in Nu_X^* , Nu-cl(\mathcal{K}) $\subseteq \mathcal{K}$. This implies Nu-cl(W_1^*) $\subseteq \mathcal{K}$. Therefore W_1^* is Nu-G*CS in Nu_X^* .

Sufficiency: Suppose that every NS in (Nu_X^*, Nu_τ) is Nu-G*CS in Nu_X^* . Let $\mathcal{K} \subseteq Nu-O(Nu_X^*)$, then

$\mathcal{K} \subseteq \text{Nu-GO}(\text{Nu}_X^*)$. Since $\mathcal{K} \subseteq \mathcal{K}$ and \mathcal{K} is Nu-OS in Nu_X^* , by hypothesis $\text{Nu-cl}(\mathcal{K}) \subseteq \mathcal{K}$. I.e., $\mathcal{K} \subseteq \text{Nu-GC}(\text{Nu}_X^*)$. Hence $\text{Nu-GO}(\text{Nu}_X^*) \subseteq \text{Nu-GC}(\text{Nu}_X^*)$. Let $W_1^* \in \text{Nu-GC}(\text{Nu}_X^*)$ then W_1^{*C} is an Nu-GOS in Nu_X^* . But $\text{Nu-GO}(\text{Nu}_X^*) \subseteq \text{Nu-GC}(\text{Nu}_X^*)$. Therefore $W_1^{*C} \subseteq \text{Nu-GC}(\text{Nu}_X^*)$. I.e., $W_1^* \in \text{Nu-GO}(\text{Nu}_X^*)$. Hence $\text{Nu-GC}(\text{Nu}_X^*) \subseteq \text{Nu-GO}(\text{Nu}_X^*)$. Thus $\text{Nu-GO}(\text{Nu}_X^*) \subseteq \text{Nu-GC}(\text{Nu}_X^*)$.

Theorem 3.22: If W_1^* is Nu-OS and an Nu-G*CS in $(\text{Nu}_X^*, \text{Nu}_\tau)$, then

W_1^* is Nu-ROS in Nu_X^*

W_1^* is Nu-RCS in Nu_X^*

Proof: (i) Let W_1^* be a Nu-OS and a Nu-G*CS in Nu_X^* . Then $\text{Nu-cl}(W_1^*) \subseteq W_1^*$. I.e., $\text{Nu-int}(\text{Nu-cl}(W_1^*)) \subseteq W_1^*$. Since W_1^* is a Nu-OS, W_1^* is Nu-POS in Nu_X^* . Hence $W_1^* \subseteq \text{Nu-int}(\text{Nu-cl}(W_1^*))$. Therefore $W_1^* = \text{Nu-int}(\text{Nu-cl}(W_1^*))$. Hence W_1^* is Nu-ROS in Nu_X^* .

(ii): Let W_1^* be a Nu-OS and an Nu-G*CS in Nu_X^* . Then $\text{Nu-cl}(W_1^*) \subseteq W_1^*$. I.e., $\text{Nu-cl}(\text{Nu-int}(W_1^*)) \subseteq W_1^*$. Since W_1^* is a Nu-OS, W_1^* is Nu-OS in Nu_X^* . Hence $W_1^* \subseteq \text{Nu-cl}(\text{Nu-int}(W_1^*))$. Therefore $W_1^* = \text{Nu-int}(\text{Nu-cl}(W_1^*))$. Hence W_1^* is Nu-RCS in Nu_X^* .

4. NEUTROSOPHIC g*-OPEN SETS

In this section we introduce Neutrosophic g*-open sets and studied some of its properties.

Definition 4.1: An NS W_1^* is said to be a Neutrosophic g*-open set (Nu-G*OS in short) in $(\text{Nu}_X^*, \text{Nu}_\tau)$ if the complement W_1^{*C} is Nu-G*CS in Nu_X^* . The family of all Nu-G*OS's of A NTS $(\text{Nu}_X^*, \text{Nu}_\tau)$ is denoted by $\text{Nu-G}^*\text{O}(\text{Nu}_X^*)$.

Theorem 4.2: A subset W_1^* of $(\text{Nu}_X^*, \text{Nu}_\tau)$ is Nu-G*OS iff $W_2^* \subseteq \text{Nu-int}(W_1^*)$ whenever W_2^* is Nu-GCS in Nu_X^* and $W_2^* \subseteq W_1^*$.

Proof: Necessity: Let W_1^* is Nu-G*OS in Nu_X^* . Let W_2^* be a Nu-GCS in Nu_X^* and $W_2^* \subseteq W_1^*$. Then W_2^{*C} is Nu-GOS in Nu_X^* such that $W_1^{*C} \subseteq W_2^{*C}$. Since W_1^{*C} is Nu-G*CS, we have $\text{Nu-cl}(W_1^{*C}) \subseteq W_2^{*C}$. Hence $\text{Nu-int}(W_1^*)^C \subseteq W_2^{*C}$. Therefore $W_2^* \subseteq \text{Nu-int}(W_1^*)$.

Sufficiency: Let $W_2^* \subseteq \text{Nu-int}(W_1^*)$ whenever W_2^* is Nu-GCS in Nu_X^* and $W_2^* \subseteq W_1^*$. Then $W_1^{*C} \subseteq W_2^{*C}$ and W_2^{*C} is Nu-GOS. By hypothesis, $(\text{Nu-int}(W_1^*))^C \subseteq W_2^{*C}$, which implies $\text{Nu-cl}(W_1^{*C}) \subseteq W_2^{*C}$. Therefore W_1^{*C} is Nu-G*CS of Nu_X^* . Hence W_1^* is Nu-G*OS in Nu_X^* .

Theorem 4.3: Every Nu-OS is Nu-G*OS .

Proof: Let W_1^* be a Nu-OS. Then W_1^{*C} is Nu-CS. By Theorem 3.3, every Nu-CS is Nu-G*CS. Therefore W_1^{*C} is Nu-G*CS. Hence W_1^* is Nu-G*OS.

Example 4.4: Let $\text{Nu}_X^* = \{w_1, w_2\}$ and let $\text{Nu}_\tau = \{0_{\text{Nu}}, \mathcal{K}, 1_{\text{Nu}}\}$ is NT on Nu_X^* , where

$$\mathcal{K} = \langle w, \left(\frac{2}{10}, \frac{5}{10}, \frac{2}{10}\right), \left(\frac{5}{10}, \frac{5}{10}, \frac{8}{10}\right) \rangle. \text{ Then NS } W_1^* = \langle w, \left(\frac{2}{10}, \frac{5}{10}, \frac{3}{10}\right), \left(\frac{7}{10}, \frac{5}{10}, \frac{8}{10}\right) \rangle$$

is Nu-G*OS but not an Nu-OS in Nu_X^* .

Theorem 4.5: Every Nu-ROS is Nu- G*OS .

Proof: Let W_1^* be a Nu-WS. Then W_1^{*C} is Nu-RCS. By Theorem 3.15, every Nu-RCS is Nu-G*CS. Therefore W_1^{*C} is Nu-G*CS. Hence W_1^* is Nu-G*OS.

Example 4.6: Let $\text{Nu}_X^* = \{w_1, w_2\}$ and let $\text{Nu}_\tau = \{0_{\text{Nu}}, \mathcal{K}, 1_{\text{Nu}}\}$ is NT on Nu_X^* , where

$$\mathcal{K} = \langle w, \left(\frac{3}{10}, \frac{5}{10}, \frac{6}{10}\right), \left(\frac{7}{10}, \frac{5}{10}, \frac{3}{10}\right) \rangle \text{ Then NS } W_1^* = \langle w, \left(\frac{3}{10}, \frac{5}{10}, \frac{7}{10}\right), \left(\frac{4}{10}, \frac{5}{10}, \frac{6}{10}\right) \rangle \text{ is Nu-G*OS but not an}$$

Nu-ROS in Nu_X^* .

Theorem 4.7: Every Nu-G*OS is Nu-GOS .

Proof: Let W_1^* be a Nu-G*OS in $(\text{Nu}_X^*, \text{Nu}_\tau)$. Then W_1^{*C} is Nu-G*CS. By Theorem 3.6, every Nu-G*CS is Nu-GCS. Therefore W_1^{*C} is Nu-GCS. Hence W_1^* is Nu-GOS.

Example 4.8: Let $Nu_X^* = \{w_1, w_2\}$ and let $Nu_\tau = \{0_{Nu}, \mathcal{K}, 1_{Nu}\}$ is NT on Nu_X^* , where

$\mathcal{K} = \langle w, (\frac{5}{10}, \frac{5}{10}, \frac{4}{10}), (\frac{2}{10}, \frac{5}{10}, \frac{2}{10}) \rangle$. Then NS $W_1^* = \langle w, (\frac{4}{10}, \frac{5}{10}, \frac{6}{10}), (\frac{2}{10}, \frac{5}{10}, \frac{6}{10}) \rangle$ is Nu-GOS but not an Nu-G*OS in Nu_X^* .

Theorem 4.9: Every Nu-G*OS is Nu- α GOS .

Proof: Let W_1^* be a Nu-G*OS in (Nu_X^*, Nu_τ) . Then W_1^{*C} is Nu-G*CS. By Theorem 3.9, every Nu-G*CS is Nu- α GCS. Therefore W_1^{*C} is Nu- α GCS. Hence W_1^* is Nu- α GOS.

Example 4.10: Let $Nu_X^* = \{w_1, w_2\}$ and let $Nu_\tau = \{0_{Nu}, \mathcal{K}, 1_{Nu}\}$ is NT on Nu_X^* , where $\mathcal{K} = \langle w, (\frac{4}{10}, \frac{5}{10}, \frac{2}{10}), (\frac{6}{10}, \frac{5}{10}, \frac{7}{10}) \rangle$. Then the NS $W_1^* = \langle w, (\frac{6}{10}, \frac{5}{10}, \frac{4}{10}), (\frac{5}{10}, \frac{5}{10}, \frac{3}{10}) \rangle$ is Nu- α GOS but not an Nu-G*OS in Nu_X^* .

Theorem 4.11: The intersection of two Nu-G*OS's is Nu-G*OS.

Proof: Let W_1^* and W_2^* be the two Nu-G*OS's in Nu_X^* , W_1^{*C} and W_2^{*C} are Nu-G*CS. By Theorem 3.28 $W_1^{*C} \cup W_2^{*C}$ is Nu-G*CS in Nu_X^* . Therefore $(W_1^* \cap W_2^*)$ is Nu-G*CS. Thus $W_1^* \cap W_2^*$ is Nu-G*OS in Nu_X^* .

Theorem 4.12: Let (Nu_X^*, Nu_τ) be a NTS. If W_1^* is NS of Nu_X^* . Then for every $W_1^* \in Nu-G^*O(Nu_X^*)$ and every $W_2^* \in (Nu_X^*), Nu-int(W_1^*) \subseteq W_2^* \subseteq W_1^*$ implies $W_2^* \in Nu-G^*O(Nu_X^*)$.

Proof: By hypothesis $Nu-int(W_1^*) \subseteq W_2^* \subseteq W_1^*$. Taking complement on both sides, we get $W_1^{*C} \subseteq W_2^{*C} \subseteq Nu-cl(W_1^{*C})$. Let $W_2^{*C} \in \mathcal{K}$ and \mathcal{K} is Nu-GOS in Nu_X^* . Since $W_1^{*C} \subseteq W_2^{*C}$, $W_1^{*C} \subseteq \mathcal{K}$. Since W_1^{*C} is Nu-G*CS, $Nu-cl(W_1^{*C}) \subseteq \mathcal{K}$. Therefore $Nu-cl(W_2^{*C}) \subseteq Nu-cl(W_1^{*C}) \subseteq \mathcal{K}$. Hence W_2^{*C} is Nu-G*CS in Nu_X^* . Therefore W_2^* is Nu-G*OS in Nu_X^* . I.e., $W_2^* \in Nu-G^*O(Nu_X^*)$

Definition: 4.13: For any Nu. set W_1^* in any NSTS,

$$Nu-g^*cl(W_1^*) = \cap \{ \mathcal{U} : \mathcal{U} \text{ is Nu-g*CS Nu. set and } W_1^* \subseteq \mathcal{U} \}$$

$$Nu-g^*int(W_1^*) = \cup \{ \mathcal{V} : \mathcal{V} \text{ is Nu-g* OS and } W_1^* \supseteq \mathcal{V} \}$$

Theorem: 4.14: In a Its (Nu_X^*, Nu_τ) a Nu. set W_1^* is Nu-g*- CS iff $W_1^* = Nu-g^*cl(W_1^*)$.

Proof: Let W_1^* be a Nu-g*CS Nu. set in NSTS (Nu_X^*, Nu_τ) . Since $W_1^* \subseteq W_1^*$ and W_1^* is Nu-g*CS , $W_1^* \in \{ \mathcal{K} : \mathcal{K} \text{ is a Nu-g*CS Nu. set and } W_1^* \subseteq \mathcal{K} \}$ and $W_1^* \subseteq \mathcal{K} \Rightarrow W_1^* = \cap \{ \mathcal{K} : \mathcal{K} \text{ is Nu-g*CS and } W_1^* \subseteq \mathcal{K} \}$ that is $W_1^* = Nu-g^*cl(W_1^*)$.

Conversely, suppose that $W_1^* = Nu-g^*cl(W_1^*)$, that is $W_1^* = \cap \{ \mathcal{K} : \mathcal{K} \text{ is a Nu-g*- CS Nu. set and } W_1^* \subseteq \mathcal{K} \}$. This denotes that $W_1^* \in \{ \mathcal{K} : \mathcal{K} \text{ is a Nu-g*CS Nu. set and } W_1^* \subseteq \mathcal{K} \}$. From now W_1^* is Nu-g*CS Nu. set.

Theorem: 4.15 In a NSTS Nu_X^* the subsequent results hold for Nu-g* - closure.

- 1) $Nu-g^*cl(0_{Nu}) = 0_{Nu}$.
- 2) $Nu-g^*cl(W_1^*)$ is Nu-g*CS Nu. set in Nu_X^* .
- 3) $Nu-g^*cl(W_1^*) \subseteq Nu-g^*cl(W_2^*)$ if $W_1^* \subseteq W_2^*$.
- 4) $Nu-g^*cl(Nu-g^*cl(W_1^*)) = Nu-g^*cl(W_1^*)$.
- 5) $Nu-g^*cl(W_1^* \cup W_2^*) \supseteq Nu-g^*cl(W_1^*) \cup Nu-g^*cl(W_2^*)$.
- 6) $Nu-g^*cl(W_1^* \cap W_2^*) \subseteq Nu-g^*cl(W_1^*) \cap Nu-g^*cl(W_2^*)$.

Proof: easy

Theorem: 4.16 In a NSTS Nu_X^* , a Nu. set W_1^* is Nu-g* OS iff $W_1^* = Nu-g^*int(W_1^*)$.

Proof: Let W_1^* be Nu-g*OS Nu. set in Nu_X^* . Since $W_1^* \subseteq W_1^*$ and W_1^* is Nu-g* OS and $W_1^* \in \{ \mathcal{K} : \mathcal{K} \text{ is a Nu-g* OS Nu. set and } W_1^* \supseteq \mathcal{K} \}$ and $W_1^* \supseteq \mathcal{K} \Rightarrow W_1^* = \cup \{ \mathcal{K} : \mathcal{K} \text{ is Nu-g*OS and } W_1^* \supseteq \mathcal{K} \}$. That is $W_1^* = Nu-g^*int(W_1^*)$.

Conversely, suppose that $W_1^* = \text{Nu-g}^*\text{-int}(W_1^*)$, that is $W_1^* = \cup \{ \mathcal{K} : \mathcal{K} \text{ is Nu-g}^*\text{-OS and } W_1^* \supseteq \mathcal{K} \}$. This implies that $W_1^* \in \{ \mathcal{K} : \mathcal{K} \text{ is Nu-g}^*\text{-OS and } W_1^* \supseteq \mathcal{K} \}$. Hence W_1^* is Nu-g^{*}-OS Nu. set.

Theorem: 4.17 In a NSTS Nu_X^* , the following hold for Nu-g^{*}-interior.

- 1) $\text{Nu-g}^*\text{-int}(0_{\text{Nu}}) = 0_{\text{Nu}}$
- 2) $\text{Nu-g}^*\text{-int}(W_1^*) \subseteq \text{Nu-g}^*\text{-int}(W_2^*)$ if $W_1^* \subseteq W_2^*$.
- 3) $\text{Nu-g}^*\text{-int}(W_1^*)$ is Nu-g^{*}-OS in Nu_X^* .
- 4) $\text{Nu-g}^*\text{-int}(\text{Nu-g}^*\text{-int}(W_1^*)) = \text{Nu-g}^*\text{-int}(W_1^*)$.
- 5) $\text{Nu-g}^*\text{-int}(W_1^* \cup W_2^*) \supseteq \text{Nu-g}^*\text{-int}(W_1^*) \cup \text{Nu-g}^*\text{-int}(W_2^*)$.
- 6) $\text{Nu-g}^*\text{-int}(W_1^* \cap W_2^*) \subseteq \text{Nu-g}^*\text{-int}(W_1^*) \cap \text{Nu-g}^*\text{-int}(W_2^*)$.

Proof: proof is as usual.

5. NEUTROSOPHIC g^{*}- CONTINUOUS

In this section we introduce Neutrosophic g^{*}-continuous and studied some properties of neutrosophic g^{*}- open map and closed map.

Definition:5.1 Let Nu_X^* and Nu_Y^* be two NTS. A function $f: \text{Nu}_X^* \rightarrow \text{Nu}_Y^*$ is said to be neutrosophic g^{*}- continuous (Nu-g^{*}- continuous) if the inverse image of every neutrosophic open set in Nu_Y^* is g^{*}- open in Nu_X^* .

Theorem:5.2 A function $f: \text{Nu}_X^* \rightarrow \text{Nu}_Y^*$ is Nu-g^{*}- continuous iff the inverse image of every Nu-closed set in Nu_Y^* is g^{*}- closed set in Nu_X^* .

Proof: Suppose the function $f: \text{Nu}_X^* \rightarrow \text{Nu}_Y^*$ is Nu-g^{*}- continuous. Let \mathcal{F} be Nu-closed set in Nu_Y^* . Then \mathcal{F}^c is Nu- open set in Nu_Y^* . Since f is Nu-g^{*}- continuous, $f^{-1}(\mathcal{F}^c)$ is Nu- g^{*}- open in Nu_X^* . But $f^{-1}(\mathcal{F}^c) = (f^{-1}(\mathcal{F}))^c$ and so $f^{-1}(\mathcal{F})$ is Nu-g^{*}- closed in Nu_X^* .

Conversely, assume that the inverse image of every Nu-closed set in Nu_Y^* is Nu-g^{*}- closed in Nu_X^* . Let \mathcal{V} be neutrosophic open set in Nu_Y^* . Then \mathcal{V}^c is Nu-closed in Nu_Y^* . By hypothesis, $f^{-1}(\mathcal{V}^c)$ is Nu-g^{*}-closed set in Nu_X^* . But $f^{-1}(\mathcal{V}^c) = (f^{-1}(\mathcal{V}))^c$ and so $f^{-1}(\mathcal{V})$ is Nu-g^{*}- open set in Nu_X^* . Hence f is Nu-g^{*}-continuous.

Theorem:5.3 Every Nu- continuous function is Nu-g^{*}- continuous.

Proof: Let $f: \text{Nu}_X^* \rightarrow \text{Nu}_Y^*$ be Nu-continuous. Let \mathcal{F} be Nu-closed set in Nu_Y^* . Then $f^{-1}(\mathcal{F})$ is Nu-closed set in Nu_X^* since f is neutrosophic continuous. And therefore $f^{-1}(\mathcal{F})$ is Nu-g^{*}- closed in Nu_X^* . Hence f is Nu-g^{*}- continuous.

Theorem:5.4 Every Nu-g^{*}-continuous function is Nu-g-continuous.

Proof: Let $f: \text{Nu}_X^* \rightarrow \text{Nu}_Y^*$ be Nu-g^{*}- continuous. Let \mathcal{F} be a Nu-closed set in Nu_Y^* . Since f is Nu-g^{*}- continuous, $f^{-1}(\mathcal{F})$ is Nu-g^{*}- closed in Nu_X^* . And therefore $f^{-1}(\mathcal{F})$ is Nu-g- closed in Nu_X^* as every Nu-g^{*}-closed set is Nu-g- closed. Hence f is Nu-g- continuous.

The converse of the above theorem need not be true as seen from the following example.

Theorem:5.5 If $f: \text{Nu}_X^* \rightarrow \text{Nu}_Y^*$ is Nu-g^{*}- continuous and Nu_X^* is neutrosophic -T^{*}_{1/2} NTS. Then f is neutrosophic -continuous.

Proof: Let $f: \text{Nu}_X^* \rightarrow \text{Nu}_Y^*$ be Nu-g^{*}- continuous . Let \mathcal{F} be Nu-closed set in Nu_Y^* . Then $f^{-1}(\mathcal{F})$ is $f^{-1}(\mathcal{F})$ Nu- g^{*}- closed in Nu_X^* since f is Nu-g^{*}- continuous. Also since Nu_X^* is neutrosophic - T^{*}_{1/2}, $f^{-1}(\mathcal{F})$ is closcl in Nu_X^* . Hence f is Nu-continuous.

Theorem:5.6 If $f: \text{Nu}_X^* \rightarrow \text{Nu}_Y^*$ is Nu-g- continuous and Nu_X^* is neutrosophic - T^{*}_{1/2} NTS. Then f is Nu-g^{*}- continuous.

Proof: Let $f: \text{Nu}_X^* \rightarrow \text{Nu}_Y^*$ be Nu-g- continuous. Let \mathcal{F} be Nu-closed set in Nu_Y^* , then $f^{-1}(\mathcal{F})$ is g-closed in Nu_X^* . Since X is neutrosophic - T^{*}_{1/2}, $f^{-1}(\mathcal{F})$ is Nu- g^{*}- closed in Nu_X^* . Hence f is Nu-g^{*}-

continuous.

Theorem:5.7 If $f: Nu_X^* \rightarrow Nu_Y^*$ is Nu-g* - continuous and $g: Nu_Y^* \rightarrow Nu_Z^*$ is Nu-continuous then $g \circ f: Nu_X^* \rightarrow Nu_Z^*$ is Nu-g* - continuous.

Proof: Let \mathcal{F} be Nu-closed set in Nu_Z^* . Then $g^{-1}(\mathcal{F})$ is closed in Nu_Y^* since g is Nu-continuous. And then $f^{-1}(g^{-1}(\mathcal{F}))$ is Nu-g* - closed in Nu_X^* since f is Nu-g* - continuous.

Now $(g \circ f)^{-1}(\mathcal{F}) = f^{-1}(g^{-1}(\mathcal{F}))$ is Nu-g* - closed in Nu_X^* . Hence $g \circ f: Nu_X^* \rightarrow Nu_Z^*$ is Nu-g* - continuous.

Theorem:5.8 If $f: Nu_X^* \rightarrow Nu_Y^*$ is Nu-g* - continuous and $g: Nu_Y^* \rightarrow Nu_Z^*$ is Nu-g* - continuous and Nu_Y^* is neutrosophic $-T_{1/2}$ space. Then $g \circ f: Nu_X^* \rightarrow Nu_Z^*$ is Nu-g* - continuous.

Proof: Let \mathcal{F} be Nu-closed set in Nu_Z^* . Then $g^{-1}(\mathcal{F})$ is Nu-g*CS in Nu_Y^* since g is Nu-g* - continuous. Since Nu_Y^* is neutrosophic $-T_{1/2}$, $g^{-1}(\mathcal{F})$ is Nu-closed in Nu_Y^* . And then $f^{-1}(g^{-1}(\mathcal{F}))$ is Nu-g*CS in Nu_X^* as f is Nu-g* - continuous. Now $(g \circ f)^{-1}(\mathcal{F}) = f^{-1}(g^{-1}(\mathcal{F}))$ is Nu-g*CS in Nu_X^* . Hence $g \circ f$ is Nu-g* - continuous.

Definition:5.9 A map $f: Nu_X^* \rightarrow Nu_Y^*$ is said to be neutrosophic g* - open if the image of every neutrosophic open set in Nu_X^* is Nu-g*-open set in Nu_Y^* .

Definition:5.10 A map $f: Nu_X^* \rightarrow Nu_Y^*$ is said to be neutrosophic g* - closed if the image of every Nu-closed set in Nu_X^* is Nu-g*-closed set in Nu_Y^* .

Theorem: 5.11 Every neutrosophic open map is neutrosophic g* - open.

Proof: Let $f: Nu_X^* \rightarrow Nu_Y^*$ be a neutrosophic open map let \mathcal{V} be an neutrosophic open set in Nu_X^* then $f(\mathcal{V})$ is Nu-open in Nu_Y^* since f is neutrosophic open map. And therefore $f(\mathcal{V})$ is Nu-g* - open in Nu_Y^* . Hence f is neutrosophic g* open map.

Theorem :5.12 If $f: Nu_X^* \rightarrow Nu_Y^*$ is Nu-g*-open map and Nu_Y^* is neutrosophic $-T_{1/2}$, then f is a Nu-open map.

Proof : Let $f: Nu_X^* \rightarrow Nu_Y^*$ is neutrosophic g* - open map. Let \mathcal{V} be neutrosophic open set in Nu_X^* . Then $f(\mathcal{V})$ is Nu-g* - open in Nu_Y^* . Since Nu_Y^* is neutrosophic $-T_{1/2}$, $f(\mathcal{V})$ is neutrosophic open set in Nu_Y^* . Hence f is Nu- open map.

Theorem:5.13 Every Nu-g* - open map is neutrosophic g - open.

Proof: Let $f: Nu_X^* \rightarrow Nu_Y^*$ be a Nu-g* - open map. Let \mathcal{V} be neutrosophic open set in Nu_X^* . Then $f(\mathcal{V})$ is Nu-g* - open in Nu_Y^* since f is Nu-g* - open map. And therefore $f(\mathcal{V})$ is Nu-g - open set in Nu_Y^* . Hence f is neutrosophic g - open map.

Theorem : 5.14 If $f: Nu_X^* \rightarrow Nu_Y^*$ is neutrosophic g - open and Nu_Y^* is neutrosophic $-T_{1/2}$ space, then f is in Nu-g* - open map.

Proof: Let \mathcal{V} be neutrosophic open set in Nu_X^* . Then $f(\mathcal{V})$ is Nu-g - open in Nu_Y^* . Since Nu_Y^* is neutrosophic $-T_{1/2}$, $f(\mathcal{V})$ is Nu-g* - open in Nu_Y^* . And hence f is Nu-g* - open map.

Theorem : 5.15 Every Nu-closed map is Nu-g* - closed map.

Proof: Let $f: Nu_X^* \rightarrow Nu_Y^*$ be Nu-closed map. Let \mathcal{F} be Nu-closed set in Nu_X^* . Then $f(\mathcal{F})$ is closed in Nu_Y^* . And therefore $f(\mathcal{F})$ is Nu-g* - closed in Nu_Y^* . And hence f is Nu-g* - closed map.

Theorem :5.16 If $f: Nu_X^* \rightarrow Nu_Y^*$ is Nu-g* - closed and Nu_Y^* is neutrosophic $-T_{1/2}$. Then f is Nu-closed map.

Proof: Let $f: Nu_X^* \rightarrow Nu_Y^*$ be Nu-g* - closed map. Let \mathcal{F} be Nu-closed set in Nu_X^* . Then $f^{-1}(\mathcal{F})$ is Nu-g* - closed in Nu_X^* . Since Nu_Y^* is neutrosophic $-T_{1/2}$, $f(\mathcal{F})$ is Nu-closed in Nu_Y^* . Hence f is neutrosophic closed map.

Theorem: 5.17 A map $f: Nu_X^* \rightarrow Nu_Y^*$ is Nu-g* - closed iff for each neutrosophic set \mathcal{S} of Nu_Y^* and for

each neutrosophic open set \mathcal{U} such that $f^{-1}(\mathcal{S}) \subseteq \mathcal{U}$ there is a Nu-g*-open set \mathcal{V} of Nu_Y^* such that $\mathcal{S} \subseteq \mathcal{V}$ and $f^{-1}(\mathcal{V}) \subseteq \mathcal{U}$.

Proof: Suppose f is Nu-g* - closed map. Let \mathcal{S} be a neutrosophic set of Nu_X^* and \mathcal{U} be a neutrosophic open set of Nu_X^* such that $f^{-1}(\mathcal{U}) \subseteq \mathcal{U}$. Then $\mathcal{V} = \text{Nu}_Y^* - f(\mathcal{U}^c)$ is a Nu-g*-open set in Nu_Y^* such that $\mathcal{S} \subseteq \mathcal{V}$ and $f^{-1}(\mathcal{V}) \subseteq \mathcal{U}$.

Conversely, suppose that \mathcal{F} is a Nu-closed set of Nu_X^* . Then $f^{-1}(f(\mathcal{F}^c)) \subseteq \mathcal{F}^c$ and \mathcal{F}^c is Nu-open. By hypothesis, there is a Nu-g*-open set \mathcal{V} of Nu_Y^* such that $f(\mathcal{F})^c \subseteq \mathcal{V}$ and $f^{-1}(\mathcal{V}) \subseteq \mathcal{F}^c$. Therefore $\mathcal{F} \subseteq f^{-1}(\mathcal{V})^c$. Hence $\mathcal{V}^c \subseteq f(\mathcal{F}) \subseteq f(f^{-1}(\mathcal{V})^c) \subseteq \mathcal{V}^c$ which implies $f(\mathcal{F}) = \mathcal{V}^c$. Since \mathcal{V}^c is Nu-g* - closed, $f(\mathcal{F})$ is Nu-g*CS and thus f is a Nu-g*- closed map.

Conclusion

In this paper, we have defined the neutrosophic g* closed sets and open sets. Then we discussed about neutrosophic g* continuity. Then, we have presented some properties of these operations. We have also investigated neutrosophic topological structures of neutrosophic sets. Hence, we hope that the findings in this paper will help researchers enhance and promote the further study on neutrosophic topology.

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