An Overview of Plithogenic Set and Symbolic Plithogenic Algebraic Structures

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Journal of Fuzzy Extension and Applications

Paper Type: Review Paper

An Overview of Plithogenic Set and Symbolic Plithogenic Algebraic Structures

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Abstract

This paper is devoted to Plithogeny, Plithogenic Set, and its extensions. These concepts are branches of uncertainty and indeterminacy instruments of practical and theoretical interest. Starting with some examples, we proceed towards general structures. Then we present definitions and applications of the principal concepts derived from plithogeny, and relate them to complex problems.

1 | Etymology of Plithogeny

Plitho-geny etymologically comes from: (Gr.) πλήθος (plithos) = crowd, large number of, multitude, plenty of, and -geny < (Gr.) -γενιά (-geniá) = generation, the production of something & γένεια (géneia) = generations, the production of something < -γένεση (-génesi) = genesis, origination, creation, development, according to Translate Google dictionaries and Webster's new world dictionary of American English, third college edition, Simon & Schuster, Inc.

Therefore, Plithogeny is the genesis or origination, creation, formation, development, and evolution of new entities from dynamics and organic fusions of contradictory and/or neutrals and/or non-contradictory multiple old entities.

Plithogeny is a dynamics of many pairs of opposites ($<A_1>$, $<antiA_1>$), …, ($<A_n>$, $<antiA_n>$), and their neutralities (indeterminacies) $<neutA_1>$, …, $<neutA_n>$.

While Neutrosophy, as a particular case of Plithogeny, is the dynamics of only one pair of opposites ($<A>$, $<antiA>$) and its neutral (indeterminacy) $<neutA>$.

Dialectics and Yin Yang are the dynamics of one pair of opposites $<A>$ and $<antiA>$, without taking into consideration their neutral (indeterminacy) $<neutA>$.

An overview of plithogenic set and symbolic plithogenic algebraic structures

Plithogenic means what is pertaining to Plithogeny.

All Plithogeny, Plithogenic set, Plithogenic logic, Plithogenic probability, Plithogenic statistics, and symbolic Plithogenic algebraic structures were [29]-[32].

This article is a synthesis of evolution of Plithogenic domain during a six-year span, with articles published by many authors around the world [1]-[28].

2 | Plithogenic Set

A Plithogenic set P is a set whose elements are characterized by one or more attributes, and each attribute may have many values. Each attribute's value v has a corresponding degree of appurtenance d(x,v) of the element x to the set P, with respect to some given criteria.

In order to obtain a better accuracy for the Plithogenic aggregation operators, a contradiction (dissimilarity) degree is defined between each attribute value and the dominant (most important) attribute value. The contradiction degrees may similarly be: either fuzzy, intuitionistic fuzzy, neutrosophic, or any fuzzy extension, yet for simplicity we'll be using the fuzzy ones.

However, there are cases when such dominant attribute value may not be taking into consideration or may not exist [therefore it is considered zero by default], or there may be many dominant attribute values. In such cases, either the contradiction degree function is suppressed, or another relationship function between attribute values should be established.

The Plithogenic aggregation operators (intersection, union, complement, inclusion, equality) are based on contradiction degrees between attributes' values, and the first two operators (intersection and union) are linear combinations of the fuzzy operators' tnorm ($\wedge_F$) and tconorm ($\vee_F$) where 'F' stands for 'fuzzy'.

Let $(\overset{0}{a_o}, \overset{c_1}{a_1}, ..., \overset{c_i}{a_i}, ..., \overset{c_n}{a_n})$ and $(\overset{0}{b_o}, \overset{c_1}{b_1}, ..., \overset{c_i}{b_i}, ..., \overset{c_n}{b_n})$ be the attribute value degrees of a generic element x from a Plithogenic set P, assigned by an Expert A, and respectively an Expert B, where the contradiction degrees of the attribute values are: $0 \le c_1 \le ... \le c_i \le ... \le c_n \le 1$, i.e. $c(a_o, a_i) = c_i$ for $1 \le i \le n$. Let $\wedge_P, \vee_P$ represent respectively the Plithogenic intersection and Plithogenic union. Then,

The Plithogenic Intersection:

$$x(\overset{0}{a_o}, \overset{c_1}{a_1}, ..., \overset{c_i}{a_i}, ..., \overset{c_n}{a_n}) \wedge_P x(\overset{0}{b_o}, \overset{c_1}{b_1}, ..., \overset{c_i}{b_i}, ..., \overset{c_n}{b_n}) = x(a_0 \wedge_F b_o, (1-c_1) \cdot (a_1 \wedge_F b_1) +$$
$$c_1 \cdot (a_1 \vee_F b_1), ..., (1-c_i) \cdot (a_i \wedge_F b_i) + c_i \cdot (a_i \vee_F b_i), ..., (1-c_n) \cdot (a_n \wedge_F b_n) + c_n \cdot (a_n \vee_F b_n)).$$

The Plithogenic Union:

$$x(\overset{0}{a_o}, \overset{c_1}{a_1}, ..., \overset{c_i}{a_i}, ..., \overset{c_n}{a_n}) \vee_P x(\overset{0}{b_o}, \overset{c_1}{b_1}, ..., \overset{c_i}{b_i}, ..., \overset{c_n}{b_n}) = x(a_0 \vee_F b_o, (1-c_1) \cdot (a_1 \vee_F b_1) +$$
$$c_1 \cdot (a_1 \wedge_F b_1), ..., (1-c_i) \cdot (a_i \vee_F b_i) + c_i \cdot (a_i \wedge_F b_i), ..., (1-c_n) \cdot (a_n \vee_F b_n) + c_n \cdot (a_n \wedge_F b_n)).$$

Plithogenic set is a generalization of the crisp set, fuzzy set, intuitionistic fuzzy set, neutrosophic set, since these these types of sets are characterized by a single attribute value (appurtenance): which has one value (membership), for the crisp set and fuzzy set, two values (membership, and nonmembership), for intuitionistic fuzzy set, or three values (membership, nonmembership and indeterminacy), for neutrosophic set.

2.1 | Example of Plithogenic Fuzzy Set

Let P be a Plithogenic set, representing the students from a college. Let x belonging to P be a generic student that is characterized by three attributes:

   I. altitude (a), whose values are {tall, short} = $\{a_1, a_2\}$.
  II. weight (w), whose values are {obese, fat, medium, thin} = $\{w_1, w_2, w_3, w_4\}$.
 III. hair color (h), whose values are {blond, reddish, brown} = $\{h_1, h_2, h_3\}$.

The uni-dimensional attribute contradiction degrees are:

$$c\left(a_1, a_2\right) = 1,$$
$$c\left(w_1, w_2\right) = 1/3; \; c\left(w_1, w_3\right) = 2/3; \; c\left(w_1, w_4\right) = 1,$$
$$c\left(h_1, h_2\right) = 1/2; \; c\left(h_1, h_3\right) = 1.$$

Dominant attribute values are: a_1, w_1, and h_1 respectively for each corresponding uni-dimensional attribute a, w, and h respectively.

The multi-attribute dimension is 9 attribute values.

P = { John(tall, short; obese, fat, medium, thin; blond, reddish, brown),

John(tall, short; obese, fat, medium, thin; blond, reddish, brown) }.

For example:

P = { John(0.8, 0.2; 0.1, 0.2, 0.4, 0.6; 0.9, 0.6, 0.0),

Richard (0.5, 0.4; 0.3, 0.21, 0.4, 0.7; 0.8, 0.5, 0.01) }.

2.2 | Example of Plithogenic Neutrosophic Set

P = { John((0.8, 0.1, 0.3), (0.2, 0.0, 0.1); (0.1, 0.5, 0.5), (0.2, 0.6, 0.6), (0.4, 0.4, 0.4),

(0.6, 0.7, 0.2); (0.9, 0.8, 0.8), (0.6, 0.3, 0.3), (0.0, 0.2, 0.2)),

Richard ((0.5, 0.2, 0.0), (0.6, 0.8, 0.8); (0.7, 0.5, 0.4), (0.1, 0.6, 0.1), (0.8, 0.4, 0.7),

(0.3, 0.4 0.2); (0.7, 0.8, 0.9), (0.5, 0.0, 0.3), (0.2, 0.8, 0.1) }.

2.3 | A Plithogenic Application to Images

A pixel x may be characterized by colors $k_1, k_2, ..., k_n$. We write $x(k_1, k_2, ..., k_n)$, where $n \geq 1$.

We may consider the degree of each color either fuzzy, intuitionistic fuzzy, or neutrosophic, or any fuzzy extension. For example:

Fuzzy degree:

$$x\left(0.4, \; 0.6, \; 0.1, \; ..., \; 0.3\right).$$

Intuitionistic fuzzy degree:

$$x\Big(\big(0.1,\ 0.2\big),\ \big(0.3,\ 0.5\big),\ \big(0.0,\ 0.6\big),\ ...,\ \big(0.8,\ 0.9\big) \Big).$$

Neutrosophic degree:

$$x\Big(\big(0.0,\ 0.3,\ 0.6\big),\ \big(0.2,\ 0.8,\ 0.9\big),\ \big(0.7,\ 0.4,\ 0.2\big),\ ...,\ \big(0.1,\ 0.1,\ 0.9\big) \Big).$$

Then, we can use the Plithogenic (fuzzy, intuitionistic fuzzy, neutrosophic, other fuzzy extension) intersection operator to combine them.

But, we first establish the degrees of contradictions $c(k_i, k_j)$ between the colors k_i and k_j in order to find the linear combinations of t-norm and t-conorm that one applies to each color (similar to the indeterminacy above). The contradiction degrees may also be: either fuzzy, intuitionistic fuzzy, neutrosophic, or other fuzzy extension. In majority of cases we choose, for simplicity, the fuzzy one.

3 | Plithogenic Logic

A Plithogenic logic proposition P is a proposition that is characterized by degrees of many truth-values with respect to the corresponding attributes' values. For each attribute's value v there is a corresponding degree of truth-value $d(P, v)$ of P with respect to the attribute value v. Plithogenic logic is a generalization of the classical logic, fuzzy logic, intuitionistic fuzzy logic, and neutrosophic logic, since these four types of logics are characterized by a single attribute value (truth-value): which has one value (truth), for the classical logic and fuzzy logic, two values (truth, and falsehood), for intuitionistic fuzzy logic, or three values (truth, falsehood, and indeterminacy), for neutrosophic logic. A Plithogenic logic proposition P, in general, may be characterized by more than four degrees of truth-values resulted from under various attributes.

3.1 | Example of Plithogenic Fuzzy Logic

Let P = "John is a knowledgeable person" be a logical proposition. It is evaluated in a fuzzy way (one component for each attribute value) by multiple experts (two) and under multiple attribute values (six), that's why it is called Plithogenic .

The three attributes under which this proposition has to be evaluated about-according to the experts-are: Science (whose attribute values are: mathematics, physics, anatomy), literature (whose attribute values are: poetry, novel), and arts (whose only attribute value is: sculpture).

According to Expert A (lexander), the fuzzy truth-values of Plithogenic proposition P are:

P_A(0.7, 0.6, 0.4; 0.9, 0.2; 0.5), which means that John's fuzzy degree of truth (knowledge) in mathematics is 0.7, degree of truth (knowledge) in physics is 0.6, degree of truth (knowledge) in anatomy is 0.4;

Degree of truth (knowledge) in poetry is 0.9, degree of truth (knowledge) in novels is 0.2;

Degree of truth (knowledge) in sculpture is 0.5.

But, according to Expert B (arbara), the truth-values of Plithogenic proposition P are:

$$P_{B\ =}\Big(0.9,\ 0.6,\ 0.2;\ 0.8,\ 0.7;\ 0.3\Big).$$

Assume that the experts consider that the attributes' values contradiction (dissimilarity) degrees are:

0	0.3	0.8	0	0.9	0
mathematics	physics	anatomy	poetry	novels	sculpture

And taking the fuzzy $t_{norm}(a, b) = a \wedge_F b = a \cdot b$

And fuzzy $t_{conorm}(a, b) = a \vee_F b = a + b - a \cdot b$

One gets:

$P_A(0.7, 0.6, 0.4; 0.9, 0.2; 0.5) \wedge_P P_B(0.9, 0.6, 0.2; 0.8, 0.7; 0.3) =$

$P_A \wedge_P {}_B(0.7 \wedge_P 0.9, 0.6 \wedge_P 0.6, 0.4 \wedge_P 0.2; 0.9 \wedge_P 0.8, 0.2 \wedge_P 0.7; 0.5 \wedge_P 0.3) =$

$P_A \wedge_P {}_B(0.7 \wedge_F 0.9, 0.6 \wedge_P 0.6, 0.4 \wedge_P 0.2; 0.9 \wedge_F 0.8, 0.2 \wedge_P 0.7; 0.5 \wedge_F 0.3) =$

$P_A \wedge_P {}_B(0.7 \cdot 0.9, (1-0.3) \cdot [0.6 \wedge_F 0.6] + 0.3 \cdot [0.6 \vee_F 0.6], (1-0.8) \cdot [0.4 \wedge_F 0.2] + 0.8 \cdot [0.4 \vee_F 0.2];$

$0.9 \cdot 0.8, (1-0.9) \cdot [0.2 \wedge_F 0.7] + 0.9 \cdot [0.2 \vee_F 0.7]; 0.5 \cdot 0.3) =$

$P_A \wedge_P {}_B(0.630, 0.7 \cdot [0.6 \cdot 0.6] + 0.3 \cdot [0.6 + 0.6 - 0.6 \cdot 0.6],$

$0.2 \cdot [0.4 \cdot 0.2] + 0.8 \cdot [0.4 + 0.2 - 0.4 \cdot 0.2];$

$0.720, 0.1 \cdot [0.2 \cdot 0.7] + 0.9 \cdot [0.2 + 0.7 - 0.2 \cdot 0.7]; 0.150) =$

$P_A \wedge_P {}_B(0.630, 0.504, 0.432; 0.720, 0.698; 0.150),$

Where $\wedge_P, \wedge_F, \vee_P, \vee_P$ represent the fuzzy intersection, fuzzy union, Plithogenic intersection, and Plithogenic union respectively.

3.2 | Example of Plithogenic Neutrosophic Logic

Let P = "John is a knowledgeable person" be a logical proposition. It is evaluated in a neutrosophic way (three components for each attribute value) by multiple attribute values (six), that's why it is called Plithogenic .

P((0.7, 0.1, 0.5), (0.6, 0.0, 0.9), (0.4, 0.4, 0.4); (0.9, 0.2, 0.3), (0.2, 0.6, 0.7); (0.5, 0.3, 0.1)), which means that John's neutrosophic degrees of truth (knowledge), indeterminate-truth, and falsehood (nonknowledge) in mathematics is 0.7, 0.1, and 0.5 respectively, and similarly for his neutrosophic degrees in physics, and anatomy.

In the same way for Richard's neutrosophic degrees.

4 | Plithogenic Probability

Since in Plithogenic probability each event E from a probability space U is characterized by many chances of the event to occur [not only one chance of the event E to occur: as in classical probability, imprecise probability, and neutrosophic probability], a Plithogenic probability distribution function, PP(x), of a random variable x, is described by many Plithogenic probability distribution sub-functions, where each sub-function represents the chance (with respect to a given attribute value) that value x occurs, and these chances of occurrence can be represented by classical, imprecise, or neutrosophic probabilities (depending on the type of degree of a chance).

4.1 | Example of Plithogenic Probabilistic

What is the Plithogenic probability that Jennifer will graduate at the end of this semester in her program of electrical engineering, given that she is enrolled in and has to pass two courses of mathematics (non-

linear differential equations, and stochastic analysis), and two courses of Mechanics (Fluid Mechanics, and Solid Mechanics)?

We have a 4 attribute-values of Plithogenic probability.

According to her adviser, Jennifer's Plithogenic single-valued fuzzy probability of graduating at the end of this semester is:

$$J\big(\ 0.5,\ 0.6;\ 0.8,\ 0.4\ \big).$$

Which means 50% chance of passing the non-linear differential equations class,

60% chance of passing the Stochastic Analysis class (as part of Mathematics), and 80% of passing the Fluid Mechanics class, and 40% of passing the Solid Mechanics class (as part of Physics).

Therefore, the Plithogenic probability in this example is composed from 4 classical probabilities.

Also, according to her adviser, Jennifer's Plithogenic single-valued neutrosophic probability of graduating at the end of this semester is:

$$J\big(\ (0.5,\ 0.1,\ 0.3),\ (0.6,\ 0.2,\ 0,7);\ (0.8,\ 0.5,\ 0.4),\ (0.4,\ 0.0,\ 0.9)\ \big).$$

Which means 50% chance of passing, 10% indeterminate-chance, and 30% chance of non-passing the non-linear differential equations class; similarly for the others.

While the Plithogenic Probability of an event E to occur is calculated with respect to many chances of the event E to occur (it is calculated with respect to each event's attribute/parameter chance of occurrence).

Therefore, the Plithogenic Probability is a Multi-Probability (i.e. multi-dimensional probability), unlike the classical, imprecise and neutrosophic probabilities that are uni-dimensional probabilities.

The Neutrosophic Probability (and similarly for classical probability, and for the imprecise probability) of an event E to occur is calculated with respect to the chance of the event E to occur (i.e. it is calculated with respect to only ONE chance of occurrence).

5 | Plithogenic Statistics

As a generalization of classical statistics and neutrosophic statistics, the Plithogenic Statistics is the analysis of events described by the Plithogenic probability.

Plithogenic statistics is a multivariate statistics, characterized by multiple random variables, whose degrees may be: classical, fuzzy, intuitioninstic fuzzy, neutrosophic, or any other fuzzy extension.

In neutrosophic statistics we have some degree of indeterminacy, incompleteness, inconsistency into the data or into the statistical inference methods.

5.1 | Example of Plithogenic Statistics

Let's consider the previous example of Plithogenic Probability that Jenifer will graduate at the end of this semester in her program of electrical engineering.

We now graph four probability distribution functions, instead of one as in the case when considering the neutrosophic distribution as a uni-dimensional neutrosophic function. Therefore, Plithogenic Statistics is an extension of the multi-variate statistics.

6 | Symbolic Plithogenic Algebraic Structures

Definitions of symbolic Plithogenic set and symbolic Plithogenic algebraic structures

Let SPS be a non-empty set, included in a universe of discourse U, defined as follows:

SPS = {x | x = $a_0 + a_1p_1 + a_2P_2 + ... + a_nP_n$, n ≥ 1, all a_i belong to R, or to C, or belong to some given algebraic structure space, for 0 ≤ i ≤ n}.

where R = the set of real numbers, C = the set of complex numbers, and all P_i are symbols (letters, or variables), and are called Symbolic (Literal) Plithogenic Components (Variables)}, where 1, P_1, P_2, …, P_n act like a base for the elements of the above set SPS. a_0, a_1, a_2, …, a_n are called coefficients.

SPS is called a Symbolic Plithogenic Set. And the algebraic structures defined on this set are called Symbolic Plithogenic Algebraic Structures.

In general, Symbolic (or Literal) Plithogenic theory is referring to the use of abstract symbols {i.e. the letters/parameters) P_1, P_2, …, P_n, representing the Plithogenic components (variables) as above} in some theory.

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