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A Neutrosophic Approach to Digital Images

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Abstract: This research paper presents a neutrosophic mathematical representation of the elements of the digital image by dividing the points of the digital picture matrix into neutrosophic sets (PNS - Picture Neutrosophic Set), and studying the degree of connection between the points of the digital image for us to reach to the connected neutrosophic sets. We have also introduced many mathematical theories and results to calculate the difference and dissimilarity between the neutrosophic sets, which contributes practically in the comparison between digital images and their different uses. Our results help mainly to upgrade and create new neutrosophic algorithms for searching inside images and videos databases**.**

Keywords: Neutrosophic set; connected neutrosophic set; picture neutrosophic set (PNS); difference measure; dissimilarity measure.

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1. Introduction

The neutrosophic logic, which resulted in a revolution in the mathematical logic world, was first introduced by Florentin in 1995 [1, 2]. It is a generalization of intuitionistic fuzzy logic. Several papers have been published in this field by Florentin and Salama et al [3-15]. It is necessary to take advantage of the features of this logic in various applied sciences.

Having studied researches related to digital image processing [16-18], we have noted that applied sciences researchers are interested in the use of fuzzy logic, first introduced by Lotfi Zadeh [19], for digital image processing because of its flexibility and appropriate features to deal with different forms of digital images. Moreover, the neutrosophic logic is a generalization and extension of fuzzy logic. It has provided many additional methods and tools, which we can be used to study digital images with greater accuracy and comprehensiveness than before.

Digital image processing is mainly based on mathematical concepts [20-26], such as mathematical logic, linear algebra (matrices), topology, statistics (especially Bayes' theory), Shannon information theory, and Fourier transform in different representations along with neural networks.

Several researchers have performed studies specifying methods to measure the dissimilarity, difference and distance between NSs. Salama, Smarandache, & Eisa, (2014) [27] have introduced image processing via neutrosophic techniques. Mohana & Mohanasundari (2019) [28] have studied some similarity measures of single valued neutrosophic rough sets. Sinha & Majumdar (2019) [29] have studied an approach to similarity measure between neutrosophic soft sets. Das, Samanta, Khan, Naseem & De (2020) [30] also have a study on discrete mathematics: sum distance in neutrosophic graphs with application.

We have organized this paper into 4 sections. In section 2, we discuss preliminaries about digital images and the neutrosophic set. In section 3, we have introduced new neutrosophic concepts, such as $K_S(\alpha)$ (the extent to which the series of points (α) belongs to the neutrosophic set S), and $C_S(p,q)$ (the connection strength between the points $p,q \in S$), based on which we have deduced connected neutrosophic sets. In addition, we have presented our vision in the field of distance and dissimilarity measures in neutrosophic sets. In section 4, we have concluded our paper.

2. Preliminaries

2.1. Digital Image:[31] It is a representation of a two-dimensional image in the form of a matrix of small squares, each image consists of thousands or millions of small squares, each of which is called the elements of the image or pixels.

When the computer starts drawing the image, it divides the screen or printed page into a grid of pixels. Then the computer uses the stored values of the digital image to give each pixel its color and brightness. The images posted on websites or by mobile phone are examples of digital images. For example, the small picture (Felix) can be represented in Figure 1:

Figure 1: Image of Cat Felix [31]

With an array (35 × 35), its elements are composed of numbers 0 and 1. Each element indicates the color of the pixel. It takes the value (0) for the black pixel and the value (1) for the white pixel. Note that digital images using two colors are called binary or Boolean images.

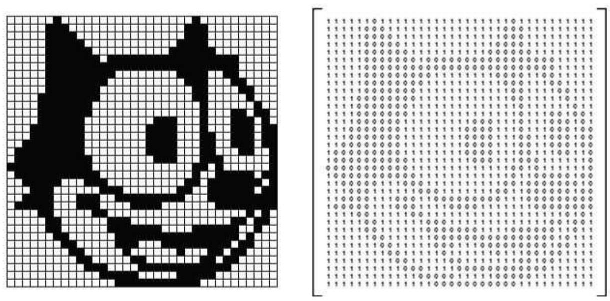

Figure 2: Matrix representing the image of Cat Felix [31]

The grayscale images are represented by a matrix, each element of which specifies the corresponding pixel intensity. For practical reasons, most of the current digital files use integers enclosed between zero-0 (for black pixels, very low color) and 255 (for the white pixel, the color is super hard).

2.2. Neutrosophic Logic:[2] Was created by professor Florentin Smarandache in 1995. It is a generalization of (fuzzy, intuitionistic, paraconsistent) logic. For any logical variable x in the neutrosophic logic A, it is described by (t, i, f), where:

$t = T_A(x)$: Truth membership function: a degree of membership function, for any x in the neutrosophic set A, and its values range in the open interval non-standard, where:

$T_A(x)$: $A \rightarrow]0^-, 1^+[$

$i = I_A(x)$: Indeterminacy membership function: a degree of indeterminacy, for any x in the neutrosophic set A, and its values range in the open interval non-standard, where:

$I_A(x)$: $A \rightarrow]0^-, 1^+[$

$f = F_A(x)$: Falsity membership function: a degree of non-membership, for any x in the neutrosophic set A, and its values range in the open interval non-standard, where:

$F_A(x)$: $A \rightarrow]0^-, 1^+[$

3. Neutrosophic Digital Image

Let M be the digital image matrix A, so any pixel (point) of image A that is expressed by the element $p\ (x, y)$ of the matrix M has four horizontal and vertical adjacent points $(x \pm 1, y)$ and $(x, y \pm 1)$ and four diagonal adjacent points $(x \pm 1, y \pm 1)$, so any point or pixel is surrounded by eight adjacent points (8-adjacent), noting the cases where the point P is present on the border of the matrix M.[18]

3.1. Connected Neutrosophic Sets:

Definition 3.1: Let S be a subset of M. For any p, q from S, they are connected in S if you find a path of points from S that connects p with q as follows:

$\alpha: p = p_0, p_1, p_2, \dots\dots, p_{n-1}, p_n = q$.

Where p_i is adjacent to p_{i-1} $(1 \leq i \leq n)$.

We denote the connection relationship between p, q by $p\rho q$.

Obviously, the relationship (ρ) represents an equivalence relationship:

$Reflexive: [p\rho p]$ & $Symmetric: [p\rho q \Rightarrow q\rho p]$ & $Transitive: [p\rho q \ \& \ q\rho z \Rightarrow p\rho z]$

Remark 3.1: By introducing the concept of non- member function and the function of indeterminacy to the neutrosophic logic, it has got more accuracy than fuzzy logic in different cases, such as an equal degree of membership. Thus, we can introduce the order relation ($\ddot{\leqq}$) between any two elements in the neutrosophic set:

Definition 3.2: $\forall\, p, q \in S$, (S is neutrosophic set), then:

$$p \ddot{\leqq} q \Leftrightarrow \begin{cases} T_S(p) < T_S(q) \\ (or)\ F_S(p) > F_S(q) \ ;\ T_S(p) = T_S(q) \\ (or)\ I_S(p) \geq I_S(q) \ ;\ T_S(p) = T_S(q), \quad F_S(p) = F_S(q) \end{cases}$$

Remark 3.2: The order relation ($\dddot{\leqq}$) maintains its consistency with fuzzy logic in the case of $T_S(p) \neq T_S(q)$, and maintains consistency with intuitionistic fuzzy logic in the case of $F_S(p) \neq F_S(q)$ & $T_S(p) = T_S(q)$.

Example 3.1: $(0,1,1) \dddot{\leqq} (0,0,0)$, $(1,1,1) \dddot{\leqq} (1,0.5,1)$

$(0.9,1,0.8) \dddot{\leqq} (1,0.3,0.1)$, $(0.7,0,0.4) \dddot{\leqq} (0.7,1,0.3)$

Definition 3.3: [2] $\forall\, p,q \in S$, then:$[p \leq q] \Leftrightarrow [T_S(p) \leq T_S(q)\ ,\ I_S(p) \geq I_S(q) \quad and \quad F_S(p) \geq F_S(q)]$

Remark 3.3: $\forall\, p,q \in S \ \Rightarrow \begin{bmatrix} [p \leq (1,0,0) = 1_N \ \ \& \ \ 0_N = (0,1,1) \ \leq p] \\ [\,p \dddot{\leqq} (1,0,0) = 1_N \ \ \& \ \ 0_N = (0,1,1) \ \dddot{\leqq} p] \end{bmatrix}$ & $[p \leq q \ \Rightarrow \ p \dddot{\leqq} q]$

Definition 3.4: Let $(\alpha: p = p_0, p_1, p_2, \ldots\ldots, p_{n-1}, p_n = q)$, series of adjacent points, between the points $,q\ :\ p_i \in S$ (S neutrosophic set). The extent to which the series of points (α) belongs to the neutrosophic set S, denote by $K_S(\alpha)$:

$K_S(\alpha) = x \quad ; \quad (x \in \alpha) \ \ and \ (\, x \dddot{\leqq} \ \ p_i \ ;\ i = 0,1, \ldots\ldots, n\,)$

$\Rightarrow \ K_S(\alpha) = \min_{\dddot{\leqq}}(p_i)$

Definition 3.5: The connect strength between the points $p,q \in S$ (S neutrosophic set).
denote by $C_S(p,q)$: $C_S(p,q) = K_S(\beta)$; $K_S(\alpha_i) \dddot{\leqq} \ K_S(\beta)$ $(\forall \alpha_i, \beta: p, \ldots, q)$

$\Rightarrow \ C_S(p,q) = \max_{\dddot{\leqq}}(K_S(\alpha_i))$

Theorem 3.1: S neutrosophic set and $\forall\, p,q \in S$, then:

1: $C_S(p,p) = p$

2: $C_S(p,q) = C_S(q,p)$

Proof:

1: α_i any path, from p to p $\quad \Rightarrow \quad K_S(\alpha_i) = \min_{\dddot{\leqq}}(p_i) \dddot{\leqq} \ p$

On the other hand:

The point p alone represents a series with a length of 0 from p to p, then:

$$\exists\ \alpha_i :\ K_S(\alpha_i) = \ p$$

Thus: $C_S(p,p) = \max_{\dddot{\leqq}}(K_S(\alpha_i)) = p$

2: Obviously. (by Definition 3.4)

Theorem 3.2: $\forall\, p,q \in S$ **(**S neutrosophic set**),** then:

$$C_S(p,q) \leq \min_{\dddot{\leqq}}(p,q)$$

Proof:

α any path, from p to q: $(\alpha: p = p_0, p_1, \ldots, p_{n-1}, p_n = q)$, then:

$K_S(\alpha) = \min_{\dddot{\leqq}}(p_i) \ \leq \min_{\dddot{\leqq}}(p_0, p_n) = \min_{\dddot{\leqq}}(p,q) \quad ; i = 0,1, \ldots\ldots, n$

$\Rightarrow \ \ C_S(p,q) = \max_{\dddot{\leqq}}\big(K_S(\alpha_i)\big) \ \leq \min_{\dddot{\leqq}}(p,q)$

Definition 3.6: $\forall\, p,q \in S,\ p$ and q is connected in S iff: $C_S(p,q) = \min_{\dddot{\leqq}}(p,q)$.

Theorem 3.3: S neutrosophic set and $\forall\, p,q \in S$, then:

p and q is connected in $S \ \ \Leftrightarrow \exists\ \alpha' : p = p_0, p_1, \ldots, p_{n-1}, p_n = q \ :\ P_i \in S$ & $P_i \geq \min_{\dddot{\leqq}}(p,q)$ (for all i)

Proof:

Let α' path from p to : $P_i \in S$ & $P_i \geq \min_{\ddot{\leqq}}(p,q)$, then:

$C_S(p,q) = \max_{\ddot{\leqq}}\big(K_S(\alpha)\big) \geq K_S(\alpha') = \min_{\ddot{\leqq}}(p_i) \geq \min_{\ddot{\leqq}}(p,q)$

And $C_S(p,q) \leq \min_{\ddot{\leqq}}(p,q)$ (Theorem 3.2)

Then: $C_S(p,q) = \min_{\ddot{\leqq}}(p,q) \Rightarrow p$ and q is connected in S

On the other hand: p and q is connected in , then:

$\exists\, \alpha'$ path from p to q : $K_S(\alpha') = \max_{\ddot{\leqq}}\big(K_S(\alpha)\big) = C_S(p,q) = \min_{\ddot{\leqq}}(p,q)$

Then for all P_i on α', we have: $P_i \geq K_S(\alpha') = \min_{\ddot{\leqq}}(p,q)$

Corollary 3.1: From the above we note that the relationship of the connection between two points is the relationship of: 1: reflexivity, 2: Symmetry, 3: not necessarily transitive.

Proof:

1: $C_S(p,p) = p = \min_{\ddot{\leqq}}(p,p)$

2: $\min_{\ddot{\leqq}}(p,q) = C_S(p,q) \Rightarrow C_S(q,p) = C_S(p,q) = \min_{\ddot{\leqq}}(p,q)$

3: Let p,q,z three points from neutrosophic set $S = [p,q,z]$ ($Matrix$ 1 × 3):

$q \ddot{\leqq} p = z$, then:

$C_S(p,q) = C_S(q,z) = q$ and $C_S(p,z) = q \neq \min_{\ddot{\leqq}}(p,z)$, thus:

(p and q is connected in S) and (q and z is connected in S), but (p and z is not connected in S)

Definition 3.7: S neutrosophic set, S is connected iff: $[\forall\, p,q \in S] \Rightarrow [C_S(p,q) = \min_{\ddot{\leqq}}(p,q)]$.

3.2. Operations on neutrosophic sets:

We will now in this section, we present our vision of distance and dissimilarity measures between two neutrosophic sets.

Definition 3.8: Let U be the set of points of the matrix M, a representative of the digital image I. denote by $PNS(U)$ for set of all neutrosophic sets in U (PNS - Picture Neutrosophic Set).

For $A,B \in PNS(U)$:

Union: $A \cup B = \big\{u\colon \big(T_{A\cup B}(u), I_{A\cup B}(u), F_{A\cup B}(u)\big); u \in U\big\}$, where:

$$T_{A\cup B}(u) = \max(T_A(u), T_B(u))$$

$$I_{A\cup B}(u) = \min(I_A(u), I_B(u))$$

$$F_{A\cup B}(u) = \min(F_A(u), F_B(u))$$

Intersection: $A \cap B = \{u\colon \big(T_{A\cap B}(u), I_{A\cap B}(u), F_{A\cap B}(u)\big); u \in U\}$, where:

$$T_{A\cap B}(u) = \min(T_A(u), T_B(u))$$

$$I_{A\cap B}(u) = \max(I_A(u), I_B(u))$$

$$F_{A\cap B}(u) = \max(F_A(u), F_B(u))$$

Example 3.2: Let us consider the following neutrosophic sets A and B in
$U = \{u_1, u_2, u_3, u_4\}$, where:
$A = \{u_1: (0,1,1),\ u_2: (1,0.2,0),\ u_3: (0.7,0.3,1),\ u_4: (0.9,0.3,1)\}$
$B = \{u_1: (0.2,1,0.2),\ u_2: (1,0.5,0.3),\ u_3: (1,0.8,1),\ u_4: (0.9,0.8,0.2)\}$
Then:
$A \cup B = \{u_1: (0.2,1,0.2),\ u_2: (1,0.2,0),\ u_3: (1,0.3,1),\ u_4: (0.9,0.3,0.2)\}$
$A \cap B = \{u_1: (0,1,1),\ u_2: (1,0.5,0.3),\ u_3: (0.7,0.8,1),\ u_4: (0.9,0.8,1)\}$

Theorem 3.4: Let $A, B \in PNS\ (U)$, then: (for all $u \in U$)
$A \subseteq B \iff [T_A(u) \leq T_B(u)\ ,\quad I_A(u) \geq I_B(u) \quad and \quad F_A(u) \geq F_B(u)]$
Proof:

$A \subseteq B \iff A \cup B = B$

$$\iff \begin{cases} \max\big(T_A(u), T_B(u)\big) = T_B(u) \\ \min\big(I_A(u), I_B(u)\big) = I_B(u) \\ \min\big(F_A(u), F_B(u)\big) = F_B(u) \end{cases} \iff \begin{cases} T_A(u) \leq T_B(u) \\ I_A(u) \geq I_B(u) \\ F_A(u) \geq F_B(u) \end{cases}$$

Definition 3.9: An operator $\backslash: PNS(U) \times PNS(U) \to PNS(U)$
Is the difference, if it satisfies for all $A, B, C \in PNS(U)$, follow properties:
DIF1: $A\backslash B \subseteq A$
DIF2: $A\backslash \emptyset = A$
DIF3: $A \subseteq B \iff A\backslash B = \emptyset$
DIF4: $if\ B \subseteq C \implies B\backslash A \subseteq C\backslash A$

Theorem 3.5: The function $\backslash: PNS(U) \times PNS(U) \to PNS(U)$ given by:

$A\backslash B = \{u: \big(T_{A\backslash B}(u), I_{A\backslash B}(u), F_{A\backslash B}(u)\big); u \in U\}$, where:

$$T_{A\backslash B}(u) = \max(0, T_A(u) - T_B(u))$$
$$I_{A\backslash B}(u) = \min(1,1 + (I_A(u) - I_B(u)))$$
$$F_{A\backslash B}(u) = \min(1,1 + (F_A(u) - F_B(u)))$$

Is the difference between $PNS(U)$ sets.
Proof:
DIF1: $A\backslash B \subseteq A$

$$\forall\ u \in U \implies \begin{cases} T_{A\backslash B}(u) = \max\big(0, T_A(u) - T_B(u)\big) \\ I_{A\backslash B}(u) = \min\Big(1,1 + \big(I_A(u) - I_B(u)\big)\Big) \\ F_{A\backslash B}(u) = \min\Big(1,1 + \big(F_A(u) - F_B(u)\big)\Big) \end{cases}$$

$[0 \leq T_A(u)] \quad and \quad [0 \leq T_B(u) \implies T_A(u) - T_B(u) \leq T_A(u)]$
Hence: $T_{A\backslash B}(u) = \max\big(0, T_A(u) - T_B(u)\big) \leq T_A(u)$

$$I_A(u) \leq 1 \quad and \quad \begin{bmatrix} I_B(u) \leq 1 & \implies I_B(u) + I_A(u) \leq 1 + I_A(u) \\ & \implies I_A(u) \leq 1 + \big(I_A(u) - I_B(u)\big) \end{bmatrix}$$

Hence: $I_{A\backslash B}(u) = \min(1,1 + (I_A(u) - I_B(u))) \geq I_A(u)$
Similarity: $F_{A\backslash B}(u) = \min(1,1 + (F_A(u) - F_B(u))) \geq F_A(u)$
Thus: $A\backslash B \subseteq A$ (by Theorem 3.4)

DIF2: $A\backslash\emptyset \;\; = A$

$\emptyset = \{u: (0,1,1) \;\; ; \forall u \in U\} \;\; \Rightarrow$

$$\forall\, u \in U \Rightarrow \begin{cases} T_{A\backslash\emptyset}(u) = \max(0, T_A(u) - 0) \;\; = \;\; T_A(u) \\ I_{A\backslash\emptyset}(u) \;= \min\big(1{,}1 + (I_A(u) - 1)\big) = \; I_A(u) \\ F_{A\backslash\emptyset}(u) = \min\big(1{,}1 + (F_A(u) - 1)\big) = \; F_A(u) \end{cases}$$

Then: $A\backslash\emptyset \;\; = A$

DIF3: $A \subseteq B \quad \Leftrightarrow \quad A\backslash B = \emptyset$

$$A \subseteq B \quad \Leftrightarrow \begin{cases} T_A(u) \;\le T_B(u) \\ I_A(u) \;\ge I_B(u) \\ F_A(u) \ge F_B(u) \end{cases} \quad \Leftrightarrow \quad \begin{cases} T_{A\backslash B}(u) = 0 \\ I_{A\backslash B}(u) = 1 \\ F_{A\backslash B}(u) = 1 \end{cases} \quad \Leftrightarrow \quad A\backslash B = \emptyset$$

DIF4: $if\; B \subseteq C \quad \Longrightarrow B\backslash A \subseteq C\backslash A$

$$B \subseteq C \quad \Longrightarrow \begin{cases} T_B(u) \;\le T_C(u) \\ I_B(u) \;\ge I_C(u) \\ F_B(u) \ge F_C(u) \end{cases} \quad \Longrightarrow \quad \begin{cases} T_B(u) - T_A(u) \le T_C(u) - T_A(u) \\ I_B(u) - I_A(u) \ge I_C(u) - I_A(u) \\ F_B(u) - F_A(u) \ge F_C(u) - F_A(u) \end{cases}$$

$$\Longrightarrow \begin{cases} T_{B\backslash A}(u) \le T_{C\backslash A}(u) \\ I_{B\backslash A}(u) \;\ge I_{C\backslash A}(u) \\ F_{B\backslash A}(u) \ge F_{C\backslash A}(u) \end{cases} \quad \Longrightarrow \quad B - A \subseteq C - A$$

Example 3.3: Let $U = \{u_1, u_2, u_3\}$, and $A, B \in PNS(U)$:

$A = \{u_1: (0.8{,}0.1{,}0.3),\; u_2: (0.9{,}0.2{,}0),\; u_3: (0.9{,}0.8{,}1)\}$

$B = \{u_1: (0.2{,}1{,}0.2),\; u_2: (1{,}0.5{,}0.3),\; u_3: (0.9{,}0.8{,}1)\}$

Then: $A\backslash B = \{u_1: (0.6{,}0.1{,}1), u_2: (0{,}0.7{,}0.7), u_3: (0{,}1{,}1)\}$

Definition 3.10: An operator $D: PNS(U) \times PNS(U) \;\to\;]^-0, 1^+[$
Is the distance measure, if it satisfies for all $A, B, C \in \; PNS(U)$, follow properties:

DIS1: $D(A, B) = 0 \;\; \Leftrightarrow \;\; A = B$

DIS2: $D(A, B) = D(B, A)$

DIS3: $D(A, C) \le D(A, B) + D(B, C)$

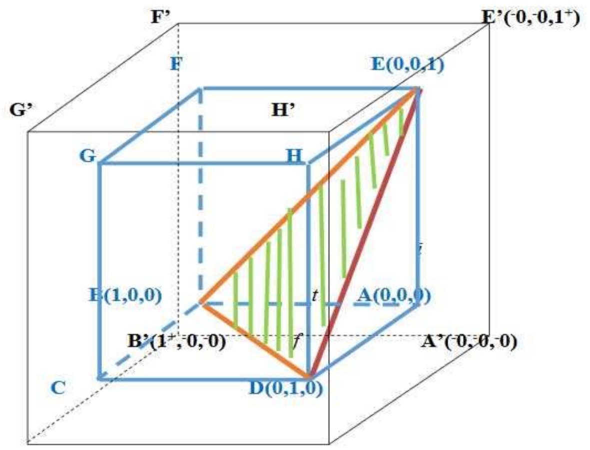

Figure 3: A three-dimension representation of a neutrosophic set [27]

Theorem 3.6: The function $D: PNS(U) \times PNS(U) \rightarrow]^{-}0, 1^{+}[$ given by: [27]

$$D(A,B) = \sqrt{\frac{1}{3n}\sum_{i=1}^{n}[(T_A(u_i) - T_B(u_i))^2 + (I_A(u_i) - I_B(u_i))^2 + (F_A(u_i) - F_B(u_i))^2]}$$

Is the distance measure between $PNS(U)$ sets.

Proof: Obviously: $D(A,B)$ is generalization of the usually used to measure the distance of objects in Euclidean geometry

Example 3.4: Let $U = \{u_1, u_2, u_3\}$, and $A, B \in INS(U)$:

$$A = \{u_1: (0.8,0.1,0.3),\ u_2: (0.4,0.5,0),\ u_3: (0.7,0.8,1)\}$$
$$B = \{u_1: (0.7,1,0.5),\ u_2: (1,0.5,0.3),\ u_3: (0.9,0.8,0.7)\}$$

Then: $D_{NE}(A,B) = \sqrt{\frac{1}{9}(0.86 + 0.45 + 0.13)} = \sqrt{\frac{1}{9}1.44} = \sqrt{0.16} = 0.4$

Definition 3.11: An operator $DM: PNS(U) \times PNS(U) \rightarrow (DM_T, DM_I, DM_F)$, where:

DM_T: denote the degree of dissimilarity. ($^{-}0 \leq DM_T \leq 1^{+}$)

DM_I: denote the degree of indeterminate dissimilarity. ($^{-}0 \leq DM_I \leq 1^{+}$)

DM_F: denote the degree of non-dissimilarity. ($^{-}0 \leq DM_F \leq 1^{+}$)

Is the dissimilarity measure, if it satisfies for all $A, B, C \in PNS(U)$, follow properties:

DISM1: $DM(A,A) = (0,1,1)$

DISM2: $DM(A,B) = DM(B,A)$

DISM3: $A \subseteq B \subseteq C \implies DM(A,B) \leq DM(A,C)$ & $DM(B,C) \leq DM(A,C)$

Remark 3.4: Let $A, B \in PNS(U)$, then:

$(A \backslash B) \cup (B \backslash A) = \{u: (T'(u), I'(u), F'(u)); u \in U\}$, where:

$T'(u) = \max\big(\max\big(0, T_A(u) - T_B(u)\big), \max\big(0, T_B(u) - T_A(u)\big)\big) = |T_A(u) - T_B(u)|$

$I'(u) = \min\Big(\min\Big(1,1 + \big(I_A(u) - I_B(u)\big)\Big), \min\Big(1,1 + \big(I_B(u) - I_A(u)\big)\Big)\Big) = 1 - |I_A(u) - I_B(u)|$

$F'(u) = \min\Big(\min\Big(1,1 + \big(F_A(u) - F_B(u)\big)\Big), \min\Big(1,1 + \big(F_B(u) - F_A(u)\big)\Big)\Big) = 1 - |F_A(u) - F_B(u)|$

Theorem 3.7: The function $DM: PNS(U) \times PNS(U) \rightarrow (DM_T, DM_I, DM_F)$

Given by, $\forall A, B \in PNS(U)$: $DM(A,B) = (DM_T(A,B), DM_I(A,B), DM_F(A,B))$, where:

$$DM_T(A,B) = \frac{1}{n}\sum_{i=1}^{n}[|T_A(u_i) - T_B(u_i)|]$$

$$DM_I(A,B) = \frac{1}{n}\sum_{i=1}^{n}[1 - |I_A(u_i) - I_B(u_i)|]$$

$$DM_F(A,B) = \frac{1}{n}\sum_{i=1}^{n}[1 - |F_A(u_i) - F_B(u_i)|]$$

Is the dissimilarity measure between $PNS(U)$ sets.

Proof:

DISM1: $DM(A,A) = (0,1,1)$

$$DM_T(A,A) = \frac{1}{n}\sum_{i=1}^{n}[|T_A(u_i) - T_A(u_i)|] = \frac{1}{n}\sum_{i=1}^{n}[0] = 0$$

$$DM_I(A,A) = \frac{1}{n}\sum_{i=1}^{n}[1 - |I_A(u_i) - I_A(u_i)|] = \frac{1}{n}\sum_{i=1}^{n}[1-0] = 1$$

$$DM_F(A,A) = \frac{1}{n}\sum_{i=1}^{n}[1 - |F_A(u_i) - F_A(u_i)|] = \frac{1}{n}\sum_{i=1}^{n}[1-0] = 1$$

DISM2: $DM(A,B) = DM(B,A)$

$$DM_T(A,B) = \frac{1}{n}\sum_{i=1}^{n}[|T_A(u_i) - T_B(u_i)|] = \frac{1}{n}\sum_{i=1}^{n}[|T_B(u_i) - T_A(u_i)|] = DM_T(B,A)$$

$$DM_I(A,B) = \frac{1}{n}\sum_{i=1}^{n}[1 - |I_A(u_i) - I_B(u_i)|] = \frac{1}{n}\sum_{i=1}^{n}[1 - |I_B(u_i) - I_A(u_i)|] = DM_I(B,A)$$

$$DM_F(A,B) = \frac{1}{n}\sum_{i=1}^{n}[1 - |F_A(u_i) - F_B(u_i)|] = \frac{1}{n}\sum_{i=1}^{n}[1 - |F_A(u_i) - F_B(u_i)|] = DM_F(B,A)$$

DISM3: $A \subseteq B \subseteq C \implies DM(A,B) \le DM(A,C) \;\&\; DM(B,C) \le DM(A,C)$

$A \subseteq B \subseteq C \implies T_A(u) \le T_B(u) \le T_C(u)$

$$\implies |T_A(u) - T_B(u)| + |T_B(u) - T_C(u)| = |T_A(u) - T_C(u)|$$

$$\implies |T_A(u) - T_B(u)| \le |T_A(u) - T_C(u)| \;\&\; |T_B(u) - T_C(u)| \le |T_A(u) - T_C(u)|$$

$$\implies DM_T(A,B) \le DM_T(A,C) \;\&\; DM_T(B,C) \le DM_T(A,C)$$

$A \subseteq B \subseteq C \implies I_A(u) \ge I_B(u) \ge I_c(u)$

$$\implies |I_A(u) - I_B(u)| + |I_B(u) - I_c(u)| = |I_A(u) - I_C(u)|$$

$$\implies |I_A(u) - I_B(u)| \le |I_A(u) - I_C(u)| \;\&\; |I_B(u) - I_c(u)| \le |I_A(u) - I_C(u)|$$

$$\implies 1 - |I_A(u) - I_B(u)| \ge 1 - |I_A(u) - I_C(u)| \;\&\; 1 - |I_B(u) - I_c(u)| \ge 1 - |I_A(u) - I_C(u)|$$

$$\implies DM_I(A,B) \ge DM_I(A,C) \;\&\; DM_I(B,C) \ge DM_I(A,C)$$

Similarity, $DM_F(A,B) \ge DM_F(A,C) \;\&\; DM_F(B,C) \ge DM_F(A,C)$.

Then: $DM(A,B) \le DM(A,C) \;\&\; DM(B,C) \le DM(A,C)$

Example 3.5: Let $U = \{u_1, u_2, u_3\}$, and $A, B \in PNS(U)$:

$$A = \{u_1: (0.1,0.1,1),\ u_2: (0,0.2,0.8),\ u_3: (1,1,0)\}$$
$$B = \{u_1: (0.5,1,0.5),\ u_2: (1,0.5,0.2),\ u_3: (0,1,1)\}$$

Then:

$$DM_T(A,B) = \frac{1}{n}\sum_{i=1}^{n}[|T_A(u_i) - T_B(u_i)|] = \frac{1}{3}(0.4 + 1 + 1) = \frac{2.4}{3} = 0.8$$

$$DM_I(A,B) = \frac{1}{n}\sum_{i=1}^{n}[1 - |I_A(u_i) - I_B(u_i)|] = \frac{1}{3}(0.1 + 0.7 + 1) = \frac{1.8}{3} = 0.6$$

$$DM_F(A,B) = \frac{1}{n}\sum_{i=1}^{n}[1 - |F_A(u_i) - F_B(u_i)|] = \frac{1}{3}(0.5 + 0.4 + 0) = \frac{0.9}{3} = 0.3$$

Thus: $DM(A,B) = (0.8,0.6,0.3)$

4. Conclusion

By combining the concepts of algebraic with the neutrosophic sets, we introduce the neutrosophic order relation ($\ddot{\leq}$), the connected points, the connection strength between the points inside the neutrosophic set and the connected neutrosophic sets. Thus, it became a new and interesting research topic on which researchers can do further studies. In addition, in this paper, we have defined the basic operations (union, intersection, difference) on the picture neutrosophic set $PNS(U)$. We have proposed a new method for dissimilarity measure between $PNS(U)$ sets. These measures and operations are used basically in image processing and comparison. In the future, we will study the properties of these measures and their applications in practical problems.

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