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Neutrosophic Statistics is an extension of Interval Statistics, while Plithogenic Statistics is the most general form of statistics (third version)

Florentin Smarandache
University of New Mexico, smarand@unm.edu

Neutrosophic Statistics is an extension of Interval Statistics, while Plithogenic Statistics is the most general form of statistics
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Florentin Smarandache
Mathematics, Physical and Natural Science Division
University of New Mexico, Gallup, NM 87301, USA
Email: smarand@unm.edu

Abstract: In this paper we prove that Neutrosophic Statistics is an extension of the Interval Statistics, since it may deal with all types of indeterminacies (with respect to the data, inferential procedures, probability distributions, graphical representations, etc.), it allows the reduction of indeterminacy, and it uses the neutrosophic probability that is more general than imprecise and classical probabilities, and has more detailed corresponding probability density functions.
While Interval Statistics *only* deals with indeterminacy that can be represented by intervals. And we respond to the arguments by Woodall et al. [1].
We show that not all indeterminacies (uncertainties) may be represented by intervals.
Also, in some applications, we should better use hesitant sets (that have less indeterminacy) instead of intervals.
We redirect the authors to the Plithogenic Probability and Plithogenic Statistics that are the most general forms of MultiVariate Probability and MultiVariate Statistics respectively (including, of course, the Imprecise Probability and Interval Statistics as subclasses).

1. Introduction

First, we present the distinctions between Neutrosophic Statistics and Interval Statistics and give conclusive examples of neutrosophic algebra that provide more accuracy than the interval algebra.
Afterwards we respond to the critics presented by Woodall et al.

Neutrosophic Statistics was first defined (book [1]) in 1998, developed (Book [3]) in 2014, related with Neutrosophic Probability (Book [9]), connected and extended to other fields (Books [2, 4-8]), a PhD Thesis on Neutrosophic Statistics in 2019 (PhD Thesis [1]), and several international seminars [S1-S5], that resulted in an explosion of articles about its applications (Articles [1 – 122]) to many fields such as: medicine, biology, economics, administration, computer science, engineering etc., regarding the decision making, rock joint roughness coefficient, repetitive sampling, indeterminate similarity coefficient, indeterminate sample/population size, individuals that only partially belong to a sample/population, indeterminate mean/variance/standard deviation, control charts, probability distributions of indeterminate or thick functions, measurement errors, tests or hypotheses under uncertainty/indeterminacy etc.

2. Neutrosophic Statistics vs. Interval Statistics

In this paper we make a comparison between Neutrosophic Statistics (NS) and Interval Statistics (IS). We show that they are different and in many cases the NS is more general than IS.

NS is not reduced to only using neutrosophic numbers in statistical applications, as Woodall et al. assert, but it is much broader. NS deals with all types of indeterminacy, while IS deals only with indeterminacy that may be represented by intervals.

Below we present several advantages of applying NS over IS:

- Neutrosophic Statistics is based on Set Analysis, while Interval Statistics on Interval Analysis, therefore the Interval Statistics is a particular case of the Neutrosophic Statistics (that uses all types of sets, not only intervals).
- The numerical neutrosophic numbers permit the reduction of indeterminacy through operations, while the intervals increase the indeterminacy (see examples below).
- Not all uncertain (indeterminate) data can be represented by intervals as in IS, while NS deals with all types of indeterminacy.
- NS deals with sample or population whose size is not well-known.
- NS deals with sample or population which contain individuals that only partially belong to the sample/population and others whose appurtenance is unknown.
- NS deals with sample or population individuals whose degree of appurtenance to the sample or population may be outside of the interval [0, 1], as in neutrosophic overset (degree > 1), underset (degree < 0), and in general neutrosophic offset (both appurtenance degrees, > 1 and < 0, for various individuals)
- Neutrosophic (or Indeterminate) Data is a vague, unclear, incomplete, partially unknown, conflicting indeterminate data.
- NS also deals with refined neutrosophic data used in the Big Data.
- Partially indeterminate curves.
- Neutrosophic Random Variable, which may not be represented as an interval sequence.
- NS also uses Thick Functions (as intersections of curves, that may not be represented by intervals) as probability distributions.
- Neutrosophic Probability Distribution (*NPD*) of an event (x) to occur is represented by three curves: $NPD(x) = (T(x), I(x), F(x))$, where $T(x)$ represent the chance that the event x occurs, $I(x)$ the indeterminate-chance that the event x occurs or not, and $F(x)$ the chance that the event x does not occur. With $T(x), I(x), F(x)$ being classical or neutrosophic (unclear, approximate, thick) functions – depending on each application, and $0 \le T(x) + I(x) + F(x) \le 3$ for all x in the given neutrosophic probability space.

  NPD is better than the classical or imprecise probability distributions, since it is a MultiVariate Probability Distribution that and presents more details about the event.
- Diagrams, histograms, pictographs, line/bar/cylinder graphs, plots with neutrosophic data (not represented by intervals).
- Not well-known (or completely unknown): the mean, variance, standard deviation, probability distribution function, and other statistic
- For example, it is no need to increase the uncertainty by extending the set of possible values, for example, {0.2, 3.7, 45.9} to the interval [0.2, 45.9] in order to be able to use the interval statistics. NS simply employs the hesitant discrete finite set {0.2, 3.7, 45.9}.

- The Qualitative Data is represented by a finite discrete neutrosophic label set, instead of a label interval.

- You cannot use Interval Statistics or Interval (Imprecise) Probability to compute the probability of a die on a cracked surface, or coin on a crack surface, or s defect die or coin.
  We deal with indeterminacy with respect to the probability or statistics space (either the surface, or the die, or the coin), indeterminacy with respect to the observer that evaluates the event, indeterminacy with respect to the event [4].
  You cannot approximate the indeterminacy from these examples by using some interval, so you need neutrosophic probability and statistics that deal with all types of indeterminacies.

- In conclusion: we cannot represent all types of indeterminacies by intervals.

For the sake of the truth, we'll respond below to the critics [1].

2.1. **Woodall at al. [1]** on their section of Neutrosophic Mathematics:

- The basic rules for arithmetic given by Smarandache [42, pp. 31-33] do not match the rules given by Zhang et al. [37].
  Smarandache [42] expressed neutrosophic numbers in the form a + bI, where a and b are real numbers, and I represents the indeterminacy interval such that I^2 = I and 0 · I = 0.

Response:

This is false, since although the book [reference 2 in this paper] contains the literal neutrosophic numbers, they were never used in the applications of neutrosophic statistics. Instead, all the times there were used the numerical neutrosophic numbers.

The authors should learn that there are two types of neutrosophic numbers of the form a + bI, where a, b are real (or complex) numbers, while "I" = indeterminacy.

(i) Literal Neutrosophic Numbers, when "I" is just a letter, where $I^2 = I$ (because: *indeterminacy* × *indeterminacy* = *indeterminacy*) and $0 \cdot I = 0$, that are used in the neutrosophic algebraic structures, but not in no paper on applications of the neutrosophic statistics - upon the best of my knowledge
The literal neutrosophic numbers were introduced and developed by Kandasamy and Smarandache starting from 2003; see several books using literal neutrosophic numbers in neutrosophic algebraic structures:
W.B.V. Kandasamy, F.Smarandache, Fuzzy Cognitive Maps and Neutrosophic Cognitive Maps, Xiquan, Phoenix, 2003, http://fs.unm.edu/NCMs.pdf
W.B.V. Kandasamy, F.Smarandache, Neutrosophic Rings, ProQuest Information & Learning, Ann Arbor, MI, USA, 2006, http://fs.unm.edu/NeutrosophicRings.pdf

Etc.

(ii) Numerical Neutrosophic Numbers, where the indeterminacy "I" is a real subset, in order to approximate the imprecise data. This is more general than the interval, since "I" may be any subset.

For example, N = 3 + 2I, where "I" is in the discrete hesitant subset {0.3, 0.9, 6.4, 45.6} of only four elements, which is not part of interval analysis (statistics). On the interval statistics, you take the interval [0.3, 45.6] in order to include the above numbers, but this this increases very much the uncertainty. Of course, there are particular cases when the "I" is an interval I = $[I_1, I_2]$,

with $I_1 \le I_2$, then N = a + bI coincides with the interval N = $[a + b\cdot I_1, a + b\cdot I_2]$.

1.2. **Woodall et al. [1]:**

- Using the approach of Zhang et al. [37] and interval arithmetic, however, the interval for the average would be [3, 5]. We consider the interval arithmetic approach to lead to the much more useful and realistic results.

Response:

Woodall et al. made a confusion, since Zhang et al. [reference 3, in this paper] paper deals with the Interval Neutrosophic Set (not within the frame of Neutrosophic Statistics), where an element

x(T, I, F), from a given neutrosophic set *A*, has degrees of membership / indeterminacy / nonmembership (T, I, F) respectively expressed under the form of an interval each of them; for example

x([0.7, 0.8], [0.2, 0.3], [0.5, 0.6]).

Therefore, the comparison with Zhang et al. interval arithmetic is irrelevant with respect to the neutrosophic statistics, since Zhang et al. only used their arithmetic on the neutrosophic sets.

Zhang et al. presented the classical operations with intervals because they need them when dealing with operations of neutrosophic triplets. For example, the addition of neutrosophic triplets

$(T_1, I_1, F_1) + (T_2, I_2, F_2) = (T_1 + T_2 - T_1T_2, I_1I_2, F_1F_2)$, where all neutrosophic components are intervals, so additions, subtractions and multiplications of intervals were needed.

See the Neutrosophic Set operations herein [127].

1.3. Wood et al. [1]

- Thus the interval neutrosophic number [4, 6] could be represented as 4 + 2I. Smarandache [42] calculated the average of two neutrosophic numbers, say a + bI and c + dI, as (a + c)/2 + [(b + d)/2]I.

As an example, consider the two neutrosophic numbers [4, 6] and [2, 4] represented as 4+2I and 4−2I, respectively.

Then using the approach of Smarandache [42], the average of these two neutrosophic numbers would be 4 + 0I, or simply the precise value 4. This result does not seem reasonable.

Response:

This just shows the advantage of the numerical neutrosophic numbers over the intervals, since they allow for the reduction of indeterminacy, while using intervals the indeterminacy increases.

For example:

$N_1 = 4 + 2I$, where $I \in [0, 1]$, shows that 2I is the indeterminate part of the number N_1, similarly for $N_2 = 4 - 2I$. If we add them, the indeterminacies of N_1 and N_2 cancel out, and for the average is:

$\frac{1}{2}(N_1 + N_2) = \frac{1}{2}(4 + 2I + 4 - 2I) = \frac{1}{2}(8) = 4$, with no indeterminacy;

while, using intervals, $N_1 = [4, 6], N_2 = [2, 4]$,

$\frac{1}{2}(N_1 + N_2) = \frac{1}{2}[6, 10] = [3, 5]$, therefore the indeterminacy is between [0, 2].

1.4. Woodall et al.

- We note that Smarandache [42] and others do not refer to interval statistical methods despite their very strong similarities with neutrosophic statistical methods.

Response

At the beginning, in the book [2], page 5, there is no reference to the interval analysis/statistics, but it is to the **set analysis/statistics** that is more general than the interval analysis/statistics:

"In most of the classical statistics equations and formulas, one simply replaces several numbers by sets. And consequently, instead of operations with numbers, one uses operations with sets. One normally replaces the parameters that are indeterminate (imprecise, unsure, and even completely unknown). "

Later on, more citations and comparisons have been presented between neutrosophic statistics vs. classical and interval statistics, watch this: http://fs.unm.edu/NS/NeutrosophicStatistics.htm

"The Neutrosophic Statistics is also a generalization of Interval Statistics, because of, among others, while Interval Statistics is based on Interval Analysis, Neutrosophic Statistics is based on Set Analysis (meaning all kinds of sets, not only intervals, for example finite discrete sets).

Also, when computing the mean, variance, standard deviation, probability distributions, and other statistics concepts in classical and interval statistics it is automatically assumed that all individuals belong 100% to the respective sample or population, but in our world, one often meet individuals that only partially belong, partially do not belong, and partially their belongness is indeterminate. The neutrosophic statistics results are more accurate/real than the classical and interval statistics, since the individuals who only partially belong do not have to be considered at the same level as those that fully belong.

The Neutrosophic Probability Distributions may be represented by three curves: one

representing the chance of the event to occur, other the chance of the event not to occur, and a third one the indeterminate chance of the event to occur or not." They provide more details than classical and interval statistics.

"Neutrosophic Statistics is the analysis of events described by the Neutrosophic Probability.

Neutrosophic Probability is a generalization of the classical probability and imprecise probability in which the chance that an event A occurs is t% true - where t varies in the subset T, i% indeterminate - where i varies in the subset I, and f% false - where f varies in the subset F. In classical probability the sum of all space probabilities is equal to 1, while in Neutrosophic Probability it is equal to 3.

In Imprecise Probability: the probability of an event is a subset T in [0, 1], not a number p in [0, 1], what's left is supposed to be the opposite, subset F (also from the unit interval [0, 1]); there is no indeterminate subset I in imprecise probability [see B9].

The function that models the Neutrosophic Probability of a random variable x is called *Neutrosophic distribution*: $NP(x) = (T(x), I(x), F(x))$, where T(x) represents the probability that value x occurs, F(x) represents the probability that value x does not occur, and I(x) represents the indeterminate / unknown probability of value x [see B3]."

Therefore, a more detailed characterization of a neutrosophic random variable, not done in classical and interval statistics.

See this book:
F. Smarandache, Introduction to Neutrosophic Measure, Neutrosophic Integral, and Neutrosophic Probability, Sitech Publishing House, Craiova, 2013,
http://fs.unm.edu/NeutrosophicMeasureIntegralProbability.pdf

1.5. Woodall et al.

- The examples involving imprecise sample sizes given in Smarandache [42] all involve attribute data without carefully expressed operational definitions. It seems impossible to have a sample of variables data without knowing the sample size. (p. 4)

Response:

We disagree. There are many frequent examples of populations and samples from our everyday life: such as school of fish in a river, flock of migratory birds, trees in a forest, plants on a given field, herd of cattle, etc. More examples below:

Indeterminate Sample Size

> "A statistician wants to analyze the reaction of the spectators to a handball match, where team A plays against team B. Suppose that about 4,000 tickets have been sold. Spectators who attend the match form a sample, whose size cannot be exactly determined, because there are also spectators who entered without tickets (as guests, or illegally), while others who had bought tickets could not come for various reasons.

Therefore, the sample size could be estimated, for example, between for example between 3,900 and 4,200."

"To estimate how many people watched the game on TV is even more vague. Electronically one finds out that about 3 million people have watched it. But this is ambiguous as well, since many people could have been watched on the same TV set, while some TVs would have been left on without anyone watching because the owners would have been busy with other things. Sample size was estimated, for example, between 2.9 – 3.2 million." [F.Smarandache, Nidus idearum. Scilogs, II: de rerum consectatione (second edition) Brussels, pages 108-109, 2016, http://fs.unm.edu/NidusIdearum2-ed2.pdf]

Comment by Woodall et al. [127]:

There is no reason to treat the sample sizes as indeterminate.

Answer:

A set of individuals may be considered a population with respect to a reference, but a sample with respect to larger reference.

A simple example when a population's size is indeterminate, but that population becomes a sample with respect to a super-population.

So, there are many cases when the sample size may not be well known.

Let's consider the population P of trees, whose size is indeterminate (between 100-120 trees), in a given park of a city. But, with respect to the trees in all 10 parks of the city, the population P is a sample (of indeterminate size: {100, 101, ..., 120}).

Notice that the sample's size is not an interval, but a discrete finite set.

Therefore, most times in the real world it is not possible to exactly estimate a sample or population size.

Woodall at al.

By the way, we spent several years studying fuzzy logic methods, finding no advantages over the use of probability and statistics.

Answer:

You have used or tried to use the fuzzy logic in statistics, I understand.

But the main distinction between fuzzy and neutrosophic logics is that

in neutrosophic logic has been introduced the indeterminacy as independent component.

Woodall et al.:

The repetitive sampling approach provides for the possibility of more than n observations to be collected at any sampling time.

Answer:

This one better falls under the Plithogenic Probability and Statistics that consider

MultiVariate Analysis of events and their statistics.

If you are interested, just see:

http://fs.unm.edu/NSS/PlithogenicProbabilityStatistics20.pdf

1.6. Mean of a Sample with partially belonging individuals

Let S = {a, b, c, d} be a sample set of four elements, such that
a = 2, b = 8, c = 5, and d = 11.
In the classical statistics it is assumed that all elements belong 100% to the sample, therefore
S = {a(1), b(1), c(1), d(1)}.

Whence the classical mean:

$$CA = \frac{2\cdot 1 \ + \ 8\cdot 1 \ + \ 5\cdot 1 \ + \ 11\cdot 1}{1+1+1+1} = \frac{26}{4} = 6.5 .$$

But, in the real world, not all elements may totally (100%) belong to the sample, for example, let's assume the neutrosophic sample be:
NS = {a(1.1), b(0.4), c(0.6), d(0.3)}, which means that:
the element *a* belongs *110%* (someone who works overtime, for example, as in the neutrosophic overset (see [B4]), *b* belongs only *40%* to the sample, *c* belongs *60%*, and *d* belongs *30%*.
Whence the neutrosophic mean (NM) is:

$$NM = \frac{2\cdot(1.1)+8\cdot(0.4)+5\cdot(0.6)+11(0.3)}{1.1+0.4+0.6+0.3} = \frac{11.7}{2.4} = 4.875$$

Clearly, the classical mean and the neutrosophic mean are different,
CM = 6.5 $\neq$ 4.875 = NM.
And consequently: the variance, standard deviation, probability distribution function and other statistics depending on them will be different as well.
But, the *neutrosophic mean* is more accurate since it reflects the real (not idealistic) mean, because it takes into account the degree of membership of each element with respect to the set." [5] And consequently the other statistics depending on them are more accurate.

1.7. The **Thick Function (Distribution)**, from the neutrosophic statistics, is defined as:
$f: R \to P(R),\ f(x) = [f_1(x), f_2(x)]$

The thick curve as the graph of a thick function [2] was introduced in 2014, and it is different from the interval functions,
because we may have a probability distribution in between two curves,
of the form f(x) = [f_1(x), f2(x)].

For example, let $f_1(x) = (x-1)^3 + 2,\ f_2(x) = 1.5x^3$,
then $f(x) = [(x-1)^3 + 2, 1.5x^3]$
which is a thick function, i.e. the zone between two below curves.

So, it is different from Interval Statistics.

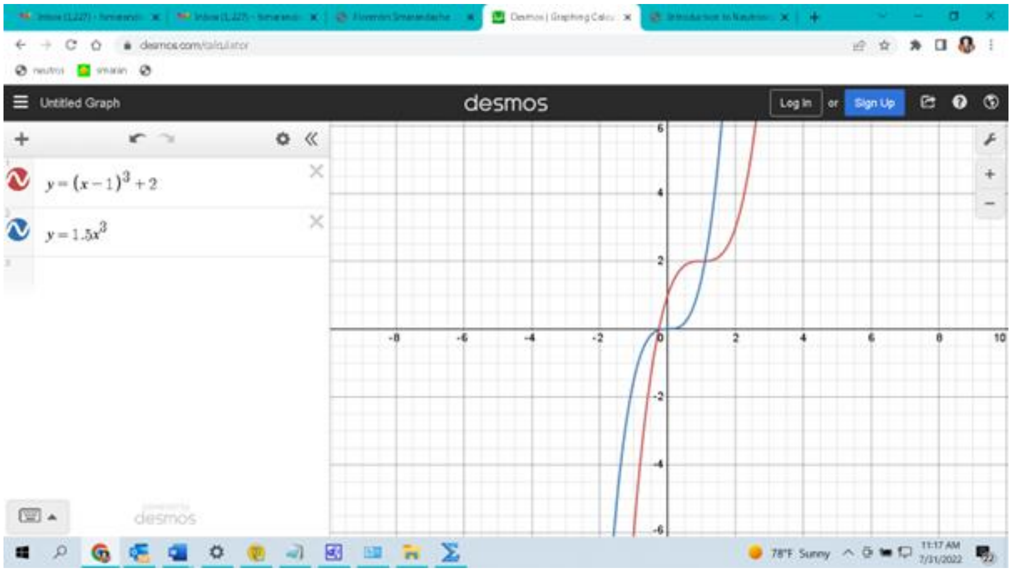

Table 1. A Thick Function used in Neutrosophic Statistics

1.7.Interval-Valued Variable vs. Neutrosophic-Number Variable

The Interval Statistics uses variables [7] of the form:
$aX + b$, where a and b are constants, and X is a set of varying intervals.
For example, $a = 2, b = 3$, and $X = [0.1, 0.3], [4, 5], [7. 9], ...$
give
$aX + b = 2[0.1, 0.3] + 3, 2[4, 5] + 3, 2[7, 9] + 3, ...$
$= [3.2, 3.6], [11, 13], [17, 21], ...$.

While the Neutrosophic Numbers have the form:
$N = a + bI$,
where "a" is the determinant (known) part of N, and "bI" is the indeterminate (unclear) part of N; I is a fixed real subset, while a and b are varying real numbers.
Example:
Let $I = [0.1, 0.2]$ be a fixed subset (we take it as an interval, although it can be any type of subset), and the initial $a = 2$ and $b = 3$, then $a = 4$ and $b = 6$, $a = 5.5$ and $b = 6.2$ etc.
The $a + bI = 2 + 3[0.1, 0.2], 4 + 6[0.1, 0.2], 7 + 3[0.1, 0.2], ...$
$= [2.3, 2.6], [4.6, 5.2], [7.3, 7.6], ...$.

So, clearly the two approaching are different, i.e. the interval-valued variable
from interval statistics is different from the neutrosophic number variable from neutrosophic statistics.

1.8. Hesitant Set vs. Interval

In neutrosophic statistics we may use all types of set, for instance the hesitant sets, that have a finite discrete number of elements. In various examples it would be advantageous to use a hesitant set instead of an interval.

Suppose the temperature, in Celsius degrees, is above *10°C*, on extreme low/high fluctuation, *18°C* low and *40°C* or *45°C* high.

In neutrosophic statistics the random variable *t* is modelled as $t = 10+I$, where $I \in \{5, 30, 35\}$, where the indeterminate part of *t* is a hesitant discrete finite set of only three elements: *5, 30,* and *35.*

In interval statistics, the random variable *t* is modelled as an interval $t = [15, 45]$, whose uncertainty is much higher than that in neutrosophic statistics, and it propagates with each new calculation.

This is another example showing the preference of using neutrosophic statistics over interval statistics.

1.9. Comparisons between interval algebra and neutrosophic algebra

a) Addition

Interval Statistics (IS)

We take the example presented by Woodall et al.

For $I \in [0, 1]$,

$N_1 = 4 + 2I = [4, 6]$

$N_2 = 4 - 2I = [2, 4]$

$N_1 + N_2 = [4 + 2, 6 + 4] = [6, 10]$

By adding $N_1 + N_2$ in interval statistics, the indeterminate part [0, 2] of each number add up, increasing the addition's uncertainty to [0, 2] + [0, 2] = [0, 4].

Neutrosophic Statistics (NS)

$N_1 = 4 + 2I$

$N_2 = 4 - 2I$

$N_1 + N_2 = 4 + 2I + 4 - 2I = 8$

The indeterminate part of N_1 is $2I = [0, 2]$ and the indeterminate part of N_2 is $-2I = [0, -2]$.

By adding $N_1 + N_2$ in the neutrosophic statistics, the indeterminate parts cancel out, $2I - 2I = 0$, and we get a determinate answer: 8.

The result [6, 10] got in interval statistics is much vaguer than 8 obtained in neutrosophic statistics. It shows the advantage of NS over IS.

While in same cases the results of the operations with neutrosophic numbers coincide with those obtained by operations with intervals, in many other cases the results are different.

Clearly, the interval statistics is different from the neutrosophic statistics.

Unlike the interval statistics that accumulates the uncertainty from an operation to another, the neutrosophic statistics diminishes or even cancels the uncertainty.

Question by Woodall et al. [127]:

I have one question about neutrosophic arithmetic. Suppose one has, as in your example, 4+2I and 4-2I. You give the sum as the constant 8. Suppose one writes the numbers equivalently as 4+2I and 2+2I, then the sum is 6+4I. why should the sum depend on how the numbers are expressed?

Answer:

The numerical neutrosophic numbers (N) are chosen by the researcher upon the parts which are considered determinate (a) and indeterminate (bI), so N = a+bI.

Therefore, they depend on what is the indeterminate/vague part of the number.

N1 = 2+2I means that the determinate part of N1 is 2, the other is indeterminate,

while N2 = 4-2I has its determinate part be 4.

N1 is different from N2 in neutrosophic statistics, but they mean the same thing

in interval statistics: [0, 4] when I = [0, 1].

This is another point to show that the Neutrosophic Statistics and Interval Statistics are different from each other.

b) Multiplication

$\mathrm{N}_1 = 4 + 2I, I \in [0, 1]$

$\mathrm{N}_2 = 4 - 2I$

<u>IS</u>

$\mathrm{N}_1 \cdot \mathrm{N}_2$ = (4 + 2I) · (4 − 2I) = (4 + 2·[0, 1]) · (4 −2·[0, 1])

= (4 + [2 · 0, 2 · 1] · (4 –[2 ·0, 2 · 1]

= (4 + [0, 2] · (4 –[·0, 2]

= [4, 6] · [2, 4] = [4 · 2, 6 · 4] = [8, 24],

the length of uncertainty is 24 – 8 = 16.

<u>NS</u>

$\mathrm{N}_1 \cdot \mathrm{N}_2$ = (4 + 2I) · (4 − 2I) = $4^2 - (2I)^2$

= 16 - $4I^2$ = 16 – 4 · $[0^2, 1^2]$

= 16 – 4 · [0, 1]

= 16 – 4 · [0, 4]

= [16 – 4, 16 – 0]

$= [12, 16]$,

length of uncertainty is $16 – 12 = 4 < 16$, therefore more accurate result from NS.

c) Subtraction

IS

$N_1 - N_2 = (4 + 2I) - (4 - 2I)$

$= [4, 6] – [2, 4]$

$= [4 – 4, 6 – 2] = [0, 4]$

NS

$N_1 - N_2 = (4 + 2I) - (4 - 2I)$

$= 4 + 2I – 4 + 2I$

$= 4I = 4 \cdot [0, 1] = [0, 4]$, the same.

But let's take other neutrosophic numbers:

$M_1 = 5 + 4I$, $I \in [2, 3]$

$M_2 = 6 + 3I$

IS

$M_1 = 5 + 4I$, $I \in [2, 3]$, hence $M_1 = 5 + [4 \cdot 2, 4 \cdot 3] = [13, 17]$;

$M_2 = 6 + 3I = 6 + 3 \cdot [2, 3] = [12, 15]$.

$M_1 - M_2 = [13, 17] – [12, 15]$

$= [13 – 15, 17 – 12] = [-2, 5]$, interval of uncertainty length $5 – (-2) = 7$.

NS

$M_1 - M_2 = (5 + 4I) - (6 + 3I) = 5 + 4I – 6 – 3I$

$= -1 + I = -1 + [2, 3] = [-1 + 2, -1 + 3]$

$= [1, 2]$, interval of uncertainty length $2 – 1 = 1$.

But $1 < 7$, so better accuracy by using the NS.

d) Division

IS

$$\frac{4+2I}{2+I}=\frac{4+2[0,1]}{2+[0,1]}=\frac{4+[0,2]}{[2+0,2+1]}=\frac{[4+0,4+2]}{[2,3]}$$

$=\frac{[4,6]}{[2,3]}=\left[\frac{4}{3},\frac{6}{2}\right]=[1.3,3]$, interval of uncertainty 3 – 1.3 = 1.7.

<u>*NS*</u>

$\frac{4+2I}{2+I}=\frac{2(2+I)}{2+I}=2$, uncertainty = 0,

which is a more accurate result, since the neutrosophic statistics permitted the simplification of the uncertainty *I*.

1.10. Refined Neutrosophic Statistics used in the Big Data

In this Big Data world, we are facing this kind of situation with more uncertainties resulted from multiple variables, leading to Refined Neutrosophy.

Thus, we may use the Refined Neutrosophic Statistics, i.e. when the indeterminacy "*I*" is split into many types of uncertainties $I_1, I_2, ..., I_s$, where $s \geq 2$, as many as needed into the application.

Refined Neutrosophic Statistics followed in the steps of the Refined Neutrosophic Set (Smarandache, 2013). Therefore, an element from a Big Data that belongs to a refined neutrosophic set, $x \in M$, may have refined neutrosophic coordinates, for example $x(T, I_1, I_2, I_3, F)$ if there are only *3* types of uncertainties. We may have as many types of uncertainties as needed into the problem.

Check first the Refined Neutrosophic Set,
where *T, I, F* can be refined/split respectively as:

$T_1, T_2, ..., T_p$,

$I_1, I_2, ..., I_r$,

and $F_1, F_2, ..., F_s$,

where *p, r, s* are integers ≥ 0, and at least one of *p, r, s* is ≥ 2 (to ensure the existence of refinement of at least one of the three neutrosophic components *T, I,* and *F*).
When *p, r*, or *s* is equal to *0*, that component is discarded. For example, T_0 means that *T* is discarded, and similarly for I_0 and F_0.

This leave room for defining the <u>Refined Fuzzy Set</u>, under the form $T_1, T_2, ..., T_p$, where *p* is an integer ≥ 2, and I_0 and F_0 are discarded.

And for <u>Refined Intuitionistic Fuzzy Set</u>, under the form $T_1, T_2, ..., T_p$, and $F_1, F_2, ..., F_s$, for integers $p, s \geq 1$, and at least one of *p* or *s* is ≥ 2 (in order to assure the refinement of a least one component *T* or *F*).

Similarly for other <u>Refined Fuzzy-Extension Sets / Logics / Probabilities / Statistics</u>.

1.10. Plithogenic Probability & Plithogenic Statistics that are generalizations of MultiVariate Probability & Statistics

The **Plithogenic Variate Analysis (PVA)** is an extension of of the classical MultiVariate Analysis, where indeterminate data or procedures, that are called neutrosophic data and respectively neutrosophic procedures, are allowed. Therefore PVA deals with neutrosophic/

indeterminate variables, neutrosophic/indeterminate subsystems, and neutrosophic/indeterminate system-of-systems as a whole.
Therefore the Plithogenic Variate Analysis studies a neutrosophic/indeterminate system as a whole, characterized by many neutrosophic/indeterminate variables (i.e. neutrosophic/indeterminate sub-systems), and many neutrosophic/indeterminate relationships. Hence many neutrosophic measurements and observations are needed.

The **Plithogenic Probability** of an event to occur is composed from the chances that the event occurs with respect to all random variables (parameters) that determine it. The Plithogenic Probability, based on Plithogenic Variate Analysis, is a multi-dimensional probability ("plitho" means "many", synonym with "multi"). We may say that it is a probability of sub-probabilities, where each sub-probability describes the behavior of one variable. We assume that the event we study is produced by one or more variables. Each variable is represented by a Probability Distribution (Density) Function (PDF).

Plithogenic Statistics (PS) encompasses the analysis and observations of the events studied by the Plithogenic Probability. Plithogenic Statistics is a generalization of classical MultiVariate Statistics, and it is a simultaneous analysis of many outcome neutrosophic/indeterminate variables, and it as well is a multi-indeterminate statistics.

Subclasses of Plithogenic Statistics are:
- Interval Statistics
- Neutrosophic Statistics
- MultiVariate Statistics
- Plithogenic Neutrosophic Statistics
- Plithogenic Indeterminate Statistics
- Plithogenic Intuitionistic Fuzzy Statistics
- Plithogenic Picture Fuzzy Statistics - Plithogenic Spherical Fuzzy Statistics
- and in general: Plithogenic (fuzzy-extension) Statistics - and Plithogenic Hybrid Statistics.

Plithogenic Refined Statistics are, similarly, the most general form of statistics that studies the analysis and observations of the events described by the Plithogenic Refined Probability.

See more development, extension of Interval Statistics and Neutrosophic Statistics to Plithogenic Probability & Plithogenic Statistics that are generalizations of MultiVariate Probability & Statistics: [6].

Conclusion

In this paper we made a comparison between Neutrosophic Statistics (NS) and Interval Statistics (IS). We showed that they are different and in many cases the NS is more general than IS.

NS is not reduced to only using neutrosophic numbers in statistical applications, as Woodall et al. assert, but it is much broader. NS deals with all types of indeterminacy, while IS deals only with indeterminacy that may be represented by intervals.

And we responded to the arguments by Woodall et al. [1].

We redirected the authors to the Plithogenic Probability and Plithogenic Statistics that are the most general forms of MultiVariate Probability and MultiVariate Statistics respectively (including, of course, the Interval Statistics as a subclass).

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