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ON CRITTENDEN AND VANDEN EYNDEN'S CONJECTURE

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It is possible to cover all (positive) integers with n geometrical progressions of integers?

Find a necessary and sufficient condition for a general class of positive integer sequences such that, for a fixed n, there are n (distinct) sequences of this class which cover all integers.

Comments:

a) No. Let $a_1, \ldots, a_n$ be respectively the first terms of each geometrical progression, and $q_1, \ldots, q_n$ respectively their ratios. Let p be a prime number different from $a_1, \ldots, a_n, q_1, \ldots, q_n$. Then p does not belong to the union of these n geometrical progressions.

b) For example, the class of progressions $A_f = \left\{ \{a_n\}_{n \geq 1} : a_n = f\left(a_{n-1}, \ldots, a_{n-i}\right) \text{ for } n \geq i+1, \text{ and } i, a_1, a_2, .. \in N^* \right\}$ with the property $\exists y \in N^*,\ \forall \left(x_1, \ldots, x_i\right) \in N^{*i} : f\left(x_1, \ldots, x_i\right) \neq y$. Does it cover all integers?

But, if $\forall y \in N^*,\ \exists \left(x_1, \ldots, x_i\right) \in N^{*i} : f\left(x_1, \ldots, x_i\right) = y$?

(Generally no.)

This (solved and unsolved) problem remembers Crittenden and Vanden Eynden's conjecture.

References:

[1] R.B. Crittenden and C. L. Vanden Eynden, *Any n arithmetic progressions covering the first 2^n integers covers all integers*, Proc. Amer. Math. Soc. 24 (1970) 475-481.

[2] R.B. Crittenden and C. L. Vanden Eynden, *The union of arithmetic progression with differences not less than k*, Amer. Math. Monthly 79 (1972) 630.

[3] R. K. Guy, *Unsolved Problem in Number Theory*, Springer-Verlag, NewYork, Heidelberg, Berlin, 1981, Problem E23, p.136.