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# Prospect-Theoretic Demand Response Management in Smart Grid Systems

Sean A. Pluemer

Sean Pluemer

*Candidate*

Electrical and Computer Engineering

*Department*

This thesis is approved, and it is acceptable in quality and form for publication:

*Approved by the Thesis Committee:*

Dr. Eirini Eleni Tsiropoulou, Chair

Dr. Lei Yang, Member

Dr. Jim Plusquellic, Member

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by

**Sean Pluemer**

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THESIS

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# Dedication

*To my family and friends, without their daily love and support I would not have been able to do this.*

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## Abstract

In smart grid systems, demand response management (DRM) permits the efficient utilization and control of electrical loads. This control is accomplished by modifying the price of electricity or by shifting peak electricity consumption to off-peak periods. This thesis proposes a DRM model composed of multiple utility companies and multiple consumers. Due to the risk caused by the users' excessive power demands, each utility company has the potential for failure. Prospect Theory principles capture the individual consumer's risk-aware behavior. As a result, a multi-leader multi-follower Stackelberg game is introduced, involving the utility companies (leaders) and consumers (followers). The Stackelberg game determines the optimal electricity rates for the utility companies and the consumers' optimal amount of energy purchases from each company. The consumers' optimal amount of energy is calculated as a Nash Equilibrium point of an n-person concave game among consumers attempting to maximize their satisfaction. The numerical results presented in this thesis demonstrate the proposed DRM framework's performance and efficiency while emphasizing the critical advantages and disadvantages of the proposed model compared to other alternative DRM approaches.

# Contents

# List of Figures

# Glossary

| | |
|---|---|
| $B$ | Set of Utility Companies where B = $\{1, ..., b, ..., \lvert B\rvert\}$ |
| $C$ | Set of Consumers where C = $\{1, ..., c, ..., \lvert C\rvert\}$ |
| $T$ | Time Horizon where T = $\{1, ..., t, ..., \lvert T\rvert\}$ |
| D | Set of consumer devices where $D_c = \{1, ..., d_c, ..., \lvert D_c\rvert\}$ |
| $P_c^{(t)}$ | Overall power demand of consumer ($c$) at time ($t$) where $P_c^{(t)} = \sum_{\forall d_c \in D_c} \delta_{d_c}^{(t)} \cdot P_{d_c}^{(t)}[B]$ |
| $P_{d_c}^{(t)}[B]$ | Power consumption of each device ($d_c$) at time ($t$) |
| $r_b^{(t)}[\$/J]$ | Utility company power price per time slot ($t$) |
| $R^{(t)}$ | Price vector of all utility companies where $R^{(t)} = \{r_1^{(t)}, ..., r_b^{(t)}, ..., r_{\lvert B\rvert}^{(t)}\}$ |
| $p_{c,b}^{(t)}$ | Amount of power ($p$) the consumer ($c$) decides to buy from each utility company ($b$) |
| $P_c^{(t)} - \sum_{\forall b \in B} p_{c,b}^{(t)}$ | Power ($p$) the consumer ($c$) decides delay |
| $C_b$ | Set of consumers ($C$) purchasing power ($p$) from utility company ($b$) |
| $\bar{p}_b^{(t)}$ | Total amount of power purchased from utility company ($b$) where $\bar{p_b^{(t)}} = \sum_{\forall c \in C_b} p_{c,b}^{(t)}$ |

*Glossary*

| | |
|---|---|
| $\tilde{p}_b$ | Maximum power ($p$) each utility company ($b$) can generate per time slot ($t$) |
| $f_b^{(t)}(\bar{p}_b^{(t)})$ | Utility companies return function |
| $F_{c,b}^{(t)}(\bar{p}_b^{(t)})$ | Each individual consumer's return function where $F_{c,b}^{(t)}(\bar{p}_b^{(t)}) = \frac{p_{c,b}^{(t)}}{\sum_{\forall c \in C_b} p_{c,b}^{(t)}} f_b^{(t)}(\bar{p}_b^{(t)})$ |
| $m_b[\$/J]$ | Cost to produce the power |
| $r_b^{(t)}$ | Utility companies' optimal energy price |
| $A_b^{(t)}$ | Companies Utility function |
| $r_b^{Max}$ | Maximum allowed power price, as defined by the power market regulations. |
| $\mathbb{J}$ | Non-cooperative game where $\mathbb{J} = [B, \{R_b\}_{\forall b \in B}, \{A_b^{(t)}\}_{\forall b \in B}]$ |
| $\Lambda_{c,b}^{(t)}(p_{c,b}^{(t)})$ | Experience cost for each consumer purching an amount of power, $p_{c,b}^{(t)}$, from each utility company $b$. |
| $\mathbb{P}(\Lambda_{c,b}^{(t)})$ | Expected cost of interacting with a utility company |
| $z_0(p_{c,b}^{(t)})$ | Reference point used to measure the consumers gains and losses |
| $S_c^{(t)}$ | Each Consumer's perceived satisfaction with purchasing electricity through the smart grid system |
| $G_c^{(t)}$ | The strategy set of consumer c in time slot t |
| $Z_x$ | Non-cooperative game among the consumers, used for solving the optimization problem. |

# Chapter 1

# Overview

## 1.1 Introduction

The next-generation electric power grid is transforming into a smart grid ecosystem that can incorporate functional capabilities such as distributed electricity resources and demand response management (DRM) processes [1,2]. Consumers can use DRM techniques to adjust their power demands over time to lower their energy consumption costs while still meeting their needs [3]. The utility companies also benefit from the DRM systems since altering the users' energy needs can reduce the peak-hour demand [4]. This reduction in peak-hour demand minimizes the risk of brownouts or even blackouts in the worst-case situation [5]. In a smart grid system with multiple utility companies and consumers, this thesis proposes a prospect-theoretic DRM framework. Within this framework consumers have the ability to purchase their power from multiple utility companies, and prospect theory principles are used to capture their behavioral traits [6]. However, each company is vulnerable to failure if overused due to high energy demand. Our goal is to reach an efficient and stable operation point by finding the ideal announced price for each company and the optimal amount of energy the consumers purchase from each company.

What sets this thesis apart is that we are considering two main theories, the tragedy of the commons and prospect theory [7]. So, each utility company has a specific electricity generation capacity per time slot. If the users request more power than the utility company can provide, the utility company will experience a blackout. This is the principle for the tragedy of the commons. Prospect theory is being used to discover how risk-averse and gain-seeking the consumers are in terms of consuming electricity versus paying for the price of the electricity.

## 1.2 Review of Related Literature

Demand Response Management (DRM) systems are not a new concept, and in fact, there is a lot of existing literature related to DRM Systems [8, 9]. In [10], a distributed Stackelberg game-theory approach was introduced. In this paper, there was a single multi-power provider with many residential users. The multi-power provider optimized its electricity price to increase the trading probability with the residential users. In response to this price, the residential users optimize their flexible loads to reduce the expected power cost. With this DRM framework, the research showed that the residential users saw a reduction in their power cost, while the multi-power providers also saw a 12.29% increase in benefit.

In [11], the combined problems of DRM and power company selection were studied. The consumers selected which power company they would purchase from based on their electricity needs and price limitations. This selection happens via a reinforcement learning mechanism [12, 13]. A two-stage game theoretic optimization framework is introduced based on this selection to solve the DRM issue. This framework optimizes the user's electricity consumption and the power companies' pricing.

In [14], the authors took a slightly different approach to the DRM problem. Because the electricity market is in studied real-time, this can result in some extreme volatility. Given that the users can shift their power demand to different times in the day, this creates a perfect opportunity to be exploited by the user. This research work used a Markov decision process to solve this problem. This DRM shows that not only can it maximize the consumer's gain, but it also significantly alleviates the supply-demand imbalance within the power grid. On average, this DRM method increased the consumer's profit by 53.8% and saved the bills of other utilities by 80.4%. However, one major issue with this DRM model was its significantly high computational complexity. In [15], the authors decided to extend their research work by decreasing the complexity of the Markov decision process (MDP) problem. With their new Markov decision process approach, they increased the power load's profit by 55.9%, maintained the bills of other utilities at 80.4%, and, most importantly, reduced the computational complexity.

An intelligent residential power management system (IREMS) is proposed in [16]. This power management system aimed to reduce the electricity bills while ensuring the power demand is below the maximum limit. The IREMS can achieve this goal by scheduling flexible loads during low pricing intervals while also monitoring the loads that can not be scheduled compared to renewable power resource availability. This IREMS also manages backup battery power storage in such a way as to reduce the dump load power dissipation whenever there is excess power available from the renewable power resources. Lastly, this IREMS uses a resource sizing algorithm to find the optimal sizes for both the renewable power resource and the battery to effectively utilize of available renewable power.

Most existing game-theoretic DRM systems assume that consumers have a rational behavior [17]. The research work in [18] aims to improve this by introducing a more realistic model. This report uses a prospect-theoretic-based model to incorporate realistic consumer behaviors, including irrationality. Furthermore, this research work also includes the physical grid constraints (in terms of voltage violations). This is in contrast to prior approaches that limit themselves to economic aspects.

## 1.3 Contributions and Outline

As seen in the research section, there has been extensive research into the demand response management problem in smart grid systems. However, there has been minimal research on the DRM problem involving multiple utility companies and multiple consumers, with the consumers having the ability to buy power from each utility company simultaneously. The main vulnerability is overloading, which occurs due to consumers' excessive power demands. Under severe conditions, this overloading can potentially result in a complete blackout.

In real life, consumers will make risk-aware decisions concerning the quantity of power that they will purchase from each utility company. They also ensure to account for operational risks, e.g., failure or uncertainty of a utility company to fulfill their power requirements [19].

This thesis introduces a prospect-theoretic DRM framework to fill in these research gaps. This prospect-theoretic DRM framework consists of multiple utility companies and multiple consumers while also accounting for the risk-aware behavior of the consumers in their purchasing decisions.

The main contributions of this paper that distinguish it from the rest of the literature are outlined below.

1. A smart grid system with multiple utility companies and multiple consumers/consumers is introduced. The key difference from prior literature is

that each consumer can purchase different amounts of power from various utility companies. Existing literature either assumes a single utility company, i.e., a monopoly, or that each consumer will only buy power from a single utility company.

2. Each consumer chooses how much electricity to buy from each provider, taking into account the risk and uncertainty of the utility companies' capacity to supply the excessive power demands. This is in contrast to previous research, which has focused solely on risk-averse clients, with uncertainty resulting from their selfish power needs. Each utility company is referred to as a Common Pool of Resources (CPR) since its generated power is available to all consumers.

3. The consumers' experience in the smart grid system is given significant consideration. Each company's perceived prospect-theoretic utility is calculated considering the consumer's cost of purchasing any amount of electricity and its perceived dissatisfaction from delaying any of its power demands. Because of the possibility of the companies failing due to the consumers' excessive power consumption, the consumer's prospect-theoretic utility is probabilistic. The chance of failure for each company is calculated using the company's power-producing characteristics and the associated power demand required by consumers. The consumer's satisfaction utility function is presented, taking into account the projected prospect-theoretic utility outcomes as well as the consumer's experienced dissatisfaction as a result of deferring some of their power demands.

4. The utility companies' optimal power prices and the consumers' optimal amount of power purchased from each company are determined by creating a multi-leader multi-follower Stackelberg game among the utility companies (leaders) and the consumers (followers). The fundamental goal of utility companies is to maximize their profit and market penetration in the smart grid

market. Thus, a non-cooperative game is devised among the utility companies, and the appropriate Nash Equilibrium (NE) is determined. The existence, uniqueness, and convergence of utility companies' NE strategies have been demonstrated. Then, a Best Response Dynamics algorithm is presented to determine the NE point [20]. As for optimizing the consumers' satisfaction utility functions among the consumers, a $n$-person non-cooperative concave game is designed and solved to identify its unique NE point, i.e., each utility company's optimal amount of power purchased.

5. To evaluate the operation and performance of the proposed framework, a detailed set of numerical results is presented. Then, taking into account the consumers' risk-aware behavioral characteristics, a sensitivity analysis is conducted. This sensitivity study aims to demonstrate the ability of the prospect-theoretic DRM framework to adjust to the realistic conditions of smart grid systems. Following that, a large-scale scalability analysis is performed to confirm the robustness of the proposed DRM system. Finally, a comparative evaluation of the proposed DRM framework against alternative DRM models and tactics is conducted. This assessment highlights the benefits and drawbacks of the proposed prospect-theoretic DRM framework.

# Chapter 2

# System Model

A smart grid system consisting of multiple utility companies and multiple consumers utilizes an energy management system (EMS) to enable the interaction between utility companies and consumers [21].

The set of utility companies is denoted as $B = \{1, ..., b, ..., |B|\}$, while the consumers are denoted as $C = \{1, ..., c, ..., |C|\}$. The time horizon for the smart grid system is denoted as $T = \{1, ..., t, ..., |T|\}$; for example a day could be $|T| = 24h$ [22]. Each consumer owns a set of devices $D_c = \{1, ..., d_c, ..., |D_c|\}$. Each device $d_c$ can be either turned on, $\delta_c^{(t)} = 1$, or turned off, $\delta_c^{(t)} = 0$, at time slot $t$. Therefore, the overall power demand of consumer $c$ at time slot $t$ is $P_c^{(t)} = \sum\limits_{\forall d_c \in D_c} \delta_{d_c}^{(t)} \cdot P_{d_c}^{(t)}[J]$ , where $P_{d_c}^{(t)}[J]$ denotes the electricity consumption of the device $d_c$ operating during time slot $t$ [23].

Each utility company discloses the price of their power $r_b^{(t)}[\$/J]$ every time slot $t$. The corresponding price vector of all the utility companies is denoted as $R^{(t)} = \{r_1^{(t)}, ..., r_b^{(t)}, ..., r_{|B|}^{(t)}\}$ . Given the utility companies announced power prices, each consumer decides to purchase a corresponding amount of power $p_{c,b}^{(t)}[J]$ from each company $b$ at time slot $t$, with $\sum_{\forall b \in B} p_{c,b}^{(t)} \leqslant P_c^{(t)}$.

The rest of the consumers power demand, i.e., $P_c^{(t)} - \sum_{\forall b \in B} p_{c,b}^{(t)}$ is delayed, to be used in a later time slot. There are a few reasons this delay may occur.

1. The price of electricity is too high, and there are no incentives for the consumer to purchase any power at the current time slot $t$.

2. The consumer has covered their essential non-shiftable power needs; these devices include essential equipment such as stoves, refrigerators, air conditioners, etc.

3. The utility companies cannot cover the consumers' excessive power demand.

$C_b$ represents the set of consumers' who power purchased from utility company $b$. Accordingly, the total power purchased from company $b$ is $\bar{p}_b^{(t)} = \sum_{\forall c \in C_b} p_{c,b}^{(t)}$. Bound by their operational characteristics and available distributed power resources, each utility company can generate a maximum amount of electricity $\tilde{p}_b$ per time slot $t$. Each utility companies return function is represented as [24, 25]:

$$f_b^{(t)}(\bar{p}_b^{(t)}) = \begin{cases} 1 - \frac{\bar{p}_b}{\tilde{p}_b}, & \text{if } \bar{p}_b \leqslant \tilde{p}_b \\ 0, & otherwise \end{cases} \tag{2.1}$$

Equation 2.1 shows the consumer's normalized satisfaction by purchasing power from company $b$. This return function, $f_b^{(t)}(\bar{p}_b^{(t)})$, is a strictly decreasing function concerning the total power purchased from company $b$ in time slot $t$. As the total power purchased approaches company $b$'s maximum power generation capacity, $\tilde{p}_b$, the utility company will begin to experience instability in its operation [26, 27]. This instability can lead to brownouts or even complete blackouts. If this happens, the consumers will receive zero satisfaction (i.e., return).

Each individual consumer's return function can be defined as:

$$F_{c,b}^{(t)}(\bar{p}_b^{(t)}) = \frac{p_{c,b}^{(t)}}{\sum\limits_{\forall c \in C_b} p_{c,b}^{(t)}} f_b^{(t)}(\bar{p}_b^{(t)}) \tag{2.2}$$

based on each utility company's return function, as defined in Equation 2.1.

This relative return function, for the consumers, harnesses proportionate fairness principles. This means that when each consumer, $c$, purchases power from company $b$, as long as the company, $b$, can meet these needs, the consumer will receive a proportional amount of satisfaction directly related to the amount of power purchased.

# Chapter 3

# Overview of Prospect-theoretic DRM Framework

This section gives an overview of the proposed prospect-theoretic demand response management framework. A multi-leader multi-follower (MLMF) Stackelberg game is introduced to represent the interactions between utility companies and consumers. Stackelberg games have been used in the literature to deal with several types of multi-layer hierarchical resource management [28–30].

A high-level graphical overview of the proposed prospect-theoretic DRM framework can be found in Fig. 3.1.

1. The utility companies, that function as market leaders, wish to maximize their profit by selling power to consumers and optimizing their market penetration. This optimization problem, framed as a non-cooperative game among the utility companies, is thoroughly examined in Chapter 4.

2. The consumers' goal is to maximize their satisfaction received by purchasing electricity, given the utility companies' announced energy prices. They must

also display risk-aware behavioral characteristics in response to the utility companies' probability of failure (due to the consumers' extreme energy demands). This optimization problem, framed as a $n$-person concave game using Prospect Theory, is thoroughly examined in Chapter 5. Chapter 5 also demonstrates the existence and uniqueness of the Nash Equilibrium as well as the computation of the optimal amount of purchased energy from each company using a distributed decision-making algorithm.

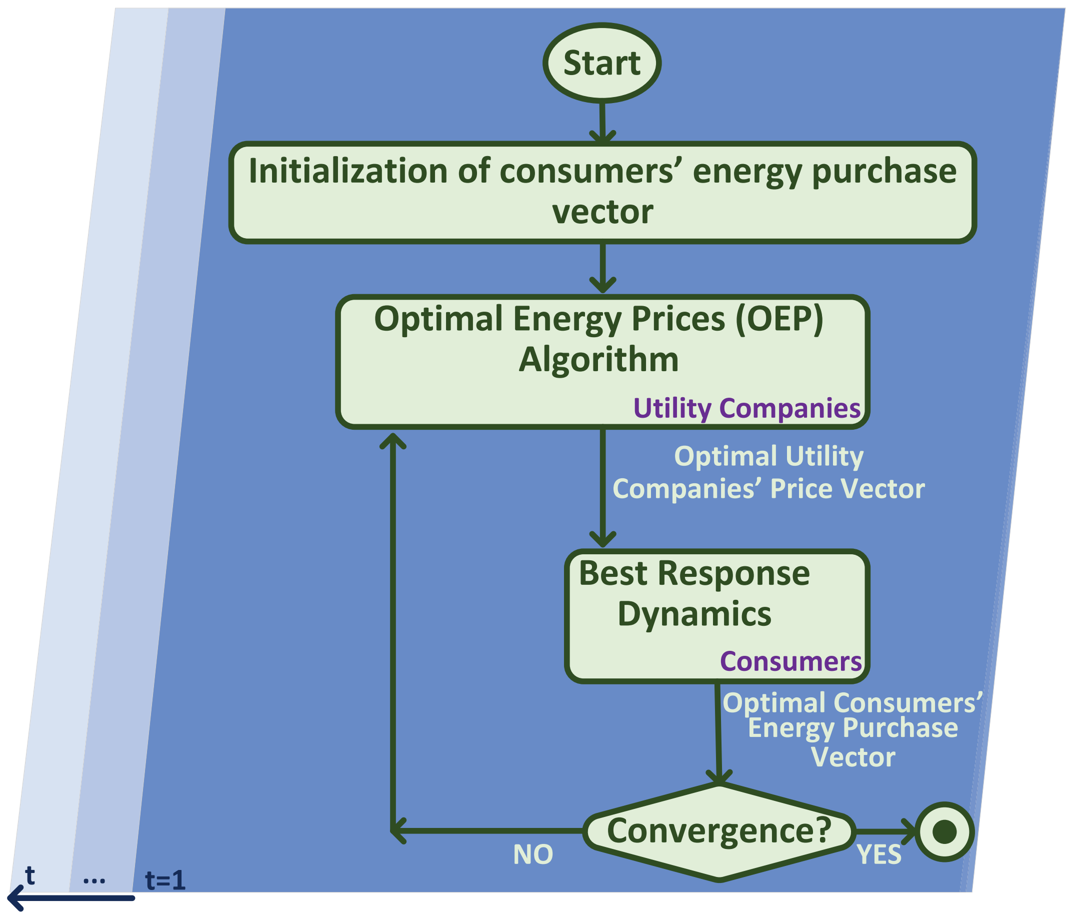


Figure 3.1: Overview of the proposed prospect-theoretic demand response management framework.

# Chapter 4

# Utility Companies Profit Optimization

This section concentrates on the utility companies' operations and perspectives. Each utility company, $b$, has a unique energy generation cost, $m_b[\$/J]$, determined by its operational characteristics and energy distribution resources (i.e., its infrastructure). In addition, each utility company can determine their optimal energy price, $r_b^{(t)}$, to maximize their profit, ( $r_b^{(t)} \sum_{\forall c \in C_b} p_{c,b}^{(t)} - c_b \sum_{\forall c \in C_b} p_{c,b}^{(t)}$).

However, the utility companies need to make sure that their electricity price is competitive compared to the other utility companies. If the companies' electricity prices are too high, fewer consumers will buy, leading to a drop in market penetration.

The utility function of each utility company is defined as:

$$A_b^{(t)}(\mathbf{p}^{(t)}, r_b^{(t)}) = \frac{r_b^{(t)} \sum\limits_{\forall c \in C_b} p_{c,b}^{(t)} - m_b \sum\limits_{\forall c \in C_b} p_{c,b}^{(t)}}{r_b^{(t)}} \tag{4.1}$$

$\mathbf{p}^{(t)} = \{p_{c,b}^{(t)}\}_{\substack{\forall c \in C \\ \forall b \in B}}$ is the consumers' purchased energy vector.

Each company's goal is to maximize its utility, $A_b^{(t)}$. This results in a maximization of profit and market penetration in the energy market. The related maximization problem is as follows:

$$\mathbf{I}_1 : \quad \max_{r_b^{(t)} \in [0, r_b^{Max}]} A_b^{(t)}(\mathbf{p}^{(t)}, r_b^{(t)}) \tag{4.2a}$$

$$s.t. \quad 0 \leqslant r_b^{(t)} \leqslant r_b^{Max} \tag{4.2b}$$

$r_b^{Max}$ is the maximum energy price, as determined by the energy market regulations [31]. The maximization problem, $\mathbf{I}_1$, is dependent on the energy prices of the remaining utility companies; changes in them will affect the amount of energy, $\sum_{\forall c \in C_b} p_{c,b}^{(t)}$, purchased from the examined utility company $b$. Also, the maximization problem, $\mathbf{I}_1$, is modeled as a non-cooperative game $\mathbb{J} = [B, \{R_b\}_{\forall b \in B}, \{A_b^{(t)}\}_{\forall b \in B}]$, where $B$ (the players) represents the set of all utility companies , $R_b = [0, r_b^{Max}]$ is the strategy set of each utility company, and $A_b^{(t)}$ is the utility function of the company.

**Theorem 1** (**Existence and Uniqueness of Nash Equilibrium**). *The non-cooperative game $\mathbb{J}$ has a unique Nash Equilibrium (NE).*

*Proof.* The strategy sets $R_1, \ldots, R_b, \ldots, R_{|B|}$ are observed to be non-empty, compact, convex subsets of finite dimensional Euclidean spaces. Also, all the utility functions $A_1^{(t)}, \ldots, A_b^{(t)}, \ldots, A_{|B|}^{(t)}$ are continuous on $\mathbb{R} = R_1 \times \cdots \times R_b \times \cdots \times R_{|B|}$. We study the utility function by taking its second order partial derivative $\frac{\partial^2 A_b^{(t)}}{\partial r_b^{(t)2}} = -2\frac{m_b \sum_{\forall c \in C_b} p_{c,b}^{(t)}}{{r_b^{(t)}}^3}$. This results in the utility function, $A_b^{(t)}$, being a quasi-concave function with respect to $r_b^{(t)}$ over $R_b$ [32]. Thus, a unique NE exists for the game $\mathbb{J}$. □

As for determining the unique NE, $\mathbf{R}^{(t)*} = [r_1^{(t)*}, \ldots, r_b^{(t)*}, \ldots, r_{|B|}^{(t)*}]$, a Best Response Dynamics algorithm is introduced [33, 34], in Algorithm 1 (Optimal Energy Prices (OEP)).

This OEP algorithm is an iterative distributed algorithm of low-complexity, $O(|B|Ctp)$, where $Ctp$ is the number of iterations in order for the OEP algorithm to converge to the NE, $\mathbf{R}^{(t)*}$. The numerical results demonstrating that the convergence of the overall Stackelberg game happens in just a few iterations is presented in Section 6.

**Algorithm 1** Optimal Energy Prices (OEP) Algorithm.

**Input:** $C$, $B$, $\{p_{c,b}^{(t)}\}_{\substack{\forall c \in C \\ \forall b \in B}}$, $\{m_b\}_{\forall b \in B}$
**Output:** Optimal Energy Price Vector $\mathbf{R}^{(t)*} = [r_1^{(t)*}, \ldots, r_b^{(t)*}, \ldots, r_{|B|}^{(t)*}]$
**Initialization:** $ctp = 0$, $Convergence = 0$, $\mathbf{R}^{(t)(ctp=0)}$
**while** $Convergence == 0$ **do**
    $ctp = ctp + 1$;
    **for** $b = 1$ to $|B|$ **do**
        *Utility company b determines* $r_b^{(t)(ctp)*}$ *based on* $\mathbf{I}_1$
    **end for**
    **if** $r_b^{(t)(ctp)*} == r_b^{(t)(ctp-1)*}$ **then**
        $Convergence = 1$
    **end if**
**end while**

# Chapter 5

# The Prospect of Energy Consumption

This section concentrates on the consumers' risk-aware behavioral characteristics regarding purchasing electricity from the utility companies while aiming to maximize their individual satisfaction.

## 5.1 Consumers' Risk-aware Behavior

At time slot $t$, each consumer, $c$, can purchase an amount of power, $p_{c,b}^{(t)}$, from each utility company $b$. As a result, the associated experience cost for each consumer by doing this action is: $\Lambda_{c,b}^{(t)}(p_{c,b}^{(t)}) = r_b^{(t)} \cdot p_{c,b}^{(t)} - w_c p_{c,b}^{(t)}$ where $w_c \in \mathbb{R}^+$ is the tailored satisfaction factor for each consumer covering their energy requirements. The first term is the consumer's financial cost of purchasing the electricity, while the second term is the satisfaction gained from using the acquired energy. The overall cost of purchasing the power is: $\Lambda_c^{(t)}(\mathbf{p}_c^{(t)}) = \sum_{\forall b \in b} p_{c,b}^{(t)}(r_b^{(t)} - w_c)$, where $\mathbf{p}_c^{(t)} = [p_{c,1}^{(t)}, \ldots, p_{c,b}^{(t)}, \ldots, p_{c,|B|}^{(t)}]$ is the consumer's $c$ energy purchase vector.

Each consumer purchases an amount of energy, $\sum_{\forall b \in B} p_{c,b}^{(t)} \leqslant P_c^{(t)}$, based on the announced energy price vector, $\mathbf{R}^{(t)*}$ (previously mentioned in Chapter 4). As a result, the remaining power $\mathcal{P}_c^{(t)} = P_c^{(t)} - \sum_{\forall b \in B} P_{c,b}^{(t)}$ is postponed to a later time slot. This delay may be because either energy prices are too expensive or because the consumer's basic energy demands have been met. The consumer feels dissatisfied by having to deffer a portion their energy demands, resulting in: $\Lambda_c^{\mathrm{dis}(t)}(\mathcal{P}_c^{(t)}) = s_c \mathcal{P}_c^{(t)}$, where $s_c \in \mathbb{R}^+$, signifies the user's individualized dissatisfaction factor.

The following is how each consumer's total experienced cost is calculated, which is the result of both purchasing energy from utility companies and deferring a portion of its energy usage for a later time slot:

$$\Lambda_c^{TOT(t)}(\mathbf{p}_c^{(t)}) = \sum_{\forall b \in B} p_{c,b}^{(t)}(r_b^{(t)} -_c) + \Lambda_c^{dis(t)}(\mathcal{P}_c^{(t)}).$$

Each utility company can generate a maximum amount of electricity, $\tilde{p}_b$, per time slot $t$. The utility companies are referred to as a Common Pool of Resources (CPR) since any consumer can purchase power from any utility company. As the total power purchased approaches the utility company's maximum power generation capacity, $\tilde{p}_b$, the utility company will begin to experience instability in its operation. This instability can lead to brownouts or even complete blackouts. If this happens, the consumers will receive zero satisfaction (i.e., return). The probability of failure (PoF) for each utility company is a strictly increasing function of the total amount of energy purchased from the company by the consumers, i.e., $\bar{p}_b = \sum_{c \in C_b} p_{c,b}^{(t)}$. For demonstrative reasons, we use a linear PoF concerning $\bar{p}_b$, as shown below:

$$E_b(\bar{p}_b^{(t)}) = \begin{cases} \frac{\bar{p}_b^{(t)}}{\tilde{p}_b}, \text{ if } \bar{p}_b^{(t)} \leqslant \tilde{p}_b \\ 1, \text{ otherwise} \end{cases} \tag{5.1}$$

If the consumers' energy demand exceeds the company's maximum generation capacity, the utility company's failure is captured in the second branch of Eq. 5.1.

The rest of this analysis can be easily replicated. This easy replication even includes other strictly increasing PoF functions, as long as they focus on the consumers' total amount of purchased energy from the utility companies. For example, a logarithmic or exponential PoF function could illustrate the utility company's heightened sensitivity to experiencing a blackout caused by the consumer's excessive energy demand.

The probability of each utility company not experiencing a failure is $(1-E_b(\bar{p}_b^{(t)}))$. Therefore, the expected cost of interacting with a utility company is:

$$\mathbb{P}(\Lambda_{c,b}^{(t)}) = (1 - E_b(\bar{p}_b^{(t)})) \cdot \Lambda_{c,b}^{(t)} + E_b(\bar{p}_b^{(t)}) \cdot s_c p_{c,b}^{(t)}.$$

However, if there is a failure, the utility company will not be able to supply any of the consumers since it will be in a non-stable operation state. This means the consumer will have to delay the corresponding amount of energy to a later time slot (second term). Based on the previous analysis, the entire estimated cost of the smart grid system for each consumer is calculated as follows:

$$\mathbb{P}(\Lambda_c^{TOT(t)}) = \sum_{\forall b \in B} \mathbb{P}(\Lambda_{c,b}^{(t)}) + \Lambda_c^{\mathrm{dis}(t)} \tag{5.2}$$

When consumers determine their ideal amount of purchased energy from each utility company, they need to consider the uncertainty of each utility company's failure. Prospect Theory, first outlined by Kahneman and Tversky [35] , can be used to capture the consumers' risk-aware decision-making process [36].

According to our suggested prospect-theoretic DRM framework, if the utility companies run properly, consumers gain personal satisfaction by meeting their energy needs. However, if a utility company fails due to overexploitation or overload, it can't meet its consumers' energy demands, resulting in consumers suffering dissatisfaction. Consumers' gains and losses are personalized and are measured relative to the reference point:

$$z_0(p_{c,b}^{(t)}) = \Lambda_c^{\mathrm{dis}(t)}(p_{c,b}^{(t)}) = s_c p_{c,b}^{(t)}.$$

This reference point reflects the consumer's dissatisfaction if the utility company fails to provide the required service [37].

By purchasing energy from a utility company, the consumer's prospect-theoretic utility function is defined as follows:

$$A_{c,b}^{PT(t)}(z_c) = \begin{cases} (z_0 - z_c)^{g_c} & \text{, if } z_c \leqslant z_0 \\ -h_c(z_c - z_0)^{\gamma_c} & \text{, otherwise} \end{cases} \tag{5.3}$$

$z_c(p_{c,b}^{(t)}) = \Lambda_{c,b}^{(t)}(p_{c,b}^{(t)}) = r_b^{(t)} p_{c,b}^{(t)} - w_c p_{c,b}^{(t)}$ denotes the consumer's experienced cost. Each consumer wishes to maximize their prospect-theoretic utility function. In the first branch of Eq. 5.3, the maximization of $A_{c,b}^{PT(t)}(z_c)$ is equivalent to the minimization of the consumer's experience cost, $\Lambda_{c,b}^{(t)}(p_{c,b}^{(t)})$. Meanwhile, in the second branch of Eq. 5.3, the maximization will reflect the minimization of any extra cost the consumer will experience if the utility company fails. The gains sensitivity parameter, $g_c \in \mathbb{R}^+$, is the consumer's personal sensitivity and interpretation of the experienced gains. On the other hand, the losses sensitivity parameter, $\gamma_c \in \mathbb{R}^+$, is the consumer's interpretation of the satisfaction losses. The loss aversion prospect-theoretic parameter, $h_c \in \mathbb{R}^+$, illustrates the weights of gains and the losses into the consumers' perception. Specifically, if $h_c \in (1, +\infty)$ the consumer cares more about the losses versus the gains of its satisfaction, while the exact opposite holds for $h_c \in [0, 1]$. For notation simplicity, without losing generality, the consumers are considered to have similar sensitivity to the losses and gains, i.e., $g_c = \gamma_c, \forall c \in C$.

In the following analysis, the properties and structure of Eq. 5.3 are explored. In the first branch,

$$A_{c,b}^{PT(t)}(z_c(p_{c,b}^{(t)})) = (s_c p_{c,b}^{(t)} - r_b^{(t)} \cdot p_{c,b}^{(t)} + w_c p_{c,b}^{(t)})^{g_c} = p_{c,b}^{(t)g_c}(s_c - r_b^{(t)} + w_c)^{g_c} = p_c^{(t)g_c} q_{c,b}^{(t)}$$

where $q_{c,b}^{(t)} = (s_c - r_b^{(t)} + w_c)^{g_c}$.

When the utility company covers the consumers' energy demand without failure, the consumers receive a positive utility, thus, $q_{c,b}^{(t)} > 0$. In the second branch of Eq.

5.3, if the utility company fails, the consumer's cost consists of the dissatisfaction of delaying its unserved energy needs. The monetary cost of the attempted purchased energy that led to the company's failure is also included in the consumer's cost.

The consumers actual cost is $z_c(p_{c,b}^{(t)}) = \Lambda_c^{dis(t)}(p_{c,b}^{(t)}) + r_b^{(t)} p_{c,b}^{(t)}$ while the corresponding prospect-theoretic utility can be written as:

$$A_{c,b}^{PT(t)}(p_{c,b}^{(t)}) = -k_c(s_c p_{c,b}^{(t)} + m_b^{(t)} p_{c,b}^{(t)} - s_c p_{c,b}^{(t)})^{g_c} = -k_c s_c^{g_c} p_{c,b}^{(t)g_c}.$$

Thus, the consumer's prospect-theoretic utility is rewritten as:

$$A_{c,b}^{PT(t)}(p_{c,b}^{(t)}) = \begin{cases} q_{c,b}^{(t)} \cdot p_{c,b}^{(t)g_c} & \text{with probability } 1 - E_b \\ -h_c s_c^{g_c} p_{c,b}^{(t)g_c} & \text{with probability } E_b \end{cases} \tag{5.4}$$

Based on Eq. 5.4, the consumer's expected prospect-theoretic utility function from purchasing energy from utility company $b$ can be derived as follows:

$$\begin{aligned} \mathbb{P}(A_{c,b}^{PT(t)}) &= q_c^{(t)} p_{c,b}^{(t)g_c}(1 - E_b) - h_c s_c^{g_c} p_{c,b}^{(t)g_c} E_b \\ &= p_{c,b}^{(t)g_c}[q_{c,b}^{(t)}(1 - E_b) - h_c s_c^{g_c} E_b] \\ &= p_{c,b}^{(t)g_c} f_{c,b}^{(t)}(\bar{p}_b^{(t)}) \end{aligned}$$

$f_{c,b}^{(t)}(\bar{p}_b^{(t)}) = q_{c,b}^{(t)}(1 - E_b) - h_c s_c^{g_c} E_b$ denotes the effective rate of return of the utility company $b$ when providing energy to consumer $c$.

## 5.2 Problem Formulation

Each consumer can dynamically purchase energy from multiple utility companies, considering the announced energy prices and the corresponding experience cost. Thus, each consumer's expected overall prospect-theoretic utility is $\sum_{\forall b \in B} \mathbb{P}(A_{c,b}^{PT(t)})$.

In addition, given the companies' declared energy prices, each consumer can choose to postpone some of their energy demands, $\mathcal{P}_c^{(t)}$, and incur associated dissatisfaction $\Lambda_c^{\text{dis}(t)}(\mathcal{P}_c^{(t)})$.

As a result, each consumer's perceived satisfaction with purchasing electricity through the smart grid system is $S_c^{(t)}(\mathbf{p}_c^{(t)}, \mathbf{p}_{-c}^{(t)}) = \sum_{\forall b \in B} \mathbb{P}(A_{c,b}^{PT(t)}) - \Lambda_c^{\text{dis}(t)}(\mathcal{P}_c^{(t)})$. $\mathbf{p}_c^{(t)}$ is the purchased energy vector of consumer, $c$, and $\mathbf{p}_{-c}^{(t)}$ is the rest of the consumers. Each consumer's goal is to optimize their perceived satisfaction while staying within their energy consumption constraints. As a result, the following is the formulation of the associated optimization problem:

$$\mathbf{P}_2 : \quad \max_{\mathbf{p}_c^{(t)} \in G_c^{(t)}} S_c^{(t)}(\mathbf{p}_c^{(t)}, \mathbf{p}_{-c}^{(t)}) \tag{5.5a}$$

$$s.t. \quad \sum_{\forall b \in B} p_{c,b}^{(t)} \leqslant P_c^{(t)} \tag{5.5b}$$

$$0 \leqslant p_{c,b}^{(t)} \leqslant P_c^{(t)} \tag{5.5c}$$

where $G_c^{(t)} = \overbrace{[0, P_c^{(t)}] \times \ldots \times [0, P_c^{(t)}]}^{|B| \text{ - times}}$ denotes the strategy set of consumer $c$ in time slot $t$. Each consumer solves the optimization problem distributedly per time slot $t$. Towards solving this problem, given the inter-dependency of the consumers' energy purchase decisions, we address it as a non-cooperative game $Z_x = [C, \{G_c^{(t)}\}_{\forall c \in C}, \{S_c^{(t)}\}_{\forall c \in C}]$ among the consumers. $C$ denotes the set of consumers, $G_c^{(t)}$ is the strategy set of each consumer at time slot $t$, and $S_c^{(t)}$ is its corresponding satisfaction function. Our goal is to show the existence and uniqueness of a NE $\mathbf{p}^{(t)*} = [\mathbf{p}_1^{(t)*}, \ldots, \mathbf{p}_c^{(t)*}, \ldots, \mathbf{p}_{|C|}^{(t)*}]$, where $\mathbf{p}_c^{(t)*} = [p_{c,1}^{(t)*}, \ldots, p_{c,b}^{(t)*}, \ldots, p_{c,|B|}^{(t)*}]$ represents the consumers' optimal amount of purchased energy from each utility company.

**Definition 1.** ***(Nash Equilibrium)*** *The vector $\mathbf{p}^{(t)*}$ is a Nash Equilibrium (NE) of the non-cooperative game $Z_x$ if for each consumer $c, \forall c \in C$ holds true that $S_c^{(t)}(\mathbf{p}_c^{(t)*}, \mathbf{p}_{-c}^{(t)*}) \geqslant S_c^{(t)}(\mathbf{p}_c^{(t)}, \mathbf{p}_{-c}^{(t)*}), \forall \mathbf{p}_c^{(t)} \in G_c^{(t)}$.*

The smart grid system's operation is stable at the NE point. Given the actions of the other consumers, no consumer can gain more satisfaction by changing their selection. Given the existing state, no user has any motivation to change their selection.

## 5.3 Problem Solution

This section demonstrates the existence and uniqueness of the NE of game $Z_x$ to identify the consumers' optimal amount of energy purchased by each utility company. $\mathbb{G}_c^{(t)} \subseteq G_c^{(t)}$ is the set of the strategies that satisfies the constraints (5.5a) and (5.5b). Therefore, the non-cooperative game $Z_x$ can be rewritten as: $\mathbb{Z}_x = [C, \{\mathbb{G}_c^{(t)}\}_{\forall c \in C}, \{S_c^{(t)}\}_{\forall c \in C}]$.

**Theorem 2.** *The non-cooperative game $\mathbb{Z}_x$ is an $|C|$-person concave game admitting at least one NE.*

To prove Theorem 2, first the following theoretical analysis and lemmas are presented.

**Lemma 1.** *When a consumer purchases energy from a utility company $b$ less than a threshold value, $p_b^{thr}, p_b^{thr} \in [0, \tilde{p}_b]$ where $f_{c,b}^{(t)}(\bar{p}_b^{(t)} = p_b^{thr}) = 0$, the consumer's expected prospect-theoretic utility function is positive.*

*Proof.* The first order derivative of each utility company's effective rate of return function $f_{c,b}^{(t)}(\bar{p}_b^{(t)})$ is determined as follows:

$$\begin{aligned}\frac{\partial f_{c,b}^{(t)}(\bar{p}_b^{(t)})}{\partial \bar{p}_b^{(t)}} &= -q_{c,b}^{(t)}\frac{\partial E_b}{\partial \bar{p}_b^{(t)}} - h_c s_c^{g_c}\frac{\partial E_b}{\partial \bar{p}_b^{(t)}} \\ &= \frac{\partial f_{c,b}^{(t)}(\bar{p}_b^{(t)})}{\partial p_{c,b}^{(t)}} \\ &= -(q_{c,b}^{(t)} + h_c s_c^{g_c})\frac{1}{\tilde{p}_b}\end{aligned}$$

Also, $\frac{\partial E_b}{\partial \bar{p}_b^{(t)}} = \frac{1}{\tilde{p}_b}$ for $\bar{p}_b^{(t)} \leqslant \tilde{p}_b$ and $\bar{p}_b^{(t)} = p_{c,b}^{(t)} + \sum\limits_{\forall c' \in C_b - \{c\}} p_{c',b}^{(t)}$.

Given that $q_{c,b}^{(t)}, h_c, s_c > 0$, it is concluded that $\frac{\partial f_{c,b}^{(t)}(\bar{p}_b^{(t)})}{\partial p_{c,b}^{(t)}} < 0$. As a result, the utility company's effective rate of return function is strictly decreasing for $p_{c,b}^{(t)} \in [0, P_c^{(t)}]$. The effective rate of return function is examined in two scenarios in the following analysis.

**CASE I:** If $f_{c,b}^{(t)}(\bar{p}_b^{(t)} = 0) \leqslant 0$, then $p_{c,b}^{(t)} \geqslant 0 \overset{f_{c,b}^{(t)}\searrow}{\Longleftrightarrow} f_{c,b}^{(t)}(p_{c,b}^{(t)}) \leqslant 0$.

**CASE II:** If $f_{c,b}^{(t)}(\bar{p}_b^{(t)} = 0) > 0$, then for $\bar{p}_b^{(t)} = \tilde{p}_b$ we have $E_b(\bar{p}_b^{(t)} = \tilde{p}_b) = 1$, thus, $f_{c,b}^{(t)}(\bar{p}_b^{(t)} = \tilde{p}_b) = -h_c s_c^{g_c} < 0$. Therefore, based on the Intermediate Value Theorem [38], $\exists p_b^{thr} \in [0, \tilde{p}_b]$, such that $f_{c,b}^{(t)}(p_b^{thr}) = 0$, and $f_{c,b}^{(t)}(\bar{p}_b^{(t)}) > 0, \forall \bar{p}_b^{(t)} \in (0, p_b^{thr})$ and $f_{c,b}^{(t)}(\bar{p}_b^{(t)}) < 0, \forall \bar{p}_b^{(t)} \in (p_b^{thr}, \tilde{p}_b)$. □

Lemma 2 studies the properties of the consumer's expected prospect-theoretic utility function

**Lemma 2.** *Concerning the amount of purchased energy, $p_{c,b}^{(t)}$, the consumer's prospect-theoretic utility function, $\mathbb{P}(A_{c,b}^{PT(t)})$, is strictly concave.*

*Proof.* The first and the second order partial derivative of the consumer's expected prospect-theoretic utility function is:

$$\frac{\partial \mathbb{P}(A_{c,b}^{PT(t)})}{\partial p_{c,b}^{(t)}} = g_c p_{c,b}^{(t)g_c-1} f_{c,b}^{(t)}(\bar{p}_b^{(t)}) + p_{c,b}^{(t)g_c} \frac{\partial f_{c,b}^{(t)}(\bar{p}_b^{(t)})}{\partial p_{c,b}^{(t)}}$$

and

$$\frac{\partial^2 \mathbb{P}(A_{c,b}^{PT(t)})}{\partial p_{c,b}^{(t)2}} = g_c(g_c-1) p_{c,b}^{(t)g_c-2} f_{c,b}^{(t)}(\bar{p}_b^{(t)}) + g_c p_{c,b}^{(t)g_c-1} \frac{\partial f_{c,b}^{(t)}(\bar{p}_b^{(t)})}{\partial p_{c,b}^{(t)}} + g_c p_{c,b}^{(t)g_c-1} \frac{\partial f_{c,b}^{(t)}(\bar{p}_b^{(t)})}{\partial p_{c,b}^{(t)}} + p_{c,b}^{(t)g_c} \frac{\partial^2 f_{c,b}^{(t)}(\bar{p}_b^{(t)})}{\partial p_{c,b}^{(t)2}}.$$

We have that $\frac{\partial f_{c,b}^{(t)}(\bar{p}_b^{(t)})}{\partial \bar{p}_b^{(t)}} = \frac{\partial f_{c,b}^{(t)}(\bar{p}_b^{(t)})}{\partial p_{c,b}^{(t)}}$, $g_c \in [0,1]$, $f_{c,b}^{(t)}(\bar{p}_b^{(t)}) \searrow$ (Lemma 1),

and $\frac{\partial^2 f_{c,b}^{(t)}(\bar{p}_b^{(t)})}{\partial \bar{p}_b^{(t)2}} = 0$. Thus $\frac{\partial^2 \mathbb{P}(A_{c,b}^{PT(t)})}{\partial p_{c,b}^{(t)2}} < 0, \forall p_{c,b}^{(t)} \in (0, p_b^{thr})$.

Therefore, we conclude that the consumer's expected prospect-theoretic utility function $\mathbb{P}(A_{c,b}^{PT(t)})$ is a strictly concave function $\forall p_{c,b}^{(t)} \in (0, p_b^{thr})$. □

In the following Lemma, the properties of the consumer's satisfaction function, $S_c^{(t)}(\mathbf{p}_c^{(t)}, \mathbf{p}_{-c}^{(t)})$, are studied.

**Lemma 3.** *The consumer's satisfaction function $S_c^{(t)}(\mathbf{p}_c^{(t)}, \mathbf{p}_{-c}^{(t)})$ is a concave function over the strategy space $\mathbb{G}_c^{(t)}$, a convex set.*

*Proof.* The constraints in (5.5a) and (5.5b) are a collection of convex functions that define a convex set $\mathbb{G}_c^{(t)} \subseteq G_c^{(t)}$. Furthermore, if and only if the corresponding Hessian matrix $V$ is negative semidefinite over the convex set $\mathbb{G}_c^{(t)}$, the consumer's satisfaction function, $S_c^{(t)}(\mathbf{p}_c^{(t)}, \mathbf{p}_{-i}^{(t)})$, is concave.

We have $\frac{\partial^2 S_c^{(t)}}{\partial p_{c,b}^{(t)} \partial p_{c,b'}^{(t)}} = \frac{\partial^2 \mathbb{P}(A_{c,b}^{PT(t)})}{\partial p_{c,b}^{(t)} \partial p_{c,b'}^{(t)}}$, thus, $\frac{\partial^2 S_c^{(t)}}{\partial p_{c,b}^{(t)2}} = \frac{\partial^2 \mathbb{P}(A_{c,b}^{PT(t)})}{\partial p_{c,b}^{(t)2}}$,

and $\frac{\partial^2 S_c^{(t)}}{\partial p_{c,b}^{(t)} \partial p_{c,b'}^{(t)}} = 0, \forall b, b' \in B, B \neq b'$ due to the fact that $\frac{\partial^2 \mathbb{P}(A_{c,b}^{PT(t)})}{\partial p_{c,b}^{(t)} \partial p_{c,b'}^{(t)}} = 0$.

Therefore, the Hessian matrix $V$ of the consumer's satisfaction function, $S_c^{(t)}$, can be represented as $V = \text{diag}(\frac{\partial^2 \mathbb{P}(A_{c,1}^{PT(t)})}{\partial p_{c,1}^{(t)2}}, \ldots, \frac{\partial^2 \mathbb{P}(A_{c,|B|}^{PT(t)})}{\partial p_{c,|B|}^{(t)2}})$. The Hessian matrix is diagonal, and each non-zero element is negative. As a result, we have $U_h = \prod_{n=1}^{h} q_{nn}$, where $U_h$ is the leading principal minor, and $q_{nn} = \frac{\partial^2 \mathbb{P}(A_{c,b}^{PT(t)})}{\partial p_{c,b}^{(t)2}}, \forall b \in B$. We have $(-1)^h U_h > 0$ for when $h$ is even and $(-1)^h U_h > 0$ for when $h$ is odd. As a result, the Hessian matrix $V$ is strictly negative definite. Thus, the consumer satisfaction function, $S_c^{(t)}(\mathbf{p}_c^{(t)}, \mathbf{p}_{-c}^{(t)})$, is determined to be a concave function over the convex set $\mathbb{G}_c^{(t)}$. □

The consumer's satisfaction function, $S_c^{(t)}(\mathbf{p}_c^{(t)}, \mathbf{p}_{-c}^{(t)})$, is concave over the convex set $\mathbb{G}_c^{(t)}$ based on the analysis in Lemmas 1-3. As a result, we can conclude that non-cooperative game $\mathbb{G}_c$ is a $|C|$-person concave game with at least one NE. In the following theorem, we demonstrate the NE's uniqueness and consumers' strategies' convergence to it [39].

**Theorem 3.** *If $Z_c + G_c^T, \forall c \in C$ is strictly negative definite, where $Z_c$ is a $|B| \times |B|$ matrix function with $(Z_c)_{bb'} = w_c \frac{\partial^2 S_c^{(t)}(\mathbf{p}_c^{(t)}, \mathbf{p}_{-c}^{(t)})}{\partial p_{c,b}^{(t)} \partial p_{c,b'}^{(t)}}, \forall b \neq b' \in B$, and the constant coefficients $w_c > 0$, then the non-cooperative game's $\mathbb{G}_c$ NE is unique and the consumers Best Response dynamics converge to it.*

*Proof.* We have: $Z_c = \text{diag}(w_c \frac{\partial^2 \mathbb{P}(A_{c,1}^{PT(t)})}{\partial p_{c,1}^{(t)2}}, \ldots, w_c \frac{\partial^2 \mathbb{P}(A_{c,|B|}^{PT(t)})}{\partial p_{c,|B|}^{(t)2}})$, based on the analysis in Lemma 3. As a result, we have $Z_c + G_c^T = 2G_c$, as $Z_c = w_c V$ and $Z_c = G_c^T$. Based on Lemma 3, we have $Z_c + G_c^T = 2w_c V$, which is strictly negative definite. Thus, we argue that the NE of the non-cooperative $\mathbb{G}_c$ is distinct and that consumers' strategies converge to it from any starting point using the Best Response Dynamics (BRD) [40, 41]. □

# Chapter 6

# Numerical Results

This section presents a numerical performance evaluation of the suggested prospect-theoretic DRM framework. Modeling and simulations will illustrate the proposed DRM framework's operation, features, and benefits. Section 6.1 demonstrates our suggested framework's process and performance from both the utility companies' and the consumers' perspectives. Section 6.2 gives a sensitivity analysis regarding the consumers' risk-aware characteristics, while section 6.3 illustrates a scalability analysis for the suggested framework to show its efficiency and robustness. Finally, Section 6.4 presents a comparative evaluation of our suggested model against alternative DRM frameworks.

In the following analysis, we consider $|B| = 5$ utility companies, $|C| = 50$ consumers, $|T| = 24h$, $t = 1h$, while the consumers' energy demand $P_c^{(t)}$ is assumed randomly distributed in the range $[0.14, 0.23]J$.

The utility companies maximum allowed energy price is 19.6 cents per KWh and the utility companies' maximum energy generation vector is $[22.16, 21.00, 20.23, 18.88, 17.73]\%$, where company $b = 1$ has the maximum energy generation capacity.

Unless otherwise explicitly stated, we also assume that: $m_b = 0.02, d_c = 0.5, w_c = 2, s_c = 1$ and $h_i = 2$. The values of the parameters considered in our simulations have been extracted from the real data for the years 2000-2021 from the U.S. Energy Information Administration considering the Southwest region of USA [31].

## 6.1 Pure Performance and Operation

This section presents the operation and performance of the proposed prospect-theoretic DRM framework from both the utility companies and the consumers' perspectives. The optimal relative power prices and overall relative amount of power purchased by utility companies are shown in Fig. 6.1a-6.1d. These figures also indicate the companies' utility and their probability of failure as a function of the total number of iterations in the Stackelberg game (lower horizontal axis) and the execution time (upper horizontal axis). The utility companies' optimal prices and total amounts of power purchased are viewed as a percentage of the all the utility companies' values per iteration.

The presented results have been derived from a Monte Carlo analysis of $10,000$ executions of our framework and capture the system's behavior in a one-time slot $t$. The results reveal that the higher the utility company's maximum power generation is, the lower the optimal price it announces (Fig. 6.1a), resulting in attracting a larger portion of power purchased from it (Fig.6.1b) and thus increasing its penetration to the market. Therefore, a higher utility is achieved (Fig.6.1c) while the corresponding probability of failure remains low (Fig.6.1d) due to its high power generation capacity that enables the utility company to cover the consumers' power needs.

Figures 6.2a-6.2b depict the amounts of power purchased from each utility company and the related anticipated cost for one time slot, respectively, as a function of our framework's iterations and corresponding real execution time.

At first glance, we notice that our suggested framework converges to a stable result very quickly (in a small number of iterations equating to only a few seconds) as shown in Figures 6.1 Furthermore, we note that the lower the power price charged by the utility company,(Fig. 6.1a) the greater the amount of power purchased by each user.(Fig. 6.2a) As a result, the lower the consumer's estimated cost, the better (Fig. 6.2b).

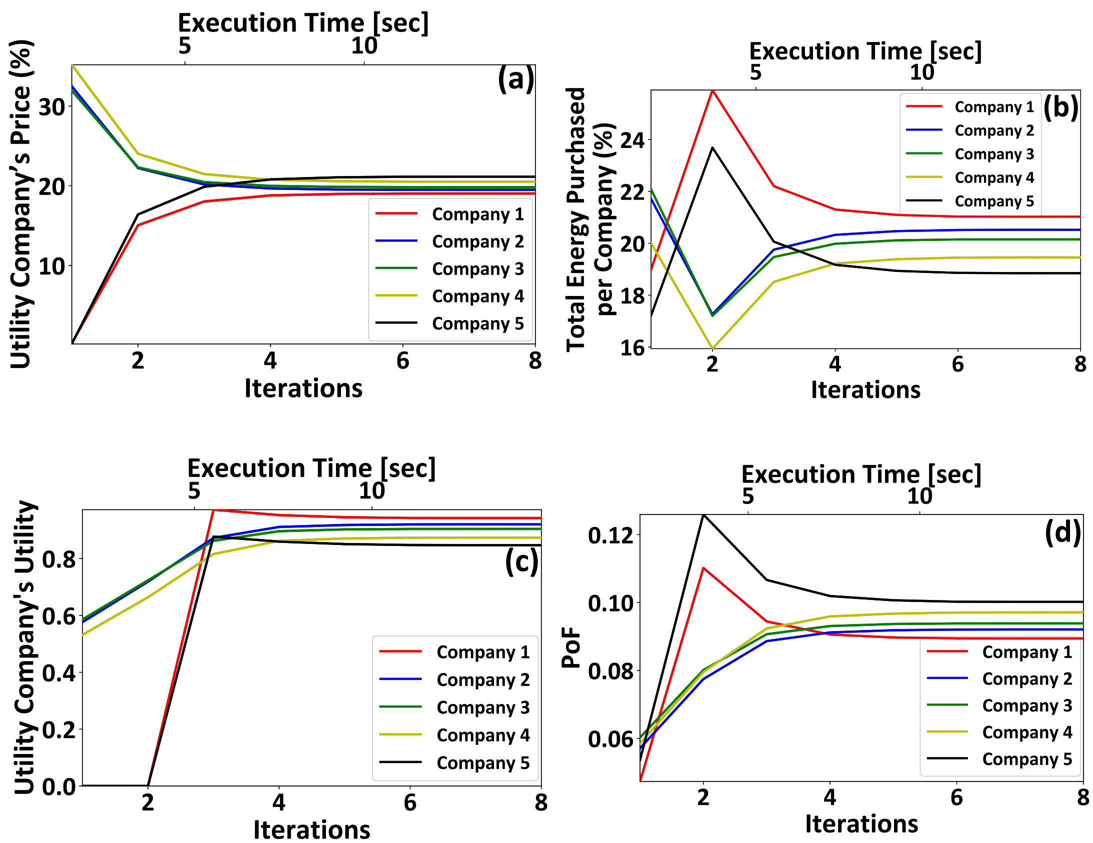


Figure 6.1: Utility companies' perspective on pure performance operations.

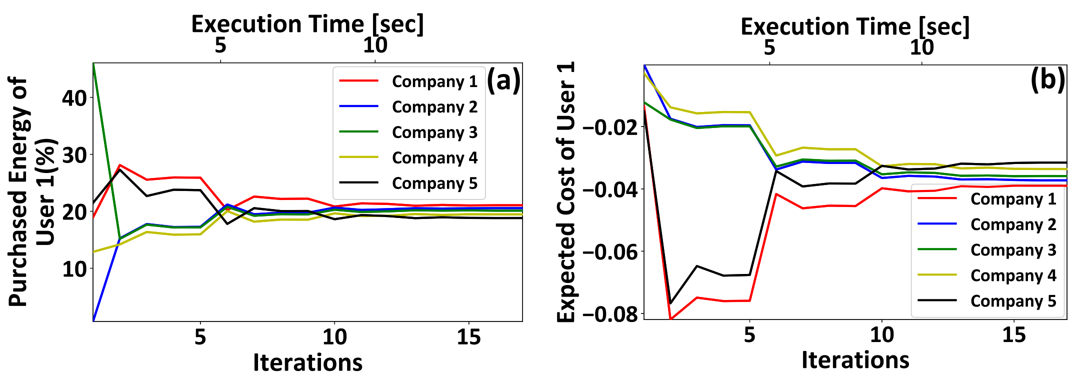


Figure 6.2: Consumers' perspective on pure performance operations.

## 6.2 Consumers Sensitivity Analysis

The impact of the consumer's behavior and risk-aware decision-making features on the operation of the proposed prospect-theoretic DRM framework is examined in this section. We consider two scenarios regarding the consumers' loss aversion prospect-theoretic behavior, as represented by the parameter $h_i, h_i \in \mathbb{R}^+$.

1. Heterogeneous: each consumer has a different loss aversion behavior.

2. Homogeneous: all consumers have the same loss aversion behavior $h_i$.

We assume that the average value of $h_i$ in the heterogeneous case is the same as the equivalent value in the homogeneous case for fairness in the comparison. For both the homogeneous and heterogeneous situations, Figures 6.3a-6.3b shows the companies' average probability of failure, consumers' average satisfaction (left vertical axis), and the companies' average utility (right vertical axis) as a function of the loss aversion parameter $h_i$. The findings show that consumers' behavioral features pull the overall system performance down in the heterogeneous scenario, which is bad for both consumers and utility companies. In particular, we find that both consumers and companies (Fig. 6.3b) obtain lower levels of satisfaction and utility. Furthermore, because consumers' decision-making processes are widely different, showing extreme behavioral patterns, companies have a higher average probability of failure due to their heterogeneous risk-aware behavioral traits.

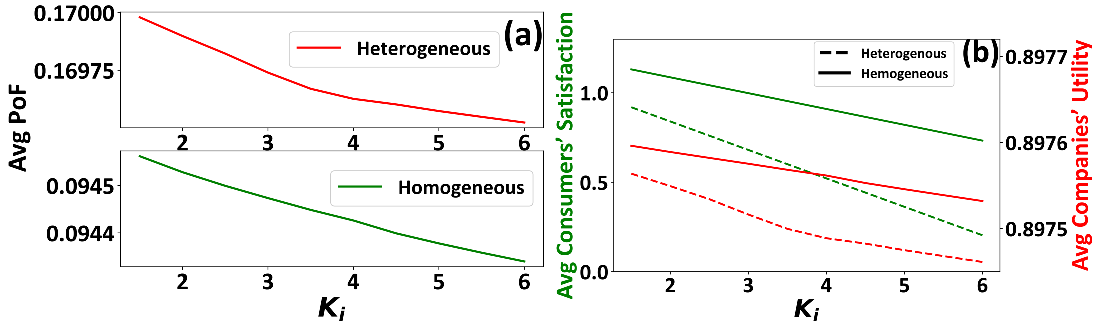


Figure 6.3: Sensitivity analysis.

## 6.3 Scalability Analysis

This section contains a comprehensive set of scalability study results for a smart grid system with a variable number of utility companies (up to 10) and consumers (up to 100). For the scalability analysis scenario under consideration, Fig 6.4a-6.4e shows the proposed framework's real execution time, the utility companies' average probability of failure, the consumers' average satisfaction, the consumers' average overall expected cost, and the utility companies' average utility, respectively. The results show that as the size of the total smart grid system grows (in terms of companies or users), our framework's execution time (Fig. 6.4a) grows in a sublinear manner, demonstrating its robustness and scalability. Furthermore, the results reveal that the suggested framework is exceptionally adaptable, as it achieves a low probability of failure (Fig. 6.4b), even in the extreme case of few companies and many consumers. This distinguishes our proposed approach from others since it considers consumers' risk-aware behavioral features, which take into account the companies' risk of failure when determining their ideal amounts of power purchased. Furthermore, when a more significant number of companies are available, consumers have a higher level of satisfaction (Fig. 6.4c). They have more flexibility regarding available power prices, and the risk of failure is significantly reduced. On the other hand, consumers pay the highest price, (Fig. 6.4d), in a crowded smart grid system supported by only a few companies because both companies' failure rates are higher, (Fig. 6.4b), and there are only a few alternatives. Likewise, as shown in Fig. 6.4e, there is a tradeoff between the number of users and the utility companies, with the latter's experienced utility being impacted.

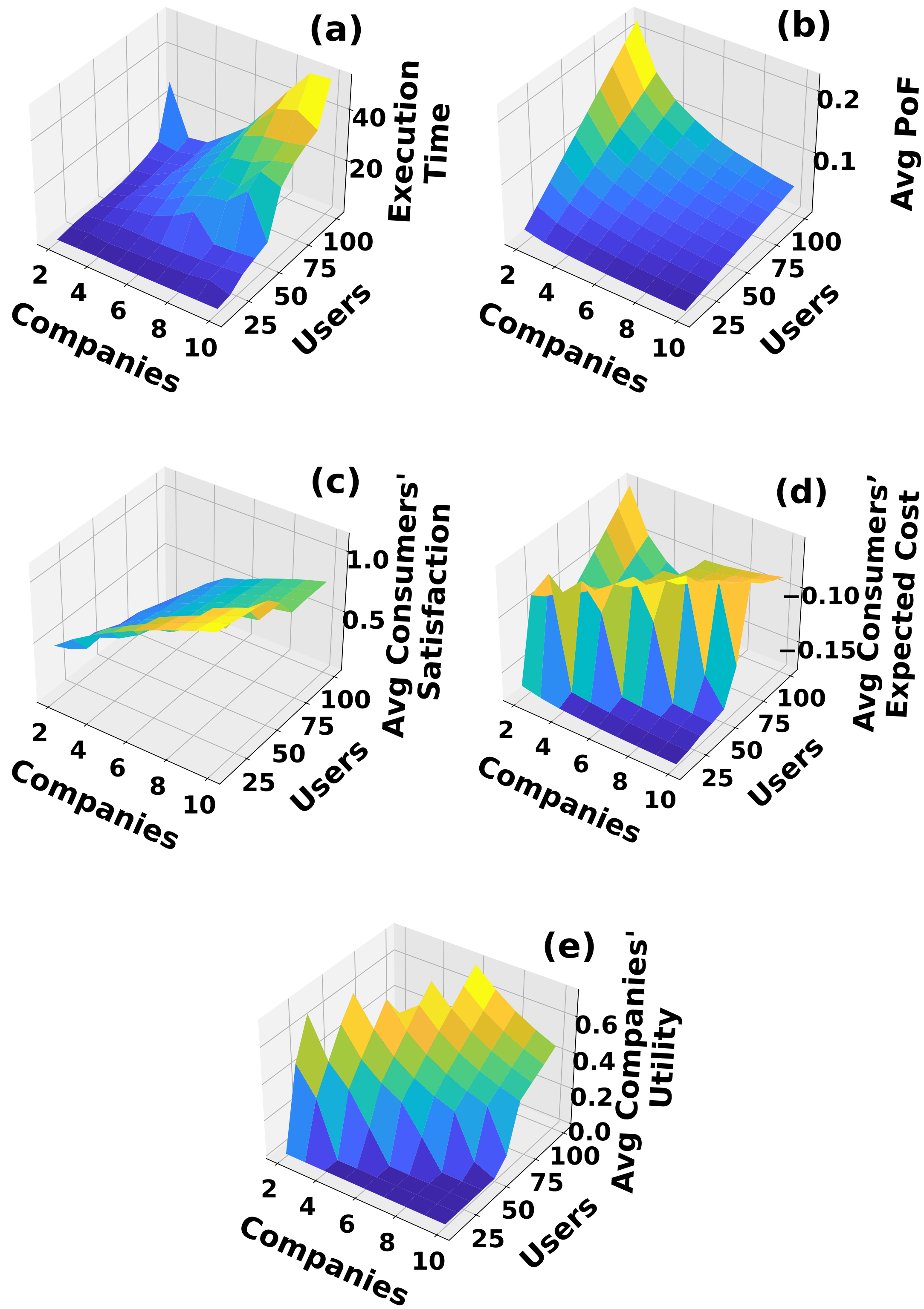


Figure 6.4: Scalability Analysis

## 6.4 Comparative Evaluation

We give a comparative assessment analysis of our proposed prospect-theoretic DRM framework (PT-DRM) versus the following alternative-mainstream DRM approaches in the following sections: (a) Time of Day (ToD) DRM: Utility companies announce a power price per time slot the day before (ahead); (b) Cap-based (Cap) DRM: Utility companies use a cap-based pricing policy, in which consumers are charged based on the price per cap; and (c) and (d) Low and High price-based DRM: Utility companies announce a fixed low and high power price, respectively, during the day. Figure 6.5a-6.5b shows the utility companies' average utility, consumers' average satisfaction, and consumers' projected cost for all of the analyzed comparable situations. The findings show that utility companies have the worst results when they announce a fixed low price, (Fig. 6.5a), while consumers have a considerable cost, (Fig. 6.5b), as they are driven to purchase vast amounts of power that cover their power demands, resulting in high consumer satisfaction, (Fig. 6.5a). High-price-based and cap-based DRM approaches, on the other hand, exhibit similar behavior from both the companies' and consumers' perspectives. Consumers buy less power in those situations than in the low-cost scenario, resulting in increased utility and consumer costs due to the higher power prices. Consumers also report a decreased overall satisfaction for the same reason. From the consumers' perspective, the ToD DRM model produces the worst outcomes since companies announce power prices the day before without accounting for the consumers' behavioral traits; as a result, they benefit only themselves while the consumers are dissatisfied, (Fig. 6.5a). Finally, the suggested prospect-theoretic DRM approach strikes a balance between companies and consumer interests. According to the PT-DRM model, companies' optimal power prices attract consumers to buy a large amount of power, resulting in high utility for companies, (Fig. 6.5a), but a high experienced cost for consumers, (Fig. 6.5b). However, the consumers cover a large portion of their power demands due to the power purchased at the associated companies' power pricing. As a result, their level of satisfaction remains high, (Fig. 6.5a)

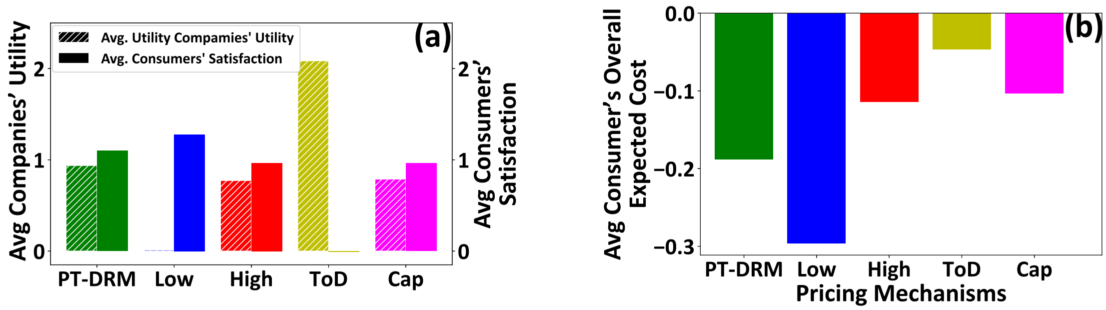


Figure 6.5: Comparative Evaluation

# Chapter 7

# Conclusion

This study describes a prospect-theoretic DRM framework for a smart grid system with multiple utility companies and multiple consumers. A multi-leader multi-follower Stackelberg game is created between the companies (leaders) and the consumers (followers). The companies find their ideal electricity price by solving a non-cooperative game and determining a single NE point. Consumers purchase their ideal amounts of power from each company while also considering the risk of the company failing (due to excessive power demand). Using Prospect Theory, we capture the consumers' risk-aware behavioral decision-making processes. A non-cooperative game is devised among the consumers and solved using the theory of a $n$-person concave game. This is done to determine its unique NE, for example, finding the optimal amounts of power each consumer purchases from each company. A comprehensive set of data is provided to illustrate the proposed framework's operation and performance and its pros and cons compared to alternative DRM frameworks.

Moving forward, we wish to extend this proposed framework by incorporating extrinsic and intrinsic motivational patterns. Companies will provide these motivations to persuade consumers to change their power purchase plans and decisions.

Contract Theory, if properly adopted, can also be used to follow the interactions between companies and their consumers.

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