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Neutrosophic Soft Fixed Points

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Abstract. In a wide spectrum of mathematical issues, the presence of a fixed point (FP) is equal to the presence of a appropriate map solution. Thus in several fields of math and science, the presence of a fixed point is important. Furthermore, an interesting field of mathematics has been the study of the existence and uniqueness of common fixed point (CFP) and coincidence points of mappings fulfilling the contractive conditions. Therefore, the existence of a FP is of significant importance in several fields of mathematics and science. Results of the FP, coincidence point (CP) contribute conditions under which maps have solutions. The aim of this paper is to explore these conditions (mappings) used to obtain the FP, CP and CFP of a neutrosophic soft set. We study some of these mappings (conditions) such as contraction map, L-lipschitz map, non-expansive map, compatible map, commuting map, weakly commuting map, increasing map, dominating map, dominated map of a neutrosophic soft set. Moreover we introduce some new points like a coincidence point, common fixed point and periodic point of neutrosophic soft mapping. We establish some basic results, particular examples on these mappings and points. In these results we show the link between FP and CP. Moreover we show the importance of mappings for obtaining the FP, CP and CFP of neutrosophic soft mapping.

Keyword. Neutrosophic set, fuzzy neutrosophic soft mapping, fixed point, coincidence point.

1. Introduction

It is well known fact that fuzzy sets (FS) [1], complex fuzzy sets (CFS) [2], intuitionistic fuzzy sets (IFSs), the soft sets [3], fuzzy soft sets (FSS) and the fuzzy parameterized fuzzy soft sets (FPFS-sets) [4], [5] have been used to model the real life problems in various fields like in medical science, environments, economics, engineering, quantum physics and psychology etc.

In 1965, L. A. Zadeh [1] introduced a FS, which is the generalization of a crisp set. A grade value of a crisp set is either 1 or 0 but a grade value of fuzzy set has all the values in closed interval $[0,1]$. A FS plays a central role in modeling of real world problems. There are a lot of applications of FS theory in various branches of science such as in engineering, economics, medical science, mathematical chemistry, image processing, non-equilibrium thermodynamics etc. The concept for IFSs is provided in [3] which are generalizations of FS. An IFS P can be expressed as $P=\{\langle \nu,, \beta_P(\nu), \gamma_P(\nu)\rangle : \nu \in X\}$, where $\beta_P(\nu)$ represents the degree of membership, $\gamma_P(\nu)$ represents the degree of non-membership of the element $\nu \in X$. FPFS-sets is the extension of a FS and soft set proposed in [4], [5]. FPFS-sets maintain a proper degree of membership to both elements and parameters.

The notion of a complex CFS, the extension of the fuszy set, was introduced by Ramot et, al., [2]. A CFS membership function has all the values in the unit disk. A complex fuzzy set is used for representing two-dimensional phenomena and plays an important role in periodic phenomena. Complex fuzzy set is used in signals and systems to identify a reference signal out of large signals detected by a digital receiver. Moreover it is used for expressing complex fuzzy solar activity (solar maximum and solar minimum) through the average number of sunspot.

Smarandache [6], [7] has given the notion of a neutrosophic set (NS). A NS is the extension of a crisp set, FS and IFS. In NS, truth membership (TM), falsity membership (FM) and indeterminacy membership (IM) are independent. In decision-making problems, the indeterminacy function is very significant. A NS and its extensions plays a vital role in many fields such as decision making problems, educational problems, image processing, medical diagnosis and conflict resolution. Moreover the field of neutrosophic probability, statistics, measures and logic have been developed in [8]. The generalization of fuzzy logic (FL) has been suggested by Smarandache in [8] and is termed as neutrosophic logic (NL). A proposition in NL is true (t), indeterminate (i) and false (f) are real values from the ranges T, I, F. T, I, F and also the sum of t, i, f are not restricted. In neutrosophic logic, there is indeterminacy term, which have no other logics, such as intuitionistic logic (IL), FL, boolean logic (BL) etc. Neutrosophic probability (NP) [8] is the extension of imprecise probability and classical probability. In NP, the chance occurs by an event is $t\%$ true, $i\%$ indeterminate and $f\%$ false where t, i, f varies in the subsets T, I and F respectively. Dynamically these subsets are functions based on parameters, but they are subsets on a static basis. In NP $n\_\sup \leq 3^{+}$, while in classical probability $n\_\sup \leq 1$. The extension of classical statistics is neutrosophic statistics [8] which is the analysis of events described by NP. There are twenty seven new definitions derived from NS, neutrosophic statistics and a neutrosophic probability. Each of these are independent. The sets derived from NS are intuitionistic set, paradoxist set, paraconsistent set, nihilist set, faillibilist set, trivialist set, and dialetheist set. Intuitionistic probability and statistics, faillibilist probability and statistics,tautological probability and statistics, dialetheist probabilityand statistics, paraconsistent probability and statistics, nihilist probability and statistics and trivialist probability and statistics are derived from neutosophic probability and statistics. N. A. Nabeeh [9] suggested a technique that would promote a personal selection process by integrating the neutrosophic analytical hierarchy process to show the ideal solution among distinct options with order preference tevhnique similar to an ideal solution (TOPSIS). M. A. Baset [10] introduced a new type of neutrosophy technique called type 2 neutrosophic numbers. By combining type 2 neutrosophic number and TOPSIS, they suggested a novel method T2NN-TOPSIS which is very useful in group decision making. They researched a multi criteria group decision making technique of the analytical network process method and Visekriterijusmska Optmzacija I Kommpromisno Resenje method under neutrosophic environment that deals high order imprecision and incomplete information [11]. M. A. Baet suggested a new strategy for estimating the smart medical device selecting process in a GDM in a vague decision environment. Neutrosophic with TOPSIS strategy is used in decision-making processes to deal with incomplete information, vagueness and uncertainty, taking into account the decision requirements in the information gathered by decision-makers [12]. They suggested the robust ranking method with NS to manage supply chain management (GSCM) performance and methods that have been widely employed to promote environmental efficiency and gain competitive benefits. The NS theory was used to manage imprecise understanding, linguistic imprecision, vague data and incomplete information [13]. Moreover M. A. Baset [14] et, al., used NS for assessment technique and decision-making to determine and evaluate the factors affecting supplier selection of supply chain management. T. Bera [15] et, al., defined a neutrosophic norm on a soft linear space known as neutrosophic soft linear space. They also modified the concept of neutrosophic soft (Ns) prime ideal over a ring. They presented the notion of Ns completely semi prime ideals, Ns completely prime ideals and Ns prime K-ideals [16]. Moreover T. Bera [17] introduced the concept of compactness and connectedness on Ns topological space along with their several characteristics. R. A. Cruz [18] et, al., discussed P-intersection, P- union, P-AND and P-OR of neutrosophic cubic sets and their related properties. N. Shah [19] et, al., studied neutrosophic soft graphs. They presented a link between neutorosophic soft sets and graphs. Moreover they also discussed the notion of strong neutosophic soft graphs.

Smarandache [20] discussed the idea of a single valued neutrosophic set (SVNS). A SVNS defined as for any space of points set U' with u in U', a SVNS W in U', the truth membership, false memebership and indeterminac membership functions denoetd as T_A, F_A and I_A respectively with $T_A, F_A, I_A \in [0,1]$ for each u in U'. A SVNS W is expressed as $W = \int_X \langle T_W(\nu), I_W(\nu), F_W(\nu)\rangle / \nu, \nu \in X$, when X is continous. For a discrete case, a SVNS can be expressed as $W = \sum_{i=1}^{n} \langle T(\nu i), I(\nu i), F(\nu i)\rangle / \nu i,\ \nu i \in X$. Later, Maji [21] gave a new concept neutrosophic soft set (NSS). For any initial universal set W and any parameters set E with $A \subset E$ and $P(W)$ represents all the NS of W. The order set (ϕ, A) is said to be the soft NS over W where $\phi : A \to P(W)$. Arockiarani et al., [22] introduced fuzzy neutrosophic soft topological space and presents main results of fuzzy neutrosophic soft topological space. Later on the researchers linked the above theories with different field of sciences.

The purpose of this paper is to study the mappings such as contraction mapping, expansive mapping, non-expansive mapping, commuting mapping, and weakly commuting mapping used to attain the FP, CP and CFP of a neutrosophic soft set. We present some basic resultsnd particular examples of fixed points, coincidence points, common fixed points in which contraction mapping, expansive mapping, non-expansive mapping, commuting mapping, and weakly commuting mapping are used.

2. Preliminaries

We will discuss here the basic notions of NS and neutrosophic soft sets. We will also discuss some new neutrosophic soft mappings such as contraction mapping, increasing mapping, dominated mapping, dominating mapping, K-lipschitz mapping, non-expansive mapping, commuting mapping, weakly compatible mapping. Moreover we will study periodic point, common fixed point, coinciding point of neutrosophic soft-mapping. Here $\hat{N}S(U'_E)$ is the collection of all neutrosophic soft points.

Definition 2.1 [7] Let U be any universal set, with generic element $\nu \in U'$. A NS $\hat{N}$ is defined by $\hat{N} = \{\langle \nu, T_{\hat{N}}(\nu), I_{\hat{N}}(\nu), F_{\hat{N}}(\nu)\rangle, \nu \in U'\}$, where $T, I, F : U \to \left]^-0, 1^+\right[$ and

$$^-0 \le T_{\hat{N}}(\nu) + I_{\hat{N}}(\nu) + F_{\hat{N}}(\nu) \le 3^+.$$

$T_{\hat{N}}(\nu), I_{\hat{N}}(\nu)$ and $F_{\hat{N}}(\nu)$ denote TM, IM and FM functions respectively. In $\left]^-0, 1^+\right[, 1^+ = 1 + \varepsilon,$ where ε is it's non-standard part and 1 is it's standard part. Likely $^-0 = 0 - \varepsilon,$ ε is it's non-standard part and 0 is it's standard part. It is difficult to employ these values in real life applications. Hence we take all the values of neutrosophic set from subset $[0,1]$.

Definition 2.2 [23] Let E and W be the set of parameters and initial universal set respectively. Let the power set of W is denoted by $P(W)$. Then a pair (β, A) is called soft set (SS) over W, where $A \subseteq E$ and $\beta : A \to P(W)$.

Definition 2.3 [21] Let E and W be the set of parameters and initial universal set respectively. Suppose that the set of all neutrosophic soft set (NSS) is denoted as $\hat{N}S(W)$. Then for $\mathrm{P} \subseteq E$, a pair (β, P) is called a $\hat{N}SS$ over W, where $\beta : \mathrm{P} \to \hat{N}S(W)$ is a mapping.

Definition 2.4 [24] Let E and W be the set of parameters and initial universal set respectively. Suppose that the set of all NSS is denoted as $\hat{N}S(W)$. A NSS $\hat{N}$ over W is a set which defined by a set valued function $\mathbf{P}_{\hat{N}}$ representing a mapping $\mathbf{P}_{\hat{N}} : E \to \hat{N}S(W)$. $\mathbf{P}_{\hat{N}}$ is known as approximate function of the $\hat{N}S(W)$. The neutrosophic soft set can be written as:

$$N=\{(e,\{\left\langle \upsilon, T_{\mathrm{P}_{\hat{N}}(e)}(\upsilon), I_{\mathrm{P}_{\hat{N}}(e)}(\upsilon), F_{\mathrm{P}_{\hat{N}}(e)}(\upsilon)\right\rangle : \upsilon\in W\}): e\in E\}$$

where $T_{N'}(\nu), I_{N'}(\nu), F_{N'}(\nu)$ represents the TM, IM and FM functions of $\mathrm{P}_{\hat{N}}(e)$ respectively and has values in $[0,1]$. Also

$$0\le T_{\mathrm{P}_{\hat{N}}(e)}(\nu), I_{\mathrm{P}_{\hat{N}}(e)}(\nu), F_{\mathrm{P}_{\hat{N}}(e)}(\nu)\le 3.$$

Definition 2.5 [22] Let U' be any universal set. The fuzzy neutrosophic set (fn-s) N' is defined as

$$N'=\{\langle \alpha, T_{N'}(\alpha), I_{N'}(\alpha), F_{N'}(\alpha)\rangle, \alpha\in X\}$$

where $T_{N'}(\alpha), I_{N'}(\alpha), F_{N'}(\alpha)$ represents the TM, IM and FM functions respectively and $T, I, F\ :\ N'\to[0,1]$. Also $0\le T_{N'}(\alpha)+I_{N'}(\alpha)+F_{N'}(\alpha)\le 3.$

Definition 2.6 [22] Let E and W be the set of parameters and initial universal set respectively. Suppose that the set of all fuzzy neutrosophic soft set (FNS-set) is denoted as $F\hat{N}S(U_E')$. Then for $\mathrm{P}\subseteq E$, a pair (β,P) is said to be a FNS-set over W, where $\beta\ :\ \mathrm{P}\to \hat{N}S(W)$ is a mapping.

Definition 2.7 [25] Let $\Lambda_{A'},\Lambda_{B'}$ be two fuzzy neutrosophic soft set. An fuzzy neutrosophic soft (FNS) relation ξ from $\Lambda_{A'}$ to $\Lambda_{B'}$ is known as FNS mapping if the two conditions are fulfilled.

(i) For every $\Lambda^{\alpha}_{A_1'}\in\Lambda_{A'}$, there exists $\Lambda^{\alpha}_{B_1'}\in\Lambda_{B'}$, where $\Omega^{\alpha}_{A_1'},\Omega^{\alpha}_{B_1'}$ are FNS elements.

(ii) For empty fuzzy FNS element in $\Lambda_{A'}$, the $\xi(\Lambda_{A'})$ is also empty FNS element.

Definition 2.8 [25] Let $\Lambda_{A'}\in FNS(W,R)$ be a FNS-set and $\phi\ :\ \Lambda_{A'}\to\Lambda_{A'}$ an FNS-mapping. A fuzzy neutrosophic element $\Lambda^{\alpha}_{A'}$ is called a fixed point of ϕ if $\phi(\Lambda^{\alpha}_{A'})=\Lambda^{\alpha}_{A'}$.

Criterion [26], [27] Let $\hat{N}S(W)$ be the set of all neutrosophic points over (W,E). Then the neutrosophic soft metric on based of neutrosophic points is defined as $d\ :\ \hat{N}S(W_E)\to\hat{N}S(W_E)$ having the following properties.

M_1). $d(\Lambda^{\alpha}_{A'},\Lambda^{\alpha}_{B'})\ge 0$ for all $\Lambda^{\alpha}_{A'},\Lambda^{\alpha}_{B'}\in\hat{N}S(W_E)$.

M_2). $d(\Lambda^{\alpha}_{A'},\Lambda^{\alpha}_{B'})=0\ \Leftrightarrow\ \Lambda^{\alpha}_{A'}=\Lambda^{\alpha}_{B'}$.

M_3). $d(\Lambda^{\alpha}_{A'},\Lambda^{\alpha}_{B'})=d(\Lambda^{\alpha}_{B'},\Lambda^{\alpha}_{A'})$.

M_4). $d(\Lambda^{\alpha}_{A'},\Lambda^{\alpha}_{B'})\le d(\Lambda^{\alpha}_{A'},\Lambda^{\alpha}_{C'})+d(\Lambda^{\alpha}_{C'},\Lambda^{\alpha}_{B'})$.

Then $(\hat{N}S(U_E'),d)$ is said to be neutrosophic soft metric space. Here $\Lambda^{\alpha}_{A'}=\Lambda^{\alpha}_{B'}$ implies $T_{\Lambda^{\alpha}_{A'}}=T_{\Lambda^{\alpha}_{B'}}, I_{\Lambda^{\alpha}_{A'}}=I_{\Lambda^{\alpha}_{B'}}$ and $F_{\Lambda^{\alpha}_{A'}}=F_{\Lambda^{\alpha}_{B'}}$.

3. Mappings on Neutrosophic Soft Set

Here, we introduced some new neutrosophic soft mappings such as contraction mapping, increasing mapping, dominated mapping, dominating mapping, K-lipschitz mapping, non-expansive mapping, commuting mapping, weakly compatible mapping. Also we introduced periodic point, common fixed point, coinciding point of neutrosophic soft-mapping. Here $\hat{N}S(U_E')$ is the collection of all neutrosophic soft points.

Definition 3.1 Let ϕ be a mapping from $\widehat{N}S(U_E^{'})$ to $\widehat{N}S(U_E^{'})$. Then ϕ is called neutrosophic soft contraction if $d(\phi(\Lambda_{A'}^{\alpha}),\phi(\Lambda_{B'}^{\alpha}))\leq kd(\Lambda_{A'}^{\alpha},\Lambda_{B'}^{\alpha})$ for all $\Lambda_{A'}^{\alpha},\Lambda_{B'}^{\alpha}\in F\widehat{N}S(U_E^{'})$ and $k\in[0,1)$. Where k is called contraction factor.

Example 3.1 Let $U'=\{\theta_1,\theta_1,\theta_3\}$ be any initial universal set and $R=A'=B'=\{\alpha_1,\alpha_2\}$. Define a NSS $\Lambda_{A'}^{\alpha}$ and $\Lambda_{B'}^{\alpha}$ as below:

$$\Lambda_{A'}^{\alpha}=\{(\alpha_1,\{\langle\theta_1,0.8,0.1,0.3\rangle,\langle\theta_2,0.6,0.7,0.4\rangle,\langle\theta_3,1,0.2,0.4\rangle\}),$$
$$(\alpha_2,\{\langle\theta_1,0.3,0.7,0.6\rangle,\langle\theta_2,0.1,0.9,0.3\rangle,\langle\theta_3,0.1,0.8,0.7\rangle\})\}$$

and

$$\Lambda_{B'}^{\alpha}=\{(\alpha_1,\{\langle\theta_1,0.9,0.7,0.1\rangle,\langle\theta_2,1,0.8,0.6\rangle,\langle\theta_3,1,0.2,0.4\rangle\}\}),$$
$$(\alpha_2,\{\langle\theta_1,0.1,0.3,0.6\rangle,\langle\theta_2,0.2,0.3,0.9\rangle,\langle\theta_3,0.1,0.8,0.7\rangle\})\}.$$

The distance defined [27] as

$$d(\phi(\Lambda_{A'}^{\alpha_1}),\phi(\Lambda_{A'}^{\alpha_2}))=\min_{\theta_i}\{(|T_{\Lambda_{B'}^{\alpha_1}}(\theta_i)-T_{\Lambda_{B'}^{\alpha_2}}(\theta_i)|^p+|I_{\Lambda_{B'}^{\alpha_1}}(\theta_i)-I_{\Lambda_{B'}^{\alpha_2}}(\theta_i)|^p+|T_{\Lambda_{B'}^{\alpha_1}}(\theta_i)-T_{\Lambda_{B'}^{\alpha_2}}(\theta_i)|^p)^{\frac{1}{p}}\}$$

$(p\geq 1)$.

In this example, we take $p=1$, now

$$\begin{aligned}
d(\phi(\Lambda_{A_1^{'}}^{\alpha_1}),\phi((\Lambda_{A_1^{'}}^{\alpha_2}))&=\min_{\theta_i}\{|T_{\Lambda_{B'}^{\alpha_1}}(\theta_i)-T_{\Lambda_{B'}^{\alpha_2}}(\theta_i)|+|I_{\Lambda_{B'}^{\alpha_1}}(\theta_i)-I_{\Lambda_{B'}^{\alpha_2}}(\theta_i)|\\
&+|F_{\Lambda_{B'}^{\alpha_1}}(\theta_i)-F_{\Lambda_{B'}^{\alpha_2}}(\theta_i)|\}\\
&=|T_{\Lambda_{B'}^{\alpha_1}}(\theta_2)-T_{\Lambda_{B'}^{\alpha_2}}(\theta_2)|+|I_{\Lambda_{B'}^{\alpha_1}}(\theta_2)-I_{\Lambda_{B'}^{\alpha_2}}(\theta_2)|\\
&+|F_{\Lambda_{B'}^{\alpha_1}}(\theta_2)-F_{\Lambda_{B'}^{\alpha_2}}(\theta_2)|\\
&=|1-0.2|+|0.8-0.3|+|0.6-0.9|\\
&=0.8+0.5+0.3\\
&=0.16\\
&=(0.2)(0.8)\\
&=0.2d(\Lambda_{A_1^{'}}^{\alpha_1},\Lambda_{A_1^{'}}^{\alpha_2}).
\end{aligned}$$

Here $k=0.2$, so ϕ is a contraction.

Definition 3.2 Let ϕ be a mapping from $\widehat{N}S(W_E)$ to $F\widehat{N}S(W_E)$. Then ϕ is called neutrosophic soft non-expansive mapping if $d(\phi(\Lambda_{A'}^{\alpha}),\phi(\Lambda_{B'}^{\alpha}))\leq kd(\Lambda_{A'}^{\alpha},\Lambda_{B'}^{\alpha})$ for all $\Lambda_{A'}^{\alpha},\Lambda_{B'}^{\alpha}\in\widehat{N}S(W_E)$ and $k=1$.

Example 3.2 Let $W=\{\upsilon_1,\upsilon_2,\upsilon_3\}$ and $R=A'=B'=\{\alpha_1,\alpha_2\}$. Define a neutrosophic soft sets $\Lambda_{A'}^{\alpha}$ and $\Lambda_{B'}^{\alpha}$ as follows:

$$\Lambda_{A'}^{\alpha}=\{(\alpha_1,\{\langle\upsilon_1,1,0.1,0.2\rangle,\langle\upsilon_2,0.6,0.7,0.4\rangle,\langle\upsilon_3,0.2,0.4,0.6\rangle\}),$$
$$(\alpha_2,\{\langle\upsilon_1,0.3,0.7,0.6\rangle,\langle\upsilon_2,0.1,0.9,0.3\rangle,\langle\upsilon_3,0.4,0.6,0.7\rangle\})\}$$

and

$$\Lambda_{B'}^{\alpha} = \{(\alpha_1, \{\langle \upsilon_1, 1, 0.5, 0.2\rangle, \langle \upsilon_2, 1, 0.5, 0.6\rangle, \langle \upsilon_3, 0.2, 0.4, 0.6\rangle\}\}),$$
$$(\alpha_2, \{\langle \upsilon_1, 0.1, 0.3, 0.6\rangle, \langle \upsilon_2, 0.2, 0.3, 0.9\rangle, \langle \upsilon_3, 0.4, 0.6, 0.7\rangle\})\}.$$

$$\begin{aligned} d(\phi(\Lambda_{A_1'}^{\alpha_1}), \phi((\Lambda_{A_1'}^{\alpha_2})) &= \min_{x_i}\{| T_{\Lambda_{B'}^{\alpha_1}}(\upsilon_i) - T_{\Lambda_{B'}^{\alpha_2}}(\upsilon_i)| + | I_{\Lambda_{B'}^{\alpha_1}}(\upsilon_i) - I_{\Lambda_{B'}^{\alpha_2}}(\upsilon_i)| \\ &\quad + | F_{\Lambda_{B'}^{\alpha_1}}(\upsilon_i) - F_{\Lambda_{B'}^{\alpha_2}}(\upsilon_i)|\} \\ &= | T_{\Lambda_{B'}^{\alpha_1}}(\upsilon_3) - T_{\Lambda_{B'}^{\alpha_2}}(\upsilon_3)| + | I_{\Lambda_{A'}^{\alpha_1}}(\upsilon_3) - I_{\Lambda_{B'}^{\alpha_2}}(\upsilon_3)| \\ &\quad + | F_{\Lambda_{A'}^{\alpha_1}}(\upsilon_3) - F_{\Lambda_{B'}^{\alpha_2}}(\upsilon_3)| \\ &= | 0.2 - 0.4| + | 0.4 - 0.6| + | 0.6 - 0.7| \\ &= 0.2 + 0.2 + 0.1 \\ &= 0.5 \\ &= (1)(0.5) \\ &= 1d(\Lambda_{A_1'}^{\alpha_1}, \Lambda_{A_1'}^{\alpha_2}). \end{aligned}$$

Here $k = 1,$ so ϕ is non-expansive.

Definition 3.3 Let ϕ be a mapping from $\breve{N}S(W_E)$ to $\breve{N}S(W_E)$. Then ϕ is called neutrosophic soft k-Lipschitz mapping if $d(\phi(\Lambda_{A'}^{\alpha}), \phi(\Lambda_{B'}^{\alpha})) \leq kd(\Lambda_{A'}^{\alpha}, \Lambda_{B'}^{\alpha})$ for all $\Lambda_{A'}^{\alpha}, \Lambda_{B'}^{\alpha} \in F\breve{N}S(W_E)$ and $k > 0.$

Example 3.3 Let $W = \{\upsilon_1, \upsilon_2, \upsilon_3\}$ and $R = A' = B' = \{\alpha_1, \alpha_2\}$. Define a NSS $\Lambda_{A'}^{\alpha}$ and $\Lambda_{B'}^{\alpha}$ as below:

$$\Lambda_{A'}^{\alpha} = \{(\alpha_1, \{\langle \upsilon_1, 0.3, 0.4, 0.3\rangle, \langle \upsilon_2, 0.6, 0.7, 0.4\rangle, \langle \upsilon_3, 0.2, 0.4, 0.6\rangle\}),$$
$$(\alpha_2, \{\langle \upsilon_1, 0.5, 0.6, 0.4\rangle, \langle \upsilon_2, 0.1, 0.9, 0.3\rangle, \langle \upsilon_3, 0.4, 0.6, 0.7\rangle\})\}$$

and

$$\Lambda_{B'}^{\alpha} = \{(\alpha_1, \{\langle \upsilon_1, 1, 0.4, 0.3\rangle, \langle \upsilon_2, 1, 0.6, 0.3\rangle, \langle \upsilon_3, 0.2, 0.4, 0.6\rangle\}\}),$$
$$(\alpha_2, \{\langle \upsilon_1, 0.5, 0.7, 0.5\rangle, \langle \upsilon_2, 0.3, 0.2, 0.9\rangle, \langle \upsilon_3, 1, 0.3, 0.9\rangle\})\}.$$

$$d(\phi(\Lambda^{\alpha_1}_{A_1'}),\phi((\Lambda^{\alpha_2}_{A_1'})) = \min_{?_i}\{|T_{\Lambda^{\alpha_1}_{B'}}(\upsilon_i)-T_{\Lambda^{\alpha_2}_{B'}}(\upsilon_i)|+|I_{\Lambda^{\alpha_1}_{B'}}(\upsilon_i)-I_{\Lambda^{\alpha_2}_{B'}}(\upsilon_i)|$$
$$+|F_{\Lambda^{\alpha_1}_{B'}}(\upsilon_i)-F_{\Lambda^{\alpha_2}_{B'}}(\upsilon_i)|\}$$
$$=|T_{\Lambda^{\alpha_1}_{B'}}(\upsilon_1)-T_{\Lambda^{\alpha_2}_{B'}}(\upsilon_1)|+|I_{\Lambda^{\alpha_1}_{A'}}(\upsilon_1)-I_{\Lambda^{\alpha_2}_{B'}}(\upsilon_1)|$$
$$+|F_{\Lambda^{\alpha_1}_{A'}}(\upsilon_1)-F_{\Lambda^{\alpha_2}_{B'}}(\upsilon_1)|$$
$$=|1-0.5|+|0.4-0.7|+|0.3-0.5|$$
$$=0.5+0.3+0.2$$
$$=1$$
$$=(2)(0.5)$$
$$=2d(\Lambda^{\alpha_1}_{A_1'},\Lambda^{\alpha_2}_{A_1'}).$$

Here $k=2,$ so ϕ is k-lipschitz.

Note: Every neutrosophic soft contraction mapping is neutrosophic soft K-lipschitz mapping but its converse does not hold.

Definition 3.4 Let ϕ be a mapping from $\hat{N}S(W_E)$ to $\hat{N}S(W_E)$. Then ϕ is said to be neutrosophic soft kanan contraction if $d(\phi(\Lambda^{\alpha}_{A'}),\phi(\Lambda^{\alpha}_{B'}))\le k[d(\Lambda^{\alpha}_{A'},\phi(\Lambda^{\alpha}_{A'}))+d(\Lambda^{\alpha}_{B'},\phi(\Lambda^{\alpha}_{B'}))]$ for all $\Lambda^{\alpha}_{A'},\Lambda^{\alpha}_{B'}\in\hat{N}S(W_E)$ and $k\in[0,\frac{1}{2})$. Where k is called contraction factor.

Definition 3.5 Let ϕ and ψ be two mappings from $\hat{N}S(U'_E)$ to $\hat{N}S(U'_E)$. Then ϕ and ψ are called neutrosophic soft commuting mapping if $\phi(\psi(\Omega^{\alpha}_{A'}))=\psi(\phi(\Omega^{\alpha}_{A'}))$ for all $\Omega^{\alpha}_{A'}\in\hat{N}S(U'_E)$.

Definition 3.6 Let ϕ and ψ be two mappings from $\hat{N}S(U'_E)$ to $\hat{N}S(U'_E)$. Then ϕ and ψ are called neutrosophic soft weakly commuting mapping if $d(\phi(\psi(\Lambda^{\alpha}_{A'})),\psi(\phi(\Lambda^{\alpha}_{A'})))\le d(\phi(\Lambda^{\alpha}_{A'}),\psi(\Lambda^{\alpha}_{A'}))$ for all $\Lambda^{\alpha}_{A'}\in\hat{N}S(U'_E)$.

Definition 3.7 Let ϕ and ψ be two mappings from $\hat{N}S(U'_E)$ to $\hat{N}S(U'_E)$. If for $\phi(\Omega^{\alpha}_{A_n'})\to\Omega^{\alpha}_{A_0'}$ and $\psi(\Omega^{\alpha}_{A_n'})\to\Omega^{\alpha}_{A_0'}$ as $n\to\infty$ and $\Omega^{\alpha}_{A_n'},\Omega^{\alpha}_{A_0'}\in\hat{N}S(U'_E)$. Then it is called neutrosophic soft compatible mapping if $\lim_{n\to\infty}d(\phi(\psi(\Omega^{\alpha}_{A'})),\psi(\phi(\Omega^{\alpha}_{A'})))\to 0.$

Definition 3.8 Let ϕ,ψ : $\hat{N}S(U'_E)\to\hat{N}S(U'_E)$ be two mappings. If there is $\Omega^{\alpha}_{A'}\in\hat{N}S(U'_E)$ such that $\phi(\Omega^{\alpha}_{A'})=\psi(\Omega^{\alpha}_{A'})=\Omega^{\alpha}_{A'}$, then $\Omega^{\alpha}_{A'}\in\hat{N}S(U'_E)$ is called common fixed point neutrosophic soft mappings.

Definition 3.9 If $\Omega^{\alpha}_{A'}$ is a fixed point of ϕ : $\hat{N}S(U'_E)\to\hat{N}S(U'_E)$, then $\Omega^{\alpha}_{A'}$ is also a fixed point ϕ^k that is $\phi^k(\Omega^{\alpha}_{A'})=\Omega^{\alpha}_{A'}$ for all $\Omega^{\alpha}_{A'}\in\hat{N}S(U'_E)$. So $\Omega^{\alpha}_{A'}$ is called periodic point of neutrosophic soft mapping ϕ and k is called period of ϕ.

Remark Every fixed point of neutrosophic soft mapping is a periodic point but every periodic point of neutrosophic soft mapping is not a fixed point.

Definition 3.9 Let ϕ, ψ be two mappings from $\hat{N}S(U'_E)$ to $\hat{N}S(U'_E)$. If $\phi(\Omega^{\alpha}_{A'})=\psi(\Omega^{\alpha}_{A'})=\Omega^{\alpha}_{B'}$, for all

$\Omega^{\alpha}_{A'}, \Omega^{\alpha}_{B'} \in F\hat{N}S(U'_E)$. Then $\Omega^{\alpha}_{A'}$ is called coincidence point of ϕ and ψ and $\Omega^{\alpha}_{B'}$ is called point of coincidence for ϕ and ψ.

Definition 3.10 Let $\phi : \hat{N}S(U'_E) \to \hat{N}S(U'_E)$ be a mapping. Then ϕ is said to be neutrosophic soft increasing map if for any $\Omega^{\alpha}_{A'} \le \Omega^{\alpha}_{B'}$ implies $\phi(\Omega^{\alpha}_{A'}) \le \phi(\Omega^{\alpha}_{B'})$ for all $\Omega^{\alpha}_{A'}, \Omega^{\alpha}_{B'} \in \hat{N}S(U'_E)$.

Definition 3.11 Let $\phi : \hat{N}S(U'_E) \to \hat{N}S(U'_E)$ be a mapping. Then ϕ is said to be neutrosophic soft dominated map if $\phi(\Omega^{\alpha}_{A'}) \le \Omega^{\alpha}_{A'}$ for all $\Omega^{\alpha}_{A'} \in \hat{N}S(U'_E)$.

Definition 3.12 Let $\phi : \hat{N}S(U'_E) \to \hat{N}S(U'_E)$ be a mapping. Then ϕ is said to be neutrosophic soft dominating map if $\Omega^{\alpha}_{A'} \le \phi(\Omega^{\alpha}_{A'})$ for all $\Omega^{\alpha}_{A'} \in \hat{N}S(U'_E)$.

4. Main Results

Banach Contraction Theorem

Proposition 1 Let $\hat{N}S(U'_E)$ be a non-empty set of neutrosophic points and $(\hat{N}S(U'_E), d)$ be a complete neutrosophic soft metric space. Suppose ϕ is a mapping from $\hat{N}S(U'_E)$ to $\hat{N}S(U'_E)$ be contraction. Then fixed point of ϕ exists and unique.

Proof Let $\Omega^{\alpha}_{A'_0} \in \hat{N}S(U'_E)$ be arbitrary. Define $\Omega^{\alpha}_{A'_1} = \phi(\Omega^{\alpha}_{A'_0})$ and by continuing we have a sequence in the form $\Omega^{\alpha}_{A'_{n+1}} = \phi(\Omega^{\alpha}_{A'_n})$. Now

$$\begin{aligned}
d(\Omega^{\alpha}_{A'_{n+1}}, \Omega^{\alpha}_{A'_n}) &= d(\phi(\Omega^{\alpha}_{A'_n}), \phi(\Omega^{\alpha}_{A'_{n-1}})) \\
&\le kd(\Omega^{\alpha}_{A'_n}, \Omega^{\alpha}_{A'_{n-1}}) \\
&= kd(\phi(\Omega^{\alpha}_{A'_{n-1}}), \phi(\Omega^{\alpha}_{A'_{n-2}})) \\
&\le k^2 d(\Omega^{\alpha}_{A'_{n-1}}, \Omega^{\alpha}_{A'_{n-2}}) \\
&= k^2 d(\phi(\Omega^{\alpha}_{A'_{n-2}}), \phi(\Omega^{\alpha}_{A'_{n-3}})) \\
&\le k^3 d(\Omega^{\alpha}_{A'_{n-2}}, \Omega^{\alpha}_{A'_{n-3}}) \\
&\quad . \\
&\quad . \\
&\quad . \\
&\le k^n d(\Omega^{\alpha}_{A'_1}, \Omega^{\alpha}_{A'_0}).
\end{aligned}$$

Now for $m, n > n_0$, we have

$$
\begin{aligned}
d(\Omega^{\alpha}_{A'_{n+1}},\Omega^{\alpha}_{A'_n}) &\le d(\Omega^{\alpha}_{A'_n},\Omega^{\alpha}_{A'_{n+1}}) + d(\Omega^{\alpha}_{A'_{n+1}},\Omega^{\alpha}_{A'_{n+2}}) + \dots + d(\Omega^{\alpha}_{A'_{m-1}},\Omega^{\alpha}_{A'_m}) \\
&\le k^n d(\Omega^{\alpha}_{A'_1},\Omega^{\alpha}_{A'_0}) + k^{n+1} d(\Omega^{\alpha}_{A'_1},\Omega^{\alpha}_{A'_0}) + \dots + k^{m-1} d(\Omega^{\alpha}_{A'_1},\Omega^{\alpha}_{A'_0}) \\
&= k^n[1 + k + k^2 + \dots + k^{m-n-1}] d(\Omega^{\alpha}_{A'_1},\Omega^{\alpha}_{A'_0}) \\
&= \frac{k^n}{1-k} d(\Omega^{\alpha}_{A'_1},\Omega^{\alpha}_{A'_0}) \\
d(\Omega^{\alpha}_{A'_{n+1}},\Omega^{\alpha}_{A'_n}) &\to 0 \text{ as } n \to \infty.
\end{aligned}
$$

So $\Omega^{\alpha}_{A'_n}$ is a cauchy sequence in $(\hat{N}S(U'_E),d)$, but $(\hat{N}S(U'_E),d)$ is complete, so there exists $\Omega^{\alpha *}_{A'} \in \hat{N}S(U'_E)$ such that $d(\Omega^{\alpha}_{A'_n},\Omega^{\alpha *}_{A'}) \to 0$ as $n \to \infty$. Now

$$
\begin{aligned}
d(\Omega^{\alpha}_{A'_{n+1}},\phi(\Omega^{\alpha *}_{A'})) &= d(\phi(\Omega^{\alpha}_{A'_n}),\phi(\Omega^{\alpha *}_{A'})) \\
&\le k d(\Omega^{\alpha}_{A'_n},\Omega^{\alpha *}_{A'}).
\end{aligned}
$$

On taking limit as $n \to \infty$, we get

$$d(\phi(\Omega^{\alpha *}_{A'}),\Omega^{\alpha *}_{A'}) \le 0.$$

But

$$d(\phi(\Omega^{\alpha *}_{A'}),\Omega^{\alpha *}_{A'}) \ge 0.$$

So

$$
\begin{aligned}
d(\phi(\Omega^{\alpha *}_{A'}),\Omega^{\alpha *}_{A'}) &= 0 \\
\phi(\Omega^{\alpha *}_{A'}) &= \Omega^{\alpha *}_{A'}.
\end{aligned}
$$

So $\Omega^{\alpha *}_{A'}$ is the FP of ϕ

Now we have to show that $\Omega^{\alpha *}_{A'}$ is unique. Suppose there exists another FP $\Omega^{\alpha *}_{B'} \in \hat{N}S(U'_E)$ such that $\phi(\Omega^{\alpha *}_{B'}) = \Omega^{\alpha *}_{B'}$. Now

$$
\begin{aligned}
d(\Omega^{\alpha *}_{A'},\Omega^{\alpha *}_{B'}) &= d(\phi(\Omega^{\alpha *}_{A'}),\phi(\Omega^{\alpha *}_{B'})) \\
&\le k d(\Omega^{\alpha *}_{A'},\Omega^{\alpha *}_{B'}) \\
(1-k) d(\Omega^{\alpha *}_{A'},\Omega^{\alpha *}_{B'}) &\le 0.
\end{aligned}
$$

Here $(1-k) \not\le 0$, so

$$d(\Omega^{\alpha ?}_{A'},\Omega^{\alpha ?}_{B'}) \le 0.$$

But

$$
\begin{aligned}
d(\Omega^{\alpha *}_{A'},\Omega^{\alpha *}_{B'}) &\ge 0 \\
d(\Omega^{\alpha *}_{A'},\Omega^{\alpha *}_{B'}) &= 0.
\end{aligned}
$$

Hence $\Omega^{\alpha *}_{A'} = \Omega^{\alpha *}_{B'}$, so the fixed point is unique.

Proposition 2 Let $(\hat{N}S(U'_E),d)$ be a complete neutrosophic soft metric space. Suppose ϕ be a mapping from $F\hat{N}S(U'_E)$ to $F\hat{N}S(U'_E)$ satisfies the contraction $d(\phi^m(\Omega^{\alpha}_{A'_1}),\phi^m(\Omega^{\alpha}_{B'_1})) \le k d(\Omega^{\alpha}_{A'_1},\Omega^{\alpha}_{B'_1})$ for all $\Omega^{\alpha}_{A'_1},\Omega^{\alpha}_{B'_1} \in \hat{N}S(U'_E)$, where $k \in [0,1)$ and m is any natural number. Then ϕ has a FP.

Proof It follows from banach contraction theorem that ϕ^m has unique a FP that is $\phi^m(\Omega^\alpha_{A_1'}) = \Omega^\alpha_{A_1'}$. Now

$$\begin{aligned}\phi^m(\phi(\Omega^\alpha_{A_1'})) &= \phi^{m+1}(\Omega^\alpha_{A_1'}) \\ &= \phi(\phi^m(\Omega^\alpha_{A_1'})) \\ &= \phi(\Omega^\alpha_{A_1'}).\end{aligned}$$

By the uniqueness of FP, we have $\phi(\Omega^\alpha_{A_1'}) = \Omega^\alpha_{A_1'}$.

Proposition 3 Let $(\widehat{N}S(U_E'), d)$ be a complete neutrosophic soft metric space. Suppose ϕ, ψ satisfy $d(\phi(\Omega^\alpha_{A_1'}), \psi(\Omega^\alpha_{B_1'})) \le \alpha d(\Omega^\alpha_{A_1'}, \phi(\Omega^\alpha_{A_1'})) + \beta d(\Omega^\alpha_{B_1'}, \psi(\Omega^\alpha_{B_1'})) + \gamma[d(\Omega^\alpha_{A_1'}, \psi(\Omega^\alpha_{B_1'})) + d(\Omega^\alpha_{B_1'}, \phi(\Omega^\alpha_{A_1'})]$ for all $\Omega^\alpha_{A_1'}, \Omega^\alpha_{B_1'} \in F\widehat{N}S(U_E')$ with α, β, γ are non-negative and $\alpha + \beta + \gamma < 1$. Then ϕ and ψ have a unique FP.

Proof Let $\Omega^\alpha_{A_1'} \in \widehat{N}S(U_E')$ be a fixed point of ϕ that is $\phi(\Omega^\alpha_{A_1'}) = \Omega^\alpha_{A_1'}$. We need to show that $\psi(\Omega^\alpha_{A_1'}) = \Omega^\alpha_{A_1'}$. Now

$$\begin{aligned}d(\Omega^\alpha_{A_1'}, \psi(\Omega^\alpha_{A_1'})) &= d(\phi(\Omega^\alpha_{A_1'}), \psi(\Omega^\alpha_{A_1'})) \\ &\le \alpha d(\Omega^\alpha_{A_1'}, \phi(\Omega^\alpha_{A_1'})) + \beta d(\Omega^\alpha_{A_1'}, \psi(\Omega^\alpha_{A_1'})) + \gamma[d(\Omega^\alpha_{A_1'}, \psi(\Omega^\alpha_{A_1'})) + d(\Omega^\alpha_{A_1'}, \phi(\Omega^\alpha_{A_1'}))] \\ &= \alpha d(\Omega^\alpha_{A_1'}, \Omega^\alpha_{A_1'}) + \beta d(\Omega^\alpha_{A_1'}, \psi(\Omega^\alpha_{A_1'})) + \gamma[d(\Omega^\alpha_{A_1'}, \psi(\Omega^\alpha_{A_1'})) + d(\Omega^\alpha_{A_1'}, \Omega^\alpha_{A_1'})] \\ &= \beta d(\Omega^\alpha_{A_1'}, \psi(\Omega^\alpha_{A_1'}) + \gamma d(\Omega^\alpha_{A_1'}, \psi(\Omega^\alpha_{A_1'}))\end{aligned}$$

$$(1 - \beta - \gamma) d(\Omega^\alpha_{A_1'}, \psi(\Omega^\alpha_{A_1'})) \le 0$$

Since $(1 - \beta - \gamma) \not\le 0$, so

$$d(\Omega^\alpha_{A_1'}, \psi(\Omega^\alpha_{A_1'})) \le 0.$$

But

$$d(\Omega^\alpha_{A_1'}, \psi(\Omega^\alpha_{A_1'})) \ge 0$$

hence

$$d(\Omega^\alpha_{A_1'}, \psi(\Omega^\alpha_{A_1'})) = 0.$$

Thus $\psi(\Omega^\alpha_{A_1'}) = \Omega^\alpha_{A_1'}$.

Proposition 4 Let $\widehat{N}S(U_E')$ be a non-empty set of neutrosophic points and $(\widehat{N}S(U_E'), d)$ be a complete neutrosophic soft metric space. Suppose ϕ is a mapping from $\widehat{N}S(U_E')$ to $\widehat{N}S(U_E')$ be kanan contraction. Then fixed point of ϕ exists and unique.

Proof Let $\Omega^\alpha_{A_0'} \in \widehat{N}S(U_E')$ be arbitrary. Define $\Omega^\alpha_{A_1'} = \phi(\Omega^\alpha_{A_0'})$ and by continuing we have a sequence in the form $\Omega^\alpha_{A_{n+1}'} = \phi(\Omega^\alpha_{A_n'})$. Now

$$\begin{aligned}
d(\Omega^{\alpha}_{A'_{n+1}},\Omega^{\alpha}_{A'_n}) &= d(\phi(\Omega^{\alpha}_{A'_n}),\phi(\Omega^{\alpha}_{A'_{n-1}}))\\
&\leq k[d(\Omega^{\alpha}_{A'_n},\phi(\Omega^{\alpha}_{A'_n}))+d(\Omega^{\alpha}_{A'_{n-1}},\phi(\Omega^{\alpha}_{A'_{n-1}}))\\
&= k[d(\Omega^{\alpha}_{A'_n},\Omega^{\alpha}_{A'_{n+1}})+d(\Omega^{\alpha}_{A'_{n-1}},\Omega^{\alpha}_{A'_n})]\\
&= kd(\Omega^{\alpha}_{A'_n},\Omega^{\alpha}_{A'_{n+1}})+kd(\Omega^{\alpha}_{A'_{n-1}},\Omega^{\alpha}_{A'_n})\\
(1-k)d(\Omega^{\alpha}_{A'_{n+1}},\Omega^{\alpha}_{A'_n}) &\leq kd(\Omega^{\alpha}_{A'_n},\Omega^{\alpha}_{A'_{n-1}})\\
d(\Omega^{\alpha}_{A'_{n+1}},\Omega^{\alpha}_{A'_n}) &\leq \frac{k}{1-k}d(\Omega^{\alpha}_{A'_n},\Omega^{\alpha}_{A'_{n-1}})\\
&= hd(\Omega^{\alpha}_{A'_n},\Omega^{\alpha}_{A'_{n-1}})
\end{aligned}$$

for $h=\frac{k}{1-k}$

$$\begin{aligned}
d(\Omega^{\alpha}_{A'_{n+1}},\Omega^{\alpha}_{A'_n}) &\leq hd(\Omega^{\alpha}_{A'_n},\Omega^{\alpha}_{A'_{n-1}})\\
&\leq h^2d(\Omega^{\alpha}_{A'_{n-1}},\Omega^{\alpha}_{A'_{n-2}})\\
&\leq h^3d(\Omega^{\alpha}_{A'_{n-2}},\Omega^{\alpha}_{A'_{n-3}})\\
&\quad .\\
&\quad .\\
&\quad .\\
&\leq h^nd(\Omega^{\alpha}_{A'_1},\Omega^{\alpha}_{A'_0}).
\end{aligned}$$

For $m>n$

$$\begin{aligned}
d(\Omega^{\alpha}_{A'_n},\Omega^{\alpha}_{A'_m}) &\leq d(\Omega^{\alpha}_{A'_n},\Omega^{\alpha}_{A'_{n+1}})+d(\Omega^{\alpha}_{A'_{n+1}},\Omega^{\alpha}_{A'_{n+2}})+\ldots+d(\Omega^{\alpha}_{A'_{m-1}},\Omega^{\alpha}_{A'_m})\\
&\leq h^nd(\Omega^{\alpha}_{A'_1},\Omega^{\alpha}_{A'_0})+h^{n+1}d(\Omega^{\alpha}_{A'_1},\Omega^{\alpha}_{A'_0})+\ldots+h^{m-1}d(\Omega^{\alpha}_{A'_1},\Omega^{\alpha}_{A'_0})\\
&= h^n[1+h+h^2+\ldots+h^{m-n-1}]d(\Omega^{\alpha}_{A'_1},\Omega^{\alpha}_{A'_0})\\
&= h^n(\frac{1}{1+h})d(\Omega^{\alpha}_{A'_1},\Omega^{\alpha}_{A'_0})\\
d(\Omega^{\alpha}_{A'_n},\Omega^{\alpha}_{A'_m}) &\to 0 \text{ as } n\to\infty.
\end{aligned}$$

The sequence $\Omega^{\alpha}_{A'_n}$ is a cauchy sequence in $(\hat{N}S(U'_E),d)$. Since $(\hat{N}S(U'_E),d)$ is complete, so $\Omega^{\alpha}_{A'_n}$ converges to any $\Omega^{\alpha*}_{A'}\in\hat{N}S(U'_E)$. Now

$$\begin{aligned}
d(\phi(\Omega^{\alpha*}_{A'}),\Omega^{\alpha}_{A'_{n+1}}) &= d(\phi(\Omega^{\alpha*}_{A'}),\phi(\Omega^{\alpha}_{A'_n}))\\
&\leq h[d(\Omega^{\alpha*}_{A'},\phi(\Omega^{\alpha*}_{A'}))+d(\Omega^{\alpha}_{A'_n},\phi(\Omega^{\alpha}_{A'_n}))].
\end{aligned}$$

Taking limit as $n\to\infty,$ we have

$$\begin{aligned}
d(\phi(\Omega^{\alpha*}_{A'}),\Omega^{\alpha*}_{A'}) &\leq h[d(\Omega^{\alpha*}_{A'},\phi(\Omega^{\alpha*}_{A'}))+d(\Omega^{\alpha*}_{A'},\phi(\Omega^{\alpha*}_{A'}))]\\
&= 2hd(\Omega^{\alpha*}_{A'},\phi(\Omega^{\alpha*}_{A'}))\\
(1-2h)d(\phi(\Omega^{\alpha*}_{A'}),\Omega^{\alpha*}_{A'}) &\leq 0
\end{aligned}$$

As $(1-2h)\not\leq 0,$ so

$$d(\phi(\Omega_{A'}^{\alpha*}),\Omega_{A'}^{\alpha*})\leq 0$$

but

$$d(\phi(\Omega_{A'}^{\alpha*}),\Omega_{A'}^{\alpha*})\geq 0$$

thus

$$d(\phi(\Omega_{A'}^{\alpha*}),\Omega_{A'}^{\alpha*})=0.$$

Hence $\Omega_{A'}^{\alpha*}\in \widehat{N}S(U_E^{'})$ is a FP of ϕ.

Suppose $\Omega_{B'}^{\alpha*}\in \widehat{N}S(U_E^{'})$ be another FP. Now

$$\begin{aligned} d(\Omega_{A'}^{\alpha*},\Omega_{B'}^{\alpha*}) &= d(\phi(\Omega_{A'}^{\alpha*}),\phi(\Omega_{B'}^{\alpha*})) \\ &\leq h[d(\Omega_{A'}^{\alpha*},\phi(\Omega_{A'}^{\alpha*}))+d(\Omega_{B'}^{\alpha*},\phi(\Omega_{B'}^{\alpha*}))] \\ &\leq h[d(\Omega_{A'}^{\alpha*},\Omega_{A'}^{\alpha*})+d(\Omega_{B'}^{\alpha*},\Omega_{B'}^{\alpha*})] \end{aligned}$$

$$d(\Omega_{A'}^{\alpha*},\Omega_{B'}^{\alpha*})\leq 0 \qquad (1)$$

but

$$d(\Omega_{A'}^{\alpha*},\Omega_{B'}^{\alpha*})\geq 0. \qquad (2)$$

From (1) and (2) we have

$$d(\Omega_{A'}^{\alpha*},\Omega_{B'}^{\alpha*})=0.$$

Hence $\Omega_{A'}^{\alpha*}=\Omega_{B'}^{\alpha*}$.

Proposition 5 Let $\phi,\psi\ :\ \widehat{N}S(U_E^{'})\to \widehat{N}S(U_E^{'})$ be weakly compatible maps. If ϕ and ψ have unique coincidence point. Then ϕ and ψ have unique common fixed point (CFP).

Proof Suppose there is $\Omega_{A_1^{'}}^{\alpha}\in \widehat{N}S(U_E^{'})$ such that $\phi(\Omega_{A_1^{'}}^{\alpha})=\psi(\Omega_{A_1^{'}}^{\alpha})=\Omega_{B_1^{'}}^{\alpha}$. Since ϕ and ψ are weakly compatible, so $\phi(\psi(\Omega_{A_1^{'}}^{\alpha}))=\psi(\phi(\Omega_{A_1^{'}}^{\alpha}))$ for all $\Omega_{A_1^{'}}^{\alpha}\in \widehat{N}S(U_E^{'})$. Now

$$\phi(\Omega_{B_1^{'}}^{\alpha})=\phi(\Omega_{B_1^{'}}^{\alpha})=\phi(\psi(\Omega_{A_1^{'}}^{\alpha}))=\psi(\phi(\Omega_{A_1^{'}}^{\alpha})).$$

So $\Omega_{B_1^{'}}^{\alpha}$ is also coincidence point (CP) of ϕ and ψ, but $\Omega_{A_1^{'}}^{\alpha}$ is the unique CP of ϕ and ψ, so

$$\phi(\Omega_{A_1^{'}}^{\alpha})=\psi(\Omega_{A_1^{'}}^{\alpha})=\phi(\Omega_{B_1^{'}}^{\alpha})=\psi(\Omega_{B_1^{'}}^{\alpha})$$

$$\Omega_{B_1^{'}}^{\alpha}=\phi(\Omega_{B_1^{'}}^{\alpha})=\psi(\Omega_{B_1^{'}}^{\alpha}).$$

So $\Omega_{B_1^{'}}^{\alpha}\in \widehat{N}S(U_E^{'})$ is CFP.

Proposition 6 Let $(\widehat{N}S(U_E^{'}),d)$ be a complete metric space and $\phi\ :\ \widehat{N}S(U_E^{'})\to \widehat{N}S(U_E^{'})$ be a mapping satisfies $d(\phi^2(\Omega_{A_1^{'}}^{\alpha}),\phi(\Omega_{A_1^{'}}^{\alpha}))\leq kd(\phi(\Omega_{A_1^{'}}^{\alpha}),\Omega_{A_1^{'}}^{\alpha})$ for all $\Omega_{A_1^{'}}^{\alpha}\in \widehat{N}S(U_E^{'})$ and $k\in[0,1)$. Then fixed point of ϕ is singleton.

Proof Let $\Omega_{A_0^{'}}^{\alpha}\in \widehat{N}S(U_E^{'})$ be arbitrary and defines $\Omega_{A_{n+1}^{'}}^{\alpha}=\phi^n(\ \Omega_{A_0^{'}}^{\alpha})=\phi(\ \Omega_{A_n^{'}}^{\alpha})$. Now

$$
\begin{aligned}
d(\phi^{n+1}(\Omega^{\alpha}_{A'_0}),\phi^{n}(\Omega^{\alpha}_{A'_0})) &\leq kd(\phi^{n}(\Omega^{\alpha}_{A'_0}),\phi^{n-1}(\Omega^{\alpha}_{A'_0}))\\
&\leq k^2 d(\phi^{n-1}(\Omega^{\alpha}_{A'_0}),\phi^{n-2}(\Omega^{\alpha}_{A'_0}))\\
&\leq k^3 d(\phi^{n-3}(\Omega^{\alpha}_{A'_0}),\phi^{n-4}(\Omega^{\alpha}_{A'_0}))\\
&\;.\\
&\;.\\
&\;.\\
&\leq k^n d(\phi(\Omega^{\alpha}_{A'_0}),\Omega^{\alpha}_{A'_0}).
\end{aligned}
$$

Now for $m > n$

$$
\begin{aligned}
d(\phi^{m}(\Omega^{\alpha}_{A'_0}),\phi^{n}(\Omega^{\alpha}_{A'_0})) &\leq d(\phi^{n}(\Omega^{\alpha}_{A'_0}),\phi^{n+1}(\Omega^{\alpha}_{A'_0})) + d(\phi^{n+1}(\Omega^{\alpha}_{A'_0}),\phi^{n+2}(\Omega^{\alpha}_{A'_0}))\\
&\quad +\ldots + d(\phi^{m-1}(\Omega^{\alpha}_{A'_0}),\phi^{m}(\Omega^{\alpha}_{A'_0}))\\
&\leq k^n d(\phi(\Omega^{\alpha}_{A'_0}),\Omega^{\alpha}_{A'_0}) + k^{n+1} d(\phi(\Omega^{\alpha}_{A'_0}),\Omega^{\alpha}_{A'_0})\\
&\quad +\ldots + k^{m-1} d(\phi(\Omega^{\alpha}_{A'_0}),\Omega^{\alpha}_{A'_0})\\
&\leq k^n[1+k+k^2+\ldots+k^{m-n-1}] d(\phi(\Omega^{\alpha}_{A'_0}),\Omega^{\alpha}_{A'_0})\\
&\leq \frac{k^n}{1-k} d(\phi(\Omega^{\alpha}_{A'_0}),\Omega^{\alpha}_{A'_0})
\end{aligned}
$$

$$d(\phi^{m}(\Omega^{\alpha}_{A'_0}),\phi^{n}(\Omega^{\alpha}_{A'_0})) \to 0 \text{ as } n \to \infty.$$

So $\phi^n(\Omega^{\alpha}_{A'_0})$ is a cauchy sequence in $(\hat{N}S(U'_E),d)$, but $(\hat{N}S(U'_E),d)$ is complete, so every cauchy sequence is convergent that is $\phi^n(\Omega^{\alpha}_{A'_0}) \to \Omega^{\alpha*}_{A'_0}$ as $n \to \infty$. Now

$$d(\phi^{n+1}(\Omega^{\alpha}_{A'_0}),\phi(\Omega^{\alpha*}_{A'_0})) \leq kd(\phi^{n}(\Omega^{\alpha}_{A'_0}),\Omega^{\alpha*}_{A'_0})$$

taking limit as $n \to \infty$, we have

$$
\begin{aligned}
&d(\Omega^{\alpha*}_{A'_0},\phi(\Omega^{\alpha*}_{A'_0})) \leq kd(\Omega^{\alpha*}_{A'_0},\Omega^{\alpha*}_{A'_0})\\
&d(\Omega^{\alpha*}_{A'_0},\phi(\Omega^{\alpha*}_{A'_0})) \leq 0
\end{aligned}
$$

but

$$
\begin{aligned}
&d(\Omega^{\alpha*}_{A'_0},\phi(\Omega^{\alpha*}_{A'_0})) \geq 0\\
&d(\Omega^{\alpha*}_{A'_0},\phi(\Omega^{\alpha*}_{A'_0})) = 0\\
&\qquad\qquad \Rightarrow \phi(\Omega^{\alpha*}_{A'_0}) = \Omega^{\alpha*}_{A'_0}.
\end{aligned}
$$

Hence $\Omega^{\alpha*}_{A'_0} \in \hat{N}S(U'_E)$ is the FP of ϕ.

Now suppose $\Omega^{\alpha*}_{B'_0} \in \hat{N}S(U'_E)$ is another FP of with $\phi(\Omega^{\alpha*}_{B'_0}) = \Omega^{\alpha*}_{B'_0}$, then

$$d(\Omega^{\alpha*}_{B_0'},\Omega^{\alpha*}_{A_0'}) = d(\phi(\Omega^{\alpha*}_{B_0'}),\phi(\Omega^{\alpha*}_{A_0'}))$$
$$= d(\phi(\phi(\Omega^{\alpha*}_{B_0'})),\phi(\Omega^{\alpha*}_{A_0'}))$$
$$\leq kd(\phi(\Omega^{\alpha*}_{B_0'})),\phi(\Omega^{\alpha*}_{A_0'}))$$
$$= kd(\Omega^{\alpha*}_{B_0'},\Omega^{\alpha*}_{A_0'})$$
$$(1-k)d(\Omega^{\alpha*}_{B_0'},\Omega^{\alpha*}_{A_0'}) \leq 0.$$

As $(1-k) \nleq 0$, so

$$d(\Omega^{\alpha ?}_{B_0'},\Omega^{\alpha ?}_{A_0'}) \leq 0 \qquad (1)$$

but

$$d(\Omega^{\alpha*}_{B_0'},\Omega^{\alpha*}_{A_0'}) \geq 0. \qquad (2)$$

From (1) and (2) we have

$$d(\Omega^{\alpha*}_{B_0'},\Omega^{\alpha*}_{A_0'}) = 0$$
$$\Rightarrow \Omega^{\alpha*}_{B_0'} = \Omega^{\alpha*}_{A_0'}.$$

Hence the FP is unique.

Proposition 7 Let ϕ,ψ : $\hat{N}S(U_E') \to \hat{N}S(U_E')$ be commuting maps. If ϕ and ψ have unique coincidence point. Then ϕ and ψ have unique common fixed point.

Proof Suppose there is $\Omega^{\alpha}_{A_1'} \in \hat{N}S(U_E')$ such that $\phi(\psi(\Omega^{\alpha}_{A_1'})) = \psi(\phi(\Omega^{\alpha}_{A_1'}))$. Since ϕ and ψ have unique coincidence point, so let $\phi(\Omega^{\alpha}_{A_1'}) = \psi(\Omega^{\alpha}_{A_1'}) = \Omega^{\alpha}_{B_1'}$. Now

$$\phi(\Omega^{\alpha}_{B_1'}) = \phi(\psi(\Omega^{\alpha}_{A_1'})) = \psi(\phi(\Omega^{\alpha}_{A_1'})) = \psi(\Omega^{\alpha}_{B_1'}).$$

Here $\Omega^{\alpha}_{B_1'} \in \hat{N}S(U_E')$ is also a coincidence point, but $\Omega^{\alpha}_{A_1'} \in \hat{N}S(U_E')$ is unique coincidence point, so

$$\phi(\Omega^{\alpha}_{B_1'}) = \psi(\Omega^{\alpha}_{B_1'}) = \psi(\Omega^{\alpha}_{A_1'}) = \phi(\Omega^{\alpha}_{A_1'}) = \Omega^{\alpha}_{B_1'}.$$

Hence $\Omega^{\alpha}_{B_1'} \in \hat{N}S(U_E')$ is also a fixed point.

Proposition 8 Every neutrosophic soft identity map is non-expansive.

Proof Suppose that I from $\hat{N}S(U_E')$ to $\hat{N}S(U_E')$ be a neutrosophic soft identity map such that $I(\Omega^{\alpha}_{A_1'}) = \Omega^{\alpha}_{A_1'}$ for all $\Omega^{\alpha}_{A_1'} \in \hat{N}S(U_E')$. Now

$$d(I(\Omega^{\alpha}_{A_1'}),I(\Omega^{\alpha}_{B_1'})) = d(\Omega^{\alpha}_{A_1'},\Omega^{\alpha}_{B_1'})$$

Here $k=1$, so I is non-expansive map.

5. Conclusion

In this paper, we have discussed some new mappings of NSS and some basic results and particular examples. Like fixed point, here also present some new concepts of points that is coincidence point, periodic point and CFP.

FP theory has a lot of applications in control and communicating system. FP theory is an important mathematical instrument used to demonstrate the existence of a solution in mathematical economics and game theory. So the notion of a neutrosophic soft fixed point can be used in these areas. For stabilization of dynamic systems, neutrosophic soft fixed point can be used. In addition, dynamic programming may employ the notion of presence and uniqueness of the common solution of neutrosophic soft set.

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