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On Neutrosophic Generalized Alpha Generalized Continuity


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Abstract: This article demonstrates a further class of neutrosophic closed sets named neutrosophic generalized αg-closed sets and discuss their essential characteristics in neutrosophic topological spaces. Moreover, we submit neutrosophic generalized αg-continuous functions with their elegant features.

Keywords: neutrosophic generalized αg-closed sets, neutrosophic generalized αg-continuous functions, and neutrosophic generalized αg-irresolute functions.

1. Introduction

Smarandache [1,2] originally handed the theory of “neutrosophic set” Recently, Abdel-Basset et al. discussed a novel neutrosophic approach [3-8] in several fields, for a few names, information and communication technology. Salama et al. [9] gave the clue of neutrosophic topological space (or simply NTS). Arokiarani et al. [10] added the view of neutrosophic α-open subsets of neutrosophic topological spaces. Imran et al. [11] presented the idea of neutrosophic semi-α-open sets in neutrosophic topological spaces. Dhavaseelan et al. [12] presented the idea of neutrosophic αm-continuity. Our aim is to introduce a new idea of neutrosophic generalized αg-closed sets and examine their vital merits in neutrosophic topological spaces. Additionally, we propose neutrosophic generalized αg-continuous functions by employing neutrosophic generalized αg-closed sets and emphasizing some of their primary characteristics.

2. Preliminaries

Everywhere of these following sections, we assume that NTSs (𝒰, 𝒱, 𝓩, 𝜉) and (𝒲, 𝜇) are briefly denoted as 𝒰, 𝒱, and 𝒲, respectively. Let 𝒞 be a neutrosophic set in 𝒰, and we are easily symbolized it by 𝑁𝑁, then the complement of 𝒞 is basically given by 𝒞̅. If 𝒞 is a neutrosophic open set in 𝒰 and shortly indicated by Ne-Os. Then, 𝒞̅ is termed a neutrosophic closed set in 𝒰 and simply referred by Ne-CS. The neutrosophic closure and the neutrosophic interior of 𝒞 are merely signified by Ne-cl(𝒞) and Ne-int(𝒞), correspondingly.

Definition 2.1 [10]: A NS 𝒞 in a NTS 𝒰 is named a neutrosophic α-open set and simply written as Ne-αOS if 𝒞 ⊆ Ne-int(Ne-cl(Ne-int(𝒞))). Besides, if Ne-cl(Ne-int(Ne-cl(𝒞))) ⊆ 𝒞, then 𝒞 is called a neutrosophic α-closed set, and we are shortly given it as Ne-αCS. The collection of all such these
Ne-αOSs (correspondingly, Ne-αCSs) in 𝑈 is referred to Ne-α0(𝑈) (correspondingly, Ne-αC(𝑈)). The intersection of all Ne-αCSs that contain 𝑋 is called the neutrosophic α-closure of 𝑋 in 𝑈 and represented by Ne-αcl(𝑋).

**Definition 2.2** [13]: A NS 𝑋 in NTS 𝑈 is so-called a neutrosophic generalized closed set and denoted by Ne-gCS if for any Ne-OS 𝑀 in 𝑈 such that 𝑋 ⊆ 𝑀, then Ne-cl(𝑋) ⊆ 𝑀. Moreover, its complement is named a neutrosophic generalized open set and referred to Ne-gOS.

**Definition 2.3** [14]: A NS 𝑋 in NTS 𝑈 is so-called a neutrosophic αg-closed set and indicated by Ne-αgCS if for any Ne-OS 𝑀 in 𝑈 such that 𝑋 ⊆ 𝑀, then Ne-αcl(𝑋) ⊆ 𝑀. Furthermore, its complement is named a neutrosophic αg-open set and symbolized by Ne-αgOS.

**Definition 2.4** [15]: A NS 𝑋 in NTS 𝑈 is so-called a neutrosophic αg-closed set and signified by Ne-αgCS if far any Ne-αOS 𝑀 in 𝑈 such that 𝑋 ⊆ 𝑀, then Ne-αcl(𝑋) ⊆ 𝑀. Besides, its complement is named a neutrosophic αg-open set and briefly written as Ne-αgOS.

**Theorem 2.5** [10,13-15]: For any NTS 𝑈, the next declarations valid and but not vice versa:
(i) for all Ne-OSs (correspondingly, Ne-CSs) are Ne-αOSs (correspondingly, Ne-αCSs).
(ii) for all Ne-OSs (correspondingly, Ne-CSs) are Ne-αgOSs (correspondingly, Ne-αgCSs).
(iii) for all Ne-αOSs (correspondingly, Ne-αCSs) are Ne-αgOSs (correspondingly, Ne-αgCSs).
(iv) for all Ne-αOS (correspondingly, Ne-αCS) are Ne-αgOSs (correspondingly, Ne-αgCSs).
(v) for all Ne-αOSs (correspondingly, Ne-αCSs) are Ne-αgOSs (correspondingly, Ne-αgCSs).

**Definition 2.6**: Let (𝑈, 𝜋) and (𝑉, 𝜃) be NTSs and 𝜂: (𝑈, 𝜋) → (𝑉, 𝜃) be a mapping, we have
(i) if for each Ne-OS (correspondingly, Ne-CS) 𝐾 in 𝑉, 𝜂⁻¹(𝐾) is a Ne-OS (correspondingly, Ne-CS) in 𝑈, then 𝜂 is known as neutrosophic continuous and denoted by Ne-continuous. [16]
(ii) if for each Ne-OS (correspondingly, Ne-CS) 𝐾 in 𝑉, 𝜂⁻¹(𝐾) is a Ne-OS (correspondingly, Ne-CS) in 𝑈, then 𝜂 is known as neutrosophic α-continuous and referred to Ne-α-continuous. [10]
(iii) if for each Ne-OS (correspondingly, Ne-CS) 𝐾 in 𝑉, 𝜂⁻¹(𝐾) is a Ne-gOS (correspondingly, Ne-gCS) in 𝑈, then 𝜂 is known as neutrosophic g-continuous and signified by Ne-g-continuous. [17]

**Remark 2.7** [17,10]: Let 𝜂: (𝑈, 𝜋) → (𝑉, 𝜃) be a map, the next declarations valid and but not vice versa:
(i) For all Ne-continuous functions are Ne-α-continuous.
(ii) For all Ne-continuous functions are Ne-g-continuous.

3. Neutrosophic Generalized αg-Closed Sets
The neutrosophic generalized $\alpha g$-closed sets and their features are studied and discussed in this part of the paper.

**Definition 3.1**: Let $C$ be a $NS$ in $NTS \ U$, then it called a neutrosophic generalized $\alpha g$-closed set and denoted by $Ne-gagCS$ if for any $Ne-\alpha gOS \ M$ in $U$ such that $C \subseteq M$, then $Ne-cl(C) \subseteq M$. We indicated the collection of all $Ne-gagCS$s in $NTS \ U$ by $Ne-gagC(U)$.

**Definition 3.2**: Let $C$ be a $NS$ in $TS \ U$, then its neutrosophic $gag$-closure is the intersection of each $Ne-gagCS$ in $U$ including $C$, and we are shortly written it as $Ne-gagcl(C)$. In other words, $Ne-gagcl(C) = \cap \{D: C \subseteq D, D$ is a $Ne-gagCS\}$.

**Theorem 3.3**: The subsequent declarations are valid in any $TS \ U$:

(i) for all $Ne-CS$s are $Ne-gagCS$s.

(ii) for all $Ne-gagCS$s are $Ne-gCS$s.

(iii) for all $Ne-gagCS$s are $Ne-\alpha gCS$s.

(iv) for all $Ne-gagCS$s are $Ne-\alpha gCS$s.

**Proof**:

(i) Suppose a $Ne-CS \ C$ is in $TS \ U$. For any $Ne-\alpha gOS \ M$, including $C$, we have $M \supseteq C = Ne-cl(C)$. Therefore, $C$ stands a $Ne-gagCS$.

(ii) Suppose $Ne-gagCS \ C$ is in $TS \ U$. For any $Ne-OS \ M$, including $C$, we have theorem (2.5) states that $M$ stands a $Ne-\alpha gOS$ in $U$. Because a $Ne-gagCS \ C$ satisfying this fact $Ne-cl(C) \subseteq M$. As a result, $C$ considers a $Ne-gCS$.

(iii) Assume $Ne-gagCS \ C$ is in $TS \ U$. For any $Ne-OS \ M$, including $C$, we have theorem (2.5) states that $M$ remains a $Ne-\alpha gOS$ in $U$. Subsequently, $Ne-\alpha gCS \ C$ satisfying this statement $Ne-\alpha cl(C) \subseteq Ne-cl(C) \subseteq M$. Therefore, $C$ becomes a $Ne-\alpha gCS$.

(iv) Assume $Ne-gagCS \ C$ is in $TS \ U$. For any $Ne-OS \ M$ including $C$, we have theorem (2.5) states that $M$ remains a $Ne-\alpha gOS$ in $U$. Subsequently, $Ne-\alpha gCS \ C$ satisfying this statement $Ne-\alpha cl(C) \subseteq Ne-cl(C) \subseteq M$. Therefore, $C$ considers a $Ne-\alpha gCS$.

The opposite conditions for this previous theorem do not look accurate by the next obvious examples.

**Example 3.4**: Suppose $U = \{p, q\}$ and let $\xi = \{0_N, A, B, 1_N\}$, such that we have the sets $A = \langle u, (0, 6,0,0.7), (0.1,0.1), (0,4,0.2) \rangle$ and $B = \langle u, (0.1,0.2), (0.1,0.1), (0,8,0.8) \rangle$, so that $(U, \xi)$ is a $NTS$. However, the $NS \ C = \langle u, (0,2,0.2), (0,1,0.1), (0,6,0.7) \rangle$ is a $Ne-gagCS$ but not a $Ne-CS$.

**Example 3.5**: Suppose $U = \{p, q, r\}$ and let $\xi = \{0_N, A, B, 1_N\}$, where such that we have the sets $A = \langle u, (0,5,0,0.4), (0.7,0,5,0,5), (0,4,0,5,0,5) \rangle$ and $B = \langle u, (0,3,0,4,0,4), (0,4,0,5,0,5), (0,3,0,4,0,6) \rangle$, so that $(U, \xi)$ is a $NTS$. However, the $NS \ C = \langle u, (0,4,0,6,0,5), (0,4,0,3,0,5), (0,5,0,6,0,4) \rangle$ is a $Ne-gCS$ but not a $Ne-gagCS$.

**Example 3.6**: Suppose $U = \{p, q\}$ and let $\xi = \{0_N, A, B, 1_N\}$, where such that we have the sets $A = \langle u, (0,5,0,6), (0,3,0,2), (0,4,0,1) \rangle$ and $B = \langle u, (0,4,0,4), (0,4,0,3), (0,5,0,4) \rangle$, so that $(U, \xi)$ is a $NTS$. 

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However, the $NS \ C = \{u, (0.5,0.4), (0.4,0.4), (0.4,0.5)\}$ is a Ne-$\alpha$gCS and hence Ne-$\gamma$gCS but not a Ne-$\gamma$gCS.

**Definition 3.7:** Let $\ C$ be any $NS$ in $TS \ U$, then it is called a neutrosophic generalized $\alpha g$-open set and referred to by Ne-$\gamma$gOS iff the set $U - C$ is a Ne-$\gamma$gCS. The collection of the whole Ne-$\gamma$gOSs in $NTS \ U$ indicated by Ne-$\gamma$gO($U$).

**Definition 3.8:** The union of the whole Ne-$\gamma$gOSs in $NTS \ U$ included in $NS \ C$ is termed neutrosophic gog-interior of $C$ and symbolized by Ne-$\gamma$gint $(C)$. In symbolic form, we have this thing $Ne$-$\gamma$gint $(C) = \bigcup(D: C \supseteq D, D$ is a Ne-$\gamma$gOS).

**Proposition 3.9:** For any $NS \ M$ in $TS \ U$, the subsequent features stand:

(i) $Ne$-$\gamma$gint $(M) = M$ iff $M$ is a Ne-$\gamma$gOS.

(ii) $Ne$-$\gamma$gcl $(M) = M$ iff $M$ is a Ne-$\gamma$gCS.

(iii) $Ne$-$\gamma$gint $(M)$ is the biggest Ne-$\gamma$gOS included in $M$.

(iv) $Ne$-$\gamma$gcl $(M)$ is the littlest Ne-$\gamma$gCS, including $M$.

**Proof:** the features (i-iv) are understandable.

**Proposition 3.10:** For any $NS \ M$ in $TS \ U$, the subsequent features stand:

(i) $Ne$-$\gamma$gint $(\overline{M}) = (Ne - \gamma$gcl $(\overline{M}))$.

(ii) $Ne$-$\gamma$gcl $(\overline{M}) = (Ne - \gamma$gint $(\overline{M})$).

**Proof:**

(i) The proof will be evident by symbolic definition, $Ne$-$\gamma$gcl $(M) = \bigcap\{D: M \subseteq D, D$ is a Ne-$\gamma$gCS\}

$$(Ne - \gamma$gcl $(\overline{M})) = \bigcap\{\overline{D}: \overline{M} \subseteq \overline{D}, \overline{D}$ is a Ne-$\gamma$gCS\}

$$= \bigcup\{\overline{D}: \overline{D} \subseteq \overline{D}, \overline{D}$ is a Ne-$\gamma$gCS\}

$$= \bigcup\{N: M \supseteq N, N$ is a Ne-$\gamma$gOS\}

$$= Ne$-$\gamma$gint $(\overline{M})$.

(ii) This feature has undeniable proof analogous to feature (i).

**Theorem 3.11:** For any Ne-OS $C$ in $TS \ U$, then this set is a Ne-$\gamma$gOS.

**Proof:** Suppose Ne-OS $C$ in $TS \ U$, so we obtain that $\overline{C}$ is a Ne-CS. Therefore, $\overline{C}$ is a Ne-$\gamma$gCS via the previous theorem (3.3), part (i). Consequently, $C$ is a Ne-$\gamma$gOS.

**Theorem 3.12:** For any Ne-$\gamma$gOS $C$ in $TS \ U$, then this set is a Ne-gOS.

**Proof:** Suppose Ne-$\gamma$gOS $C$ in $TS \ U$, so we obtain that $\overline{C}$ is a Ne-$\gamma$gCS. Therefore, $\overline{C}$ is a Ne-gCS via the previous theorem (3.3), part (ii). Consequently, $C$ is a Ne-gOS.

**Lemma 3.13:** For any Ne-$\gamma$gOS $C$ in $TS \ U$, then this set is Ne-$\gamma$gOS (correspondingly, Ne-gOS).

**Proof:** The proof of this lemma is similar to one of the previous theorem.

**Proposition 3.14:** For any two Ne-$\gamma$gCSs $C$ and $D$ in $TS \ U$, then the set $C \cup D$ is a Ne-$\gamma$gCS.
Proof: Suppose any two Ne-gagCSs \( E \) and \( D \) in \( NTS \ U \) and \( M \) is a Ne-agOS, including \( E \) and \( D \). In other words, we have \( E \cup D \subseteq M \). So, \( E, D \subseteq M \). Because \( E \) and \( D \) are Ne-gagCSs, we get that Ne-cl(\( E \)), Ne-cl(\( D \)) \( \subseteq M \). Therefore, Ne-cl(\( E\cup D \)) = Ne-cl(\( E \)) \cup Ne-cl(\( D \)) \( \subseteq M \). Then Ne-cl(\( E\cup D \)) \( \subseteq M \). Thus, \( E \cup D \) stands a Ne-gagCS.

Proposition 3.15: For any two Ne-gagOSs \( E \) and \( D \) in \( TS \ U \), then the set \( E \cap D \) is a Ne-gagOS.

Proof: Suppose any two Ne-gagOSs \( E \) and \( D \) in \( TS \ U \). Subsequently, we have that \( E \) and \( D \) are Ne-gagCSs. So, we reach to this fact \( E \cap D \) is a Ne-gagCS by proposition (3.14). Because \( E \cup D = \overline{(E \cap D)} \), we obtain to our final result \( E \cap D \) is a Ne-gagOS.

Proposition 3.16: Let Ne-gagCS \( E \) be in \( TS \ U \), then Ne-cl(\( E \)) - \( E \) does not include non-empty Ne-CS in \( U \).

Proof: Assume we have Ne-gagCS \( E \) and Ne-CS \( F \) in \( NTS \ U \) so as \( F \subseteq Ne-cl(\( E \)) - \( E \). Because \( E \) stands a Ne-gagCS, this gives us the fact Ne-cl(\( E \)) \( \subseteq \overline{F} \). The latter means \( F \subseteq Ne - cl(\( E \)) \). Subsequently, we arrive to \( F \subseteq Ne-cl(\( E \)) \cap (Ne - cl(\( E \))) = 0_N \). Therefore, \( F = 0_N \) and so, we reach to our final result Ne-cl(\( E \)) - \( E \) does not include non-empty Ne-CS.

Proposition 3.17: Let Ne-gagCS \( E \) be in \( NTS \ U \) iff Ne-cl(\( E \)) - \( E \) does not include non-empty Ne-agCS in \( U \).

Proof: Assume we have Ne-gagCS \( E \) and Ne-agCS \( G \) in \( NTS \ U \) so as \( G \subseteq Ne-cl(\( E \)) - \( E \). Because \( E \) considers a Ne-gagCS, this gives us the fact Ne-cl(\( E \)) \( \subseteq \overline{G} \). The latter means \( G \subseteq Ne - cl(\( E \)) \). Subsequently, we arrive to \( G \subseteq Ne-cl(\( E \)) \cap (Ne - cl(\( E \))) = 0_N \). Therefore, \( G \) is empty. On The Other Hand, let us assume that Ne-cl(\( E \)) - \( E \) does not include non-empty Ne-agCS in \( U \). Suppose \( M \) is Ne-agOS so as \( E \subseteq M \). If we have this truth Ne-cl(\( E \)) \( \subseteq M \) but then we get this fact Ne-cl(\( E \)) \( \cap (\overline{M}) \) is non-empty. Meanwhile, we know that Ne-cl(\( E \)) is Ne-CS and at the same time, we have \( \overline{M} \) is Ne-agOS, so Ne-cl(\( E \)) \( \cap (\overline{M}) \) is non-empty Ne-agOS included Ne-cl(\( E \)) - \( E \). This leads us to a contradiction. Consequently Ne-cl(\( E \)) \( \not\subseteq M \). Therefore, \( E \) considers a Ne-gagCS.

Theorem 3.18: Let Ne-agOS and Ne-gagCS \( E \) be in \( TS \ U \), then \( E \) considers a Ne-CS in \( U \).

Proof: Assume we have Ne-agOS and Ne-gagCS \( E \) is in \( TS \ U \), so we get that Ne-cl(\( E \)) \( \subseteq E \) and subsequently, we reach to \( E \subseteq Ne-cl(\( E \)) \). Consequently, Ne-cl(\( E \)) = \( E \). Therefore, \( E \) stands a Ne-CS.

Theorem 3.19: Let Ne-gagCS \( E \) be in \( NTS \ U \) so as \( E \subseteq D \subseteq Ne-cl(\( E \)), but then again \( D \) considers a Ne-gagCS in \( U \).

Proof: Assume we have Ne-gagCS \( E \) and Ne-agOS \( M \) are in \( NTS \ U \) so as \( D \subseteq M \). Later, \( E \subseteq M \). Subsequently, \( E \) stands a Ne-gagCS; this fact pursues Ne-cl(\( E \)) \( \subseteq M \). So, \( D \subseteq Ne-cl(\( E \)) \) infers Ne-cl(\( D \)) \( \subseteq Ne-cl(\( Ne-cl(\( E \))) = Ne-cl(\( E \)) \). Consequently, Ne-cl(\( D \)) \( \subseteq M \). Therefore, \( D \) exists a Ne-gagCS.

Theorem 3.20: Let Ne-gagOS \( E \) be in \( NTS \ U \) so as Ne-int(\( E \)) \( \subseteq D \subseteq E \), but then again \( D \) considers a Ne-gagOS in \( U \).
Proof: Assume we have Ne-\(\lambda\)-gOS \(C\) is in \(NTS\ \mathcal{U}\) so as Ne-int(\(C\)) \(\subseteq \mathcal{D} \subseteq \mathcal{C}\). After that, \(\mathcal{U} - \mathcal{C}\) stands a Ne-\(\lambda\)-gOS as well as \(\mathcal{E} \subseteq \mathcal{D} \subseteq Ne-cl(\mathcal{E})\). But then again, we depend on theorem (3.19) to get \(\mathcal{U} - \mathcal{D}\) is a Ne-\(\lambda\)-gOS. Therefore, \(\mathcal{D}\) exists a Ne-\(\lambda\)-gOS.

Theorem 3.21: A \(NS\ \mathcal{C}\) is Ne-\(\lambda\)-gOS iff \(\mathcal{P} \subseteq Ne-int(\mathcal{C})\) so as \(\mathcal{P} \subseteq \mathcal{C}\) and \(\mathcal{P}\) considers a Ne-\(\lambda\)-gOS.

Proof: Assume we have that Ne-\(\lambda\)-gOS \(\mathcal{P}\) satisfying \(\mathcal{P} \subseteq \mathcal{C}\) and \(\mathcal{P} \subseteq Ne-int(\mathcal{C})\). Afterward, \(\mathcal{E} \subseteq \mathcal{P}\) and we have by lemma (3.13), \(\mathcal{P}\) remains a Ne-\(\lambda\)-gOS. Accordingly, Ne-cl(\(\mathcal{E}\)) = Ne-int(\(\mathcal{E}\)) \(\subseteq \mathcal{P}\). Subsequently, \(\mathcal{E}\) stands a Ne-\(\lambda\)-gOS. Therefore, \(\mathcal{E}\) stands a Ne-\(\lambda\)-gOS.

On the contrary, we assume Ne-\(\lambda\)-gOS \(\mathcal{E}\) and Ne-\(\lambda\)-gOS \(\mathcal{P}\) is so as \(\mathcal{P} \subseteq \mathcal{C}\). Subsequently, \(\mathcal{E} \subseteq \mathcal{P}\). While \(\mathcal{E}\) exists a Ne-\(\lambda\)-gOS and \(\mathcal{P}\) remains a Ne-\(\lambda\)-gOS, we reach to that Ne-cl(\(\mathcal{E}\)) \(\subseteq \mathcal{P}\). Therefore, \(\mathcal{P} \subseteq Ne-int(\mathcal{C})\).

Remark 3.22: The next illustration demonstrates the relative among the distinct kinds of Ne-CS:

![Diagram](fig_31.png)

4. Neutrosophic Generalized \(\alpha\)-\(g\)-Continuous Functions

In this part of this paper, the neutrosophic generalized \(\alpha\)-\(g\)-continuous functions are performed and examined their fundamental features.

Definition 4.1: Let \(\eta; (\mathcal{U}, \zeta) \rightarrow (\mathcal{V}, \gamma)\) be a map so as \(\mathcal{U}\) and \(\mathcal{V}\) are \(NTS\), then:
(i) \(\eta\) is named a neutrosophic \(\alpha\)-\(g\)-continuous and signified by Ne-\(\alpha\)-\(g\)-continuous if for every Ne-OS (correspondingly, Ne-CS) \(\mathcal{K}\) in \(\mathcal{V}\), \(\eta^{-1}(\mathcal{K})\) is a Ne-\(\alpha\)-\(g\)-OS (correspondingly, Ne-\(\alpha\)-\(g\)-CS) in \(\mathcal{U}\).
(ii) \(\eta\) is named a neutrosophic \(g\)-\(a\)-continuous and signified by Ne-\(g\)-\(a\)-continuous if for every Ne-OS (correspondingly, Ne-CS) \(\mathcal{K}\) in \(\mathcal{V}\), \(\eta^{-1}(\mathcal{K})\) is a Ne-\(g\)-\(a\)-OS (correspondingly, Ne-\(g\)-\(a\)-CS) in \(\mathcal{U}\).

Theorem 4.2: Let \(\eta\) be a function on \(NTS\ \mathcal{U}\) and valued in \(TS\ \mathcal{V}\). So, we have the following:
(i) all Ne-\(g\)-continuous functions are Ne-\(\alpha\)-\(g\)-continuous.
(ii) all Ne-\(\alpha\)-continuous functions are Ne-\(g\)-\(a\)-continuous.
(iii) all Ne-\(g\)-\(a\)-continuous functions are Ne-\(\alpha\)-\(g\)-continuous.

Proof:
(i) Let Ne-CS \(\mathcal{K}\) be in \(NTS\ \mathcal{V}\) and Ne-\(g\)-continuous function \(\eta\) defined on \(NTS\ \mathcal{U}\) and valued in \(TS\ \mathcal{V}\). By definition of Ne-\(g\)-continuous, \(\eta^{-1}(\mathcal{K})\) remains a Ne-gCS in \(\mathcal{U}\). So, we have \(\eta^{-1}(\mathcal{K})\) is a Ne-\(\alpha\)-gCS in \(\mathcal{U}\) because of theorem (2.5) part (iii). As a result, \(\eta\) stands a Ne-\(\alpha\)-\(g\)-continuous.
(ii) Let Ne-CS $\mathcal{K}$ be in $NTS\ V$ and Ne-$\alpha$-continuous function $\eta$ defined on $NTS\ U$ and valued in $NTS\ V$. By definition of Ne-$\alpha$-continuous, $\eta^{-1}(\mathcal{K})$ remains a Ne-$\alpha$CS in $U$. So, we have $\eta^{-1}(\mathcal{K})$ is a Ne-gaCS in $U$ because of theorem (2.5) part (iv). As a result, $\eta$ stands a Ne-ga-continuous.

(iii) Let Ne-CS $\mathcal{K}$ be in $NTS\ V$ and Ne-ga-continuous function $\eta$ defined on $NTS\ U$ and valued in $TS\ V$. So, we have $\eta^{-1}(\mathcal{K})$ is a Ne-gaCS and then $\eta^{-1}(\mathcal{K})$ is a Ne-$\alpha$CS in $U$ because of theorem (2.5) part (v). Therefore, $\eta$ stands a Ne-$\alpha$-continuous.

The reverse of the beyond proposition does not become valid as shown in the next examples.

**Example 4.3:** (i) Assume $U = \{p, q\}$ and $\xi = \{0_N, \mathcal{A}, \mathcal{B}, 1_N\}$ and $\varrho = \{0_N, \mathcal{B}, \mathcal{C}, 1_N\}$, where $\mathcal{A} = \{u, (0.6,0.7), (0.4,0.3), (0.5,0.2)\}$, $\mathcal{B} = \{u, (0.5,0.5), (0.5,0.4), (0.6,0.5)\}$ and $\mathcal{C} = \{u, (0.5,0.5), (0.6,0.4), (0.7,0.5)\}$ are the neutrosophic sets, then $(U, \xi)$ and $(U, \varrho)$ are NTs. Define $\eta: (U, \xi) \rightarrow (U, \varrho)$ as a $\eta(p) = q$ and $\eta(q) = p$. Then $\eta$ is Ne-$\alpha$-continuous. But $\tilde{\mathcal{C}} = \{u, (0.7,0.5), (0.6,0.4), (0.5,0.5)\}$ is a Ne-CS in $(U, \varrho)$, $\eta^{-1}(\tilde{\mathcal{C}})$ is not a Ne-gaCS in $(U, \xi)$. Thus $\eta$ is not a Ne-ga-continuous.

(ii) Let $U = \{p, q\}$ and let $\xi = \{0_N, \mathcal{A}, \mathcal{B}, 1_N\}$ and $\varrho = \{0_N, \mathcal{B}, \mathcal{C}, 1_N\}$, where $\mathcal{A} = \{u, (0.6,0.7), (0.4,0.3), (0.5,0.2)\}$, $\mathcal{B} = \{u, (0.5,0.5), (0.5,0.4), (0.6,0.5)\}$ and $\mathcal{C} = \{u, (0.5,0.5), (0.6,0.5), (0.7,0.5)\}$ are the neutrosophic sets, then $(U, \xi)$ and $(U, \varrho)$ are NTs. Define $\eta: (U, \xi) \rightarrow (U, \varrho)$ as a $\eta(p) = p$ and $\eta(q) = q$. Then $\eta$ is Ne-$\alpha$-continuous. But $\tilde{\mathcal{C}} = \{u, (0.4,0.5), (0.5,0.5), (0.5,0.5)\}$ is a Ne-CS in $(U, \varrho)$, $\eta^{-1}(\tilde{\mathcal{C}})$ is not a Ne-$\alpha$CS in $(U, \xi)$. Thus $\eta$ is not a Ne-$\alpha$-continuous.

(iii) Let $U = \{p, q\}$ and let $\xi = \{0_N, \mathcal{A}, \mathcal{B}, 1_N\}$ and $\varrho = \{0_N, \mathcal{B}, \mathcal{C}, 1_N\}$, where $\mathcal{A} = \{u, (0.6,0.7), (0.4,0.3), (0.5,0.2)\}$, $\mathcal{B} = \{u, (0.5,0.5), (0.5,0.4), (0.6,0.5)\}$ and $\mathcal{C} = \{u, (0.5,0.5), (0.6,0.4), (0.7,0.5)\}$ are the neutrosophic sets, then $(U, \xi)$ and $(U, \varrho)$ are NTs. Define $\eta: (U, \xi) \rightarrow (U, \varrho)$ as a $\eta(p) = q$ and $\eta(q) = p$. Then $\eta$ is Ne-$\alpha$-continuous. But $\tilde{\mathcal{C}} = \{u, (0.5,0.5), (0.5,0.5), (0.6,0.4)\}$ is a Ne-CS in $(U, \varrho)$, $\eta^{-1}(\tilde{\mathcal{C}})$ is not a Ne-ga-CS in $(U, \xi)$. Thus $\eta$ is not a Ne-ga-continuous.

**Definition 4.4:** Let $\eta$ be a function on $NTS\ U$ and valued in $TS\ V$. Then, we named $\eta$ as neutrosophic generalized $\alpha$-continuous and shortly wrote it as Ne-ga-$\alpha$-continuous if for each Ne-CS $\mathcal{K}$ in $V$, $\eta^{-1}(\mathcal{K})$ is a Ne-gaCS in $U$.

**Theorem 4.5:** Let $\eta$ be a function on $NTS\ U$ and valued in $TS\ V$. Afterward, $\eta$ remains a Ne-ga-continuous function iff for each Ne-OS $\mathcal{K}$ in $V$, $\eta^{-1}(\mathcal{K})$ is a Ne-gaOS in $U$.

**Proof:** Let Ne-OS $\mathcal{K}$ and Ne-CS $\overline{\mathcal{K}}$ are in $V$. Therefore, $\eta^{-1}(\mathcal{K}) = (\eta^{-1}(\overline{\mathcal{K}}))$ remains a Ne-gaCS in $U$. Consequently, $\eta^{-1}(\mathcal{K})$ exists a Ne-gaOS in $U$. The reverse proof is evident.

**Proposition 4.6:** For all Ne-ga-continuous functions are Ne-$\alpha$-continuous.

**Proof:** Let Ne-CS $\mathcal{K}$ be in $NTS\ V$ and Ne-ga-continuous function $\eta$ defined on $NTS\ U$ and valued in $TS\ V$. By definition of Ne-ga-continuous function, $\eta^{-1}(\mathcal{K})$ stands a Ne-gaCS in $U$. So, we have $\eta^{-1}(\mathcal{K})$ remains a Ne-$\alpha$CS in $U$ because of theorem (3.3) part (iii). As a result, $\eta$ exists a Ne-$\alpha$-continuous.
Proposition 4.7: For all Ne-gg-continuous functions are Ne-ga-continuous.

Proof: Let Ne-CS $\mathcal{K}$ be in $NTS$ $\mathcal{V}$ and Ne-gg-continuous function $\eta$ defined on $NTS$ $\mathcal{U}$ and valued in $TS$ $\mathcal{V}$. By definition of Ne-gg-continuous function, $\eta^{-1}(\mathcal{K})$ stands a Ne-ggCS in $\mathcal{U}$. So, we have $\eta^{-1}(\mathcal{K})$ remains a Ne-gaCS in $\mathcal{U}$ because of theorem (3.3) part (iv). As a result, $\eta$ exists a Ne-ga-continuous.

The reverse of the beyond proposition does not become valid as shown in the next examples.

Example 4.8: Let $\mathcal{U} = \{p,q\}$ and let $\xi = \{0_{N},\mathcal{A},\mathcal{B},1_{N}\}$ and $\varphi = \{0_{N},\mathcal{C},1_{N}\}$, where $\mathcal{A} = \langle u, (0,5,0.6), (0,3,0.2), (0,4,0.1) \rangle$, $\mathcal{B} = \langle u, (0,4,0.4), (0,4,0.3), (0,5,0,4) \rangle$ and $\mathcal{C} = \langle u, (0,5,0,4), (0,4,0,4), (0,4,0,5) \rangle$ are the neutrosophic sets, then $(\mathcal{U},\xi)$ and $(\mathcal{U},\varphi)$ are NTSs. Define $\eta: (\mathcal{U},\xi) \rightarrow (\mathcal{U},\varphi)$ as a $\eta(p) = q$ and $\eta(q) = p$. Then $\eta$ is Ne- $\alpha$ g-continuous. But $\mathcal{C} = \langle u, (0,4,0,5), (0,4,0,4), (0,5,0,4) \rangle$ is a Ne-CS in $(\mathcal{U},\varphi)$, $\eta^{-1}(\mathcal{C})$ is a Ne-gaCS but not a Ne-ggCS in $(\mathcal{U},\xi)$. Thus $\eta$ is not a Ne-ga-continuous.

Example 4.9: Let $\mathcal{U} = \{p,q\}$ and let $\xi = \{0_{N},\mathcal{A},\mathcal{B},1_{N}\}$ and $\varphi = \{0_{N},\mathcal{C},1_{N}\}$, where $\mathcal{A} = \langle u, (0,5,0,6), (0,3,0,2), (0,4,0,1) \rangle$, $\mathcal{B} = \langle u, (0,4,0,4), (0,4,0,3), (0,5,0,4) \rangle$ and $\mathcal{C} = \langle u, (0,5,0,4), (0,4,0,4), (0,4,0,5) \rangle$ are the neutrosophic sets, then $(\mathcal{U},\xi)$ and $(\mathcal{U},\varphi)$ are NTSs. Define $\eta: (\mathcal{U},\xi) \rightarrow (\mathcal{U},\varphi)$ as a $\eta(p) = q$ and $\eta(q) = p$. Then $\eta$ is Ne-$\alpha$ g-continuous. But $\mathcal{C} = \langle u, (0,4,0,5), (0,4,0,4), (0,5,0,4) \rangle$ is a Ne-CS in $(\mathcal{U},\varphi)$, $\eta^{-1}(\mathcal{C})$ is a Ne-gaCS but not a Ne-ggCS in $(\mathcal{U},\xi)$. Thus $\eta$ is not a Ne-ga-continuous.

Definition 4.10: Let $\eta$ be a function on $NTS$ $\mathcal{U}$ and valued in $TS$ $\mathcal{V}$. Then, we named $\eta$ as neutrosophic generalized $\alpha$-g-irresolute and shortly wrote it as Ne-$g\alpha$-g-irresolute if for each Ne-ggCS $\mathcal{K}$ in $\mathcal{V}$, $\eta^{-1}(\mathcal{K})$ is a Ne-ggCS in $\mathcal{U}$.

Theorem 4.11: Let $\eta$ be a function on $NTS$ $\mathcal{U}$ and valued in $TS$ $\mathcal{V}$. Afterward, $\eta$ remains a Ne-gg-irresolute function iff for each Ne-ggOS $\mathcal{K}$ in $\mathcal{V}$, $\eta^{-1}(\mathcal{K})$ is a Ne-ggOS in $\mathcal{U}$.

Proof: Let Ne-ggOS $\mathcal{K}$ and Ne-ggCS $\mathcal{K}$ are in $\mathcal{V}$. Therefore, $\eta^{-1}(\mathcal{K}) = (\eta^{-1}(\mathcal{K}))$ remains a Ne-ggCS in $\mathcal{U}$. Consequently, $\eta^{-1}(\mathcal{K})$ exists a Ne-ggOS in $\mathcal{U}$. The reverse proof is evident.

Proposition 4.12: For all Ne-gg-irresolute functions are Ne-gg-continuous.

Proof: Let Ne-CS $\mathcal{K}$ be in $NTS$ $\mathcal{V}$ and Ne-gg-irresolute function $\eta$ defined on $NTS$ $\mathcal{U}$ and valued in $TS$ $\mathcal{V}$. So, we have $\mathcal{K}$ stands a Ne-ggCS in $\mathcal{V}$ by theorem (3.3) part (i). By definition of Ne-$g\alpha$ g-irresolute function, $\eta^{-1}(\mathcal{K})$ stands a Ne-$g\alpha$ gCS in $\mathcal{U}$. As a result, $\eta$ exists a Ne-gg-continuous.

The subsequent example explains that the inverse of the overhead proposition does not work.

Example 4.13: Suppose $\mathcal{U} = \{p,q\}$ and let $\xi = \{0_{N},\mathcal{B},1_{N}\}$ and $\varphi = \{0_{N},\mathcal{A},\mathcal{B},1_{N}\}$, where $\mathcal{A} = \langle u, (0,6,0,7), (0,4,0,3), (0,5,0,2) \rangle$ and $\mathcal{B} = \langle u, (0,5,0,5), (0,5,0,4), (0,6,0,5) \rangle$ are the neutrosophic sets, then $(\mathcal{U},\xi)$ and $(\mathcal{U},\varphi)$ are NTSs. Define $\eta: (\mathcal{U},\xi) \rightarrow (\mathcal{U},\varphi)$ as a $\eta(p) = q$ and $\eta(q) = p$. Then $\eta$ is Ne-gg-continuous. But $\mathcal{C} = \langle u, (0,5,0,5), (0,6,0,4), (0,5,0,7) \rangle$ is a Ne-ggCS in $(\mathcal{U},\varphi)$, $\eta^{-1}(\mathcal{C})$ is not a Ne-ggCS in $(\mathcal{U},\xi)$. Thus $\eta$ is not a Ne-gg-irresolute.
Definition 4.14: We called a NTS $\mathcal{U}$ with a neutrosophic $T_2$-space if for each Ne-gCS in $\mathcal{U}$ is a Ne-CS and we denoted it by Ne-$T_2$-space.

Definition 4.15: We called a NTS $\mathcal{U}$ with a neutrosophic $T_{gag}$-space if for each Ne-gagCS in $\mathcal{U}$ is a Ne-CS and we denoted by Ne-$T_{gag}$-space.

Proposition 4.16: Every Ne-$T_2$-space stands a Ne-$T_{gag}$-space.

Proof: Let $\mathcal{C}$ be a Ne-gagCS in Ne-$T_2$-space $\mathcal{U}$. By theorem (3.3) part (ii), we obtain $\mathcal{C}$ is a Ne-gCS. By definition of Ne-$T_2$-space, we reach to that $\mathcal{C}$ is a Ne-CS in $\mathcal{U}$. Therefore, $\mathcal{U}$ endures a Ne-$T_{gag}$-space.

Theorem 4.17: Let $\eta_1$ be a Ne-gag-continuous function on NTS $\mathcal{U}$ and valued in NTS $\mathcal{V}$ and let $\eta_2$ be a Ne-g-continuous function on NTS $\mathcal{V}$ and valued in TS $\mathcal{W}$. If $\mathcal{V}$ is a Ne-$T_2$-space, then $\eta_2 \circ \eta_1$ is a Ne-gag-continuous function.

Proof: Assume Ne-CS $\mathcal{K}$ is in $\mathcal{W}$. Meanwhile, we have a Ne-g-continuous function $\eta_2$ defined on a Ne-$T_2$-space $\mathcal{V}$, then $\eta_2^{-1}(\mathcal{K})$ stands a Ne-CS in $\mathcal{V}$. Subsequently, we also see a Ne-gag-continuous function $\eta_1$ defined on $\mathcal{U}$, then $\eta_1^{-1}(\eta_2^{-1}(\mathcal{K}))$ stands a Ne-gagCS in $\mathcal{U}$. Therefore, $\eta_2 \circ \eta_1$ stands a Ne-gag-continuous.

Theorem 4.18: Let $\eta$ be a function on NTS $\mathcal{U}$ and valued in TS $\mathcal{V}$, we have the following results:
(i) If NTS $\mathcal{U}$ stands a Ne-$T_2$-space then the function $\eta$ becomes a Ne-g-continuous iff it considers a Ne-gag-continuous.
(ii) If NTS $\mathcal{U}$ stands a Ne-$T_{gag}$-space then the function $\eta$ becomes a Ne-continuous iff it considers a Ne-gag-continuous.

Proof: 
(i) Let Ne-CS $\mathcal{K}$ be in $\mathcal{V}$ and $\eta$ be a Ne-g-continuous function. By definition of Ne-g-continuous, $\eta^{-1}(\mathcal{K})$ is a Ne-gCS in $\mathcal{U}$. Besides, the definition of Ne-$T_2$-space states $\eta^{-1}(\mathcal{K})$ is a Ne-CS. So, $\eta^{-1}(\mathcal{K})$ is a Ne-gagCS in $\mathcal{U}$ by theorem (3.3) part (i). Therefore, $\eta$ is a Ne-gag-continuous.
On the contrary, let Ne-CS $\mathcal{K}$ be in $\mathcal{V}$ and let $\eta$ be a Ne-gag-continuous. By definition of Ne-gag-continuous, $\eta^{-1}(\mathcal{K})$ is a Ne-gagCS in $\mathcal{U}$. Besides, we have $\eta^{-1}(\mathcal{K})$ is a Ne-gCS in $\mathcal{U}$ by theorem (3.3) part (ii). Therefore, $\eta$ is a Ne-continuous.
(ii) Let Ne-CS $K$ be in $V$ and let $\eta$ be a Ne-continuous. By definition of Ne-continuous, $\eta^{-1}(K)$ is a Ne-CS in $U$. So, we have $\eta^{-1}(K)$ is a Ne-gagCS in $U$ by theorem (3.3) part (i). Therefore, $\eta$ is a Ne-gag-continuous.

On the contrary, let Ne-CS $K$ be in $V$ and let $\eta$ be a Ne-gag-continuous. Besides, we have $\eta^{-1}(K)$ is a Ne-gagCS in $U$. Furthermore, the definition of Ne-$T_{\text{gag}}$-space gives $\eta^{-1}(K)$ is a Ne-CS in $U$. Therefore, $\eta$ is a Ne-continuous.

**Remark 4.19:** The subsequent illustration indicates the relative among the various kinds of Ne-continuous functions:

![Fig. 4.1](image_url)

5. Conclusion

The class of Ne-gagCS described employing Ne-agCS structures a neutrosophic topology and deceptions between the classes of Ne-CS and Ne-gCS. We as well illustration Ne-gag-continuous functions by applying Ne-gagCS. The Ne-gagCS know how to be developed to establish another neutrosophic homeomorphism.

**Funding:** This work does not obtain any external grant.

**Acknowledgments:** The authors are highly grateful to the Referees for their constructive suggestions.

**Conflicts of Interest:** The authors declare no conflict of interest.

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Received: Apr 25, 2020. Accepted: July 5 2020