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A new distance measure for trapezoidal fuzzy neutrosophic numbers based on the centroids

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Abstract: Distance measure is a numerical measurement of the distance between any two objects. The aim of this paper is to propose a new distance measure for trapezoidal fuzzy neutrosophic numbers based on the centroids with graphical representation. In addition, the metric properties of the proposed measure are examined in detail. A decision making problem also has been solved using the proposed distance measure for a software selection process. comparative analysis has been done with the existing methods to show the potential of the proposed distance measure and various forms of trapezoidal fuzzy neutrosophic number have been listed out to show the uniqueness of the proposed graphical representation. Further, advantages of the proposed distance measure have been given.

Keywords: trapezoidal fuzzy neutrosophic numbers; centroids; distance measure

1-Introduction

Zadeh introduced a mathematical frame work called fuzzy set [43] which plays a very significant role in many aspects of science. Intuitionistic fuzzy set is the generalization of the Zadeh's fuzzy set which was presented by Atanassov [3]. Later, triangular intuitionistic fuzzy sets was developed by Liu and Yuan [22] which is based on the combination of triangular fuzzy numbers and intuitionistic fuzzy sets. The fundamental characteristic of the triangular intuitionistic fuzzy set is that the values of its membership function and non-membership function are triangular fuzzy numbers rather than exact numbers. Furthermore, Ye [38] extended the triangular intuitionistic fuzzy set to the trapezoidal intuitionistic fuzzy set, where its fundamental characteristic is that the values of its membership function and non-membership function are trapezoidal fuzzy numbers rather than triangular fuzzy numbers, and proposed the trapezoidal intuitionistic fuzzy prioritized weighted averaging (TIFPWA) operator and trapezoidal intuitionistic fuzzy prioritized weighted geometric (TIFPWG) operator and their multi-criteria decision-making method, in which the criteria are in different priority level. Recently, Wang et al. [35] introduced a single-valued neutrosophic set, which is a subclass of a neutrosophic set presented by Smarandache [30], as a generalization of the classic set, fuzzy set and intuitionistic fuzzy set. The single-valued neutrosophic set can independently express truth-membership degree, indeterminacy-membership degree and falsity-membership degree and deal with incomplete, indeterminate and inconsistent information. All the factors described by the single-valued neutrosophic set are very suitable for human thinking due to the imperfection of knowledge that human receives or observes from the external world. For example, for a given proposition ''Movie X would be hit,'' in this situation human brain certainly cannot generate precise answers in terms of yes or no, as indeterminacy is the sector of unawareness of a proposition's value between truth and falsehood. Obviously, the neutrosophic components are best fit in the representation of indeterminacy and inconsistent information, while the intuitionistic fuzzy set cannot represent and handle indeterminacy and inconsistent information. Hence, the single-valued neutrosophic set has been a rapid development and a wide range of applications [39, 40]. Ye [42] introduced the trapezoidal neutrosophic set and its application to multiple attribute decision-making. Cui and Ye [10], Donghai et al. [16], Ebadi et al. [17], Guha and Chakraborty [18], Hajjari [19], Nayagam et al. [25], Rouhparvar et al. [29], Wu [37], Ye [40], Zou et al. [45] and more researchers have shown interest on decision making problem using distance measures. Weighted projection measure, the combination of angle cosine and weighted projection measure,similarity measure, hybrid vector similarity measure of single valued neutrosophic set and interval valued neutrosophic set, outranking strategy, complete ranking, new ranking function have been introduced so far under fuzzy, intuitionistic fuzzy and neutrosophic environments and applied in decision making problem. The rest of the paper is organized as follows. In section 2, literature review is given. In section 3, basic concepts are presented for better understanding. In section 4, proposed a new distnace measure and its graphical representation, and derived its properties in detail. In section 5, new methodology is described for a decision making process using the proposed measure. In section 6, a numerical example is using the proposed methodology to choose the best software system. In section 7, comparative analysis has been done with the existing methods and various forms of trapezoidal fuzzy neutrosophic numbers have been listed out to ahow the uniqueness of the proposed graphical representation. In section 8, advantages of the proposed measure are given. In section 9, conclusion of the present work is given with the future direction.

2-Literature Review

The authors of, Ahmad et al. [1] proposed a similarity measure based on the distance and set theory for generalized trapezoidal fuzzy numbers. Allahviranloo et al. [2] contributed a new distance measure and ranking method for generalized trapezoidal fuzzy numbers. Atanassov [3] introduced intuitionistic fuzzy sets. Azman and Abdullah [4] proposed a novel centroid method for trapezoidal fuzzy numbers for ranking. Biswas et al. [6] solved a decision making problem using expected value of neutrosophic trapezoidal numbers. Biswas et al. [6] solved a decision making problem using distance measure under interval trapezoidal neutrosophic numbers. Bolos et al. [7] designed the performance indicators of financial assets using neutrosophic fuzzy numbers. Bora and Gupta [8] studied the reaction of distance measure on the work of K-Means algorithm Matlab. Chakraborty et al. [9] presented different forms of trapezoidal neutrosophic number and deneutrosophication techniques. Cui and Ye [10] proposed logarithmic similarity measure and applied in medical diagnosis under dynamic neutrosophic cubic setting. Darehmiraki [11] introduced a new ranking methodology to solve linear programming problem. Das and De [12] introduced a new distance measure for the ranking IFNs. Das and Guha [13] introduced a ranking method for IFN using the point of centroid. Deli and Oztaurk [14] introduced a defuzzification method and applied in a decision-making problem for single valued trapezoidal neutrosophic numbers. Dhar et al. [15] indicated square neutrosophic fuzzy matrices. Donghai et al. [16] proposed a new similarity measure and distance measure between hesitant linguisticterm sets and applied the proposed concepts in a decision making problem. Ebadi et al. [17] proposed a novel distance measure for trapezoidal fuzzy numbers. Guha and Chakraborty [18] contributed a theoretical development of distance measure for intuitionistic fuzzy numbers (IFNs). Hajjari [19] conferred a new distance measure for Trapezoidal fuzzy numbers. Huang and Wu [20] presented equivalent forms of the triangle inequalities in fuzzy metric spaces. Liang et al. [21] proposed an integrated approach under a single valued trapezoidal neutrosophic environment. Liu and Yuan [22] prospected fuzzy number of intuitionistic fuzzy set. Llopis and Micheli [23] rectified a state of conflict in the sequence of input images. Minculete and Paltanea [24] introduced an enhanced estimates for the triangle inequality. Nayagam et al. [25] contributed a complete ranking of IFNs. Pardha Saradhi et al. [26] presented ordering of IFNs using centroids of centroids. Ravi Shankar et al. [27] developed a new ranking formula using centroid of centroids for fuzzy numbers and applied in a fuzzy critical path method. Rezvani [28] proposed a new ranking exponential formula using median value for trapezoidal fuzzy numbers. Rouhparvar et al. [29] introduced a novel fuzzy distance measure. Uppada [31] examined clustering algorithm using centroid clearly. Varghese and Kuriakose [32] proposed a formula to find the centroid of the fuzzy number. Wang [33] introduced geometric aggregation operator and applied in a decision making problem under intuitionistic fuzzy environment. Wang [34] proposed arithmetic aggregation operators. Wang et al. [35] introduced single valued neutrosophic sets. Wei et al. [36] introduced some persuaded aggregation operators under intuinistic fuzzy setting and applied in a group decision making problem. Wu [37] explained about distance metrics and their role in data transformations.Ye [38] proposed prioritized aggregation operators based on trapezoidal intuitionistic fuzzy concept and applied in a multi-criteria decision making problem. Ye [39] solved minimum spanning tree problem under single valued neutrosophic setting and its clustering method. Ye [40] proposed single valued neutrosophic cross entropy measure and applied in a decision making problem. Ye [41] introduced the expected Dice similarity measure and applied in a decision making problem. Ye [42] projected trapezoidal neutrosophic set and applied in a multiple attribute decision making. Zhang et al. [44] introduced interval neutrosophic sets and used in multi criteria decision making problem. Zou et al. [45] introduced a distance measure between neutrosophic sets as an evidential approach. From the literature, it is found that distance measure for trapezoidal neutrosophic numbers using centroids with its properties has not yet been studied so far. Hence the motivation of the present study.

Hence, in this paper a new distance measure for trapezoidal fuzzy neutrosophic numbers based on centroids has been proposed with its metric properties in detail. Also the graphical representation is presented for trapezoidal fuzzy neutrosophic number. Comparative study also have been made with the existing cases for both proposed distance measure and proposed graphical representation. Further advantages of the proposed distance measure are presented.

3-Preliminaries

Definition 1. [38] Let X be a space of discourse, a trapezoidal intuitionistic fuzzy set B in X is defined as: $B=\{\langle y,\alpha_B(y),\beta_B(y)\rangle | y\in X\}$, where $\alpha_B(y)\subset[0,1]$ and $\beta_B(y)\subset[0,1]$ are two trapezoidal fuzzy numbers $\alpha_B(y)=(\alpha_B^1(y),\alpha_B^2(y),\alpha_B^3(y),\alpha_B^4(y)):Y\to[0,1]$ and $\beta_B(y)=(\beta_B^1(y),\beta_B^2(y),\beta_B^3(y),\beta_B^4(y)):Y\to[0,1]$ with the condition that $0\le\alpha_B^4(y)+\beta_B^4(y)\le 1,\ \forall y\in Y$.

For Convenience, let $\alpha_B(y)=(a,b,c,d)$ and $\beta_B(y)=(e,f,g,h)$ be two trapezoidal fuzzy numbers, thus a trapezoidal intuitionistic fuzzy number (TrIFN) can be denoted by $j=\langle(a,b,c,d),(e,f,g,h)\rangle$, which is basic element in a trapezoidal intuitionistic fuzzy set.

If $b=c$ and $f=g$ hold in a TrIFN j, which is a special case of the TrIFN.

Definition 2. [38] Let $j_1=\langle(a_1,b_1,c_1,d_1),(e_1,f_1,g_1,h_1)\rangle$ and $j_2=\langle(a_2,b_2,c_2,d_2),(e_2,f_2,g_2,h_2)\rangle$, be two TrIFNs. Then there are the following operational rules:

1. $$j_1\oplus j_2=\left\langle\begin{array}{l}(a_1+a_2-a_1a_2,b_1+b_2-b_1b_2,c_1+c_2-c_1c_2,d_1+d_2-d_1d_2),\\(e_1e_2,f_1f_2,g_1g_2,h_1h_2)\end{array}\right\rangle$$

2. $$j_1\otimes j_2=\left\langle\begin{array}{l}(a_1a_2,b_1b_2,c_1c_2,d_1d_2),\\(e_1+e_2-e_1e_2,f_1+f_2-f_1f_2,g_1+g_2-g_1g_2,h_1+h_2-h_1h_2)\end{array}\right\rangle$$

3. $$\lambda j_1=\left\langle\begin{array}{l}\left(1-(1-a_1)^\lambda,1-(1-b_1)^\lambda,1-(1-c_1)^\lambda,1-(1-d_1)^\lambda\right),\\(e_1^\lambda,f_1^\lambda,g_1^\lambda,h_1^\lambda)\end{array}\right\rangle,\lambda>0;$$

4. $$m_1^\lambda=\left\langle\begin{array}{l}(a_1^\lambda,b_1^\lambda,c_1^\lambda,d_1^\lambda),\left(1-(1-e_1)^\lambda,1-(1-f_1)^\lambda,1-(1-g_1)^\lambda,1-(1-h_1)^\lambda\right),\\\left(1-(1-i_1)^\lambda,1-(1-j_1)^\lambda,1-(1-k_1)^\lambda,1-(1-l_1)^\lambda\right)\end{array}\right\rangle,\lambda\ge 0$$

Definition 3. [30] From philosophical point of view, Smarandache [30] originally presented the concept of a neutrosophic set B in a universal set Y, which is characterized independently by a truth-membership function $T_B(y)$, an indeterminacy membership function $I_B(y)$ and a falsity-membership function $F_B(y)$. The function $T_B(y)$, $I_B(y)$ and $F_B(y)$ in Y are real standard or nonstandard subsets of $]^-0,1^+[$, such that $T_B(y):Y \to]^-0,1^+[$, $I_B(y):Y \to]^-0,1^+[$, and $F_B(y):Y \to]^-0,1^+[$ .Then, the sum of $T_B(y), I_B(y)$ and $F_B(y)$ satisfies the condition $^-0 \le \sup T_B(y) + \sup I_B(y) + \sup F_B(y) \le 3^+$. Obviously, it is difficult to apply the neutrosophic set to practical problems. To easily apply it in science and engineering fields, Wang et al. [35] introduced the concept of a single-valued neutrosophic set as a subclass of the neutrosophic set and gave the following definition.

Definition 4. [35] A single-valued neutrosophic set B in a universal set Y is characterized by a truth-membership function $T_B(y)$, an indeterminacy-membership function $I_B(y)$ and a falsity-membership function $F_B(y)$. Then, a single-valued neutrosophic set B can be denoted by $B = \{\langle y, T_B(y), I_B(y), F_B(y)\rangle | y \in Y\}$

where, $T_B(y), I_B(y), F_B(y) \in [0,1]$ for each $y \in Y$. Therefore, the sum of $T_B(y), I_B(y)$ and $F_B(y)$ satisfies $0 \le T_B(y) + I_B(y) + F_B(y) \le 3$.

Let $M = \{\langle y, T_M(y), I_M(y), F_M(y)\rangle | y \in Y\}$ and $N = \{\langle y, T_N(y), I_N(y), F_N(y)\rangle | y \in Y\}$ be two single-valued neutrosophic sets, then we the following relations [8,11]:

1. Complement: $M^C = \{\langle y, F_M(y), 1 - I_M(y), T_M(y)\rangle | y \in Y\}$;
2. Inclusion: $M \subseteq N$ if and only if $T_M(y) \le T_N(y)$, $I_M(y) \ge I_N(y)$ and $F_M(y) \ge F_N(y)$ for any $y \in Y$;
3. Equality: $M = N$ if and only if $M \subseteq N$ and $N \subseteq M$;
4. Union: $M \cup N = \{\langle y, T_M(y) \vee T_N(y), I_M(y) \wedge I_N(y), F_M(y) \wedge F_N(y)\rangle | y \in Y\}$;
5. Intersection: $M \cap N = \{\langle y, T_M(y) \wedge T_N(y), I_M(y) \vee I_N(y), F_M(y) \vee F_N(y)\rangle | y \in Y\}$;

6. Addition: $M \oplus N = \left\{ \left\langle \begin{array}{l} y, T_M(y) + T_N(y) - T_M(y)T_N(y), I_M(y)I_N(y), \\ F_M(y)F_N(y) \end{array} \right\rangle \middle| y \in Y \right\}$;

7. Multiplication: $M \otimes N = \left\{ \left\langle \begin{array}{l} y, T_M(y)T_N(y), I_M(y) + I_N(y) - I_M(y)I_N(y), \\ F_M(y) + F_N(y) - F_M(y)F_N(y) \end{array} \right\rangle \middle| y \in Y \right\}$.

Definition 5. [42] Let Y be a space of discourse, a trapezoidal neutrosophic set H in Y is defined as follow:

$H = \{\langle y, T_H(y), I_H(y), F_H(y)\rangle | y \in Y\}$, where $T_H(y) \subset [0,1]$, $I_H(y) \subset [0,1]$ and $F_H(y) \subset [0,1]$ are three trapezoidal fuzzy numbers $T_H(y) = \left(t_H^1(y), t_H^2(y), t_H^3(y), t_H^4(y)\right) : Y \to [0,1]$,

$I_H(y) = \left(i_H^1(y), i_H^2(y), i_H^3(y), i_H^4(y)\right) : Y \to [0,1]$ and

$F_H(y) = \left(f_H^1(y), f_H^2(y), f_H^3(y), f_H^4(y)\right) : Y \to [0,1]$ with the condition

$0 \le t_H^4(y) + i_H^4(y) + f_H^4(y) \le 3,\ y \in Y$.

For convenience, the three trapezoidal fuzzy numbers are denoted by $T_H(y) = (a,b,c,d)$, $I_H(y) = (e,f,g,h)$ and $F_H(y) = (i,j,k,l)$. Thus, a trapezoidal neutrosophic numbers is denoted by $m = \langle (a,b,c,d),(e,f,g,h),(i,j,k,l)\rangle$, which is a basic element in the trapezoidal neutrosophic set.

If $b = c$, $f = g$ and $j = k$ hold in a trapezoidal neutrosophic number j_1, it reduces to the triangular neutrosophic number, which is considered as a special case of the trapezoidal neutrosophic number.

Definition 6. [42] Let $m_1 = \langle (a_1,b_1,c_1,d_1),(e_1,f_1,g_1,h_1),(i_1,j_1,k_1,l_1)\rangle$, and

$m_2 = \langle (a_2,b_2,c_2,d_2),(e_2,f_2,g_2,h_2),(i_2,j_2,k_2,l_2)\rangle$ be two trapezoidal neutrosophic numbers. Then there are the following operational rules:

1. $m_1 \oplus m_2 = \left\langle \begin{array}{l} (a_1 + a_2 - a_1a_2, b_1 + b_2 - b_1b_2, c_1 + c_2 - c_1c_2, d_1 + d_2 - d_1d_2), \\ (e_1e_2, f_1f_2, g_1g_2, h_1h_2), (i_1i_2, j_1j_2, k_1k_2, l_1l_2) \end{array} \right\rangle$,

2. $$m_1 \otimes m_2 = \left\langle \begin{array}{l} (a_1 a_2, b_1 b_2, c_1 c_2, d_1 d_2), \\ (e_1 + e_2 - e_1 e_2, f_1 + f_2 - f_1 f_2, g_1 + g_2 - g_1 g_2, h_1 + h_2 - h_1 h_2), \\ (i_1 + i_2 - i_1 i_2, j_1 + j_2 - j_1 j_2, k_1 + k_2 - k_1 k_2, l_1 + l_2 - l_1 l_2) \end{array} \right\rangle;$$

3. $$\lambda m_1 = \left\langle \begin{array}{l} \left(1-(1-a_1)^{\lambda}, 1-(1-b_1)^{\lambda}, 1-(1-c_1)^{\lambda}, 1-(1-d_1)^{\lambda}\right), \\ \left(e_1^{\lambda}, f_1^{\lambda}, g_1^{\lambda}, h_1^{\lambda}\right), \left(i_1^{\lambda}, j_1^{\lambda}, k_1^{\lambda}, l_1^{\lambda}\right) \end{array} \right\rangle, \lambda > 0;$$

4. $$m_1^{\lambda} = \left\langle \begin{array}{l} \left(a_1^{\lambda}, b_1^{\lambda}, c_1^{\lambda}, d_1^{\lambda}\right), \\ \left(1-(1-e_1)^{\lambda}, 1-(1-f_1)^{\lambda}, 1-(1-g_1)^{\lambda}, 1-(1-h_1)^{\lambda}\right), \\ \left(1-(1-i_1)^{\lambda}, 1-(1-j_1)^{\lambda}, 1-(1-k_1)^{\lambda}, 1-(1-l_1)^{\lambda}\right) \end{array} \right\rangle, \lambda \geq 0$$

Definition 7. [18] Let P and Q be the intuitionistic fuzzy sets with membership functions $\mu_P(x), \mu_Q(x)$, non-membership functions $\nu_P(x), \nu_Q(x)$ and hesitation degree $\pi_P(x), \pi_Q(x)$. Then the normalized Hamming distance is

$$D(P,Q) = \frac{1}{2n}\sum_{i=1}^{n}\left[\left|\mu_P(x_i) - \mu_Q(x_i)\right| + \left|\nu_P(x_i) - \nu_Q(x_i)\right| + \left|\pi_P(x_i) - \pi_Q(x_i)\right|\right]$$

And the normalized Euclidean distance is

$$D_E(P,Q) = \sqrt{\frac{1}{2n}\sum_{i=1}^{n}\left[\left(\mu_P(x_i) - \mu_Q(x_i)\right)^2 + \left(\nu_P(x_i) - \nu_Q(x_i)\right)^2 + \left(\pi_P(x_i) - \pi_Q(x_i)\right)^2\right]}$$

Definition 8. [17] Consider the real values r_i, $i = 1, 2, 3, \ldots, 6$ and if $r_1 \leq r_2, r_3 \leq r_4, r_5 \leq r_6$ then the following results are true.

1. $\max\{r_1, r_3, r_5\} \leq \max\{r_2, r_4, r_6\}$

2. $\max\{r_1 + r_2, r_3 + r_4, r_5 + r_6\} \leq \max\{r_1, r_3, r_5\} + \max\{r_2, r_4, r_6\}$

Definition 9. [34] For any real numbers $r_i, s_i \geq 0, i = 1, 2, \ldots, d$, the Euclidean distance is defined as, $D(r,s) = \sqrt{\sum_{i=1}^{d}(r_i - s_i)^2}$ and satisfies the condition that $\left[\sum_{i=1}^{d}(r_i + s_i)^p\right]^{1/p} \leq \left(\sum_{i=1}^{d}(r_i)^p\right)^{1/p} + \left(\sum_{i=1}^{d}(s_i)^p\right)^{1/p}$.

Definition 10. [42] Let $m_p = \langle (a_p, b_p, c_p, d_p), (e_p, f_p, g_p, h_p), (i_p, j_p, k_p, l_p) \rangle, p = 1,2,3,...,n$ be the trapezoidal fuzzy neutrosophic numbers then the trapezoidal fuzzy neurosophic weighted geometric operator is defined by

$$TFNWG(m_1, m_2, ..., m_n) = m_1^{\omega_1} \otimes m_2^{\omega_2} \otimes m_3^{\omega_3} \otimes ... \otimes m_n^{\omega_n}$$

$$= \left\langle \left(\prod_{p=1}^{n} a_p^{\omega_p}, \prod_{p=1}^{n} b_p^{\omega_p}, \prod_{p=1}^{n} c_p^{\omega_p}, \prod_{p=1}^{n} d_p^{\omega_p} \right), \left(1 - \prod_{p=1}^{n} (1 - e_p)^{\omega_p}, 1 - \prod_{p=1}^{n} (1 - f_p)^{\omega_p}, 1 - \prod_{p=1}^{n} (1 - g_p)^{\omega_p}, 1 - \prod_{p=1}^{n} (1 - h_p)^{\omega_p} \right), \right.$$

$$\left. \left(1 - \prod_{p=1}^{n} (1 - i_p)^{\omega_p}, 1 - \prod_{p=1}^{n} (1 - j_p)^{\omega_p}, 1 - \prod_{p=1}^{n} (1 - k_p)^{\omega_p}, 1 - \prod_{p=1}^{n} (1 - l_p)^{\omega_p} \right) \right\rangle$$

where, $\omega_1, \omega_2, ..., \omega_n$ are the weight vectors and the sum of the weight vectors is 1.

Definition 11. [9] Graphical representation of trapezoidal neutrosophic number

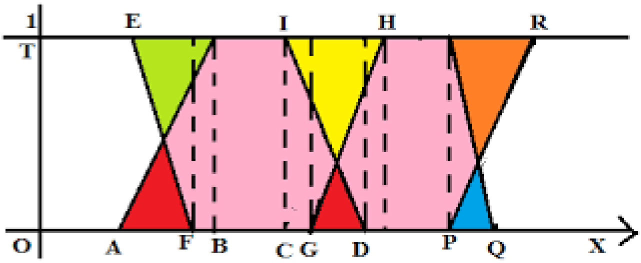


Figure 1. Graphical representation of Trapezoidal neutrosophic number

Figure 1 shows that graphical representation of trapezoidal fuzzy neutrosophic number can be done in different ways. It is a linear trapezoidal neutrosophic number.

4-Proposed Distance Measure for Trapezoidal Fuzzy Neutrosophic Number

Here we propose a new distance measure for trapezoidal fuzzy neutrosophic number based on centroids. Firstly, individual graphical representation proposed measure is presented here with the individual representation of truth, indeterminacy, falsity membership functions and trapezoidal fuzzy neutrosophic fuzzy number described by Figure 2-Figure 6.

Centre point of the object is called centroid. It should lie inside the object. At this point, the three medians of the triangle intersect and is termed point of intersection. Centroid is the average of coordinate points in X axis and Y axis of each vertex of the triangle. Centroid is the fixed point of all linear transformation which maintains length in translation, rotation, glides and reflection.

The centroid of the truth, indeterminacy and falsity trapezoid is treated as a balance point for the trapezoid. The centroid of each part are estimated using the calculation of centroid and the simple area and this combination will generate a triangle. Also the distance is measured from the centroid of all the parts to X axis and Y axis. Here the area of all the parts are multiplied by the distance and find their sum to get the total value. And the sum of the products of the area and distances is divided by the total area and obtain the centroid of circumcentre described by x and y point. Since centroid based distance measure may be derived using Euclidean measure, here it is obtained from the circumcentre of the centroids and the authentic point for the trapezoidal fuzzy neutrosophic number.

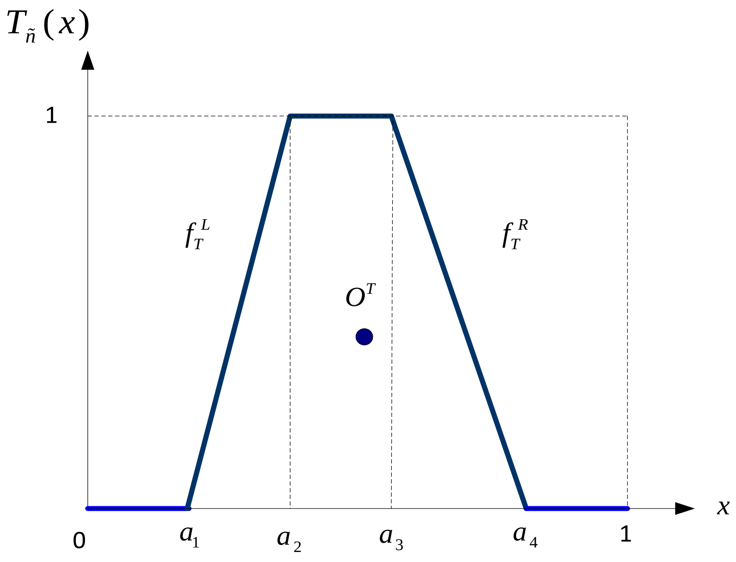


Figure 2. Truth membership function of trapezoidal fuzzy neutrosophic set with centroid

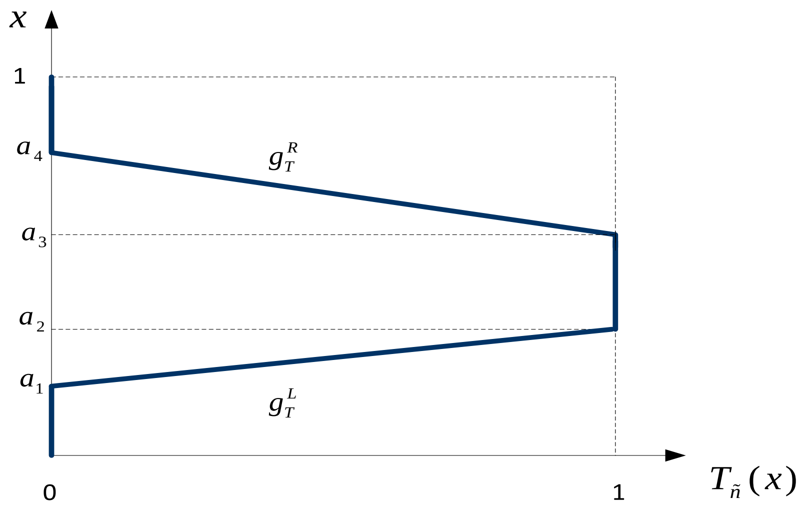


Figure 3. Truth membership function of trapezoidal fuzzy neutrosophic set

Suppose $\tilde{n} = \langle (a_1, a_2, a_3, a_4), (b_1, b_2, b_3, b_4), (c_1, c_2, c_3, c_4) \rangle$ be a trapezoidal fuzzy neutrosophic number. Based on the literature (Y. M. Wang et al. On the centroids of fuzzy numbers), we can get the centroid point $O^T = (x_0^T(\tilde{n}), y_0^T(\tilde{n}))$ of the truth membership function of trapezoidal fuzzy neutrosophic number $\tilde{n}$.

$$x_o^T(\tilde{n}) = \frac{\int_{a_1}^{a_2} x f_T^L dx + \int_{a_2}^{a_3} x \cdot 1 dx + \int_{a_3}^{a_4} x f_T^R dx}{\int_{a_1}^{a_2} f_T^L dx + \int_{a_2}^{a_3} 1 dx + \int_{a_3}^{a_4} f_T^R dx} = \frac{1}{3}[a_1 + a_2 + a_3 + a_4 - \frac{a_4 a_3 - a_1 a_2}{(a_4 + a_3) - (a_1 + a_2)}],$$

$$y_o^T(\tilde{n}) = \frac{\int_0^1 y(g_T^L - g_T^R) dy}{\int_0^1 (g_T^L - g_T^R) dy} = \frac{1}{3}[1 + \frac{a_3 - a_2}{(a_4 + a_3) - (a_1 + a_2)}].$$

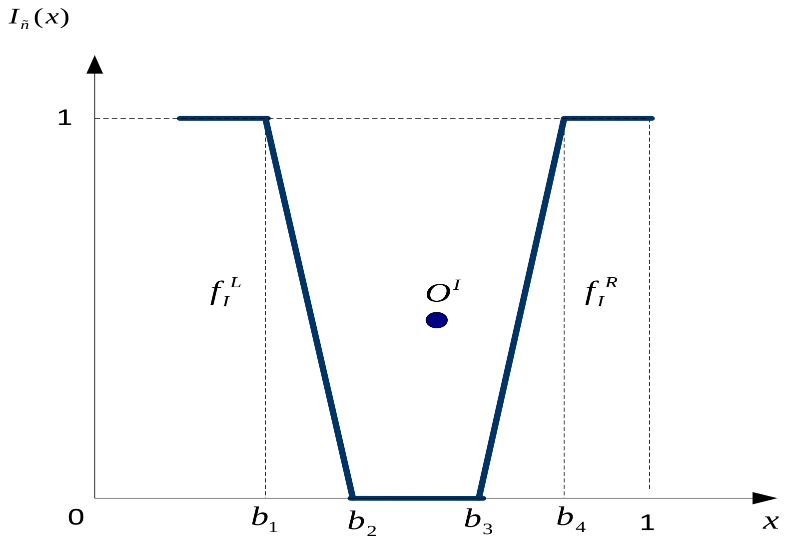


Figure 4. Indeterminate membership function of trapezoidal fuzzy neutrosophic set with centroid

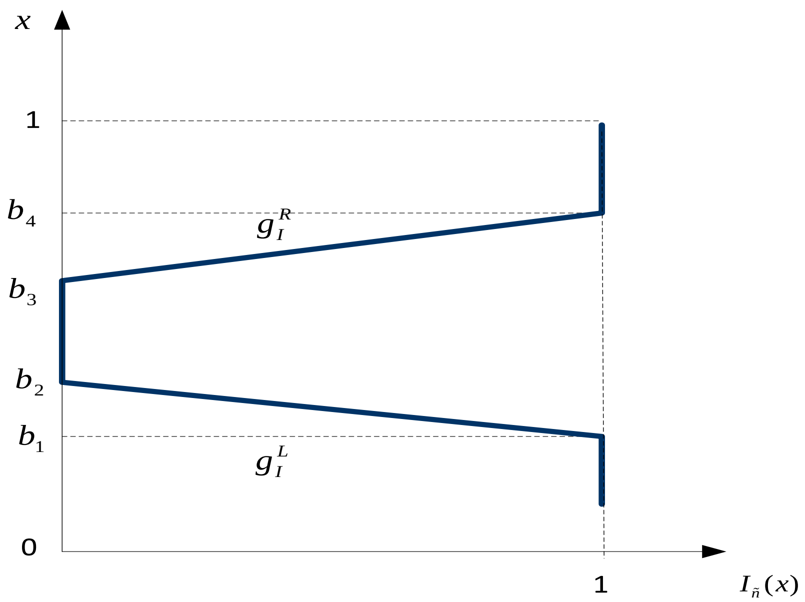


Figure 5. Indeterminate membership function of trapezoidal fuzzy neutrosophic number

we can get the centroid point $O^I = (x_o^I(\tilde{n}), y_o^I(\tilde{n}))$ of indeterminacy membership function of trapezoidal fuzzy neutrosophic number $\tilde{n}$.

$$x_o^I(\tilde{n}) = \frac{\int_{b_1}^{b_2} x f_I^L dx + \int_{b_2}^{b_3} x \cdot 1 dx + \int_{b_3}^{b_4} x f_I^R dx}{\int_{b_1}^{b_2} f_I^L dx + \int_{b_2}^{b_3} 1 dx + \int_{b_3}^{b_4} f_I^R dx} = \frac{1}{3}[b_1 + b_2 + b_3 + b_4 - \frac{b_4 b_3 - b_1 b_2}{(b_4 + b_3) - (b_1 + b_2)}],$$

$$y_o^I(\tilde{n}) = \frac{\int_0^1 y(g_I^L - g_I^R) dy}{\int_0^1 (g_I^L - g_I^R) dy} = \frac{1}{3}[1 + \frac{b_3 - b_2}{(b_4 + b_3) - (b_1 + b_2)}].$$

Similarly, we can get the centroid point $O^F = (x_o^F(\tilde{n}), y_o^F(\tilde{n}))$ of falsity membership function of trapezoidal fuzzy neutrosophic number $\tilde{n}$.

$$x_o^F(\tilde{n}) = \frac{\int_{c_1}^{c_2} x f_F^L dx + \int_{c_2}^{c_3} x \cdot 1 dx + \int_{c_3}^{c_4} x f_F^R dx}{\int_{c_1}^{c_2} f_F^L dx + \int_{c_2}^{c_3} 1 dx + \int_{c_3}^{c_4} f_F^R dx} = \frac{1}{3}[c_1 + c_2 + c_3 + c_4 - \frac{c_4 c_3 - c_1 c_2}{(c_4 + c_3) - (c_1 + c_2)}],$$

$$y_o^F(\tilde{n}) = \frac{\int_0^1 y(g_F^L - g_F^R) dy}{\int_0^1 (g_F^L - g_F^R) dy} = \frac{1}{3}[1 + \frac{c_3 - c_2}{(c_4 + c_3) - (c_1 + c_2)}].$$

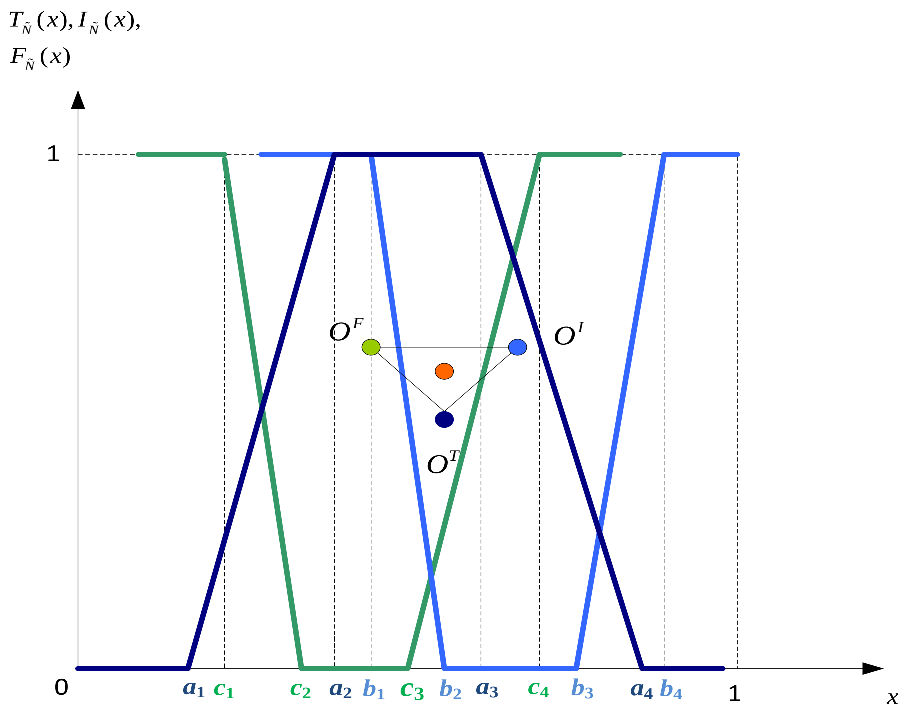


Figure 6. Trapezoidal fuzzy neutrosophic number with circumcentre of Centroids

In the above figure 5, the red dot represents the center of gravity of the triangle consisting of O^T, O^I, and O^F. According to the coordinate formula of the center of gravity of the triangle, we can get the coordinates of red dots $O=(x(\tilde{n}), y(\tilde{n}))$.

$$x(\tilde{n})=\frac{x_o^T(\tilde{n})+x_o^I(\tilde{n})+x_o^F(\tilde{n})}{3}$$

$$=\frac{1}{3}\left\{\begin{array}{l}\frac{1}{3}[a_1+a_2+a_3+a_4-\frac{a_4a_3-a_1a_2}{(a_4+a_3)-(a_1+a_2)}] \\ +\frac{1}{3}[b_1+b_2+b_3+b_4-\frac{b_4b_3-b_1b_2}{(b_4+b_3)-(b_1+b_2)}]+\frac{1}{3}[c_1+c_2+c_3+c_4-\frac{c_4c_3-c_1c_2}{(c_4+c_3)-(c_1+c_2)}]\end{array}\right\}$$

$$=\frac{1}{9}[\sum_{i=1}^{4}a_i+\sum_{i=1}^{4}b_i+\sum_{i=1}^{4}c_i-\frac{a_4a_3-a_1a_2}{(a_4+a_3)-(a_1+a_2)}-\frac{b_4b_3-b_1b_2}{(b_4+b_3)-(b_1+b_2)}-\frac{c_4c_3-c_1c_2}{(c_4+c_3)-(c_1+c_2)}]$$

$$y(\tilde{n})=\frac{y_o^T(\tilde{n})+y_o^I(\tilde{n})+y_o^F(\tilde{n})}{3}$$

$$=\frac{\frac{1}{3}[1+\frac{a_3-a_2}{(a_4+a_3)-(a_1+a_2)}]+\frac{1}{3}[1+\frac{b_3-b_2}{(b_4+b_3)-(b_1+b_2)}]+\frac{1}{3}[1+\frac{c_3-c_2}{(c_4+c_3)-(c_1+c_2)}]}{3}$$

$$=\frac{1}{9}[3+\frac{a_3-a_2}{(a_4+a_3)-(a_1+a_2)}+\frac{b_3-b_2}{(b_4+b_3)-(b_1+b_2)}+\frac{c_3-c_2}{(c_4+c_3)-(c_1+c_2)}]$$

Definition1: Let $\tilde{n}_1=\langle(a_1,a_2,a_3,a_4),(b_1,b_2,b_3,b_4),(c_1,c_2,c_3,c_4)\rangle$ and $\tilde{n}_2=\langle(e_1,e_2,e_3,e_4),(f_1,f_2,f_3,f_4),(g_1,g_2,g_3,g_4)\rangle$ be two trapezoidal fuzzy neutrosophic numbers, and their centroids are $O_1=(x(\tilde{n}_1), y(\tilde{n}_1))$, $O_2=(x(\tilde{n}_2), y(\tilde{n}_2))$ respectively, then the distance between $\tilde{n}_1$ and $\tilde{n}_2$ is

$$D(\tilde{n}_1,\tilde{n}_2)=\frac{1}{9}\sqrt{\begin{array}{l}[\sum_{i=1}^{4}a_i+\sum_{i=1}^{4}b_i+\sum_{i=1}^{4}c_i-\sum_{i=1}^{4}e_i-\sum_{i=1}^{4}f_i-\sum_{i=1}^{4}g_i-(\frac{a_4a_3-a_1a_2}{(a_4+a_3)-(a_1+a_2)}-\frac{e_4e_3-e_1e_2}{(e_4+e_3)-(e_1+e_2)}) \\ -(\frac{b_4b_3-b_1b_2}{(b_4+b_3)-(b_1+b_2)}-\frac{f_4f_3-f_1f_2}{(f_4+f_3)-(f_1+f_2)})-(\frac{c_4c_3-c_1c_2}{(c_4+c_3)-(c_1+c_2)}-\frac{g_4g_3-g_1g_2}{(g_4+g_3)-(g_1+g_2)})]^2 \\ +[(\frac{a_4a_3-a_1a_2}{(a_4+a_3)-(a_1+a_2)}-\frac{e_4e_3-e_1e_2}{(e_4+e_3)-(e_1+e_2)})+(\frac{b_4b_3-b_1b_2}{(b_4+b_3)-(b_1+b_2)}-\frac{f_4f_3-f_1f_2}{(f_4+f_3)-(f_1+f_2)}) \\ +(\frac{c_4c_3-c_1c_2}{(c_4+c_3)-(c_1+c_2)}-\frac{g_4g_3-g_1g_2}{(g_4+g_3)-(g_1+g_2)})]^2\end{array}}$$

Theorem 1: This distance $D(\tilde{n}_1,\tilde{n}_2)$ of $\tilde{n}_1$ and $\tilde{n}_2$ fulfills the following properties:

1. $0 \le D(\tilde{n}_1, \tilde{n}_2) \le 1$;

2. $D(\tilde{n}_1, \tilde{n}_2) = 0$ if and only if $\tilde{n}_1 = \tilde{n}_2$, i.e., $a_i = e_i$, $b_i = f_i$ and $c_i = g_i$ hold for $i = 1, 2, 3, 4$;

3. $D(\tilde{n}_1, \tilde{n}_2) = D(\tilde{n}_2, \tilde{n}_1)$.

4. If $\tilde{n}_1, \tilde{n}_2 \,\&\, \tilde{n}_3$ are the trapezoidal fuzzy neutrosophic numbers then

$$D(\tilde{n}_1, \tilde{n}_3) \le D(\tilde{n}_1, \tilde{n}_2) + D(\tilde{n}_2, \tilde{n}_3)$$

Proof

1. It is easy to prove $0 \le D(\tilde{n}_1, \tilde{n}_2)$. In addition, it can be seen from figure 1, the maximum distance is the distance between the point $(0,0)$ and the point $(1,1)$, or the point $(0,1)$ and the point $(1,0)$, assume the coordinates of centroids of $\tilde{n}_1$ and $\tilde{n}_2$ are O_1 and O_2, and $O_1 = (0,1)$ and $O_2 = (1,0)$, or $O_1 = (1,0)$ and $O_2 = (0,1)$, or $O_1 = (0,0)$ and $O_2 = (1,1)$, or $O_1 = (1,1)$ and $O_2 = (0,0)$, then the $D(\tilde{n}_1, \tilde{n}_2) = 1$, otherwise, $D(\tilde{n}_1, \tilde{n}_2) < 1$, thus $0 \le D(\tilde{n}_1, \tilde{n}_2) \le 1$.

2. if $\tilde{n}_1 = \tilde{n}_2$, i.e., $a_i = e_i$, $b_i = f_i$ and $c_i = g_i$, then

$$D(\tilde{n}_1,\tilde{n}_2)=\frac{1}{9}\sqrt{\begin{array}{l}[\sum_{i=1}^{4}a_i+\sum_{i=1}^{4}b_i+\sum_{i=1}^{4}c_i-\sum_{i=1}^{4}a_i-\sum_{i=1}^{4}b_i-\sum_{i=1}^{4}c_i-(\frac{a_4a_3-a_1a_2}{(a_4+a_3)-(a_1+a_2)}-\frac{a_4a_3-a_1a_2}{(a_4+a_3)-(a_1+a_2)})\\ -(\frac{b_4b_3-b_1b_2}{(b_4+b_3)-(b_1+b_2)}-\frac{b_4b_3-b_1b_2}{(b_4+b_3)-(b_1+b_2)})-(\frac{c_4c_3-c_1c_2}{(c_4+c_3)-(c_1+c_2)}-\frac{c_4c_3-c_1c_2}{(c_4+c_3)-(c_1+c_2)})]^2\\ +[(\frac{a_4a_3-a_1a_2}{(a_4+a_3)-(a_1+a_2)}-\frac{a_4a_3-a_1a_2}{(a_4+a_3)-(a_1+a_2)})+(\frac{b_4b_3-b_1b_2}{(b_4+b_3)-(b_1+b_2)}-\frac{b_4b_3-b_1b_2}{(b_4+b_3)-(b_1+b_2)})\\ +(\frac{c_4c_3-c_1c_2}{(c_4+c_3)-(c_1+c_2)}-\frac{c_4c_3-c_1c_2}{(c_4+c_3)-(c_1+c_2)})]^2\end{array}}$$

$$=0.$$

if $D\left(\tilde{n}_1,\tilde{n}_2\right)=0$, then

$$\begin{array}{l}[\sum_{i=1}^{4}a_i+\sum_{i=1}^{4}b_i+\sum_{i=1}^{4}c_i-\sum_{i=1}^{4}e_i-\sum_{i=1}^{4}f_i-\sum_{i=1}^{4}g_i-(\frac{a_4a_3-a_1a_2}{(a_4+a_3)-(a_1+a_2)}-\frac{e_4e_3-e_1e_2}{(e_4+e_3)-(e_1+e_2)})\\ -(\frac{b_4b_3-b_1b_2}{(b_4+b_3)-(b_1+b_2)}-\frac{f_4f_3-f_1f_2}{(f_4+f_3)-(f_1+f_2)})-(\frac{c_4c_3-c_1c_2}{(c_4+c_3)-(c_1+c_2)}-\frac{g_4g_3-g_1g_2}{(g_4+g_3)-(g_1+g_2)})]^2\\ +[(\frac{a_4a_3-a_1a_2}{(a_4+a_3)-(a_1+a_2)}-\frac{e_4e_3-e_1e_2}{(e_4+e_3)-(e_1+e_2)})+(\frac{b_4b_3-b_1b_2}{(b_4+b_3)-(b_1+b_2)}-\frac{f_4f_3-f_1f_2}{(f_4+f_3)-(f_1+f_2)})\\ +(\frac{c_4c_3-c_1c_2}{(c_4+c_3)-(c_1+c_2)}-\frac{g_4g_3-g_1g_2}{(g_4+g_3)-(g_1+g_2)})]^2=0,\end{array}$$

Thus,

$$\begin{array}{l}\sum_{i=1}^{4}a_i+\sum_{i=1}^{4}b_i+\sum_{i=1}^{4}c_i-\sum_{i=1}^{4}e_i-\sum_{i=1}^{4}f_i-\sum_{i=1}^{4}g_i-(\frac{a_4a_3-a_1a_2}{(a_4+a_3)-(a_1+a_2)}-\frac{e_4e_3-e_1e_2}{(e_4+e_3)-(e_1+e_2)})\\ -(\frac{b_4b_3-b_1b_2}{(b_4+b_3)-(b_1+b_2)}-\frac{f_4f_3-f_1f_2}{(f_4+f_3)-(f_1+f_2)})-(\frac{c_4c_3-c_1c_2}{(c_4+c_3)-(c_1+c_2)}-\frac{g_4g_3-g_1g_2}{(g_4+g_3)-(g_1+g_2)})\\ =0,\end{array}$$

$$\begin{array}{l}(\frac{a_4a_3-a_1a_2}{(a_4+a_3)-(a_1+a_2)}-\frac{e_4e_3-e_1e_2}{(e_4+e_3)-(e_1+e_2)})+(\frac{b_4b_3-b_1b_2}{(b_4+b_3)-(b_1+b_2)}-\frac{f_4f_3-f_1f_2}{(f_4+f_3)-(f_1+f_2)})\\ +(\frac{c_4c_3-c_1c_2}{(c_4+c_3)-(c_1+c_2)}-\frac{g_4g_3-g_1g_2}{(g_4+g_3)-(g_1+g_2)})\\ =0,\end{array}$$

thus

$$\frac{a_4a_3-a_1a_2}{(a_4+a_3)-(a_1+a_2)}-\frac{e_4e_3-e_1e_2}{(e_4+e_3)-(e_1+e_2)}=0,$$
$$\frac{b_4b_3-b_1b_2}{(b_4+b_3)-(b_1+b_2)}-\frac{f_4f_3-f_1f_2}{(f_4+f_3)-(f_1+f_2)}=0,$$
$$\frac{c_4c_3-c_1c_2}{(c_4+c_3)-(c_1+c_2)}-\frac{g_4g_3-g_1g_2}{(g_4+g_3)-(g_1+g_2)}=0,$$

thus

$a_i=e_i$, $b_i=f_i$, $c_i=g_i$, that is $\tilde{n}_1=\tilde{n}_2$.

3. Since,

$$\begin{aligned}&[\sum_{i=1}^{4}a_i+\sum_{i=1}^{4}b_i+\sum_{i=1}^{4}c_i-\sum_{i=1}^{4}e_i-\sum_{i=1}^{4}f_i-\sum_{i=1}^{4}g_i-(\frac{a_4a_3-a_1a_2}{(a_4+a_3)-(a_1+a_2)}-\frac{e_4e_3-e_1e_2}{(e_4+e_3)-(e_1+e_2)})\\&-(\frac{b_4b_3-b_1b_2}{(b_4+b_3)-(b_1+b_2)}-\frac{f_4f_3-f_1f_2}{(f_4+f_3)-(f_1+f_2)})-(\frac{c_4c_3-c_1c_2}{(c_4+c_3)-(c_1+c_2)}-\frac{g_4g_3-g_1g_2}{(g_4+g_3)-(g_1+g_2)})]^2\\&+[(\frac{a_4a_3-a_1a_2}{(a_4+a_3)-(a_1+a_2)}-\frac{e_4e_3-e_1e_2}{(e_4+e_3)-(e_1+e_2)})+(\frac{b_4b_3-b_1b_2}{(b_4+b_3)-(b_1+b_2)}-\frac{f_4f_3-f_1f_2}{(f_4+f_3)-(f_1+f_2)})\\&+(\frac{c_4c_3-c_1c_2}{(c_4+c_3)-(c_1+c_2)}-\frac{g_4g_3-g_1g_2}{(g_4+g_3)-(g_1+g_2)})]^2\end{aligned}$$

$$\begin{aligned}&=[\sum_{i=1}^{4}e_i+\sum_{i=1}^{4}f_i+\sum_{i=1}^{4}g_i-\sum_{i=1}^{4}a_i-\sum_{i=1}^{4}b_i-\sum_{i=1}^{4}c_i-(\frac{e_4e_3-e_1e_2}{(e_4+e_3)-(e_1+e_2)}-\frac{a_4a_3-a_1a_2}{(a_4+a_3)-(a_1+a_2)})\\&-(\frac{f_4f_3-f_1f_2}{(f_4+f_3)-(f_1+f_2)}-\frac{b_4b_3-b_1b_2}{(b_4+b_3)-(b_1+b_2)})-(\frac{g_4g_3-g_1g_2}{(g_4+g_3)-(g_1+g_2)}-\frac{c_4c_3-c_1c_2}{(c_4+c_3)-(c_1+c_2)})]^2\\&+[(\frac{e_4e_3-e_1e_2}{(e_4+e_3)-(e_1+e_2)}-\frac{a_4a_3-a_1a_2}{(a_4+a_3)-(a_1+a_2)})+(\frac{f_4f_3-f_1f_2}{(f_4+f_3)-(f_1+f_2)}-\frac{b_4b_3-b_1b_2}{(b_4+b_3)-(b_1+b_2)})\\&+(\frac{g_4g_3-g_1g_2}{(g_4+g_3)-(g_1+g_2)}-\frac{c_4c_3-c_1c_2}{(c_4+c_3)-(c_1+c_2)})]^2\end{aligned}$$

then $D(\tilde{n}_1,\tilde{n}_2)=D(\tilde{n}_2,\tilde{n}_1)$.

4. Using Def. 8, we can prove (4).

Let $\tilde{n}_1=\langle(a_1,a_2,a_3,a_4),(b_1,b_2,b_3,b_4),(c_1,c_2,c_3,c_4)\rangle$,

$\tilde{n}_2=\langle(e_1,e_2,e_3,e_4),(f_1,f_2,f_3,f_4),(g_1,g_2,g_3,g_4)\rangle$ and

$\tilde{n}_3 = \langle (j_1, j_2, j_3, j_4), (k_1, k_2, k_3, k_4), (l_1, l_2, l_3, l_4) \rangle$ are the three trapezoidal fuzzy neutrosophic numbers then $D(\tilde{n}_1, \tilde{n}_3) \le D(\tilde{n}_1, \tilde{n}_2) + D(\tilde{n}_2, \tilde{n}_3)$

Using the results we have,

$$D(\tilde{n}_1, \tilde{n}_3) = \frac{1}{9}\sqrt{\begin{array}{l}[\sum_{i=1}^{4} a_i + \sum_{i=1}^{4} b_i + \sum_{i=1}^{4} c_i - \sum_{i=1}^{4} j_i - \sum_{i=1}^{4} k_i - \sum_{i=1}^{4} l_i - (\frac{a_4 a_3 - a_1 a_2}{(a_4 + a_3) - (a_1 + a_2)} - \frac{j_4 j_3 - j_1 j_2}{(j_4 + j_3) - (j_1 + j_2)}) \\ -(\frac{b_4 b_3 - b_1 b_2}{(b_4 + b_3) - (b_1 + b_2)} - \frac{k_4 k_3 - k_1 k_2}{(k_4 + k_3) - (k_1 + k_2)}) - (\frac{c_4 c_3 - c_1 c_2}{(c_4 + c_3) - (c_1 + c_2)} - \frac{l_4 l_3 - l_1 l_2}{(l_4 + l_3) - (l_1 + l_2)})]^2 \\ +[(\frac{a_4 a_3 - a_1 a_2}{(a_4 + a_3) - (a_1 + a_2)} - \frac{j_4 j_3 - j_1 j_2}{(j_4 + j_3) - (j_1 + j_2)}) + (\frac{b_4 b_3 - b_1 b_2}{(b_4 + b_3) - (b_1 + b_2)} - \frac{k_4 k_3 - k_1 k_2}{(k_4 + k_3) - (k_1 + k_2)}) \\ +(\frac{c_4 c_3 - c_1 c_2}{(c_4 + c_3) - (c_1 + c_2)} - \frac{l_4 l_3 - l_1 l_2}{(l_4 + l_3) - (l_1 + l_2)})]^2\end{array}}$$

$$
\leq \frac{1}{9}\sqrt{\begin{aligned}
&\{\sum_{i=1}^{4}a_i+\sum_{i=1}^{4}b_i+\sum_{i=1}^{4}c_i-\sum_{i=1}^{4}e_i-\sum_{i=1}^{4}f_i-\sum_{i=1}^{4}g_i+\sum_{i=1}^{4}e_i+\sum_{i=1}^{4}f_i+\sum_{i=1}^{4}g_i-\sum_{i=1}^{4}j_i-\sum_{i=1}^{4}k_i-\sum_{i=1}^{4}l_i\\
&-[(\frac{a_4a_3-a_1a_2}{(a_4+a_3)-(a_1+a_2)}-\frac{e_4e_3-e_1e_2}{(e_4+e_3)-(e_1+e_2)})-(\frac{e_4e_3-e_1e_2}{(e_4+e_3)-(e_1+e_2)}-\frac{j_4j_3-j_1j_2}{(j_4+j_3)-(j_1+j_2)})]\\
&-[(\frac{b_4b_3-b_1b_2}{(b_4+b_3)-(b_1+b_2)}-\frac{f_4f_3-f_1f_2}{(f_4+f_3)-(f_1+f_2)})-(\frac{f_4f_3-f_1f_2}{(f_4+f_3)-(f_1+f_2)}-\frac{k_4k_3-k_1k_2}{(k_4+k_3)-(k_1+k_2)})]\\
&-[(\frac{c_4c_3-c_1c_2}{(c_4+c_3)-(c_1+c_2)}-\frac{g_4g_3-g_1g_2}{(g_4+g_3)-(g_1+g_2)})-(\frac{g_4g_3-g_1g_2}{(g_4+g_3)-(g_1+g_2)}-\frac{l_4l_3-l_1l_2}{(l_4+l_3)-(l_1+l_2)})]\}^2\\
&+\{[(\frac{a_4a_3-a_1a_2}{(a_4+a_3)-(a_1+a_2)}-\frac{e_4e_3-e_1e_2}{(e_4+e_3)-(e_1+e_2)})+(\frac{e_4e_3-e_1e_2}{(e_4+e_3)-(e_1+e_2)}-\frac{j_4j_3-j_1j_2}{(j_4+j_3)-(j_1+j_2)})]\\
&-[(\frac{b_4b_3-b_1b_2}{(b_4+b_3)-(b_1+b_2)}-\frac{f_4f_3-f_1f_2}{(f_4+f_3)-(f_1+f_2)})+(\frac{f_4f_3-f_1f_2}{(f_4+f_3)-(f_1+f_2)}-\frac{k_4k_3-k_1k_2}{(k_4+k_3)-(k_1+k_2)})]\\
&-[(\frac{c_4c_3-c_1c_2}{(c_4+c_3)-(c_1+c_2)}-\frac{g_4g_3-g_1g_2}{(g_4+g_3)-(g_1+g_2)})+(\frac{g_4g_3-g_1g_2}{(g_4+g_3)-(g_1+g_2)}-\frac{l_4l_3-l_1l_2}{(l_4+l_3)-(l_1+l_2)})]\}^2
\end{aligned}}
$$

$$
\leq \frac{1}{9}\left[\sqrt{\begin{aligned}
&\{\sum_{i=1}^{4}a_i+\sum_{i=1}^{4}b_i+\sum_{i=1}^{4}c_i-\sum_{i=1}^{4}e_i-\sum_{i=1}^{4}f_i-\sum_{i=1}^{4}g_i+\sum_{i=1}^{4}e_i+\sum_{i=1}^{4}f_i+\sum_{i=1}^{4}g_i-\sum_{i=1}^{4}j_i-\sum_{i=1}^{4}k_i-\sum_{i=1}^{4}l_i\\
&-[(\frac{a_4a_3-a_1a_2}{(a_4+a_3)-(a_1+a_2)}-\frac{e_4e_3-e_1e_2}{(e_4+e_3)-(e_1+e_2)})-(\frac{e_4e_3-e_1e_2}{(e_4+e_3)-(e_1+e_2)}-\frac{j_4j_3-j_1j_2}{(j_4+j_3)-(j_1+j_2)})]\\
&-[(\frac{b_4b_3-b_1b_2}{(b_4+b_3)-(b_1+b_2)}-\frac{f_4f_3-f_1f_2}{(f_4+f_3)-(f_1+f_2)})-(\frac{f_4f_3-f_1f_2}{(f_4+f_3)-(f_1+f_2)}-\frac{k_4k_3-k_1k_2}{(k_4+k_3)-(k_1+k_2)})]\\
&-[(\frac{c_4c_3-c_1c_2}{(c_4+c_3)-(c_1+c_2)}-\frac{g_4g_3-g_1g_2}{(g_4+g_3)-(g_1+g_2)})-(\frac{g_4g_3-g_1g_2}{(g_4+g_3)-(g_1+g_2)}-\frac{l_4l_3-l_1l_2}{(l_4+l_3)-(l_1+l_2)})]\}^2\\
&+\{[(\frac{a_4a_3-a_1a_2}{(a_4+a_3)-(a_1+a_2)}-\frac{e_4e_3-e_1e_2}{(e_4+e_3)-(e_1+e_2)})+(\frac{e_4e_3-e_1e_2}{(e_4+e_3)-(e_1+e_2)}-\frac{j_4j_3-j_1j_2}{(j_4+j_3)-(j_1+j_2)})]\\
&-[(\frac{b_4b_3-b_1b_2}{(b_4+b_3)-(b_1+b_2)}-\frac{f_4f_3-f_1f_2}{(f_4+f_3)-(f_1+f_2)})+(\frac{f_4f_3-f_1f_2}{(f_4+f_3)-(f_1+f_2)}-\frac{k_4k_3-k_1k_2}{(k_4+k_3)-(k_1+k_2)})]\\
&-[(\frac{c_4c_3-c_1c_2}{(c_4+c_3)-(c_1+c_2)}-\frac{g_4g_3-g_1g_2}{(g_4+g_3)-(g_1+g_2)})+(\frac{g_4g_3-g_1g_2}{(g_4+g_3)-(g_1+g_2)}-\frac{l_4l_3-l_1l_2}{(l_4+l_3)-(l_1+l_2)})]\}^2
\end{aligned}}\right]^2
$$

$$
\leq \frac{1}{9}\left(\begin{aligned}
&[\sum_{i=1}^{4}a_i+\sum_{i=1}^{4}b_i+\sum_{i=1}^{4}c_i-\sum_{i=1}^{4}e_i-\sum_{i=1}^{4}f_i-\sum_{i=1}^{4}g_i-(\frac{a_4a_3-a_1a_2}{(a_4+a_3)-(a_1+a_2)}-\frac{e_4e_3-e_1e_2}{(e_4+e_3)-(e_1+e_2)})\\
&-(\frac{b_4b_3-b_1b_2}{(b_4+b_3)-(b_1+b_2)}-\frac{f_4f_3-f_1f_2}{(f_4+f_3)-(f_1+f_2)})-(\frac{c_4c_3-c_1c_2}{(c_4+c_3)-(c_1+c_2)}-\frac{g_4g_3-g_1g_2}{(g_4+g_3)-(g_1+g_2)})]^2\\
&+[(\frac{a_4a_3-a_1a_2}{(a_4+a_3)-(a_1+a_2)}-\frac{e_4e_3-e_1e_2}{(e_4+e_3)-(e_1+e_2)})+(\frac{b_4b_3-b_1b_2}{(b_4+b_3)-(b_1+b_2)}-\frac{f_4f_3-f_1f_2}{(f_4+f_3)-(f_1+f_2)})\\
&+(\frac{c_4c_3-c_1c_2}{(c_4+c_3)-(c_1+c_2)}-\frac{g_4g_3-g_1g_2}{(g_4+g_3)-(g_1+g_2)})]^2
\end{aligned}\right)^{1/2}
$$

$$+\frac{1}{9}\left(\begin{array}{l}[\sum_{i=1}^{4}e_i+\sum_{i=1}^{4}f_i+\sum_{i=1}^{4}g_i-\sum_{i=1}^{4}j_i-\sum_{i=1}^{4}k_i-\sum_{i=1}^{4}l_i-(\frac{e_4e_3-e_1e_2}{(e_4+e_3)-(e_1+e_2)}-\frac{j_4j_3-j_1j_2}{(j_4+j_3)-(j_1+j_2)})\\ -(\frac{f_4f_3-f_1f_2}{(f_4+f_3)-(f_1+f_2)}-\frac{k_4k_3-k_1k_2}{(k_4+k_3)-(k_1+k_2)})-(\frac{g_4g_3-g_1g_2}{(g_4+g_3)-(g_1+g_2)}-\frac{l_4l_3-l_1l_2}{(l_4+l_3)-(l_1+l_2)})]^2\\ +[(\frac{e_4e_3-e_1e_2}{(e_4+e_3)-(e_1+e_2)}-\frac{j_4j_3-j_1j_2}{(j_4+j_3)-(j_1+j_2)})+(\frac{f_4f_3-f_1f_2}{(f_4+f_3)-(f_1+f_2)}-\frac{k_4k_3-k_1k_2}{(k_4+k_3)-(k_1+k_2)})\\ +(\frac{g_4g_3-g_1g_2}{(g_4+g_3)-(g_1+g_2)}-\frac{l_4l_3-l_1l_2}{(l_4+l_3)-(l_1+l_2)})]^2\end{array}\right)^{1/2}$$

Using Def.9 we have,

$$\leq\frac{1}{9}\sqrt{\begin{array}{l}[\sum_{i=1}^{4}a_i+\sum_{i=1}^{4}b_i+\sum_{i=1}^{4}c_i-\sum_{i=1}^{4}e_i-\sum_{i=1}^{4}f_i-\sum_{i=1}^{4}g_i-(\frac{a_4a_3-a_1a_2}{(a_4+a_3)-(a_1+a_2)}-\frac{e_4e_3-e_1e_2}{(e_4+e_3)-(e_1+e_2)})\\ -(\frac{b_4b_3-b_1b_2}{(b_4+b_3)-(b_1+b_2)}-\frac{f_4f_3-f_1f_2}{(f_4+f_3)-(f_1+f_2)})-(\frac{c_4c_3-c_1c_2}{(c_4+c_3)-(c_1+c_2)}-\frac{g_4g_3-g_1g_2}{(g_4+g_3)-(g_1+g_2)})]^2\\ +[(\frac{a_4a_3-a_1a_2}{(a_4+a_3)-(a_1+a_2)}-\frac{e_4e_3-e_1e_2}{(e_4+e_3)-(e_1+e_2)})+(\frac{b_4b_3-b_1b_2}{(b_4+b_3)-(b_1+b_2)}-\frac{f_4f_3-f_1f_2}{(f_4+f_3)-(f_1+f_2)})\\ +(\frac{c_4c_3-c_1c_2}{(c_4+c_3)-(c_1+c_2)}-\frac{g_4g_3-g_1g_2}{(g_4+g_3)-(g_1+g_2)})]^2\end{array}}$$

$$+\frac{1}{9}\sqrt{\begin{array}{l}[\sum_{i=1}^{4}e_i+\sum_{i=1}^{4}f_i+\sum_{i=1}^{4}g_i-\sum_{i=1}^{4}j_i-\sum_{i=1}^{4}k_i-\sum_{i=1}^{4}l_i-(\frac{e_4e_3-e_1e_2}{(e_4+e_3)-(e_1+e_2)}-\frac{j_4j_3-j_1j_2}{(j_4+j_3)-(j_1+j_2)})\\ -(\frac{f_4f_3-f_1f_2}{(f_4+f_3)-(f_1+f_2)}-\frac{k_4k_3-k_1k_2}{(k_4+k_3)-(k_1+k_2)})-(\frac{g_4g_3-g_1g_2}{(g_4+g_3)-(g_1+g_2)}-\frac{l_4l_3-l_1l_2}{(l_4+l_3)-(l_1+l_2)})]^2\\ +[(\frac{e_4e_3-e_1e_2}{(e_4+e_3)-(e_1+e_2)}-\frac{j_4j_3-j_1j_2}{(j_4+j_3)-(j_1+j_2)})+(\frac{f_4f_3-f_1f_2}{(f_4+f_3)-(f_1+f_2)}-\frac{k_4k_3-k_1k_2}{(k_4+k_3)-(k_1+k_2)})\\ +(\frac{g_4g_3-g_1g_2}{(g_4+g_3)-(g_1+g_2)}-\frac{l_4l_3-l_1l_2}{(l_4+l_3)-(l_1+l_2)})]^2\end{array}}$$

$\leq D(\tilde{n}_1,\tilde{n}_2)+D(\tilde{n}_2,\tilde{n}_3)$ and hence the result (4).

5- Decision Making method based on new distance measure based on centroids

In this section, we establish an approach based an trapezoidal fuzzy neutrosophic number weighted geometric arithmetic operator and a new distance measure based on centroid to deal with trapezoidal fuzzy neutrosophic information. The proposed approach is described as follows.

Step 1: Apply trapezoidal fuzzy neutrosophic number weighted geometric arithmetic operator [39] to find the aggregated trapezoidal fuzzy neutrosophic numbers for all the alternatives.

Step 2: Use the proposed distance measure, find the distances between all the alternatives and the ideal trapezoidal fuzzy neutrooshic number

Step 3: Rank the alternatives in which smaller value of distance indicate the best one.
Step 4: End

6- Numerical Example for the application of the proposed distance measure

In this section, a numerical example of a software selection problem and the aggregation operator called trapezoidal neutrosophic number weighted geometric averaging operator are get used from Ye [39] for a multiple attribute decision making problem is contributed to exhibit the application and effectiveness of the proposed distance measure under trapezoidal fuzzy neutrosophic environment. For a software selection process, consider candidate software systems are given as the set of five alternatives s_1, s_2, s_3, s_4, s_5 and the investment company need to take a decision according to four criteria: (i). the contribution to organization performance, (ii). The effort totranform from current system, (iii). The costs of hardware/software investment, (iv). The outsourcing software deneloper reliability denoted by C_1, C_2, C_3, C_4 respectively with the weight vector $\omega = (0.25, 0.25, 0.3, 0.2)^T$. The experts evaluate the five alternatives with repect to the four criteions under trapezoidal fuzzy neutrosophic environment and thus we can form the trapezoidal fuzzy neutrosophic decision matrix:

Table 1: Decision matrix using trapezoidal fuzzy neutrosophic numbers

$$D = \begin{bmatrix} \langle (0.4,0.5,0.6,0.7),(0.0,0.1,0.2,0.3),(0.1,0.1,0.1,0.1) \rangle & \langle (0.0,0.1,0.2,0.3),(0.0,0.1,0.2,0.3),(0.2,0.3,0.4,0.5) \rangle & \langle (0.3,0.4,0.5,0.6),(0.0,0.1,0.2,0.3),(0.1,0.1,0.1,0.1) \rangle & \langle (0.3,0.4,0.5,0.6),(0.1,0.1,0.1,0.1),(0.1,0.2,0.3,0.4) \rangle \\ \langle (0.3,0.4,0.5,0.5),(0.1,0.2,0.3,0.4),(0.0,0.1,0.1,0.1) \rangle & \langle (0.2,0.3,0.4,0.5),(0.0,0.1,0.2,0.3),(0.0,0.1,0.2,0.3) \rangle & \langle (0.0,0.1,0.1,0.2),(0.1,0.1,0.1,0.1),(0.5,0.6,0.7,0.8) \rangle & \langle (0.3,0.4,0.5,0.6),(0.0,0.1,0.2,0.3),(0.1,0.1,0.1,0.2) \rangle \\ \langle (0.1,0.1,0.1,0.1),(0.1,1.1,0.1,0.1),(0.6,0.7,0.8,0.9) \rangle & \langle (0.0,0.1,0.1,0.2),(0.0,0.1,0.2,0.3),(0.3,0.4,0.5,0.6) \rangle & \langle (0.2,0.3,0.4,0.5),(0.0,0.1,0.2,0.3),(0.1,0.2,0.2,0.3) \rangle & \langle (0.1,0.2,0.3,0.4),(0.1,0.1,0.1,0.1),(0.3,0.4,0.5,0.6) \rangle \\ \langle (0.7,0.7,0.7,0.7),(0.0,0.1,0.2,0.3),(0.1,0.1,0.1,0.1) \rangle & \langle (0.4,0.5,0.6,0.7),(0.1,0.1,0.1,0.1),(0.0,0.1,0.2,0.2) \rangle & \langle (0.2,0.3,0.4,0.5),(0.0,0.1,0.2,0.3),(0.1,0.2,0.3,0.3) \rangle & \langle (0.1,0.2,0.3,0.4),(0.1,0.1,0.1,0.1),(0.1,0.1,0.1,0.1) \rangle \\ \langle (0.0,0.1,0.2,0.2),(0.1,0.1,0.1,0.1),(0.5,0.6,0.7,0.8) \rangle & \langle (0.4,0.4,0.4,0.4),(0.0,0.1,0.2,0.3),(0.0,0.1,0.2,0.3) \rangle & \langle (0.6,0.7,0.7,0.8),(0.1,0.1,0.1,0.1),(0.0,0.1,0.1,0.2) \rangle & \langle (0.1,0.2,0.3,0.3),(0.1,0.2,0.3,0.4),(0.2,0.3,0.4,0.5) \rangle \end{bmatrix}$$

Here we used the developed method to obtain the best software system(s) and it is described as follows:

Step 1: Using trapezoidal fuzzy neutrosophic weighted geometric operator in Definition 10, get the aggregated trapezoidal fuzzy neutrosophic numbers of $n_i,\ i = 1,2,3,4,5$ for the software system $S_i,\ i = 1,2,3,4,5$ as follows:

$$n_1 = \langle (0.0000, 0.2985, 0.4162, 0.5244), (0.0209, 0.1003, 0.1809, 0.2639), (0.1261, 0.1745, 0.2266, 0.2836) \rangle$$
$$n_2 = \langle (0.0000, 0.2458, 0.2919, 0.3798), (0.0563, 0.1262, 0.1984, 0.2739), (0.1879, 0.2944, 0.3717, 0.4743) \rangle$$
$$n_3 = \langle (0.0000, 0.1599, 0.1888, 0.2545), (0.0464, 0.1000, 0.1566, 0.2162), (0.3437, 0.4502, 0.5424, 0.6655) \rangle$$
$$n_4 = \langle (0.2833, 0.3885, 0.4807, 0.5658), (0.0464, 0.1000, 0.1566, 0.2162), (0.1480, 0.2276, 0.3109, 0.3109) \rangle$$
$$n_5 = \langle (0.0000, 0.2912, 0.3756, 0.3910), (0.0760, 0.1210, 0.1690, 0.2208), (0.1958, 0.3012, 0.3877, 0.5020) \rangle$$

Step 2: Use the proposed distance measure and find the distance between all $n_i,\ i = 1, 2, 3, 4, 5$ and the ideal trapezoidal fuzzy neutrosophic number $n_{Ideal} = \langle (1,1,1,1), (0,0,0,0), (0,0,0,0) \rangle$.

The obtained distances are as follows:

$$D(n_1, I) = 0.1712 = D_1$$
$$D(n_2, I) = 0.1276 = D_2$$
$$D(n_3, I) = 0.1000 = D_3$$
$$D(n_4, I) = 0.1280 = D_4$$
$$D(n_5, I) = 0.1246 = D_5$$

Step 3: Find the best alternative by considering the smaller value of the distance as the smaller value of distance indicates the best one.

Using step 2 it is found that, $D_3 > D_5 > D_2 > D_4 > D_1$ and from the ranking order, S_3 is the best is the best software system.

7- Comparative analysis for the proposed distance measure and graphical representation

In this section, a comparative study is made to show the effectiveness of the proposed distance measure with the existing methods and to show the uniqueness of the proposed graphical representation.

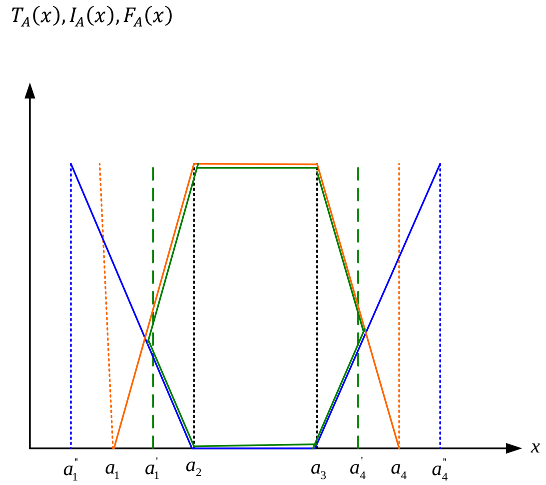
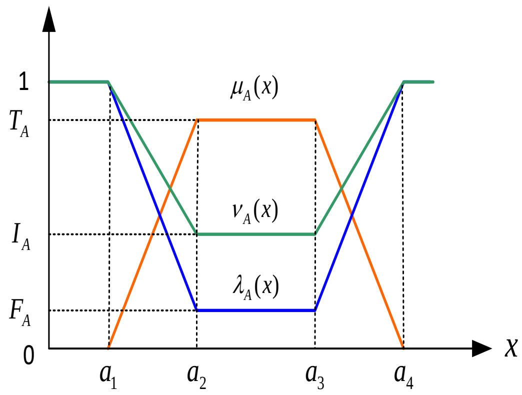
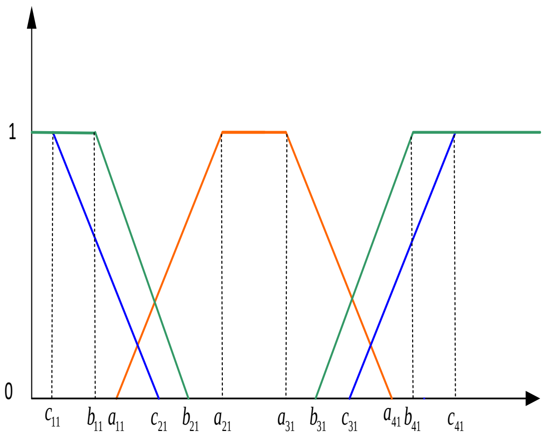
Table 2: Comparative analysis with the existing methods

Existing Methods	Score/ distance values					Ranking
	D_1	D_2	D_3	D_4	D_5	
[6]	0.6092	0.4512	0.6039	0.6121	0.6321	$S_2 > S_3 > S_1 > S_4 > S_5$
[16]	0.2788	0.6790	0.9394	0.6564	0.4014	$S_3 > S_2 > S_4 > S_5 > S_1$
[42]	0.6553	0.5779	0.5069	0.6835	0.5904	$S_4 > S_1 > S_5 > S_2 > S_3$
[45]	0.7716	0.7798	0.7349	0.8124	0.8201	$S_3 > S_1 > S_2 > S_4 > S_5$

From the Table 2, it is found that, the third software system is the best one among the five alternatives. The results in the existing methods overlaps the proposed result. Theresore the proposed methodology using the proposed under trapezoidal fuzzy neutrosophic environment to solve the decision making problem suitably in comparision with the existing methods.

Table 3 represents the various forms of trapezoidal fuzzy neutrosophic numbers (TrFNN) have been listed out and it shows the uniqueness of the proposed graphical representation among the existing graphical representations.

Table 3: Comparative analysis with the existing graphical representation

Trapezoidal fuzzy neutrosophic forms	Graphical representation
Darehmiraki [11]; A is a TrFNN, $a_1^{"}, a_1, a_1^{'}, a_2, a_3, a_4^{'}, a_4, a_4^{"} \in R$ such that $a_1^{"} \le a_1 \le a_1^{'} \le a_2 \le a_3 \le a_4^{'} \le a_4 \le a_4^{"}$ $A = \left\langle \left(a_1^{"}, a_1, a_1^{'}, a_2, a_3, a_4^{'}, a_4, a_4^{"} \right), T_A, I_A, F_A \right\rangle$	
Liang [21]; A is a TrFNN, $a_1, a_2, a_3, a_4 \in [0,1]$ such that $0 \le a_1 \le a_2 \le a_3 \le a_4 \le 1$ $A = \left\langle [a_1, a_2, a_3, a_4], (T_A, I_A, F_A) \right\rangle$	
Biswas [5]; A is a TpFNN, $(a_{41}, a_{21}, a_{31}, a_{41}), (b_{41}, b_{21}, b_{31}, b_{41}),$ $(c_{41}, c_{21}, c_{31}, c_{41}) \in R$ such that $c_{11} \le b_{11} \le a_{11} \le c_{21} \le b_{21} \le a_{21}$ $\le a_{31} \le b_{31} \le c_{31} \le a_{41} \le b_{41} \le c_{41}$ and $A = \left\langle (a_{11}, a_{21}, a_{31}, a_{41}), (b_{11}, b_{21}, b_{31}, b_{41}),\right.$ $\left.(c_{11}, c_{21}, c_{31}, c_{41}) \right\rangle$	

8-Advantages of the proposed measure

An efficient distance measure boosts the performance of task analysis or clustering. Also centroid method is specific and location based one and acquire the best geographical location in consideration of the distance between all the competences. Though the existing methods namely Euclidean measure, Manhattan measure Minkowski measure and Hamming distance measure have been applied in many real time problems they could not provide good results for the indeterminate data. Hence in this paper, we proposed a new distance measure for trapezoidal neutrosophic fuzzy numbers based on centroids and the significant advantages of the proposed measure are given as follows.

(i). Trapezoidal fuzzy neutrosophic number is a simple design of arithmetic operations and easy and perceptive interpretation as well. Therefore the proposed measure is an easy and effective one under neutrosophic environment.

(ii). Distance measure can be estimated with simple algorithm and significant level of accuracy can be acquired as well.

(iii). While taking the important decision of choosing the method to measure a distance it can be used due its simplicity.

(iv). The proposed distance measure is based on centroid and hence estimation of the distance between all objects of the data set is possible and indeterminacy also can be addressed.

(v). It is derived using Euclidean distance and hence it is very useful in correlation analysis.

(vi). Also it can be applied in location planning, operations management, Neutrosophic Statistics, clustering, medical diagnosis, Optimization and image processing to get more accurate results without any computational complexity.

9-Conclusion and Future Research

The concept of distance measure of trapezoidal fuzzy neutrosophic number has sufficient scope of utilization in different studies in various domain. In this paper, we proposed a new distance measure for the trapezoidal fuzzy neutrosophic number based on centroid with the graphical representation. Also, the properties of the proposed measure have been derived in detail. In addition, a decision making problem has been solved using the proposed measure as a numerical example. Further, comparative analysis has been done with the existing methods to show the potential of the proposed distance measure and various forms of trapezoidal fuzzy neutrosophic number have been listed and shown the uniqueness of the proposed graphical representation. Furthermore, advantages of the proposed measure are given. In future, the present work may be extended to other special types of neutrosophic set like pentagonal neutrosophic set, neutrosophic rough set, interval valued neutrosophic set and plithogeneic environments.

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