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A Novel Approach for Pairwise Separation Axioms on Bi-Soft Topology Using Neutrosophic Sets and An Output Validation in Real Life Application

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Abstract: The set that lightens the vagueness stage more energetically than fuzzy sets are neutrosophic sets. Bi-soft topological space is a space which goes for two different topologies with certain parameters. This work carries out, construction of such type of topology on neutrosophic. Besides by means of this, separation axioms are extended to pairwise separation axioms by using neutrosophic and to analyze the relationship among the class of such spaces. Here some of their properties are discussed with illustrative examples. In addition to it, we initiate the matrix form of neutrosophic soft sets in such space. Here problems deal to take a decision in life by the choice of two different groups. The aim of this decision making problem is to determine the unique thing or person from the universe by giving marks depending on parameters. Step by step process of solving the problem is explained in algorithm, also formulae given to determine their values with illustrative examples.

Keywords: Neutrosophic sets (NSs); neutrosophic soft sets (NSSs); neutrosophic soft topological spaces (NSTSs); neutrosophic soft $T_{i=0,1,2,3,4}$ -spaces (NS $T_{i=0,1,2,3,4}$ -spaces); neutrosophic bitopological spaces (NBTSs); neutrosophic bi-soft topological spaces (NBSTSs); pairwise neutrosophic soft $T_{i=0,1,2,3,4}$ -spaces (pairwise NS $T_{i=0,1,2,3,4}$ -spaces); decision making (DM).

1. Introduction

Zadeh [54], evaluated a fuzzy set (1965) to explore the situations like risky, unclear, erratic and distortion occurs in our life cycle. Fuzzy sets simplify classical sets and are unique cases of the membership functions. It has been used in a spacious collection of domains. This set extended to develop intuitionistic fuzzy set (IFS) theory (1986) by Atanassov [47]. Smarandache [7] originated a set which forecast the indeterminancy part along with truth and false statements, called NS (1998), such as blending of network arises to unpredictable states. It is a dynamic structure which postulates the concept of all other sets introduced before. Later, he generalized the NS on IFS [8] and recently proposed his work on attributes valued set, plithogenic set (PS) [9]. In day-to-day life decisions taken to diagnostic the problems either positive or negative even not both. Such types of problems are key role in all fields and so most of the researchers studied DM problem. In recent times various works
have been done on these sets by Salma and Alblowi [33] and on extension of neutrosophic analysis on DM by Abdel et al. [1-6].

Soft set (1999) is a broad mathematical gadget which accord with a group of objects based on fairly accurate descriptions with orientation to elements of a parameter set was projected by Molodtsov [46]. Topological structure on this set explored by Shabir & Naz [38] as soft topological spaces (2011). Anon this thought were developed by Ali et al. [35, 40], Bayramov and Gunduz [22, 29], Cagman et al. [37], Chen [43], Feng et al. [41], Hussain and Ahmad [36], Maji et al. [44, 45], Min [39], Nazmul and Samanta [32], Pie and Miao [42], Tantawy et al. [26], Varol and Aygun [31], Zorlutuna et al. [34]. Maji [30] presented the binding of neutrosophic with soft set termed as NSSs (2013). Bera & Mahapatra [23] defined such type of set on topological structure, named as NSTSs (2017). Using these concepts, Deli & Broumi [27], Bera & Mahapatra [10, 24], on separation axioms by Cigdem et al. [20, 21] have done some research works. Mostly DM applied on these sets related to fuzzy with multicriteria by Chinnadurai et al. [13, 14 & 19], Abishek et al. [12], Muhammad et al. [16], Mehmood et al. [17], Evanzalin Ebenanjar et al. [18] and Faruk [25].

Kelly [55] imported the approach of a set equipped with two topologies, named as bitopological space (BTS) (1963), which is the generic system of topological space. Also it was carried out by Lane [53], Patty [52], Kalaiselvi and Sindhu [15] and pairwise concepts by Kim [51], Singal and Asha [50], Lal [48], Reilly [49]. Naz, Shabir and Ali [28] introduced bi-soft topological spaces (BSTSs) (2015) and studied the separation axioms on it. Taha and Alkan [11] presented BTS on neutrosophic structure as NBSTSs (2019) which is engaged with two neutrosophic topologies (NTs).

The intension of this paper is to initiate the idea of NS on BSTS and to study some essential properties of such spaces. Also, the pairwise concept on separation axioms implemented in NBSTS. In addition, the NSSs referred as matrix form on NBSTS. As real life application, decisions made to select the one among the universe based on its parameters by considering two different groups as neutrosophic soft topologies (NSTs).

The arrangements made in this paper are as follows. Some basic definitions related to NS are in segment 2. The results of NBSTS are proved and disproved by counter examples in segment 3. The bonding among the pairwise separation axioms on NBSTS are stated with illustrative examples in segment 4. In segment 5, the method of evaluating DM problems are described in algorithm and formula specified to calculate the scores of universe set, to choose the best among them with illustrative examples. Finally, concluded with future work in segment 6.

2. Preamble

In this segment, we evoke few primary definitions associated to NSS, NSTS, BSTS and NBSTS.

**Definition 2.1** [30] Let $V$ be the set of universe and $E$ be a set of parameters. Let $NS(V)$ denote the set of all NSs of $V$. Then a estimated function of NSS $K$ over $V$ is a set defined by a mapping $f_K : E \rightarrow NS(V)$. The NSS is a parameterized family of the set $NS(V)$ which can be written as a set of ordered pairs,

$$K = \{\{e, (v, T_{f_K(e)}(v), I_{f_K(e)}(v), F_{f_K(e)}(v)) : v \in V\} : e \in E\}$$
where \( T_{f_k(e)}(v), I_{f_k(e)}(v), F_{f_k(e)}(v) \in [0, 1] \) respectively called the truth-membership, indeterminacy-membership and false-membership functions of \( f_k(e) \) and the inequality

\[
0 \leq T_{f_k(e)}(v) + I_{f_k(e)}(v) + F_{f_k(e)}(v) \leq 3
\]

is obvious.

**Definition 2.2** [23] Let \( \text{NSS}(V) \) denote the set of all NSSs of \( V \) through all \( e \in E \) and \( \tau_u \subset \text{NSS}(V,E) \). Then \( \tau_u \) is called NST on \((V, E)\) if the following conditions are satisfied.

(i) \( \phi_u, 1_u \in \tau_u \), where null NSS \( \phi_u = \{ (e, \{(v,0,0,1) : v \in V\}) : e \in E \} \) and absolute NSS \( 1_u = \{ (e, \{(v,1,1,0) : v \in V\}) : e \in E \} \).

(ii) the intersection of any finite number of members of \( \tau_u \) belongs to \( \tau_u \).

(iii) the union of any collection of members of \( \tau_u \) belongs to \( \tau_u \).

Then the triplet \((V, E, \tau_u)\) is called a NBTS.

Every member of \( \tau_u \) is called \( \tau_u \)-open NSS, whose complement is \( \tau_u \)-closed NSS.

**Definition 2.3** [21] Let \( \text{NSS}(V,E) \) denote the family of all NSSs over \( V \). The NSS \( u_{e}^{(\alpha, \beta, \gamma)} \) is called a NSP, for every \( u \in V, 0 < \alpha, \beta, \gamma \leq 1, e \in E \) and is defined as follows:

\[
u_{e}^{(\alpha, \beta, \gamma)}(e')(v) = \begin{cases} (\alpha, \beta, \gamma), & \text{if } e' = e \text{ and } v = u \\ (0,0,1), & \text{if } e' \neq e \text{ and } v \neq u \end{cases} \]

Obviously, every NSS is the union of its NSPs.

**Definition 2.4** [11] Let \((V, \tau_{u1})\) and \((V, \tau_{u2})\) be the two different NTs on \( V \). Then \((V, \tau_{u1}, \tau_{u2})\) is called a NBTS.

### 3. NBSTS

In this segment, the conception of NBSTS is defined and some key resources of topology are studied on it. The theoretical results are supported by some significant descriptive examples.

**Definition 3.1** The quadruple \((V, E, \tau_{u1}, \tau_{u2})\) is called a NBSTS over \((V, E)\), where \( \tau_{u1} \) and \( \tau_{u2} \) are NSTs independently satisfy the axioms of NSTS.

The elements of \( \tau_{u1} \) are \( \tau_{u1} \)-neutrosophic soft open sets (\( \tau_{u1} \)-NSOSs) and the complement of it are \( \tau_{u1} \)-neutrosophic soft closed sets (\( \tau_{u1} \)-NSCSs).

**Example 3.2** Let \( V = \{v_1, v_2\} \), \( E = \{e_1, e_2\} \) and \( \tau_{u1} = \{ \phi_u, 1_u, K_1 \} \) and \( \tau_{u2} = \{ \phi_u, 1_u, L_1, L_2 \} \) where \( K_1, L_1, L_2 \) are NSSs over \((V, E)\), defined as follows

\[
f_{K_1}(e_1) = \langle v_1, (1,1,0) >, < v_2, (0,0,1) > \rangle,
\]

\[
f_{K_1}(e_2) = \langle v_1, (0,0,1) >, < v_2, (1,1,0) > \rangle
\]

and

\[
f_{L_1}(e_1) = \langle v_1, (1,0,1) >, < v_2, (0,0,1) > \rangle,
\]

\[
f_{L_1}(e_2) = \langle v_1, (0,0,1) >, < v_2, (1,0,1) > \rangle.
\]
Thus $\mathcal{r}_{u1}$ and $\mathcal{r}_{u2}$ are NSTs on $(V, E)$ and so $(V, E, \mathcal{r}_{u1}, \mathcal{r}_{u2})$ is a NBSTS over $(V, E)$.

**Example 3.3** Let the neutrosophic soft indiscrete (trivial) topology $\mathcal{r}_{u1} = \{\phi_v, 1_v\}$ and neutrosophic soft discrete topology $\mathcal{r}_{u2} = NSS(V, E)$.

Then $(V, E, \mathcal{r}_{u1}, \mathcal{r}_{u2})$ is a NBSTS over $(V, E)$.

**Definition 3.4** Let $(V, E, \mathcal{r}_{u1}, \mathcal{r}_{u2})$ be a NBSTS over $(V, E)$, where $\mathcal{r}_{u1}$ and $\mathcal{r}_{u2}$ are NSTs on $(V, E)$ and $P, Q \in NSS(V, E)$ be any two arbitrary NSSs. Suppose $\mathcal{r}_{u1} = \{P \cap K_i / K_i \in \mathcal{r}_{u1}\}$ and $\mathcal{r}_{u2} = \{Q \cap L_i / L_i \in \mathcal{r}_{u2}\}$. Then $\mathcal{r}_{u1}$ and $\mathcal{r}_{u2}$ are also NSTs on $(V, E)$. Thus $(V, E, \mathcal{r}_{u1}, \mathcal{r}_{u2})$ is a NBST subspace of $(V, E, \mathcal{r}_{u1}, \mathcal{r}_{u2})$.

**Theorem 3.5** Let $(V, E, \mathcal{r}_{u1}, \mathcal{r}_{u2})$ be a NBSTS over $(V, E)$, where $\mathcal{r}_{u1}(e)$ and $\mathcal{r}_{u2}(e)$ are defined as

$\mathcal{r}_{u1}(e) = \{f_{K}(e) / K \in \mathcal{r}_{u1}\}$

$\mathcal{r}_{u2}(e) = \{f_{L}(e) / L \in \mathcal{r}_{u2}\}$ for each $e \in E$.

Then $(V, E, \mathcal{r}_{u1}(e), \mathcal{r}_{u2}(e))$ is a NBTS over $(V, E)$.

Proof. Follows from the fact that $\mathcal{r}_{u1}$ and $\mathcal{r}_{u2}$ are NTs on $V$.

**Example 3.6** Let $V = \{v_1, v_2, v_3\}$, $E = \{e_1, e_2\}$ and $\mathcal{r}_{u1} = \{\phi_v, 1_v, K_1, K_2\}$ and $\mathcal{r}_{u2} = \{\phi_v, 1_v, L_1, L_2, L_3, L_4\}$ where $K_1, K_2, L_1, L_2, L_3, L_4$ are NSSs over $(V, E)$, defined as follows

$f_{K_1}(e_1) = \{< v_1, (1, .5, 4)>, < v_2, (6, 6, 6)>, < v_3, (3, 4, 9) > \}$,

$f_{K_1}(e_2) = \{< v_1, (8, 4, 5)>, < v_2, (7, 7, 3)>, < v_3, (7, 5, 6) > \}$ ;

$f_{K_2}(e_1) = \{< v_1, (8, .5, 1)>, < v_2, (8, 6, 5)>, < v_3, (5, 6, 4) > \}$,

$f_{K_2}(e_2) = \{< v_1, (9, 7, 1)>, < v_2, (9, 9, 2)>, < v_3, (8, 6, 3) > \}$

and

$f_{L_1}(e_1) = \{< v_1, (3, 7, 6)>, < v_2, (4, 3, 8)>, < v_3, (6, 4, 5) > \}$,

$f_{L_1}(e_2) = \{< v_1, (4, 6, 8)>, < v_2, (3, 7, 2)>, < v_3, (3, 3, 7) > \}$ ;

$f_{L_2}(e_1) = \{< v_1, (6, 6, 8)>, < v_2, (2, 9, 3)>, < v_3, (1, 2, 4) > \}$,
Thus \( r_{u1} \) and \( r_{u2} \) are NSTs on \((V, E)\) and so \((V, E, r_{u1}, r_{u2})\) is a NBSTS over \((V, E)\).

Now,

\[
\begin{align*}
\tau_{u1}(e_1) &= \left\{ \phi_1, L_1, \{ v_1, (1.5, 4), v_2, (6, 6.6), v_3, (3, 4.9) \} \right\}, \\
\tau_{u2}(e_1) &= \left\{ \phi_1, L_1, \{ v_1, (3, 7.6), v_2, (4, 3.8), v_3, (6, 4.5) \} \right\},
\end{align*}
\]

and

\[
\begin{align*}
\tau_{u1}(e_2) &= \left\{ \phi_1, L_1, \{ v_1, (8, 4.5), v_2, (7, 7.3), v_3, (7, 5.6) \} \right\}, \\
\tau_{u2}(e_2) &= \left\{ \phi_1, L_1, \{ v_1, (4, 6.8), v_2, (3, 7.2), v_3, (3, 3.7) \} \right\},
\end{align*}
\]

are NTs on \( V \).

Thus \((V, E, \tau_{u1}(e), \tau_{u2}(e))\) is a NBTS over \((V, E)\).

**Definition 3.7** Let \((V, E, \tau_{u1}, \tau_{u2})\) be a NBSTS over \((V, E)\). Then the supremum NST is \( \tau_{u1} \lor \tau_{u2} \), which is the smallest NST on \( V \) that contains \( \tau_{u1} \cup \tau_{u2} \).

**Example 3.8** Let us consider 3.5 example, where \( \tau_{u1} \) and \( \tau_{u2} \) are NSTs on \((V, E)\).

Then,

\[
K_1 \cup L_4 = P = \left\{ \begin{array}{l} f_P(e_1) = \{ v_1, (3, 7.4), v_2, (6, 6.6), v_3, (6, 4.5) \} \\ f_P(e_2) = \{ v_1, (8, 6.5), v_2, (7, 7.2), v_3, (7, 5.6) \} \end{array} \right. 
\]

and

\[
K_1 \lor L_4 = Q = \left\{ \begin{array}{l} f_Q(e_1, e_2) = \{ v_1, (3, 7.4), v_2, (6, 6.6), v_3, (6, 4.5) \} \\ f_Q(e_1, e_2) = \{ v_1, (8, 4.5), v_2, (7, 7.3), v_3, (7, 5.4) \} \\ f_Q(e_1, e_2) = \{ v_1, (1, 6.4), v_2, (6, 7.2), v_3, (3, 4.7) \} \\ f_Q(e_2, e_2) = \{ v_1, (8, 6.5), v_2, (7, 7.2), v_3, (7, 5.6) \} \end{array} \right. 
\]

Thus \( K_1 \lor L_4 \) is the smallest NSS on \( V \) that contains \( K_1 \cup L_4 \).
**Theorem 3.9** If \((V, E, \tau_{u1}, \tau_{u2})\) is a NBSTS over \((V, E)\), then \(\tau_{u1} \cap \tau_{u2}\) is a NST over \((V, E)\).

**Proof.** Let \((V, E, \tau_{u1}, \tau_{u2})\) be a NBSTS over \((V, E)\).

(i) Since \(\phi_u, 1_u \in \tau_{u1}\) and \(\phi_u, 1_u \in \tau_{u2}\), it follows that \(\phi_u, 1_u \in \tau_{u1} \cap \tau_{u2}\).

(ii) Suppose that \(\{K_i | i \in I\}\) is a family of NSSs in \(\tau_{u1} \cap \tau_{u2}\). Then \(K_i \in \tau_{u1}\) and \(K_i \in \tau_{u2}\) for all \(i \in I\).

Thus \(\bigcup_{i \in I} K_i \in \tau_{u1}\) and \(\bigcup_{i \in I} K_i \in \tau_{u2}\).

Therefore \(\bigcup_{i \in I} K_i \in \tau_{u1} \cap \tau_{u2}\).

(iii) Let \(K, L \in \tau_{u1} \cap \tau_{u2}\).

Then \(K, L \in \tau_{u1}\) and \(K, L \in \tau_{u2}\).

Since \(K \cap L \in \tau_{u1}\) and \(K \cap L \in \tau_{u2}\), we have \(K \cap L \in \tau_{u1} \cap \tau_{u2}\).

Hence \(\tau_{u1} \cap \tau_{u2}\) is a NST over \((V, E)\).

**Remark 3.10** If \((V, E, \tau_{u1}, \tau_{u2})\) is a NBSTS over \((V, E)\), then \(\tau_{u1} \cup \tau_{u2}\) need not be a NST over \((V, E)\).

**Example 3.11** Let us consider 3.5 example where \(\tau_{u1}\) and \(\tau_{u2}\) are NSTs on \((V, E)\).

Then,

\[
K_1 \cup L_4 = P = \begin{cases}
   f_{p_1}(e_1) = \{< v_1, (0.3, 0.7, 0.4) >, < v_2, (0.6, 0.6, 0.6) >, < v_3, (0.6, 0.4, 0.5) >
   
   f_{p_2}(e_2) = \{< v_1, (0.8, 0.6, 0.5) >, < v_2, (0.7, 0.7, 0.2) >, < v_3, (0.7, 0.5, 0.6) >
\end{cases}
\]

Thus \(K_1 \cup L_4 \notin \tau_{u1} \cup \tau_{u2}\).

Hence \(\tau_{u1} \cup \tau_{u2}\) is not a NST over \((V, E)\).

### 4. Neutrosophic bi-soft separation axioms

In this segment, the separation of NBSTS is explored. The pairwise NS \(T_{i=0,1,2,3,4}\)-spaces on NBSTS are introduced and the relationships among them are examined with relevant examples.

**Definition 4.1** A NBSTS \((V, E, \tau_{u1}, \tau_{u2})\) over \((V, E)\) is called a pairwise NS \(T_0\)-space, if \(u^{(\alpha, \beta, \gamma)}_{(c)}\) and \(v^{(\alpha, \beta, \gamma)}_{(c)}\) are distinct NSPs then there exist \(\tau_{u1}\)-NSOS \(K\) and \(\tau_{u2}\)-NSOS \(L\) such that

\[
u^{(\alpha, \beta, \gamma)}_{(c)} \in K \quad ; \quad u^{(\alpha, \beta, \gamma)}_{(c)} \cap L = \phi_u
\]

or \(v^{(\alpha, \beta, \gamma)}_{(c)} \in L \quad ; \quad v^{(\alpha, \beta, \gamma)}_{(c)} \cap K = \phi_u\).
Example 4.2 Consider neutrosophic soft indiscrete (trivial) topology \( \tau_{u1} = \{ \phi, u \} \) and neutrosophic soft discrete topology \( \tau_{u2} = \text{NSS}(U, E) \). Thus \((V, E, \tau_{u1}, \tau_{u2})\) is a pairwise NS \( T_0 \)-space.

Theorem 4.3 Let \((V, E, \tau_{u1}, \tau_{u2})\) be a NBSTS over \((V, E)\). If \((V, E, \tau_{u1}, \tau_{u2})\) is a pairwise NS \( T_0 \)-space then \((V, E, \tau_{u1} \vee \tau_{u2})\) is a NS \( T_0 \)-space.

Proof. Let \((V, E, \tau_{u1}, \tau_{u2})\) be a NBSTS over \((V, E)\).

Suppose that \((V, E, \tau_{u1}, \tau_{u2})\) is a pairwise NS \( T_0 \)-space.

Let \( u^{(\alpha, \beta, \gamma)}_{(e)} \) and \( v^{(\alpha, \beta, \gamma)}_{(e)} \) be any two distinct NSPs.

Then there exist \( \tau_{u1} \)-NSOS K and \( \tau_{u2} \)-NSOS L such that

\[
u^{(\alpha, \beta, \gamma)}_{(e)} \in K \ ; \ u^{(\alpha, \beta, \gamma)} \cap L = \phi_u
\]

or

\[
v^{(\alpha, \beta, \gamma)}_{(e)} \in L \ ; \ v^{(\alpha, \beta, \gamma)} \cap K = \phi_u
\]

In either case \( K, L \in \tau_{u1} \cup \tau_{u2} \).

Hence \((V, E, \tau_{u1} \cup \tau_{u2})\) is a NS \( T_0 \)-space.

Theorem 4.4 Let \((V, E, \tau_{u1}, \tau_{u2})\) be a NBSTS over \((V, E)\). If \((V, E, \tau_{u1}, \tau_{u2})\) is a pairwise NS \( T_0 \)-space then \((V, E, \tau_{u1}, \tau_{u2})\) is also a pairwise NS \( T_0 \)-space.

Proof. Let \((V, E, \tau_{u1}, \tau_{u2})\) be a NBSTS over \((V, E)\).

Let \( u^{(\alpha, \beta, \gamma)}_{(e)} \) and \( v^{(\alpha, \beta, \gamma)}_{(e)} \) be any two distinct NSPs and \( P, Q \in \text{NSS}(U, E) \).

Suppose that \((V, E, \tau_{u1}, \tau_{u2})\) is a pairwise NS \( T_0 \)-space.

Then there exist \( \tau_{u1} \)-NSOS K and \( \tau_{u2} \)-NSOS L such that

\[
u^{(\alpha, \beta, \gamma)}_{(e)} \in K \ ; \ u^{(\alpha, \beta, \gamma)} \cap L = \phi_u
\]

or

\[
v^{(\alpha, \beta, \gamma)}_{(e)} \in L \ ; \ v^{(\alpha, \beta, \gamma)} \cap K = \phi_u
\]

Now \( u^{(\alpha, \beta, \gamma)}_{(e)} \in P \) and \( u^{(\alpha, \beta, \gamma)}_{(e)} \in K \).

Then \( u^{(\alpha, \beta, \gamma)}_{(e)} \in P \cap K \), where \( K \in \tau_{u1} \).

Consider \( u^{(\alpha, \beta, \gamma)}_{(e)} \cap L = \phi_u \).

\[
\Rightarrow u^{(\alpha, \beta, \gamma)}_{(e)} \cap L \cap Q = \phi_u \cap Q.
\]

\[
\Rightarrow u^{(\alpha, \beta, \gamma)}_{(e)} \cap (Q \cap L) = \phi_u.
\]

Thus \( u^{(\alpha, \beta, \gamma)}_{(e)} \in P \cap K \); \( u^{(\alpha, \beta, \gamma)}_{(e)} \cap (Q \cap L) = \phi_u \), where \( P \cap K \in \tau_{u1} \), \( Q \cap L \in \tau_{u2} \).
Or if $v_{(c)}^{(\alpha, \beta, \gamma)} \in L$ ; $v_{(c')}^{(\alpha, \beta, \gamma)} \cap K = \phi$, it can be proved that

$v_{(c)}^{(\alpha, \beta, \gamma)} \in Q \cap L$ ; $v_{(c')}^{(\alpha, \beta, \gamma)} \cap (Q \cap L) = \phi$, where $P \cap K \in \tau_{u_1}$, $Q \cap L \in \tau_{u_2}$.

Hence $(V, E, \tau_{u_1}, \tau_{u_2})$ is also a pairwise NS $T_0$-space.

**Definition 4.5** A NBSTS $(V, E, \tau_{u_1}, \tau_{u_2})$ over $(V, E)$ is called pairwise NS $T_1$-space, if $u_{(c)}^{(\alpha, \beta, \gamma)}$ and $v_{(c')}^{(\alpha, \beta, \gamma)}$ are distinct NSPs then there exist $\tau_{u_1}$-NSOS $K$ and $\tau_{u_2}$-NSOS $L$ such that

$u_{(c)}^{(\alpha, \beta, \gamma)} \in K$ ; $u_{(c)}^{(\alpha, \beta, \gamma)} \cap L = \phi$.

and $v_{(c')}^{(\alpha, \beta, \gamma)} \in L$ ; $v_{(c')}^{(\alpha, \beta, \gamma)} \cap K = \phi$.

**Example 4.6** Let $V = \{v_1, v_2\}$, $E = \{e\}$, and $v_{(c)}^{(2,3,7)}$ and $v_{(c')}^{(9,4,1)}$ be NSPs. Let $\tau_{u_1} = \{\phi_1, L, K\}$ and $\tau_{u_2} = \{\phi_1, L\}$ where $K$ and $L$ are NSSs over $(V, E)$, defined as

$K = v_{(c)}^{(2,3,7)} = f_{K}(e) = \{< v_1, (2,3,7), v_2, (0,0,1) >\}$

and

$L = v_{(c')}^{(9,4,1)} = f_{L}(e) = \{< v_1, (0,0,1), v_2, (9,4,1) >\}$.

Thus $(V, E, \tau_{u_1}, \tau_{u_2})$ is a NBSTS over $(V, E)$.

Hence $(V, E, \tau_{u_1}, \tau_{u_2})$ is a pairwise NS $T_1$-space, also a pairwise NS $T_{0}$-space.

**Theorem 4.7** Every pairwise NS $T_1$-space is also a pairwise NS $T_{0}$-space.

Proof. Follows from the Definitions 4.1 and 4.3.

**Remark 4.8** The converse of the 4.7 theorem is not true, which is shown in the following example.

**Example 4.9** Let $V = \{v_1, v_2\}$, $E = \{e_1, e_2\}$, and $v_{(c)}^{(2,5,7)}$, $v_{(c')}^{(8,2,3)}$, $v_{(c')}^{(2,7,5)}$ and $v_{(c')}^{(1,1,9)}$ be NSPs. Let $\tau_{u_1} = \{\phi_1, L, K_1, K_2, K_3\}$ and $\tau_{u_2} = \{\phi_1, L, L_1, L_2\}$ where $K_1, K_2, K_3, L_1, L_2$ are NSSs over $(V, E)$, defined as

$K_1 = v_{(c)}^{(2,5,7)} = f_{K_{1}}(e_1) = \{< v_1, (2,5,7), v_2, (0,0,1) >\}$

and

$K_2 = v_{(c')}^{(2,5,7)} = f_{K_{1}}(e_2) = \{< v_1, (0,0,1), v_2, (0,0,1) >\}$.
\[ K_2 = V^{(1,1,9)}_{2(e_1)} = \begin{cases} f_{K_2}(e_1) = \{<v_1,(0,0,1)>,<v_2,(0,0,1)>,<v_3,1.1,9>\} \\ f_{K_2}(e_2) = \{<v_1,(0,0,1)>,<v_2,(1,1,9)>,<v_3,0,0,1>\} \end{cases} \]

\[ K_3 = K_1 \cup K_2 \]

and

\[ L_1 = V^{(2,7,5)}_{2(e_1)} = \begin{cases} f_{L_1}(e_1) = \{<v_1,(0,0,1)>,<v_2,(2,7,5)>,<v_3,0,0,1>\} \\ f_{L_1}(e_2) = \{<v_1,(0,0,1)>,<v_2,(0,0,1)>,<v_3,1,1,9>\} \end{cases} \]

\[ L_2 = \{\{2,7,5\},\{2,8,2\},\{2,7,5\},\{1,1,9\}\} = \begin{cases} f_{L_2}(e_1) = \{<v_1,(2,5,7)>,<v_2,(2,7,5)>,<v_3,0,0,1>\} \\ f_{L_2}(e_2) = \{<v_1,(2,8,2)>,<v_2,(1,1,9)>,<v_3,0,0,1>\} \end{cases} \]

Thus \((V, E, \tau_{u_1}, \tau_{u_2})\) is a NBSTS over \((V, E)\).

Hence \((V, E, \tau_{u_1}, \tau_{u_2})\) is a pairwise NS \(T_0\)-space, but not a pairwise NS \(T_1\)-space since for NSPs
\[ v^{(2,5,7)}_{(e_1)} \quad \text{and} \quad v^{(1,1,9)}_{(e_2)} \quad \text{,} \quad (V, E, \tau_{u_1}, \tau_{u_2}) \quad \text{is not a pairwise NS \(T_1\)-space.} \]

**Theorem 4.10** Let \((V, E, \tau_{u_1}, \tau_{u_2})\) be a NBSTS over \((V, E)\). If \((V, E, \tau_{u_1})\) or \((V, E, \tau_{u_2})\) is not a NS \(T_0\)-space, then \((V, E, \tau_{u_1}, \tau_{u_2})\) is a pairwise NS \(T_0\)-space but not a pairwise NS \(T_1\)-space.

Proof. Let \(K \in \tau_{u_1}\) and \(L \in \tau_{u_2}\), also \(u^{(\alpha,\beta,\gamma)}_{(e)}\) and \(v^{(\alpha,\beta,\gamma)}_{(e')}\) be any two distinct NSPs.

Suppose \((V, E, \tau_{u_1})\) is a NS \(T_0\)-space and \((V, E, \tau_{u_2})\) is not a NS \(T_0\)-space.

Then, \(u^{(\alpha,\beta,\gamma)}_{(e)} \in K\); \(u^{(\alpha,\beta,\gamma)}_{(e)} \cap L = \phi_u\)

and \(v^{(\alpha,\beta,\gamma)}_{(e')} \in L\); \(v^{(\alpha,\beta,\gamma)}_{(e')} \cap K = \phi_u\)

Thus by Definitions 4.1 and 4.3, \((V, E, \tau_{u_1}, \tau_{u_2})\) is a pairwise NS \(T_0\)-space but not a pairwise NS \(T_1\)-space.

**Theorem 4.11** Let \((V, E, \tau_{u_1}, \tau_{u_2})\) be a NBSTS over \((V, E)\). Then \((V, E, \tau_{u_1}, \tau_{u_2})\) is a pairwise NS \(T_1\)-space if and only if \((V, E, \tau_{u_1})\) and \((V, E, \tau_{u_2})\) are NS \(T_1\)-spaces.

Proof. Let \((V, E, \tau_{u_1}, \tau_{u_2})\) be a NBSTS over \((V, E)\).

Let \(u^{(\alpha,\beta,\gamma)}_{(e)}\) and \(v^{(\alpha,\beta,\gamma)}_{(e')}\) be any two distinct NSPs.

Suppose that \((V, E, \tau_{u_1})\) and \((V, E, \tau_{u_2})\) are NS \(T_1\)-spaces.

Then there exist \(\tau_{u_1}\)-NSOS \(K\) and \(\tau_{u_2}\)-NSOS \(L\) such that

\[ u^{(\alpha,\beta,\gamma)}_{(e)} \in K \quad ; \quad u^{(\alpha,\beta,\gamma)}_{(e)} \cap L = \phi_u \]

and \(v^{(\alpha,\beta,\gamma)}_{(e')} \in L \quad ; \quad v^{(\alpha,\beta,\gamma)}_{(e')} \cap K = \phi_u \)

In either case the result follows immediately.

Thus \((V, E, \tau_{u_1}, \tau_{u_2})\) is a pairwise NS \(T_1\)-space.

Conversely, assume that \((V, E, \tau_{u_1}, \tau_{u_2})\) is a pairwise NS \(T_1\)-space.
Then there exist some \( \tau_{u1} \)-NSOS \( K_1 \) and \( \tau_{u2} \)-NSOS \( L_1 \) such that
\[
u^{(a, \beta, \gamma)}_e \in K_1 \; ; \; \nu^{(a, \beta, \gamma)}_e \cap L_1 = \phi_u
\]
and
\[
u^{(a, \beta, \gamma)}_{e'} \in L_1 \; ; \; \nu^{(a, \beta, \gamma)}_{e'} \cap K_1 = \phi_u
\]
Also there exist some \( \tau_{u1} \)-NSOS \( K_2 \) and \( \tau_{u2} \)-NSOS \( L_2 \) such that
\[
u^{(a, \beta, \gamma)}_e \in K_2 \; ; \; \nu^{(a, \beta, \gamma)}_e \cap L_2 = \phi_u
\]
and
\[
u^{(a, \beta, \gamma)}_{e'} \in L_2 \; ; \; \nu^{(a, \beta, \gamma)}_{e'} \cap K_2 = \phi_u
\]
Hence \((V, E, \tau_{u1})\) and \((V, E, \tau_{u2})\) are NS \( T_1 \)-spaces.

**Theorem 4.12** Let \((V, E, \tau_{u1}, \tau_{u2})\) be a NBSTS over \((V, E)\). If \((V, E, \tau_{u1}, \tau_{u2})\) is a pairwise NS \( T_1 \)-space then \((V, E, \tau_{u1} \vee \tau_{u2})\) is a NS \( T_1 \)-space.

Proof. Let \((V, E, \tau_{u1}, \tau_{u2})\) be a NBSTS over \((V, E)\).

Suppose that \((V, E, \tau_{u1}, \tau_{u2})\) is a pairwise NS \( T_1 \)-space.

Let \( u^{(a, \beta, \gamma)}_e \) and \( v^{(a, \beta, \gamma)}_{e'} \) be any two distinct NSPs.

Then there exist \( \tau_{u1} \)-NSOS \( K \) and \( \tau_{u2} \)-NSOS \( L \) such that
\[
u^{(a, \beta, \gamma)}_e \in K \; ; \; \nu^{(a, \beta, \gamma)}_e \cap L = \phi_u
\]
and
\[
u^{(a, \beta, \gamma)}_{e'} \in L \; ; \; \nu^{(a, \beta, \gamma)}_{e'} \cap K = \phi_u
\]
In either case \( K, L \in \tau_{u1} \vee \tau_{u2} \).

Hence \((V, E, \tau_{u1} \vee \tau_{u2})\) is a NS \( T_1 \)-space.

**Theorem 4.13** Let \((V, E, \tau_{u1}, \tau_{u2})\) be a NBSTS over \((V, E)\). If \((V, E, \tau_{u1}, \tau_{u2})\) is a pairwise NS \( T_1 \)-space then \((V, E, \tau_{u1}, \tau_{u2})\) is also a pairwise NS \( T_1 \)-space.

Proof. Let \((V, E, \tau_{u1}, \tau_{u2})\) be a NBSTS over \((V, E)\).

Let \( u^{(a, \beta, \gamma)}_e \) and \( v^{(a, \beta, \gamma)}_{e'} \) be any two distinct NSPs and \( P, Q \in \text{NSS}(U, E) \).

Suppose that \((V, E, \tau_{u1}, \tau_{u2})\) is a pairwise NS \( T_1 \)-space.

Then there exist \( \tau_{u1} \)-NSOS \( K \) and \( \tau_{u2} \)-NSOS \( L \) such that
\[
u^{(a, \beta, \gamma)}_e \in K \; ; \; \nu^{(a, \beta, \gamma)}_e \cap L = \phi_u
\]
and
\[
u^{(a, \beta, \gamma)}_{e'} \in L \; ; \; \nu^{(a, \beta, \gamma)}_{e'} \cap K = \phi_u
\]
Now \( u^{(a, \beta, \gamma)}_e \in P \) and \( u^{(a, \beta, \gamma)}_e \in K \).

Then \( u^{(a, \beta, \gamma)}_e \in P \cap K \), where \( K \in \tau_{u1} \).
Consider \( u^{(a,\beta,\gamma)}_c \cap L = \phi_u \).

\[
\Rightarrow u^{(a,\beta,\gamma)}_c \cap L \cap Q = \phi_u \cap Q.
\]

\[
\Rightarrow u^{(a,\beta,\gamma)}_c \cap (Q \cap L) = \phi_u.
\]

Thus \( u^{(a,\beta,\gamma)}_c \in P \cap K \); \( u^{(a,\beta,\gamma)}_c \cap (Q \cap L) = \phi_u \), where \( P \cap K \in \tau_1 \), \( Q \cap L \in \tau_2 \).

Further if \( v^{(a,\beta,\gamma)}_e \cap L = \phi_u \), it can be proved that

\[
v^{(a,\beta,\gamma)}_e \cap Q = \phi_u \), where \( P \cap K \in \tau_1 \), \( Q \cap L \in \tau_2 \).

Hence \((V, E, \tau_1, \tau_2)\) is also a pairwise NS-\(T_1\)-space.

**Theorem 4.14** Let \((V, E, \tau_1, \tau_2)\) be a NBST over \((V, E)\). For each pair of distinct NSPs \( u^{(a,\beta,\gamma)}_c \) and \( v^{(a,\beta,\gamma)}_e \), \( u^{(a,\beta,\gamma)}_c \) is a \( \tau_{a2} \)-NSCS and \( v^{(a,\beta,\gamma)}_e \) is a \( \tau_{a1} \)-NSCS, then \((V, E, \tau_{a1}, \tau_{a2})\) is a pairwise NS-\(T_1\)-space.

**Proof.** Let \((V, E, \tau_{a1}, \tau_{a2})\) be a NBST over \((V, E)\).

Suppose that for each pair of distinct NSPs \( u^{(a,\beta,\gamma)}_c \) and \( v^{(a,\beta,\gamma)}_e \), \( u^{(a,\beta,\gamma)}_c \) is a \( \tau_{a2} \)-NSCS.

Then \( \left( u^{(a,\beta,\gamma)}_c \right)^c \) is a \( \tau_{a2} \)-NSOS.

Let \( u^{(a,\beta,\gamma)}_c \) and \( v^{(a,\beta,\gamma)}_e \) be any two distinct NSPs.

(i.e.,) \( u^{(a,\beta,\gamma)}_c \cap v^{(a,\beta,\gamma)}_e = \phi_u \).

Thus

\[
v^{(a,\beta,\gamma)}_e \in \left( u^{(a,\beta,\gamma)}_c \right)^c \text{ and } u^{(a,\beta,\gamma)}_c \cap \left( u^{(a,\beta,\gamma)}_c \right)^c = \phi_u
\] (1)

Similarily assume that for each NSP, \( v^{(a,\beta,\gamma)}_e \) is a \( \tau_{a1} \)-NSCS.

Then \( \left( v^{(a,\beta,\gamma)}_e \right)^c \) is a \( \tau_{a1} \)-NSOS such that

\[
u^{(a,\beta,\gamma)}_c \in \left( v^{(a,\beta,\gamma)}_e \right)^c \text{ and } v^{(a,\beta,\gamma)}_e \cap \left( v^{(a,\beta,\gamma)}_e \right)^c = \phi_u
\] (2)

From (1) and (2),

\[
u^{(a,\beta,\gamma)}_c \in \left( v^{(a,\beta,\gamma)}_e \right)^c \text{ and } v^{(a,\beta,\gamma)}_e \cap \left( v^{(a,\beta,\gamma)}_e \right)^c = \phi_u
\]

and

\[
v^{(a,\beta,\gamma)}_e \in \left( u^{(a,\beta,\gamma)}_c \right)^c \text{ and } u^{(a,\beta,\gamma)}_c \cap \left( u^{(a,\beta,\gamma)}_c \right)^c = \phi_u
\]
Hence \((V, E, \tau_{u1}, \tau_{u2})\) is a pairwise NS \(T_1\)-space.

**Definition 4.15** A NBSTS \((V, E, \tau_{u1}, \tau_{u2})\) over \((V, E)\) is called pairwise NS \(T_2\)-space or pairwise NS Hausdorff space, if \(u^{(a, \beta, \gamma)}_{(e)}\) and \(v^{(a, \beta, \gamma)}_{(e)}\) are distinct NSPs then there exist \(\tau_{u1}\)-NSOS \(K\) and \(\tau_{u2}\)-NSOS \(L\) such that \(u^{(a, \beta, \gamma)}_{(e)} \in K\), \(v^{(a, \beta, \gamma)}_{(e)} \in L\) and \(K \cap L = \phi_u\).

**Example 4.16** Let \(V = \{v_1, v_2\}\), \(E = \{e_1, e_2\}\), and \(v^{(2, 5, 7)}_{(1(e_1))}\), \(v^{(2, 8, 2)}_{(1(e_2))}\), \(v^{(2, 7, 5)}_{(2(e_1))}\) and \(v^{(1, 1, 9)}_{(2(e_2))}\) be NSPs. Let \(\tau_{u1} = \{\phi_u, 1_u, K_1, K_2, K_3\}\) and \(\tau_{u2} = \{\phi_u, 1_u, L_1, L_2, L_3\}\) where \(K_1, K_2, K_3, L_1, L_2, L_3\) are NSSs over \((V, E)\), defined as follows

\[
K_1 = v^{(2, 5, 7)}_{(1(e_1))} = \begin{cases} 
 f_{K_1}(e_1) = \langle v_1, (2, 5, 7) \rangle \rangle, v_2, (0, 0, 1) \rangle \\
 f_{K_1}(e_2) = \langle v_1, (0, 0, 1) \rangle \rangle, v_2, (0, 0, 1) \rangle 
\end{cases};
\]

\[
K_2 = v^{(1, 1, 9)}_{(2(e_2))} = \begin{cases} 
 f_{K_2}(e_1) = \langle v_1, (0, 0, 1) \rangle \rangle, v_2, (0, 0, 1) \rangle \\
 f_{K_2}(e_2) = \langle v_1, (0, 0, 1) \rangle \rangle, v_2, (1, 1, 9) \rangle 
\end{cases};
\]

\[
K_3 = K_1 \cup K_2
\]

Then \((V, E, \tau_{u1}, \tau_{u2})\) is a NBSTS over \((V, E)\).

Hence \((V, E, \tau_{u1}, \tau_{u2})\) is a pairwise NS \(T_2\)-space.

**Theorem 4.17** Let \((V, E, \tau_{u1}, \tau_{u2})\) be a NBSTS over \((V, E)\). If \((V, E, \tau_{u1}, \tau_{u2})\) is a pairwise NS \(T_2\)-space then \((V, E, \tau_{u1} \lor \tau_{u2})\) is a NS \(T_2\)-space.

Proof. Let \((V, E, \tau_{u1}, \tau_{u2})\) be a NBSTS over \((V, E)\).

Suppose that \((V, E, \tau_{u1}, \tau_{u2})\) is a pairwise NS \(T_2\)-space.

Let \(u^{(a, \beta, \gamma)}_{(e)}\) and \(v^{(a, \beta, \gamma)}_{(e)}\) be any two distinct NSPs.

Then there exist \(\tau_{u1}\)-NSOS \(K\) and \(\tau_{u2}\)-NSOS \(L\) such that \(u^{(a, \beta, \gamma)}_{(e)} \in K\), \(v^{(a, \beta, \gamma)}_{(e)} \in L\) and \(K \cap L = \phi_u\).

In either case \(K, L \in \tau_{u1} \lor \tau_{u2}\).

Hence \((V, E, \tau_{u1} \lor \tau_{u2})\) is a NS \(T_2\)-space.

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Theorem 4.18 Let \((V, E, r_{u1}, r_{u2})\) be a NBSTS over \((V, E)\). If \((V, E, r_{u1}, r_{u2})\) is a pairwise NS \(T_2\)-space then \((V, E, r_{u1}, r_{u2})\) is also a pairwise NS \(T_2\)-space.

Proof. Let \((V, E, r_{u1}, r_{u2})\) be a NBSTS over \((V, E)\).

Let \(u_{(e)}^{(\alpha, \beta, \gamma)}\) and \(v_{(e')}^{(\alpha, \beta, \gamma)}\) be any two distinct NSPs and \(P, Q \in \text{NSS}(U, E)\).

Suppose that \((V, E, r_{u1}, r_{u2})\) is a pairwise NS \(T_2\)-space.

Then there exist \(r_{u1}\)-NSOS \(K\) and \(r_{u2}\)-NSOS \(L\) such that

\[ u_{(e)}^{(\alpha, \beta, \gamma)} \in K, \quad v_{(e')}^{(\alpha, \beta, \gamma)} \in L \quad \text{and} \quad K \cap L = \phi_u. \]

Now \(u_{(e)}^{(\alpha, \beta, \gamma)} \in P\) and \(v_{(e')}^{(\alpha, \beta, \gamma)} \in Q\) and \(v_{(e')}^{(\alpha, \beta, \gamma)} \in L\)

Then \(u_{(e)}^{(\alpha, \beta, \gamma)} \in P \cap K, \quad v_{(e')}^{(\alpha, \beta, \gamma)} \in Q \cap L\) where \(K \in r_{u1}, \quad L \in r_{u2}\).

Consider \(K \cap L = \phi_u\).

\[ \Rightarrow (P \cap K) \cap (L \cap Q) = P \cap \phi_u \cap Q. \]

\[ \Rightarrow (P \cap K) \cap (Q \cap L) = \phi_u. \]

Thus \(u_{(e)}^{(\alpha, \beta, \gamma)} \in P \cap K, \quad v_{(e')}^{(\alpha, \beta, \gamma)} \in Q \cap L\) and \((P \cap K) \cap (Q \cap L) = \phi_u\).

Hence \((V, E, r_{u1}, r_{u2})\) is also a pairwise NS \(T_2\)-space.

Theorem 4.19 Every pairwise NS \(T_2\)-space is also a pairwise NS \(T_1\)-space.

Proof. Follows from Definitions 4.3 and 4.15.

Theorem 4.20 Let \((V, E, r_{u1}, r_{u2})\) be a NBSTS over \((V, E)\). \((V, E, r_{u1}, r_{u2})\) is a pairwise NS \(T_2\)-space if and only if for any two distinct NSPs \(u_{(e)}^{(\alpha, \beta, \gamma)}\) and \(v_{(e')}^{(\alpha, \beta, \gamma)}\), there exist \(r_{u1}\)-NSOS \(K\) containing \(u_{(e)}^{(\alpha, \beta, \gamma)}\) but not \(v_{(e')}^{(\alpha, \beta, \gamma)}\) such that \(v_{(e')}^{(\alpha, \beta, \gamma)} \notin K\).

Proof. Let \((V, E, r_{u1}, r_{u2})\) be a NBSTS over \((V, E)\).

Let \(u_{(e)}^{(\alpha, \beta, \gamma)}\) and \(v_{(e')}^{(\alpha, \beta, \gamma)}\) be any two distinct NSPs.

Suppose that \((V, E, r_{u1}, r_{u2})\) is a pairwise NS \(T_2\)-space.

Then there exist \(r_{u1}\)-NSOS \(K\) and \(r_{u2}\)-NSOS \(L\) such that

\[ u_{(e)}^{(\alpha, \beta, \gamma)} \in K, \quad v_{(e')}^{(\alpha, \beta, \gamma)} \in L \quad \text{and} \quad K \cap L = \phi_u. \]

Since \(u_{(e)}^{(\alpha, \beta, \gamma)} \cap v_{(e')}^{(\alpha, \beta, \gamma)} = \phi_u\) and \(K \cap L = \phi_u, \quad v_{(e')}^{(\alpha, \beta, \gamma)} \notin K\).

Thus \(v_{(e')}^{(\alpha, \beta, \gamma)} \notin K\).
Conversely, assume that for any two distinct NSPs $u_{(c)}^{(α,β,γ)}$ and $v_{(c')}^{(α,β,γ)}$, there exist $τ_{u_1}$-NSOS $K$ containing $u_{(c)}^{(α,β,γ)}$ but not $v_{(c')}^{(α,β,γ)}$ such that $v_{(c')}^{(α,β,γ)} ∈ K$.

Then $v_{(c')}^{(α,β,γ)} \notin (K)^c$.

Thus $K$ and $(K)^c$ are disjoint $τ_{u_1}$-NSOS and $τ_{u_1}$-NSOS containing $u_{(c)}^{(α,β,γ)}$ and $v_{(c')}^{(α,β,γ)}$ respectively.

**Theorem 4.21** Let $(V, E, τ_{u_1}, τ_{u_2})$ be a NBSTS over $(V, E)$ and $(V, E, τ_{u_1}, τ_{u_2})$ be a pairwise NS $T_1$-space for every NSP $u_{(c)}^{(α,β,γ)} ∈ K ∈ τ_{u_1}$. If there exist $τ_{u_2}$-NSOS $L$ such that $u_{(c)}^{(α,β,γ)} ∈ L ⊆ L ⊆ K$, then $(V, E, τ_{u_1}, τ_{u_2})$ is a pairwise NS $T_2$-space.

Proof. Let $(V, E, τ_{u_1}, τ_{u_2})$ be a NBSTS over $(V, E)$ and let it be a pairwise NS $T_1$-space.

Suppose that $u_{(c)}^{(α,β,γ)} ∩ v_{(c')}^{(α,β,γ)} = φ_a$.

Let $u_{(c)}^{(α,β,γ)}$ be a $τ_{u_1}$-NSCS and $v_{(c')}^{(α,β,γ)}$ be a $τ_{u_2}$-NSCS.

Then $v_{(c')}^{(α,β,γ)} ∈ τ_{u_2}$-NSOS such that

$u_{(c)}^{(α,β,γ)} ∈ v_{(c')}^{(α,β,γ)} ∈ τ_{u_2}$

Then there exist a $τ_{u_2}$-NSOS $L$ such that

$u_{(c)}^{(α,β,γ)} ∈ L ⊆ L ⊆ v_{(c')}^{(α,β,γ)} ∈ (L)^c$.

Thus $v_{(c')}^{(α,β,γ)} ∈ (L)^c$, $u_{(c)}^{(α,β,γ)} ∈ L$ and $L ∩ (L)^c = φ_a$.

Hence $(V, E, τ_{u_1}, τ_{u_2})$ is a pairwise NS $T_2$-space.

**Remark 4.22** Let $(V, E, τ_{u_1}, τ_{u_2})$ be a NBSTS over $(V, E)$. For any NSS $K$ over $(V, E)$, $(K)^{τ_{u_2}}$ denotes the NS closure of $K$ with respect to $τ_{u_2}$-NST over $(V, E)$.

**Theorem 4.23** Let $(V, E, τ_{u_1}, τ_{u_2})$ be a NBSTS over $(V, E)$. Then the following are equivalent:

1. $(V, E, τ_{u_1}, τ_{u_2})$ is a pairwise NS Hausdorff space over $(V, E)$.
2. If $u_{(c)}^{(α,β,γ)}$ and $v_{(c')}^{(α,β,γ)}$ are distinct NSPs, there exist $τ_{u_1}$-NSOS $K$ such that

$u_{(c)}^{(α,β,γ)} ∈ K$ and $v_{(c')}^{(α,β,γ)} ∈ (K)^{τ_{u_2}}$.

Proof. (1) $⇒$ (2). Suppose that $(V, E, τ_{u_1}, τ_{u_2})$ is a pairwise NS Hausdorff space over $(V, E)$.

Then there exist $τ_{u_1}$-NSOS $K$ and $τ_{u_2}$-NSOS $L$ such that

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\( u_{(e)}^{(\alpha, \beta, \gamma)} \in K \), \( v_{(e')}^{(\alpha, \beta, \gamma)} \in L \) and \( K \cap L = \phi_u \).

So that \( K \subseteq L^c \).

Since \( (K)_{v_2}^c \) is the smallest \( \tau_{v_2} \)-NSCS that contains \( K \) and \( L^c \) is a \( \tau_{v_2} \)-NSCS, then \( (K)_{v_2}^c \subseteq L^c \).

\[ \Rightarrow L \subseteq (K)_{v_2}^c. \]

Thus \( v_{(e')}^{(\alpha, \beta, \gamma)} \in L \subseteq (K)_{v_2}^c \).

Hence \( v_{(e')}^{(\alpha, \beta, \gamma)} \in \left( (K)_{v_2}^c \right)^c \).

(2) \( \Rightarrow \) (1). Let \( u_{(e)}^{(\alpha, \beta, \gamma)} \) and \( v_{(e')}^{(\alpha, \beta, \gamma)} \) be any two distinct NSPs.

By assumption, there exist \( \tau_{u_1} \)-NSOS \( K \) such that \( u_{(e)}^{(\alpha, \beta, \gamma)} \in K \) and \( v_{(e')}^{(\alpha, \beta, \gamma)} \in \left( (K)_{v_2}^c \right)^c \).

As \( (K)_{v_2}^c \) is a \( \tau_{v_2} \)-NSCS so \( L = \left( (K)_{v_2}^c \right)^c \in \tau_{v_2} \).

Now \( u_{(e)}^{(\alpha, \beta, \gamma)} \in K \), \( v_{(e')}^{(\alpha, \beta, \gamma)} \in L \) and

\[ K \cap L = K \cap \left( (K)_{v_2}^c \right)^c \]

\[ \Rightarrow K \cap \left( (K)_{v_2}^c \right)^c \]

\[ = \phi_u. \]

Thus \( K \cap L = \phi_u \).

Hence \( (V, E, \tau_{u_1}, \tau_{u_2}) \) is a pairwise NS Hausdorff space over \( (V, E) \).

**Definition 4.24** Let NSS \( (V, E) \) be the family of all NSSs over the universe \( V \) and \( u \in V \). Then \( u_{E}^{(\alpha, \beta, \gamma)} \) denotes the NSS over \( (V, E) \) for which \( u_{(e)}^{(\alpha, \beta, \gamma)} = \left[ u^{(\alpha, \beta, \gamma)} \right]_{e} \) for all \( e \in E \).

**Corollary 4.25** Let \( (V, E, \tau_{u_1}, \tau_{u_2}) \) be a pairwise NS \( T_2 \)-space over \( (V, E) \). Then for each NSP \( u_{(e)}^{(\alpha, \beta, \gamma)} \),

\[ u_{E}^{(\alpha, \beta, \gamma)} = \bigcap \left\{ (K)_{v_2}^c : u_{(e)}^{(\alpha, \beta, \gamma)} \in K \in \tau_{u_1} \right\}. \]

Proof. Let \( (V, E, \tau_{u_1}, \tau_{u_2}) \) be a pairwise NS \( T_2 \)-space over \( (V, E) \) and \( u_{(e)}^{(\alpha, \beta, \gamma)} \) be a NSP.

Then there exist a NSOS \( u_{(e)}^{(\alpha, \beta, \gamma)} \in K \in \tau_{u_1} \).

If \( u_{(e)}^{(\alpha, \beta, \gamma)} \) and \( v_{(e')}^{(\alpha, \beta, \gamma)} \) are distinct NSPs, by 4.24 theorem, there exist \( \tau_{u_1} \)-NSOS \( K \) such that
\[ u^{(\alpha, \beta, \gamma)}_e \in K \] and \[ v^{(\alpha, \beta, \gamma)}_{(e')} \in (\mathbb{R}^e)^2 \] .

\[ \Rightarrow v^{(\alpha, \beta, \gamma)}_{(e')} \notin f_{(e')} (K) \] ,

\[ \Rightarrow v^{(\alpha, \beta, \gamma)}_{(e')} \notin \bigcap_{u^{(\alpha, \beta, \gamma)}_e \in K \in \tau_{u_1}} \left( f_{(e')} (K) \right) \] for all \( e' \in E \).

Thus

\[ \bigcap_{u^{(\alpha, \beta, \gamma)}_e \in K \in \tau_{u_1}} \left( f_{(e')} (K) \right) \subseteq u^{(\alpha, \beta, \gamma)}_e \] (1)

Also it is obvious that \( u^{(\alpha, \beta, \gamma)}_e \in K \subseteq (\mathbb{R}^e)^2 \).

Thus

\[ u^{(\alpha, \beta, \gamma)}_e \subseteq \bigcap_{u^{(\alpha, \beta, \gamma)}_e \in K \in \tau_{u_1}} \left( f_{(e')} (K) \right) \] (2)

Hence from (1) and (2),

\[ u^{(\alpha, \beta, \gamma)}_e = \bigcap_{u^{(\alpha, \beta, \gamma)}_e \in K \in \tau_{u_1}} \left( f_{(e')} (K) \right) \]

**Corollary 4.26** Let \( (V, E, \tau_{u_1}, \tau_{u_2}) \) be a pairwise NS \( T_2 \)-space over \( (V, E) \). Then for each NSP \( u^{(\alpha, \beta, \gamma)}_e \),

\[ \left( u^{(\alpha, \beta, \gamma)}_e \right)^c \in \tau_{u_i} \] for \( i = 1, 2 \).

**Proof.** Let \( (V, E, \tau_{u_1}, \tau_{u_2}) \) be a pairwise NS \( T_2 \)-space over \( (V, E) \) and \( u^{(\alpha, \beta, \gamma)}_e \) be a NSP.

By 4.25 corollary,

\[ \left( u^{(\alpha, \beta, \gamma)}_e \right)^c = \bigcup \left( \left( f_{(e')} (K) \right)^c : u^{(\alpha, \beta, \gamma)}_e \in K \in \tau_{u_1} \right) \]

Since \( \left( f_{(e')} (K) \right)^c \) is a \( \tau_{u_2} \)-NSCS, then \( \left( f_{(e')} (K) \right)^c \in \tau_{u_2} \).

By the axioms of a NS topological space,

\[ \bigcup \left( \left( f_{(e')} (K) \right)^c : u^{(\alpha, \beta, \gamma)}_e \in K \in \tau_{u_1} \right) \in \tau_{u_2} \] .

Thus \( \left( u^{(\alpha, \beta, \gamma)}_e \right)^c \in \tau_{u_2} \). Similarly it can be proved that \( \left( u^{(\alpha, \beta, \gamma)}_e \right)^c \in \tau_{u_1} \).

Hence \( \left( u^{(\alpha, \beta, \gamma)}_e \right)^c \in \tau_{u_i} \) for \( i = 1, 2 \).
Definition 4.27 A NBSTS \((V, E, \tau_{u_1}, \tau_{u_2})\) over \((V, E)\) is called pairwise NS regular space, if \(K\) is a \(\tau_{u_1}\)-NSCS and \(u^{(\alpha, \beta, \gamma)}_e \cap \tau = \phi_u\) then there exist \(\tau_{u_2}\)-NSOSs \(L_1\) and \(L_2\) such that \(u^{(\alpha, \beta, \gamma)}_e \subseteq L_1, K \subseteq L_2\) and \(L_1 \cap L_2 = \phi_u\).

A NBSTS \((V, E, \tau_{u_1}, \tau_{u_2})\) over \((V, E)\) is called pairwise NS \(T_3\)-space, if it is both a pairwise NS regular space and a pairwise NS \(T_3\)-space.

Theorem 4.28 Let \((V, E, \tau_{u_1}, \tau_{u_2})\) be a NBTS over \((V, E)\). Then \((V, E, \tau_{u_1}, \tau_{u_2})\) is a pairwise NS \(T_3\)-space if and only if for every \(u^{(\alpha, \beta, \gamma)}_e \in \tau_{u_1}\), there exists \(L \in \tau_{u_2}\) such that \(u^{(\alpha, \beta, \gamma)}_e \subseteq L \subseteq \bar{L} \subseteq K\).

Proof. Let \((V, E, \tau_{u_1}, \tau_{u_2})\) be a NBTS over \((V, E)\).

Suppose that \((V, E, \tau_{u_1}, \tau_{u_2})\) is a pairwise NS \(T_3\)-space and \(u^{(\alpha, \beta, \gamma)}_e \in \tau_{u_1}\).

Since \((V, E, \tau_{u_1}, \tau_{u_2})\) is a pairwise NS \(T_3\)-space for the NSP \(u^{(\alpha, \beta, \gamma)}_e\) and \(\tau_{u_1}\)-NSCS \(K\), there exist \(\tau_{u_2}\)-NSOSs \(L_1\) and \(L_2\) such that \(u^{(\alpha, \beta, \gamma)}_e \subseteq L_1, K \subseteq L_2\) and \(L_1 \cap L_2 = \phi_u\).

Thus \(u^{(\alpha, \beta, \gamma)}_e \subseteq L_1 \subseteq \bar{L}_1 \subseteq K\).

Since \((L_2)^c\) is a \(\tau_{u_2}\)-NSCS, \(\bar{L}_1 \subseteq (L_2)^c\).

Hence \(u^{(\alpha, \beta, \gamma)}_e \subseteq L_1 \subseteq \bar{L}_1 \subseteq K\).

Conversely, let \(u^{(\alpha, \beta, \gamma)}_e \cap \tau = \phi_u\) and \(K\) be a \(\tau_{u_1}\)-NSCS.

Thus \(u^{(\alpha, \beta, \gamma)}_e \subseteq K\).

From the condition of the theorem,
\[u^{(\alpha, \beta, \gamma)}_e \subseteq L \subseteq \bar{L} \subseteq K^c\]

Then \(u^{(\alpha, \beta, \gamma)}_e \subseteq L, K \subseteq \bar{L}^c\) and \(L \cap \bar{L}^c = \phi_u\).

Hence \((V, E, \tau_{u_1}, \tau_{u_2})\) is a pairwise NS \(T_3\)-space.

Definition 4.29 A NBSTS \((V, E, \tau_{u_1}, \tau_{u_2})\) over \((V, E)\) is called pairwise NS normal space, if for every pair of disjoint \(\tau_{u_1}\)-NSCSs \(K_1\) and \(K_2\), there exists disjoint \(\tau_{u_2}\)-NSOSs \(L_1\) and \(L_2\) such that \(K_1 \subseteq L_1\) and \(K_2 \subseteq L_2\).

A NBSTS \((V, E, \tau_{u_1}, \tau_{u_2})\) over \((V, E)\) is called pairwise NS \(T_4\)-space, if it is both a pairwise NS normal space and a pairwise NS \(T_4\)-space.
Theorem 4.30 Let \((V, E, \tau_{u1}, \tau_{u2})\) be a NBSTS over \((V, E)\). Then \((V, E, \tau_{u1}, \tau_{u2})\) is a pairwise NS \(T_4\)-space if and only if for each \(\tau_{u1}\)-NSCS \(K\) and \(\tau_{u1}\)-NSOS \(L\) with \(K \subseteq L\), there exists a \(\tau_{u2}\)-NSOS \(P\) such that \(K \subseteq P \subseteq \overline{P} \subseteq L\).

Proof. Let \((V, E, \tau_{u1}, \tau_{u2})\) be a NBSTS over \((V, E)\). Suppose that \((V, E, \tau_{u1}, \tau_{u2})\) is a pairwise NS \(T_4\)-space and \(K\) be \(\tau_{u1}\)-NSCS and \(K \subseteq L \subseteq \tau_{u1}\).

Then \(L^c\) is a \(\tau_{u1}\)-NSCS and \(K \cap L^c = \phi_u\).

Since \((V, E, \tau_{u1}, \tau_{u2})\) is a pairwise NS \(T_4\)-space, there exist \(\tau_{u2}\)-NSOSs \(P_1\) and \(P_2\) such that \(K \subseteq P_1, M^c \subseteq P_2\) and \(P_1 \cap P_2 = \phi_u\).

Thus \(K \subseteq P_2 \subseteq (P_2)^c \subseteq L\).

Since \((P_2)^c\) is a \(\tau_{u2}\)-NSCS, \(\overline{P_1} \subseteq (P_2)^c\).

Hence \(K \subseteq P_1 \subseteq \overline{P_1} \subseteq L\).

Conversely, let \(K_1\) and \(K_2\) be any two disjoint \(\tau_{u1}\)-NSCSs.

Then \(K_1 \subseteq \overline{(K_2)^c}\).

From the condition of the theorem, there exists a \(\tau_{u2}\)-NSOS \(P\) such that \(K_1 \subseteq P \subseteq \overline{P} \subseteq (K_2)^c\).

Thus \(P\) and \((\overline{P})^c\) are \(\tau_{u2}\)-NSOSs.

Then \(K_1 \subseteq P\), \(K_2 \subseteq \overline{P}^c\) and \(P \cap (\overline{P})^c = \phi_u\).

Hence \((V, E, \tau_{u1}, \tau_{u2})\) is a pairwise NS \(T_4\)-space.

Example 4.31 Let \(V = \{v_1, v_2\}\), \(E = \{e_1, e_2, e_3\}\), and \(v_{1(1,2,3)}(4), v_{2(1,3,2)}(5,7), v_{3(1,2,5)}(6,7)\) and \(v_{4(1,1,9)}\) be NSPs.

Then \(\tau_{u1} = \{\phi_u, l_u, K_1, K_2, K_3, K_4, K_5, K_6, K_7\}\) and \(\tau_{u2} = \{\phi_u, l_u, l_1, l_2, l_3, l_4, l_5, l_6, l_7\}\) where \(K_1, K_2, K_3, K_4, K_5, K_6, K_7, L_1, L_2, L_3, L_4, L_5, L_6, L_7\) are NSSs over \((V, E)\), defined as follows

\[
K_1 = \begin{cases} 
  f_{K_1}(e_1) = \{< v_1, (2, 4, 3)>, < v_2, (1, 1, 0) > \} \\
  f_{K_1}(e_2) = \{< v_1, (1, 1, 0) >, < v_2, (1, 1, 0) > \} \\
  f_{K_1}(e_3) = \{< v_1, (1, 1, 0) >, < v_2, (1, 1, 0) > \}
\end{cases}
\]

\[
K_2 = \begin{cases} 
  f_{K_2}(e_1) = \{< v_1, (1, 1, 0) >, < v_2, (1, 1, 0) > \} \\
  f_{K_2}(e_2) = \{< v_1, (1, 1, 0) >, < v_2, (2, 7, 5) > \} \\
  f_{K_2}(e_3) = \{< v_1, (1, 1, 0) >, < v_2, (1, 1, 0) > \}
\end{cases}
\]

\[
K_3 = \begin{cases} 
  f_{K_3}(e_1) = \{< v_1, (1, 1, 0) >, < v_2, (1, 1, 0) > \} \\
  f_{K_3}(e_2) = \{< v_1, (1, 1, 0) >, < v_2, (1, 1, 0) > \} \\
  f_{K_3}(e_3) = \{< v_1, (2, 5, 7) >, < v_2, (1, 1, 0) > \}
\end{cases}
\]
\[ K_4 = K_1 \cap K_2 \; ; \]
\[ K_5 = K_1 \cap K_3 \; ; \]
\[ K_6 = K_2 \cap K_3 \; ; \]
\[ K_7 = K_1 \cap K_2 \cap K_3 \]

and

\[
L_4 = \begin{cases} 
 f_{L_4}(e_1) = \langle v_1, (5, 7, 1) >, \langle v_2, (0, 0, 1) > \\
 f_{L_4}(e_2) = \langle v_1, (0, 0, 1) >, \langle v_2, (0, 0, 1) > \\
 f_{L_4}(e_3) = \langle v_1, (0, 0, 1) >, \langle v_2, (0, 0, 1) > 
\end{cases} 
\]

\[
L_2 = \begin{cases} 
 f_{L_2}(e_1) = \langle v_1, (0, 0, 1) >, \langle v_2, (0, 0, 1) > \\
 f_{L_2}(e_2) = \langle v_1, (0, 0, 1) >, \langle v_2, (8, 6, 1) > \\
 f_{L_2}(e_3) = \langle v_1, (0, 0, 1) >, \langle v_2, (0, 0, 1) > 
\end{cases} 
\]

\[
L_3 = \begin{cases} 
 f_{L_3}(e_1) = \langle v_1, (0, 0, 1) >, \langle v_2, (0, 0, 1) > \\
 f_{L_3}(e_2) = \langle v_1, (0, 0, 1) >, \langle v_2, (0, 0, 1) > \\
 f_{L_3}(e_3) = \langle v_1, (7, 5, 2) >, \langle v_2, (0, 0, 1) > 
\end{cases} 
\]

\[
L_4 = L_2 \cup L_3; \\
L_5 = L_1 \cup L_3; \\
L_6 = L_2 \cup L_3; \\
L_7 = L_1 \cup L_2 \cup L_3;
\]

Thus \((V, E, S_1, S_2)\) is a NBSTS over \((V, E)\).

Consider \((S_1)\) = \{(\phi_a, \lambda_u, (K_1)^c, (K_2)^c, (K_3)^c, (K_4)^c, (K_5)^c, (K_6)^c, (K_7)^c)\}

where \((K_1)^c, (K_2)^c, (K_3)^c, (K_4)^c, (K_5)^c, (K_6)^c, (K_7)^c\) are \(S_1\)-NSCSs over \((V, E)\), defined as follows

\[
(K_1)^c = \begin{cases} 
 f_{(K_1)^c}(e_1) = \langle v_1, (3, 6, 2) >, \langle v_2, (0, 0, 1) > \\
 f_{(K_1)^c}(e_2) = \langle v_1, (0, 0, 1) >, \langle v_2, (0, 0, 1) > \\
 f_{(K_1)^c}(e_3) = \langle v_1, (0, 0, 1) >, \langle v_2, (0, 0, 1) > 
\end{cases} 
\]

\[
(K_2)^c = \begin{cases} 
 f_{(K_2)^c}(e_1) = \langle v_1, (0, 0, 1) >, \langle v_2, (0, 0, 1) > \\
 f_{(K_2)^c}(e_2) = \langle v_1, (0, 0, 1) >, \langle v_2, (5, 3, 2) > \\
 f_{(K_2)^c}(e_3) = \langle v_1, (0, 0, 1) >, \langle v_2, (0, 0, 1) > 
\end{cases} 
\]

\[
(K_3)^c = \begin{cases} 
 f_{(K_3)^c}(e_1) = \langle v_1, (0, 0, 1) >, \langle v_2, (0, 0, 1) > \\
 f_{(K_3)^c}(e_2) = \langle v_1, (0, 0, 1) >, \langle v_2, (0, 0, 1) > \\
 f_{(K_3)^c}(e_3) = \langle v_1, (7, 5, 2) >, \langle v_2, (0, 0, 1) > 
\end{cases} 
\]

\[
(K_4)^c = (K_1)^c \cup (K_2)^c \; ; \\
(K_5)^c = (K_3)^c \cup (K_4)^c \; ; \\
(K_6)^c = (K_1)^c \cup (K_3)^c \; ; \\
(K_7)^c = (K_2)^c \cup (K_3)^c \; ;
\]
\[(K_{\phi})^c = (K_2)^c \cup (K_3)^c \quad ;\]

\[(K_7)^c = (K_1)^c \cup (K_2)^c \cup (K_3)^c \]

Hence \((V, E, \tau_{u_1}, \tau_{u_2})\) is a pairwise NS \( T_4 \)-space, also a pairwise NS \( T_3 \)-space.

5. DM Problem in NBSTS

In this segment, measured the output of problem and evaluated the decision on NBSTS.

Definition 5.1 Let \( V \) be the set of universal set, \( E \) be its parameter and \( \tau_{u_1} = [\phi_u, 1_u, P] \) and \( \tau_{u_2} = [\phi_u, 1_u, Q] \) be two NSTs. Then NSSs \( P \) and \( Q \) in NBSTS \((V, E, \tau_{u_1}, \tau_{u_2})\) over \((V, E)\) are defined by \( k \times l \) matrix where every entries are marks of \( v_k \) based on each parameters \( e_j \).

\[
[p]_{k \times l} = \begin{bmatrix}
<T_{f\phi(e_1)}(v_1), F_{f\phi(e_1)}(v_1), I_{f\phi(e_1)}(v_1) > & <T_{f\phi(e_2)}(v_1), F_{f\phi(e_2)}(v_1), I_{f\phi(e_2)}(v_1) > & \cdots & <T_{f\phi(e_i)}(v_1), F_{f\phi(e_i)}(v_1), I_{f\phi(e_i)}(v_1) > \\
<T_{f\phi(e_1)}(v_2), F_{f\phi(e_1)}(v_2), I_{f\phi(e_1)}(v_2) > & <T_{f\phi(e_2)}(v_2), F_{f\phi(e_2)}(v_2), I_{f\phi(e_2)}(v_2) > & \cdots & <T_{f\phi(e_i)}(v_2), F_{f\phi(e_i)}(v_2), I_{f\phi(e_i)}(v_2) > \\
\vdots & \vdots & \ddots & \vdots \\
<T_{f\phi(e_1)}(v_k), F_{f\phi(e_1)}(v_k), I_{f\phi(e_1)}(v_k) > & <T_{f\phi(e_2)}(v_k), F_{f\phi(e_2)}(v_k), I_{f\phi(e_2)}(v_k) > & \cdots & <T_{f\phi(e_i)}(v_k), F_{f\phi(e_i)}(v_k), I_{f\phi(e_i)}(v_k) >
\end{bmatrix}
\]

and

\[
[q]_{k \times l} = \begin{bmatrix}
<T_{f\phi(e_1)}(v_1), F_{f\phi(e_1)}(v_1), I_{f\phi(e_1)}(v_1) > & <T_{f\phi(e_2)}(v_1), F_{f\phi(e_2)}(v_1), I_{f\phi(e_2)}(v_1) > & \cdots & <T_{f\phi(e_i)}(v_1), F_{f\phi(e_i)}(v_1), I_{f\phi(e_i)}(v_1) > \\
<T_{f\phi(e_1)}(v_2), F_{f\phi(e_1)}(v_2), I_{f\phi(e_1)}(v_2) > & <T_{f\phi(e_2)}(v_2), F_{f\phi(e_2)}(v_2), I_{f\phi(e_2)}(v_2) > & \cdots & <T_{f\phi(e_i)}(v_2), F_{f\phi(e_i)}(v_2), I_{f\phi(e_i)}(v_2) > \\
\vdots & \vdots & \ddots & \vdots \\
<T_{f\phi(e_1)}(v_k), F_{f\phi(e_1)}(v_k), I_{f\phi(e_1)}(v_k) > & <T_{f\phi(e_2)}(v_k), F_{f\phi(e_2)}(v_k), I_{f\phi(e_2)}(v_k) > & \cdots & <T_{f\phi(e_i)}(v_k), F_{f\phi(e_i)}(v_k), I_{f\phi(e_i)}(v_k) >
\end{bmatrix}
\]

where \( v_1, v_2, \ldots, v_k \in V \) and \( e_1, e_2, \ldots, e_l \in E \).

Clearly \( \tau_{u_1} = [\phi_u, 1_u, [P]_{k \times l}] \) and \( \tau_{u_2} = [\phi_u, 1_u, [Q]_{k \times l}] \) are also NSTs in NBSTS \((V, E, \tau_{u_1}, \tau_{u_2})\) over \((V, E)\).

Thus the outcome result (OR) of \( v \in V \) is given by the formula

\[
OR(v)^{c_j} = \left[ \frac{T_{f\phi(e_j)}(v) - F_{f\phi(e_j)}(v)}{2} \right] + \left[ \frac{I_{f\phi(e_j)}(v) - F_{f\phi(e_j)}(v)}{2} \right]
\]

where \( e_j \in E \).

The Net Result (NR) of each \( v_1, v_2, \ldots, v_k \in V \) is

\[
NR(v_i)^{c_j} = \sum_{j=1}^{l} R(v_i)^{c_j}
\]

for all \( i = 1 \) to \( k \).

Example 5.2 Let \( V = \{v_1, v_2\} \), \( E = \{e_1, e_2\} \) and \( \tau_{u_1} = [\phi_u, 1_u, [K_1]_{2 \times 2}, [K_2]_{2 \times 2}, [K_3]_{2 \times 2}, [K_4]_{2 \times 2}] \) and \( \tau_{u_2} = [\phi_u, 1_u, [L_1]_{2 \times 2}, [L_2]_{2 \times 2}] \) where \( [K_1]_{2 \times 2}, [K_2]_{2 \times 2}, [K_3]_{2 \times 2}, [K_4]_{2 \times 2}, [L_1]_{2 \times 2}, [L_2]_{2 \times 2} \) are NSSs over \((V, E)\), defined as follows.
\[
\begin{align*}
[K_1]_{2 \times 2} &= \begin{bmatrix}
<1.1,2.3> & <4.5,6> \\
<9.4,7> & <2.6,3>
\end{bmatrix}, \\
[K_2]_{2 \times 2} &= \begin{bmatrix}
<2.3,1> & <4.7,2> \\
<5.7,6> & <3.5,2>
\end{bmatrix}, \\
[K_3]_{2 \times 2} &= \begin{bmatrix}
<2.3,1> & <4.7,2> \\
<9.7,6> & <3.6,2>
\end{bmatrix}, \\
[K_4]_{2 \times 2} &= \begin{bmatrix}
<1.2,3> & <4.5,6> \\
<5.4,7> & <2.5,3>
\end{bmatrix}.
\end{align*}
\]
and
\[
\begin{align*}
[L_1]_{2 \times 2} &= \begin{bmatrix}
<5.8,1> & <3.4,2> \\
<3.1,2> & <6.7,2>
\end{bmatrix}, \\
[L_2]_{2 \times 2} &= \begin{bmatrix}
<4.7,2> & <2.3,5> \\
<2.1,6> & <3.6,9>
\end{bmatrix}.
\end{align*}
\]
Thus \((V, E, \tau_{u1}, \tau_{u2})\) is a NBSTS over \((V, E)\).

**Algorithm**

**Step 1:** List the set of things or person \(v \in V\) with their parameters \(e \in E\).

**Step 2:** Go through the records of the particulars.

**Step 3:** Collect the data for each \(v \in V\) according to all \(e \in E\).

**Step 4:** Define NSSs.

**Step 5:** Define two different topologies \(\tau_{u1}\) and \(\tau_{u2}\) where each satisfies the condition of NST and so \((V, E, \tau_{u1}, \tau_{u2})\) is a NBSTS over \((V, E)\).

**Step 6:** Form \(NSSs \in \tau_{u1}, \tau_{u2}\) matrix with collected data where \(v_k\) as rows and \(e_l\) as columns.

**Step 7:** Calculate the OR for all \(v \in V\).

**Step 8:** Calculate the NR for all \(v \in V\).

**Step 9:** Select a highest value among all the calculated NR.

**Step 10:** If two or more NR are identical, add one more parameter and repeat the process.

**Step 11:** End the process while we acquire the unique NR of \(v_k\).

**Problem 5.3** Let us suppose that there are two groups of women. First group consists of young age women (YAW, aging 20-25), say \(\tau_{u1}\), and second group consists of middle age women (MAW, aging 30-35), say \(\tau_{u2}\). Our aim is to insist both groups of women to select a saree together according to their desire and choice.

1. Let \(V = \{sr_1, sr_2, sr_3, sr_4, sr_5\}\) be the set of sample sarees and selection done by the set of parameters let it be \(E = \{c, q, d, p\}\) where is \(c =\) colour, \(q =\) quality, \(d =\) design and \(p =\) price.
2. Both groups are analyzing the sarees collections.
3. Data are collected for each sarees according to its paramaters given.
4. Convert these data as NSSs, say YAW and MAW.
5. Let \(\tau_{u1} = \{\phi_{u1}, 1u, YAW\}\) and \(\tau_{u2} = \{\phi_{u2}, 1u, MAW\}\) be two NSTs and so \((V, E, \tau_{u1}, \tau_{u2})\) is a NBSTS over \((V, E)\).
6. The matrix form of NSSs YAW and MAW are as follows:
7. The Table 5.3.1 is obtained by using the formula (5.1.1),

<table>
<thead>
<tr>
<th></th>
<th>sr1</th>
<th>sr2</th>
<th>sr3</th>
<th>sr4</th>
<th>sr5</th>
</tr>
</thead>
<tbody>
<tr>
<td>c</td>
<td>.105</td>
<td>.3575</td>
<td>-.08</td>
<td>-.225</td>
<td>.21</td>
</tr>
<tr>
<td>q</td>
<td>.375</td>
<td>.21</td>
<td>.045</td>
<td>.15</td>
<td>-.085</td>
</tr>
<tr>
<td>d</td>
<td>.06</td>
<td>.04</td>
<td>.22</td>
<td>.45</td>
<td>-.1125</td>
</tr>
<tr>
<td>p</td>
<td>.09</td>
<td>.0975</td>
<td>.0425</td>
<td>.125</td>
<td>-.2925</td>
</tr>
</tbody>
</table>

8. The Table 5.3.2 is obtained by using the formula (5.1.2),

<table>
<thead>
<tr>
<th></th>
<th>sr1</th>
<th>sr2</th>
<th>sr3</th>
<th>sr4</th>
<th>sr5</th>
</tr>
</thead>
<tbody>
<tr>
<td>c</td>
<td>.105</td>
<td>.3575</td>
<td>-.08</td>
<td>-.225</td>
<td>.21</td>
</tr>
<tr>
<td>q</td>
<td>.375</td>
<td>.21</td>
<td>.045</td>
<td>.15</td>
<td>-.085</td>
</tr>
<tr>
<td>d</td>
<td>.06</td>
<td>.04</td>
<td>.22</td>
<td>.45</td>
<td>-.1125</td>
</tr>
<tr>
<td>p</td>
<td>.09</td>
<td>.0975</td>
<td>.0425</td>
<td>.125</td>
<td>-.2925</td>
</tr>
<tr>
<td>NR</td>
<td>.63</td>
<td>.705</td>
<td>.2275</td>
<td>.2</td>
<td>-.28</td>
</tr>
</tbody>
</table>

Thus the second saree has selected by both the categories of women.

**Problem 5.4** Consider the situation of problem 5.3.
1. Let \( V = \{ sr_1, sr_2, sr_3, sr_4, sr_5 \} \) be the set of sample sarees and selection done by the set of parameters let it be \( E = \{ c, q, d, p \} \) where is \( c = \) colour, \( q = \) quality, \( d = \) design and \( p = \) price.
2. Both groups are analyzing the sarees collections.
3. Data are collected for each sarees according to its paramaters given.
4. Convert these data as NSSs, say YAW and MAW.
5. Let \( \tau_{u_1} = \{ \phi_{u_1}, I_u, YAW \} \) and \( \tau_{u_2} = \{ \phi_{u_2}, I_u, MAW \} \) be two NSTs and so \((V, E, \tau_{u_1}, \tau_{u_2})\) is a NBSTS over \((V, E)\).
6. The matrix form of NSSs YAW and MAW are as follows:
MAW, and ϕ are as follows:

\[
[YAW]_{5x4} = \begin{bmatrix}
<.5,7,2> & <.5,3,1> & <.4,2,7> & <.8,4,5>
<.9,3,4> & <.3,4,2> & <.8,7,5> & <.9,3,6>
<.1,3,4> & <.7,4,8> & <.2,7,1> & <.3,2,5>
<.2,4,6> & <.4,6,8> & <.7,2,2> & <.7,1,3>
<.7,5,2> & <.1,2,3> & <.3,6,9> & <.6,3,9>
\end{bmatrix}
\]

and

\[
[MAW]_{5x4} = \begin{bmatrix}
<.6,4,3> & <.7,2,1> & <.3,6,2> & <.4,7,3>
<.8,4,2> & <.8,4,2> & <.5,9,4> & <.4,4,4>
<.2,4,8> & <.6,7,3> & <.8,2,1> & <.5,1,2>
<.2,6,7> & <.3,4,5> & <.7,3,2> & <.2,9,1>
<.3,1,2> & <.2,1,2> & <.3,5,2> & <.1,9,2>
\end{bmatrix}
\]

7. The Table 5.4.1 is obtained by using the formula (5.1.1),

<table>
<thead>
<tr>
<th>sr₁</th>
<th>sr₂</th>
<th>sr₃</th>
<th>sr₄</th>
<th>sr₅</th>
</tr>
</thead>
<tbody>
<tr>
<td>c</td>
<td>.105</td>
<td>.3575</td>
<td>-.2925</td>
<td>-.225</td>
</tr>
<tr>
<td>q</td>
<td>.45</td>
<td>.21</td>
<td>.045</td>
<td>.15</td>
</tr>
<tr>
<td>d</td>
<td>.06</td>
<td>.04</td>
<td>.22</td>
<td>.375</td>
</tr>
<tr>
<td>p</td>
<td>.09</td>
<td>.0975</td>
<td>.0425</td>
<td>.125</td>
</tr>
</tbody>
</table>

Table 5.4.2. NR table.

<table>
<thead>
<tr>
<th>sr₁</th>
<th>sr₂</th>
<th>sr₃</th>
<th>sr₄</th>
<th>sr₅</th>
</tr>
</thead>
<tbody>
<tr>
<td>c</td>
<td>.105</td>
<td>.3575</td>
<td>-.2925</td>
<td>-.225</td>
</tr>
<tr>
<td>q</td>
<td>.45</td>
<td>.21</td>
<td>.045</td>
<td>.15</td>
</tr>
<tr>
<td>d</td>
<td>.06</td>
<td>.04</td>
<td>.22</td>
<td>.375</td>
</tr>
<tr>
<td>p</td>
<td>.09</td>
<td>.0975</td>
<td>.0425</td>
<td>.125</td>
</tr>
<tr>
<td>NR</td>
<td>.705</td>
<td>.705</td>
<td>.015</td>
<td>.125</td>
</tr>
</tbody>
</table>

Thus first and second sarees have selected by both categories of women.

In this situation, we just add a parameter f = fabric in E and repeat the process.

4. After adding one more parameter, convert these data as NSSs, say YAW* and MAW*.

5. Let \( \tau₁ = \{ ϕ₁,1_u, YAW^* \} \) and \( \tau₂ = \{ ϕ₂,1_u, MAW^* \} \) be two NSTs and so \( (V, E, \tau₁, \tau₂) \) is a NBSTS over \( (V, E) \).

6. The matrix form of NSSs YAW* and MAW* are as follows:
7. The Table 5.4.3 is obtained by using the formula (5.1.1),

<table>
<thead>
<tr>
<th></th>
<th>sr1</th>
<th>sr2</th>
<th>sr3</th>
<th>sr4</th>
<th>sr5</th>
</tr>
</thead>
<tbody>
<tr>
<td>c</td>
<td>.105</td>
<td>.3575</td>
<td>-.2925</td>
<td>-.225</td>
<td>.21</td>
</tr>
<tr>
<td>q</td>
<td>.45</td>
<td>.21</td>
<td>.045</td>
<td>.15</td>
<td>-.085</td>
</tr>
<tr>
<td>d</td>
<td>.06</td>
<td>.04</td>
<td>.22</td>
<td>.375</td>
<td>-.1125</td>
</tr>
<tr>
<td>p</td>
<td>.09</td>
<td>.0975</td>
<td>.0425</td>
<td>.125</td>
<td>-.08</td>
</tr>
<tr>
<td>f</td>
<td>.175</td>
<td>.22</td>
<td>.1</td>
<td>.0225</td>
<td>-.225</td>
</tr>
</tbody>
</table>

8. The Table 5.4.4 is obtained by using the formula (5.1.2),

<table>
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<th>sr2</th>
<th>sr3</th>
<th>sr4</th>
<th>sr5</th>
</tr>
</thead>
<tbody>
<tr>
<td>c</td>
<td>.105</td>
<td>.3575</td>
<td>-.2925</td>
<td>-.225</td>
<td>.21</td>
</tr>
<tr>
<td>q</td>
<td>.45</td>
<td>.21</td>
<td>.045</td>
<td>.15</td>
<td>-.085</td>
</tr>
<tr>
<td>d</td>
<td>.06</td>
<td>.04</td>
<td>.22</td>
<td>.375</td>
<td>-.1125</td>
</tr>
<tr>
<td>p</td>
<td>.09</td>
<td>.0975</td>
<td>.0425</td>
<td>.125</td>
<td>-.08</td>
</tr>
<tr>
<td>f</td>
<td>.175</td>
<td>.22</td>
<td>.1</td>
<td>.0225</td>
<td>-.225</td>
</tr>
<tr>
<td>NR</td>
<td>.88</td>
<td>.925</td>
<td>.115</td>
<td>.1475</td>
<td>-.2925</td>
</tr>
</tbody>
</table>

Thus the second saree has been selected by both categories of women.

**Problem 5.5** Consider the situation that there are six students on the main stage for Quiz Finale. There are two teams, each team consists of three students, one is Winner (W) and other is Runner (R). Let F1 and F2 be two final authorities to judge the event. Our problem is to find the best player in the winning team whose teammates are not mentioned here.

1. Let \( V = \{s_1, s_2, s_3, s_4, s_5, s_6\} \) be the set of students and judgement is based on the set of parameters let it be \( E = \{ra, eff, ca, mr, gp\} \) where \( ra = \) right answers, \( eff = \) effectiveness, \( ca = \) complex analysis, \( mr = \) memory, \( gp = \) grasping power.

Chinnadurai V and Sindhu M P. A Novel Approach for Pairwise Separation Axioms on Bi-Soft Topology Using Neutrosophic Sets and An Output Validation in Real Life Application
2. First of all these final authorities will go through the records of the students.

3. They will collect student’s data according to their paramaters given.

4. These data are converted into two different NSSs, say FA1 and FA2.

5. Let \( r_{st1} = \{ \phi_{st11}, FA1 \} \) and \( r_{st2} = \{ \phi_{st21}, FA2 \} \) be two NSTs and so \((V, E, r_{st1}, r_{st2})\) is a NBTS over \((V, E)\).

6. The matrix form of NSSs FA1 and FA2 are as follows:

\[
[FA1]_{5x5} = \begin{bmatrix}
<.4,.2,.7> & <.6,.3,.1> & <.2,.4,.8> & <.2,.9,.1> & <.6,.5,.3> \\
<.7,.3,.2> & <.8,.6,.1> & <.5,.4,.3> & <.9,.7,.2> & <.2,.7,.5> \\
<.3,.6,.6> & <.3,.5,.4> & <.6,.4,.2> & <.1,.2,.3> & <.5,.4,.6> \\
<.2,.6,.3> & <.7,.5,.4> & <.8,.6,.1> & <.4,.2,.7> & <.7,.3,.4> \\
<.6,.5,.4> & <.9,.2,.1> & <.7,.3,.4> & <.3,.5,.4> & <.4,.1,.4>
\end{bmatrix}
\]

and

\[
[FA2]_{5x5} = \begin{bmatrix}
<.4,.7,.3> & <.2,.3,.4> & <.5,.7,.2> & <.3,.4,.2> & <.2,.7,.1> \\
<.5,.1,.2> & <.6,.7,.3> & <.9,.3,.4> & <.7,.4,.8> & <.7,.2,.2> \\
<.7,.8,.1> & <.9,.3,.6> & <.1,.3,.4> & <.4,.6,.8> & <.3,.6,.9> \\
<.2,.6,.7> & <.7,.4,.8> & <.2,.4,.6> & <.1,.2,.3> & <.8,.4,.5> \\
<.5,.9,.4> & <.1,.2,.3> & <.7,.5,.2> & <.4,.2,.7> & <.3,.2,.5> \\
<.8,.4,.5> & <.9,.6,.3> & <.5,.3,.1> & <.1,.3,.4> & <.7,.3,.2>
\end{bmatrix}
\]

7. The Table 5.5.1 is obtained by using the formula (5.1.1),

<table>
<thead>
<tr>
<th></th>
<th>sti</th>
<th>stj</th>
<th>stk</th>
<th>stk</th>
<th>stl</th>
<th>stm</th>
</tr>
</thead>
<tbody>
<tr>
<td>ra</td>
<td>.11</td>
<td>.32</td>
<td>.045</td>
<td>-.12</td>
<td>.045</td>
<td>.195</td>
</tr>
<tr>
<td>eff</td>
<td>.105</td>
<td>.175</td>
<td>.26</td>
<td>.055</td>
<td>.24</td>
<td>.175</td>
</tr>
<tr>
<td>ca</td>
<td>.0675</td>
<td>.2275</td>
<td>.0325</td>
<td>.075</td>
<td>.24</td>
<td>.12</td>
</tr>
<tr>
<td>mr</td>
<td>.035</td>
<td>.135</td>
<td>-.018</td>
<td>-.02</td>
<td>-.13</td>
<td>.24</td>
</tr>
<tr>
<td>gp</td>
<td>.08</td>
<td>.055</td>
<td>-.175</td>
<td>.195</td>
<td>-.085</td>
<td>.18</td>
</tr>
</tbody>
</table>

8. The Table 5.5.2 is obtained by using the formula (5.1.2),

<table>
<thead>
<tr>
<th></th>
<th>sti</th>
<th>stj</th>
<th>stk</th>
<th>stk</th>
<th>stl</th>
<th>stm</th>
</tr>
</thead>
<tbody>
<tr>
<td>ra</td>
<td>.11</td>
<td>.32</td>
<td>.045</td>
<td>-.12</td>
<td>.045</td>
<td>.195</td>
</tr>
<tr>
<td>eff</td>
<td>.105</td>
<td>.175</td>
<td>.26</td>
<td>.055</td>
<td>.24</td>
<td>.175</td>
</tr>
<tr>
<td>ca</td>
<td>.0675</td>
<td>.2275</td>
<td>.0325</td>
<td>.075</td>
<td>.24</td>
<td>.12</td>
</tr>
<tr>
<td>mr</td>
<td>.035</td>
<td>.135</td>
<td>-.018</td>
<td>-.02</td>
<td>-.13</td>
<td>.24</td>
</tr>
<tr>
<td>gp</td>
<td>.08</td>
<td>.055</td>
<td>-.175</td>
<td>.195</td>
<td>-.085</td>
<td>.18</td>
</tr>
<tr>
<td>NR</td>
<td>.3975</td>
<td>.9125</td>
<td>-.0375</td>
<td>.005</td>
<td>.31</td>
<td>.91</td>
</tr>
</tbody>
</table>

Here both \( st_2 \) and \( st_6 \) got high score from judges, so they both does not belongs to \( R \).
Case (i). If $s_{t_2} \in W$ and $s_{t_6} \in R$, then the best player award goes to $s_{t_2}$.

Case (ii). If $s_{t_2} \in R$ and $s_{t_6} \in W$, then the best player award goes to $s_{t_6}$.

Case (iii). If $s_{t_2} \in W$ and $s_{t_6} \in W$, then we just add a parameter $ld = \text{leadership}$.

4. After adding one more parameter, convert these data as NSSs, say $FA_1^*$ and $FA_2^*$.

5. Let $\tau_{u_1} = \left\{ \phi_{u_1}, I_{u_1}, FA_1^* \right\}$ and $\tau_{u_2} = \left\{ \phi_{u_2}, I_{u_2}, FA_2^* \right\}$ be two NSTs and so $(V, E, \tau_{u_1}, \tau_{u_2})$ is a NBSTS over $(V, E)$.

6. The matrix form of NSSs $FA_1^*$ and $FA_2^*$ are as follows:

$$
[FA_1^*]_{6 \times 6} =
\begin{bmatrix}
<.4,.2,.7> & <.6,.3,.1> & <.2,.4,.8> & <.2,.9,.1> & <.6,.5,.3> & <.3,.2,.4>
\end{bmatrix}
\begin{bmatrix}
<.7,.3,.2> & <.8,.6,.1> & <.5,.4,.3> & <.9,.7,.2> & <.2,.7,.5> & <.9,.1,.1>
\end{bmatrix}
\begin{bmatrix}
<.3,.6,.6> & <.3,.5,.4> & <.6,.4,.2> & <.1,.2,.3> & <.5,.4,.6> & <.7,.5,.3>
\end{bmatrix}
\begin{bmatrix}
<.2,.6,.3> & <.7,.5,.4> & <.8,.6,.1> & <.4,.2,.7> & <.7,.3,.4> & <.8,.2,.1>
\end{bmatrix}
\begin{bmatrix}
<.6,.5,.4> & <.9,.2,.1> & <.7,.3,.4> & <.3,.5,.4> & <.4,.1,.4> & <.3,.1,.5>
\end{bmatrix}
\begin{bmatrix}
<.7,.3,.4> & <.6,.7,.2> & <.8,.9,.6> & <.3,.5,.4> & <.3,.5,.2> & <.6,.2,.6>
\end{bmatrix}
$$

and

$$
[FA_2^*]_{6 \times 6} =
\begin{bmatrix}
<.4,.7,.3> & <.2,.3,.4> & <.5,.7,.2> & <.3,.4,.2> & <.2,.7,.1> & <.7,.5,.4>
\end{bmatrix}
\begin{bmatrix}
<.5,.1,.2> & <.6,.7,.3> & <.9,.3,.4> & <.7,.4,.8> & <.7,.2,.2> & <.6,.9,.1>
\end{bmatrix}
\begin{bmatrix}
<.7,.8,.1> & <.9,.3,.6> & <.1,.3,.4> & <.4,.6,.8> & <.3,.6,.9> & <.8,.4,.4>
\end{bmatrix}
\begin{bmatrix}
<.2,.6,.7> & <.7,.4,.8> & <.2,.4,.6> & <.1,.2,.3> & <.8,.4,.5> & <.3,.4,.5>
\end{bmatrix}
\begin{bmatrix}
<.5,.9,.4> & <.1,.2,.3> & <.7,.5,.2> & <.4,.2,.7> & <.3,.2,.5> & <.7,.3,.2>
\end{bmatrix}
\begin{bmatrix}
<.8,.4,.5> & <.9,.6,.3> & <.5,.3,.1> & <.1,.3,.4> & <.7,.3,.2> & <.5,.9,.4>
\end{bmatrix}
$$

7. The Table 5.5.3 is obtained by using the formula (5.1.1),

<table>
<thead>
<tr>
<th></th>
<th>$s_{t_1}$</th>
<th>$s_{t_2}$</th>
<th>$s_{t_3}$</th>
<th>$s_{t_4}$</th>
<th>$s_{t_5}$</th>
<th>$s_{t_6}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$ra$</td>
<td>.11</td>
<td>.32</td>
<td>.045</td>
<td>-.12</td>
<td>.045</td>
<td>.195</td>
</tr>
<tr>
<td>$eff$</td>
<td>.105</td>
<td>.175</td>
<td>.24</td>
<td>.055</td>
<td>.24</td>
<td>.175</td>
</tr>
<tr>
<td>$ca$</td>
<td>.0675</td>
<td>.2275</td>
<td>.0325</td>
<td>.075</td>
<td>.24</td>
<td>.12</td>
</tr>
<tr>
<td>$mr$</td>
<td>.035</td>
<td>.135</td>
<td>-.018</td>
<td>-.02</td>
<td>-.13</td>
<td>.24</td>
</tr>
<tr>
<td>$gp$</td>
<td>.08</td>
<td>.055</td>
<td>-.175</td>
<td>.195</td>
<td>-.085</td>
<td>.18</td>
</tr>
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<td>.175</td>
<td>.12</td>
<td>.0675</td>
</tr>
</tbody>
</table>

8. The Table 5.5.4 is obtained by using the formula (5.1.2),

<table>
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</tr>
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</tr>
<tr>
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<td>-.02</td>
<td>-.13</td>
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<td>.055</td>
<td>-.175</td>
<td>.195</td>
<td>-.085</td>
<td>.18</td>
</tr>
</tbody>
</table>
Thus the best player award goes to $st_2$.

6. Conclusion

The main involvement of this paper is to preface the definition of NBSTSTs and the study of some important properties of such spaces including separation axioms and the relationship between $T_{0,1,2,3,4}$-spaces. The key of this paper is to apply NBSTST in real life problems to take a decision, which might be positive or negative. In our problems two different types of NSTs are combined together to choose a unique decision according to the algorithm and calculation made by the formulae given here. Subsequently, NBSTST can be built up to pairwise NS separated sets, pairwise NS connected spaces, pairwise NS connected sets, pairwise NS disconnected spaces, pairwise NS disconnected sets and so on. We look forward to encourage this type of NBSTST will find a way to other types of topological structures. In future, some case studies which we mention in this paper need to develop on multicriteria structures DM also.

References


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