Neutrosophic Crisp Open Set and Neutrosophic Crisp Continuity via Neutrosophic Crisp Ideals

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Abstract—The focus of this paper is to propose a new notion of neutrosophic crisp sets via neutrosophic crisp ideals and to study some basic operations and results in neutrosophic crisp topological spaces. Also, neutrosophic crisp L-openness and neutrosophic crisp L- continuity are considered as a generalizations for a crisp and fuzzy concepts. Relationships between the above new neutrosophic crisp notions and the other relevant classes are investigated. Finally, we define and study two different types of neutrosophic crisp functions.

Index Terms—Neutrosophic Crisp Set; Neutrosophic Crisp Ideals; Neutrosophic Crisp L-open Sets; Neutrosophic Crisp L- Continuity; Neutrosophic Sets.

I. INTRODUCTION

The fuzzy set was introduced by Zadeh [20] in 1965, where each element had a degree of membership. In 1983 the intuitionstic fuzzy set was introduced by K. Atanassov [1, 2, 3] as a generalization of fuzzy set, where besides the degree of membership and the degree of non- membership of each element. Salama et al [11] defined intuitionistic fuzzy ideal and neutrosophic ideal for a set and generalized the concept of fuzzy ideal concepts, first initiated by Sarker [19]. Smarandache [16, 17, 18] defined the notion of neutrosophic sets, which is a generalization of Zadeh's fuzzy set and Atanassov's intuitionistic fuzzy set. Neutrosophic sets have been investigated by Salama et al. [4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15]. In this paper is to introduce and study some new neutrosophic crisp notions via neutrosophic crisp ideals. Also, neutrosophic crisp L-openness and neutrosophic crisp L- continuity are considered. Relationships between the above new neutrosophic crisp notions and the other relevant classes are investigated. Recently, we define and study two different types of neutrosophic crisp functions.

The paper unfolds as follows. The next section briefly introduces some definitions related to neutrosophic set theory and some terminologies of neutrosophic crisp set and neutrosophic crisp ideal. Section 3 presents neutrosophic crisp L- open and neutrosophic crisp L- closed sets. Section 4 presents neutrosophic crisp L–continuous functions. Conclusions appear in the last section.

II. PRELIMINARIES

We recollect some relevant basic preliminaries, and in particular, the work of Smarandache in [16, 17, 18], and Salama et al. [4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15].

2.1 Definitions [9].

1) Let X be a non-empty fixed set. A neutrosophic crisp set (NCS for short) A is an object having the form $A=\langle A_1, A_2, A_3\rangle$ where A_1, A_2 and A_3 are subsets of X satisfying $A_1 \cap A_2 = \phi$, $A_1 \cap A_3 = \phi$ and $A_2 \cap A_3 = \phi$.

2) Let $A=\langle A_1, A_2, A_3\rangle$, be a neutrosophic crisp set on a set X, then $p=\langle\{p_1\},\{p_2\},\{p_3\}\rangle$, $p_1 \neq p_2 \neq p_3 \in X$ is called a neutrosophic crisp point. A neutrosophic crisp point (NCP for short) $p=\langle\{p_1\},\{p_2\},\{p_3\}\rangle$, is said to be belong to a neutrosophic crisp set $A=\langle A_1, A_2, A_3\rangle$, of X, denoted by $p \in A$, if may be defined by two types

i) **Type 1:** $\{p_1\} \subseteq A_1, \{p_2\} \subseteq A_2$ and $\{p_3\} \subseteq A_3$,

ii) **Type 2:** $\{p_1\} \subseteq A_1, \{p_2\} \supseteq A_2$ and $\{p_3\} \subseteq A_3$.

3) Let X be non-empty set, and L a non–empty family of NCSs. We call L a neutrosophic crisp ideal (NCL for short) on X if

i. $A \in L$ and $B \subseteq A \Rightarrow B \in L$ [heredity],
ii. $A \in L$ and $B \in \mathrm{L} \Rightarrow A \vee B \in \mathrm{L}$ [Finite additivity].

A neutrosophic crisp ideal L is called a σ - neutrosophic crisp ideal if $\{A_j\}_{j\in \mathrm{N}} \subseteq L$, implies $\bigcup_{j\in J} \mathrm{A}_j \in \mathrm{L}$ (countable additivity).

The smallest and largest neutrosophic crisp ideals on a non-empty set X are $\{\phi_N\}$ and the NSs on X. Also, NCL_f, NCL_c are denoting the neutrosophic crisp ideals (NCL for short) of neutrosophic crisp subsets having finite and countable support of X respectively. Moreover, if A is a nonempty NS in X, then $\{B \in NCS : B \subseteq A\}$ is an NCL on X. This is called the principal NCL of all NCSs, denoted by NCL$\langle A \rangle$.

2.1 Proposition [9]

Let $\{L_j : j \in J\}$ be any non - empty family of neutrosophic crisp ideals on a set X. Then $\underset{j\in J}{\cap} L_j$ and $\underset{j\in J}{\cup} L_j$ are neutrosophic crisp ideals on X, where

$\underset{j\in J}{\cap} L_j = \left\langle \underset{j\in J}{\cap} A_{j1}, \underset{j\in J}{\cap} A_{j2}, \underset{j\in J}{\cup} A_{j3} \right\rangle$ or

$\underset{j\in J}{\cap} L_j = \left\langle \underset{j\in J}{\cap} A_{j1}, \underset{j\in J}{\cup} A_{j2}, \underset{j\in J}{\cup} A_{j3} \right\rangle$ and

$\underset{j\in J}{\cup} L_j = \left\langle \underset{j\in J}{\cup} A_{j1}, \underset{j\in J}{\cup} A_{j2}, \underset{j\in J}{\cap} A_{j3} \right\rangle$ or

$\underset{j\in J}{\cup} L_j = \left\langle \underset{j\in J}{\cup} A_{j1}, \underset{j\in J}{\cap} A_{j2}, \underset{j\in J}{\cap} A_{j3} \right\rangle$.

2.2 Proposition [9]

A neutrosophic crisp set $A = \langle A_1, A_2, A_3 \rangle$ in the neutrosophic crisp ideal L on X is a base of L iff every member of L is contained in A.

2.1 Theorem [9]

Let $A = \langle A_1, A_2, A_3 \rangle$, and $B = \langle B_1, B_2, B_3 \rangle$, be neutrosophic crisp subsets of X. Then $A \subseteq B$ iff $p \in A$ implies $p \in B$ for any neutrosophic crisp point p in X.

2.2 Theorem [9]

Let $A = \langle A_1, A_2, A_3 \rangle$, be a neutrosophic crisp subset of X. Then $A = \cup\{p : p \in A\}$.

2.3 Proposition [9]

Let $\{A_j : j \in J\}$ is a family of NCSs in X. Then

(a_1) $p = \langle \{p_1\}, \{p_2\}, \{p_3\} \rangle \in \underset{j\in J}{\cap} A_j$ iff $p \in A_j$ for each $j \in J$.

(a_2) $p \in \underset{j\in J}{\cup} A_j$ iff $\exists j \in J$ such that $p \in A_j$.

2.4 Proposition [9]

Let $A = \langle A_1, A_2, A_3 \rangle$ and $B = \langle B_1, B_2, B_3 \rangle$ be two neutrosophic crisp sets in X. Then

a) $A \subseteq B$ iff for each p we have $p \in A \Leftrightarrow p \in B$ and for each p we have $p \in A \Rightarrow p \in B$.

b) $A = B$ iff for each p we have $p \in A \Rightarrow p \in B$ and for each p we have $p \in A \Leftrightarrow p \in B$.

2.5 Proposition[9]

Let $A = \langle A_1, A_2, A_3 \rangle$ be a neutrosophic crisp set in X. Then $A = \cup < \{p_1 : p_1 \in A_1\}, \{p_2 : p_2 \in A_2\}, \{p_3 : p_3 \in A_3\}$.

2.2 Definition [9]

Let $f : X \to Y$ be a function and p be a neutrosophic crisp point in X. Then the image of p under f, denoted by $f(p)$, is defined by $f(p) = \langle \{q_1\}, \{q_2\}, \{q_3\} \rangle$, where $q_1 = f(p_1), q_2 = f(p_2)$ and $q_3 = f(p_3)$.

It is easy to see that $f(p)$ is indeed a NCP in Y, namely $f(p) = q$, where $q = f(p)$, and it is exactly the same meaning of the image of a NCP under the function f.

2.3 Definition [9]

Let p be a neutrosophic crisp point of a neutrosophic crisp topological space $(X, NC\tau)$. A neutrosophic crisp neighbourhood (NCNBD for short) of a neutrosophic crisp point p if there is a neutrosophic crisp open set(NCOS for short) B in X such that $p \in B \subseteq A$.

2.3 Theorem [9]

Let $(X, NC\tau)$ be a neutrosophic crisp topological space (NCTS for short) of X. Then the neutrosophic crisp set A of X is NCOS iff A is a NCNBD of p for every neutrosophic crisp set $p \in A$.

2.4 Definition [9]

Let (X, τ) be a neutrosophic crisp topological spaces (NCTS for short) and L be neutrosophic crisp ideal (NCL, for short) on X. Let A be any NCS of X. Then the neutrosophic crisp local function $NCA^*(L, \tau)$ of A is the union of all neutrosophic crisp point NCTS(NCP, for short) $P = \langle \{p_1\}, \{p_2\}, \{p_3\} \rangle$, such that if $U \in N((p))$ and $NA^*(L, \tau) = \cup\{p \in X : A \wedge U \notin L$ for every U nbd of N(P)$\}$, $NCA^*(L, \tau)$ is called a neutrosophic crisp local function of A with respect to τ and L which it will be denoted by $NCA^*(L, \tau)$, or simply $NCA^*(L)$. The neutrosophic crisp topology generated by $NCA^*(\mathrm{L})$ in [9] we will be denoted by NC*.

2.5 Theorem [9]

Let (X,τ) be a NCTS and L_1, L_2 be two neutrosophic crisp ideals on X. Then for any neutrosophic crisp sets *A*, *B* of X. then the following statements are verified

i) $A \subseteq B \Rightarrow NCA^*(L,\tau) \subseteq NCB^*(L,\tau)$,
ii) $L_1 \subseteq L_2 \Rightarrow NCA^*(L_2,\tau) \subseteq NCA^*(L_1,\tau)$,
iii) $NCA^* = NCcl(A^*) \subseteq NCcl(A)$,
iv) $NCA^{**} \subseteq NCA^*$,
v) $NC(A \cup B)^* = NCA^* \cup NCB^*$,
vi) $NC(A \cap B)^*(L) \subseteq NCA^*(L) \cap NCB^*(L)$
vii) $\ell \in L \Rightarrow NC(A \cup \ell)^* = NCA^*$
viii) $NCA^*(L,\tau)$ be a neutrosophic crisp closed set.

2.6 Theorem [9]

Let $NC\tau_1, NC\tau_2$ be two neutrosophic crisp topologies on X. Then for any neutrosophic crisp ideal L on X, $NC\tau_1 \subseteq NC\tau_2$ implies $NCA^*(L, NC\tau_2) \subseteq NCA^*(NCL, NC\tau_1)$, for every $A \in L$ then $NC\tau^*{}_1 \subseteq NC\tau^*{}_2$. A basis $NC\beta(L,\tau)$ for $NC\tau^*(L)$ can be described as follows:
$NC\beta(L,\tau) = \{A - B : A \in NC\tau, B \in NCL\}$. Then we have the following theorem.

2.7 Theorem [9]

$NC\beta(L,\tau) = \{A - B : A \in \tau, B \in L\}$ forms a basis for the generated NCTS of the NCT (X,τ) with neutrosophic crisp ideal L on X.

2.8 Theorem [9]

Let $NC\tau_1, NC\tau_2$ be two neutrosophic crisp topologies on X. Then for any topological neutrosophic crisp ideal L on X, $NC\tau_1 \subseteq NC\tau_2$ implies $NC\tau^*{}_1 \subseteq NC\tau^*{}_2$.

2.9 Theorem [9]

Let (X,τ) be a NCTS and L_1, L_2 be two neutrosophic crisp ideals on X. Then for any neutrosophic crisp set A in X, we have

i) $NCA^*(L_1 \cup L_2,\tau) = NCA^*(L_1, NC\tau^*(L_1)) \cap NCA^*(L_2, NC\tau^*(L_2))$
ii) $NC\tau^*(L_1 \cup L_2) = (NC\tau^*(L_1))^*(L_2) \cap (NC\tau^*(L_2))^*(L_1)$

2.1 Corollary [9]

Let (X,τ) be a NCTS with topological neutrosophic crisp ideal L on X. Then

i) $NCA^*(L,\tau) = NCA^*(L,\tau^*)$ and $NC\tau^*(L) = NC(NC\tau^*(L))^*(L)$,
ii) $NC\tau^*(L_1 \cup L_2) = (NC\tau^*(L_1)) \cup (NC\tau^*(L_2))$.

III. Neutrosophic crisp L- open and neutrosophic crisp L- closed sets

Definition 3.1

Given (X,τ) be a NCTS with neutrosophic crisp ideal L on X, and A is called a neutrosophic crisp L–open set iff there exists $\zeta \in \tau$ such that $A \subseteq \zeta \subseteq NCA^*$.

We will denote the family of all neutrosophic crisp L–open sets by NCLO(X).

Theorem 3.1

Let (X, τ) be a NCTS with neutrosophic crisp ideal L, then $A \in$ NCLO(X) iff $A \subseteq$ NCint(NCA*).

Proof

Assume that $A \in$ NCLO(X) then by Definition 3.1there exists $\zeta \in \tau$ such that $A \subseteq \zeta \subseteq NCA^*$. But NCint(NCA*) $\subseteq$ NCA*, put ζ = NCint (NCA*). Hence $A \subseteq$ NCint(NCA*). Conversely $A \subseteq$ NCint (NCA*) $\subseteq$ NCA*. Then there exists ζ = NCint (NCA*) $\in \tau$. Hence $A \in$ NCLO(X).

Remark 3.1

For a NCTS (X,τ) with neutrosophic crisp ideal L and A be a neutrosophic crisp set on X, the following holds: If $A \in$ NCLO (X) then NCint (A) $\subseteq$ NCA*.

Theorem 3.2

Given (X,τ) be a NCTS with neutrosophic crisp ideal L on X and A, B are neutrosophic crisp sets such that $A \in$ NCLO(X), $B \in \tau$ then $A \cap B \in$ NCLO(X)

Proof

From the assumption $A \cap B \subseteq$ NCint (NCA*) $\cap$ B = NCint (NCA$^* \cap$ B), we have $A \cap B \subseteq$ NCintNC(A $\cap$ B)* and this complete the proof.

Corollary 3.1

If $\{A_j\}_{j\in J}$ is a neutrosophic crisp L–open set in NCTS (X,τ) with neutrosophic crisp ideal L. Then $\cup\{A_j\}_{j\in J}$ is neutrosophic crisp L–open sets.

Corollary 3.2

For a NCTS (X,τ) with neutrosophic crisp ideal L, and neutrosophic crisp set A on X and A ∈ NCLO(X), then $NCA^* = NC(NCintNC(NCA^{*}))^{*}$ and $NCcl^*(A)) = NCint(NCA^*)$.

Proof: It's clear.

Definition 3.2

Given a NCTS (X,τ) with neutrosophic crisp ideal L on X and neutrosophic crisp set A. Then A is said to be:

(i) Neutrosophic crisp τ* – closed (or NC*-closed) if $NCA^* \leq A$

(ii) Neutrosophic crisp L–dense – in – itself (or NC*– dense – in – itself) if $A \subseteq NCA^*$.

(iii) Neutrosophic crisp * – perfect if A is NC* – closed and NC* – dense – in – itself.

Theorem 3.3

Given a NCTS (X,τ) with neutrosophic crisp ideal L and A is a neutrosophic crisp set on X, then

(i) NC* – closed iff NCcl*(A) = A .

(ii) NC* – dense – in – itself iff NCcl*(A) $=NCA^*$.

(iii) NC* – perfect iff NCcl*(A) = NCA^* = A .

Proof: Follows directly from the neutrosophic crisp closure operator NCcl* for a neutrosophic crisp topology τ*(L) (NCτ* for short).

Remark 3.2

One can deduce that

(i) Every NC*–dense – in – itself is neutrosophic crisp dense set.

(ii) Every neutrosophic crisp closed (resp. neutrosophic crisp open) set is N*–closed (resp. NCτ∗–open).

(iii) Every neutrosophic crisp L–open set is NC∗ – dense – in – itself.

Corollary 3.3

Given a NCTS (X,τ) with neutrosophic crisp ideal L on X and A ∈ τ then we have:

(i) If A is NC* –closed then $A^* \subseteq$ NCint(A) $\subseteq NC\,Cl$ (A).

(ii) If A is NC* –dense – in – itself then Nint(A) $\subseteq NCA^*$.

(iii) If A is NC* –perfect then NCint(A) = NCcl(A) = NCA∗.

Proof: Obvious.

we give the relationship between neutrosophic crisp L–open set and some known neutrosophic crisp openness.

Theorem 3.4

Given a NCTS (X,τ) with neutrosophic crisp ideal L and neutrosophic crisp set A on X then the following holds:

(i) If A is both neutrosophic crisp L – open and NC∗ – erfect then A is neutrosophic crisp open.

(ii) If A is both neutrosophic crisp open and NC∗– dense–in – itself then A is neutrosophic crisp L–open.

Proof. Follows from the definitions.

Corollary 3.4

For a neutrosophic crisp subset A of a NCTS (X,τ) with neutrosophic crisp ideal L on X, we have:

(i) If A is NC∗–closed and NL–open then NCint (A) = $NCint(NCA^*)$.

(ii) If A is NC∗–perfect and NL–open then A =NC int (NCA^*).

Remark 3.3

One can deduce that the intersection of two neutrosophic crisp L–open sets is neutrosophic crisp L–open.

Corollary 3.5

Given (X,τ) be a NCTS with neutrosophic crisp ideal L and neutrosophic crisp set A on X. The following hold: If L= $\{N^x\}$, then $NCA^*(L) = \phi_N$ and hence A is neutrosophic crisp L–open iff A = ϕ_N .

Proof: It's clear.

Definition 3.5

Given a NCTS (X,τ) with neutrosophic crisp ideal L and neutrosophic crisp set A then neutrosophic crisp ideal interior of A is defined as largest neutrosophic crisp L–open set contained in A , we denoted by NCL–NCint(A).

Theorem 3.5

If (X,τ) is a NCTS with neutrosophic crisp ideal L and neutrosophic crisp set A then

(i) $A \wedge$ Nint (NCA^*) is neutrosophic crisp L–open set.
(ii) NL–Nint (A) $=0_N$ iff Nint $(NCA^*) = 0_N$.

Proof

(i) Since NCint $NCA^* = NCA^* \cap$ NCint (NCA^*), then NCint $NCA^* = NCA^* \cap$ NCint $(NCA^*) \subseteq NC(A \cap NCA^*)^*$. Thus $A \cap NC A^* \subseteq (A \cap (A \cap$ NCint $NC(NCA^*))^* \subseteq$ NCintNC($A \cap$ NCint $NC(NCA^*)^*$. Hence $A \cap$ NCint $NCA^* \in$ NCLO(X).
(ii) Let NCL–NCint(A) = ϕ_N, then $A \cap A^* = \phi_N$, implies NCcl (A ∩ NCint$(NCA^*) = \phi_N$ and so $A \cap$ Nint $A^* = \phi_N$. Conversely assume that NCint $NCA^* = \phi_N$, then A ∩ NC int(NC A^*)= ϕ_N. Hence NCL–NCint (A) = ϕ_N.

Theorem 3.6

If (X,τ) be a NCTS with neutrosophic crisp ideal L and A is aneutrosophic crisp set on X, then NCL–NCint(A) = A ∩ NCint(NCA^*).

Proof. The first implication follows from Theorem 3.4, that is $A \cap NCA^* \subseteq$ NCL-NCint(A) (1)

For the reverse inclusion, if $\zeta \in$ NCLO(X) and $\zeta \subseteq$ A then $NC\zeta^* \subseteq NCA^*$ and hence NC $int(NC\zeta^*) \subseteq$ NCint(NCA^*). This implies $\zeta = \zeta \cap$ NCint$(NC\zeta^*) \subseteq A \cap NCA^*$.

Thus NCL–NCint(A) ⊆ A ∩ NCint(NCA^*) (2)

From (1) and (2) we have the result.

Corollary 3.6

For a NCTS (X,τ) with neutrosophic crisp ideal L and neutrosophic crisp set A on X then the following holds:

(i) If A is NC∗ – closed then NL–Nint (A) ⊆ A.
(ιι) If A is NC∗ – dense – in– itself then NL – Nint (A) ⊆ A^*.
(iii) If A is NC∗ – perfect set then NCL – NCint (A) ⊆ NCA^*.

Definition 3.6

Given (X,τ) be a NCTS with neutrosophic crisp ideal L and ζ be a neutrosophic crisp set on X, ζ is called neutrosophic crisp L–closed set if its complement is neutrosophic crisp L–open set . We will denote the family of neutrosophic crisp L–closed sets by NLCC(X).

Theorem 3.7

Given (X,τ) be a NCTS with neutrosophic crisp ideal L and ζ be a neutrosophic crisp set on X. ζ is neutrosophic crisp L closed, then $NC(NCint\zeta)^* \leq \zeta$.

Proof: It's clear.

Theorem 3.8

Let (X,τ) be a NCTS with neutrosophic crisp ideal L on X and ζ be a neutrosophic crisp set on X such that $NC(\text{NCint } \zeta)^{*c} = \text{NCint } \zeta^{c*}$ then $\zeta \in$ NLC(X) iff $NC(\text{NCint } \zeta)^* \subseteq \zeta$.

Proof

(Necessity) Follows immedially from the above theorem (Sufficiency). Let $NC(\text{NCint}\zeta)^* \subseteq \zeta$ then $\zeta^c \subseteq NC(\text{NCint } \zeta)^{*c} = \text{NCint } (NC\zeta)^{c*}$. from the hypothesis. Hence $\zeta^c \in$ NCLO(X), Thus $\zeta \in$ NLCC(X).

Corollary 3.7

For a NCTS (X,τ) with neutrosophic crisp ideal L on X the following holds:

(i) The union of neutrosophic crisp L – closed set and neutrosophic crisp closed set is neutrosophic crisp L–closed set.
(ii) The union of neutrosophic crisp L – closed and neutrosophic crisp L–closed is neutrosophic crisp perfect.

IV. NEUTROSOPHIC CRISP L–CONTINUOUS FUNCTIONS

By utilizing the notion of NL – open sets, we establish in this article a class of neutrosophic crisp L- continuous function. Many characterizations and properties of this concept are investigated.

Definition 4.1

A function $f : (X,\tau) \rightarrow (Y,\sigma)$ with neutrosophic crisp ideal L on X is said to be neutrosophic crisp L–continuous if for every $\zeta \in \sigma$, $f^{-1}(\zeta) \in$ NCLO(X).

Theorem 4.1

For a function $f : (X,\tau) \rightarrow (Y,\sigma)$ with neutrosophic crisp ideal L on X the following are equivalent:

(i.) f is neutrosophic crisp L-continuous. For a neutrosophic crisp point p in X and each $\zeta \in \sigma$ containing f (p), there exists $A \in$ NCLO(X) containing p such that $f(A) \subseteq \sigma$.

(ii.) For each neutrosophic crisp point p in X and $\zeta \in \sigma$ containing f (p), $(f^{-1}(\zeta))^*$ is neutrosophic crisp nbd of p.

(iii.) The inverse image of each neutrosophic crisp closed set in Y is neutrosophic crisp L–closed.

Proof

(i) → (ii).Since $\zeta \in \sigma$ containing f (p), then by (i), $f^{-1}(\zeta) \in$ NCLO(X), by putting A = $f^{-1}(\zeta)$ which containing p, we have f (A) $\subseteq \sigma$

(ii) → (iii). Let $\zeta \in \sigma$ containing f (p). Then by (ii) there exists A ∈ NCLO(X) containing p such that f (A) $\leq \sigma$, so p ∈ A ⊆ NCint(NCA*)≤ NCint $(f^{-1}(\zeta))^* \subseteq (f^{-1}(\zeta))^*$. Hence $(f^{-1}(\zeta))^*$ is neutrosophic crisp nbd of p.

(iii) → (i) Let $\zeta \in \sigma$, since $(f^{-1}(\zeta))$ is neutrosophic crisp nbd of any point $f^{-1}(\zeta)$, every point $x_\varepsilon \in (f^{-1}(\zeta))^*$ is a neutrosophic crisp interior point of $f^{-1}(\zeta)^*$. Then $f^{-1}(\zeta) \subseteq$ NCint NC $(f^{-1}(\zeta))^*$ and hence f is neutrosophic crisp L–continuous

(i)→ (iv) Let $\xi \in$ y be a neutrosophic crisp closed set. Then ξ^c is neutrosophic crisp open set, by $f^{-1}(\xi^c) = (f^{-1}(\xi))^c \in$ NCLO(X). Thus $f^{-1}(\xi)$ is neutrosophic crisp L–closed set.

The following theorem establish the relationship between neutrosophic crisp L-continuous and neutrosophic crisp continuous by using the previous neutrosophic crisp notions.

Theorem 4.2

Given f : (X,τ) → (Y,σ) is a function with a neutrosophic crisp ideal L on X then we have. If f is neutrosophic crisp L– continuous of each neutrosophic crisp*– perfect set in X, then f is neutrosophic crisp continuous.

Proof: Obvious.

Corollary 4.1

Given a function f : (X,τ) → (Y,σ) and each member of X is neutrosophic crisp NC*–dense – in – itself. Then we have every neutrosophic crisp continuous function is neutrosophic crisp NCL–continuous.

Proof: It's clear.

We define and study two different types of neutrosophic crisp functions.

Definition 4.2

A function $f:(X,\tau)\to(Y,\sigma)$ with neutrosophic crisp ideal L on Y is called neutrosophic crisp L-open (resp. neutrosophic crisp NCL- closed), if for each $A \in \tau$ (resp. A is neutrosophic crisp closed in X), $f(A) \in NCLO(Y)$ (resp. $f(A)$ is NCL-closed).

Theorem 4.3

Let a function $f:(X,\tau)\to(Y,\sigma)$ with neutrosophic crisp ideal L on Y. Then the following are equivalent:

(i.) f is neutrosophic crisp L-open.

(ii.) For each p in X and each neutrosophic crisp ncnbd A of p, there exists a neutrosophic crisp L-open set $B \in I^Y$ containing $f(p)$ such that $B \subseteq f(A)$.

Proof: Obvious.

Theorem 4.4

A neutrosophic crisp function $f:(X,\tau)\to(Y,\sigma)$ with neutrosophic crisp ideal L on Y be a neutrosophic crisp L-open (resp.neutrosophic crisp L-closed), if A in Y and B in X is a neutrosophic crisp closed (resp. neutrosophic crisp open) set C in Y containing A such that $f^{-1}(C) \subseteq B$.

Proof

Assume that $A = 1_Y - (f(1_X - B))$, since $f^{-1}(C) \leq B$ and $A \leq C$ then C is neutrosophic crisp L-closed and $f^{-1}(C) = 1_X - f^{-1}(f(1_X - A)) \leq B$.

Theorem 4.5

If a function $f:(X,\tau)\to(Y,\sigma)$ with neutrosophic crisp ideal L on Y is a neutrosophic crisp L-open, then $f^{-1}NC(NC\,\mathrm{int}(A))^* \leq NC\left(f^{-1}(A)\right)^*$ such that $f^{-1}(A)$ is neutrosophic crisp*-dense-in-itself and A in Y.

Proof

Since A in Y, $NC\left(f^{-1}(A)\right)^*$ is neutrosophic crisp closed in X containing $f^{-1}(A)$, f is neutrosophic crisp L-open then by using Theorem 4.4 there is a neutrosophic crisp L-closed set $A \subseteq B$ suchthat, $\left(f^{-1}(A)\right)^* \supseteq f^{-1}(B) \geq f^{-1}NC(\mathrm{int}(B))^* \supseteq f^{-1}NC(NC\,\mathrm{int}(\mu))^*$.

Corollary 4.2

For any bijective function $f:(X,\tau)\to(Y,\sigma)$ with neutrosophic crisp ideal L on Y , the following are equivalent:

(i.) $f^{-1}:(Y,\sigma)\to(X,\tau)$ is neutrosophic crisp L-continuous.

(ii.) f is neutrosophic crisp L-open.

(iii.) f is neutrosophic crisp L-closed.

Proof: Follows directly from Definitions.

V. CONCLUSION

In our work, we have put forward some new concepts of neutrosophic crisp open set and neutrosophic crisp continuity via neutrosophic crisp ideals. Some related properties have been established with example. It 's hoped that our work will enhance this study in neutrosophic set theory.

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