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Abstract: In this manuscript, we introduce the notion of neutrosophic soft rough topology (NSR-topology) defined on neutrosophic soft rough set (NSR-set). We define certain properties of NSR-topology including NSR-interior, NSR-closure, NSR-exterior, NSR-neighborhood, NSR-limit point, and NSR-bases. Furthermore, we aim to develop some multi-criteria decision-making (MCDM) methods based on NSR-set and NSR-topology to deal with ambiguities in the real-world problems. For this purpose, we establish algorithm 1 for suitable brand selection and algorithm 2 to determine core issues to control crime rate based on NSR-lower approximations, NSR-upper approximations, matrices, core, and NSR-topology.

Keywords: Neutrosophic soft rough (NSR) set, NSR-topology, NSR-interior, NSR-closure, NSR-exterior, NSR-neighborhood, NSR-limit point, NSR-bases, Multi-criteria group decision making.

1. Introduction

The limitations of existing research are recognized in the field of management, social sciences, operational research, medical, economics, artificial intelligence, and decision-making problems. These limitations can be dealt with the Fuzzy set [1], rough set [2, 3], neutrosophic set [4, 5], soft set [6], and different hybrid structures of these sets. Rough set theory was initiated by Pawlak [2], which is an effective mathematical model to deal with vagueness and imprecise knowledge. Its boundary region gives the concept of vagueness, which can be interpreted by using the vagueness of Frege's idea. He invented that vagueness can be dealt with the upper and lower approximations of precise set using any equivalence relation. In the real life, rough set theory has many applications in different fields such as social sciences, operational research, medical, economics, and artificial intelligence, etc. Many real-world problems have neutrosophy in their nature and cannot handle by using fuzzy or intuitionistic fuzzy set theory. For example, when we are dealing with conductors and non-conductors there must be a possibility having insulators. For this purpose, Smarandache [4, 5] inaugurated the neutrosophic set theory as a generalization of fuzzy and intuitionistic fuzzy set theory. The neutrosophic set yields the value from real standard or non-standard subsets of $]^{-}0, 1^{+}[$. It is difficult to utilize these values in daily life science and technology problems. Therefore, the concept of a single-valued neutrosophic set, which takes value from the subset of [0,1], as defined by Wang et al. [7]. The beauty of this set is that it gives the membership grades for truth, indeterminacy and falsity for the corresponding attribute. All the grades are independent of each other and provide information about the three shades of an arbitrary attribute. Smarandache [8] extended the neutrosophic set respectively to neutrosophic Overset (when some neutrosophic components are >1), Neutrosophic Underset (when some neutrosophic components are <0), and to Neutrosophic Offset (when some neutrosophic components are off the interval $[0,1]$, i.e. some neutrosophic components are >1 and other neutrosophic components <0). In 2016, Smarandache introduced the Neutrosophic Tripolar Set and Neutrosophic Multipolar Set, also the Neutrosophic Tripolar Graph and Neutrosophic Multipolar Graph [8].

The soft set is a mathematical model to deal ambiguities and imprecisions in parametric manners. This is another abstraction of the crisp set theory. In 1999, Molodtsov [9] worked on parametrizations of the universal set and invented a parameterized family of subsets of the universal set called soft set. In recent years, many mathematicians worked on different hybrid structures of the fuzzy and rough sets. Ali et al. [10, 11] established some novel operations in the soft sets, rough soft sets and, fuzzy soft set theory. Aktas and Çağman [12] introduced various results on soft sets and soft groups. Bakier et al. [13] introduced the idea of soft rough topology. Çağman et al. [14] introduced various results on soft topology. Chen [15] worked on parametrizations reduction of soft sets and gave its applications in decision-making. Feng et al. [16, 17] established various results on soft set, fuzzy set, rough set and soft rough sets with the help of illustrations. Hashmi et al. [18] introduced the notion of m-polar neutrosophic set and m-polar neutrosophic topology and their applications to multi-criteria decision-making (MCDM) in medical diagnosis and clustering analysis. Hashmi and Riaz [19] introduced a novel approach to the census process by using Pythagorean m-polar fuzzy Dombi's aggregation operators. Kryskiewicz [20] introduced the rough set approach to incomplete information systems. Karaaslan and Çağman [21] introduced bipolar soft rough sets and presented their applications in decision-making. Kumar and Garg [22] introduced the TOPSIS method based on the connection number of set pair analyses under an interval-valued intuitionistic fuzzy set environment. Maji et al. [23, 24, 25] worked on some results of a soft set and gave its applications in decision-making problems. He also invented the idea of a neutrosophic soft set and gave various results to intricate the concept with numerous applications. Naeem et al. [26] introduced the novel concept of Pythagorean m-polar fuzzy sets and the TOPSIS method for the selection of advertisement mode. Peng and Garg [27] introduced algorithms for interval-valued fuzzy soft sets in emergency decision making based on WDBA and CODAS with new information measures. Peng and Yang [28] presented some results for Pythagorean fuzzy sets. Peng et al. [29] introduced Pythagorean fuzzy information measures and their applications. Peng et al. [30] introduced a Pythagorean fuzzy soft set and its application. Peng and Dai [31] introduced certain approaches to single-valued neutrosophic MADM based on MABAC, TOPSIS and, new similarity measure with score function. Marei [32] invented some more results on neutrosophic soft rough sets and worked on its modifications. Pei and Miao [33] worked on the information system using the idea of a soft set. Quran et al. [34] introduced a novel approach to neutrosophic soft rough set under uncertainty. Riaz et al. [35] introduced soft rough topology with its applications to group decision making.

Riaz and Hashmi [36] introduced the notion of linear Diophantine fuzzy Set (LDFS) and its applications towards the MCDM problem. Linear Diophantine fuzzy Set (LDFS) is superior to IFS, PFS and, q-ROFS. Riaz and Hashmi [37] introduced novel concepts of soft rough Pythagorean m-Polar fuzzy sets and Pythagorean m-polar fuzzy soft rough sets with application to decision-making. Riaz and Tehrim [38] established the idea of cubic bipolar fuzzy ordered weighted geometric aggregation operators and, their application using internal and external cubic bipolar fuzzy data. They presented various illustrations and decision-making applications of these concepts by using different algorithms. Roy and Maji [39] introduced a fuzzy soft set-theoretic approach to decision-making problems. Salama [40] investigated some topological properties of rough sets with tools for data mining. Shabir and Naz [41] worked on soft topological spaces and presented their applications. Thivagar et al. [42] presented some mathematical innovations of a modern topology in medical events. Xueling et al. [43] presented some decision-making methods based on certain hybrid soft set models. Zhang et al. [44, 45, 46] established fuzzy soft β-covering based fuzzy rough sets, fuzzy soft coverings based fuzzy rough sets and, covering on generalized intuitionistic fuzzy rough sets with their applications to multi-attribute decision-making (MADM) problems. Broumi et al. [47] established the concept of rough neutrosophic sets. Christianto et al. [48] introduced the idea about the extension of standard deviation notion with neutrosophic interval and quadruple neutrosophic numbers. Adeleke et al. [49, 50] invented the concepts of refined eutrosophic rings I and refined neutrosophic rings II. Parimala et al. [51] worked on $\alpha\omega$-closed sets and its connectedness in terms of neutrosophic topological spaces. Ibrahim et al. [52] introduced the neutrosophic subtraction algebra and neutrosophic subraction semigroup.

The neutrosophic soft rough set and neutrosophic soft rough topology have many applications in MCDM problems. This hybrid erection is the most efficient and flexible rather than other constructions. It is constructed with a combination of neutrosophic, soft and, rough set theory. The interesting point in this structure is that by using this idea, we can deal with those type of models which have roughness, neutrosophy and, parameterizations in their nature.

The motivation of this extended and hybrid work is presented step by step in the whole manuscript. This model is generalized form and use to collect data at a large scale and applicable in medical, engineering, artificial intelligence, agriculture and, other daily life problems. In the future, this work can be gone easily for other approaches and different types of hybrid structures.

The layout of this paper is systematized as follows. Section 2, implies some basic ideas including soft set, rough set, neutrosophic set, neutrosophic soft set and, neutrosophic soft rough set. We elaborate on these ideas with the help of illustrations. In Section 3, we establish neutrosophic soft rough topology (NSR-topology) with some examples. We introduce some topological structures on NSR-topology named NSR-interior, NSR-closure, NSR-exterior, NSR-neighborhood, NSR-limit point and, NSR-bases. In Section 4 and 5, we present multi-criteria decision-making problems by using two different algorithms on NSR-set and NSR-topology. We use the idea of upper and lower approximations for NSR-set and construct algorithms using NSR-sets and NSR-bases We discuss the optimal results obtained from both algorithms and present a comparitive analysis of proposed approach with some existing approaches. Finally, the conclusion of this research is summarized in section 6.

2. Preliminaries

This section presents some basic definitions including soft set, rough set, neutrosophic soft set, and neutrosophic soft rough set .

Definition 2.1 [18]

Let U be the universal set. Let $I(U)$ is collection of subsets of U. A pair $(\Theta, \mathfrak{A})$ is said to be a soft set over the universe U, where $\mathfrak{A} \subseteq E$ and $\Theta: \mathfrak{A} \to I(U)$ is a set-valued function. We denote soft set as $(\Theta, \mathfrak{A})$ or $\Theta_{\mathfrak{A}}$ and mathematically write it as

$$\Theta_{\mathfrak{A}} = \{(\xi, \Theta(\xi)): \xi \in \mathfrak{A}, \Theta(\xi) \in I(U)\}.$$

For any $\xi \in \mathfrak{A}$, $\Theta(\xi)$ is ξ-approximate elements of soft set $\Theta_{\mathfrak{A}}$.

Definition 2.2 [21]

Let U be the initial universe and $Y \subseteq U$. Then, lower, upper, and boundary approximations of Y are defined as

$$\mathfrak{R}_{å}(Y) = \bigcup_{g \in U} \{\mathfrak{R}(g): \mathfrak{R}(g) \subseteq Y\},$$

$$\mathfrak{R}^{å}(Y) = \bigcup_{g \in U} \{\mathfrak{R}(g): \mathfrak{R}(g) \cap Y \neq \emptyset\},$$

and

$$B_{\mathfrak{R}}(Y) = \mathfrak{R}^{å}(Y) - \mathfrak{R}_{å}(Y),$$

respectively. Where $\mathfrak{R}$ is an indiscernibility relation $\mathfrak{R} \subseteq U \times U$ which indicates our information about elements of U. The set Y is said to be defined if $\mathfrak{R}^{å}(Y) = \mathfrak{R}_{å}(Y)$. If $\mathfrak{R}^{å}(Y) \neq \mathfrak{R}_{å}(Y)$ i.e $B_R(Y) \neq \emptyset$, the set Y is rough set w.r.t $\mathfrak{R}$.

Definition 2.3 [41] Let U be the initial universe. Then, a neutrosophic set N on the universe U is defined as

$$N = \{< g, \mathfrak{T}_N(g), \mathfrak{I}_N(g), \mathfrak{F}_N(g) >: g \in U\}, \text{where}$$

$$^{-}0 \leq \mathfrak{T}_N(g) + \mathfrak{I}_N(g) + \mathfrak{F}_N(g) \leq 3^{+}, \text{where}$$

$$\mathfrak{T}, \mathfrak{I}, \mathfrak{F}: U \to]^{-}0, 1^{+}[.$$

Where $\mathfrak{T}, \mathfrak{I}$ and $\mathfrak{F}$ represent the degree of membership, degree of indeterminacy and degree of non-membership for some $g \in U$, respectively.

Definition 2.4 [16] Let U be an initial universe and E be a set of parameters. Suppose $\mathfrak{A} \subset E$, and let $\mathcal{I}(U)$ represents the set of all neutrosophic sets of U. The collection $(\Phi, \mathfrak{A})$ is said to be the neutrosophic soft set over U, where Φ is a mapping given by

$$\Phi: \mathfrak{A} \to \mathcal{I}(U).$$

The set containing all neutrosophic soft sets over U is denoted by NS_U.

Example 2.5 Consider $U = \{g_1, g_2, g_3, g_4, g_5\}$ be set of objects and attribute set is given by $\mathfrak{A} = \{\xi_1, \xi_2, \xi_3, \xi_4\} = E = \mathfrak{A}$, where

The neutrosophic soft set represented as $\Phi_{\mathfrak{A}}$. Consider a mapping $\Phi: \mathfrak{A} \to I(U)$ such that

$$\Phi(\xi_1) = \{< g_1, 0.7,0.7,0.3 >, < g_2, 0.5,0.7,0.7 >, < g_3, 0.7,0.5,0.2 >, < g_4, 0.7,0.4,0.4 >, < g_5, 0.9,0.3,0.4 >\},$$
$$\Phi(\xi_2) = \{< g_1, 0.9,0.5,0.4 >, < g_2, 0.7,0.3,0.5 >, < g_3, 0.9,0.2,0.4 >, < g_4, 0.9,0.3,0.3 >, < g_5, 0.9,0.4,0.3 >\},$$
$$\Phi(\xi_3) = \{< g_1, 0.8,0.5,0.4 >, < g_2, 0.7,0.5,0.4 >, < g_3, 0.8,0.3,0.6 >, < g_4, 0.6,0.3,0.7 >, < g_5, 0.8,0.4,0.5 >\},$$
$$\Phi(\xi_4) = \{< g_1, 0.9,0.7,0.5 >, < g_2, 0.8,0.7,0.7 >, < g_3, 0.8,0.7,0.5 >, < g_4, 0.8,0.6,0.7 >, < g_5, 1.0,0.6,0.7 >\}.$$

The tabular representation of neutrosophic soft set $K = (\Phi, \mathfrak{A})$ is given in Table 1.

$(\Phi,\mathfrak{A})$	g_1	g_2	g_3	g_4	g_5
ξ_1	(0.7,0.7,0.3)	(0.5,0.7,0.7)	(0.7,0.5,0.2)	(0.7,0.4,0.4)	(0.9,0.3,0.4)
ξ_2	(0.9,0.5,0.4)	(0.7,0.3,0.5)	(0.9,0.2,0.4)	(0.9,0.3,0.3)	(0.9,0.4,0.3)
ξ_3	(0.8,0.5,0.4)	(0.7,0.5,0.4)	(0.8,0.3,0.6)	(0.6,0.3,0.7)	(0.8,0.4,0.5)
ξ_4	(0.9,0.7,0.5)	(0.8,0.7,0.7)	(0.8,0.7,0.5)	(0.8,0.6,0.7)	(1.0,0.6,0.7)

Table 1: Neutrosophic soft set $(\Phi,\mathfrak{A})$

Definition 2.6 Let $(\Phi,\mathfrak{A})$ be a neutrosophic soft set on a universe U. For some elements $g \in U$, a neutrosophic right neighborhood, regarding $\xi \in \mathfrak{A}$ is interpreted as follows;

$$g_\xi = \{g_i \in U: \mathfrak{T}_\xi(g_i) \geq \mathfrak{T}_\xi(g), \mathfrak{I}_\xi(g_i) \geq \mathfrak{I}_\xi(g), \mathfrak{F}_\xi(g_i) \leq \mathfrak{F}_\xi(g)\}.$$

Definition 2.7 Let $(\Phi,\mathfrak{A})$ be a neutrosophic soft set over a universe U. For some elements $g \in U$, a neutrosophic right neighborhood regarding all parameters $\mathfrak{A}$ is interpreted as follows;

$$g]_{\mathfrak{A}} = \cap \{g_{\xi_i}: \xi_i \in \mathfrak{A}\}.$$

Example 2.8 Consider Example 2.5 then we find the following neutosophic right neighborhood regarding all parameters $\mathfrak{A}$ as

$$g_{1\xi_1} = g_{1\xi_2} = g_{1\xi_3} = g_{1\xi_4} = \{g_1\}, g_{2\xi_1} = g_{2\xi_3} = \{g_1, g_2\}, g_{2\xi_2} = \{g_1, g_2, g_4, g_5\}, g_{2\xi_4} = \{g_1, g_2, g_3\}, g_{3\xi_1} = g_{3\xi_4} = \{g_1, g_3\}, g_{3\xi_2} = \{g_1, g_3, g_4, g_5\}, g_{3\xi_3} = \{g_1, g_3, g_5\}, g_{4\xi_1} = \{g_1, g_3, g_4\}, g_{4\xi_2} = \{g_4, g_5\}, g_{4\xi_3} = U, g_{4\xi_4} = U, g_{5\xi_1} = g_{5\xi_2} = g_{5\xi_4} = \{g_5\}, g_{5\xi_3} = \{g_1, g_5\}.$$

It follows that,

$g_1]_{\mathfrak{A}} = \{g_1\}$,
$g_2]_{\mathfrak{A}} = \{g_1, g_2\}$,
$g_3]_{\mathfrak{A}} = \{g_1, g_3\}$,
$g_4]_{\mathfrak{A}} = \{g_4\}$,
$g_5]_{\mathfrak{A}} = \{g_5\}$.

Definition 2.9 Let $(\Phi,\mathfrak{A})$ be a neutrosophic soft set over U. For any $X \subseteq U$, neutrosophic soft lower ($\underline{apr}_{NSR}$) approximation, neutrosophic soft upper ($\overline{apr}_{NSR}$) approximation, and neutrosophic soft boundary (B_{NSR}) approximation of X are defined as

$$\underline{apr}_{NSR}(X) = \cup \{g]_{\mathfrak{A}}: g \in U, g]_{\mathfrak{A}} \subseteq X\}$$

$$\overline{apr}_{NSR}(X) = \cup \{g]_{\mathfrak{A}}: g \in U, g]_{\mathfrak{A}} \cap X \neq \emptyset\}$$

$$B_{NSR}(X) = \overline{apr}_{NSR}(X) - \underline{apr}_{NSR}(X),$$

respectively. If $\underline{apr}_{NSR}(X) = \overline{apr}_{NSR}(X)$ then X is neutrosophic soft definable set.

Example 2.10 Consider Example 2.5, If $X = \{g_1\} \subseteq U$, then $\underline{apr}_{NSR}(X) = \{g_1\}$ and $\overline{apr}_{NSR}(X) = \{g_1, g_2, g_3\}$. Since its clear $\underline{apr}_{NSR}(X) \neq \overline{apr}_{NSR}(X)$, so X is neutrosophic soft rough set on U.

3 Neutrosophic Soft Rough Topology

In this section, we introduce and study the idea of neutrosophic soft rough topology and its related properties. Concepts of (NSR)-open set, (NSR)-closed set, (NSR)-closure, (NSR)-interior, (NSR)-exterior, (NSR)-neighborhood, (NSR)-limit point, and (NSR)-bases are defined.

Definition 3.1 Let U be the initial space, $\mathfrak{Y} \subseteq \mathrm{U}$ and $\mathrm{G} = (\mathrm{U}, \mathrm{K})$ be a neutrosophic soft approximation space, where $\mathrm{K} = (\Phi, \mathfrak{A})$ is a neutrosophic soft set. The upper and lower approximations are calculated on the basis of neutrosophic soft approximation space and neighborhoods. Then, the collection

$$\tau_{\mathrm{NSR}}(\mathfrak{Y}) = \{\mathrm{U}, \emptyset, \underline{\mathrm{apr}}_{\mathrm{NSR}}(\mathfrak{Y}), \overline{\mathrm{apr}}_{\mathrm{NSR}}(\mathfrak{Y}), \mathrm{B}_{\mathrm{NSR}}(\mathfrak{Y})\}$$

is called neutrosophic soft rough topology (NSR-topology) which guarantee the following postulates:

- U and $\emptyset$ belongs to $\tau_{\mathrm{NSR}}(\mathfrak{Y})$.
- Union of members of $\tau_{\mathrm{NSR}}(\mathfrak{Y})$ belongs to $\tau_{\mathrm{NSR}}(\mathfrak{Y})$.
- Finite Intersection of members of $\tau_{\mathrm{NSR}}(\mathfrak{Y})$ belongs to $\tau_{\mathrm{NSR}}(\mathfrak{Y})$.

Then $(\mathrm{U}, \tau_{\mathrm{NSR}}(\mathfrak{Y}), \mathrm{E})$ is said to be NSR-topological space, if $\tau_{\mathrm{NSR}}(\mathfrak{Y})$ is Neutrosophic soft rough topology.

Note that Neutrosophic soft rough topology is based on lower and upper approximations of neutrosophic soft rough set.

Example 3.2 From Example 2.5, if $\mathfrak{Y} = \{g_2, g_4\} \subseteq \mathrm{U}$, we obtain $\underline{\mathrm{apr}}_{\mathrm{NSR}}(\mathfrak{Y}) = \{g_4\}$, $\overline{\mathrm{apr}}_{\mathrm{NSR}}(\mathfrak{Y}) = \{g_1, g_2, g_4\}$ and $\mathrm{B}_{\mathrm{NSR}}(\mathfrak{Y}) = \{g_1, g_2\}$. Then,

$$\tau_{\mathrm{NSR}}(\mathfrak{Y}) = \{\mathrm{U}, \emptyset, \{g_4\}, \{g_1, g_2, g_4\}, \{g_1, g_2\}\}$$

is a NSR-topology.

Definition 3.3 Let $(\mathrm{U}, \tau_{\mathrm{NSR}}(\mathfrak{Y}), \mathrm{E})$ be an NSR-topological space. Then, the members of $\tau_{\mathrm{NSR}}(\mathfrak{Y})$ are called NSR-open sets. An NSR-set is said to be an NSR-closed set if its complement belongs to $\tau_{\mathrm{NSR}}(\mathfrak{Y})$.

Proposition 3.4 Consider $(\mathrm{U}, \tau_{\mathrm{NSR}}(\mathfrak{Y}), \mathrm{E})$ as NSR-space over U. Then,

- U and $\emptyset$ are NSR-closed sets.
- The intersection of any number of NSR-closed sets is an NSR-closed set over U.
- The finite union of NSR-closed sets is an NSR-closed set over U.

Proof. The proof is straightforward.

Definition 3.5 Let $(\mathrm{U}, \tau_{\mathrm{NSR}}(\mathfrak{Y}), \mathrm{E})$ be an NSR-space over U and $\tau_{\mathrm{NSR}}(\mathfrak{Y}) = \{\mathrm{U}, \emptyset\}$. Then, τ_{NSR} is called NSR-indiscrete topology on U w.r.t $\mathfrak{Y}$ and corresponding space is said to be an NSR-indiscrete space over U.

Definition 3.6 Let $(\mathrm{U}, \tau_{\mathrm{NSR}}(\mathfrak{Y}), \mathrm{E})$ is an NSR-topological space and $\mathrm{A} \subseteq \mathrm{B} \subseteq \mathrm{U}$. Then, the collection $\tau_{\mathrm{NSR}_\mathrm{A}} = \{\mathrm{B_i} \cap \mathrm{A} : \mathrm{B_i} \in \tau_{\mathrm{NSR}}, \mathrm{i} \in \mathrm{L} \subseteq \mathrm{N}\}$ is called NSR-subspace topology on A. Then, $(\mathrm{A}, \tau_{\mathrm{NSR}_\mathrm{A}})$ is called an NSR-topological subspace of $(\mathrm{B}, \tau_{\mathrm{NSR}})$.

Definition 3.7 Let $(\mathrm{U}, \tau_{\mathrm{NSR}'}(\mathfrak{Y}), \mathrm{E})$ and $(\mathrm{U}, \tau_{\mathrm{NSR}}(\mathfrak{X}), \mathrm{E})$ be two NSR-topological spaces. $\tau_{\mathrm{NSR}'}(\mathfrak{Y})$ is finer than $\tau_{\mathrm{NSR}}(\mathfrak{X})$, if $\tau_{\mathrm{NSR}'}(\mathfrak{Y}) \supseteq \tau_{\mathrm{NSR}}(\mathfrak{X})$.

Definition 3.8 Let $(\mathrm{U}, \tau_{\mathrm{NSR}}(\mathfrak{Y}), \mathrm{E})$ be a NSR-topological space and $\beta_{\mathrm{NSR}} \subseteq \tau_{\mathrm{NSR}}$. If we can write members of τ_{NSR} as the union of members of β_{NSR}, then β_{NSR} is called NSR-basis for the NSR-topology τ_{NSR}.

Proposition 3.9 If $\tau_{\mathrm{NSR}}(\mathfrak{Y})$ is an NSR-topology on U w.r.t $\mathfrak{Y}$ the the collection

$$\beta_{\mathrm{NSR}} = \{\mathrm{U}, \underline{\mathrm{apr}}_{\mathrm{NSR}}(\mathfrak{Y}), \mathrm{B}_{\mathrm{NSR}}(\mathfrak{Y})\}$$

is a base for $\tau_{\mathrm{NSR}}(\mathfrak{Y})$

Theorem 3.10 Let $(U, \tau_{NSR}(\mathfrak{Y}), E)$ and $(U, \tau_{NSR'}(\mathfrak{Y}'), E)$ be two NSR-topological spaces w.r.t $\mathfrak{Y}$ and $\mathfrak{Y}'$ respectively. Let β_{NSR} and $\beta_{NSR'}$ be NSR-bases for τ_{NSR} and $\tau_{NSR''}$ respectively. If $\beta_{NSR'} \subseteq \beta_{NSR}$ then τ_{NSR} is finer than $\tau_{NSR'}$ and $\tau_{NSR'}$ is weaker than τ_{NSR}.

Theorem 3.11 Let $(U, \tau_{NSR}(\mathfrak{Y}), E)$ be an NSR-topological space. If β_{NSR} is an NSR-basis for τ_{NSR}. Then, the collection $\beta_{NSR_B} = \{A_i \cap B : A_i \in \beta_{NSR}, i \in I \subseteq \mathbb{N}\}$ is an NSR-basis for the NSR-subspace topology on B.

Proof. Consider $A_i \in \tau_{NSR_B}$. By definition of NSR-subspace topology, $C = D \cap B$,where $D \in \tau_{NSR}$. Since $D \in \tau_{NSR}$, it follows that $D = \cup_{A_i \in \beta_{NSR}} A_i$. Therefore,

$$C = (\cup_{A_i \in \beta_{NSR}} A_i) \cap B = \cup_{A_i \in \beta_{NSR}} (A_i \cap B).$$

3.1 Main Results

We present some results of neutrosophic soft rough topology including NSR-interior, NSR-exterior, NSR-closure, NSR-frontier, NSR-neighbourhood and NSR-limit point. These are some topological properties of NSR-topology and can be used to prove various results related to NSR-topological spaces.

Definition 3.12 Let $(U, \tau_{NSR}(\mathfrak{Y}), E)$ be an NSR-topological space w.r.t $\mathfrak{Y}$, where $T \subseteq U$ be an arbitrary subset. The NSR-interior of T is union of all NSR-open subsets of T and we denote it as $Int_{NSR}(T)$.

We verify that $Int_{NSR}(T)$ is the largest NSR-open set contained by T.

Theorem 3.13 Let $(U, \tau_{NSR}(\mathfrak{Y}), E)$ be a NSR-topological space over U w.r.t $\mathfrak{Y}$, S and T are NSR-sets over U. Then

- $Int_{NSR}(\emptyset) = \emptyset$ and $Int_{NSR}(U) = U$,
- $Int_{NSR}(S) \subseteq S$,
- S is NSR-open set $\Leftrightarrow Int_{NSR}(S) = S$,
- $Int_{NSR}(Int_{NSR}(S)) = Int_{NSR}(S)$,
- $S \subseteq T$ implies $Int_{NSR}(S) \subseteq Int_{NSR}(T)$,
- $Int_{NSR}(S) \cup Int_{NSR}(T) \subseteq Int_{NSR}(S \cup T)$,
- $Int_{NSR}(S) \cap Int_{NSR}(T) = Int_{NSR}(S \cap T)$.

Proof. (i) and (ii) are obvious.

(iii) First, suppose that $Int_{NSR}(S) = S$. Since $Int_{NSR}(S)$ is an NSR-open set, it follows that S is NSR-open set. For the converse, if S is a NSR-open set, then the largest NSR-open set that is contained in S is S itself. Thus, $Int_{NSR}(S) = S$.

(iv) Since $Int_{NSR}(S)$ is an NSR-open set, by part (iii) we get $Int_{NSR}(Int_{NSR}(S)) = Int_{NSR}(S)$.

(v) Suppose that $S \subseteq T$. By (ii) $Int_{NSR}(S) \subseteq S$. Then $Int_{NSR}(S) \subseteq T$. Since $Int_{NSR}(S)$ is NSR-open set contained by T. So by definition of NSR-interior $Int_{NSR}(S) \subseteq Int_{NSR}(T)$.

(vi) By using (ii) $Int_{NSR}(S) \subseteq S$ and $Int_{NSR}(T) \subseteq T$. Then, $Int_{NSR}(S) \cup Int_{NSR}(T) \subseteq S \cup T$. Since $Int_{NSR}(S) \cup Int_{NSR}(T)$ is an NSR-open, it follows that $Int_{NSR}(S) \cup Int_{NSR}(T) \subseteq Int_{NSR}(S \cup T)$.

(vii) By using (ii) $Int_{NSR}(S) \subseteq S$ and $Int_{NSR}(T) \subseteq T$. Then, $Int_{NSR}(S) \cap Int_{NSR}(T) \subseteq S \cap T$. Since $Int_{NSR}(S) \cap Int_{NSR}(T)$ is NSR-open, it follows that $Int_{NSR}(S) \cap Int_{NSR}(T) \subseteq Int_{NSR}(S \cap T)$. For the converse, $S \cap T \subseteq S$ also $S \cap T \subseteq T$. Then, $Int_{NSR}(S \cap T) \subseteq Int_{NSR}(S)$ and $Int_{NSR}(S \cap T) \subseteq Int_{NSR}(T)$. Hence $Int_{NSR}(S \cap T) \subseteq Int_{NSR}(S) \cap Int_{NSR}(T)$.

Definition 3.14 Let $(U, \tau_{NSR}(\mathfrak{Y}), E)$ be an NSR-topological space w.r.t $\mathfrak{Y}$, where $\mathfrak{Y} \subseteq U$. Let $T \subseteq U$. Then, NSR-exterior of T is defined as $Int_{NSR}(T^c)$, where T^c is complement of T. NSR-exterior of T is denoted by $Ext_{NSR}(T)$.

Definition 3.15 Let $(U, \tau_{NSR}(\mathfrak{Y}), E)$ be an NSR-topological space w.r.t $\mathfrak{Y}$, where $\mathfrak{Y} \subseteq U$. Let $T \subseteq U$. Then, NSR-closure of T is defined to be intersection of all NSR-closed supersets of T and is denoted by $Cl_{NSR}(T)$.

Example 3.16 Consider the NSR-topology given in Example 3.2, taking $T = \{g_1, g_2, g_3\}$, so $T^c = \{g_4, g_5\}$. Then $Int_{NSR}(T) = \{g_1, g_2\}$, $Ext_{NSR(T)} = Int_{NSR}(T^c) = \{g_4\}$ and $Cl_{NSR}(\mathrm{T}) = \{g_1, g_2, g_3, g_5\}$.

Theorem 3.17 Let $(U, \tau_{NSR}(\mathfrak{Y}), E)$ be a NSR-topological space over U w.r.t $\mathfrak{Y}$, S and T are NSR-sets over U. Then

- $Cl_{NSR}(\emptyset) = \emptyset$ and $Cl_{NSR}(U) = U$,
- $S \subseteq Cl_{NSR}(S)$,
- S is NSR-closed set $\Leftrightarrow S = Cl_{NSR}(S)$,
- $Cl_{NSR}(Cl_{NSR}(S)) = Cl_{NSR}(S)$,
- $S \subseteq T$ implies $Cl_{NSR}(S) \subseteq Cl_{NSR}(T)$,
- $Cl_{NSR}(S \cup T) = Cl_{NSR}(S) \cup Cl_{NSR}(T)$,
- $Cl_{NSR}(S \cap \mathrm{T}) \subseteq Cl_{NSR}(S) \cap Cl_{NSR}(T)$.

Proof. (i) and (ii) are straightforward.

(iii) First, consider $S = Cl_{NSR}(S)$. Since $Cl_{NSR}(S)$ is an NSR-closed set, so S is an NSR-closed set over U. For the converse, suppose that S be an NSR-closed set over U. Then, S is NSR-closed superset of S. So that $S = Cl_{NSR}(S)$.

(iv) By definition $Cl_{NSR}(S)$ is always NSR-closed set. Therefore, by part (iii) we have
$Cl_{NSR}(Cl_{NSR}(S)) = Cl_{NSR}(S)$.

(v) Let $S \subseteq T$. By (ii) $T \subseteq Cl_{NSR}(T)$. Then, $S \subseteq Cl_{NSR}(T)$. Since $Cl_{NSR}(T)$ is a NSR-closed superset of S, it follows that $Cl_{NSR}(S) \subseteq Cl_{NSR}(T)$.

(vi) Since $S \subseteq S \cup T$ and $T \subseteq S \cup T$, by part (v), $Cl_{NSR}(S) \subseteq Cl_{NSR}(S \cup T)$ and $Cl_{NSR}(T) \subseteq Cl_{NSR}(S \cup T)$. Hence $Cl_{NSR}(S) \cup Cl_{NSR}(S) \subseteq Cl_{NSR}(S \cup T)$. For the converse, let $S \subseteq Cl_{NSR}(S)$ and $T \subseteq Cl_{NSR}(T)$. Then, $S \cup T \subseteq Cl_{NSR}(S) \cup CL_{NSR}(T)$. Since $Cl_{NSR}(S) \cup Cl_{NSR}(T)$ is a NSR-closed superset of $S \cup T$. Thus, $Cl_{NSR}(S \cup T) = Cl_{NSR}(S) \cup Cl_{NSR}(T)$.

(vii) Since $S \cap T \subseteq S$ and $S \cap T \subseteq T$, by part(5) $Cl_{NSR}(S \cap T) \subseteq Cl_{NSR}(S)$ and $Cl_{NSR}(S \cap T) \subseteq Cl_{NSR}(T)$. Thus, we obtain $Cl_{NSR}(S \cap T) \subseteq Cl_{NSR}(S) \cap Cl_{NSR}(T)$.

Definition 3.18 Let $(U, \tau_{NR}(\mathfrak{Y}), E)$ be a NSR-topological space w.r.t $\mathfrak{Y}$, where $\mathfrak{Y} \subseteq U$. Let $T \subseteq U$. Then, NSR-frontier or NSR-boundary of T is denoted by $F_{r_{NSR}}(T)$ or $b_{NSR}(T)$ and mathematically defined as

$$F_{r_{NSR}}(T) = Cl_{NSR}(T) \cap Cl_{NSR}(T^c).$$

Clearly NSR-frontier $F_{r_{NSR}}(T)$ is an NSR-closed set.

Example 3.19 Consider the NSR-topology given in Example 3.2, taking $T = \{g_1, g_2, g_3\}$, so $T^c = \{g_4, g_5\}$. Then, $Cl_{NSR}(T) = \{g_1, g_2, g_3, g_5\}$ and $Cl_{NSR}(T^c) = \{g_3, g_4, g_5\}$.

$$F_{r_{NSR}}(T) = Cl_{NSR}(T) \cap Cl_{NSR}(T^c) = \{g_3, g_5\}$$

Definition 3.20 Let $(U, \tau_{NR}(\mathfrak{Y}), E)$ be an NSR-topological space. A subset X of U is said to be NSR-neighborhood of $g \in U$ if there exist an NSR-open set W_g containing g so that

$$g \in W_g \subseteq X.$$

Definition 3.21 The set of all the NSR-limit points of S is known as NSR-derived set of S and is denoted by S^d_{NSR}.

4 NSR-set in multi-criteria decision-making

In this section, we present an idea for multi-criteria decision-making method based on the neutrosophic soft rough sets $NSR-set$.

Let $U = \{g_1, g_2, g_3, \ldots, g_m\}$ is the set of objects under observation, E be the set of criteria to analyze the objects in U. Let $\mathfrak{A} = \{\xi_1, \xi_2, \xi_3, \ldots, \xi_n\} \subseteq E$ and $(\Phi, \mathfrak{A})$ be a neutrosophic soft set over U. Suppose that $H = \{P_1, P_2, \ldots, P_k\}$ be a set of experts, $\mathfrak{Y}_1, \mathfrak{Y}_2, \ldots, \mathfrak{Y}_k$ are subsets of U which indicate results of initial evaluations of experts $P_1, P_2, \ldots, P_k$, respectively and $\mathfrak{T}_1, \mathfrak{T}_2, \ldots \mathfrak{T}_r \in NS_U$ are real results that previously obtained for same or similar problems in different times or different places.

Definition 4.1 Let $\underline{apr}_{NSR_{\mathfrak{T}_q}}(\mathfrak{Y}_j), \overline{apr}_{NSR_{\mathfrak{T}_q}}(\mathfrak{Y}_j)$ be neutrosophic soft lower and upper approximations of $\mathfrak{Y}_j (j = 1,2, \ldots, k)$ related to $\mathfrak{T}_q (q = 1,2, \ldots, r)$. Then,

$$\underline{a} = \begin{pmatrix} \underline{n}_1^1 & \underline{n}_2^1 & \cdots & \underline{n}_k^1 \\ \underline{n}_1^2 & \underline{n}_2^2 & \cdots & \underline{n}_k^2 \\ \vdots & \vdots & \ddots & \vdots \\ \underline{n}_1^r & \underline{n}_2^r & \cdots & \underline{n}_k^r \end{pmatrix} \quad (1)$$

$$\overline{a} = \begin{pmatrix} \overline{n}_1^1 & \overline{n}_2^1 & \cdots & \overline{n}_k^1 \\ \overline{n}_1^2 & \overline{n}_2^2 & \cdots & \overline{n}_k^2 \\ \vdots & \vdots & \ddots & \vdots \\ \overline{n}_1^r & \overline{n}_2^r & \cdots & \overline{n}_k^r \end{pmatrix} \quad (2)$$

are called neutrosophic soft lower and neutrosophic upper approximations matrices, respectively, and represented by $\underline{a}$ and $\overline{a}$. Here

$$\underline{n}_j^q = (\underline{g}_{1j}^q, \underline{g}_{2j}^q, \ldots, \underline{g}_{nj}^q) \quad (3)$$

$$\overline{n}_j^q = (\overline{g}_{1j}^q, \overline{g}_{2j}^q, \ldots, \overline{g}_{nj}^q) \quad (4)$$

Where

$$\underline{g}_{ij}^q = \begin{cases} 1, g_i \in \underline{apr}_{NSR_{\mathfrak{T}_q}}(\mathfrak{Y}_j) \\ 0, g_i \notin \underline{apr}_{NSR_{\mathfrak{T}_q}}(\mathfrak{Y}_j) \end{cases}$$

and

$$\overline{g}_{ij}^q = \begin{cases} 1, g_i \in \overline{apr}_{NSR_{\mathfrak{T}_q}}(\mathfrak{Y}_j) \\ 0, g_i \notin \overline{apr}_{NSR_{\mathfrak{T}_q}}(\mathfrak{Y}_j) \end{cases}$$

Definition 4.2 Let $\underline{n}$ and $\overline{n}$ be neutrosophic soft lower and neutrosophic upper approximations matrices based on $\underline{apr}_{NSR_{\mathfrak{T}_q}}(\mathfrak{Y})_j$, $\overline{apr}_{NSR_{\mathfrak{T}_q}}(\mathfrak{Y})_j$ for $q = 1,2,\dots r$ and $j = 1,2,\dots,k$. Neutrosophic soft lower approximation vector represented by $(\underline{n})$ and neutrosophic soft upper approximation vector represented by $(\overline{n})$ are defined by, respectively,

$$\underline{n} = \bigoplus_{j=1}^{k} \bigoplus_{q=1}^{r} \underline{n}_j^q \tag{5}$$

$$\overline{n} = \bigoplus_{j=1}^{k} \bigoplus_{q=1}^{r} \overline{n}_j^q \tag{6}$$

Here the operation $\oplus$ represents the vector summation.

Definition 4.3 Let $\underline{n}$ and $\overline{n}$ be neutrosophic soft $\mathfrak{T}_q -$ lower approximation vector and neutrosophic soft $\mathfrak{T}_q -$ upper approximation vector, respectively. Then, vector summation $\underline{n} \oplus \overline{n} = (w_1, w_2, \dots, w_n)$ is called decision vector.

Definition 4.4 Let $\underline{n} \oplus \overline{n} = (w_1, w_2, \dots, w_n)$ be the decision vector. Then, each w_i is called a weighted number of $g_i \in U$ and g_i is called an optimum element of U if it weighted number is maximum of $w_i \forall i \in I_n$. In this case, if there are more then one optimum elements of U, select one of them.

Algorithm 1 for neutrosophic soft rough set:

Input

Step-1: Take initial evaluations $\mathfrak{Y}_1, \mathfrak{Y}_2, \dots, \mathfrak{Y}_k$ of experts $P_1, P_2, \dots, P_k$.

Step-2: Construct $\mathfrak{T}_1, \mathfrak{T}_2, \dots \mathfrak{T}_r$ neutrosophic soft sets using real results.

Step-3: Compute $\underline{apr}_{NSR_{\mathfrak{T}_q}}(\mathfrak{Y}_j)$ and $\overline{apr}_{NSR_{\mathfrak{T}_q}}(\mathfrak{Y}_j)$ for each $q = 1,2,\dots,r$ and $j = 1,2,\dots,k$.

Step-4: Construct neutrosophic soft lower and neutrosophic soft upper approximations matrices $\underline{a}$ and $\overline{a}$.

Step-5: Compute $\underline{n}$ and $\overline{n}$,

Step-6: Compute $\underline{n} \oplus \overline{n}$,

Output

Step-7: Select $max_{i \in I_n} w_i$.

The flow chart of proposed algorithm 1 is represented in Figure.1

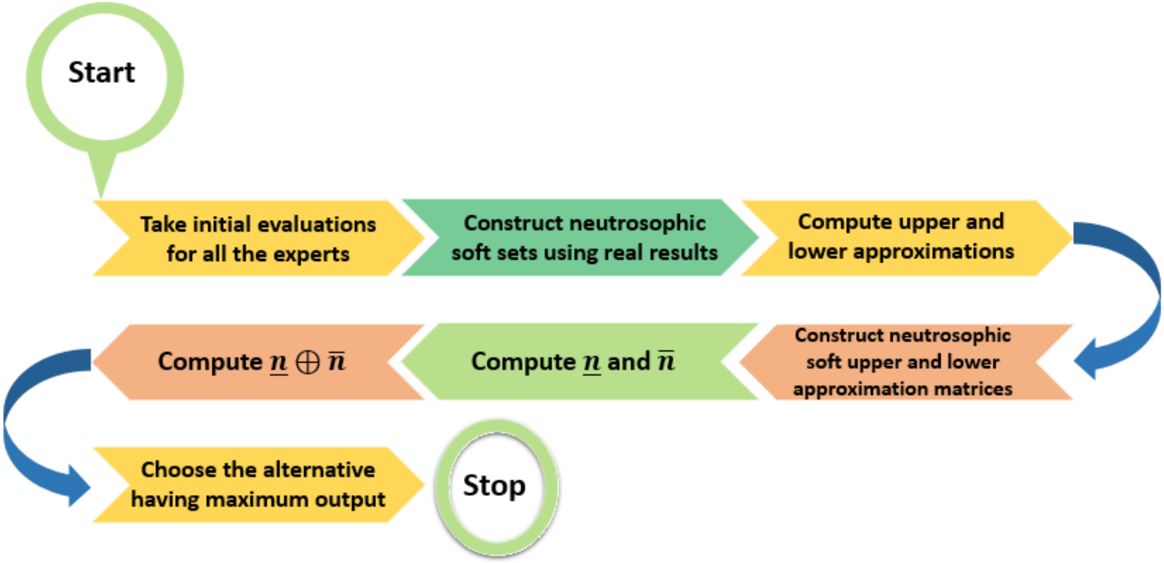

Fig 1: Flow chart diagram of proposed algorithm 1 for NSR-set.

Example 4.5 In finance company three finance experts P_1, P_2, P_3 want to make investment one of the clothing brand

$$\{g_1 = Jor, g_2 = Aero, g_3 = Chan, g_4 = Li, g_5 = Srk\}.$$

The set of parameters include the following parameters

$$\mathfrak{A} = \{\xi_1 = Market\ Share, \xi_2 = Acknowledgement, \xi_3 = Uniqueness, \xi_4 = Economical\ Magnification\}$$

$Step1$: $\mathfrak{Y}_1 = \{g_1, g_2, g_4\}, \mathfrak{Y}_2 = \{g_1, g_3, g_5\}, \mathfrak{Y}_3 = \{g_2, g_4, g_5\}$ are primary evaluations of experts P_1, P_2, P_3, respectively.

$Step2$: Neutrosophic soft sets $\mathfrak{T}_1, \mathfrak{T}_2, \mathfrak{T}_3$ are the actual results in individual three periods and tabular representations of these neutrosophic soft sets are given in Table 2, Table 3 and Table 4, respectively.

$\mathfrak{T}_1$	ξ_1	ξ_2	ξ_3	ξ_4
g_1	(0.6,0.6,0.2)	(0.8,0.4.0.3)	(0.7,0.4,0.3)	(0.8,0.6,0.4)
g_2	(0.4,0.6,0.6)	(0.6,0.2,0.4)	(0.6,0.4,0.3)	(0.7,0.6,0.6)
g_3	(0.6,0.4,0.2)	(0.8,0.1,0.3)	(0.7,0.2,0.5)	(0.7,0.6,0.4)
g_4	(0.6,0.3,0.3)	(0.8,0.2,0.2)	(0.5,0.2,0.6)	(0.7,0.5,0.6)
g_5	(0.8,0.2,0.3)	(0.8,0.3,0.2)	(0.7,0.3,0.4)	(0.9,0.5,0.7)

Table 2: Neutrosophic soft set $\mathfrak{T}_1$

$\mathfrak{T}_2$	ξ_1	ξ_2	ξ_3	ξ_4
g_1	(0.6,0.4,0.2)	(0.8,0.1,0.3)	(0.7,0.2,0.5)	(0.7,0.6,0.4)
g_2	(0.4,0.6,0.6)	(0.6,0.2,0.4)	(0.6,0.4,0.3)	(0.7,0.6,0.6)
g_3	(0.8,0.2,0.3)	(0.8,0.3,0.2)	(0.7,0.3,0.4)	(0.9,0.5,0.7)
g_4	(0.6,0.3,0.3)	(0.8,0.2,0.2)	(0.5,0.2,0.6)	(0.7,0.5,0.6)
g_5	(0.6,0.6,0.2)	(0.8,0.4.0.3)	(0.7,0.4,0.3)	(0.8,0.6,0.4)

Table 3: Neutrosophic soft set $\mathfrak{T}_2$

$\mathfrak{T}_3$	ξ_1	ξ_2	ξ_3	ξ_4
g_1	(0.6,0.6,0.2)	(0.8,0.4.0.3)	(0.7,0.4,0.3)	(0.8,0.6,0.4)
g_2	(0.6,0.3,0.3)	(0.8,0.2,0.2)	(0.5,0.2,0.6)	(0.7,0.5,0.6)
g_3	(0.6,0.4,0.2)	(0.8,0.1,0.3)	(0.7,0.2,0.5)	(0.7,0.6,0.4)
g_4	(0.4,0.6,0.6)	(0.6,0.2,0.4)	(0.6,0.4,0.3)	(0.7,0.6,0.6)
g_5	(0.8,0.2,0.3)	(0.8,0.3,0.2)	(0.7,0.3,0.4)	(0.9,0.5,0.7)

Table 4: Neutrosophic soft set $\mathfrak{T}_3$

The tabular representation of the neutrosophic right neighborhoods of $\mathfrak{T}_1, \mathfrak{T}_2, \mathfrak{T}_3$ are given in Table5, Table 6 and Table 7 respectively.

Neighborhoods of $\mathfrak{T}_1$	
$g_1]_{\mathfrak{A}}$	$\{g_1\}$
$g_2]_{\mathfrak{A}}$	$\{g_1, g_2\}$
$g_3]_{\mathfrak{A}}$	$\{g_1, g_3\}$
$g_4]_{\mathfrak{A}}$	$\{g_4\}$
$g_5]_{\mathfrak{A}}$	$\{g_5\}$

Table 5: Neutrosophic right neighborhoods of $\mathfrak{T}_1$ w.r.t set $\mathfrak{A}$

Neighborhoods of $\mathfrak{T}_2$	
$g_1]_{\mathfrak{A}}$	$\{g_1, g_5\}$
$g_2]_{\mathfrak{A}}$	$\{g_2, g_5\}$
$g_3]_{\mathfrak{A}}$	$\{g_3\}$
$g_4]_{\mathfrak{A}}$	$\{g_4\}$
$g_5]_{\mathfrak{A}}$	$\{g_5\}$

Table 6: Neutrosophic right neighborhoods of $\mathfrak{T}_2$ w.r.t set $\mathfrak{A}$

Neighborhoods of $\mathfrak{T}_3$	
$g_1]_{\mathfrak{A}}$	$\{g_1\}$
$g_2]_{\mathfrak{A}}$	$\{g_2\}$
$g_3]_{\mathfrak{A}}$	$\{g_1, g_3\}$
$g_4]_{\mathfrak{A}}$	$\{g_1, g_4\}$
$g_5]_{\mathfrak{A}}$	$\{g_5\}$

Table 7: Neutrosophic right neighborhoods of $\mathfrak{T}_3$ w.r.t set $\mathfrak{A}$

$Step3$: Next we find $\underline{apr}_{NSR_{\mathfrak{T}_1}}$ and $\overline{apr}_{NSR_{\mathfrak{T}_1}}$ for each $\mathfrak{Y}_j$, where $j = 1,2,3$.

$$\underline{apr}_{NSR_{\mathfrak{T}_1}}(\mathfrak{Y}_1) = \{g_1, g_2, g_4\},$$
$$\overline{apr}_{NSR_{\mathfrak{T}_1}}(\mathfrak{Y}_1) = \{g_1, g_2, g_3, g_4\},$$
$$\underline{apr}_{NSR_{\mathfrak{T}_1}}(\mathfrak{Y}_2) = \{g_1, g_3, g_5\},$$
$$\overline{apr}_{NSR_{\mathfrak{T}_1}}(\mathfrak{Y}_2) = \{g_1, g_2, g_3, g_5\},$$
$$\underline{apr}_{NSR_{\mathfrak{T}_1}}(\mathfrak{Y}_3) = \{g_4, g_5\},$$
$$\overline{apr}_{NSR_{\mathfrak{T}_1}}(\mathfrak{Y}_3) = \{g_1, g_2, g_3, g_4, g_5\}$$

Similarly we find $\underline{apr}_{NSR_{\mathfrak{T}_2}}, \overline{apr}_{NSR_{\mathfrak{T}_2}}$ and $\underline{apr}_{NSR_{\mathfrak{T}_3}}$, $\overline{apr}_{NSR_{\mathfrak{T}_3}}$ corresponding to each $\mathfrak{Y}_j$, where $j = 1,2,3$.

$$\underline{apr}_{NSR_{\mathfrak{T}_2}}(\mathfrak{Y}_1) = \{g_4\},$$
$$\overline{apr}_{NSR_{\mathfrak{T}_2}}(\mathfrak{Y}_1) = \{g_1, g_2, g_4, g_5\},$$
$$\underline{apr}_{NSR_{\mathfrak{T}_2}}(\mathfrak{Y}_2) = \{g_1, g_3, g_5\},$$
$$\overline{apr}_{NSR_{\mathfrak{T}_2}}(\mathfrak{Y}_2) = \{g_1, g_2, g_3, g_5\},$$
$$\underline{apr}_{NSR_{\mathfrak{T}_2}}(\mathfrak{Y}_3) = \{g_4, g_5\},$$
$$\overline{apr}_{NSR_{\mathfrak{T}_2}}(\mathfrak{Y}_3) = \{g_1, g_2, g_4, g_5\}$$

and

$$\underline{apr}_{NSR_{\mathfrak{T}_3}}(\mathfrak{Y}_1) = \{g_1, g_2, g_4\},$$
$$\overline{apr}_{NSR_{\mathfrak{T}_3}}(\mathfrak{Y}_1) = \{g_1, g_2, g_3, g_4\},$$
$$\underline{apr}_{NSR_{\mathfrak{T}_3}}(\mathfrak{Y}_2) = \{g_1, g_3, g_5\},$$
$$\overline{apr}_{NSR_{\mathfrak{T}_3}}(\mathfrak{Y}_2) = \{g_1, g_3, g_4, g_5\},$$
$$\underline{apr}_{NSR_{\mathfrak{T}_3}}(\mathfrak{Y}_3) = \{g_2, g_5\},$$
$$\overline{apr}_{NSR_{\mathfrak{T}_3}}(\mathfrak{Y}_3) = \{g_1, g_2, g_4, g_5\}$$

$Step4$: Neutrosophic soft lower approximation matrix and neutrosophic soft upper approximation matrix are obtained as follows:

$$\underline{a} = \begin{pmatrix} (1,1,0,1,0) & (1,0,1,0,1) & (0,0,0,1,1) \\ (0,0,0,1,0) & (1,0,1,0,1) & (0,0,0,1,1) \\ (1,1,0,1,0) & (1,0,1,0,1) & (0,1,0,0,0) \end{pmatrix} \tag{7}$$

$$\overline{a} = \begin{pmatrix} (1,1,1,1,0) & (1,1,1,0,1) & (1,1,1,1,1) \\ (1,1,0,1,1) & (1,1,1,0,1) & (1,1,0,1,1) \\ (1,1,1,1,0) & (1,0,1,1,1) & (1,1,0,1,1) \end{pmatrix} \tag{8}$$

$Step5$: Using Eqs. 7 and 8, neutrosophic soft lower approximation vector and neutrosophic soft upper approximation vector are obtained as follows:

$$\underline{n} = (5,3,3,5,5)$$

$$\overline{n} = (9,8,6,7,7)$$

$Step6$: Decision vector is obtained as $\underline{n} \oplus \overline{n} = (14,11,9,12,12)$.

$Step7$: Since $max_{i \in I_n} w_i = w_1 = 14$, optimal clothing brand is $g_1 = Jor$.

5 NSR-topology in multi-criteria decision-making

In this section, we use the concept of NSR-topology in multi-criteria decision-making. The idea of core in the picking of attributes to the rough set was introduced by Thivagar in [45]. In the following definition, we develop this idea of core to the NSR-set.

Definition 5.1 Let U be the set of objects, $K = (\Phi, \mathfrak{A})$ is the neutrosophic soft set and $G = (U, K)$ is the the corresponding neutrosophic soft approximation space. Let $\mathfrak{R}$ be an indiscernibility relation. Let τ_{NSR} be an NSR-topology on U and β_{NSR} be the basis defined for τ_{NSR}. Let $\mathfrak{N}$ be the subset of $\mathfrak{A}$, is said to be core of $\mathfrak{R}$ if $\beta_{\mathfrak{N}} \neq \beta_{NSR-(s)}$ for each 's' in $\mathfrak{N}$. i.e. a core of $\mathfrak{R}$ is the subset of attributes with the condition that if we remove any element from $\mathfrak{N}$ it will affect the classification power of the attributes.

Algorithm 2 for neutrosophic soft rough topology:

Input

Step-1: Consider initial universe U, set of attributes $\mathfrak{A}$ which can be classified into division $\mathbb{D}$ of decision attributes, $\mathbb{C}$ of condition attributes and an indiscernibility relation $\mathfrak{R}$ on U. Construct the neutrosophic soft set in tabular form corresponding to $\mathbb{C}$ condition attributes and a subset $\mathfrak{Y}$ of U. The columns indicate the elements of universe, rows represent the attributes and entries of table give attribute values.

Output

Step-2: Classify set $\mathfrak{Y}$ and find the NSR-approximation subsets $\left(\underline{\mathfrak{R}}_G(\mathfrak{Y}), \overline{\mathfrak{R}}_G(\mathfrak{Y})\right) and\ B_G(\mathfrak{Y})$

w.r.t $\mathfrak{R}$.

Step-3: Define Neutrosophic Soft Rough Topology $\tau_{\mathfrak{R}}$ on U and find basis β_{NSR}.

Step-4: By removing an attribute ξ from $\mathbb{C}$, find again the NSR-approximation subsets $(\underline{\mathfrak{R}}_G(\mathfrak{Y}), \overline{\mathfrak{R}}_G(\mathfrak{Y})), B_G(\mathfrak{Y}))$ w.r.t $\mathfrak{R} on \mathbb{C} - (\xi)$.

Step-5: Generate $NSR - topology\ \tau_{NSR-(\xi)}$ on U,define its basis $\beta_{NSR-(\xi)}$.

Step-6: Repeat step 4 and step 5 for each attribute in $\mathbb{C}$.

Step-7: The attributes for which $\beta_{NSR-(\xi)} \neq \beta_{NSR}$ gives the $core(\mathfrak{R})$.

The flow chart diagram of proposed algorithm 2 is represented as Figure 2.

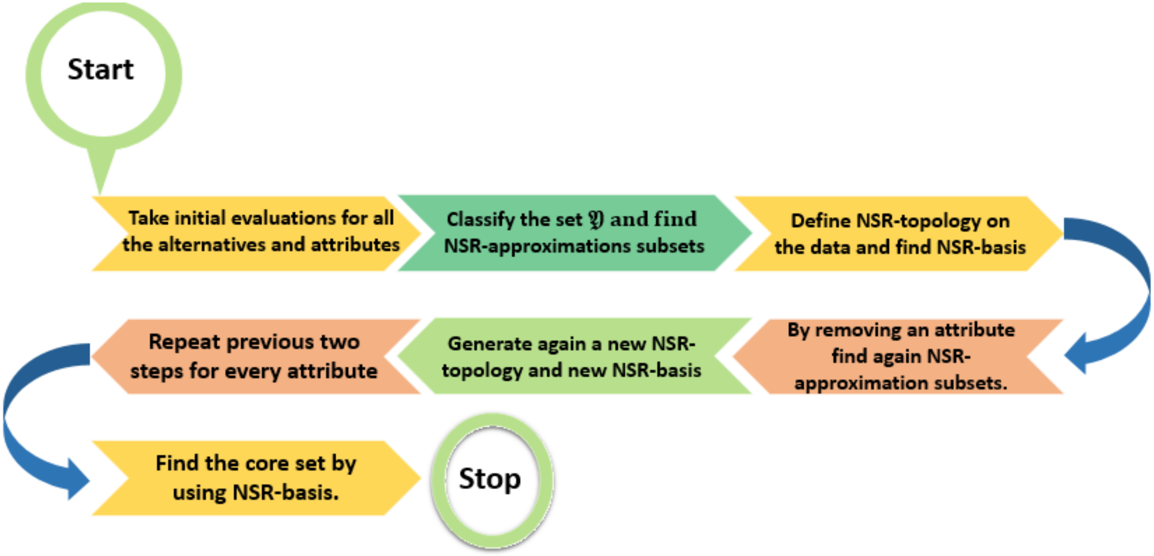

Fig 2: The flow chart diagram of algorithm 2 for NSR-topology.

Example 5.2 Here we consider the problem of Crime rate in developing countries of Asia, Crime is an unlawful act punishable by a state or other authority. In other words, we can say that a crime is an act harmful not only to some individual but also to a community, society or the state. A developing country is a country with a less developed industrial base and a low Human Development Index (HDI) relative to other countries. Developing countries are facing so many issues including high crime rate. This is the fundamental reason of emerging questions in our mind, that why the crime rate is higher in developing countries?

We apply the concept of NSR-topology in Crime rate of developing countries of Asia. Consider the following information table which shows data about 5 developing countries. The rows of the table represent the objects(countries). Let $U = \{g_1 = Bangladesh, g_2 = Afghanistan, g_3 = SriLanka, g_4 = Nepal, g_5 = Pakistan\}$ be the set of developing countries and $\mathfrak{A} = \{\xi_1, \xi_2, \xi_3, \xi_4\}$, where ξ_1 stands for `corruption', $\xi_2$ stands for `poverty ', ξ_3 stands for `self actualization' and $\xi_4$ stands for `lack of education'. Let $K = (\Phi, \mathfrak{A})$ is the neutrosophic soft set over U shown by Table 8,corresponding soft approximation space $G = (U, K)$.

K	ξ_1	ξ_2	ξ_3	ξ_4	Crime Rate
g_1	(0.6,0.6,0.2)	(0.8,0.4.0.3)	(0.7,0.4,0.3)	(0.8,0.6,0.4)	High
g_2	(0.4,0.6,0.6)	(0.6,0.2,0.4)	(0.6,0.4,0.3)	(0.7,0.6,0.6)	Medium
g_3	(0.6,0.4,0.2)	(0.8,0.1,0.3)	(0.7,0.2,0.5)	(0.7,0.6,0.4)	Medium
g_4	(0.6,0.3,0.3)	(0.8,0.2,0.2)	(0.5,0.2,0.6)	(0.7,0.5,0.6)	High
g_5	(0.8,0.2,0.3)	(0.8,0.3,0.2)	(0.7,0.3,0.4)	(0.9,0.5,0.7)	High

Table 8: Neutrosophic soft set $K = (\Phi, \mathfrak{A})$

The tabular representation of neutrosophic right neighborhoods of K w.r.t set $\mathfrak{A}$ is given Table 9.

Neighborhoods of K	
$g_1]_{\mathfrak{A}}$	$\{g_1\}$
$g_2]_{\mathfrak{A}}$	$\{g_1, g_2\}$
$g_3]_{\mathfrak{A}}$	$\{g_1, g_3\}$

$g_4]_{\mathfrak{A}}$	$\{g_4\}$
$g_5]_{\mathfrak{A}}$	$\{g_5\}$

Table 9: Neutrosophic right neighborhoods of K w.r.t set $\mathfrak{A}$

For $\mathfrak{Y} = \{g_1, g_3, g_5\}$ and indiscernibility relation 'Crime rate' we have $\underline{\mathfrak{R}}_G(\mathfrak{Y}) = \{g_1, g_3, g_5\}$, $\overline{\mathfrak{R}}_G(\mathfrak{Y}) = \{g_1, g_2, g_3, g_5\}$ and $B_G(\mathfrak{Y}) = \{g_2\}$.

So we define NSR-topology as $\tau_{NSR}(\mathfrak{Y}) = \{U, \emptyset, \{g_1, g_3, g_5\}, \{g_1, g_2, g_3, g_5\}, \{\mathrm{g}_2\}\}$ and its basis $\beta_{NSR} = \{U, \{g_1, g_3, g_5\}, \{g_2\}\}$.

If we remove the attribute `Corruption', then the tabular representation of neutrosophic right neighborhoods of K w.r.t set $\mathfrak{A} - \xi_1$ is given Table 10.

Neighborhoods of K	
$g_1]_{\mathfrak{A}-\xi_1}$	$\{g_1\}$
$g_2]_{\mathfrak{A}-\xi_1}$	$\{g_1, g_2\}$
$g_3]_{\mathfrak{A}-\xi_1}$	$\{g_1, g_3\}$
$g_4]_{\mathfrak{A}-\xi_1}$	$\{g_4\}$
$g_5]_{\mathfrak{A}-\xi_1}$	$\{g_5\}$

Table 10: Neutrosophic right neighborhoods of K w.r.t set $\mathfrak{A} - \xi_1$

we have

$$\tau_{NSR-\xi_1}(\mathfrak{Y}) = \{U, \emptyset, \{g_1, g_3, g_5\}, \{g_1, g_2, g_3, g_5\}, \{g_2\}\}$$

is a NSR-topology and its basis is

$$\beta_{NSR} - \xi_1 = \{U, \{g_1, g_3, g_5\}, \{g_2\}\} = \beta_{NSR}.$$

If we remove the attribute `poverty', then the tabular representation of neutrosophic right neighborhoods of K w.r.t set $\mathfrak{A} - \xi_2$ is given Table 11.

Neighborhoods of K	
$g_1]_{\mathfrak{A}-\xi_2}$	$\{g_1\}$
$g_2]_{\mathfrak{A}-\xi_2}$	$\{g_1, g_2\}$
$g_3]_{\mathfrak{A}-\xi_2}$	$\{g_1, g_3\}$
$g_4]_{\mathfrak{A}-\xi_2}$	$\{g_1, g_3, g_4\}$
$g_5]_{\mathfrak{A}-\xi_2}$	$\{g_5\}$

Table 11: Neutrosophic right neighborhoods of K w.r.t set $\mathfrak{A} - \xi_2$

We have an NSR-topology and its base as follows:

$$\tau_{NSR-\xi_2}(Y) = \{U, \emptyset, \{g_1, g_3, g_5\}, \{g_2, g_4\}\}$$

and $$\beta_{NSR} - \xi_2 = \{U, \{g_1, g_3, g_5\}, \{g_2, g_4\}\} \neq \beta_{NSR},$$

respectively. If we remove the attribute 'self actualization', then the tabular representation of neutrosophic right neighborhoods of K w.r.t set $\mathfrak{A} - \xi_3$ is given Table 12.

Neighborhoods of K	
$g_1]_{\mathfrak{A}-\xi_3}$	$\{g_1\}$
$g_2]_{\mathfrak{A}-\xi_3}$	$\{g_1, g_2\}$

$g_3]_{\mathfrak{A}-\xi_3}$	$\{g_1, g_3\}$
$g_4]_{\mathfrak{A}-\xi_3}$	$\{g_4\}$
$g_5]_{\mathfrak{A}-\xi_3}$	$\{g_5\}$

Table 12: Neutrosophic right neighborhoods of K w.r.t set $\mathfrak{A}-\xi_3$

We have an NSR-topology and its base as follows:

$$\tau_{NSR-\xi_3}(Y)=\{U,\emptyset,\{g_1,g_3,g_5\},\{g_1,g_2,g_3,g_5\},\{g_2\}\}$$

and

$$\beta_{NSR}-\xi_3=\{U,\emptyset,\{g_1,g_3,g_5\},\{g_2\}=\beta_{NSR}\},$$

respectively. If we remove the attribute `lack of education', then the tabular representation of neutrosophic right neighborhoods of K w.r.t set $\mathfrak{A}-\xi_4$ is given Table 13.

Neighborhoods of K	
$g_1]_{\mathfrak{A}-\xi_4}$	$\{g_1\}$
$g_2]_{\mathfrak{A}-\xi_4}$	$\{g_1, g_2\}$
$g_3]_{\mathfrak{A}-\xi_4}$	$\{g_1, g_3\}$
$g_4]_{\mathfrak{A}-\xi_4}$	$\{g_4\}$
$g_5]_{\mathfrak{A}-\xi_4}$	$\{g_5\}$

Table 13: Neutrosophic right neighborhoods of K w.r.t set $\mathfrak{A}-\xi_4$

We have an NSR-topology and its base as follows:

$$\tau_{NSR-\xi_4}(Y)=\{U,\emptyset,\{g_1,g_3,g_5\},\{g_1,g_2,g_3,g_5\},\{g_2\}\}$$

and

$$\beta_{NSR}-\xi_4=\{U,\emptyset,\{g_1,g_3,g_5\},\{g_2\}=\beta_{NSR}\},$$

respectively. Thus, $CORE(NSR)=\{\xi_2\}$, i.e., `poverty' is the deciding attributes of the Crime Rate in developing countries of Asia.

Discussion and comparitive analysis 5.3 In this section, we discuss our results obtained from both numerical examples and present a comparative analysis of proposed topological space to some existing topological spaces. Table 14 describes the comparison of both proposed algorithms based on NSR-sets and NSR-topology. The algorithm 1 is used to find the optimal decision about the set of alternatives and establish the ranking order between them. We can choose the best and worst alternative from the given input information. While algorithm 2 is used to choose the most relevant and significant attribute to which one can observe the specific characteristic of the alternatives. This is called the CORE of the problem, which is an essential part of the decision-making difficulty. Both algorithms have their own merits and can be used to solve decision-making problems in medical, artificial intelligence, business, agriculture, engineering, etc.

Proposed Algorithms	Choice values	Final Decision	Selection criteria
Algorithm 1 (NSR-sets)	$g_1 \succ g_4 \succ g_5 \succ g_2 \succ g_3$	g_1	Based on alternatives
Algorithm 2 (NSR-topology)	$CORE(NSR)=\{\xi_2\}$	$\xi_2=$ poverty	Based on attributes

Table 14: Comparison of prooposed algorithms

Now we present a soft comparative analysis of proposed approach with some existing approaches. In Table 15, we describe the comparison and discuss about their advantages and limitations.

Set theories	Information about Indeterminacy part	Upper and lower approximations with boundary region	Parameterizations	Advantages	Limitations
Fuzzy sets [1]	No	No	No	Deal with the hesitations.	Do not collect any information about the indeterminacy of input data.
Neutrosophic sets [4, 5]	Yes	No	No	Deal with the data having indeterminacy information.	Do not deal with the roughness and parameterizations.
Rough sets [2, 3]	No	Yes	No	Deal with the roughness of input information and create upper, lower and boundary regions.	Do not give any information about the parameterizations.
Soft sets [6]	No	No	Yes	Deal with the uncertainity with parameterizations.	Do not provide information about the roughness of data.
Soft rough sets [17]	No	Yes	Yes	Deal with uncertainities and roughness of data.	Do not give information about the indeterminacy part of problem.
Rough neutrosophic sets [47]	Yes	Yes	No	Deal with the roughness having indeterminacy information.	Do not deal with the parameterizations.
Neutrosophic soft rough sets and topology (proposed)	Yes	Yes	Yes	Provide the data of indeterminacy part and remove roughness under parameterizations without any loss of information.	Effective but heavy calculations as compared to above existing theories.

Table 15: Comparitive analysis of proposed approach with some exsting theories.

6. Conclusion

Most of the issues in decision-making problems are associated with uncertain, imprecise and, multipolar information, which cannot be tackled properly through the fuzzy set. So to overcome this particular deficiency rough set was introduced by Pawlak, which deals with the vagueness of input data. This research implies the novel approach of neutrosophic soft rough set (NSR-set) with neutrosophic soft rough topology (NSR-topology). We presented various topological structures of NSR-topology named as NSR-interior, NSR-closure, NSR-exterior, NSR-neighborhood, NSR-limit point and, NSR-bases with numerous examples. We established two novel algorithms to deal with multi-criteria decision-making (MCDM) problems under NSR-data. One is based on NSR-sets and the other is based on NSR-topology with NSR-bases. This research is more efficient and flexible than the other approaches. The proposed algorithms are simple and easy to understand which can be applied easily on whatever type of alternatives and measures. Both algorithms are flexible and easily altered according to the different situations, inputs and, outputs. In the future, we will extend our work to solve the MCDM problems by using TOPSIS, AHP, VIKOR, ELECTRE family and, PROMETHEE family using different hybrid structures of fuzzy and rough sets.

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