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Neutrosophic Multiset Topological Space

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Abstract: In this article we have investigated some properties of neutrosophic multiset topology. The behavior of compactness and connectedness in neutrosophic multiset topology, continuous function on neutrosophic multiset topology etc have been examined. Neutrosophic multiset is a generalization of multisets and neutrosophic sets. Several properties of neutrosophic topological space in view of neutrosophic multiset topological space have been studied.

Keywords: Neutrosophic Multiset; Neutrosophic Minimal set; Neutrosophic Maximal set; Neutrosophic Multiset topology; Compactness, Connectedness; Continuous Neutrosophic Multiset; Separation axioms; Distance function.

1. Introduction

In recent years, multisets and neutrosophic sets have become a subject of great interest for researchers. Mathematicians always like to solve a complicated problem in a simple way and to find out the most feasible solution. Neutrosophy has been introduced and studied by Smarandache [13, 15] as a new branch of philosophy. Recently various papers published on neutrosophic topology and many researchers doing very well, neutrosophic decision making had been studied in [15, 17]. Algebraic properties of neutrosophic set studied in [9, 13], Neutrosophic Bipolar Vague Soft Set, and its property studied in [9]. Smarandache generalizes intuitionistic fuzzy sets (IFSs) and other kinds of sets to neutrosophic sets (NSs). In Smarandache [12, 13], some distinctions between NSs and IFSs are underlined. decision-making problem, algebraic property one can analysis by topological property connectedness and compactness property that property can help to take the decision into a more reliable way. Smarandache [13, 14, 15] also defined various notions of neutrosophic topologies on the non-standard interval. The logic of the neutrosophic set is very clear and its utilization on topology is very beneficial for many standard problems like diagnosis of bipolar disorder diseases group decision making and analytical property and evaluation Hospital medical care systems etc. [1, 9, 13]. The relation between the intuitionistic fuzzy topology (IFT) on an IFS and the neutrosophic topology are also analyzed by Smarandache.

Multiset theory was introduced by Bilzard [3]. Later on multiset topological space was studied by many researcher Shravan and Tripathy [17, 18, 19]. The purpose of this paper is to construct a new
generalization of topological space called the neutrosophic multiset topological space. The possible
application of neutrosophic multiset topological space has been studied. For the different types of
behavior of objects in nature sometimes set theory and multiset theory fails to describe some
particular situation. Sometimes it is observed that Neutrosophic Multiset can be described in an
easier way to handle such cases. Neutrosophic set and topological space have been studied by
Salama and Albrowi [10, 11]. The concept of multiset topological space has been applied for studying
different properties of spatial objects. In this article we have used multiset neutrosophic topological
space for studying various spatial topological properties, like closeness connectedness and the
completeness property and its application further in various fields.

2. Materials and Methods

We procure some existing definitions in this paper, one may refer to Smarandache ([13], [15]) and S.
Alias, et.al [2].

We define functions \( T, F \) and \( I \) from \( X \) to \([0, 1]\). Where \( T \) is membership value, \( F \) fails membership
value and \( I \) is the indeterminacy value.

The definition of neutrosophic multiset was first define by Smarandache [12] as follows.

**Definition 2.1.** [12] A Neutrosophic Multiset is a neutrosophic set where one or more elements are
repeated with the same neutrosophic components, or with different neutrosophic components.

**Definition 2.2.** The Empty neutrosophic multiset is denoted by \( N_0 \) and define by
\[ N_0 = \{ \langle x(0,1,1) \rangle : \forall x \in X \} \text{ where } x \text{ can be repeated.} \]

**Definition 2.3.** The Whole neutrosophic multiset is denoted by \( W_X \) and define by
\[ W_X = \{ \langle x(1,0,0) \rangle : \forall x \in X \} \text{ where } x \text{ can be repeated.} \]

The power set of neutrosophic multiset is denoted by \( P(X) \).

The collection of all possible subsets of \( X \) is called the power set of the netrosophic multiset.

**Definition 2.4.** Let \( A = \{ \langle x \in T_A(x), I_A(x), F_A(x) \rangle : x \in X \} \) be a neutrosophic multiset on \( X \) then the
compliment of \( A \) is denoted by \( A^c \) and define by
\[ A^c = \{ \langle x \in F_A(x), 1-I_A(x), T_A(x) \rangle : x \in X \} . \]

Where \( x \), can be repeated based on its multiplicity and the corresponding \( T, F, I \) values may or may
not be equal.
Definition 2.5. The intersection of NM sets are defined by \( A \cap B = \{x: x \in A \text{ and } x \in B\} \).

Definition 2.6. The union of NM sets are defined by \( A \cup B = \{x: x \in A \text{ or } x \in B\} \).

Definition 2.7. In the NM sets \( A \subset B \) if \( x \in A \) implies that \( x \in B \).

Definition 2.8. Cardinality of a NM set \( A \) is denote the number of elements in a set \( A \) which is define by \( \text{card}(A) \).

Definition 2.9. The Cartesian product of two neutrosophic multiset is defined by \( A \times B = \{(x, y): x \in A \text{ and } y \in B\} \).

Definition 2.10. The difference of two NM sets \( A \) and \( B \) is the collection of members such that all members belong to \( A \) but not in \( B \).

Now we introduce two new types of operation maximal union NM set and minimal intersection NM set.

Definition 2.11. Let \( X \) be a non-empty set, and neutrosophic multiset \( A \) and \( B \) in the form \( A = \{(x < T_A(x), I_A(x), F_A(x)): x \in X\} \) and \( B = \{(x < T_B(x), I_B(x), F_B(x)): x \in X\} \), then the operations of maximal union and minimal intersection NM set relation are defined as follows:

1. \( (A \cup B)_{\max} = \{(x < T_{(A \cup B)_{\max}}(x), F_{(A \cup B)_{\max}}(x)): x \in X\} \), where \( T_{(A \cup B)_{\max}}(x) = \max(T_A(x), T_B(x)) \) and \( F_{(A \cup B)_{\max}}(x) = \min(I_A(x), I_B(x)) \).

2. \( (A \cap B)_{\min} = \{(x < T_{(A \cap B)_{\min}}(x), F_{(A \cap B)_{\min}}(x)): x \in X\} \), where \( T_{(A \cap B)_{\min}}(x) = \min(T_A(x), T_B(x)) \) and \( F_{(A \cap B)_{\min}}(x) = \max(I_A(x), I_B(x)) \).

Example 2.1. Let \( X = \{x, y, z, t\} \) and \( A = \{x < 0.1, 0.5, 0.3\}, \{x < 0.7, 0.2, 0.5\}, \{y < 0.3, 0.2, 0.7\}, \{y < 0.9, 0.3, 0.1\}, \{z < 0.1, 1.0\}, \{t < 0.3, 0.2, 0.7\}\), \( B = \{x < 0.2, 0.3\}, \{x < 0.8, 0.5, 0.2\}, \{y < 0.3, 0.2, 0.7\}, \{y < 0.3, 0.2, 0.7\}, \{z < 0.7, 0.8, 0.3\}, \{t < 0.8, 1.0\}\) be neutrosophic multisets, then the maximal union and minimal intersection are \( (A \cup B)_{\max} = \{x < 0.8, 0.2, 0.2\}, \{y < 0.9, 0.2, 0.1\}, \{z < 0.7, 0.7, 0.3\}, \{t < 0.5, 0.7, 0.5\}\) and
We formulate the following results without proof.

**Result 2.1.** Union of any family of neutrosophic multisets is always a neutrosophic multiset.

**Result 2.2.** Intersection of any family of neutrosophic multisets is always a neutrosophic multiset.

**Result 2.3.** The complement of a neutrosophic multiset is always a neutrosophic multiset.

**Result 2.4.** Every neutrosophic set is a neutrosophic multiset but not necessarily conversely.

**Example 2.2.** Let \( A = \{ 8 < 0.6, 0.3, 0.2 >, 8 < 0.6, 0.3, 0.2 >, 8 < 0.4, 0.1, 0.3 >, 7 < 0.2, 0.7, 0.0 > \} \).

Here \( A \) is a neutrosophic multiset but not a neutrosophic set.

**Result 2.5.** Let \( \{ A_j : j \in \Lambda \} \) be an arbitrary family of NM set in \( X \), then the arbitrary maximal union and arbitrary minimal intersection is also a NM set.

**Remark 2.1.** A neutrosophic multiset is a natural generalization of multiset as well as Cantor set.

We introduced neutrosophic multiset topological space and study some of its properties.

**Definition 2.12.** Let \( X \) be neutrosophic multiset and a non-empty family \( \tau \) subsets of \( W_x \) is said to be neutrosophic multiset topological space if the following axioms hold:

1. \( N_\emptyset, W_x \in \tau \).
2. \( A \cap B \in \tau \), for \( A, B \in \tau \).
3. \( \bigcup_{i \in \Lambda} A_i \in \tau \), for \( \forall [A_i : i \in \Lambda] \in \tau \)

In this case the pair \( (W_x, \tau) \) is called a neutrosophic multiset topological space (NMTS in short) and any neutrosophic multiset in \( \tau \) is known as open neuterosophic multiset (ONMS in short) in \( W_x \).

The elements of \( \tau^c \) are called closed neutrosophic multisets, otherwise a neutrosophic set \( F \) is closed if and only if its complement \( F^c \) is an open neutrosophic multiset.
**Definition 2.13.** Let \((W_x, \mathcal{T}_1)\) and \((W_x, \mathcal{T}_2)\) be two neutrosophic multiset topological spaces on \(W_x\). Then \(\mathcal{T}_1\) is said be contained in \(\mathcal{T}_2\) that is if \(\mathcal{T}_1 \subseteq \mathcal{T}_2\) i.e, \(A \in \mathcal{T}_2\) for each \(A \in \mathcal{T}_1\). In this case, we also say that \(\mathcal{T}_1\) is coarser than \(\mathcal{T}_2\).

**Definition 2.14.** Let \((W_x, \mathcal{T})\) be a neutrosophic multiset topological space on \(W_x\). A non-empty family of subsets \(\beta\) of \(X\) is called neutrosophic multiset basis of the neutrosophic multiset topological space \(W_x\) if any element of \(\mathcal{T}\) can be express as the union of the element of \(\beta\).

**Remark 2.2.** As usual, basis of a neutrosophic multiset topological space is not unique.

**Definition 2.15.** Let \((W_x, \mathcal{T})\) be a neutrosophic multiset topological space with base \(\beta\). The interior of the neutrosophic multiset \(A\) is the union of basis element of \(\mathcal{T}\) which is contained in \(A\) and it is denoted by \(NM_int(A)\), i.e, \(NM_int(A) = \{ \cup \beta_i : \beta_i \subseteq A \text{ and } \beta_i \in \beta \}\).

**Definition 2.16.** Let \((W_x, \mathcal{T})\) be a neutrosophic multiset topological space. The closure of the neutrosophic multiset \(A\) is the intersection of all closed neutrosophic multiset containing the set \(A\) it is denoted by \(NM_cl(A)\), i.e, \(NM_cl(A) = \{ \cap \mathcal{F}_i : A \subseteq \mathcal{F}_i \text{ and } \mathcal{F}_i \in \mathcal{T} \}\).

In view of the definitions, we formulate the following result.

**Proposition 2.1.** Let \((W_x, \mathcal{T})\) be a neutrosophic multiset topological space and \(A, B\) be two neutrosophic multiset on \(W_x\), then the following property hold:

1. \(NM_int(A) \subseteq A\).
2. \(A \subseteq B \Rightarrow NM_int(A) \subseteq NM_int(B)\).
3. \(A \subseteq NM_cl(A)\).
4. \(A \subseteq B \Rightarrow NM_cl(A) \subseteq NM_cl(B)\)
5. \(NM_int(NM_int(A)) = NM_int(A)\).
6. \(NM_cl(NM_cl(A)) = NM_cl(A)\).
7. \(NM_cl(A \cup B) = NM_cl(A) \cup NM_cl(B)\).
8. \(NM_int(W_x) = W_x\).
9. \(NM_cl(\emptyset) = \emptyset\).
Definition 2.17. Let \((W_x, \tau)\) be a neutrosophic multiset topological space a non-empty set \(S\) is called a subbasis if the finite intersection of the elements of \(S\) can form a basis for \(\tau\).

Definition 2.18. Let \((W_x, \tau)\) be a neutrosophic multiset topological space a point \(p \in A \subseteq W_x\) is said to be a limit point of \(A\) if for every basis element \(\beta\) containing \(p\) contains one element of \(A\) other than \(p\), i.e. \(\beta \cap A \neq \emptyset\).

3. Results

3.1. Compactness, Connectedness and Continuous map.

Definition 3.1.1. Let \((W_x, \tau)\) be a neutrosophic multiset topological space. A neutrosophic multiset \(A\) is said to be disjoint if \(\exists\) two neutrosophic multisets \(B, C\) such that \(B \cap C = \emptyset\) and \(A = B \cup C\).

Definition 3.1.2. Let \((W_x, \tau)\) be a neutrosophic multiset topological space. The space \(W_x\) is said to be connected if \(W_x\) cannot be express as the union of two disjoint neutrosophic multisets.

Definition 3.1.3. Let \((W_x, \tau)\) be a neutrosophic multiset topological space. The space \(W_x\) is said to be compact if every open cover of \(W_x\) has a finite subcover.

Proposition 3.1.1. Every finite neutrosophic multiset topological space is compact.

Definition 3.1.4. Let \((W_x, \tau_1)\) and \((W_x, \tau_2)\) be two neutrosophic multiset topological space. The NMS function \(f: (W_x, \tau_1) \rightarrow (W_x, \tau_2)\) is said to be continuous if for each open neutrosophic multiset \(V\) of \(\tau_2\) the neutrosophic multiset \(f^{-1}(V)\) is an open submset of \(\tau_1\).

Proposition 3.1.2. Let \(f\) be a continuous function from a NMS topological space \((W_x, \tau_1)\) to another NMS topological space \((W_x, \tau_2)\), the function \(f\) is said to be a homomorphism if \(f(A \cup B) = f(A) \cup f(B)\) where \(A, B \in \tau_1\) and \(f(A), f(B) \in \tau_2\).

3.2. Separation axioms on neutrosophic multiset.
We have defined disjoint neutrosophic multiset, connectedness, compactness and the continuous image of neutrosophic multiset topological space. In this section we define separation axioms on NMS topological space.

In the NMS a singleton set \([p]\) is define by \([p] = \{x \in T_{(x)} \cup \overline{T_{(x)}}, \overline{\overline{T_{(x)}}} \} \neq 0\), when \(x = p\) otherwise \(T_{(x)} = 0\), \(I_{(x)} = 1 - T_{(x)}\) and \(F_{(x)} = 1 - T_{(x)}\) for all \(x \in W\).

Where \(x\) can be occurs more than one times it’s depends on its multiplicity and then \(T, F, I\) value may or may not be equal.

**Definition 3.2.1.** Let \((W_x, \tau)\) be a neutrosophic multiset topological space. If there exist only two open neutrosophic multiset in \((W_x, \tau)\) is called indiscrete NMS topological space.

**Definition 3.2.2.** Let \((W_x, \tau)\) be a neutrosophic multiset topological space. If every singleton neutrosophic multiset is an open NMS set then \((W_x, \tau)\) is called discrete NMS topological space.

**Definition 3.2.3.** Let \((W_x, \tau)\) be a neutrosophic multiset topological space. If for every two distinct NMS singleton sets, \(\{x_1\}; \{x_2\}\) then there exist \(V, U \in \tau\) such that \(\{x_1\} \subseteq V\) and \(\{x_2\} \subseteq U\) and \(\{x_1\} \not\subseteq U\). Hence, \((W_x, \tau)\) is NMST-\(1\)-space. i.e., there exists \(\tau\)-open NMS which contains one of them but not the other.

**Definition 3.2.4.** Let \((W_x, \tau)\) be a neutrosophic multiset topological space. If for every two distinct NMS singleton sets, \(\{x_1\}; \{x_2\}\) then there exist \(V, U \in \tau\) such that \(\{x_1\} \subseteq V\) and \(\{x_2\} \not\subseteq V\) and \(\{x_2\} \subseteq U\) and \(\{x_1\} \not\subseteq U\). Hence, \((W_x, \tau)\) is NMST-\(2\)-space.

**Definition 3.2.5.** Let \((W_x, \tau)\) be a neutrosophic multiset topological space. If for every two distinct NMS singleton sets, \(\{x_1\}; \{x_2\}\) then there exist \(V, U \in \tau\) such that \(\{x_1\} \subseteq V\) and \(\{x_2\} \not\subseteq V\) and \(\{x_2\} \subseteq U\) and \(\{x_1\} \not\subseteq U\) and \(U \cap V = N_\emptyset\). Hence, \((W_x, \tau)\) is NMST-\(3\)-space.
**Proposition 3.2.1.** Every NMST$_2$-space is NMST$_1$-space but it is not necessarily conversely.

**Example 3.2.1.** In co-finite neutrosophic multiset topological space is not a NMST$_2$-space but when the space has the finite neutrosophic multiset topology then it is NMST$_1$-space.

So when we consider the infinite neutrosophic multiset topology we can get our desire result.

**Proposition 3.2.2.** Every NMST$_1$-space is NMST$_0$-space but it is not necessarily conversely.

**Example 3.2.2.** Let $W_{x} = \{x<0.5, 0.7, 0.5>, x<0.5, 0.7, 0.5>, y<0.3, 0.4, 0.7>\}$ and $\tau = \{ W_{x}, N_{\emptyset}, \{ y \} \}$.

Here $(W_{x}, \tau)$ is a NMST$_0$-space but it is not a NMST$_1$.

**Proposition 3.2.3.** Every NMST$_2$-space is NMST$_0$-space but it is not necessarily conversely.

**Example 3.2.3.** Since every NMST$_0$-space is not a NMST$_1$-space and every NMST$_1$-space is not a NMST$_2$-space so every NMST$_0$-space is not a NMST$_2$-space.

**Proposition 3.2.4.** Every discrete NMS topological space is NMST$_2$-space.

### 3.3. Distance function on NMS.

In this section we are going to define a distance function on Neutrosophic set. Since in Neutrosophic set we have defined Neutrosophic elements, Neutrosophic subset so it is natural to ask, can we measure the distance between two Neutrosophic points or two Neutrosophic sets or is there any distance between a Neutrosophic point to a Neutrosophic set?

The distance function between multiset points is defined by Shravan and Tripathy [12], based on the multiplicity and the elements.

The Neutrosophic point $p$ of a Neutrosophic multiset $W_{x}$ is define by $p = \{(T_{(x)}), I_{(x)}, F_{(x)}\}: T_{(x)} \neq \emptyset$ when $x=p$, otherwise $T_{(x)}=0$, $I_{(x)} = 1-T_{(x)}$ and $F_{(x)} = 1-T_{(x)}$ for all $x \in W_{x}$.

**Note:** The Neutrosophic point $p$ can have multiple time it’s depends on its multiplicity.

**Definition 3.3.1.** Let $x$, $y$ be two Neutrosophic points on a Neutrosophic set $W_{x}$. The distance between the points is denoted by $d_{\chi}(x, y)$ and is define by $d_{\chi}(x, y) = \sup\{ |x-y|, |T_{(x)} - T_{(y)}|, \}$, where the distance function $d_{\chi}$ is define by, $d_{\chi}:W_{x} \rightarrow \mathbb{R} \cup \{0\}$.
**Definition 3.3.2.** Let $x$ be a Neutrosophic point and $A$ be a subset on a Neutrosophic set $W$. The distance between the point $x$ and set the $A$ is denoted by $d(x, A)$ and is define by $d(x, B) = \inf \sup \{ |x - y_i|, |T(x) - T(y_i)|, |I(x) - I(y_i)|, |F(x) - F(y_i)| : \text{for all } y_i \in A \}.$

**Definition 3.3.3.** Let $A, B$ be two Neutrosophic subset of a Neutrosophic set $W$. The distance between the sets $A$ and set $B$ is denoted by $d(A, B)$ and is define by $d(A, B) = \inf \sup \{ |x_i - y_i|, |T(x_i) - T(y_i)|, |I(x_i) - I(y_i)|, |F(x_i) - F(y_i)| : \forall x_i \in A, \text{and } y_i \in B \}.$

From the definition 5.1, 5.2 and 5.3 we can define another definition of matric space on a Neutrosophic multiset.

**Definition 3.3.4.** A non-empty Neutrosophic set $W$ is said to be a Neutrosophic metric space with the distance function $d: W 	imes W \rightarrow R \cup \{0\}$, if $W$ satisfy following:

1. $d(x, y) \geq 0, \forall x, y \in W.$
2. $d(x, y) = 0, \text{iff } x = y \text{ and } T(x) = T(y), F(x) = F(y), I(x) = I(y).$
3. $d(x, y) = d(y, x), \forall x, y \in W.$
4. $d(x, z) \leq d(x, y) + d(y, z), \forall x, y, z \in W.$

**Theorem 3.3.1.** If $d^1$ and $d^2$ be two Neutrosophic metric spaces then $\bar{d} = \max\{d^1, d^2\}$ is also a Neutrosophic metric space.

**Theorem 3.3.2.** If $d^1$ and $d^2$ be two Neutrosophic metric space then $\bar{d} = \min\{d^1, d^2\}$ is not a Neutrosophic metric space.

The proof of the above two theorem is obvious using the concept of general matric space.

4. **Applications**

The work done in this paper is based on the application of neutrosophic sets in multiset topological space. These can be further applicable for the development of neutrosophic topology separation axioms on neutrosophic multiset topology and neutrosophic multisets.

5. **Conclusions**
In this paper we have established some properties of the neutrosophic multiset topological space such as compactness and connectedness, continuous function on neutrosophic multiset topology, separation axioms on neutrosophic multiset topology. Also we have introduced the notion of the distance function in neutrosophic multiset and examined some properties. This paper can be useful for further development of neutrosophic multiset theory and neutrosophic topology.

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