A Numerical Circuit Simulation Study of High Voltage, Multi-Pulse Transmission Lines and Spark Gap Switches

Joe Ming Ju Chen
University of New Mexico

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Joe Chen
Candidate

Electrical and Computer Engineering
Department

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Approved by the Dissertation Committee:

Dr. Salvador Portillo, Chairperson

Dr. Edl Schamiloglu

Dr. Gregory Dale
A NUMERICAL CIRCUIT SIMULATION STUDY OF
HIGH VOLTAGE, MULTI-PULSE TRANSMISSION LINES
AND SPARK GAP SWITCHES

by

JOE CHEN

B.S., UNIVERSITY OF NEW MEXICO 2018

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A NUMERICAL CIRCUIT SIMULATION STUDY OF HIGH VOLTAGE, MULTI-PULSE TRANSMISSION LINES AND SPARK GAP SWITCHES

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M.S., Electrical Engineering, University of New Mexico, 2021

Abstract

The design and development of high voltage, multi-pulse transmission line generators is of high interest for linear accelerator and other pulse power applications. Los Alamos National Laboratory and L3 Applied Technologies are in development and testing of a multi-pulse series pulse forming water transmission line system dubbed the Series Pulse-Line Integrated Test Stand (SPLITS). The system consists of up to four water lines in series charged by a Marx bank of total capacitance and voltage of 39nF and -600kV, respectively, to produce multiple 120ns, -300kV pulses on a matched resistive load of 5.5 Ω. There is a need for a predictive model of SPLITS which encapsulates the reflections due to mismatched impedances and the time-varying nature of a spark gap switch. The research describe in this thesis develops a circuit simulation for SPLITS along with a physical, parameterized spark gap switch model. Along with the simulation, a detailed analysis of the output pulses of this circuit is performed to understand and optimize the pulse parameters, such as risetime and percentage flattop, required for linear accelerator applications. The resultant data is compared to experimental data taken from SPLITS. Additionally, this thesis develops an analytical solution to the charging circuit of the water line to characterize the response of the system.
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Chapter 1

Introduction

1.1 Introduction

As part of the nuclear stockpile stewardship program of the National Nuclear Security Agency (NNSA), Los Alamos National Laboratory (LANL) is developing advanced accelerators based on advanced pulsed power technologies for multi-pulse hydrodynamic experiments [1]. These accelerators must deliver MeV pulses with stringent risetime, pulse width, and voltage flattop parameters in order to meet radiographic diode requirements. Presently at LANL, the Dual Axis Radiographic Hydrodynamic Test (DARHT) Facility is one of the few facilities that provides high penetration multi-axis radiographic capabilities for hydrodynamic tests [2]. DARHT, which consists of two induction linear accelerators oriented orthogonal to each other, provides multiple electron pulses along two axes. These electron beams can have energies up to 19 MeV and are directed towards specially designed high Z (atomic number) targets. These convert the kinetic energy of the electrons into x-rays which are then used to image hydrodynamic test objects. DARHT axis 1 provides a single, high resolution radiograph with a 3 MV injector and 64 induction cells. This axis produces a 19 MeV, 2 kA, 60 ns electron beam. DARHT axis 2 consists of a 2.5 MV injector, 6 injection cells, and 68 induction cells. The second axis can reach energies of up to 18 MeV with a pulse length in excess of 1 μs. An electromagnetic kicker is used to break up the beam into four distinct pulses before being focused on the Bremsstrahlung converter and thus creating time resolved radiographic images.

To expand on the experimental capabilities of hydrodynamic testing, NNSA has issued a requirement for a facility that is able to perform these tests underground. And thus, project Scorpius was formed [3]. The initial design of Scorpius is based on DARHT axis 1 because of the high-resolution images it can provide. However, DARHT axis 1 is not time resolved but
time-integrated since it is a single short pulse. Therefore, Scorpius is designed to produce multiple electron bunches at the injector stage instead of producing a single long pulse and ‘chopping it up’ into multiple pulses similar to the downstream of DARHT axis 2. The production of multiple electron bunches calls for induction cells that are capable of handling multiple driving high voltage pulses that can fit underground in addition to the required high pulse power needed to create these pulses [4]. A fast risetime on the pulse reduces the size linear induction cells. Figure 1.1 shows a general layout of the Scorpius accelerator.

![Figure 1.1: Schematic layout of Scorpius system [3].](image)

The overview of Scorpius (Figure 1.1) starts with the inductive voltage adder (IVA) where multiple high voltage pulses drive induction cells which act as a 1:1 transformer that delivers energy to an electron source, which for these cases would be through a thermionic cathode. This source produces multiple bunches of electrons which then travel through the linear induction accelerator line. This line has multiple induction cells and has similar operating principles as the induction cells in the IVA, which transfer energy and accelerate the electron bunches. Due to the multiple electron bunches, these induction cells require multiple high voltage pulses to drive the cell. An important aspect of these cells are the magnetic cores which allow for the acceleration of the electron bunches. These cores must be able to support multiple pulses without saturation of the magnetic material. The electron bunches then travel through the downstream electron beam transport section where the beam position is monitored and focused before hitting the Bremsstrahlung target. The ultimate goal of Scorpius Linear induction accelerator is to have a > 20 MeV final beam energy with 80 ns beam pulse and a low percentage flattop across the square pulse.
1.2 Project Overview

An early concept for Scorpius was to power the linear induction cells of Scorpius with multi pulse transmission lines [3]. This is one of the key aspects into delivering energy into the accelerating electron beam. This transmission line is in development and being tested at L3-Harris in San Leandro on the Series Pulsed-Line Integrated Test Stand (SPLITS) [5]. The test bed comprises of multiple coaxial water pulse forming lines in series charged with a Marx bank that will allow for independent pulse timing. Each of the water line sections has its own laser triggered spark gap (LTSG) system that is pressured with SF₆ gas and triggered by a 532 nm laser beam focused near the anode. The water line will be used to drive cables that will go into the linear induction cells of Scorpius. This line is crucial in characterizing the effects for the induction cells.

A two-electrode coaxial transmission line consists of two nested coaxial cylinder filled with a dielectric material. In this design it is water. Coaxial water transmission lines have been used for a long time in the field of pulsed power due to the high dielectric constant of water during pulsed operation [6]. This allows the transmission line to operate at higher maximum voltage as well as reducing the size of the system. The transient voltage and output waveform for a coaxial transmission line with a matched load, would have an output of half the maximum voltage and twice the electrical length (Figure 1.2).

![Figure 1.2: A transmission line pulser circuit with its output voltage waveform. When the load has a matched impedance to the transmission line, it produces a voltage pulse with amplitude of \( \frac{V_0}{2} \) and a pulse width of twice the electrical length [7].](image-url)
In most pulsed power applications, the transmission line can be represented and modeled by its equivalent lumped circuit elements (Figure 1.3) which consist of a series of self-inductance per unit length \( L \), series resistance per unit length \( R \), a shunt capacitance per unit length \( C \), and a resistive shunt conductance per unit length \( G \). Two important factors derived from lumped circuit elements are the characteristic impedance of the line and the electrical length. Timing in pulsed power systems is important, especially in a multi pulse system because, in such a system knowing the timing of the switches and the electrical lengths of the system allows for proper pulse separation and pulse arrival to the load. This is critical for accelerating electron bunches where the high voltage pulses must be delivered at the exact time when the bunches arrive.

![Figure 1.3: Lumped element equivalent resistance of a transmission line [8].](image)

The characteristic impedance of a lossless transmission line can be represented by \( Z_0 = \sqrt{\frac{L}{C}} \) and it’s critical to design around this factor because of mismatched system impedance. A matched impedance allows for complete power deposition into the load without any reflected waves. Any small mismatch in impedance will create reflected waves which can influence critical pulse parameters such as voltage flattop. A low percentage flattop across the pulse reduces chromatic aberration which lead to focal spot blur in accelerating electron bunches. For a multi pulse series water line, a mismatch can cascade and sum up reflection to latter pulses downstream. A concern for this system, is the impedance of the switches that creates the transient pulses. These switches, when closed, create an arc across two electrodes which deliver the pulse to the next section of the line. The arc has time-varying elements such as
inductance and resistance, which causes the arc to have a time-varying impedance. Understanding the switch impedance is important in managing mismatch on the transmission line due to the reasons above.

1.3 Thesis Goal and Organization

The goal of this project is to create a SPICE model which emulates the experimental test bed developed by L3. The main SPICE tool used is Micro-Cap 12 [9]. This model, along with data taken from SPLITS, is important as a predictive tool of a water transmission line’s performance. The important aspect of this SPICE model is the spark gap switch because a non-ideal spark gap can add impedance mismatch into the system. One of the important characteristics is the resistance of the plasma arc formed when the pulse travels through the water line. To model the resistance, we used the Braginskii transport equations [10] to develop the arc resistance because it represents the channel as energy transport rather than an ideal switch which is represented by a Heaviside function. This resistance equation has many parameters that are useful for our purpose, such as the gas type, gap distance, gas pressure, and the heavy dependence on current and time. The other characteristic that defines the switch is the capacitance and inductance. In this thesis, the static capacitance and static inductance are solved numerically. It is crucial that this model captures the anomalies imparted onto the pulses as it cascades down multiple water lines.

The remainder of this thesis is organized as follows. Chapter 2 presents a derivation of the Braginskii resistance expression, as well as analytical calculations of inductance and capacitance. Chapter 3 describes the design and construction of the SPICE circuit model of SPLITS and various numerical simulations of the spark gap. Chapter 4 presents an overview of the results. Chapter 5 concludes the thesis with final statements and future work.
Chapter 2

Theoretical Analysis

2.1 Introduction

In order to understand pulse propagation through a coaxial transmission line under fast transient conditions, it is beneficial to develop an analytical model of the line to determine the current and voltage waveforms along with the risetime of the pulse. For a dynamic system, a lumped circuit model of the transmission line, shown in chapter 1, is used to understand the response of the line under an applied source which can determine electrical length and impedance, which is crucial for systems that require minimum impedance mismatching and precise timing. The following section will study the transmission line in a general lumped element case and in a transient case to determine these parameters. The transient model will be for the case of a capacitive source with an initial charge and capacitive load as well as a lossless coaxial transmission line, which will have arbitrary values of capacitance, inductance, and resistance. This model needs to be able to determine time-dependent current, voltage, and risetime of the pulse across the load. Additionally, this thesis will develop an explicit model for pulse prediction as the analytical model is unwieldy for general use.

2.2 Lumped Element Analysis of a Coaxial Transmission Line

Figure 1.3, repeated below, shows the equivalent circuit of a general transmission line. As stated in section one, each lumped element represents a physical aspect of the transmission line. The series resistance represents the losses in the conductors due to heat. The series inductance represents the self-inductance of the total conducting lines. The parallel capacitance represents the capacitance between inner and outer conductor. The parallel conductance represents the leakage current between the inner and outer conductor due to the resistivity of the medium.
Figure 2.1: Lumped element equivalent resistance of a transmission line [8].

Using Kirchhoff’s voltage law on the circuit and taking the limit as $\Delta z \to 0$ gives the following differential equation

\[
\frac{\partial v(z, t)}{\partial z} = -R_i(z, t) - L \frac{\partial i(z, t)}{\partial t}.
\] (2.1)

Kirchhoff’s current law gives

\[
\frac{\partial i(z, t)}{\partial z} = -G_i(z, t) - C \frac{\partial v(z, t)}{\partial t}.
\] (2.2)

These equations can be reduced from partial differential equations to a cosine-based phasor notation by applying the following equations

\[
v(z, t) = \Re\left[V(z)e^{j\omega t}\right]
\] (2.3)

\[
i(z, t) = \Re\left[I(z)e^{j\omega t}\right].
\] (2.4)

This simplifies the differential equations 2.1 and 2.2 as follows
\[- \frac{dV(z)}{dz} = (R + jwL)I(z) \quad (2.5)\]

\[- \frac{dI(z)}{dz} = (G + jwC). \quad (2.6)\]

These two equations can then be solved simultaneously to yield the wave equation.

\[\frac{d^2V(z)}{dz^2} - \gamma^2V(z) = 0 \quad (2.7)\]

\[\frac{d^2I(z)}{dz^2} - \gamma^2I(z) = 0, \quad (2.8)\]

where

\[\gamma = \sqrt{(R + jwL)(G + jwC)} \quad (2.9)\]

is the complex propagation constant. The general solution for the second order differential equation of a traveling sinusoidal wave is given by:

\[V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{-\gamma z} \quad (2.10)\]

\[I(z) = I_0^+ e^{-\gamma z} + I_0^- e^{-\gamma z}. \quad (2.11)\]

Solving for current and voltage can be done simultaneously using equations 2.10, 2.11 and 2.7, 2.8 to give, respectively:

\[I(z) = \frac{\gamma}{(R + jwL)}(V_0^+ e^{-\gamma z} + V_0^- e^{-\gamma z}) \quad (2.12)\]

\[V(z) = \frac{G + jwC}{\gamma}(I_0^+ e^{-\gamma z} + I_0^- e^{-\gamma z}), \quad (2.13)\]
resulting in the characteristic impedance of the transmission line

\[ Z = \sqrt{\frac{(R + jwL)}{(G + jwC)}} \]  
(2.14)

The lossless electrical length of the line Td is:

\[ Td = \sqrt{LC} = \frac{\text{length}}{v_p}, \]  
(2.15)

where \( v_p \) is the lossless propagation velocity. The electrical length is the one-way propagation time down the transmission which relates the physical length to the pulse length.

For the geometry in Figure 2.2, the line parameters for this coaxial line are commonly given by

![Figure 2.2: Coaxial line geometry with \( a \) being the inner radius and \( b \) being the outer radius.](image)

\[ R = \frac{R_s}{2\pi} \left( \frac{1}{a} - \frac{1}{b} \right) \]  
(2.16)

\[ L = \frac{\mu}{2\pi} \ln \left( \frac{b}{a} \right) \]  
(2.17)

\[ G = \frac{2\pi\varepsilon_0}{\ln \left( \frac{b}{a} \right)} \]  
(2.18)
\[ C = \frac{2\pi \omega e'}{\ln \left(\frac{b}{a}\right)}, \]  

where \( R_s \) is the surface resistivity of the conductor and the material between the conductor has a complex permittivity of \( \epsilon = \epsilon' - j\epsilon'' \).

### 2.3 Analytical Analysis of Transmission Line Circuit

For the transient case, we would like to determine the voltage, current, and risetime of the pulse using a lumped circuit model. Figure 2.3 is a reduced circuit model of SPLITS. A Marx bank is used as the source, then discharged into a capacitive load, which represents a pulse forming line, and is then carried through a lossless T model transmission line. A lossless T-model defines the lumped circuit transmission as a series inductance split in two with the parallel capacitor in-between. A Marx bank is a parallel configuration of capacitors charged to voltage \( V_0 \) and discharged in series that then multiplies the initial voltage by the number of capacitors in parallel. The series discharge of the parallel stages is accomplished via pressurized gas switches and the resultant ‘erected’ voltage is driven onto the transmission line. The Marx bank, in this circuit, is characterized by the total parallel capacitance \( C_g \), the total series inductance \( L_g \), and the total resistance \( R_g \). This leads to a slowing inductor which is used to extend the output pulse length from the Marx. The coaxial transmission line is laid in a T-model. In the case of a lossless line the shunt conductance and series resistance are zero and all that is left is the series inductance and the shunt capacitance \( C \). The transmission line is connected to a charge isolation inductor. \( L_1 \) and \( L_2 \) represent the series inductance of the slowing inductor with half the transmission line inductance and the isolation inductor with half the transmission line inductance, respectively. The capacitive transmission line load is defined by a capacitance \( C_l \). All of this is being switch with an ideal time-dependent single input single output switch.

The initial states of these components are static except for the Marx voltage \( V_0 \).
Figure 2.3. Circuit diagram of reduced SPLITS model.

This type of circuit can be solved by two different methods; it can be solved by the differential method or solved using the Laplace transform method. The Laplace transform method is used in this thesis because the method is able to reduce this circuit down to a set of linear polynomial equations which are simpler to solve.

Applying the Laplace transform to the time-domain circuit elements produces an equivalent s-domain circuit. The table below shows the equivalent circuit elements in time domain and its s-domain counterpart, where V is the voltage in volts, I is the current in Amperes, R is the resistance in Ω, C is the capacitance in Farads, and L is the inductance in Henries.
Table 2.1: Time-domain and S-Domain representation of principle circuit components.

<table>
<thead>
<tr>
<th>Circuit Element</th>
<th>Time-Domain</th>
<th>S-Domain</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resistor</td>
<td>$v = i \cdot R$</td>
<td>$V = I \cdot R$</td>
</tr>
<tr>
<td>Capacitor</td>
<td>$v = \frac{1}{C} \int_{t_0}^{t} i \cdot dt + V_0$</td>
<td>$V = \frac{I}{sC} + \frac{V_0}{s}$ [I = sCV - CV_0]</td>
</tr>
<tr>
<td></td>
<td>$i = C \cdot \frac{dv}{dt}$</td>
<td>$I = sCV - CV_0$</td>
</tr>
<tr>
<td>Inductor</td>
<td>$v = L \cdot \frac{di}{dt}$</td>
<td>$V = sLI - LI_0$</td>
</tr>
<tr>
<td></td>
<td>$i = \frac{1}{L} \int_{t_0}^{t} v \cdot dt + I_0$</td>
<td>$I = \frac{V}{sL} + \frac{I_0}{s}$</td>
</tr>
</tbody>
</table>

With the circuit elements transformed into its $s$-domain equivalent forms, one can simply apply Kirchhoff’s laws to the circuit. Starting off we define two current loops (see Figure 2.3), loop 1 and loop 2. Loop 1 is the sum of voltages on the generator and the transmission line and loop 2 is the sum of voltages on the transmission line and the load.

The initial conditions for this case are all to be zero except the initial charge voltage on the Marx generator. Thus, the resultant equations for the capacitance and inductance reduce to

Resistor:

$$V = I \cdot R \quad (2.20)$$

Capacitor:

$$V = \frac{I}{sC} + \frac{V_0}{s} \quad (2.21)$$

Inductor:

$$V = sLI. \quad (2.22)$$

Using Kirchhoff’s current loop. We can form both our loop equations as
Loop 1: \[
\frac{V_0}{s} = I_1(s) \left[ \frac{1}{s} \left( \frac{C_g + C_1}{C_g C_1} \right) + s(L_g + L_1) + R_g \right] - I_2(s) \left[ \frac{1}{s C_1} \right]
\] (2.23)

Loop 2: \[
0 = -I_1(s) \left[ \frac{1}{s C_1} \right] + I_2(s) \left[ \frac{1}{s} \left( \frac{C_L + C_1}{C_L C_1} \right) + s L_2 \right].
\] (2.24)

Multiplying both sides by \( s \) and rewriting the loop one and two equations, we get

Rewrite Loop 1: \[
V_0 = I_1(s) \left[ s^2(L_g + L_1) + s R_g + \left( \frac{C_g + C_1}{C_g C_1} \right) \right] - I_2(s) \left[ \frac{1}{C_1} \right] \] (2.25)

Rewrite Loop 2: \[
I_1(s) = I_2(s) \left[ s^2 L_2 + \left( \frac{C_L + C_1}{C_L C_1} \right) \right].
\] (2.26)

Substitute loop 2, 2.26, into loop 1, 2.25, in terms of \( I_2(s) \):

\[
V_0 = I_2(s) \left[ s^2 L_2 + \left( \frac{C_L + C_1}{C_L C_1} \right) \right] \left[ s^2(L_g + L_1) + s R_g + \left( \frac{C_g + C_1}{C_g C_1} \right) \right] - I_2(s) \left[ \frac{1}{C_1} \right].
\] (2.27)

This resultant equation is analytically intractable in its present form. You will notice that the equation becomes rather involved and is a quartic polynomial function. Usually, the next step of using the Laplace transform requires finding the roots of the polynomial. Once roots are found this equation can be reduced to a form where an inverse Laplace transform can be applied. This will then become an equation similar to \( I_2(t) = V_0*(....) \) which is the analytical equation we want but it is quite the unsightly mess to be useful. The general formula for solving the roots of a quartic equation can be seen below. The inverse solution to this is as unsightly. By following the differential equation approach we also reached the same stale mate of finding the roots.
Figure 2.4: Solution to quartic root function [11], see Appendix B.

A more practical approach would rely on a computational solver such as MATLAB and explicitly define the variable values can solve for the solution to this. From MATLAB we can see the solution to this circuit comparatively to the same circuit using SPICE program, Micro-Cap 12, later on in this chapter. For now, we continue to write an analytical solution for Figure 2.3.

Let \( K \) be the solution to the roots of the quartic function in Figure 2.4. The characteristic polynomial is in the form \( ax^4 + bx^3 + cx^2 + dx + e = 0 \) and the variables be

\[
K = \text{root}(a,b,c,d,e)
\]

\[
a = c1*cg*cl*l2*lg + c1*cg*cl*l1*l2
\]

\[
b = c1*cg*cl*l2
\]

\[
c = cg*cl*lg + cg*cl*l2 + cg*cl*l1 + c1*cg*lg + c1*cl*l2 + c1*cg*l1
\]

\[
d = cg*cl*rg + c1*cg*rg
\]

\[
e = cl + cg + cl.
\]

The inverse Laplace solution for the current \( I_2 \) is then, where \( K(i) \) is the \( i^{th} \) root:

\[
i_2(t) = C_g C_L V_0 \sum_{i=1}^{4} \frac{e^{K(i) \cdot t}}{[4aK(i)^3 + 3c1C_gC_L2R_gK(i)^2 + 2cK(i) + d]'}, \tag{2.28}
\]

while the voltage is represented as

\[
v_2(t) = \frac{1}{C_L} \int_{0}^{t_0} i_2(t) \cdot dt + V_{Load}(0). \tag{2.29}
\]
2.4 SPICE Model

This is the analytical solution for the reduced SPLITS circuit and is compared to the solution from SPICE models for three cases seen below.

Figure 2.5: SPICE model from Micro-Cap 12 representing the reduced SPLITS model.

From top to bottom, Figure 2.5 corresponds to case 1, case 2, and case 3, respectively.

Case 1: The Transmission line was represented as a lumped T-model with C2, L8, and L9. L8 and L9 are the inherent inductances and C2 is the inherent capacitance from the charging cable and L2 and L4 are the slowing inductor and isolation inductors, respectively.
Case 2: Using the built in Transmission Line Model, Z₀=60 Ω, T₀ = 44 ns.
Case 3: Built in Transmission Line Model, Length = 12.2 m, C = 0.0598 nF, L = 0.215 μH.

Figure 2.6: Analytical solution for three cases of Micro-Cap SPICE simulations comparing the current waveform.
Figure 2.7: Peak current difference, (left to right peaks) -15 ns (-0.045 mA), -37 ns (+0.004 mA), -52 ns (+0.129 mA).

From here we can observe that the analytical solution from the Laplace transform is similar comparatively to the Micro-Cap 12 simulation.

When the output of a Marx is switched on, the transmission line adds a resonating frequency to the current delivered to the capacitive load. This ‘wiggle’ in the current, for the application of pulsed switching, is an issue because it will disturb the output waveform. For a coaxial water line creating distinctive square pulses, if the wiggle is on the same time scale as the width of the pulse, it can constructively or destructively interfere with the maximum voltage. In the case where the resonant frequency is smaller than the width of the pulse, it will cause overshoots on the flattop of the pulse. For many pulsed power applications, the flattop of the pulse, ideally, needs to be constant. Adding overshoots and higher order frequencies to the pulse is unwanted. In some cases, the higher order resonant frequency can be used as an advantage due to the fact that it can give an extra boost into the peak voltage of the square wave when switched at the right time. Whether or not this is a concern depends on the amplitude of the resonant wave.
Now that we have a current equation the voltage is solved simply by applying the current to the shunt capacitor on the transmission line, equation 2.29, to obtain the voltage waveform. The time it takes to reach peak voltage can be solved for by taking the time derivative of the voltage and equating it to 0.

Figure 2.8: Analytical solution for three cases of Micro-Cap SPICE simulations comparing the voltage waveform.
In summary, the analytical form for the current and voltage waveforms aligns with the SPICE model for case 1. There is a slight time difference in the transmission line model provided using Micro-Cap 12 and the analytical model. This is due to the fact that the transmission line model is described as an impedance and time delay in Micro-Cap, while case 1 and the analytical solution add additional elements which change the resonant frequency of the system. The analytical model is useful in its explicit form to determine and design for a desired response (risetime, falltime, peak voltage, peak current, pulse width, etc.) of the reduced SPLITS system.
2.5 **Principle of a Spark Gap**

Spark gaps are commonly used devices in applications where high voltages and currents must be switched in short time periods [7,12]. This high voltage operation requires the spark gap to have a high holdoff voltage, which is the amount of electric potential the device can manage before creating a self-sustaining arc capable of delivering up to megaamperes of current. A common configuration of a spark gap comprises two electrodes separated by an insulating medium which can be a gas or liquid. The hold off voltage of the spark gap is dependent on the dielectric strength of the medium between the gap, the length of the gap, and the geometry of the electrodes. The gas breakdown through a spark gap can be characterized by Paschen’s law [13]. When the spark gap transitions from a non-conducting state to a conducting state through a breakdown processes, it forms a self-sustaining plasma channel. A common theory that describes the breakdown process was developed by Townsend [14]. This theory describes the formation of a plasma channel as an avalanche effect of cascading electrons. Starting with an applied electric field between the cathode and anode electrode, an ejected electron gets accelerated from the cathode towards the anode. This electron has a probability distribution of ionizing a particle through collision. This then produces a secondary electron which then follows the electric field, creating an avalanche of electrons through continuous collisions until a channel is formed. This gives the spark gap channel a time-dependent behavior due to the probabilistic nature of the collision. The resistance of the channel goes from a high impedance state to a low impedance state as the Townsend breakdown develops a wider plasma channel; this also gives the channel a time-dependent inductance due to its dependence on radius. Most importantly, for a self-breaking switch, only the high field strength is required to initiate the breakdown process. This adds a statistical quality to the breakdown and causes a variance in the timing of channel creation, which is called jitter. In many instances it is desirable that the switching spark gap operates with a minimum of jitter, and that the resistance of the spark gap during operation is at minimum. A method to minimize the jitter of a spark gap is to initiate the electrical breakdown externally with a laser or inducing a higher field gradient.

Early during the conduction phase of a spark gap, whether it is laser preionization triggered, electrically triggered, or self-breaking, the resistance of the arc between the spark gap
electrodes is large compared to the load. If the duration of the current pulse delivered to the load is comparable to the time required for the arc to expand, resistive losses in the switch will be non-negligible. The same applies to the inductance of the arc that is created. The time-varying resistance and time-varying inductance of the spark gap can drastically influence the behavior of a system. The resistance reduces the voltage flattop of the pulse and the inductance influences the risetime and induces oscillatory behavior to the system. In the following section a circuit model and equation for the time-dependent behavior will described for then an ideal spark gap.

2.6 Circuit Representation of a Spark Gap Switch

![Circuit diagram of a spark gap switch](image)

Figure 2.10: Circuit representation of a spark gap switch.

A circuit model of a spark gap switch is represented as a resistor in series with an inductor and in parallel with a capacitor. An ideal time-dependent switch (SW) is used to initiate the discharge. These components are related to the inductance of the arc (L), resistance of the arc (R), as well as the capacitance of the electrodes (C) shown above as an example.

An approximate model of a spark gap switch can be made using resistance and inductance in series, a capacitance in parallel along with an ideal time-dependent switch. Normally the ideal switch ignores series capacitance and inductance, and only uses resistance and time. The resistance can be defined as a Heaviside step function of some initial resistance close to 1 MΩ down to a final resistance of 1 μΩ. For pulsed power systems, this is not the case. The final resistance is heavily dependent on the current in the system, which can bring the final stagnated resistance to the range of several Ω to mΩ. To describe the time-varying nature of the channel
arc we start off with the famous Braginskii plasma resistance. Over the years there has been considerable research describing the resistance of the plasma channel, such as Toepler’s formula [15], given by

\[ R = \frac{K_T \cdot d}{Q}. \]  

(2.30)

The value R is resistance, \( K_T \) is the Toepler constant, d is the length of the spark gap, and Q is the charged transferred.

A few other resistance equations are developed based off of Braginskii’s equation to find the plasma radius [15,16,17]. In this this we will derive and discuss the resistance equation from Braginskii’s radially-dependent equation [10]. This method is superior in the sense that the final equation incorporates parameters such as gap distance, pressure, and gas type, which are parameters in pressurized switches that we want to incorporate.

The Braginskii resistance equation needed to be incorporated into the SPICE model and this was done via mathematical or parametrized functions. The Braginskii equation is used to represent the energy loss in the arc and the time-dependence radial growth of the plasma channel closing the gap.

### 2.6.1 Derivation of Braginskii Resistance for a Spark Gap

From the Braginskii equation [10], we obtain the relationship describing the Ohmic heating and radial expansion of the plasma column as a function of time. This is described in equation 2.30, which has parameters of current, density conductivity, and radius. Note that the Braginskii equation is written using Gaussian units, and is given by.

\[ 2 \pi^2 \rho \cdot r(t)^3 \frac{dr(t)}{dt} \cdot \xi = \frac{I_{cgs}(t)^2}{\sigma_{cgs}}, \]  

(2.31)
where \( \rho \) = Density, \( \frac{g}{cm^3} \); \( r \) = Channel radius, cm; \( I_{cgs} \) = Current, \( \frac{cm^2 g}{sec^2} \); \( \sigma_{cgs} \) = Conductivity, \( \frac{1}{s} \);

and where \( \xi \) represents the coefficient for a strong shock. The coefficient is also provided by Braginskii and is based on the specific heat of the gas in the switch and the velocity of the gas as it expands, and is given by

\[
\xi = K_p [1 + (\gamma - 1)^{-1}(2 \cdot \frac{t}{r(t)} \cdot \frac{dr(t)}{dt})^{-1}]. \tag{2.32}
\]

The other terms in equation 2.32 are given in CGS units and are \( \xi = 4.5 \) (Dimensionless); \( \gamma \) = Specific heat ratio (Dimensionless); and \( K_p = 0.9 \) (coefficient of resistance – constant) (Dimensionless).

Rearranging equation 2.31 to a more applicable form, we obtain

\[
r(t) \cdot \frac{dr(t)}{dt} = \frac{I_{cgs}(t)^{\frac{2}{3}}}{(\sigma_{cgs} \cdot \rho_0 \cdot \xi \cdot 2 \cdot \pi^2)^{\frac{1}{3}}} \tag{2.33}
\]

Taking the second derivative of the radius in equation 2.33, we obtain

\[
\frac{d(r(t)^2)}{dt} = 2 \cdot r(t) \cdot \frac{dr(t)}{dt}. \tag{2.34}
\]

Substituting equation 2.33 into equation 2.32 yields

\[
\frac{I_{cgs}(t)^{\frac{2}{3}}}{(\sigma_{cgs} \cdot \rho_0 \cdot \xi \cdot 2 \cdot \pi^2)^{\frac{1}{3}}} \int_0^t \frac{I_{cgs}(t)^{\frac{1}{3}}}{\sigma_{cgs}} dt. \tag{2.35}
\]

Equation 2.35 yields the radius as a function of current and conductivity.
To stay consistent with the units in SPICE, the current and conductivity are converted from Gaussian units to Amperes and Siemens/m, respectively. They are given by

\[
\frac{I_{c\,gs}^2}{\sigma_{c\,gs}} = \frac{I_{si}^2}{\sigma_{si}} \times 10^9. \tag{2.36}
\]

The resistance of a solid cylinder is given by

\[
R(t) = \frac{d}{\sigma_{si} \times \pi \times r(t)^2}, \tag{2.37}
\]

where R is the resistance in Ω, d is the gap distance in cm, and conductivity in S/cm. Substitute into equation 2.35 to get the general form

\[
R(t) = \frac{d}{10^3 \times \sigma_{si}^2 \times \left(\frac{4\pi}{\rho + \xi}\right)^{\frac{1}{3}} \times \int_0^t I_{si} \frac{2}{3} \, dt}. \tag{2.38}
\]

This resulting relationship can then be further expanded to include pressure terms beginning with the ideal gas law, which is a critical function for laser triggered spark gaps that are dependent on the pressure of the insulating gas. The expression for density becomes

\[
\rho = \frac{P \times M}{Rg \times T} \times 10^{-6}, \tag{2.39}
\]

where P = Pressure, Pascals; M = Molar mass, g/mol; and Rg = Ideal gas constant, Joules/mol*Kelvin. Equation 2.38 was substituted in equation 2.37 to obtain the final equation 2.39. This is then implemented in the analog circuit switch model as the new resistance of the switch. This form is now complete and incorporates the parameters of the experiment, such as gap distance d, pressure P, gas conductivity σ, and it is dynamic with both current and time, and is given as
\[ R = \frac{d}{10^{\frac{11}{3}} \sigma_{si}^{\frac{2}{3}} \left( \frac{4\pi \cdot 10^6}{P \cdot M} \right)^{\frac{1}{3}} \cdot \int_0^t I_{si}^{\frac{2}{3}} \, dt} \] (2.40)

Figure 2.11: A piecewise linear square current pulse of varying amplitudes as an input test function of equation 2.39.
Figure 2.12: A Braginskii resistance parametric model output with respect to a square current pulse and given current amplitude.

Figure 2.11 and Figure 2.12 show the Braginskii model running with a piecewise linear pulse of a 120 ns pulse at varying currents to get a sense of the resistance curve decay of the model. This is used comparatively to the SPICE model. The approximate resistance is calculated as the current is increase from 30 kA to 300 kA.

### 2.6.2 Switch Inductance Calculation

The calculation of the switch inductance is from Grover [18], which for a coaxial transmission line is

\[
L = 0.002 \times d \left[ \ln \left( \frac{r_2}{r_1} \right) \right],
\]

(2.41)

where \(L\) = Inductance, \(\mu\)H; \(d\) = Gap distance, cm; \(r_1\) = Inner Conductor Outer radius, cm; and \(r_2\) = Outer Conductor Inner radius, cm.
Since the inner conductor is the spark gap and the outer conductor is the outer wall, those parameters are used for \( r_1 \) and \( r_2 \), respectively. The inductance is calculated from the electrode radius to the arc radius with respect to the length of the spark gap. The arc radius is approximately 0.1 cm [19]. A total radial outline of can now be formed and inductance of the spark gap can be solved. For the SPLITS test stand the inductance calculated is 73.3 nH. This is found using by using a gap distance of 2.54 cm. The figures below show the electrode geometry used in the inductance calculation.

![Graph showing radius vs. distance](image)

**Figure 2.13**: Radius of the electrode with a constant arc and the line of symmetry cross at 6 cm.
The inductance of an arc channel is difficult to know precisely due to the changing and nonuniform arc channel. It is generally thought to be negligibly small before the arc is fully formed and is modeled as a conducting wire once the arc has been established.

An issue with this method is that the arc is time-dependent, and the radius grows as energy travels. In the static induction calculation, the assumption was made that the arc has fully expanded and is stagnated. A method to solve this is to incorporate the Braginskii radius equation into the equation provided in Grover, but convergence errors occur, and this is discussed in Chapter 5.

**2.6.3 Switch Capacitance Calculation**

The switch geometry is crucial for the operation of pulsed power due to the effects it has on the hold off voltage and the static capacitance and inductance of the switch. To calculate the spark gap capacitance, the finite element method code from Field Precision’s suite Amaze was used. For the LTSG a pair of hemispherical tungsten copper alloy was used with a hole drilled
through for the flow of gas and a laser path. The radius of the switch is approximately 4.7 cm. The spacing of the electrodes can be varied from 1.27 cm to 3.81 cm. The necessary hold off voltages for this switch is around -650 kV.

![Figure 2.15: HiPhi model of the LTSG showing the x-y contour of the field distribution in the spark gap.](image)

The calculation of capacitance value is obtained using the Field Precision Suite HiPhi [20], a three-dimensional electrostatic solver. This simulation solves for the field energy stored between the spark gap electrodes. From there the capacitance can be calculated using

\[
U = 0.5CV^2, \quad (2.42)
\]

where \( U \) = Stored Field Energy, Joules; \( C \) = Capacitance, Farad; and \( V \) = Voltage, V.

For a 2.54 cm gap, the capacitance of the switch is 5.09 pF.

### 2.7 Conclusion

In conclusion the goal of chapter 2 was to develop an analytical equation for a reduced transmission line of SPLITS circuit. This goal was not fully achieved due to the complexity and messiness of the solution. An explicit result would be a better fit for this rather than a
purely analytical result. An equation for the impedance of the line and the electrical length is found where it will be used in the SPICE model.

The other goal of this chapter was to introduce the importance of the spark gap when it is nonideal where the resistance, capacitance, and inductance matter. We have successfully derived a resistance equation from Braginskii’s radius equation. We have also finished an inductance calculation to solve for the inductance of the switch for various gap distances and determined a static capacitance the switch geometry.
Chapter 3

Numerical Analysis

3.1 Introduction

In the previous chapter we focused on deriving the parameters needed for the spark gap model. This chapter will focus on applying these analytical results into a multipulse circuit representation of the SPLITS transforming line system. Later on in this chapter we will use numerical simulations to verify our calculation of inductance and capacitance using an electrostatic solver.

SPLITS, shown in Figure 3.1, currently consists of two 60-ns water line sections in series and charged up to nominally -600 kV. Each water line section is independently charged with a Marx charging unit through an inductor. This is then discharged with a LTGS and produces two independent 120 ns, -300 kV pulses. Each water line section has an independent LTGS filled with high-pressure SF₆ gas, allowing for independent timing of the pulses. Typical pulse separations vary between 200 ns to 1000 ns. The laser used to trigger the spark gaps is 45 mJ, 266 nm. The SPLITS test bed contains several voltage and current monitors at various axial and azimuthal points on each of the water transmission lines.

SPLITS is now in its four-line configuration and initial data is being obtained. At the time of the writing of this thesis, the SPLITS system was comprised of only the two, and so this thesis will focus only on the modeling of the two-pulse system compared and its comparison with experimental results. If the model can accurately portray the two-pulse line, it will represent a critical steppingstone into creating a four-pulse line model which will be discuss later in this thesis. In this present model, the focus is on the time-varying, current-dependent resistance of the LTGS. This will be represented through the use of the Braginskii model derived in the previous chapter.
3.2 SPICE Model of Two Pulse Line SPLITS

The following model is based on SPLITS at L3 San Leandro. The purpose of this model to see if it can replicate and predict output current waveforms from the actual experiment.

Figure 3.2: Full two water line model of SPLITS with matched load.
The SPICE circuit simulation incorporates equation 3.1 into its spark gap model for the SPLITS circuit model in Figure 3.1. This model is comprised of the charging unit and two pulse forming lines delivering two 120 ns pulses into a 5.5 Ω load. The charging unit consists of a Marx at 39 nF (C1) capacitor set to -610 kV initial condition and has an equivalent resistance of 28.5 Ω (R1) and an equivalent inductance of 1.2 μH (L1). This charges two transmission lines (T1, T2) through a slowing inductor in series with the line. The purpose of the slowing inductor is to limit the current through the Marx switch thus widening the pulse width of the current output from the Marx. The slowing inductor has an inductance of 200 μH (L2, L3).

Figure 3.3: Comparison of the charge cables and Marx voltage to the SPLITS circuit model.

Figure 3.3 shows the voltage on the Marx and on cable 1 and 2 which are right after the slowing inductor before the transmission line T1 and T2 compared to the measured values using a D-dot sensor. Overall, the SPICE model follows the experimental data. The model captures the major trends and dips compared to the experimental data, but it does not track the higher frequency oscillations.
Figure 3.4: Close up of voltage on charge cable 1 and charge cable 2 at the time of triggering.

Figure 3.4 shows the close-up on the transient switching the energy from the Marx into the charge cable 1 and charge cable 2 and displays the timing from the model compared to experimental data. This also shows that there is some reflection of voltage back into cable 2 after PFL1 is discharged.

The charge cables (T1 and T2) themselves have a characteristic impedance of 60 $\Omega$ and an electrical length of 44 ns. This is then attached to an isolation inductor of 21 $\mu$H (L4, L5) in parallel to a 90 $\Omega$ resistor (R2, R3) before connecting to the waterlines through a switch. The purpose of the isolation set-up is to prevent any voltage reflection back into the Marx bank and power supplies. The time-controlled switches (SW3, SW4) are used as an isolation to prevent any leakage current before the actual discharge. A time-controlled switch in SPICE is an open to close switch where it closes for a defined length of time. The first pulse at switch SW4 is triggered, and the second pulse is triggered at switch SW3; the pulse separation can be varied. This then reaches two transmission lines (T3, T5) with peakers (T4, T6) at the end. The lines (T3, T5) have characteristic impedances of 5.5 $\Omega$ and electrical lengths of 54.7 ns. The peakers (T4, T6) have impedances of 4.6 $\Omega$ and electrical lengths of 3.2 ns for both. This peaker section is used to reduce the risetime of the output pulse. In the most recent design, snubber hardware
was implemented in efforts to reduce impedance mismatch and pulse reflections downstream. PFL1 is terminated with an open load which is represented by a high value resistor of 1MΩ (R4).

![MicorCap Sim](image)

Figure 3.5: Comparison of voltage waveforms of PFL1 and PFL2 from model to experimental data.

Figure 3.5 shows the PFL switching using the Marx voltage. A single Marx voltage is used to switch into two pulses. The Marx charges the two PFLs in parallel, so they follow the output voltage of the Marx. It is required to have the pulse-to-pulse separation be variable. In order to create the two pulses and make it variable, it is required to switch the PFLs at a time of equidistant spacing from the peak voltage of the Marx output. The first pulse is created on the rise of the Marx output and the second pulse is created on the fall of the Marx output. This will create two pulses at the cost of switching below the peak voltage of the Marx. The further apart the pulses are, the further away from the peak voltage of the Marx output it is. Another cost to this method is that the second pulse will have a slightly lower voltage flattop then the first pulse due to the natural damping coefficient of an RLC circuit.
Figure 3.6 shows the nuances in the formation of the pulse. The timing of the model is near perfect for PL1 and PL2 compared to experimental data. The higher frequency oscillations are still missing here as well.

### 3.2.1 Spark Gap Circuit Implementation

The inductance and capacitance of the switch is a set value of 73.3 nH and 5.09 pF. The calculated values were performed in the previous section using a gap distance of 2.54 cm. The resistance of the switch is calculated using the Braginskii plasma resistance through equation 3.1 which was derived in Chapter 2

\[
R = \frac{d}{10^3 \sigma_{si}^{2/3} \left(4\pi+10^6 \frac{76^2}{10^3} \xi \right)^{1/3} \int_0^t l_{si}^{2/3} dt}.
\]  

(3.1)
The implementation of this equation in Micro-Cap for a single switch as follows:

Integrated Current:

\[ iB1 = \text{SUM}((I(L7)\times2))^{(1/3)}(T) \]

Constant:

\[ \text{con1} = \left(\frac{4}{\text{sigcm}\times100\times\rho} \times \text{xi} \times \pi \times 2\right)^{1/3} \]

Braginskii Radius Squared:

\[ \text{BragR1} = 10^{3}\times\text{con1}\times iB1 \]

Switch Resistance:

\[ \text{Rsw3val} = \text{IF}(\text{logic1}, R\text{int}, d/(\text{sigcm}\times\pi\times\text{BragR1})) \]

The If statement is used to define an initial resistance value for the switch, initially set to 1 MΩ. The logic value is determined by time - if the time before SW3, SW4 is considered TRUE, then the switch resistance is 1 MΩ. Once the SW3, SW4 is latched, then the logic is FALSE, which then switches to the modified Braginskii calculated resistance. This sets the starting resistance to a more physical initial value, which is required by the integration of the current. If left to calculate initial resistance, the simulation will assume there is already a channel formed and have a dynamic high valued resistance before the switch closes. The logic statement sets a boundary on when the channel of the arc is formed.
Figure 3.7: Resistance of spark gap from the Braginski model.

Figure 3.7 shows the resistance of the switches SW1 and SW2 as the discharges occur. The open state of the switch rapidly decays down and plateaus around ~100 mΩ. The 1st switch plateaus to a lower value than the second switch.

Figure 3.8: The radius of the arc of the spark gap as a function of time.
We can see that both switches show that the arc radius is around 0.1 cm in Figure 3.8. An interesting note on the SPICE spark gap resistance in Figure 3.8 is that, after the 1st switch is turned on, it decays and then plateaus. When the 2nd switch is turned on, it influences the resistance of the first switch. This leads to a small secondary drop in resistance of the first switch.

The integration of current is carried out on I(L6), I(L7) as these are equal to the current flowing through the plasma arc. By using the SUM function in Micro-Cap, this allows numerical integration through a sum over the area of the curve. After the last switch, it goes to another transmission line (T7) which has an impedance of 5.5 Ω and electrical length of 79.2 ns, which then gets delivered to the load.

Table 3.1: LTGS parameters.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Quantity</th>
<th>Value</th>
</tr>
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<tr>
<td>d</td>
<td>Gap Distance (cm)</td>
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<tr>
<td>ξ</td>
<td>Constant for a Strong Shock</td>
<td>4.5 [10]</td>
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<tr>
<td>P</td>
<td>Pressure (psi)</td>
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<tr>
<td>σ</td>
<td>Conductivity (S/cm)</td>
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<td>I</td>
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<tr>
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<td>Ideal Gas Constant</td>
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</tbody>
</table>

The parameters in Table 3.1 are determined by the configuration of the LTGS. For the following results, the gap distance is set to 2.5 cm, pressure is set to 160 psi, and the gas used is SF₆. The properties of SF₆ are a conductivity of 1200 S/cm, molar mass 146 g/mol, and accounting for room temperature (293 K). These determine the time-dependent resistivity of this spark gap model. The Braginskii model uses a constant conductivity. For now, this is acceptable to use, but developing a time-dependent conductivity model of SF₆ will be key in calculating the resistance through Braginskii.
3.2.2 **SPICE Two-Pulse Simulation Results**

The following figures are the resultant voltage plots from the SPICE simulations.

![Figure 3.9: Voltage signal across a matched load for a 200 ns pulse separation.](image)

![Figure 3.10: Voltage signal across a matched load for a 400 ns pulse separation.](image)
Figure 3.11: Voltage signal across a matched load for a 1000 ns pulse separation.

These figures show the output voltage across the matched load at 200, 400, 1000 ns pulse-to-pulse separation with parameters for the Braginskii equation as shown in Table 3.1 to test the variability of the pulses. The red and blue outlines dictate pulse 1 and pulse 2 through a pulse identification routine written in Python.

The crucial parameters for of these results are tabulated in Table 3.2. Note that the flattop we considered is a constant 80 ns time window from the center of the full width at half max (FWHM) which is shown in the cyan bar. The goal for SPLITS was to have the capability to vary the pulse separation. The flattop requirements are 5% for any single pulse and 1.5% for the beam energy at the end of the machine. The percentage flattop is the variance of the flattop voltage over the average flattop voltage in the 80 ns window. This factor is critical because in charged particle accelerators, the charge particle bunch is accelerated by the voltage pulses. The flattop is kept constant to uniformly accelerate the charge particles. If the flattop were not uniform, there will be an undesired distortion to the charged particle beam bunch.
Table 3.2: Pulse Parameters for two-line model with a matched load of 5.5 Ω.

<table>
<thead>
<tr>
<th>Pulse Separation</th>
<th>Pulse Number</th>
<th>FWHM</th>
<th>Risetime</th>
<th>Half Voltage</th>
<th>Flattop Voltage</th>
<th>Flattop Peak Voltage</th>
<th>Percentage Flattop</th>
</tr>
</thead>
<tbody>
<tr>
<td>200 ns</td>
<td>1</td>
<td>124.0 ns</td>
<td>26 ns</td>
<td>−150.5 kV</td>
<td>−298.4 kV</td>
<td>−304.5 kV</td>
<td>±2.3%</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>125.0 ns</td>
<td>70 ns</td>
<td>−150.3 kV</td>
<td>−291.2 kV</td>
<td>−303.8 kV</td>
<td>±4.9%</td>
</tr>
<tr>
<td>400 ns</td>
<td>1</td>
<td>124.0 ns</td>
<td>26 ns</td>
<td>−150.1 kV</td>
<td>−297.8 kV</td>
<td>−303.9 kV</td>
<td>±2.4%</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>124.0 ns</td>
<td>33 ns</td>
<td>−147.4 kV</td>
<td>−290.1 kV</td>
<td>−301.6 kV</td>
<td>±3.0%</td>
</tr>
<tr>
<td>1000 ns</td>
<td>1</td>
<td>125.0 ns</td>
<td>27 ns</td>
<td>−147.0 kV</td>
<td>−291.5 kV</td>
<td>−297.9 kV</td>
<td>±2.5%</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>125.0 ns</td>
<td>32 ns</td>
<td>−144.2 kV</td>
<td>−285.0 kV</td>
<td>−294.5 kV</td>
<td>±2.6%</td>
</tr>
</tbody>
</table>

From the results, the FWHM and the risetime were relatively consistent for the various pulse separations. The FWHM was ~125 ns for pulse 1 and 2 at the three pulse separation scenarios. The risetime for pulse 1 is consistent at 26 ns for the various pulse separation, but pulse 2 risetime was 32 ns. For the 200 ns pulse 2 the risetime was 70 ns, this is due to the ‘knee’ of pulse 1 affecting the rise of pulse 2. The reflections from the line can be seen as the oscillations on the baseline; these reflections are amplified the more water lines that are added in series. This affects the flattop and the average flattop voltage of the pulse which is shown in the figures.
Figure 3.12: Comparison of peak and average voltage across the load for 1000 ns, 500 ns, and 200 ns pulse timing separations.

From Figure 3.12, pulse 1 is shown to have a higher average and peak voltage than pulse 2 for all timing variation cases. The difference for the average flattop voltage from pulse 1 and 2 is ~7 kV and the flattop peak voltage varies around a few kVs as well. The flattop average voltage decreases on pulse 1 and 2 for increasing pulse separation. This is expected due to the pulse triggering timing of the PFLs, since the longer the separation the further away from the Marx peak output voltage. There is also a difference of pulse 1 and pulse 2 flattop average voltages; this is also due to using the same Marx output voltage. The output voltage of the Marx is not perfectly symmetric on upswing and downswing of the voltage, which is expected due to the nature of the RLC circuit.
Figure 3.13. Comparison of percentage flattop of 1000 ns, 500 ns, and 200 ns pulse timing separations.

From Figure 3.13, as the pulse separation increases the percentage flattop of the second pulse converges to a value similar to the first pulse. For the first pulse, the percentage flattop is around ±2.4%. The second pulse is expected to have a higher percentage flattop due to reflections. The closer the second pulse is to the first pulse, the more it is affected by the reflections. As shown in the 200 ns pulse separation case, the second pulse has a percentage flattop of ±4.6%. The requirement for the flattop is 5% for any single pulse; however, as long as it is less than 5%, it is acceptable. This model will be compared against experimental results and an ideal switch model in Chapter 4.
3.3 SPICE Model of Four Pulse Line SPLITS

This section will describe an expansion on the previous two-line model to a four-line model. In chapter 4 we will show how closely the two-line model compares the experimental data. The final product goal will be a four-water transmission line device and the four-line SPICE model is a steppingstone to developing the device.

Figure 3.14. Four line SPLITS model, elaborated on in Appendix A.

The four-line model in Figure 3.14 is practically the same as the two-pulse model but now doubled in series with two Marx bank instead of one. The other difference is that the Marx voltage is charged to -650 kV to reach an average flattop voltage of around -300 kV. There are a few new additions to this model, though. One is that T116 and T117 are transmission lines attached before the load section. These transmission lines have an impedance of 22 Ω and line
length 75 ns, and impedance of 7.33 Ω and line length 75ns for T116 and T117, respectively, to match to the new load section. This represents the output cables that will be connected to the test stand. Each output cable has a resistance of 44 Ω. The SPLITS system is designed to drive 8 of these cables with a total parallel resistance of 5.5 Ω. The load connected to T117 is a resistor of 7.33 Ω. The load connected to T116 is parallel of R12 and R19, R12 being 22 Ω and R19 being an electron diode. Two of the 8 cables are attached to the electron diode while the other 6 are terminated in a dummy load. Two configurations were tested, one with a matched load with just 22 Ω and the other with an electron diode attached in parallel to the 22 Ω load. For the matched scenario, the total parallel resistance is 5.5 Ω which is the same resistance as the two-line model. The electron diode was modeled using the Child-Langmuir law.

3.3.1 Child Langmuir Electron Diode Model

The Child-Langmuir space charge limited current expression for two planar electrodes separated by gap spacing d is given by

\[ I = J \cdot d^2 = K \cdot V^{3/2} , \tag{3.2} \]

where \( d = \) Distance from anode to cathode (mm); \( J = \) Current density \((\frac{A}{mm^2})\); \( V = \) Potential difference (V); and \( I = \) Current (A). To determine the constant \( K \), we define the optimal parameters for the electron diode. For the parameters of \( I = 1000 \) A and \( V = 300 \) kV, we obtain \( K = 6.08e-6 \). Using Ohm’s law, we obtain \( R = \frac{1}{K\cdot V^{1/2}} \), where \( K \) was given above and \( V \) is the input pulse voltage from the simulation. This is used as the electron diode R12.
Figure 3.15: Output current across the electron diode.

Figure 3.15 shows the current produced from this cathode for a 1000 ns pulse-to-pulse separation. The peak current is shown to be ~900 A on the electron diode using the Child-Langmuir Law for the resistance of R12.
3.3.2 SPICE Four-Pulse with Electrode Diode Simulation Results

The following shows the output voltage waveforms across the electron diode load.

![Figure 3.16: The output voltage of the across the electron diode at 1000 ns pulse-to-pulse separation for the four-line model.](image)

From the four-line model, the SPICE simulations were able to produce similar results for pulse 1 and pulse 2. As expected, the reflections sum up as more pulses are added. Pulse 3 and pulse 4 have a sharper flattop. In addition, the baseline of this output waveform is growing in intensity as more pulses are added on. It is shown that there is reflected noise after each pulse that grows in length after the end of each pulse. The noise after the first pulse last around ~500 ns while the next few pulses have noise past 500 ns. To note, the red and blue are the determined pulses using the pulse identification routine. The two colors are used to show better distinction of pulses from the neighboring pulse and have no other meaning.
Figure 3.17: The output voltage of the across the electron diode at 500 ns pulse-to-pulse separation for the four-line model.

The 500 ns pulse-to-pulse output waveform shows similar characteristics as the 1000 ns pulse-to-pulse separation with pulse 3 and pulse 4 having a sharper flattop. The noise after each pulse is starting to affect the flattop of the pulses following the first pulse.
Figure 3.18: The output voltage of the across the electron diode at 200 ns pulse-to-pulse separation for the four-line model.

Figure 3.18 has shown that the pulses are starting to slightly overlap with one another.

Table 3.3: Pulse parameters for the four-line model with an electron diode load.

<table>
<thead>
<tr>
<th>Pulse Separation</th>
<th>Pulse Number</th>
<th>FWHM</th>
<th>Risetime</th>
<th>Half Voltage</th>
<th>Flattop Average Voltage</th>
<th>Flattop Peak Voltage</th>
<th>Percentage Flattop</th>
</tr>
</thead>
<tbody>
<tr>
<td>200 ns</td>
<td>1</td>
<td>124.0 ns</td>
<td>27 ns</td>
<td>−153.1 kV</td>
<td>−303.4 kV</td>
<td>−310.3 kV</td>
<td>±2.6%</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>126.0 ns</td>
<td>44 ns</td>
<td>−152.8 kV</td>
<td>−301.7 kV</td>
<td>−311.5 kV</td>
<td>±3.7%</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>127.0 ns</td>
<td>61 ns</td>
<td>−150.7 kV</td>
<td>−296.0 kV</td>
<td>−307.2 kV</td>
<td>±4.6%</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>129.0 ns</td>
<td>59 ns</td>
<td>−150.3 kV</td>
<td>−292.7 kV</td>
<td>−308.6 kV</td>
<td>±5.2%</td>
</tr>
<tr>
<td>500 ns</td>
<td>1</td>
<td>124.0 ns</td>
<td>27 ns</td>
<td>−152.6 kV</td>
<td>−302.4 kV</td>
<td>−309.3 kV</td>
<td>±2.6%</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>124.0 ns</td>
<td>33 ns</td>
<td>−149.8 kV</td>
<td>−295.1 kV</td>
<td>−305.8 kV</td>
<td>±2.8%</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>126.0 ns</td>
<td>46 ns</td>
<td>−150.4 kV</td>
<td>−288.5 kV</td>
<td>−307.0 kV</td>
<td>±4.4%</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>122.0 ns</td>
<td>49 ns</td>
<td>−156.0 kV</td>
<td>−289.0 kV</td>
<td>−318.0 kV</td>
<td>±4.0%</td>
</tr>
<tr>
<td>1000 ns</td>
<td>1</td>
<td>125.0 ns</td>
<td>28 ns</td>
<td>−149.9 kV</td>
<td>−303.8 kV</td>
<td>−297.0 kV</td>
<td>±2.6%</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>125.0 ns</td>
<td>33 ns</td>
<td>−146.8 kV</td>
<td>−300.1 kV</td>
<td>−300.1 kV</td>
<td>±2.5%</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>125.0 ns</td>
<td>46 ns</td>
<td>−148.9 kV</td>
<td>−284.7 kV</td>
<td>−305.9 kV</td>
<td>±3.8%</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>127.0 ns</td>
<td>53 ns</td>
<td>−151.4 kV</td>
<td>−289.1 kV</td>
<td>−311.8 kV</td>
<td>±4.0%</td>
</tr>
</tbody>
</table>

Table 3.3: Pulse parameters for four-line model with an electron diode load.
It is observed that the FWHM stays relatively constant for the three pulse separation cases. The risetime is slower on pulse 3 and 4 compared to pulse 1 and 2. Nominally, the risetime for pulse 1 and 2 stays around 30 ns expect for the case of the 200 ns pulse separation due to the slight overlay of the preceding pulse. For pulse 3 and 4, the risetime is nominally around 50 ns with the exception of the 200 ns pulse separation. It is expected that the average flattop voltage is lower for longer pulse separations due to the equidistant switching from the Marx output voltage. For all three pulse separation cases, pulse 3 and 4 have a lower average voltage flattop then pulse 1 and 2. This is due to the reflections that add noise to the flattop. We know that pulse 3 and 4 are more susceptible to the reflection noise.

Figure 3.19: Comparison of peak and average voltage across the electron diode for the 1000 ns, 500 ns, and 200 ns separation cases.

In Figure 3.19, looking at the flattop peak voltage of all four pulses, they are relatively the same. The average flattop voltage, however, shows that pulse 3 and 4 have a lower average
voltage than pulse 1 and 2, which means that pulse 3 and 4 are more volatile than pulse 1 and 2. This can also be seen on the percentage flattop, which increases from pulse 1 to pulse 4.

Figure 3.20: Comparison of percentage flattop across the electron diode for 1000 ns, 500 ns, and 200ns separation cases.

From Figure 3.20, for 1000 ns pulse separation, it is shown that the percentage flattop is around $\pm 2.6\%$ for pulse 1 and 2 and $\pm 3.9\%$ for pulse 3 and 4. The closer the pulses are to each other, the more effect the reflections have on the flattop. For 500 ns pulse separation, pulse 1 and 2 have a $\pm 2.7\%$ flattop and pulse 3 and 4 have a $\pm 4.2\%$ flattop. For the 200 ns case, the percentage flattop is the worst. Pulse 1 should not be affected by the reflections since it is the first to go and the percentage flattop shows $\pm 2.6\%$. However, pulse 2,3,4 have been greatly affected by the reflections and the slight overlap of the neighboring pulse with flattop percentage of $\pm 3.7\%$, $\pm 4.6\%$, and $\pm 5.2\%$, respectively.
3.3.3 SPICE Four-Pulse with 5.5 Ohm Load Simulation Results

The following waveforms show the voltage output across a matched load.

Figure 3.21 The output voltage across the matched loaded at 1000 ns pulse-to-pulse separation for the four-line model.
Figure 3.22. The output voltage across the matched loaded at 500 ns pulse-to-pulse separation for the four-line model.
Figure 3.23: The output voltage across the matched loaded at 200 ns pulse-to-pulse separation for the four-line model.

Similarly, for the electron diode load, the same characteristic applies to the matched load case, but with a higher average voltage.
Table 3.4: Pulse parameters for the four-line model with a matched load.

<table>
<thead>
<tr>
<th>Pulse Separation</th>
<th>Pulse Number</th>
<th>FWHM</th>
<th>Risetime</th>
<th>Half Voltage</th>
<th>Flattop Average Voltage</th>
<th>Flattop Peak Voltage</th>
<th>Percentage Flattop</th>
</tr>
</thead>
<tbody>
<tr>
<td>200 ns</td>
<td>1</td>
<td>124.0 ns</td>
<td>27 ns</td>
<td>−159.6 kV</td>
<td>−315.1 kV</td>
<td>−322.3 kV</td>
<td>±2.7%</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>125.0 ns</td>
<td>43 ns</td>
<td>−157.6 kV</td>
<td>−309.8 kV</td>
<td>−318.1 kV</td>
<td>±4.5%</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>126.0 ns</td>
<td>54 ns</td>
<td>−155.8 kV</td>
<td>−304.4 kV</td>
<td>−317.8 kV</td>
<td>±5.2%</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>128.0 ns</td>
<td>61 ns</td>
<td>−155.5 kV</td>
<td>−300.3 kV</td>
<td>−321.4 kV</td>
<td>±6.1%</td>
</tr>
<tr>
<td>500 ns</td>
<td>1</td>
<td>124.0 ns</td>
<td>28 ns</td>
<td>−159.1 kV</td>
<td>−315.1 kV</td>
<td>−322.3 kV</td>
<td>±2.7%</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>124.0 ns</td>
<td>33 ns</td>
<td>−155.9 kV</td>
<td>−307.0 kV</td>
<td>−318.4 kV</td>
<td>±2.8%</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>125.0 ns</td>
<td>47 ns</td>
<td>−156.9 kV</td>
<td>−300.8 kV</td>
<td>−319.9 kV</td>
<td>±4.3%</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>121.0 ns</td>
<td>49 ns</td>
<td>−162.7 kV</td>
<td>−300.6 kV</td>
<td>−332.0 kV</td>
<td>±4.1%</td>
</tr>
<tr>
<td>1000 ns</td>
<td>1</td>
<td>124.0 ns</td>
<td>29 ns</td>
<td>−156.1 kV</td>
<td>−308.8 kV</td>
<td>−316.5 kV</td>
<td>±2.8%</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>125.0 ns</td>
<td>33 ns</td>
<td>−152.8 kV</td>
<td>−302.0 kV</td>
<td>−312.3 kV</td>
<td>±2.6%</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>125.0 ns</td>
<td>46 ns</td>
<td>−154.9 kV</td>
<td>−296.7 kV</td>
<td>−318.5 kV</td>
<td>±3.8%</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>127.0 ns</td>
<td>53 ns</td>
<td>−157.5 kV</td>
<td>−300.8 kV</td>
<td>−324.5 kV</td>
<td>±4.0%</td>
</tr>
</tbody>
</table>

Figure 3.24: The differences from the electron diode case and the matched case for the average and peak voltages.
Figure 3.25: The differences from the electron diode case and the matched case for the percentage flattop.

Similar patterns are shown in Figures 3.24 and 3.25 with the matched load compared to the electron diode load. Notably, there is more of a voltage drop across the matched load than for the electron diode. The percentage flattop for the 200 ns case is higher for the later pulses for the matched load compared to the electron diode.
### 3.3.4 SPICE Four-Pulse Electrical Length Sweeps

Figure 3.26: Variation of Percentage Flattop on all Four Pulses as Pulse 4 is Varied.

Figure 3.26 shows the percentage flattop of the four pulses as the electrical length of pulse 4 is swept to find the optimal minimum point. This shows that the electrical length for pulse 4, has a minimum around 52.5ns length with a flattop percentage of 3.6%. Decreasing and increasing the electrical length from this minimum will increase the percentage flattop. As expected, pulse 1 and are unaffected to barely affected by the change of the electrical length on pulse 4. This is due to the fact that pulse 4 is the last to be switched. Pulse 3 although is slight variations in the percentage flattop, this is due to the fact that pulse 3 is using the same Marx output as pulse 4 and the voltage on pulse 3 decreases as pulse 4 electrical length increases. This causes the deviations in the percentage flattop of pulse 3.
Figure 3.27: (Left) Comparison of Pulse 4 with varying Electrical Length. (Right) The two minimum and two maximum percentage flattop pulses.

Figure 3.27 shows pulse 4 variation as the electrical length is increased. For the extreme cases, the optimal point for the electrical length of the pulse 4 is 52-53 ns with a percentage flattop of 3.8%. For 45 ns electrical length, the pulse is too short, and it has a percentage flattop of 5.6%. For the 65 ns case, there is a peak at the end of the flattop and a percentage flattop of 5.3%.

Figure 3.28: Comparison of the four pulse with voltage scaling on the Marx charge.
Figure 3.28 shows the average voltage flattop with respect to Marx charge voltage. This also shows a linear relation between the average flattop voltage and the Marx charge voltage.

### 3.4 Conclusion

The goal of chapter 3 was to develop a SPICE model of the two-water line system and the four-water line system. The Braginskii equation from Chapter 2 was fully implemented into the SPICE model and the results were presented. In Chapter 4, there will be a comparison of experimental data with the SPICE model with Braginskii resistance and with an ideal resistance switch. Furthermore, this chapter has developed a four-line model with two load scenarios, one being an electron diode developed using the Child-Langmuir law and a matched load of 5.5 Ω. We have also discussed the importance of reducing the percentage flattop and how the reflections cause a great deal of noise in pulse 3 and 4.
Chapter 4

Experimental Analysis

4.1 Introduction

Chapter 3 focused on the development of a SPICE model for the two water line SPLITS. This chapter focuses on the comparison of the results taken from the experimental setup and the SPICE simulation. We use the two-line model as a basis for comparison because it allows us to compare the pulse-to-pulse separation and reflections on the second pulse. A single line model was also used to see variation of the first pulse due to different switch parameters, but due to limited experimental data only a few cases were used.

4.2 Two Pulse SPLITS

As a reminder, Figure 4.1 presents the circuit model of the two-pulse line. In this case, the Marx voltage is charged to -600 kV and the rest of the parameters are the same as in Table 3.1 for the Braginskii parameters. We compare the output of this model with Braginskii resistance, a constant resistance of 1 mΩ, and the experimental data from SPLITS.

Figure 4.1: Two-line circuit model of SPLITS.
The results from the SPICE modeling matched the experimental results taken at L3, albeit with some variation. The voltage across the load is compared to the real data from the L3 line. The data from the L3 line was taken across the resistor, which is measured using a D-dot sensor. The data collected had pulse separation of 1000 ns, 400 ns, and 200 ns pulse separation. This were compared with Micro-Cap modeling results.

Figure 4.2: Comparison of the Micro-Cap model with a constant resistance and Braginskii resistance switch to experimental data. A 1000 ns rising edge-rising edge pulse separation, full pulse, was used.
Figure 4.3: Comparison of the Micro-Cap model with a constant resistance and Braginskii resistance switch to experimental data. A 1000 ns rising edge-rising edge pulse separation was used. A close up of the first pulse is presented.
Figure 4.4: Comparison of the Micro-Cap model with a constant resistance and Braginskii resistance switch to experimental data. A 1000 ns rising edge-rising edge pulse separation was used. A close up of the second pulse is presented.

From the comparison in Figures 4.2-4.4, it is clear that the SPICE circuit model of SPLITS was able to properly capture the experimental results. It was able to capture the peaks and oscillation from the reflections in the falling trail of the pulse. The Braginskii model was able to match the experiment closer than the ideal resistance model. From the first pulse and second pulse, the Braginskii model tracked the rising edge, falling edge, and reflections closer to the experimental results than the constant resistance model. The following figure of 400 ns and 200 ns pulse separation tells a similar story to the 1000 ns separation. The close up of the first pulse in these cases will be omitted since it is very similar to the 1000 ns case.
Figure 4.5: Comparison of the Micro-Cap model with a constant resistance and Braginskii resistance switch to experimental data. A 400 ns rising edge-rising edge pulse separation, full pulse was used.
Figure 4.6: Comparison of the Micro-Cap model with a constant resistance and Braginskii resistance switch to experimental data. A 400 ns rising edge-rising edge pulse separation was used. A close up of the second pulse is presented.
Figure 4.7: Comparison of the Micro-Cap model with a constant resistance and Braginskii resistance switch to experimental data. A 200 ns rising edge-rising edge pulse separation, full pulse was used.
Figure 4.8: Comparison of the Micro-Cap model with a constant resistance and Braginskii resistance switch to experimental data. A 200 ns rising edge-rising edge pulse separation was used. A close up of the second pulse is presented.

The results from the 1000ns, 400ns, 200ns pulse separation compares well with the data from experiment. The entirety of the pulses, with the Braginskii model, is more in-line with the experimental data than with a constant resistance model. Notably the rising and falling edge of the pulse follows the sloped trend from experimental data, rather than straight upwards with the constant resistance, this is due the time-varying nature of the resistance within the LTSG. If we look at the rising and falling edge of the pulse, the constant resistance model has a faster risetime while Braginskii model is slower. This is due to the fact, that resistance changes with the amount of current flowing through the spark gap. While the constant resistance model changes from high to low resistance in an instant. The reflections on the transmission line also fits well within the model, as shown in Figure 4.8 where the prominent oscillation due to impedance mismatch can be seen on the trail of the pulse.
Table 4.1: Ideal resistance pulse parameters.

<table>
<thead>
<tr>
<th>Pulse Separation</th>
<th>Pulse Number</th>
<th>FWHM</th>
<th>Risetime</th>
<th>Half Voltage</th>
<th>Flattop Average Voltage</th>
<th>Flattop Peak Voltage</th>
<th>Percentage Flattop</th>
</tr>
</thead>
<tbody>
<tr>
<td>200 ns</td>
<td>1</td>
<td>122.0 ns</td>
<td>11 ns</td>
<td>−152.7 kV</td>
<td>−306.5 kV</td>
<td>−307.3 kV</td>
<td>±0.2%</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>122.0 ns</td>
<td>19 ns</td>
<td>−154.7 kV</td>
<td>−302.8 kV</td>
<td>−319.5 kV</td>
<td>±3.9%</td>
</tr>
<tr>
<td>400 ns</td>
<td>1</td>
<td>122.0 ns</td>
<td>11 ns</td>
<td>−152.4 kV</td>
<td>−305.8 kV</td>
<td>−306.7 kV</td>
<td>±0.2%</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>123.0 ns</td>
<td>18 ns</td>
<td>−150.7 kV</td>
<td>−300.9 kV</td>
<td>−310.5 kV</td>
<td>±1.5%</td>
</tr>
<tr>
<td>1000 ns</td>
<td>1</td>
<td>122.0 ns</td>
<td>11 ns</td>
<td>−149.1 kV</td>
<td>−300.5 kV</td>
<td>−300.5 kV</td>
<td>±0.3%</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>123.0 ns</td>
<td>18 ns</td>
<td>−147.7 kV</td>
<td>−295.6 kV</td>
<td>−302.9 kV</td>
<td>±0.5%</td>
</tr>
</tbody>
</table>

Table 4.2: Braginskii resistance pulse parameters.

<table>
<thead>
<tr>
<th>Pulse Separation</th>
<th>Pulse Number</th>
<th>FWHM</th>
<th>Risetime</th>
<th>Half Voltage</th>
<th>Flattop Average Voltage</th>
<th>Flattop Peak Voltage</th>
<th>Percentage Flattop</th>
</tr>
</thead>
<tbody>
<tr>
<td>200 ns</td>
<td>1</td>
<td>123.8 ns</td>
<td>28 ns</td>
<td>−147.9 kV</td>
<td>−293.1 kV</td>
<td>−299.4 kV</td>
<td>±2.5%</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>125.6 ns</td>
<td>52 ns</td>
<td>−147.9 kV</td>
<td>−299.4 kV</td>
<td>−298.8 kV</td>
<td>±5.2%</td>
</tr>
<tr>
<td>400 ns</td>
<td>1</td>
<td>124.0 ns</td>
<td>28 ns</td>
<td>−147.5 kV</td>
<td>−292.4 kV</td>
<td>−298.8 kV</td>
<td>±2.6%</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>124.0 ns</td>
<td>34 ns</td>
<td>−145.0 kV</td>
<td>−284.5 kV</td>
<td>−296.7 kV</td>
<td>±3.3%</td>
</tr>
<tr>
<td>1000 ns</td>
<td>1</td>
<td>123.0 ns</td>
<td>28 ns</td>
<td>−144.5 kV</td>
<td>−286.4 kV</td>
<td>−292.9 kV</td>
<td>±2.6%</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>124.0 ns</td>
<td>32 ns</td>
<td>−141.9 kV</td>
<td>−279.7 kV</td>
<td>−289.7 kV</td>
<td>±2.8%</td>
</tr>
</tbody>
</table>

Table 4.3: Experimental pulse parameters.

<table>
<thead>
<tr>
<th>Pulse Separation</th>
<th>Pulse Number</th>
<th>FWHM</th>
<th>Risetime</th>
<th>Half Voltage</th>
<th>Flattop Average Voltage</th>
<th>Flattop Peak Voltage</th>
<th>Percentage Flattop</th>
</tr>
</thead>
<tbody>
<tr>
<td>200 ns</td>
<td>1</td>
<td>123.8 ns</td>
<td>22 ns</td>
<td>−150.7 kV</td>
<td>−299.3 kV</td>
<td>−306.4 kV</td>
<td>±2.1%</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>125.6 ns</td>
<td>37.6 ns</td>
<td>−146.9 kV</td>
<td>−289.1 kV</td>
<td>−299.1 kV</td>
<td>±3.3%</td>
</tr>
<tr>
<td>400 ns</td>
<td>1</td>
<td>124.2 ns</td>
<td>22.8 ns</td>
<td>−149.5 kV</td>
<td>−296.8 kV</td>
<td>−304.1 kV</td>
<td>±2.1%</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>124.2 ns</td>
<td>26.8 ns</td>
<td>−146.1 kV</td>
<td>−287.2 kV</td>
<td>−299.3 kV</td>
<td>±2.8%</td>
</tr>
<tr>
<td>1000 ns</td>
<td>1</td>
<td>124.4 ns</td>
<td>24.8 ns</td>
<td>−146.3 kV</td>
<td>−288.3 kV</td>
<td>−298.5 kV</td>
<td>±2.8%</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>125.4 ns</td>
<td>24 ns</td>
<td>−143.1 kV</td>
<td>−285.0 kV</td>
<td>−293.8 kV</td>
<td>±1.2%</td>
</tr>
</tbody>
</table>

From the three tables above, the ideal resistance has half the risetime as the experimental and the Braginskii model. This is expected since there is an instant transient for the ideal switch rather than a rapid decay such as Braginskii. These tables also show that the Braginskii switch model, for these parameters, matches closer to the experimental data. One interesting note is
that pulse 1 has a higher percentage flattop for the 1000 ns case compared to the 400 ns and 200 ns cases. The reason for this is how the pulses are triggered, see Figure 4.9.

Figure 4.9: Overlay of 1000 (blue), 400 (orange), 200 (green) ns pulse separation output with their respective Marx voltage position. The black line is the overlaid output voltage waveform for the three cases.

Figure 4.9 shows the triggering scheme for each pulse separation. Pulse 1 for each pulse separation cases does not start at the same time. The pulse separation distance starts from the center of the Marx output and then spreads out. The flattop and peak voltage of the pulses are directly related to where on the Marx output it was triggered. Thus, there are small discrepancies in each of the pulse separation cases since the position where they were triggered on the Marx output is different.
Figure 4.10: Percentage flattop of each pulse: (blue) 1000 ns, (orange) 500 ns, (green) 200 ns pulse separation. Also shown are experiment (solid), Braginskii (dash), and ideal (dash-dot).

The trend continues as the percentage flattop is shown to be larger in pulse 2 than pulse 1. It is surprising. In the experimental data, pulse 2 was shown to have a lower percentage flattop than pulse 1. The Braginskii resistance model and the experimental data’s percentage flattop for pulse 1 are nearly 2.5%, but the constant resistance model shows it to be around 0.3%.
Figure 4.11: The average voltage of each pulse: (blue) 1000 ns, (orange) 500 ns, (green) 200 ns pulse separation. Also shown are experiment (solid), Braginskii (dash), and ideal (dash-dot).

The average voltage for the constant resistance case shows a trend upwards from pulse 1 to pulse 2 for all three pulse separation scenarios, while for the experimental and Braginskii model, the average voltage trends down from pulse 1 to pulse 2.

4.3 Single Pulse Testing

In this section we explored the single pulse model to validate the spark gap switch model further. Due to limited data, we could not explore all the parametric values of the switch, but we were able to explore various gap distances and pressures.
Table 4.4: LTSG and Marx Parameters for Single Pulse

<table>
<thead>
<tr>
<th>Shot #</th>
<th>Pressure (Psi)</th>
<th>Gap Distance (Inch)</th>
<th>Marx Charge (kV)</th>
<th>Trigger Type</th>
<th>%Laser Power</th>
</tr>
</thead>
<tbody>
<tr>
<td>529</td>
<td>90</td>
<td>0.5</td>
<td>50</td>
<td>Self-Break</td>
<td></td>
</tr>
<tr>
<td>548</td>
<td>90</td>
<td>0.5</td>
<td>60</td>
<td>Self-Break</td>
<td></td>
</tr>
<tr>
<td>593</td>
<td>123</td>
<td>0.5</td>
<td>60</td>
<td>Laser</td>
<td></td>
</tr>
<tr>
<td>722</td>
<td>140</td>
<td>0.5</td>
<td>50</td>
<td>Laser</td>
<td>100</td>
</tr>
<tr>
<td>735</td>
<td>140</td>
<td>0.5</td>
<td>50</td>
<td>Laser</td>
<td>50</td>
</tr>
<tr>
<td>747</td>
<td>140</td>
<td>0.5</td>
<td>50</td>
<td>Laser</td>
<td>30</td>
</tr>
<tr>
<td>1149</td>
<td>140</td>
<td>0.7</td>
<td>65</td>
<td>Laser</td>
<td>100</td>
</tr>
<tr>
<td>1155</td>
<td>140</td>
<td>0.7</td>
<td>65</td>
<td>Laser</td>
<td>100</td>
</tr>
</tbody>
</table>

Table 4.4 shows a few shots that we have analyzed. The circuit for the shots is similar to the two-pulse model minus one of the PFL and again into a matched load of 5.5 Ω. The shots 529, 548, 593 will is grouped together denoted the 500s shots and 722, 735, 747 group denoted 700s shots. The 500s shots were single PFL with gap distance of 1.27 cm and with various pressure. These three shots were chosen to compare the difference in trigger type, Marx charge voltage, and pressure. With a new gap distance, we modified the inductance and capacitance of the switch to 58.5 nH and 4.3 pF, respectively.
4.3.1 500s Shots

Figure 4.12: Normalized single PFL overlaid 500s shots.

These shots were normalized and time-shifted to see the difference in pulse structure between different PFL schemes. From Figure 4.12, we can see that there are little differences in the shots, but shot 593 shows a slightly lower front end of the pulse compared to the other two shots.
Figure 4.13: Comparison of Micro-Cap circuit model to shot 548 with charge cable electrical length of 44 ns and 74 ns, respectively.

Figure 4.13 shows that the Micro-Cap model with Braginskii spark gap captures the output pulse of this scheme quite well. We have noticed that there is a slight dip in the peak of the pulse shown in Figure 4.14. This is due to the charge cable’s electrical length. In the two line and four-line model, the charge cable length is 44 ns. We can correct for this dip in the peak by making the charge cable electrical length to be 74 ns. The side effect to this is that the trailing end becomes blockier in some parts and does not display the small features compared to the 44 ns length, see Figure 4.15.
Figure 4.14: Comparison of Micro-Cap circuit model to Shot 548 flat top.

Figure 4.15: Comparison of Micro-Cap circuit model to shot 548, trailing end.
4.3.2  700s Shots

The 700s shots varied the laser power used to trigger the LTSG. The shots showed that there are little to no differences between the 700 shots when the LTSG is triggered with varying laser power. We compare shot 722 in the figure below. Surprisingly, the Micro-Cap circuit model has a sharper rise in the front end of the pulse than the experimental data.

Figure 4.16: Comparison of a single PFL circuit model to shot 722.
Figure 4.17: Comparison of shot 529 and 735.

In the previous comparisons we have shown that Marx charge voltage and the trigger scheme have little effect on the pulse structure. In Figure 4.17 we compare shots 529 and 735 since the only relevant difference in parameter between the two is the pressure. Shot 529 and 735 have a switch pressure of 90 psi and 140 psi, respectively. The pulse structure shows that the front end of the pulse is lower and the trailing end broadens out more when the pressure is higher. This aligns well with the pressure sweep in Figure 4.18. This figure shows PFL1 switch pressure swept from 1 psi to 150 psi. As pressure goes from low to high, we observe that the front end of the pulse has a broader slope and the trailing end is also broader. The effect of the pressure can be seen in Figure 4.18.
Figure 4.18: The effect of pressure sweep of Braginskii resistance on Pulse 1 with 1000 ns separation.
Figure 4.19: Preliminary snubber shots with pulse separation of 200 ns and 400 ns, respectively.

Figure 4.19 shows the two-line voltage pulses with the snubber addition onto the switch. This snubber circuit dampens the reflected waves from the cascading PFLs. We can observe that the peaks and the oscillations on trailing end on pulse 2 have been dampened. Present efforts are made to add the snubber into the two-line circuit model.

4.4 Conclusion

The goal of Chapter 4 is to present the experimental data and compare the data to the SPICE model. The SPICE model included two switch models, one being a modified switch resistance using Braginskii equation described in chapter 2 and 3 and the other with a constant resistance.
The simulation vs. experimental measurement results comparison showed that the SPICE modeled the experimental measurements show good agreement. Overall, the timing and oscillations on the pulse are captured in the SPICE model. The Braginskii model outperformed the constant resistance model. While the constant resistance model captured the pulse correctly, it overshot/undershot the rising edge, falling edge, and some of the finer detail. The Braginskii model was able to capture dips and peaks more in line with the experiment then the constants resistance model. This chapter also explore a single PFL with various system scheme, Table 4.4, to further test the spark gap model.
Chapter 5

Future Work and Conclusion

5.1 Conclusion of Results

LANL and L3 Harris have designed and constructed a Marx charged multi-pulse water transmission line as a driver for the induction cells of a new radiographic accelerator. This transmission line is a modular design and is designed to handle up to four multiple pulses. This thesis develops a circuit based parametric model for a two-pulse line and four pulse line configuration, and implements a time-dependent Braginskii resistance, a static capacitance, and static inductance into a spark gap model.

The main results from this thesis are as follows:

- Derivation of an analytical solution for a reduced transmission line circuit as shown in Figure 2.3.
- Design of a switch model in SPICE with the actual physical parameters.
- Design of a circuit model for the SPLITS system in two- and four-line configuration with a matched load and an electron diode load.

One of the first major results from this thesis is the derivation of an analytical function for the reduced transmission line circuit. With this function, it provides us a method of designing and optimizing SPLITS Marx output waveforms. From the SPLITS circuit, the output pulses of the line are directly influenced by the Marx output. It is shown that charging and the switching time of the PFL affect pulse formation. The Marx output pulse width must be wider than the PFL’s electrical length for the output pulse to have a relatively constant flattop. The analytical function is developed to optimize the Marx output pulse for the PFL. With this function, we have the capability to adjust the parameters of the circuit to provide desired output such as peak voltage, peak current, pulse width, risetime, fall time and minimization of higher order oscillations.
The second major result is the design of a two-line SPICE circuit that matches very well with experimental data. Along with this circuit, we have derived and implemented a time-varying spark gap switch model that outperforms an ideal switch model which improves the SPLITS circuit model. This spark gap model incorporates Braginskii resistance, which was derived, the model also has a static inductance which was calculated from Grover coaxial equation and a static capacitance from 3d electrostatic simulation. In the Appendix C, we have greatly explored the impact of various switch parameter including but not limited to gap distance and pressure. This provides valuable information on the impact a spark gap switch can have on this circuit.

The third major results were taking a step further and producing a four-line SPICE circuit of SPLITS. This will help greatly and provide feedback to the current testing of the four-line experiment. The four-line model suffers from the impedance mismatch reflection more than the two-line model. The feedback from this circuit simulation will predict results from experiment as well as distinguishing issue beforehand.

5.2 Discussion of Results

Further discussion of each chapter result is discussed below along with future work that can be performed to improve on this work.

Chapter 2 focuses on the derivation of the analytical solution for a reduced transmission line circuit. The derivation focused on three cases of a lumped parameter circuit and compared with several transmission line analytical models. The first uses a lumped T-model circuit using a capacitor and inductors. The second and third cases used the standard model found in the code. The model is parameterized with either the input impedance and time delay or the length, capacitance, and inductance. Parameterization was compared for the second and third cases, respectively. We observe that the analytical solution of this circuit perfectly matches the lumped T-model while both the Micro-Cap transmission line model had a lagging time shift compared to the analytical solution. This is due the how the transmission line model is
represented in SPICE. The transmission line model in SPICE, is implemented as a matched resistance with time delays of the current and voltage. The difference is due to the lumped model having additional capacitive and inductive elements resulting in a different resonant frequency. We have chosen to use the provided ideal transmission line model for the SPICE due the simplicity, the ease of implementation in the SPLITS circuit, and that the output voltage pulse’s frequency is far greater than the high order frequency from the transmission line which suffers from frequency difference.

The latter half of chapter 2 focuses on the derivation of a physical switch model, the second of the main results. The main goal of this model was to derive a resistance of the arc based on the Braginskii equation which captures the temporal evolution both axially and radially of the arc expansion along with implementation of a static inductance and capacitance. The derived Braginskii equation shows that the resistance is temporally evolving. The Braginskii resistance equation provided the needed parameterization of the LTSG such as the gap distance, pressure, and gas type. Implementation of Braginskii resistance into the spark gap switch model was an improvement over the ideal resistance switch. The inductance of the spark gap is calculated as a coaxial line. This inductance is static, and the inner radius is tracked from the surface of the one electrode to the arc length and the end of the second electrode. The radius although is time-varying, is kept constant for this case which to make the inductance static. The value chosen for the radius of the arc is 1 mm, which is a good estimate for an arc operating in this regime [19]. The case for a time-varying inductance will be discussed in the future work section. The static capacitance is solved using a 3D electrostatic solver which calculates the electric field energy stored between the electrodes and from which the capacitance can be extracted. The switching mechanism of the spark gap is through an ideal time controlled switched. One of the downsides of using an ideal switch is that it does not account for prefire breakdown and jitter of the switch. This can be incorporated into the spark gap model through static breakdown formulas and random timing elements.

Chapter 3 and 4 focus on the design of the SPLITS lumped parameter SPICE model using a two-line and four-line system with various loads such as a matched resistive and electron diode. The two-line model is compared to the experimental results provided by L3. Overall, the model was satisfactory when compared to the experiment. The model with the physical switch was
able to capture the experimental pulse closer than the ideal switch. One of the downfalls of this model was that there was a time delay needed in the second PFL of -66.8 ns to match the timing of the experimental data. This might be caused by the jitter of the spark gap plus other timing factors.

5.3 Future Experiments

While the overall goal of this project was achieved, the model still needs fine tuning, and adjustments and the further experiments can be performed to further improve knowledge of the physical processes in a LTSG. Below provides a list of extended work.

1. Implementation of the time-dependent conductance and inductance nature of the switch. One of the crucial parameters in the Braginskii resistance is the conductivity of the ionized gas. From Reference [17], it shows the conductivity of SF$_6$ of an arc as it varies with time. We used the peak value of conductivity from that reference and kept it constant within the spark gap model. We need to better understand the conductivity via measurements of SF$_6$ arcs and develop a time-dependent function for conductivity. Exploration of methods to represent conductivity is needed for various gases such as SF$_6$ at various pressures. The time-dependent inductance is another crucial parameter. This can be incorporated into the spark gap model by using Braginskii radius equation into the Grover equation. The issue lies in the implementation into SPICE. When implemented into SPICE we get a convergence error as we close the spark gap. Even when setting the initial condition to be a value, SPICE cannot accurately calculate the radius with the inductance since the radius equation require a current value that is determined with the inductance. This creates a loop where the resistance equation and the inductance equation rely on each other at the same time stamp. This creates an unknown which causes an error. This will help improve the spark gap switch model if implemented successfully.

2. Variation in the parameters of the experiment compared to the circuit model. The two-line SPICE model was only tested with varying pulse separation and a constant gap distances and pressure it did not include variations of this parameter. The model needs
to be better tested with varying gap distances and pressures compared to experiment. The current SPLITS incorporates the water lines with snubber hardware to reduce the reflected wave. This needs to be updated in the SPICE model.

3. Experimental image measurements of the spark radius. This is useful to validate the Braginskii radius equation. This can be done initial with an open shutter camera to get the maximum radius of the arc. Then we need a time resolved measurement of the radius, this will track the growth and the arc and will validate the Braginskii equation.

4. Particle-in-Cell (PIC) modeling of the LTSG. This will simulate the same conditions as the spark gap and help determine a function for conductivity and inductance which can be used in the SPICE model. LSP is a good PIC code for this because you can add in a Marx or transmission line circuit as driver for the spark gap.
Appendix A

Full SPICE Model of Four-Pulse SPLITS Circuit
Appendix B

Quartic Roots
Appendix C

Parameter Sweep of Switch Parameters for Two-Line Model
Pressure Sweep Full Pulse 1000ns

Pressure Sweep 1st Pulse 1000ns
REFERENCES


