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Neutrosophic interrelationship of Key Performance Indicators in an accounting process

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Abstract. This paper is an investigation on the interrelationship among some financial performance indicators of the company “Zambrano Loor Naira Narcisa” located in the city of Santo Domingo, Ecuador. The study is conducted because this company presents some difficulties in this regard. In order to guarantee the profundity of the analysis, we apply the technique of dynamic study of Neutrosophic Cognitive Maps, where the cause-effect relationship among the indicators is described and established; in addition it is provided the causal connecting weights between each pair of indicators. Four experts have made the evaluations. The advantage of neutrosophic cognitive maps over fuzzy cognitive maps is that the former maintains the modeling of the uncertainty of the latter, and also offers the possibility of incorporating indetermination, thus constituting a more realistic source to assess cause-effect relationships.

Keywords: Key Performance Indicator, financial accounting, neutrosophic cognitive map, neutrosophic number.

1 Introduction

The company “Zambrano Loor Naira Narcisa” is located in the city of Santo Domingo, Ecuador. It is dedicated to the wholesale and retail sale of lubricants and refrigerants for all types of motor vehicles. In an interview with its manager, he alleged that they are obliged to keep accounts according to the Ecuadorian categorization. This investigation aims to achieve an adequate accounting process, due to they do not have appropriate financial information for decision making according to the needs of the company, see [1].

The accounting is essential because it provides financial information of the company, to people or entities interested in knowing the results on the profitability and solvency of the business. The users can be internal or external; the most important external user is the State, which is interested in the collection of income taxes. The internal users are the owners of the companies that we call shareholders, the administrators and the employees; see [2-6].

The objective of this paper is to study the key financial performance indicators of the company “Zambrano Loor Naira Narcisa”. Similar studies can be found in [7, 8]. For this purpose, Neutrosophic Cognitive Maps are applied; see [9-11]. Cognitive maps are basically directed graphs, where their nodes represent concepts and their edges represent causal relationships among these concepts, see [12]. Additionally, each edge is associated with a weight that can be null, which means that there is no relationship between the concepts; if it is a positive value it means that if one concept increases (decreases) its value the other will also tend to increase (decrease); while if it is negative it means that if one concept tends to increase (decrease), the other will tend to decrease (increase).

Both, fuzzy cognitive maps ([13, 14]) and neutrosophic cognitive maps incorporate uncertainty in causal relationships, and the latter ones are based on Neutrosophy. Neutrosophy is the branch of philosophy that deals with the phenomena of neutrality, see [15]. Neutralities are the consequence of contradictions, ignorance, paradoxes, inconsistencies, among other reasons. Neutrosophic sets generalize fuzzy, intuitionist fuzzy, interval-valued fuzzy sets, among others. Neutrosophic sets include for the first time a membership function of indeterminacy, aside from the membership function and the non-membership function.

Another generalization is that these three membership functions are independent each other, unlike the
intuitionistic fuzzy sets. On the other hand, neutrosophic sets are defined in subsets of the non-standard interval \([0,1]^*\).

The definition of neutrosophic sets is only of philosophical interest, that is why the interval-valued neutrosophic sets and the simplest single-valued neutrosophic sets are defined, which are subsets or elements of the interval\([0, 1]\), respectively. For convenience, single-valued neutrosophic numbers are defined, which is the simplest form of single-valued neutrosophic sets.

It is worth noting that neutrosophy has opened a large number of practical applications within dissimilar fields ([16]) as decision-making ([17-19]), digital image processing ([20-23]), time series forecasting ([24-26]), artificial intelligence ([27]), game theory ([28]), recommender systems ([29]), learning management systems ([30]), biomedical diagnosis ([31]), among others. Additionally, rings can be defined where the main element is the symbol "I" of indeterminacy, and where the neutrosophic number is divided into a determinate part "d" and an indeterminate part "I", while the neutrosophic number is defined as the sum of these two elements, which is convenient for defining operations between elements of neutrosophic cognitive maps. Therefore, neutrosophic cognitive maps include the possible weights of fuzzy cognitive maps, and also a symbolic value denoted by I representing the indetermination.

In this article a dynamic study of the causal relationship among the different financial indicators is carried out. For this, we incorporate the complexity of cause-effect relations, where two-way relations can exist, that is to say, a concept can be cause and at the same time effect of another one.

Accounting processes are usually viewed objectively, based on financial measures. This makes that the managers and decision makers lose the holistic view of these indicators, which depends on both economic and social variables such as the satisfaction of the company’s customers or the education level of the employees, among others. On the other hand, up to the authors’ knowledge, the tool known as dynamic study of neutrosophic cognitive maps has been applied in exclusively social phenomena such as the problem of a minority who are the Hijras in India or the migrant workers sick with HIV, see [9, 10]. The most important purpose of this paper is to study the key financial performance indicators of the company “Zambrano Loor Naira Narcisa”, which will contribute to solving a major problem for this company.

The main contribution of this paper is the demonstration through this real case, that it is possible to use the dynamic study of neutrosophic cognitive maps to measure a financial situation of a company, where subjective criteria based on objective measures of accounting are associated with subjective measures like the customer’s satisfaction. Additionally, mathematical modeling based on neutrosophic logic instead of fuzzy logic, allows a more accurate study, since the relationship between some variables can be unknown. Thus, it is possible to model the indeterminacy existing between them, which is not possible by using fuzzy set theory or intuitionistic fuzzy set theory.

The paper is divided into the following sections. Section 2 contains a summary of the main concepts of the Neutrosophic Cognitive Maps method. Section 3 contains the results of applying NCMs for the analysis of the performance of key indicators in the company “Zambrano Loor Naira Narcisa”. The last section contains the conclusions.

### 2 A summary on Neutrosophic Cognitive Maps

To begin this section we have the following basic definitions regarding neutrosophy:

**Definition 1.** ([15]) Let \(X\) be a universe of discourse. A **Neutrosophic Set** (NS) is characterized by three membership functions, \(u_A(x), r_A(x), v_A(x) : X \rightarrow \mathbb{I} = [0, 1]^*\), which satisfy the condition \(0 \leq \inf u_A(x) + \inf r_A(x) + \inf v_A(x) \leq \sup u_A(x) + \sup r_A(x) + \sup v_A(x) \leq 3\) for all \(x \in X\). \(u_A(x), r_A(x)\) and \(v_A(x)\) are the membership functions of truthfulness, indeterminacy and falseness of \(x\) in \(A\), respectively, and their images are standard or non-standard subsets of \([0, 1]^*\).

**Definition 2.** ([15]) Let \(X\) be a universe of discourse. A **Single-Valued Neutrosophic Set** (SVNS) \(A\) on \(X\) is a set of the form:

\[
A = \{ (x, u_A(x), r_A(x), v_A(x)) : x \in X \}  
\]

Where \(u_A(x), r_A(x), v_A(x) : X \rightarrow [0,1]\) satisfy the condition \(0 \leq u_A(x) + r_A(x) + v_A(x) \leq 3\) for all \(x \in X\). \(u_A(x), r_A(x)\) and \(v_A(x)\) denote the membership functions of truthfulness, indeterminacy and falseness of \(x\) in \(A\), respectively. For convenience a **Single-Valued Neutrosophic Number** (SVNN) will be expressed as \(A = (a, b, c)\), which \(a, b, c \in [0,1]\) and satisfy \(0 \leq a + b + c \leq 3\).

Other important definitions are related to the graphs. See [9-11, 15].

**Definition 3.** A neutrosophic graph is a graph containing at least one indeterminate edge, which is represented by dotted lines.

**Definition 4.** A neutrosophic directed graph is a directed graph containing at least one indeterminate edge, which is represented by dotted lines.

**Definition 5.** A **Neutrosophic Cognitive Map** (NCM) is a neutrosophic directed graph, whose nodes represent concepts and whose edges represent causal relationships among the edges.

If \(C_1, C_2, ..., C_k\) are \(k\) nodes, each of the \(C_i\) \((i = 1, 2, ..., k)\) can be represented by a vector \((x_1, x_2, ..., x_k)\)
where \( x_i \in \{0, 1, I\} \). \( x_i = 0 \) means that the node \( C_i \) is in an activated state, \( x_i = 1 \) means that the node \( C_i \) is in a deactivated state and \( x_i = I \) means that the node \( C_i \) is in an indeterminate state, in a specific time or in a specific situation.

If \( C_m \) and \( C_n \) are two nodes of the NCM, an edge directed from \( C_m \) to \( C_n \) is called a connection and represents the causality from \( C_m \) to \( C_n \). Each node in the NCM is associated with a weight within the set \([-1, 0, 1, I]\). If \( \alpha_{mn} \) denotes the weight of the edge \( C_mC_n \), \( \alpha_{mn} \in [-1, 0, 1, I] \) then we have the following:

\[
\begin{align*}
\alpha_{mn} &= 0 & \text{if } C_m \text{ has no effect on } C_n, \\
\alpha_{mn} &= 1 & \text{if an increase (decrease) in } C_m \text{ produces an increase (decrease) in } C_n, \\
\alpha_{mn} &= -1 & \text{if an increase (decrease) in } C_m \text{ produces a decrease (increase) in } C_n, \\
\alpha_{mn} &= I & \text{if the effect of } C_m \text{ on } C_n \text{ is indeterminate.}
\end{align*}
\]

**Definition 6.** A NCM having edges with weights in \([-1, 0, 1, I]\) is called Simple Neutrosophic Cognitive Map.

**Definition 7.** If \( C_1, C_2, \ldots, C_k \) are the nodes of a NCM. The neutrosophic matrix \( N(E) \) is defined as \( N(E) = (\alpha_{mn}) \), where \( \alpha_{mn} \) denotes the weight of the directed edge \( C_mC_n \), such that \( \alpha_{mn} \in [-1, 0, 1, I] \). \( N(E) \) is called the neutrosophic adjacency matrix of the NCM.

**Definition 8.** Let \( C_1, C_2, \ldots, C_k \) be the nodes of a NCM. Let \( A = (a_1, a_2, \ldots, a_k) \), where \( a_m \in [-1, 0, 1, I] \). \( A \) is called instantaneous state neutrosophic vector and means a position of on-off-indeterminate state of the node in a given instant.

\[
\begin{align*}
a_m &= 0 & \text{if } C_m \text{ is deactivated (has no effect)}, \\
a_m &= 1 & \text{if } C_m \text{ is activated (has an effect)}, \\
a_m &= I & \text{if } C_m \text{ is indeterminate (its effect cannot be determined).}
\end{align*}
\]

**Definition 9.** Let \( C_1, C_2, \ldots, C_k \) be the nodes of a NCM. Let \( C_1C_2, C_2C_3, C_3C_4, \ldots, C_mC_n \) be the edges of the NCM, then the edges constitute a directed cycle.

The NCM is called cyclic if it has a directed cycle. It is said acyclic if it has not a directed cycle.

**Definition 10.** A NCM containing cycles is said to have feedback. When there is feedback in the NCM, it is said that it is a dynamic system.

**Definition 11.** Let \( C_1C_2, C_2C_3, C_3C_4, \ldots, C_{k-1}C_k \) be a cycle. When \( C_m \) is activated and its causality flows through the edges of the cycle and then it is the cause of \( C_m \) itself, then the dynamic system circulates. This is fulfilled for each node \( C_m \) with \( m = 1, 2, \ldots, k \). The equilibrium state for this dynamic system is called the hidden pattern.

**Definition 12.** If the equilibrium state of a dynamic system is a single state, then it is called a fixed point.

An example of a fixed point is when a dynamic system starts by being activated by \( C_1 \). If it is assumed that the NCM sits on \( C_1 \) and \( C_k \), i.e. the state remains as \( (1, 0, \ldots, 0, 1) \), then this vector of neutrosophic state is called fixed point.

**Definition 13.** If the NCM is established with a neutrosophic state-vector that repeats itself in the form:

\[
A_1 \to A_2 \to \cdots \to A_m \to A_1,
\]

then the equilibrium is called a limit cycle of the NCM.

**Method for Determining the Hidden Patterns**

Let \( C_1, C_2, \ldots, C_k \) be the nodes of the NCM with feedback. Assume that \( E \) is the associated adjacency matrix. A hidden pattern is found when \( C_1 \) is activated and a vector input \( A_1 = (1, 0, 0, \ldots, 0) \) is given. The data must pass through the neutrosophic matrix \( N(E) \), which is obtained by multiplying \( A_1 \) by the matrix \( N(E) \).

Let \( A_1N(E) = (a_1, a_2, \ldots, a_k) \) with the threshold operation of replacing \( m \) by \( I \) if \( a_m > p \) and \( a_m \) by \( 0 \) if \( a_m < p \) (\( p \) is a suitable positive integer) and \( a_m \) is replaced by \( I \) if this is not an integer. The resulting concept is updated; vector \( C_1 \) is included in the updated vector by transforming the first coordinate of the resulting vector into \( 1 \).

If \( A_1N(E) \to A_2 \) is assumed then \( A_2N(E) \) is considered and the same procedure is repeated. This procedure is repeated until a limit cycle or fixed point is reached.

**Definition 14.** A neutrosophic number \( N \) is defined as a number as follows, see \([15, 32-34]:(\ref{eq:2})\):

\[
N = d + I
\]

(\ref{eq:2})

Where \( d \) is called determinate part and \( I \) is called indeterminate part.

Given \( N_1 = a_1 + b_1I \) and \( N_2 = a_2 + b_2I \) two neutrosophic numbers, some operations between them are defined as follows:

\[
N_1 + N_2 = a_1 + a_1 + (b_1 + b_2)I \quad \text{(Addition)};
\]

\[
N_1 - N_2 = a_1 - a_1 + (b_1 - b_2)I \quad \text{(Difference)}.
\]
\[ N_1 \times N_2 = a_1 a_2 + (a_1 b_2 + b_1 a_2 + b_1 b_2) \] (Product).

\[ \frac{N_1}{N_2} = \frac{a_1 + b_1 I}{a_2 + b_2} = \frac{a_1}{a_2} + \frac{a_2 b_1 - a_1 b_2}{(a_2 + b_2)} \] (Division).

3 Results

First of all, we describe the selected key performance indicators of the financial accounting:

I. **Liquidity**: This indicator reflects the relationship between the financial resources that a company has in the short-term to face the payment obligations contracted in the same period, which makes it possible to determine whether it has sufficient resources to cover its commitments. The greater the result of the current ratio, the greater the possibility that liabilities will be paid, since there are sufficient assets that can be converted into cash when required.

II. **Efficiency**: It measures the effectiveness with which the company’s resources are being used, measuring the degree of efficiency with which the assets are used in its operations, and their speed of recovery, expressing the result by index or number of times.

III. **Indebtedness**: It aims to measure the extent to which and how creditors participate in the financing of the company. In the same way, it tries to establish the risk that such creditors run, the risk of the owners and the convenience or inconvenience of a certain level of indebtedness of the company.

IV. **Return on equity**: This indicator measures the return on net investment, i.e. stockholders’ equity. It relates the net profit that an organization has earned during a period and compares it with the investment that corresponds to the shareholders. Return on stockholders’ equity is a key indicator of the extent to which a company has generated a return on the resources that shareholders have entrusted to manage.

V. **Leverage**: is the relationship between credit and equity invested in a financial transaction. The higher the credit, the higher the leverage and the lower the equity investment. In other words, leverage is simply using debt to finance an operation. By reducing the initial capital that needs to be provided, there is an increase in the return obtained. Increased leverage also increases the risks of the operation, as it results in less flexibility or greater exposure to insolvency or inability to meet payments.

VI. **Customer satisfaction**: It is a measure of how products and services provided by a company meet or exceed customer expectations.

Let us note that I_6 is not precisely a financial indicator, however, it is a key factor to study the good performance of the enterprise, and thus we decided to include it.

Another important aspect to remark is that experts’ evaluations depend on the context of company “Zambrano Loor Naira Narcisa” in the year 2017, i.e., they are not only generic. Thus, first experts studied the financial situation of this company and later they evaluated the relationship among the indicators.

For group evaluation, each expert evaluates according to a value in the set \{-1, 0, 1, I\}, next the median per causal-effect weight is calculated. In case that one expert evaluates with I, then the aggregated results of this causal-effect weight is taken as I.

Figure 1 is the graphical representation of the NCM tree corresponding to the company, where the dotted red lines symbolize the indeterminate weights.

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Figure 1: Graphical representation of the tree of NCM.

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Accordingly, the matrix $N(E)$ corresponding to the NCM shown in Figure 1 is the following:

$$
N(E) = \begin{pmatrix}
0 & 1 & 0 & -1 & 1 & 1 \\
0 & 0 & -1 & 1 & -1 & 0 \\
1 & 1 & 0 & -1 & 1 & 1 \\
0 & 1 & 1 & 0 & 0 & 1 \\
0 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0
\end{pmatrix}
$$

When the method for determining hidden patterns is applied, if $i_3$ is activated with the vector $A_1 = (0 \ 1 \ 0 \ 0 \ 0 \ 0)$ the following results are obtained:

$A_1N(E) = (0 \ 0 \ -1 \ 1 \ -1 \ 1) \rightarrow (0 \ 1 \ 0 \ 1 \ 0 \ 1) = A_2,$

$A_2N(E) = (0 \ 1 \ -2 \ 1 \ -1 \ 2) \rightarrow (0 \ 1 \ 0 \ 1 \ 0 \ 1) = A_2.$

We support the calculations on the software Octave 4.2.1, see [35].

If $i_3$ is activated with the vector $B_1 = (0 \ 0 \ 1 \ 0 \ 0 \ 0)$ the following results are obtained:

$B_1N(E) = (1 \ 1 \ 0 \ -1 \ 1 \ 1) \rightarrow (1 \ 1 \ 1 \ 0 \ 1 \ 1) = B_2,$

$B_2N(E) = (1 \ 1 + 2I \ 1 + 2I \ - 1 \ 2 + I \ 4I) \rightarrow (1 \ 1 \ 1 \ 0 \ 1 \ 1) = B_2.$

If $i_3$ is activated with the vector $D_1 = (0 \ 0 \ 0 \ 0 \ 0 \ 1)$ the following results are obtained:

$D_1N(E) = (0 \ 0 \ -1 \ 0 \ 0 \ 0) \rightarrow (0 \ 0 \ 0 \ 0 \ 0 \ 1) = D_2,$

$D_2N(E) = (0 \ 0 \ -1 \ 0 \ 0 \ 0) \rightarrow (0 \ 0 \ 0 \ 0 \ 0 \ 1) = D_2.$

The other indicators have not fixed points when the algorithm was applied.

Thus, the conclusion is that when “efficiency” state is activated then “Return on equity” and “Customer satisfaction” states are also activated. Whereas, when “indebtedness” state is activated, then, the other indicators except “Return on equity” are activated. Let us note that the state corresponding to “Customer satisfaction” is a non influential fixed point over the other indicators.

These results show which are the fundamental aspects that must be strengthened in the company. Firstly, efficiency is considered a key financial performance indicator of the company. To improve this indicator in a company dedicated to sales, it is necessary to strengthen some weak points that the company has, and not all of them are purely financial. For example, it is essential to have a sufficiently trained and motivated staff to increase sales, so that without changing the products to sale and their quality, the number of sales would increase, which is why the company must try to put their more talented salespeople in the most important places of the enterprise, and that will increase the number of customers loyal to the company.

On the other hand, the control of the indebtedness indicator is of vital importance, because it is necessary to find the precise balance between the debt that should be contracted so that it is sufficient to perform the business and also it should be payable in a reasonable amount and term, without ruining the business. This will also lead to customer satisfaction. That is to say, a sufficiently reasonable risk of the creditors towards the company will produce positive results for the quality of the service.

Additionally, it is shown that an improvement in any of the financial indicators of the company will produce greater customer satisfaction, which is logical. Therefore it will increase quality, and in turn this will increase the number of customers loyal to the company, which will contribute to improving the indicators, making this a loop with positive feedback.

**Conclusion**

This paper was dedicated to study the causal-effect relationship among the key performance indicators in the accounting process of the company "Zambrano Loor Naira Narcisa" located in the city of Santo Domingo, Ecuador. This is because the company has been shown some difficulties in its financial performance. Thus, we applied the neutrosophic cognitive map technique and our conclusion is that it is necessary to improve the company’s efficiency and to correctly deal with the indebtedness, consequently, these two aspects shall contribute to the progress of the enterprise.

In this paper we solved an accounting problem of a company supported by the dynamic analysis of neutrosophic cognitive maps, which is unusual, considering that the analysis of accounting is fundamentally based on objective measures, see [36, 37]. The use of this tool allowed the objective variables to be associated with a subjective aspect of the company such as “customer satisfaction”, which cannot be directly measured as an accounting variable.
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