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Distance and Similarity Measures of Interval Neutrosophic Soft Sets

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Distance and Similarity Measures of Interval Neutrosophic Soft Sets

Said Broumi, Irfan Deli and Florentin Smarandache

Abstract: In this paper several distance and similarity measures of interval neutrosophic soft sets are introduced. The measures are examined based on the geometric model, the set theoretic approach and the matching function. Finally, we have successfully shown an application of this similarity measure of interval neutrosophic soft sets.

Keywords: Distance, Similarity Measure, Neutrosophic set, Interval Neutrosohic sets, Interval Neutrosohic Soft sets.

1. Introduction

In 1965, fuzzy set theory was firstly given by Zadeh [2] which is applied in many real applications to handle uncertainty.Then,interval-valued fuzzy set [3],intuitionisticfuzzy set theory[4] and interval valued intuitionistic fuzzy sets[5] was introduced by Türkşen, Atanassov and Atanassov and Gargov, respectively. This theories can only handle incomplete information not the indeterminate information and inconsistent information which exists commonly in belief systems. So, Neutrsophic sets, founded by F.Smarandache [1], has capapility to deal with uncertainty, imprecise, incomplete and inconsistent information which exist in real world from philosophical point of view. The theory is a powerful tool formal framework which generalizes the concept of the classic set, fuzzy set [2], interval-valued fuzzy set [3], intuitionistic fuzzy set [4] interval-valued intuitionistic fuzzy set [5], and so on.

In the actual applications, sometimes, it is not easy to express the truth-membership, indeterminacy-membership and falsity-membership by crisp value, and they may be easier to expressed by interval numbers. The neutrosophic set and their operators need to be specified from scientific or engineering point of view. So, after the pioneering work of Smarandache, in 2005, Wang [6] proposed the notion of interval neutrosophic set (INS for short) which is another extension of neutrosophic sets. INS can be described by a membership interval, a non-membership interval and indeterminate interval, thus the interval value (INS) has the virtue of complementing NS, which is more flexible and practical than neutrosophic set. The sets provides a more reasonable mathematical framework to deal with indeterminate and inconsistent information.A lot of works about neutrosophic set theory have been studied by several researches [7,11,13,14,15,16,17,18,19,20].

In 1999, soft theory was introduced byMolodtsov[45] as a completely new mathematical toolfor modeling uncertainties. After Molodsov, based on the several operations on softsets introduced in [33,34,35,36,46],some more properties and algebra may be found in [32,34]. We can found some new conceptc ombined with fuzzy set in [28,29,37,39,42], interval-valued fuzzy set in [38], intuitionistic fuzzy set in [50], rough set in [43,47], interval-valued intuitionistic fuzzy set in [45], neutrosophic set in [8,9,27], interval neutrosophic set [31].

Also in some problems it is often needed to compare two sets such as fuzzy, soft, neutrosophic etc. Therefore, some researchers has studied of similarity measurement between fuzzysets in [24,48], interval valued fuzzy in [48], neutrosophic set in [23,26], interval neutrosophic set in [10,12].Recently similarity measure of softsets [40,49], intuitionistic fuzzy soft sets [30]was studied. Similarity measure between two sets such as fuzzy, soft has been defined by many authors which are based on both distances and matching function. The significant differences between similarity measure based on matching function and similarity measure based on distance is that if intersection of the two sets equals empty, then between similarity measure based on matching function the two sets is zero in but similarity measure based on distance may not be equal to zero. Distance-based measures are also popular because it is easier to calculate the intermediate distance between two fuzzy sets or soft sets. It’s mentioned in [40]. In this paper several distance and similarity measures of interval neutrosophic soft sets are introduced. The measures are examined based on the geometric model, the set-theoretic approach and the matching function. Finally, we give an application for similarity measures of interval neutrosophic soft sets

2. Prelimiairies

This section gives a brief overview of concepts of neutrosophic set [1], and interval valued neutrosophic set [6], soft set [41],neutrosophic soft set [27] and interval valued neutrosophic soft set [31]. More detailed explanations related to this subsection may be found in [8,9,27,31,36].

Definition 2.1[1]Neutrosophic Sets

Let X be an universe of discourse, with a generic element in X denoted by x, the neutrosophic (NS) set is an object having the form $A = \{< x: T_A(x), I_A(x), F_A(x)>, x \in X\}$, where the functions T, I, F : X→ $]^-0, 1^+[$ define respectively the degree of membership (or Truth) , the degree of indeterminacy, and the degree of non-membership (or Falsehood) of the element $x \in X$ to the set A with the condition.

$$^-0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+. \quad (1)$$

From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of $]^-0, 1^+[$. So instead of $]-0, 1^+[$ we need to take the interval [0, 1] for technical applications, because $]^-0, 1^+[$ will be difficult to apply in the real applications such as in scientific and engineering problems.

For two NS ,A_{NS} ={ <x , $T_A(x)$, $I_A(x)$, $F_A(x)$> | x ∈ X } (2)

And B_{NS}={<x , $T_B(x)$, $I_B(x)$, $F_B(x)$> | x ∈ X } the two relations are defined as follows:

(1)$A_{NS} \subseteq B_{NS}$if and only if $T_A(x) \leq T_B(x)$,$I_A(x) \geq I_B(x)$,$F_A(x) \geq F_B(x)$

(2)$A_{NS} = B_{NS}$ if and only if , $T_A(x) = T_B(x)$,$I_A(x) = I_B(x)$,$F_A(x) = F_B(x)$

Definition 2.2 [6] Interval Valued Neutrosophic Sets

Let X be a universe of discourse, with generic element in X denoted by x. An interval valued neutrosophic set (for short IVNS) A in X is characterized by truth-membership function $T_A(x)$, indeteminacy-membership function $I_A(x)$ and falsity-membership function $F_A(x)$. For each point x in X, we have that $T_A(x), I_A(x), F_A(x) \in [0,1]$.

For two IVNS , A_{IVNS} ={ <x , $[T_A^L(x), T_A^U(x)]$, $[I_A^L(x), I_A^U(x)]$, $[F_A^L(x), F_A^U(x)]$> | x ∈ X } (3)

And B_{IVNS} ={<x , $[T_B^L(x), T_B^U(x)]$, $[I_B^L(x), I_B^U(x)]$, $[F_B^L(x), F_B^U(x)]$ > | x ∈ X } the two relations are defined as follows:

(1)$A_{IVNS} \subseteq B_{IVNS}$if and only if $T_A^L(x) \leq T_B^L(x)$,$T_A^U(x) \leq T_B^U(x)$, $I_A^L(x) \geq I_B^L(x)$,$I_A^U(x) \geq I_B^U(x)$, $F_A^L(x) \geq F_B^L(x)$,$F_A^U(x) \geq F_B^U(x)$

(2)$A_{IVNS} = B_{IVNS}$ if and only if , $T_A^L(x_i) = T_B^L(x_i)$,$T_A^U(x_i) = T_B^U(x_i)$,$I_A^L(x_i) = I_B^L(x_i)$, $I_A^U(x_i) = I_B^U(x_i)$,$F_A^L(x_i) = F_B^L(x_i)$ and $F_A^U(x_i) = F_B^U(x_i)$ for any x ∈ X.

The complement of A_{IVNS} is denoted by A_{IVNS}^o and is defined by

A_{IVNS}^o={<x , $[F_A^L(x), F_A^U(x)]$> , $[1 - I_A^U(x), 1 - I_A^L(x)]$, $[T_A^L(x), T_A^U(x)]$| x ∈ X }

Definition 2.3 [41] Soft Sets

Let U be an initial universe set and E be a set of parameters. Let P(U) denotes the power set of U. Consider a nonempty set A, A ⊂ E. A pair (F, A) is called a soft set over U, where F is a mapping given by F: A → P (U).
It can be written a set of ordered pairs(F, A) = {(x, F (x)): x∈A}.

As an illustration, let us consider the following example.
Example 1 Suppose that U is the set of houses under consideration, say U = {h_1, h_2, . . ., h_5}. Let E be the set of some attributes of such houses, say E = {e_1, e_2, . . ., e_6}, where e_1, e_2, . . ., e_6 stand for the attributes “expensive”, “beautiful”, “wooden”, “cheap”, “modern”, and “in bad repair”, respectively.
In this case, to define a soft set means to point out expensive houses, beautiful houses, and so on. For example, the soft set (F,A) that describes the “attractiveness of the houses” in the opinion of a buyer, say Thomas, may be defined like this:
A={e_1,e_2,e_3,e_4,e_5};
F(e_1) = {h_2, h_3, h_5}, F(e_2) = {h_2, h_4}, F(e_3) = {h_1}, F(e_4) = U, F(e_5) = {h_3, h_5}.

Definition 2.4 Neutrosophic soft Sets [27]

Let U be an initial universe set and E be a set of parameters. Consider A⊆E. LetN(U) denotes the set of all neutrosophic sets of U. Thecollection (F,A) is termed to be the soft neutrosophic set over Udenoted by N, where F is amappinggivenbyF : A→P(U).

It can be written a set of ordered pairs N= {(x, F (x)): x∈A}.

Definition 2.5 Interval Valued Neutrosophic Soft Sets [31]

Let U be an universe set, IVN(U) denotes the set of all interval valued neutrosophic sets of U and E be a set of parameters that are describe the elements of U. Thecollection (K, E) is termed to be the interval valued neutrosophic soft sets (ivn-soft sets)over U denoted byϒ, where K is a mapping given by K : E→IVN(U).

It can be written a set of ordered pairs

ϒ= {(x, K(x)): x∈E}

Here, K which is interval valued neutrosophic sets, is called approximate function of the ivn-soft sets ϒ and K(x) is called x-approximate value of x ∈E.

Generally, K, L, M,... will be used as an approximate functions of ϒ, Ψ , Ω… respectively.

Note that the sets of all ivn-soft sets over U will be denoted by IVNS(U).

Then a relation form of ϒ is defined by R_K= { (r_K(e,u)/(e, u)) : u∈U, e∈E}
where
r_K: ExU→IV NS(U) and r_K (e_i,u_j)=$a_{ij}$for all e_i ∈ E and u_j ∈U.

Here,
1. ϒ is an ivn-soft subset of Ψ, denoted by ϒ ⋐ Ψ, if K(e) ⊆L(e) for alle∈E.
2. ϒ is an ivn-soft equals toΨ, denoted by ϒ = Ψ, if K(e)=L(e) for all e∈E.
3. The complement of ϒ is denoted by$ϒ^c$, and is defined by $ϒ^c$ = {(x, K^o (x)): x∈E}

As an illustration for ivn-soft, let us consider the following example.

Example 2. Suppose that U is the set of houses under consideration, say U = {h_1, h_2, h_3}. Let E be the set of some attributes of such houses, say E = {e_1, e_2, e_3, e_4}, where e_1, e_2, . . ., e_6 stand for the attributes “expensive”, “beautiful”, “wooden”, “cheap”, “modern”, and “in bad repair”, respectively.
In this case we give an ivn-soft set as;

ϒ= { (e_1,{<h_1,[0.5, 0.6], [0.6 ,0.7],[0.3,0.4]> ,<h_2, [0.5, 0.6], [0.6 ,0.7],[0.3,0.4]> ,

<h_3, [0.5, 0.6], [0.6,0.7],[0.3,0.4]> }),(e_2,{<h_1,[0.2, 0.3], [0.5 ,0.6],[0.3,0.6]> ,

<h_2, [0.4, 0.6], [0.2 ,0.3],[0.2,0.3]> , <h_3, [0.5, 0.6], [0.6 ,0.7],[0.3,0.4]> }),

(e_3,{<h_1,[0.3, 0.4], [0.1 ,0.5],[0.2,0.4]>,<h_2, [0.2, 0.5], [0.3,0.4],[0.4,0.5]>,

<h_3, [0.5, 0.6], [0.6,0.7],[0.3,0.4]> }),(e_4,{<h_1,[0.4, 0.6], [0.3 ,0.5],[0.3,0.4]>,

<h_2, [0.4, 0.6], [0.2 ,0.3],[0.2,0.3]>) ,<h_3, [0.3, 0.4], [0.2,0.7],[0.1,0.4]>}) }

Definition 2.6 (*Distance axioms)*

Let E be a set of parameters. Suppose that ϒ = <K,E>, Ψ = <L,E> and Ω = <M,E>; are three ivn-soft sets in universe U. Assume d is a mapping,

d :IVNS(U) x IVNS(U) ⟶ [0, 1].If d satisfies the following properties ((1)-(4)) :

(1) d (ϒ, Ψ) ≥ 0;

(2) d (ϒ, Ψ) = d (Ψ, ϒ) ;

(3) d (ϒ, Ψ) = 0 iff Ψ = ϒ;

(4) d (ϒ,Ψ) + d (Ψ, Ω) ≥ d (ϒ,Ω) .

Hence d(ϒ,Ψ) is called a distance measure between ivn-soft sets ϒ and Ψ.

Definition 2.7(*similarity axioms)*

A real function S: INS(U) x INS(U) ⟶ [0, 1] is named a similarity measure between two ivn-soft set ϒ=(K,E) and Ψ =(M,E) if S satisfies all the following properties:

(1) S (ϒ, Ψ) ∈ [0, 1];

(2) S(ϒ, ϒ)=S(Ψ, Ψ) = 1;

(3) S(ϒ, Ψ) = S(Ψ, ϒ);

(4) S (ϒ, Ω) ≤ S (ϒ,Ψ) and S (ϒ, Ω) ≤ S (Ψ,Ω) if ϒ ⊆ Ψ ⊆ Ω

Hence S(ϒ,Ψ) is called a similarity measure between ivn-soft setsϒ and Ψ.

For more details on the algebra and operations on interval neutrosophic set and soft set and interval neutrosophic soft set, the reader may refer to [5,6,8,9,12, 31,45,52].

3. Distance Measure between Interval Valued Neutrosophic Soft Sets

In this section, we present the definitions of the Hamming and Euclidean distances between ivn-soft sets and the similarity measures between ivn-soft sets based on the distances, which can be used in real scientific and engineering applications.

Based on Hamming distance between two interval neutrosophic set proposed by Ye[12] as follow:

$$D\,(A,B)=\frac{1}{6}\sum_{i=1}^{n}\Big[\big|\mathrm{T}_{\mathrm{A}}^{\mathrm{L}}(\mathrm{x_i})-\mathrm{T}_{\mathrm{B}}^{\mathrm{L}}(\mathrm{x_i})\big|+\big|\mathrm{T}_{\mathrm{A}}^{\mathrm{U}}(\mathrm{x_i})-\mathrm{T}_{\mathrm{B}}^{\mathrm{U}}(\mathrm{x_i})\big|+\big|\mathrm{I}_{\mathrm{A}}^{\mathrm{L}}(\mathrm{x_i})-\mathrm{I}_{\mathrm{B}}^{\mathrm{L}}(\mathrm{x_i})\big|+\big|\mathrm{I}_{\mathrm{A}}^{\mathrm{U}}(\mathrm{x_i})-\mathrm{I}_{\mathrm{B}}^{\mathrm{U}}(\mathrm{x_i})\big|+\big|\mathrm{F}_{\mathrm{A}}^{\mathrm{L}}(\mathrm{x_i})-\mathrm{F}_{\mathrm{B}}^{\mathrm{L}}\big|+\big|\mathrm{F}_{\mathrm{A}}^{\mathrm{L}}(\mathrm{x_i})-\mathrm{F}_{\mathrm{B}}^{\mathrm{U}}(\mathrm{x_i})\big|\Big]$$

We extended it to the case of ivn-soft sets as follows:

Definition 3.1 Let ϒ = (K,E) $=[a_{ij}]_{mxn}$ and Ψ = (M,E)$=[b_{ij}]_{mxn}$ be two ivn-soft sets.

$K(e) = \{<x, [\mathrm{T}_{\mathrm{K(e)}}^{\mathrm{L}}(x),\mathrm{T}_{\mathrm{K(e)}}^{\mathrm{U}}(x)], [\mathrm{I}_{\mathrm{K(e)}}^{\mathrm{L}}(x),\mathrm{I}_{\mathrm{K(e)}}^{\mathrm{U}}(x)], [\mathrm{F}_{\mathrm{K(e)}}^{\mathrm{L}}(x),\mathrm{T}_{\mathrm{K(e)}}^{\mathrm{U}}(x)]>: x \in X\}$

$M(e) = \{<x, [\mathrm{T}_{\mathrm{M(e)}}^{\mathrm{L}}(x),\mathrm{T}_{\mathrm{M(e)}}^{\mathrm{U}}(x)], [\mathrm{I}_{\mathrm{M(e)}}^{\mathrm{L}}(x),\mathrm{I}_{\mathrm{M(e)}}^{\mathrm{U}}(x)], [\mathrm{F}_{\mathrm{M(e)}}^{\mathrm{L}}(x),\mathrm{F}_{\mathrm{M(e)}}^{\mathrm{U}}(x)]>: x \in X\}$

Then we define the following distances for ϒ and Ψ

(1) The Hamming distance $d_{IVNSS}^{H}(\Upsilon, \Psi)$,

$$d_{IVNSS}^{H}(\Upsilon, \Psi)=\sum_{j=1}^{n}\sum_{i=1}^{m}\frac{\big[|\Delta_{ij}^{L}T|+|\Delta_{ij}^{U}T|+|\Delta_{ij}^{L}I|+|\Delta_{ij}^{U}I|+|\Delta_{ij}^{L}F|+|\Delta_{ij}^{U}F|\big]}{6}$$

Where $\Delta_{ij}^{L}T=\mathrm{T}_{\mathrm{K(e)}}^{\mathrm{L}}(\mathrm{x_i})-\mathrm{T}_{\mathrm{M(e)}}^{\mathrm{L}}(\mathrm{x_i})$,$\Delta_{ij}^{U}T=\mathrm{T}_{\mathrm{K(e)}}^{\mathrm{U}}(\mathrm{x_i})-\mathrm{T}_{\mathrm{M(e)}}^{\mathrm{U}}(\mathrm{x_i})$,$\Delta_{ij}^{L}I=\mathrm{I}_{\mathrm{K(e)}}^{\mathrm{L}}(\mathrm{x_i})-\mathrm{I}_{\mathrm{M(e)}}^{\mathrm{L}}(\mathrm{x_i})$,$\Delta_{ij}^{U}I=\mathrm{I}_{\mathrm{K(e)}}^{\mathrm{U}}(\mathrm{x_i})-\mathrm{I}_{\mathrm{M(e)}}^{\mathrm{U}}(\mathrm{x_i})$,$\Delta_{ij}^{L}F=\mathrm{F}_{\mathrm{K(e)}}^{\mathrm{L}}(\mathrm{x_i})-\mathrm{F}_{\mathrm{M(e)}}^{\mathrm{L}}(\mathrm{x_i})$ and $\Delta_{ij}^{U}F=\mathrm{F}_{\mathrm{K(e)}}^{\mathrm{U}}(\mathrm{x_i})-\mathrm{F}_{\mathrm{M(e)}}^{\mathrm{U}}(\mathrm{x_i})$

(2) The normalized Hamming distance $d_{IVNSS}^{nH}(\Upsilon, \Psi)$,

$$d_{IVNSS}^{nH}(\Upsilon, \Psi)=\frac{d_{IVNSS}^{H}(\Upsilon, \Psi)}{mn}$$

(3)The Euclidean distance$d_{IVNSS}^{E}(\Upsilon, \Psi)$,

$$d_{IVNSS}^{E}(\Upsilon, \Psi)=\sqrt{\sum_{j=1}^{n}\sum_{i=1}^{m}\frac{(\Delta_{ij}^{L}T)^2+(\Delta_{ij}^{U}T)^2+(\Delta_{ij}^{L}I)^2+(\Delta_{ij}^{U}I)^2+(\Delta_{ij}^{L}F)^2+(\Delta_{ij}^{U}F)^2}{6}}$$

Where $\Delta_{ij}^{L}T=\mathrm{T}_{\mathrm{K(e)}}^{\mathrm{L}}(\mathrm{x_i})-\mathrm{T}_{\mathrm{M(e)}}^{\mathrm{L}}(\mathrm{x_i})$,$\Delta_{ij}^{U}T=\mathrm{T}_{\mathrm{K(e)}}^{\mathrm{U}}(\mathrm{x_i})-\mathrm{T}_{\mathrm{M(e)}}^{\mathrm{U}}(\mathrm{x_i})$,$\Delta_{ij}^{L}I=\mathrm{I}_{\mathrm{K(e)}}^{\mathrm{L}}(\mathrm{x_i})-\mathrm{I}_{\mathrm{M(e)}}^{\mathrm{L}}(\mathrm{x_i})$,$\Delta_{ij}^{U}I=\mathrm{I}_{\mathrm{K(e)}}^{\mathrm{U}}(\mathrm{x_i})-\mathrm{I}_{\mathrm{M(e)}}^{\mathrm{U}}(\mathrm{x_i})$,$\Delta_{ij}^{L}F=\mathrm{F}_{\mathrm{K(e)}}^{\mathrm{L}}(\mathrm{x_i})-\mathrm{F}_{\mathrm{M(e)}}^{\mathrm{L}}(\mathrm{x_i})$ and $\Delta_{ij}^{U}F=\mathrm{F}_{\mathrm{K(e)}}^{\mathrm{U}}(\mathrm{x_i})-\mathrm{F}_{\mathrm{M(e)}}^{\mathrm{U}}(\mathrm{x_i})$

(4)The normalized Euclidean distance $d_{IVNSS}^{nE}(\Upsilon, \Psi)$,

$$d_{IVNSS}^{nE}(\Upsilon, \Psi)=\frac{d_{IVNSS}^{E}(\Upsilon, \Psi)}{\sqrt{mn}}$$

Here, it is clear that the following properties hold:

(1) $0 \le d_{IVNSS}^{H}(\Upsilon, \Psi) \le m\,n$ and $0 \le d_{IVNSS}^{nH}(\Upsilon, \Psi) \le 1$;
(2) $0 \le d_{IVNSS}^{E}(\Upsilon, \Psi) \le \sqrt{mn}$ and $0 \le d_{IVNSS}^{nE}(\Upsilon, \Psi) \le 1$;

Example 3. Assume that two interval neutrosophic soft sets ϒ and Ψ are defined as follows

K (e1) =(<x_1,[0.5, 0.6],[0.6 ,0.7],[0.3,0.4]>,<x_2, [0.5, 0.6], [0.6 ,0.7],[0.3,0.4]>), K (e2) =(<x_1,[0.2, 0.3], [0.5 ,0.6],[0.3,0.6]> ,<x_2, [0.4, 0.6], [0.2 ,0.3],[0.2,0.3]>),

M (e1) =(<x_1,[0.3, 0.4], [0.1 ,0.5],[0.2,0.4]> ,<x_2, [0.2, 0.5], [0.3 ,0.4],[0.4,0.5]>),

M (e2) =(<x_1,[0.4, 0.6], [0.3 ,0.5],[0.3,0.4]> ,<x_2, [0.3, 0.4], [0.2,0.7],[0.1,0.4]>),

$$d^{H}_{IVNSS}(\Upsilon\ ,\Psi) = \sum_{j=1}^{2}\sum_{i=1}^{2}\frac{\left[\left|\Delta^{\mathrm{L}}_{\mathrm{ij}}\mathrm{T}\right|+\left|\Delta^{\mathrm{U}}_{\mathrm{ij}}\mathrm{T}\right|+\left|\Delta^{\mathrm{L}}_{\mathrm{ij}}\mathrm{I}\right|+\left|\Delta^{\mathrm{U}}_{\mathrm{ij}}\mathrm{I}\right|+\left|\Delta^{\mathrm{L}}_{\mathrm{ij}}\mathrm{F}\right|+\left|\Delta^{\mathrm{U}}_{\mathrm{ij}}\mathrm{F}\right|\right]}{6}$$

$$=\frac{|0.5-0.3|+|0.6-0.4|+|0.6-0.1|+|0.7-0.5|+|0.3-0.2|+|0.4-0.4|}{6}$$

$$+\frac{|0.5-0.2|+|0.6-0.5|+|0.6-0.3|+|0.7-0.4|+|0.3-0.4|+|0.4-0.5|}{6}+$$

$$\frac{|0.2-0.4|+|0.3-0.6|+|0.5-0.3|+|0.6-0.5|+|0.3-0.3|+|0.6-0.4|}{6}+$$

$$\frac{|0.4-0.3|+|0.6-0.4|+|0.2-0.2|+|0.3-0.7|+|0.2-0.1|+|0.3-0.4|}{6}$$

$d^{H}_{IVNSS}(\Upsilon\ ,\Psi)$ =0.71

Theorem 3.2 The functions $d^{H}_{IVNSS}(\Upsilon\ ,\Psi)$, $d^{nH}_{IVNSS}(\Upsilon\ ,\Psi)$, $d^{E}_{IVNSS}(\Upsilon\ ,\Psi)$, $d^{nE}_{IVNSS}(\Upsilon\ ,\Psi)$: *IVNS*(*U*) Ÿ R^{+}given by Definition 3.1 respectively are metrics, where R^{+} is the set of all non-negative real numbers.

Proof. The proof is straightforward.

4. Generalized weighted distance measure between two interval valued nutrosophic soft sets.

Let A and B be two interval neutrosophic sets, then S.Broumi and F.Smarandache[11] proposed a generalized interval valued neutrosophic weighted distance measure between A and B as follows:

$$d_{\lambda}(A\,,B) = \left\{\frac{1}{6}\sum_{j=1}^{m}\quad \sum_{i=1}^{n} w_i\left[|T^{L}_{A}(x_i)-T^{L}_{B}(x_i)|^{\lambda}+|T^{U}_{A}(x_i)-T^{U}_{B}(x_i)|^{\lambda}+|I^{L}_{A}(x_i)-I^{L}_{B}(x_i)|^{\lambda}+|I^{U}_{A}(x_i)-I^{U}_{B}(x_i)|^{\lambda}+|F^{L}_{A}(x_i)-F^{L}_{B}(x_i)|^{\lambda}+|F^{U}_{A}(x_i)-F^{U}_{B}(x_i)|^{\lambda}\right]\right\}^{\frac{1}{\lambda}} \quad (4)$$

where

$\lambda>$ 0and $T^{L}_{A}(x_i)$,$T^{U}_{A}(x_i)$,$I^{L}_{A}(x_i)$,$I^{U}_{A}(x_i)$, ,$F^{L}_{A}(x_i)$,$F^{U}_{A}(x_i)$, $T^{L}_{B}(x_i)$,$T^{U}_{B}(x_i)$, $I^{L}_{B}(x_i)$,$I^{U}_{B}(x_i)$, $F^{L}_{B}(x_i)$,$F^{U}_{B}(x_i)$, ∈ [0, 1]

,we extended the above equation (4) distance to the case of interval valued neutrosophic soft set between ϒ and Ψ as follow:

$$d_{\lambda}(\Upsilon\,,\Psi) = \left\{\frac{1}{6}\sum_{j=1}^{m}\quad \sum_{i=1}^{n} w_i\left[\left|\Delta^{\mathrm{L}}_{\mathrm{ij}}\mathrm{T}\right|^{\lambda}+\left|\Delta^{\mathrm{U}}_{\mathrm{ij}}\mathrm{T}\right|^{\lambda}+\left|\Delta^{\mathrm{L}}_{\mathrm{ij}}\mathrm{I}\right|^{\lambda}+\left|\Delta^{\mathrm{U}}_{\mathrm{ij}}\mathrm{I}\right|^{\lambda}+\left|\Delta^{\mathrm{L}}_{\mathrm{ij}}\mathrm{F}\right|^{\lambda}+\left|\Delta^{\mathrm{U}}_{\mathrm{ij}}\mathrm{F}\right|^{\lambda}\right]\right\}^{\frac{1}{\lambda}}(5)$$

Where $\Delta^{\mathrm{L}}_{\mathrm{ij}}\mathrm{T}= \mathrm{T}^{\mathrm{L}}_{\mathrm{K(e)}}(\mathrm{x_i})-\mathrm{T}^{\mathrm{L}}_{\mathrm{M(e)}}(\mathrm{x_i})$,$\Delta^{\mathrm{U}}_{\mathrm{ij}}\mathrm{T}= \mathrm{T}^{\mathrm{U}}_{\mathrm{K(e)}}(\mathrm{x_i})-\mathrm{T}^{\mathrm{U}}_{\mathrm{M(e)}}(\mathrm{x_i})$,$\Delta^{\mathrm{L}}_{\mathrm{ij}}\mathrm{I}= \mathrm{I}^{\mathrm{L}}_{\mathrm{K(e)}}(\mathrm{x_i})-\mathrm{I}^{\mathrm{L}}_{\mathrm{M(e)}}(\mathrm{x_i})$,$\Delta^{\mathrm{U}}_{\mathrm{ij}}\mathrm{I}= \mathrm{I}^{\mathrm{U}}_{\mathrm{K(e)}}(\mathrm{x_i})-\mathrm{I}^{\mathrm{U}}_{\mathrm{M(e)}}(\mathrm{x_i})$,$\Delta^{\mathrm{L}}_{\mathrm{ij}}\mathrm{F}= \mathrm{F}^{\mathrm{L}}_{\mathrm{K(e)}}(\mathrm{x_i})-\mathrm{F}^{\mathrm{L}}_{\mathrm{M(e)}}(\mathrm{x_i})$ and $\Delta^{\mathrm{U}}_{\mathrm{ij}}\mathrm{F}= \mathrm{F}^{\mathrm{U}}_{\mathrm{K(e)}}(\mathrm{x_i})-\mathrm{F}^{\mathrm{U}}_{\mathrm{M(e)}}(\mathrm{x_i})$.

Normalized generalized interval neutrosophic distance is

$$d_{\lambda}^{n}(\Upsilon,\Psi) = \left\{\frac{1}{6n}\sum_{j=1}^{m} \sum_{i=1}^{n} w_i \left[\left|\Delta_{\mathrm{ij}}^{\mathrm{L}}\mathrm{T}\right|^{\lambda} + \left|\Delta_{\mathrm{ij}}^{\mathrm{U}}\mathrm{T}\right|^{\lambda} + \left|\Delta_{\mathrm{ij}}^{\mathrm{L}}\mathrm{I}\right|^{\lambda} + \left|\Delta_{\mathrm{ij}}^{\mathrm{U}}\mathrm{I}\right|^{\lambda} + \left|\Delta_{\mathrm{ij}}^{\mathrm{L}}\mathrm{F}\right|^{\lambda} + \left|\Delta_{\mathrm{ij}}^{\mathrm{U}}\mathrm{F}\right|^{\lambda}\right]\right\}^{\frac{1}{\lambda}} \quad (6)$$

If w=$\{\frac{1}{n},\frac{1}{n},\dots,\frac{1}{n}\}$,the Eq. (6) is reduced to the following distances:

$$d_{\lambda}(\Upsilon,\Psi) = \left\{\frac{1}{6}\sum_{j=1}^{m} \sum_{i=1}^{n} \left[\left|\Delta_{\mathrm{ij}}^{\mathrm{L}}\mathrm{T}\right|^{\lambda} + \left|\Delta_{\mathrm{ij}}^{\mathrm{U}}\mathrm{T}\right|^{\lambda} + \left|\Delta_{\mathrm{ij}}^{\mathrm{L}}\mathrm{I}\right|^{\lambda} + \left|\Delta_{\mathrm{ij}}^{\mathrm{U}}\mathrm{I}\right|^{\lambda} + \left|\Delta_{\mathrm{ij}}^{\mathrm{L}}\mathrm{F}\right|^{\lambda} + \left|\Delta_{\mathrm{ij}}^{\mathrm{U}}\mathrm{F}\right|^{\lambda}\right]\right\}^{\frac{1}{\lambda}} \quad (7)$$

$$d_{\lambda}(\Upsilon,\Psi) = \left\{\frac{1}{6n}\sum_{j=1}^{m} \sum_{i=1}^{n} \left[\left|\Delta_{\mathrm{ij}}^{\mathrm{L}}\mathrm{T}\right|^{\lambda} + \left|\Delta_{\mathrm{ij}}^{\mathrm{U}}\mathrm{T}\right|^{\lambda} + \left|\Delta_{\mathrm{ij}}^{\mathrm{L}}\mathrm{I}\right|^{\lambda} + \left|\Delta_{\mathrm{ij}}^{\mathrm{U}}\mathrm{I}\right|^{\lambda} + \left|\Delta_{\mathrm{ij}}^{\mathrm{L}}\mathrm{F}\right|^{\lambda} + \left|\Delta_{\mathrm{ij}}^{\mathrm{U}}\mathrm{F}\right|^{\lambda}\right]\right\}^{\frac{1}{\lambda}} \quad (8)$$

Particular case

(i) if λ =1 then the equation (7), (8) is reduced to the following hamming distance and normalized hamming distance between interval valued neutrosophic soft set

$$d_{IVNSS}^{H}(\Upsilon,\Psi) = \sum_{j=1}^{n}\sum_{i=1}^{m}\frac{\left[\left|\Delta_{\mathrm{ij}}^{\mathrm{L}}\mathrm{T}\right|+\left|\Delta_{\mathrm{ij}}^{\mathrm{U}}\mathrm{T}\right|+\left|\Delta_{\mathrm{ij}}^{\mathrm{L}}\mathrm{I}\right|+\left|\Delta_{\mathrm{ij}}^{\mathrm{U}}\mathrm{I}\right|+\left|\Delta_{\mathrm{ij}}^{\mathrm{L}}\mathrm{F}\right|+\left|\Delta_{\mathrm{ij}}^{\mathrm{U}}\mathrm{F}\right|\right]}{6} \quad (9)$$

$$d_{IVNSS}^{nH}(\Upsilon,\Psi) = \frac{d_{IVNSS}^{H}(\Upsilon,\Psi)}{mn} \quad (10)$$

(ii) If λ =2 then the equation (7) , (8) is reduced to the following Euclidean distance and normalized Euclidean distance between interval valued neutrosophic soft set

$$d_{IVNSS}^{E}(\Upsilon,\Psi) = \sqrt{\sum_{j=1}^{n}\sum_{i=1}^{m}\frac{(\Delta_{\mathrm{ij}}^{\mathrm{L}}\mathrm{T})^2+(\Delta_{\mathrm{ij}}^{\mathrm{U}}\mathrm{T})^2+(\Delta_{\mathrm{ij}}^{\mathrm{L}}\mathrm{I})^2+(\Delta_{\mathrm{ij}}^{\mathrm{U}}\mathrm{I})^2+(\Delta_{\mathrm{ij}}^{\mathrm{L}}\mathrm{F})^2+(\Delta_{\mathrm{ij}}^{\mathrm{U}}\mathrm{F})^2}{6}} \quad (11)$$

$$d_{IVNSS}^{NE}(\Upsilon,\Psi) = \frac{d_{IVNSS}^{E}(\Upsilon,\Psi)}{\sqrt{mn}} \quad (12)$$

5. Similarity Measures between Interval Valued Neutrosophic Soft Sets

This section proposes several similarity measures of interval neutrosophic soft sets.

It is well known that similarity measures can be generated from distance measures. Therefore, we may use the proposed distance measures to define similarity measures. Based on the relationship of similarity measures and distance measures, we can define some similarity measures between IVNSSs Υ = (K,E) and Ψ = (M,E) as follows:

5.1. Similarity measure based on the geometric distance model

Now for each $e_i \in$E , K(e_i) and M(e_i) are interval neutrosophic set. To find similarity between Υ and Ψ. We first find the similarity between K(e_i) and M(e_i).

Based on the distance measures defined above the similarity as follows:

$S_{IVNSS}^{H}(\Upsilon,\Psi)= \frac{1}{1+d_{IVNSS}^{H}(\Upsilon,\Psi)}$ and $S_{IVNSS}^{E}(\Upsilon,\Psi)=\frac{1}{1+d_{IVNSS}^{E}(\Upsilon,\Psi)}$

$S^{nH}_{IVNSS}(\Upsilon,\Psi)=\frac{1}{1+d^{nH}_{IVNSS}(\Upsilon,\Psi)}$ and $S^{nE}_{IVNSS}(\Upsilon,\Psi)=\frac{1}{1+d^{nE}_{IVNSS}(\Upsilon,\Psi)}$

Example 4 : Based on example 3,then

$S^{H}_{IVNSS}(\Upsilon,\Psi)=\frac{1}{1+0.71}=\frac{1}{1.71}=0.58$

Based on (4), we define the similarity measure between the interval valued neutrosophic soft sets $\Upsilon$and Ψ as follows:

$$\mathrm{S_{DM}}(\Upsilon,\Psi)= 1-\left\{\frac{1}{6n}\sum_{i=1}^{n}\left[\left|T^{L}_{K(e)}(x_i)-T^{L}_{M(e)}(x_i)\right|^{\lambda}+\left|T^{U}_{K(e)}(x_i)-T^{U}_{M(e)}(x_i)\right|^{\lambda}+\left|I^{L}_{K(e)}(x_i)-I^{L}_{M(e)}(x_i)\right|^{\lambda}+\left|I^{U}_{K(e)}(x_i)-I^{U}_{M(e)}(x_i)\right|^{\lambda}+\left|F^{L}_{K(e)}(x_i)-F^{L}_{M(e)}(x_i)\right|^{\lambda}+\left|F^{U}_{K(e)}(x_i)-F^{U}_{M(e)}(x_i)\right|^{\lambda}\right]\right\}^{\frac{1}{\lambda}} \quad (13)$$

Where $\lambda > 0$ *and* $\mathrm{S_{DM}}(\Upsilon,\Psi)$ is the degree of similarity of A and B .

If we take the weight of each element $x_i \in$ X into account, then

$$\mathrm{S^{w}_{DM}}(\Upsilon,\Psi)= 1-\left\{\frac{1}{6}\sum_{i=1}^{n}w_i\left[\left[\left|T^{L}_{K(e)}(x_i)-T^{L}_{M(e)}(x_i)\right|^{\lambda}+\left|T^{U}_{K(e)}(x_i)-T^{U}_{M(e)}(x_i)\right|^{\lambda}+\left|I^{L}_{K(e)}(x_i)-I^{L}_{M(e)}(x_i)\right|^{\lambda}+\left|I^{U}_{K(e)}(x_i)-I^{U}_{M(e)}(x_i)\right|^{\lambda}+\left|F^{L}_{K(e)}(x_i)-F^{L}_{M(e)}(x_i)\right|^{\lambda}+\left|F^{U}_{K(e)}(x_i)-F^{U}_{M(e)}(x_i)\right|^{\lambda}\right]\right]\right\}^{\frac{1}{\lambda}} \quad (14)$$

If each elements has the same importance ,i.e w =$\{\frac{1}{n},\frac{1}{n},\dots,\frac{1}{n}\}$, then (14) reduces to (13)

By definition 2.7 it can easily be known that $\mathrm{S_{DM}}(\Upsilon,\Psi)$ satisfies all the properties of definition..

$$\left[\left|\Delta^{L}_{ij}T\right|+\left|\Delta^{U}_{ij}T\right|+\left|\Delta^{L}_{ij}I\right|+\left|\Delta^{U}_{ij}I\right|+\left|\Delta^{L}_{ij}F\right|+\left|\Delta^{U}_{ij}F\right|\right]$$

Similarly , we define another similarity measure of Υ and Ψ as:

$$S(\Upsilon,\Psi) = 1 - \left[\frac{\sum_{i=1}^{n}\left(\left|\Delta^{L}_{ij}T\right|^{\lambda}+\left|\Delta^{U}_{ij}T\right|^{\lambda}+\left|\Delta^{L}_{ij}I\right|^{\lambda}+\left|\Delta^{U}_{ij}I\right|^{\lambda}+\left|\Delta^{L}_{ij}F\right|^{\lambda}+\left|\Delta^{U}_{ij}F\right|^{\lambda}\right)}{\sum_{i=1}^{n}\left(\left|T^{L}_{K}(x_i)+T^{L}_{M}(x_i)\right|^{\lambda}+\left|T^{U}_{K}(x_i)+T^{U}_{M}(x_i)\right|^{\lambda}+\left|I^{L}_{K}(x_i)+I^{L}_{M}(x_i)\right|^{\lambda}+\left|I^{U}_{K}(x_i)+I^{U}_{M}(x_i)\right|^{\lambda}+\left|F^{L}_{K}(x_i)+F^{L}_{M}(x_i)\right|^{\lambda}+\left|F^{U}_{K}(x_i)+F^{U}_{M}(x_i)\right|^{\lambda}\right)}\right]^{\frac{1}{\lambda}} \quad (15\ )$$

If we take the weight of each element $x_i \in$ X into account, then

$$S(\Upsilon,\Psi) = 1 - \left[\frac{\sum_{i=1}^{n} w_i\left(\left|\Delta_{ij}^{L}T\right|^{\lambda}+\left|\Delta_{ij}^{U}T\right|^{\lambda}+\left|\Delta_{ij}^{L}I\right|^{\lambda}+\left|\Delta_{ij}^{U}I\right|^{\lambda}+\left|\Delta_{ij}^{L}F\right|^{\lambda}+\left|\Delta_{ij}^{U}F\right|^{\lambda}\right)}{\sum_{i=1}^{n} w_i\left(\left|T_K^L(x_i)+T_M^L(x_i)\right|^{\lambda}+\left|T_K^U(x_i)+T_M^U(x_i)\right|^{\lambda}+\left|I_K^L(x_i)+I_M^L(x_i)\right|^{\lambda}+\left|I_K^U(x_i)+I_M^U(x_i)\right|^{\lambda}+\left|F_K^L(x_i)+F_M^L(x_i)\right|^{\lambda}+\left|F_K^U(x_i)+F_M^U(x_i)\right|^{\lambda}\right)}\right]^{\frac{1}{\lambda}} \quad (16)$$

This also has been proved that all the properties of definition are satisfied, If each elements has the same importance, and then (16) reduces to (15)

5.2.Similarity measure based on the interval valuedneutrosophic theoretic approach:

In this section, following the similarity measure between two interval neutrosophic sets defined by S.Broumi and F.Samarandache in [11], we extend this definition to interval valued neutrosophic soft sets.

Let $S_i(\Upsilon,\Psi)$ indicates the similarity between the interval neutrosophic soft sets Υ and Ψ .To find the similarity between Υ and Ψ first we have to find the similarity between their e - approximations. Let $S_i(\Upsilon,\Psi)$ denote the similarity between the two e_i- approximations K(e_i) and M(e_i).

Let Υ and Ψ be two interval valued neutrosophic soft **sets**, then we define a similarity measure between K(e_i) and M(e_i) as follows:

$$S_i(\Upsilon,\Psi)= \frac{\sum_{i=1}^{n}\{\min\{T_{K(e_i)}^L(x_j),T_{M(e_i)}^L(x_j)\}+\min\{T_{K(e_i)}^U(x_j),T_{M(e_i)}^U(x_j)\}+\min\{I_{K(e_i)}^L(x_j),I_{M(e_i)}^L(x_j)\}+\min\{I_{K(e_i)}^U(x_j),I_{M(e_i)}^U(x_j)\}+\min\{F_{K(e_i)}^L(x_j),F_{M(e_i)}^L(x_j)\}+\min\{F_{K(e_i)}^U(x_j),F_{M(e_i)}^U(x_j)\}}{\sum_{i=1}^{n}\{\max\{T_{K(e_i)}^L(x_j),T_{M(e_i)}^L(x_j)\}+\max\{T_{K(e_i)}^U(x_j),T_{M(e_i)}^U(x_j)\}+\max\{I_{K(e_i)}^L(x_j),I_{M(e_i)}^L(x_j)\}+\max\{I_{K(e_i)}^U(x_j),I_{M(e_i)}^U(x_j)\}+\max\{F_{K(e_i)}^L(x_j),F_{M(e_i)}^L(x_j)\}+\max\{F_{K(e_i)}^U(x_j),F_{M(e_i)}^U(x_j)\}} (17))$$

Then $S(\Upsilon,\Psi) = \max_{i} S_i(\Upsilon,\Psi)$

The similarity measure has the following proposition

Proposition 4.2

Let Υ and Ψ be interval valued neutrosophic soft sets then

i. **$0 \leq S(\Upsilon,\Psi) \leq 1$**
ii. $S(\Upsilon,\Psi) = S(\Psi,\Upsilon)$
iii. $S(\Upsilon,\Psi) = 1$ if $\Upsilon = \Psi$

$T_{K(e)}^L(x_i) = T_{M(e)}^L(x_i)$, $T_{K(e)}^U(x_i) = T_{M(e)}^U(x_i)$, $I_{K(e)}^L(x_i) = I_{M(e)}^L(x_i)$, $I_{K(e)}^U(x_i) = I_{M(e)}^U(x_i)$ and$F_{K(e)}^L(x_i) = F_{M(e)}^L(x_i)$, $F_{K(e)}^U(x_i) = F_{M(e)}^U(x_i)$for i=1,2,…., n

Iv .$\Upsilon \subseteq \Psi \subseteq \Omega \Rightarrow S(\Upsilon,\Psi) \leq \min(S(\Upsilon,\Psi), S(\Psi,\Omega))$

Proof. Properties (i) and(ii) follows from definition

(iii) it is clearly that if $\Upsilon = \Psi \Rightarrow S(\Upsilon,\Psi) = 1$

$\Rightarrow \sum_{i=1}^{n}\{\min\{T^L_{K(e)}(x_i), T^L_{M(e)}(x_i)\} + \min\{T^U_{K(e)}(x_i), T^U_{M(e)}(x_i)\} + \min\{I^L_{K(e)}(x_i), I^L_{M(e)}(x_i)\} + \min\{I^U_{K(e)}(x_i), I^U_{M(e)}(x_i)\} + \min\{F^L_{K(e)}(x_i), F^L_{M(e)}(x_i)\} + \min\{F^U_{K(e)}(x_i), F^U_{M(e)}(x_i)\}$
$= \sum_{i=1}^{n}\{\max\{T^L_{K(e)}(x_i), T^L_{M(e)}(x_i)\} + \max\{T^U_{K(e)}(x_i), T^U_{M(e)}(x_i)\} + \max\{I^L_{K(e)}(x_i), I^L_{M(e)}(x_i)\} + \max\{I^U_{K(e)}(x_i), I^U_{M(e)}(x_i)\} + \max\{F^L_{K(e)}(x_i), F^L_{M(e)}(x_i)\} + \max\{F^U_{K(e)}(x_i), F^U_{M(e)}(x_i)\}$

$\Rightarrow \sum_{i=1}^{n}\{[\min\{T^L_{K(e)}(x_i), T^L_{M(e)}(x_i)\} - \max\{T^L_{K(e)}(x_i), T^L_{M(e)}(x_i)\}] + [\min\{T^U_{K(e)}(x_i), T^U_{M(e)}(x_i)\} - \max\{T^U_{K(e)}(x_i), T^U_{M(e)}(x_i)\}] + [\min\{I^L_{K(e)}(x_i), I^L_{M(e)}(x_i)\} - \max\{I^L_{K(e)}(x_i), I^L_{M(e)}(x_i)\}] + [\min\{I^U_{K(e)}(x_i), I^U_{M(e)}(x_i)\} - \max\{I^U_{K(e)}(x_i), I^U_{M(e)}(x_i)\}] + [\min\{F^L_{K(e)}(x_i), F^L_{M(e)}(x_i)\} - \max\{F^L_{K(e)}(x_i), F^L_{M(e)}(x_i)\}] + [\min\{F^U_{K(e)}(x_i), F^U_{M(e)}(x_i)\} - \max\{F^U_{K(e)}(x_i), F^U_{M(e)}(x_i)]\} = 0$

Thus for each x,

$[\min\{T^L_{K(e)}(x_i), T^L_{M(e)}(x_i)\} - \max\{T^L_{K(e)}(x_i), T^L_{M(e)}(x_i)\}] = 0$

$[\min\{T^U_{K(e)}(x_i), T^U_{M(e)}(x_i)\} - \max\{T^U_{K(e)}(x_i), T^U_{M(e)}(x_i)\}] = 0$

$[\min\{I^L_{K(e)}(x_i), I^L_{M(e)}(x_i)\} - \max\{I^L_{K(e)}(x_i), I^L_{M(e)}(x_i)\}] = 0$

$[\min\{I^U_{K(e)}(x_i), I^U_{M(e)}(x_i)\} - \max\{I^U_{K(e)}(x_i), I^U_{M(e)}(x_i)\}] = 0$

$[\min\{F^L_{K(e)}(x_i), F^L_{M(e)}(x_i)\} - \max\{F^L_{K(e)}(x_i), F^L_{M(e)}(x_i)\}] = 0$

$[\min\{F^U_{K(e)}(x_i), F^U_{M(e)}(x_i)\} - \max\{F^U_{K(e)}(x_i), F^U_{M(e)}(x_i)]\} = 0$ holds

Thus $T^L_{K(e)}(x_i) = T^L_{M(e)}(x_i)$, $T^U_{K(e)}(x_i) = T^U_{M(e)}(x_i)$, $I^L_{K(e)}(x_i) = I^L_{M(e)}(x_i)$, $I^U_{K(e)}(x_i) = I^U_{M(e)}(x_i)$, $F^L_A(x_i) = F^L_B(x_i)$ and $F^U_{K(e)}(x_i) = F^U_{M(e)}(x_i) \Rightarrow \Upsilon = \Psi$

(iv) now we prove the last result.

Let $\Upsilon \subseteq \Psi \subseteq \Omega$,then we have

$T^L_{K(e)}(x_i) \leq T^L_{M(e)}(x_i) \leq T^L_C(x)$, $T^U_{K(e)}(x_i) \leq T^U_{M(e)}(x_i) \leq T^L_C(x)$, $I^L_{K(e)}(x) \geq I^L_{M(e)}(x) \geq I^L_C(x)$, $I^U_A(x) \geq I^U_{M(e)}(x) \geq I^U_C(x)$, $F^L_{K(e)}(x) \geq F^L_{M(e)}(x) \geq F^L_C(x)$, $F^U_{K(e)}(x) \geq F^U_{M(e)}(x) \geq F^U_C(x)$ for all $x \in X$ .Now

$T^L_K(x) + T^U_K(x) + I^L_K(x) + I^U_K(x) + F^L_M(x) + F^U_M(x) \geq T^L_K(x) + T^U_K(x) + I^L_K(x) + I^U_K(x) + F^L_C(x) + F^U_C(x)$

And

$T^L_M(x) + T^U_M(x) + I^L_M(x) + I^U_M(x) + F^L_K(x) + F^U_K(x) \geq T^L_C(x) + T^U_C(x) + I^L_C(x) + I^U_C(x) + F^L_K(x) + F^U_K(x)$

$$S(\Upsilon,\Psi) = \frac{T_K^L(x) + T_K^U(x) + I_K^L(x) + I_K^U(x) + F_M^L(x) + F_M^U(x)}{T_M^L(x) + T_M^U(x) + I_M^L(x) + I_M^U(x) + F_K^L(x) + F_K^U(x)} \geq \frac{T_K^L(x) + T_K^U(x) + I_K^L(x) + I_K^U(x) + F_C^L(x) + F_C^U(x)}{T_C^L(x) + T_C^U(x) + I_C^L(x) + I_C^U(x) + F_K^L(x) + F_K^U(x)} = S(\Upsilon,\Omega)$$

Again similarly we have

$$T_M^L(x) + T_M^U(x) + I_M^L(x) + I_M^U(x) + F_C^L(x) + F_C^U(x) \geq T_K^L(x) + T_K^U(x) + I_K^L(x) + I_K^U(x) + F_C^L(x) + F_C^U(x)$$

$$T_C^L(x) + T_C^U(x) + I_C^L(x) + I_C^U(x) + F_K^L(x) + F_K^U(x) \geq T_C^L(x) + T_C^U(x) + I_C^L(x) + I_C^U(x) + F_M^L(x) + F_M^U(x)$$

$$S(\Psi,\Omega) = \frac{T_M^L(x) + T_M^U(x) + I_M^L(x) + I_M^U(x) + F_C^L(x) + F_C^U(x)}{T_C^L(x) + T_C^U(x) + I_C^L(x) + I_C^U(x) + F_M^L(x) + F_M^U(x)} \geq \frac{T_K^L(x) + T_K^U(x) + I_K^L(x) + I_K^U(x) + F_C^L(x) + F_C^U(x)}{T_C^L(x) + T_C^U(x) + I_C^L(x) + I_C^U(x) + F_K^L(x) + F_K^U(x)} = S(\Upsilon,\Omega)$$

$\Rightarrow S(\Upsilon,\Omega) \leq \min(S(\Upsilon,\Psi)\,,\, S(\Psi,\Omega))$

Hence the proof of this proposition

If we take the weight of each element $x_i \in X$ into account, then

$$S(\Upsilon,\Psi) = \frac{\sum_{i=1}^{n} w_i\{\min\{T_A^L(x_i),T_B^L(x_i)\} + \min\{T_A^U(x_i),T_B^U(x_i)\} + \min\{I_A^L(x_i),I_B^L(x_i)\} + \min\{I_A^U(x_i),I_B^U(x_i)\} + \min\{F_A^L(x_i),F_B^L(x_i)\} + \min\{F_A^U(x_i),F_B^U(x_i)\}}{\sum_{i=1}^{n} w_i\{\max\{T_A^L(x_i),T_B^L(x_i)\} + \max\{T_A^U(x_i),T_B^U(x_i)\} + \max\{I_A^L(x_i),I_B^L(x_i)\} + \max\{I_A^U(x_i),I_B^U(x_i)\} + \max\{F_A^L(x_i),F_B^L(x_i)\} + \max\{F_A^U(x_i),F_B^U(x_i)\}}$$

(18)

Particularly ,if each element has the same importance, then (18) is reduced to (17) ,clearly this also satisfies all the properties of definition .

Theorem Υ = <K,E>, Ψ = <L,E> and Ω = <M,E>; are three ivn-soft sets in universe U such that $\Upsilon$is a ivn-soft subset of Ψ and $\Psi$is a soft subset of Ω then, $S(\Upsilon, \Omega) \leq S(\Psi, \Omega)$.

Proof. The proof is straightforward.

5.3. Similarity measure based for matching function by using interval neutrosophic sets:

Chen [24] and Chen et al. [25]) introduced a matching function to calculate the degree of similarity between fuzzy sets. In the following, we extend the matching function to deal with the similarity measure of interval valued neutrosophic soft sets.

Let $\Upsilon = A$ and Ψ=B be two interval valued neutrosophic soft sets, then we define a similarity measure between Υ and Ψ as follows:

$$S_{MF}(\Upsilon\,,\Psi) = \frac{\sum_{i=1}^{n}\left(\left(T_A^L(x_i)\cdot\ T_B^L(x_i)\right) + \left(T_A^U(x_i)\cdot\ T_B^U(x_i)\right) + \left(I_A^L(x_i)\cdot\ I_B^L(x_i)\right) + \left(I_A^U(x_i)\cdot\ I_B^U(x_i)\right) + \left(F_A^L(x_i)\cdot\ F_B^L(x_i)\right) + \left(F_A^U(x_i)\cdot\ F_B^U(x_i)\right)\right)}{\max\left(\sum_{i=}^{n}(T_A^L(x_i)^2 + T_A^U(x_i)^2 + I_A^L(x_i)^2 + I_A^U(x_i)^2 + F_A^L(x_i)^2 + F_A^U(x_i)^2),\ \sum_{i=}^{n}(T_B^L(x_i)^2 + T_B^U(x_i)^2 + I_B^L(x_i)^2 + I_B^U(x_i)^2 + F_B^L(x_i)^2 + F_B^U(x_i)^2)\right)}$$

$T_{K(e)}^{L}(x_i) = T_{M(e)}^{L}(x_i),\ T_{K(e)}^{U}(x_i) = T_{M(e)}^{U}(x_i)\ ,\ I_{K(e)}^{L}(x_i) = I_{M(e)}^{L}(x_i),\ I_{K(e)}^{U}(x_i) = I_{M(e)}^{U}(x_i)$ and $F_{K(e)}^{L}(x_i) = F_{M(e)}^{L}(x_i)\ ,\ F_{K(e)}^{U}(x_i) = F_{M(e)}^{U}(x_i)$

(19)

Proof.

i. $\mathbf{0 \le S_{MF}(\Upsilon, \Psi) \le 1}$

The inequality $S_{MF}(\Upsilon, \Psi) \ge 0$ is obvious. Thus, we only prove the inequality $S(\Upsilon, \Psi) \le 1$.

$$S_{MF}(\Upsilon,\Psi)=\sum_{\mathbf{i=1}}^{\mathbf{n}}\Big(\big(\mathrm{T}_{\mathrm{K(e)}}^{\mathrm{L}}(\mathrm{x_i})\cdot\ T_{M(e)}^{L}(\mathrm{x_i})\big)+\big(\mathrm{T}_{\mathrm{K(e)}}^{\mathrm{U}}(\mathrm{x_i})\cdot\ T_{M(e)}^{U}(\mathrm{x_i})\big)+\big(I_{K(e)}^{L}(x_i)\cdot\ I_{M(e)}^{L}(\mathrm{x_i})\big)+\big(I_{K(e)}^{U}(x_i)\cdot\ I_{M(e)}^{U}(\mathrm{x_i})\big)+\big(F_{K(e)}^{L}(x_i)\cdot\ F_{M(e)}^{L}(\mathrm{x_i})\big)+\big(F_{K(e)}^{U}(x_i)\cdot\ F_{M(e)}^{U}(\mathrm{x_i})\big)\Big)$$

$$=T_{K(e)}^{L}(x_1)\cdot\ T_{M(e)}^{L}(x_1)+T_{K(e)}^{L}(x_2)\cdot\ T_{M(e)}^{L}(x_2)+\ldots+T_{K(e)}^{L}(x_n)\cdot\ T_{M(e)}^{L}(x_n)+T_{K(e)}^{U}(x_1)\cdot\ T_{M(e)}^{U}(x_1)+T_{K(e)}^{U}(x_2)\cdot\ T_{M(e)}^{U}(x_2)+\ldots+T_{K(e)}^{U}(x_n)\cdot\ T_{M(e)}^{U}(x_n)+$$

$$I_{K(e)}^{L}(x_1)\cdot\ I_{M(e)}^{L}(x_1)+I_{K(e)}^{L}(x_2)\cdot\ I_{M(e)}^{L}(x_2)+\ldots+I_{K(e)}^{L}(x_n)\cdot\ I_{M(e)}^{L}(x_n)+I_{K(e)}^{U}(x_1)\cdot\ I_{M(e)}^{U}(x_1)+I_{K(e)}^{U}(x_2)\cdot\ I_{M(e)}^{U}(x_2)+\ldots+I_{K(e)}^{U}(x_n)\cdot\ I_{M(e)}^{U}(x_n)+$$

$$F_{K(e)}^{L}(x_1)\cdot\ F_{M(e)}^{L}(x_1)+F_{K(e)}^{L}(x_2)\cdot\ F_{M(e)}^{L}(x_2)+\ldots+F_{K(e)}^{L}(x_n)\cdot\ F_{M(e)}^{L}(x_n)+F_{K(e)}^{U}(x_1)\cdot\ F_{M(e)}^{U}(x_1)+F_{K(e)}^{U}(x_2)\cdot\ F_{M(e)}^{U}(x_2)+\ldots+F_{K(e)}^{U}(x_n)\cdot\ F_{M(e)}^{U}(x_n)+$$

According to the Cauchy–Schwarz inequality:

$$(x_1\cdot y_1+x_2\cdot y_2+\cdots+x_n\cdot y_n)^2\le({x_1}^2+{x_2}^2+\cdots+{x_n}^2)\ \cdot({y_1}^2+{y_2}^2+\cdots+{y_n}^2)$$

where $(x_1, x_2, é\ , x_n) \in R^n$*and* $(y_1, y_2, é\ , y_n)\ \in R^n$ *we can obtain*

$$[S_{MF}(\Upsilon,\Psi)]^2\le\sum_{\boldsymbol{i=1}}^{\boldsymbol{n}}\big(T_{K(e)}^{L}(x_i)^2+T_{K(e)}^{U}(x_i)^2+I_{K(e)}^{L}(x_i)^2+I_{K(e)}^{U}(x_i)^2+F_{K(e)}^{L}(x_i)^2+F_{K(e)}^{U}(x_i)^2\big)\cdot$$

$$\sum_{\boldsymbol{i=1}}^{\boldsymbol{n}}\big(T_{M(e)}^{L}(x_i)^2+T_{M(e)}^{U}(x_i)^2+I_{M(e)}^{L}(x_i)^2+I_{M(e)}^{U}(x_i)^2+F_{M(e)}^{L}(x_i)^2+F_{M(e)}^{U}(x_i)^2\big)=\mathrm{S}(\Upsilon,\Upsilon)\cdot\mathrm{S}(\Psi,\Psi)$$

Thus $S_{MF}(\Upsilon,\Psi)\le[S(\Upsilon,\Upsilon)]^{\frac{1}{2}}\cdot[S(\Psi,\Psi)]^{\frac{1}{2}}$

Then $S_{MF}(\Upsilon,\Psi)\le\max\{\mathrm{S}(\Upsilon,\Upsilon),\ \mathrm{S}(\Psi,\Psi)]$

Therefore,$S_{MF}(\Upsilon,\Psi)\le 1$.

If we take the weight of each element $x_i \in$ X into account, then

$$S_{MF}^{w}(\Upsilon,\Psi)=\frac{\sum_{\boldsymbol{i=1}}^{\boldsymbol{n}}w_i\Big(\big(T_A^L(x_i)\cdot\ T_B^L(x_i)\big)+\big(T_A^U(x_i)\cdot\ T_B^U(x_i)\big)+\big(I_A^L(x_i)\cdot\ I_B^L(x_i)\big)+\big(I_A^U(x_i)\cdot\ I_B^U(x_i)\big)+\big(F_A^L(x_i)\cdot\ F_B^L(x_i)\big)+\big(F_A^U(x_i)\cdot\ F_B^U(x_i)\big)\Big)}{\max(\sum_{i=}^{n}w_i\ (\mathrm{T_A^L(x_i)^2+T_A^U(x_i)^2+\ I_A^L(x_i)^2+I_A^U(x_i)^2+\ F_A^L(x_i)^2+F_A^U(x_i)^2}),\ \ \sum_{i=}^{n}w_i\ (\mathrm{T_B^L(x_i)^2+T_B^U(x_i)^2+\ I_B^L(x_i)^2+I_B^U(x_i)^2+\ F_B^L(x_i)^2+F_B^U(x_i)^2}))}\qquad(20)$$

Particularly ,if each element has the same importance, then (20) is reduced to (19) ,clearly this also satisfies all the properties of definition .

The larger the value of S(Υ,Ψ) ,the more the similarity between Υ and Ψ.

Majumdar and Samanta [40] compared the properties of the two measures of soft sets and proposed α-similar of two soft sets. In the following, we extend to interval valued neutrosophic soft sets as;

Let $X_{\Upsilon,\Psi}$ denote the similarity measure between two ivn-soft sets Υ and Ψ Table compares the properties of the two measures of similarity of ivn-soft soft sets discussed here. It can be seen that most of the properties are common to both.and few differences between them do exist.

Property	S (**geometric**)	*S* (**theoretic**)	*S* (**matching)**
S(Υ, Ψ) = S(Ψ, Υ)	Yes	Yes	Yes
0 ÒS(Υ, Ψ) Ò1	Yes	Yes	Yes
Υ = Ψ⇒S(Υ, Ψ) = 1	Yes	No	?
S(Υ, Ψ) = 1 ⇒Υ = Ψ	Yes	Yes	?
Υ ∩ Ψ = ∅⇒S(Υ, Ψ) = 0	No	No	?
S(Υ, Υ^c) = 0	No	No	?

Definition A relation α≈ on IVNS(U), called α-similar, as follows:two inv-soft sets Υ and Ψ are said to be α-similar, denoted as Υα≈Ψ iff S(Υ, Ψ) ≥ α for α ∈(0, 1).

Here, we call the two ivn-soft sets significantly similar if S(Υ, Ψ) >0.5

Lemma [40] α≈is reflexive and symmetric, but not transitive.

Majumdar and Samanta [40] introduced a technique of similarity measure of two soft sets which can be applied to detect whether an ill person is suffering from a certain disease or not. In a example, they was tried to estimate the possibility that an ill person having certain visible symptoms is suffering from pneumonia.Therefore, they were given an example by using similarity measure of two soft sets. In the following application, similarly we will try for ivn-soft sets in same example. Some of it is quoted from [40] .

6. An Application

This technique of similarity measure of two inv-soft sets can be applied to detect whether an ill person is suffering from a certain disease or not. In the followinge xample, we will try to estimate the possibility that an ill person having certain visible symptoms is suffering from pneumonia. For this, we first construct a model inv-soft set for pneumonia and the inv-soft set for the ill person. Next we find the similarity measure of these two sets. If they are significantly similar, then we conclude that the person is possibly suffering from pneumonia.

Let our universal set contain only two elements yes and no, i.e. *U* = {yes=h_1, no=h_2}. Here the set of parameters *E* is the set of certain visible symptoms. Let E = {e1, e2, e3, e4, e5, e6}, where *e*1 = high body temperature, *e*2 = cough with chest congestion, *e*3 = body ache, *e*4 = headache, *e*5 = loose motion, and *e*6 = breathing trouble. Our model inv-soft for pneumoniaΥis given below and this can be prepared with the help of a medical person:

Υ= { (e_1,{<h_1,[0.5, 0.6], [0.6 ,0.7],[0.3,0.4]> ,<h_2, [0.5, 0.6], [0.6 ,0.7],[0.3,0.4]>}),

(e_2,{< h_1, [0.5, 0.6], [0.6,0.7],[0.3,0.4]>,<h_2,[0.2, 0.3], [0.5 ,0.6],[0.3,0.6]>}),

(e_3,{<h_1, [0.4, 0.6], [0.2 ,0.3],[0.2,0.3]> , <h_2,[0.5, 0.6], [0.6 ,0.7],[0.3,0.4]> }),

(e_4,{<h_1,[0.3, 0.4], [0.1 ,0.5],[0.2,0.4]>,<h_2, [0.2, 0.5],[0.3,0.4],[0.4,0.5]> }),

(e_5,{<h_1,[0.5, 0.6],[0 .6,0.7],[0.3,0.4]>,<h_2,[0.4, 0.6], [0.3 ,0.5],[0.3,0.4]>}),

(e_6,{<h_1, [0.4, 0.6], [0.2 ,0.3],[0.2,0.3]>) , <h_2, [0.3, 0.4], [0.2,0.7],[0.1,0.4]>}) }

Now the ill person is having fever, cough and headache. After talking to him, we can construct his ivn-soft Ψ as follows:

Ψ = { (e_1,{<h_1,[0.1, 0.2], [0.1 ,0.2],[0.8,0.9]> ,<h_2, [0.1, 0.2], [0.0 ,0.1],[0.8,0.9]>}),

(e_2,{< h_1, [0.8, 0.9], [0.1,0.2],[0.2,0.9]> ,<h_2,[0.8, 0.9], [0.2 ,0.9],[0.8,0.9]>}),

(e_3,{<h_1,[0.1, 0.9], [0.7 ,0.8],[0.6,0.9]> , <h_2, [0.1, 0.8], [0.6 ,0.7],[0.8,0.7]> }),

(e_4,{<h_1,[0.8, 0.8], [0.1 ,0.9],[0.3,0.3]>,<h_2, [0.6, 0.9],[0.5,0.9],[0.8,0.9]> }),

(e_5,{<h_1, [0.3, 0.4],[0 .1,0.2],[0.8,0.8]> ,<h_2,[0.5, 0.9], [0.8 ,0.9],[0.1,0.2]>}),

(e_6,{<h_1, [0.1, 0.2], [0.8 ,0.9],[0.7,0.7]>) , <h_2, [0.7, 0.8], [0.8,0.9],[0.0,0.4]>}) }

Then we find the similarity measure of these two ivn-soft sets as:
$S^{H}_{IVNSS}(\Upsilon ,\Psi)= \frac{1}{1+d^{H}_{IVNSS}(\Upsilon ,\Psi)} =0.17$

Hence the two ivn-softsets, i.e. two symptoms Υ and $\Psi$are not significantly similar. Therefore, we conclude that the person is not possibly suffering from pneumonia. A person suffering from the following symptoms whose corresponding ivn-soft set Ω is given below:

Ω= { (e_1,{<h_1,[0.5, 0.7], [0.5 ,0.7],[0.3,0.5]> ,<h_2, [0.6, 0.6], [0.6 ,0.8],[0.3,0.5]>}),

(e_2,{< h_1, [0.5, 0.7], [0.5,0.7],[0.3,0.4]> ,<h_2,[0.2, 0.4], [0.6 ,0.7],[0.2,0.7]>}),

(e_3,{<h_1, [0.4, 0.7], [0.2 ,0.2],[0.1,0.3]> , <h_2, [0.4, 0.8], [0.2 ,0.8],[0.2,0.8]> }),

(e_4,{<h_1,[0.3, 0.4], [0.1 ,0.5],[0.2,0.6]>,<h_2, [0.2, 0.5],[0.3,0.4],[0.4,0.5]> }),

(e_5,{<h_1, [0.5, 0.6],[0 .6,0.7],[0.3,0.4]> ,<h_2,[0.4, 0.6], [0.3 ,0.5],[0.1,0.8]>}),

(e_6,{<h_1, [0.4, 0.7], [0.3 ,0.7],[0.2,0.8]>) , <h_2, [0.5, 0.2], [0.3,0.5],[0.2,0.5]>}) }

Then,

$S^{H}_{IVNSS}(\Upsilon ,\Omega)= \frac{1}{1+d^{H}_{IVNSS}(\Upsilon ,\Psi)} = 0.512$

Here the two ivn-soft sets, i.e. two symptoms Υ and Ω are significantly similar. Therefore, we conclude that the person is possibly suffering from pneumonia. This is only a simple example to show the possibility of using this method for diagnosis of diseases which could be improved by incorporating clinical results and other competing diagnosis.

Conclusions

In this paper we have defined, for the first time, the notion of distance and similarity measures between two interval neutrosophic soft sets. We have studied few properties of distance and similarity measures. The similarity measures have natural applications in the field of pattern recognition, feature extraction, region extraction, image processing, coding theory etc. The results of the proposed similarity measure and existing similarity measure are compared. We also give an application for similarity measures of interval neutrosophic soft sets.

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