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Shawkat Alkhazaleh

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Shawkat Alkhazaleh

Department of Mathematics, Faculty of Science and Information Technology, Jadara University, Irbid, Jordan

shmk79@gmail.com

Abstract. In 1995, Smarandache initiated the theory of neutrosophic set as new mathematical tool for handling problems involving imprecise, indeterminacy, and inconsistent data. Molodtsov initiated the theory of soft set as a new mathematical tool for dealing with uncertainties, which traditional mathematical tools cannot handle. He has showed several applications of this theory for solving many practical problems in economics, engineering, social science, medical science, etc. In 2017 Smarandache initiated the theory of Plithogenic Set and their properties. He also generalized the soft set to the hypersoft set by transforming the function F into a multi-attribute function and introduced the hybrids of Crisp, Fuzzy, Intuitionistic Fuzzy, Neutrosophic, and Plithogenic Hypersoft Set. In this research, for the first time we define the concept of Plithogenic soft set, and give it some generalizations and study some of its operations. Furthermore, We give examples for these concepts and operations. Finally, the similarities between two Plithogenic soft sets are also given.

Keywords: Soft set; Neutrosophic set; Neutrosophic soft set; Plithogenic set; Plithogenic soft set.

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1. Introduction

In our life not everything non vague or certain where many things around us are surrounded by uncertainty and vagueness , Zadeh [13] was successfully fulfilled the need to represent uncertain data by introducing the concept of fuzzy sets. Also Atanassov [4] extend Zadehs notion of fuzzy set to Intuitionistic fuzzy sets which proved to be a better model of uncertainty. The words neutrosophy and neutrosophic were introduced for the first time by Smarandache [9,10] as a more general platform, which extends the concepts of the fuzzy set and intuitionistic fuzzy set. In 1999 Molodtsov [8] proposed a parameterised family of sets named ”soft set”, to deal with uncertainty in a parametric manner. Smarandache defined a concept of plithogenic set, where a plithogenic set P is a set whose elements are characterized by one or more attributes, and each attribute may have many values. Each attribute's value v has a corresponding degree of appurtenance $d(x, v)$ of the element x, to the set P, with respect to some given criteria. In order to obtain better accuracy for the plithogenic aggregation operators, a contradiction

(dissimilarity) degree is defined between each attribute value and the dominant (most important) attribute value. However, there are cases when such dominant attribute value may not be taking into consideration or may not exist [therefore it is considered zero by default], or there may be many dominant attribute values. In such cases, either the contradiction degree function is suppressed, or another relationship function between attribute values should be established. The plithogenic aggregation operators (intersection, union, complement, inclusion and equality) are based on contradiction degrees between attributes values and the first two are linear combinations of the fuzzy operators t-norm and t-conorm. Plithogenic set is a generalization of the crisp set, fuzzy set, intuitionistic fuzzy set, and neutrosophic set, since these four types of sets are characterized by a single attribute value (appurtenance): which has one value (membership)for the crisp set and fuzzy set, two values (membership, and non-membership)for intuitionistic fuzzy set, or three values(membership, non-membership, and indeterminacy) for neutrosophic set. A plithogenic set, in general, may have elements characterized by attributes with four or more attributes. In 2019 Abdel-Basset et al. [1] suggested an approach constructed on the connotation of plithogenic theory technique to come up with a methodical procedure to assess the infirmary serving under a framework of plithogenic theory. In this research, they gave some definitions of the plithogenic environment, which is more general and comprehensive than fuzzy, intuitionistic fuzzy and neutrosophic ones. Abdel-Basset et al. [3] proposed method to increase the accuracy of the evaluation. This method is a combination of quality function deployment with plithogenic aggregation operations. They applied the aggregation operation to aggregate the decision makers opinions of requirements that are needed to evaluate the supply chain sustainability and the evaluation metrics based on the requirements. Also they applied the aggregation operation to aggregate the evaluation of information gathering difficulty. In 2020 Abdel-Basset and Rehab [2] proposed a methodology as a combination of plithogenic multi-criteria decision-making approach based on the Technique in Order of Preference by Similarity to Ideal Solution and Criteria Importance Through Inter-criteria Correlation methods to estimation of sustainable supply chain risk management.

2. **Preliminary**

In this section, we recall some definitions and properties required in this paper.

Definition 2.1. [9,10] A neutrosophic set A on the universe of discourse X is defined as

$$A = \{< x; T_A(x); I_A(x); F_A(x) >; x \in X\}$$

where T; I; $F : X \rightarrow]^{-}0; 1^{+}[$ and $^{-}0 \leqslant T_A(x) + I_A(x) + F_A(x) \leq 3^{+}$.

Remark 2.2. In a more general way, the summation can be less than 1 (for incomplete neutrosophic information), equal to 1 (for complete neutrosophic information), or greater than 1 (for paraconsistent/conflicting neutrosophic information).

Molodtsov defined soft set in the following way. Let U be a universe and E be a set of parameters. Let $P(U)$ denote the power set of U and $A \subseteq E$.

Definition 2.3. [8] A pair (F, A) is called a *soft set* over U, where F is a mapping

$$F : A \to P(U).$$

In other words, a soft set over U is a parameterized family of subsets of the universe U. For $\varepsilon \in A, F(\varepsilon)$ may be considered as the set of ε-approximate elements of the soft set (F, A).

Definition 2.4. [6] Let $P(U)$ denotes the set of all fuzzy sets of U. Let $A_i \subseteq E$ .A pair$(F_i, A_i)$is called a fuzzy soft set over U , where F_i is a mapping given by $F_i : A_i \longrightarrow P(U)$

Definition 2.5. [7] Let U be an initial universal set and let E be set of parameters. Let $P(U)$ denotes the set of all intuitionistic fuzzy sets of U. A pair (F, A) is called an intuitionistic fuzzy soft set over U if F is a mapping given by $F : A \longrightarrow P(U)$. We write an Intuitionistic fuzzy soft set shortly as IF soft set

Definition 2.6. [5] Let U be an initial universe set and E be a set of parameters. Consider $A \subset E$. Let $P(U)$ denotes the set of all neutrosophic sets of U. The collection (F, A) is termed to be the neutrosophic soft set over U, where F is a mapping given by $F : A \to P(U)$.

3. **Formal Definition of Single (Uni-Dimensional) Attribute Plithogenic Set**

In this section we recall the definition of plithogenic set given by Smarandache [11, 12] and some definitions related to this concept as follows:
Let U be a universe of discourse, and P a non-empty set of elements, $P \subseteq U$.

3.1. *Attribute Value Spectrum*

Definition 3.1. Let $\mathfrak{A}$ be a non-empty set of uni-dimensional attributes $\mathfrak{A} = \{\alpha_1, \alpha_2, ..., \alpha_m\}$, $m \geq 1$; and $\alpha_i \in \mathfrak{A}$ be a given attribute whose spectrum of all possible values (or states) is the non-empty set S, where S can be a finite discrete set, $S = \{s_1, s_2, ..., s_l\}$,$1 \leq l \leq \infty$, or infinitely countable set $S = \{s_1, s_2, ..., s_\infty\}$, or infinitely uncountable (continuum) set $S =]a, b[, a < b$, where $]...[$ is any open, semi-open, or closed interval from the set of real numbers or from other general set.

3.2. *Attribute Value Range*

Definition 3.2. Let V be a non-empty subset of S, where V is the range of all attribute's values needed by the experts for their application. Each element $x \in P$ is characterized by all attribute's values in $V = \{v_1, v_2, ..., v_n\}$, for $n \geq 1$.

3.3. *Dominant Attribute Value*

Definition 3.3. Into the attributes value set V, in general, there is a dominant attribute value, which is determined by the experts upon their application. Dominant attribute value is defined as the most important attribute value that the experts are interested in.

Remark 3.4. There are cases when such dominant attribute value may not be taking into consideration or not exist, or there may be many dominant (important) attribute values - when different approach should be employed.

3.4. *Attribute Value Truth-Value Degree Function*

Definition 3.5. The attribute value truth-value degree function is: $\forall P \in U, d : U \times V \longrightarrow \wp([0,1]^z)$, so $d(P, v)$ is a subset of $[0,1]^z$, where $\wp([0,1]^z)$ is the power set of the $[0,1]^z$ such that
$z = F$ (for fuzzy degree of truth-value),
$z = IF$ (for intuitionistic fuzzy degree of truth-value), or
$z = N$ (for neutrosophic degree de truth-value).

3.5. *Attribute Value Contradiction Degree Function*

Definition 3.6. The attribute value contradiction degree function between any two attribute values v_1 and v_2, denoted by $c(v_1, v_2)$ is a function define by $c : V \times V \longrightarrow [0,1]$ such that the cardinal $|V| \geq 1$ and satisfying the following axioms:

(1) $c(v_1, v_1) = 0$, the contradiction degree between the same attribute values is zero;
(2) $c(v_1, v_2) = c(v_2, v_1)$, commutativity.

3.6. *plithogenic set*

Definition 3.7. Let U be a universe of discourse, and P a non-empty set of elements, $P \subseteq U$. a is a (multi-dimensional in general) attribute, V is the range of the attributes values, d is the degree of appurtenance of each element xs attribute value to the set P with respect to some given criteria ($x \in P$),and c stands for the degree of contradiction between attribute values. Then (P, a, V, d, c) is called a plithogenic set.

Remark 3.8. (1) d may stand for d_F , d_{IF} or d_N, when dealing with fuzzy degree of appurtenance, intuitionistic fuzzy degree of appurtenance, or neutrosophic degree of appurtenance respectively of an element x to the plithogenic set P;

(2) c may stand for c_F , c_{IF} or c_N, when dealing with fuzzy degree of contradiction, intuitionistic fuzzy degree of contradiction, or neutrosophic degree of contradiction between attribute values respectively;

(3) The functions $d(.,.)$ and $c(.,.)$ are defined in accordance with the applications the experts need to solve;

(4) One uses the notation: $x(d(x,V))$, where $d(x,v) = \{d(x,v)\forall v \in V\}$, $\forall x \in P$;

The plithogenic set is an extension of all: crisp set, fuzzy set, intuitionistic fuzzy set, and neutrosophic set. In this Paper we will study the plithogenic set as an extension of fuzzy set, intuitionistic fuzzy set, and neutrosophic set.

In **Single-Valued Fuzzy Set (SVFS)**, the attribute is $\alpha =$ "*appurtenance*"; the set of attribute values

$V = \{T\}$, whose cardinal $|V| = 1$; the dominant attribute value $= T$; the attribute value appurtenance degree function: $d : P \times V \longrightarrow [0,1]$, $d(x,T) \in [0,1]$

and the attribute value contradiction degree function:

$c : V \times V \longrightarrow [0,1]$, $c(T,T) = 0$,

In **Single-Valued intuitionistic fuzzy set (SVIFS)**, the attribute is $\alpha =$ "*appurtenance*"; the set of attribute values $V = \{T, F\}$, whose cardinal $|V| = 2$;

the dominant attribute value $= T$; the attribute value appurtenance degree function:

$d : P \times V \longrightarrow [0,1]$, $d(x,T) \in [0,1]$, $d(x,F) \in [0,1]$ with $0 \leq d(x,T) + d(x,F) \leq 1$; and

the attribute value contradiction degree function: $c : V \times V \longrightarrow [0,1]$, $c(T,T) = c(F,F) = 0$, $c(T,F) = 1$.

In **Single-Valued Neutrosophic Set (SVNS)**, the attribute is $\alpha =$ "*appurtenance*"; the set of attribute values

$V = \{T, I, F\}$, whose cardinal $|V| = 3$; the dominant attribute value $= T$; the attribute value appurtenance degree function:

$d : P \times V \longrightarrow [0,1]$, $d(x,T) \in [0,1]$, $d(x,I) \in [0,1]$, $d(x,F) \in [0,1]$,

with $0 \leq d(x,T) + d(x,I) + d(x,F) \leq 3$;

and the attribute value contradiction degree function:

$c : V \times V \longrightarrow [0,1]$,

$c(T,T) = c(I,I) = c(F,F) = 0$,

$c(T,F) = 1$,

$c(T,I) = c(F,I) = 0.5$.

4. **Plithogenic Intersection**

In this section we recall the definition of plithogenic intersection over three cases: fuzzy, intuitionistic fuzzy and neutrosophic set which are defined for the first time by Smarandache in 2017 and 2018 [11, 12].

4.1. *Plithogenic Fuzzy Intersection*

Definition 4.1. Let $U = \{u_1, u_2, ..., u_n\}$. $A = \left\{\frac{u_1}{\langle\mu(u_1)}, \frac{u_2}{\mu(u_2)}, ..., \frac{u_n}{\mu(u_n)}\right\}$, and $B = \left\{\frac{u_1}{\nu(u_1)}, \frac{u_2}{\nu(u_2)}, ..., \frac{u_n}{\nu(u_n)}\right\}$ be any two plithogenic fuzzy sets over U. Then the plithogenic fuzzy intersection between A and B define as follows:

$$A \wedge_{FP} B = \left\{\frac{u_i}{(1-c_v)(\mu(u_i) \wedge_F \nu(u_i)) + (c_v)(\mu(u_i) \vee_F \nu_F(u_i))}\right\}, \forall i = 1, 2, ..., n$$

Where $\wedge_F$ and $\vee_F$ represent fuzzy t-norm and fuzzy t-conorm (s-norm) respectively and c_v represent the degrees of contradictions.

4.2. *Plithogenic Intuitionistic Fuzzy Intersection*

Definition 4.2. Let $U = \{u_1, u_2, ..., u_n\}$. Suppose

$A = \left\{\frac{u_1}{(\mu_T(u_1), \mu_F(u_1))}, \frac{u_2}{(\mu_T(u_2), \mu_F(u_2))}, ..., \frac{u_n}{(\mu_T(u_n), \mu_F(u_n))}\right\}$, and

$B = \left\{\frac{u_1}{(\nu_T(u_1), \nu_F(u_1))}, \frac{u_2}{(\nu_T(u_2), \nu_F(u_2))}, ..., \frac{u_n}{(\nu_T(u_n), \nu_F(u_n))}\right\}$

be any two plithogenic intuitionistic fuzzy sets over U. Then the plithogenic intuitionistic fuzzy intersection between A and B is defined as follows:

$$A \wedge_{IP} B = \left\{\frac{u_i}{((1-c_v)(\mu(u_i) \wedge_F \nu(u_i)) + (c_v)(\mu(u_i) \vee_F \nu_F(u_i)), (c_v)(\mu(u_i) \vee_F \nu(u_i)) + (1-c_v)(\mu(u_i) \wedge_F \nu_F(u_i)))}\right\}, \forall i = 1, 2, ..., n$$

Where $\wedge_F$ and $\vee_F$ represent fuzzy t-norm and fuzzy t-conorm (s-norm) respectively and c_v represent the degrees of contradictions.

4.3. *Plithogenic Neutrosophic Intersection*

Definition 4.3. Let $U = \{u_1, u_2, ..., u_n\}$. Suppose

$A = \left\{\frac{u_1}{\langle\mu_T(u_1), \mu_I(u_1), \mu_F(u_1)\rangle}, \frac{u_2}{\langle\mu_T(u_2), \mu_I(u_2), \mu_F(u_2)\rangle}, ..., \frac{u_n}{\langle\mu_T(u_n), \mu_I(u_n), \mu_F(u_n)\rangle}\right\}$, and

$B = \left\{\frac{u_1}{\langle\nu_T(u_1), \nu_I(u_1), \nu_F(u_1)\rangle}, \frac{u_2}{\langle\nu_T(u_2), \nu_I(u_2), \nu_F(u_2)\rangle}, ..., \frac{u_n}{\langle\nu_T(u_n), \nu_I(u_n), \nu_F(u_n)\rangle}\right\}$

be any two plithogenic neutrosophic set over U. Then the plithogenic neutrosophic intersection between A and B is defined as follows:

$$A \wedge_{NP} B = \left\{\frac{u_i}{\langle(1-c_v)(\mu(u_i) \wedge_F \nu(u_i)) + (c_v)(\mu(u_i) \vee_F \nu_F(u_i)), \frac{1}{2}[(\mu_I(u_1) \wedge_F \nu_T(u_1)) + (\mu_I(u_1) \vee_F \nu_T(u_1))], (c_v)(\mu(u_i) \vee_F \nu(u_i)) + (1-c_v)(\mu(u_i) \wedge_F \nu_F(u_i))\rangle}\right\},$$

$\forall i = 1, 2, ..., n$, where $\wedge_F$ and $\vee_F$ represent fuzzy t-norm and fuzzy t-conorm (s-norm) respectively and c_v represent the degrees of contradictions.

5. Plithogenic Union

In this section we recall the definition of plithogenic union over three cases: fuzzy, intuitionistic fuzzy and neutrosophic set which are defined for the first time by Smarandache in 2017 and 2018 [11, 12].

5.1. *Plithogenic Fuzzy Union*

Definition 5.1. Let U, A and B as defined in 4.1. Then the plithogenic fuzzy union between A and B is defined as follows:

$$A \vee_{FP} B = \left\{ \frac{u_i}{(1-c_v)(\mu(u_i) \vee_F \nu(u_i)) + (c_v)(\mu(u_i) \wedge_F \nu_F(u_i))} \right\}, \forall i = 1, 2, ..., n$$

Where $\wedge_F$ and $\vee_F$ represent fuzzy t-norm and fuzzy t-conorm (s-norm) respectively and c_v represent the degrees of contradictions.

5.2. *Plithogenic Intuitionistic Fuzzy Union*

Definition 5.2. Let U, A and B as defined in 4.2. Then the plithogenic intuitionistic fuzzy union between A and B is defined as follows:

$$A \vee_{IP} B = \left\{ \frac{u_i}{((1-c_v)(\mu(u_i) \vee_F \nu(u_i)) + (c_v)(\mu(u_i) \wedge_F \nu_F(u_i)), (c_v)(\mu(u_i) \wedge_F \nu(u_i)) + (1-c_v)(\mu(u_i) \vee_F \nu_F(u_i)))} \right\}, \forall i = 1, 2, ..., n$$

Where $\wedge_F$ and $\vee_F$ represent fuzzy t-norm and fuzzy t-conorm (s-norm) respectively and c_v represent the degrees of contradictions.

5.3. *Plithogenic Neutrosophic Union*

Definition 5.3. Let U, A and B as defined in 4.3. Then the plithogenic neutrosophic intersection between A and B is defined as follows:

$$A \vee_{NP} B = \left\{ \frac{u_i}{\langle (1-c_v)(\mu(u_i) \vee_F \nu(u_i)) + (c_v)(\mu(u_i) \wedge_F \nu_F(u_i)), \frac{1}{2}[(\mu_I(u_1) \wedge_F \nu_T(u_1)) + (\mu_I(u_1) \vee_F \nu_T(u_1))], (c_v)(\mu(u_i) \wedge_F \nu(u_i)) + (1-c_v)(\mu(u_i) \vee_F \nu_F(u_i)) \rangle} \right\},$$

$\forall i = 1, 2, ..., n$, where $\wedge_F$ and $\vee_F$ represent fuzzy t-norm and fuzzy t-conorm (s-norm) respectively and c_v represent the degrees of contradictions.

5.4. *Hypersoft set*

Definition 5.4. [11] Let U be a universe of discourse, $\wp(U)$ the power set of U. Let $a_1, a_2, \cdots, a_n$, for $n \geq 1$, be n distinct attributes, whose corresponding attribute values are respectively the sets $A_1, A_2, \cdots, A_n$, with $A_i \cap A_j = \varnothing$, for $j \neq i$, and $i, j \in \{1, 2, \cdots, n.\}$. Then the pair

$$(F, A_1 \times A_2 \times \cdots \times A_n)$$

where:

$$F : A_1 \times A_2 \times \cdots \times A_n \longrightarrow \wp(U)$$

is called a Hypersoft Set over U.

6. **Plithogenic Soft Set**

In this section we define the concept of plithogenic soft set(In General) as a generalization of soft set. We also, define its basic operations namely, union and intersection and study their properties.

Definition 6.1. Let U be a universe of discourse, $(U)^z$ the $z-$power set of U such that
$z = C$ (Power set of U),
$z = F$ (The set of all fuzzy set of U),
$z = IF$ (The set of all intuitionistic fuzzy set of U), or
$z = N$ (The set of all neutrosophic set of U). Let $a_1, a_2, \cdots, a_n$, for $n \geq 1$, be n distinct attributes, whose corresponding attribute values are respectively the sets $V_1, V_2, \cdots, V_n$, with $V_i \cap V_j = \varnothing$, for $j \neq i$, and $i, j \in \{1, 2, \cdots, n.\}$. Suppose $V_i = \{v_{i1}, v_{i2}, \cdots, v_{im_i}\}$ and let $\Upsilon = V_1 \times V_2 \times \cdots \times V_n$. Let $D = (D_1, D_2, \cdots D_n)$ the dominant attribute element of A_i $\forall i$ and $c\left(D_i, v_{ij}\right), i \in \{1, 2, \cdots n\}, j \in \{1, 2, \cdots m_i\}$ the attribute value contradiction degree function such that $c_i : V_i \times V_i \longrightarrow [0, 1]$, Then the pair (F_P^z, Υ), where:

$$F_P^z : \Upsilon \longrightarrow [0, 1]_D \times (U)^z$$

is called a Plithogenic Soft Set (P-SS In short) over U.

To illustrate the above definition we give the following examples for all cases: crisp, fuzzy, intuitionistic and neutrosophic:

6.1. *Plithogenic Crisp Soft Set*

Example 6.2. Let $U = \{c_1, c_2, c_3, c_4\}$. Here U represents the set of cars. Let $a_1 = speed, a_2 = color, a_3 = model, a_4 = manufacturingyear$. Suppose their attributes values respectively:
Speed $\equiv A_1 = \{slow, fast, veryfast\}$,
Color $\equiv A_2 = \{white, yellow, red, black\}$,
Model $\equiv A_3 = \{model_1, model_2, model_3, model_4\}$,
manufacturing year $\equiv A_4 = \{2015\ and\ befor, 2016, 2017, 2018, 2019\}$.
Where the dominant attribute's value $D = (slow, red, model_3, 2019)$. Let $H \subseteq \Upsilon$ such that $H = \{\epsilon_1 = (slow, white, model_1, 2015\ and\ befor), \epsilon_2 = (slow, yellow, model_3, 2017),$
$\epsilon_3 = (fast, red, model_4, 2018)\}$. Define a function $F_P^z : H \longrightarrow [0, 1]_D \times (U)^C$ as follows:

$$F_P^C(\epsilon_1) = \left\{ \frac{(c_1,(0,0.4,1,1)_D)}{(1,1,1,1)}, \frac{(c_4,(0,0.4,1,1)_D)}{(1,1,1,1)} \right\},$$

$$F_P^C(\epsilon_2) = \left\{ \frac{(c_2,(0,0.5,0,1)_D)}{(1,1,1,1)} \right\},$$

$$F_P^C(\epsilon_3) = \left\{ \frac{(c_3,(0.5,0,1,1)_D)}{(1,1,1,1)} \right\},$$

Then we can find the plithogenic crisp soft set (P-CSS) $\left(F_P^C, H\right)$ as consisting of the following collection of approximations:

$$(F_P^C, H) = \left\{ \left(\epsilon_1, \left\{ \frac{(c_1,(0,0.4,1,1)_D)}{(1,1,1,1)}, \frac{(c_4,(0,0.4,1,1)_D)}{(1,1,1,1)} \right\}\right), \left(\epsilon_2, \left\{ \frac{(c_2,(0,0.5,0,1)_D)}{(1,1,1,1)} \right\}\right), \left(\epsilon_3, \left\{ \frac{(c_3,(0.5,0,1,1)_D)}{(1,1,1,1)} \right\}\right) \right\}.$$

6.2. *Plithogenic Fuzzy Soft Set*

Example 6.3. Consider Example 6.2 and suppose that the

$$F_P^F(\epsilon_1) = \left\{ \frac{(c_1,(0,0.4,1,1)_D)}{(0.6,0.7,1,1)}, \frac{(c_2,(0,0.4,1,1)_D)}{(0.7,0.3,0,0)}, \frac{(c_3,(0,0.4,1,1)_D)}{(0.1,0.3,0,0)}, \frac{(c_4,(0,0.4,1,1)_D)}{(0.7,0.6,1,1)} \right\},$$

$$F_P^F(\epsilon_2) = \left\{ \frac{(c_1,(0,0.5,0,1)_D)}{(0.6,0.2,0,0)}, \frac{(c_2,(0,0.5,0,1)_D)}{(0.5,0.7,1,1)}, \frac{(c_3,(0,0.5,0,1)_D)}{(0.1,0.1,0,0)}, \frac{(c_4,(0,0.5,0,1)_D)}{(0.7,0.4,0,0)} \right\},$$

$$F_P^F(\epsilon_3) = \left\{ \frac{(c_1,(0.5,0,1,1)_D)}{(0.1,0.2,0,0)}, \frac{(c_2,(0.5,0,1,1)_D)}{(0.1,0.3,0,0)}, \frac{(c_3,(0.5,0,1,1)_D)}{(0.8,0.7,1,1)}, \frac{(c_4,(0.5,0,1,1)_D)}{(0.1,0.1,0,0)} \right\},$$

Then we can find the plithogenic fuzzy soft set (P-FSS) $\left(F_P^F, H\right)$ as consisting of the following collection of approximations:

$$\begin{aligned}(F_P^F, H) = \Bigg\{ & \left(\epsilon_1, \left\{ \frac{(c_1,(0,0.4,1,1)_D)}{(0.6,0.7,1,1)}, \frac{(c_2,(0,0.4,1,1)_D)}{(0.7,0.3,0,0)}, \frac{(c_3,(0,0.4,1,1)_D)}{(0.1,0.3,0,0)}, \frac{(c_4,(0,0.4,1,1)_D)}{(0.7,0.6,1,1)} \right\}\right), \\ & \left(\epsilon_2, \left\{ \frac{(c_1,(0,0.5,0,1)_D)}{(0.6,0.2,0,0)}, \frac{(c_2,(0,0.5,0,1)_D)}{(0.5,0.7,1,1)}, \frac{(c_3,(0,0.5,0,1)_D)}{(0.1,0.1,0,0)}, \frac{(c_4,(0,0.5,0,1)_D)}{(0.7,0.4,0,0)} \right\}\right), \\ & \left(\epsilon_3, \left\{ \frac{(c_1,(0.5,0,1,1)_D)}{(0.1,0.2,0,0)}, \frac{(c_2,(0.5,0,1,1)_D)}{(0.1,0.3,0,0)}, \frac{(c_3,(0.5,0,1,1)_D)}{(0.8,0.7,1,1)}, \frac{(c_4,(0.5,0,1,1)_D)}{(0.1,0.1,0,0)} \right\}\right) \Bigg\}.\end{aligned}$$

6.3. *Plithogenic Intuitionistic Fuzzy Soft Set*

Example 6.4. Consider Example 6.2 and suppose that the

$$F_P^{IF}(\epsilon_1) = \left\{ \frac{(c_1,(0,0.4,1,1)_D)}{((0.6,0.2),(0.7,0.1),(1,0),(1,0))}, \frac{(c_2,(0,0.4,1,1)_D)}{((0.7,0.1),(0.3,0.5),(0,1),(0,1))}, \frac{(c_3,(0,0.4,1,1)_D)}{((0.1,0.8),(0.3,0.5),(0,1),(0,1))}, \frac{(c_4,(0,0.4,1,1)_D)}{((0.7,0.2),(0.6,0.3),(1,0),(1,0))} \right\},$$

$$F_P^{IF}(\epsilon_2) = \left\{ \frac{(c_1,(0,0.5,0,1)_D)}{((0.6,0.3),(0.2,0.6),(0,1),(0,1))}, \frac{(c_2,(0,0.5,0,1)_D)}{((0.5,0.4),(0.7,0.2),(1,0),(1,0))}, \frac{(c_3,(0,0.5,0,1)_D)}{((0.1,0.7),(0.1,0.8),(0,1),(0,1))}, \frac{(c_4,(0,0.5,0,1)_D)}{((0.7,0.2),(0.4,0.4),(0,1),(0,1))} \right\},$$

$$F_P^{IF}(\epsilon_3) = \left\{ \frac{(c_1,(0.5,0,1,1)_D)}{((0.1,0.7),(0.2,0.7),(0,1),(0,1))}, \frac{(c_2,(0.5,0,1,1)_D)}{((0.1,0.8),(0.3,0.5),(0,1),(0,1))}, \frac{(c_3,(0.5,0,1,1)_D)}{((0.8,0.1),(0.7,0.2),(1,0),(1,0))}, \frac{(c_4,(0.5,0,1,1)_D)}{((0.1,0.8),(0.1,0.7),(0,1),(0,1))} \right\},$$

Then we can find the plithogenic intuitionistic fuzzy soft set (P-IFSS) $\left(F_P^{IF}, H\right)$ as consisting of the following collection of approximations:

$$\left(F_P^{IF}, H\right) = \Big\{ \Big(\epsilon_1, \Big\{ \frac{(c_1, (0, 0.4, 1, 1)_D)}{((0.6, 0.2), (0.7, 0.1), (1, 0), (1, 0))}, \frac{(c_2, (0, 0.4, 1, 1)_D)}{((0.7, 0.1), (0.3, 0.5), (0, 1), (0, 1))}, \frac{(c_3, (0, 0.4, 1, 1)_D)}{((0.1, 0.8), (0.3, 0.5), (0, 1), (0, 1))}, \frac{(c_4, (0, 0.4, 1, 1)_D)}{((0.7, 0.2), (0.6, 0.3), (1, 0), (1, 0))} \Big\} \Big),$$

$$\Big(\epsilon_2, \Big\{ \frac{(c_1, (0, 0.5, 0, 1)_D)}{((0.6, 0.3), (0.2, 0.6), (0, 1), (0, 1))}, \frac{(c_2, (0, 0.5, 0, 1)_D)}{((0.5, 0.4), (0.7, 0.2), (1, 0), (1, 0))}, \frac{(c_3, (0, 0.5, 0, 1)_D)}{((0.1, 0.7), (0.1, 0.8), (0, 1), (0, 1))}, \frac{(c_4, (0, 0.5, 0, 1)_D)}{((0.7, 0.2), (0.4, 0.4), (0, 1), (0, 1))} \Big\} \Big),$$

$$\Big(\epsilon_3, \Big\{ \frac{(c_1, (0.5, 0, 1, 1)_D)}{((0.1, 0.7), (0.2, 0.7), (0, 1), (0, 1))}, \frac{(c_2, (0.5, 0, 1, 1)_D)}{((0.1, 0.8), (0.3, 0.5), (0, 1), (0, 1))}, \frac{(c_3, (0.5, 0, 1, 1)_D)}{((0.8, 0.1), (0.7, 0.2), (1, 0), (1, 0))}, \frac{(c_4, (0.5, 0, 1, 1)_D)}{((0.1, 0.8), (0.1, 0.7), (0, 1), (0, 1))} \Big\} \Big) \Big\}.$$

6.4. *Plithogenic Neutrosophic Soft Set*

Example 6.5. Consider Example 6.2 and $U = \{c_1, c_2, c_3\}$, suppose that the

$$F_P^N(\epsilon_1) = \left\{ \frac{(c_1,(0,0.4,1,1)_D)}{((0.6,0.3,0.1),(0.7,0.2,0.1),(1,0,0),(1,0,0))}, \frac{(c_2,(0,0.4,1,1)_D)}{((0.7,0.1,0.2),(0.3,0.5,0.2),(0,0,1),(0,0,1))}, \frac{(c_3,(0,0.4,1,1)_D)}{((0.1,0.1,0.8),(0.3,0.6,0.1),(0,0,1),(0,0,1))} \right\},$$

$$F_P^N(\epsilon_2) = \left\{ \frac{(c_1,(0,0.5,0,1)_D)}{((0.6,0.3,0.1),(0.2,0.6,0.2),(0,0,1),(0,0,1))}, \frac{(c_2,(0,0.5,0,1)_D)}{((0.5,0.2,0.3),(0.7,0.2,0.1),(1,0,0),(1,0,0))}, \frac{(c_3,(0,0.5,0,1)_D)}{((0.1,0.2,0.7),(0.1,0.1,0.8),(0,0,1),(0,0,1))} \right\},$$

$$F_P^N(\epsilon_3) = \left\{ \frac{(c_1,(0.5,0,1,1)_D)}{((0.1,0.2,0.7),(0.2,0.1,0.7),(0,0,1),(0,0,1))}, \frac{(c_2,(0.5,0,1,1)_D)}{((0.1,0.1,0.8),(0.3,0.2,0.5),(0,0,1),(0,0,1))}, \frac{(c_3,(0.5,0,1,1)_D)}{((0.8,0.1,0.1),(0.7,0.1,0.2),(1,0,0),(1,0,0))} \right\},$$

Then we can find the plithogenic neutrosophic soft set (P-NSS) $\left(F_P^N, H\right)$ as consisting of the following collection of approximations:

$$\left(F_P^{N}, H\right) = \Big\{ \Big(\epsilon_1, \Big\{ \frac{(c_1, (0, 0.4, 1, 1)_D)}{((0.6, 0.3, 0.1), (0.7, 0.2, 0.1), (1, 0, 0), (1, 0, 0))}, \frac{(c_2, (0, 0.4, 1, 1)_D)}{((0.7, 0.1, 0.2), (0.3, 0.5, 0.2), (0, 0, 1), (0, 0, 1))}, \frac{(c_3, (0, 0.4, 1, 1)_D)}{((0.1, 0.1, 0.8), (0.3, 0.6, 0.1), (0, 0, 1), (0, 0, 1))} \Big\} \Big),$$

$$\Big(\epsilon_2, \Big\{ \frac{(c_1, (0, 0.5, 0, 1)_D)}{((0.6, 0.3, 0.1), (0.2, 0.6, 0.2), (0, 0, 1), (0, 0, 1))}, \frac{(c_2, (0, 0.5, 0, 1)_D)}{((0.5, 0.2, 0.3), (0.7, 0.2, 0.1), (1, 0, 0), (1, 0, 0))}, \frac{(c_3, (0, 0.5, 0, 1)_D)}{((0.1, 0.2, 0.7), (0.1, 0.1, 0.8), (0, 0, 1), (0, 0, 1))} \Big\} \Big),$$

$$\Big(\epsilon_3, \Big\{ \frac{(c_1, (0.5, 0, 1, 1)_D)}{((0.1, 0.2, 0.7), (0.2, 0.1, 0.7), (0, 0, 1), (0, 0, 1))}, \frac{(c_2, (0.5, 0, 1, 1)_D)}{((0.1, 0.1, 0.8), (0.3, 0.2, 0.5), (0, 0, 1), (0, 0, 1))}, \frac{(c_3, (0.5, 0, 1, 1)_D)}{((0.8, 0.1, 0.1), (0.7, 0.1, 0.2), (1, 0, 0), (1, 0, 0))} \Big\} \Big) \Big\}.$$

7. **Union and Intersection**

In this section, we introduce the definitions of union and intersection of plithogenic soft sets, derive their properties, and give some examples.

Definition 7.1. The *union* of two plithogenic soft sets (F_P^z, A) and (G_P^z, B) over U, denoted by
$(F_P^z, A) \bigvee_P^z (G_P^z, B)$, is the plithogenic soft set (H_P^z, Ω) where $\Omega = A \cup B$, and $\forall\, \varepsilon \in \Omega$,

$$H_P^z(\varepsilon) = \begin{cases} F_P^z(\varepsilon), & \text{if } \varepsilon \in A - B \\ G_P^z(\varepsilon), & \text{if } \varepsilon \in B - A \\ F_P^z(\varepsilon) \vee_P^z G_P^z(\varepsilon), & \text{if } \varepsilon \in A \cap B \end{cases}$$

where $\vee_P^z$ is a z-plithogenic union.

Definition 7.2. The *intersection* of two plithogenic soft sets (F_P^z, A) and (G_P^z, B) over U, denoted by

$(F_P^z, A) \bigwedge_P^z (G_P^z, B)$, is the plithogenic soft set (H_P^z, Ω) where $\Omega = A \cup B$, and $\forall \varepsilon \in \Omega$,

$$H_P^z(\varepsilon) = \begin{cases} F_P^z(\varepsilon), & \text{if } \varepsilon \in A - B \\ G_P^z(\varepsilon), & \text{if } \varepsilon \in B - A \\ F_P^z(\varepsilon) \wedge_P^z G_P^z(\varepsilon), & \text{if } \varepsilon \in A \cap B \end{cases}$$

where $\wedge_P^z$ is a z-plithogenic intersection.

7.1. *Plithogenic Fuzzy Soft Union*

Example 7.3. Consider Example 6.2 Let

$$A = \Big\{ a_1 = (slow, white, model_1, 2015 \ and \ befor), a_2 = (slow, yellow, model_3, 2017),$$
$$a_3 = (fast, red, model_4, 2018) \Big\} and$$
$$B = \Big\{ b_1 = (slow, white, model_1, 2015 \ and \ befor), b_2 = (slow, yellow, model_3, 2017),$$
$$b_3 = (fast, red, model_4, 2018), b_4 = (slow, red, model_1, 2019) \Big\}$$

Suppose (F_P^F, A) and (G_P^F, A) are two plithogenic fuzzy soft sets over U such that

$$(F_P^F, A) = \Bigg\{ \left(a_1, \left\{ \frac{(c_1, (0, 0.4, 1, 1)_D)}{(0.6, 0.7, 1, 1)}, \frac{(c_2, (0, 0.4, 1, 1)_D)}{(0.7, 0.3, 0, 0)}, \frac{(c_3, (0, 0.4, 1, 1)_D)}{(0.1, 0.3, 0, 0)}, \frac{(c_4, (0, 0.4, 1, 1)_D)}{(0.7, 0.6, 1, 1)} \right\} \right),$$
$$\left(a_2, \left\{ \frac{(c_1, (0, 0.5, 0, 1)_D)}{(0.6, 0.2, 0, 0)}, \frac{(c_2, (0, 0.5, 0, 1)_D)}{(0.5, 0.7, 1, 1)}, \frac{(c_3, (0, 0.5, 0, 1)_D)}{(0.1, 0.1, 0, 0)}, \frac{(c_4, (0, 0.5, 0, 1)_D)}{(0.7, 0.4, 0, 0)} \right\} \right),$$
$$\left(a_3, \left\{ \frac{(c_1, (0.5, 0, 1, 1)_D)}{(0.1, 0.2, 0, 0)}, \frac{(c_2, (0.5, 0, 1, 1)_D)}{(0.1, 0.3, 0, 0)}, \frac{(c_3, (0.5, 0, 1, 1)_D)}{(0.8, 0.7, 1, 1)}, \frac{(c_4, (0.5, 0, 1, 1)_D)}{(0.1, 0.1, 0, 0)} \right\} \right) \Bigg\}.$$

$$(G_P^F, B) = \Bigg\{ \left(b_1, \left\{ \frac{(c_1, (0, 0.4, 1, 1)_D)}{(0.5, 0.6, 1, 1)}, \frac{(c_2, (0, 0.4, 1, 1)_D)}{(0.8, 0.4, 0, 0)}, \frac{(c_3, (0, 0.4, 1, 1)_D)}{(0.2, 0.2, 0, 0)}, \frac{(c_4, (0, 0.4, 1, 1)_D)}{(0.7, 0.5, 1, 1)} \right\} \right),$$
$$\left(b_2, \left\{ \frac{(c_1, (0, 0.5, 0, 1)_D)}{(0.5, 0.3, 0, 0)}, \frac{(c_2, (0, 0.5, 0, 1)_D)}{(0.7, 0.7, 1, 1)}, \frac{(c_3, (0, 0.5, 0, 1)_D)}{(0.3, 0.2, 0, 0)}, \frac{(c_4, (0, 0.5, 0, 1)_D)}{(0.8, 0.5, 0, 0)} \right\} \right),$$
$$\left(b_3, \left\{ \frac{(c_1, (0.5, 0, 1, 1)_D)}{(0.5, 0.6, 0, 0)}, \frac{(c_2, (0.5, 0, 1, 1)_D)}{(0.8, 0.3, 0, 0)}, \frac{(c_3, (0.5, 0, 1, 1)_D)}{(0.2, 0.7, 1, 1)}, \frac{(c_4, (0.5, 0, 1, 1)_D)}{(0.6, 0.3, 0, 0)} \right\} \right),$$
$$\left(b_4, \left\{ \frac{(c_1, (0, 0, 1, 1)_D)}{(0.2, 0.3, 1, 0)}, \frac{(c_2, (0, 0, 1, 1)_D)}{(0.1, 0.3, 0, 0)}, \frac{(c_3, (0, 0, 1, 1)_D)}{(0.8, 0.7, 1, 1)}, \frac{(c_4, (0, 0, 1, 1)_D)}{(0.1, 0.1, 1, 0)} \right\} \right) \Bigg\}.$$

By using basic fuzzy union (maximum) and basic fuzzy intersection (minimum) we have:

$\left(F_P^F, A\right) \bigvee_P^F \left(G_P^F, B\right) = \left(H_P^F, K\right)$ where

$$\begin{aligned}\left(H_P^F, K\right) = \Bigg\{ & \left(k_1, \left\{\frac{(c_1,(0,0.4,1,1)_D)}{(0.6,0.66,1,1)}, \frac{(c_2,(0,0.4,1,1)_D)}{(0.8,0.36,0,0)}, \frac{(c_3,(0,0.4,1,1)_D)}{(0.2,0.26,0,0)}, \frac{(c_4,(0,0.4,1,1)_D)}{(0.7,0.56,1,1)}\right\}\right), \\ & \left(k_2, \left\{\frac{(c_1,(0,0.5,0,1)_D)}{(0.6,0.25,0,0)}, \frac{(c_2,(0,0.5,0,1)_D)}{(0.7,0.7,1,1)}, \frac{(c_3,(0,0.5,0,1)_D)}{(0.3,0.15,0,0)}, \frac{(c_4,(0,0.5,0,1)_D)}{(0.8,0.45,0,0)}\right\}\right), \\ & \left(k_3, \left\{\frac{(c_1,(0.5,0,1,1)_D)}{(0.3,0.6,0,0)}, \frac{(c_2,(0.5,0,1,1)_D)}{(0.45,0.3,0,0)}, \frac{(c_3,(0.5,0,1,1)_D)}{(0.5,0.7,1,1)}, \frac{(c_4,(0.6,0,1,1)_D)}{(0.35,0.3,0,0)}\right\}\right), \\ & \left(k_4, \left\{\frac{(c_1,(0,0,1,1)_D)}{(0.2,0.3,1,0)}, \frac{(c_2,(0,0,1,1)_D)}{(0.1,0.3,0,0)}, \frac{(c_3,(0,0,1,1)_D)}{(0.8,0.7,1,1)}, \frac{(c_4,(0,0,1,1)_D)}{(0.1,0.1,1,0)}\right\}\right) \Bigg\}.\end{aligned}$$

To describe the result in Example above let's compute $\frac{(c_1,(0,0.4,1,1)_D)}{(0.6,0.7,1,1)} \vee_P^F \frac{(c_1,(0,0.4,1,1)_D)}{(0.5,0.6,1,1)}$ as follows:

$(1-0)*max(0.6, 0.5)+0*min(0.6, 0.5) = 0.6$, $(1-0.4)*max(0.7, 0.6)+0.4*min(0.7, 0.6) = 0.66$, $(1-1)*max(1, 1) + 1*min(1, 1) = 1$ and $(1-1)*max(1, 1) + 1*min(1, 1) = 1$

7.2. *Plithogenic Fuzzy Soft Intersection*

Example 7.4. Consider Example 7.3 By using basic fuzzy union (maximum) and basic fuzzy intersection (minimum) we have: $\left(F_P^F, A\right) \bigwedge_P^F \left(G_P^F, B\right) = \left(H_P^F, K\right)$ where

$$\begin{aligned}\left(H_P^F, K\right) = \Bigg\{ & \left(k_1, \left\{\frac{(c_1,(0,0.4,1,1)_D)}{(0.5,0.64,1,1)}, \frac{(c_2,(0,0.4,1,1)_D)}{(0.7,0.34,0,0)}, \frac{(c_3,(0,0.4,1,1)_D)}{(0.1,0.18,0,0)}, \frac{(c_4,(0,0.4,1,1)_D)}{(0.7,0.52,1,1)}\right\}\right), \\ & \left(k_2, \left\{\frac{(c_1,(0,0.5,0,1)_D)}{(0.5,0.25,0,0)}, \frac{(c_2,(0,0.5,0,1)_D)}{(0.5,0.7,1,1)}, \frac{(c_3,(0,0.5,0,1)_D)}{(0.1,0.15,0,0)}, \frac{(c_4,(0,0.5,0,1)_D)}{(0.7,0.45,0,0)}\right\}\right), \\ & \left(k_3, \left\{\frac{(c_1,(0.5,0,1,1)_D)}{(0.3,0.2,0,0)}, \frac{(c_2,(0.5,0,1,1)_D)}{(0.45,0.3,0,0)}, \frac{(c_3,(0.5,0,1,1)_D)}{(0.5,0.7,1,1)}, \frac{(c_4,(0.5,0,1,1)_D)}{(0.35,0.1,0,0)}\right\}\right), \\ & \left(k_4, \left\{\frac{(c_1,(0,0,1,1)_D)}{(0.2,0.3,1,0)}, \frac{(c_2,(0,0,1,1)_D)}{(0.1,0.3,0,0)}, \frac{(c_3,(0,0,1,1)_D)}{(0.8,0.7,1,1)}, \frac{(c_4,(0,0,1,1)_D)}{(0.1,0.1,1,0)}\right\}\right) \Bigg\}.\end{aligned}$$

To describe the result in Example above let's compute $\frac{(c_1,(0,0.4,1,1)_D)}{(0.6,0.7,1,1)} \wedge_P^F \frac{(c_1,(0,0.4,1,1)_D)}{(0.5,0.6,1,1)}$ as follows:

$(1-0)*min(0.6, 0.5)+0*max(0.6, 0.5) = 0.5$, $(1-0.4)*min(0.7, 0.6)+0.4*max(0.7, 0.6) = 0.64$, $(1-1)*min(1, 1) + 1*max(1, 1) = 1$ and $(1-1)*min(1, 1) + 1*max(1, 1) = 1$

7.3. *Plithogenic Intuitionistic Fuzzy Soft Union*

Example 7.5. Consider Example 7.3. Suppose $\left(F_P^{IF}, A\right)$ and $\left(G_P^{IF}, A\right)$ are two plithogenic intuitionistic fuzzy soft sets over U such that

$$\begin{aligned}\left(F_P^{IF}, A\right) = \Bigg\{ & \left(a_1, \left\{\frac{(c_1,(0,0.4,1,1)_D)}{((0.6,0.3),(0.7,0.3),(1,0),(1,0))}, \frac{(c_2,(0,0.4,1,1)_D)}{((0.7,0.2),(0.3,0.5),(0,1),(0,1))}, \frac{(c_3,(0,0.4,1,1)_D)}{((0.1,0.8),(0.3,0.5),(0,1),(0,1))}, \frac{(c_4,(0,0.4,1,1)_D)}{((0.7,0.2),(0.6,0.2),(1,0),(1,0))}\right\}\right), \\ & \left(a_2, \left\{\frac{(c_1,(0,0.5,0,1)_D)}{((0.6,0.3),(0.2,0.7),(0,1),(0,1))}, \frac{(c_2,(0,0.5,0,1)_D)}{((0.5,0.4),(0.7,0.3),(1,0),(1,0))}, \frac{(c_3,(0,0.5,0,1)_D)}{((0.1,0.8),(0.1,0.9),(0,1),(0,1))}, \frac{(c_4,(0,0.5,0,1)_D)}{((0.7,0.3),(0.4,0.5),(0,1),(0,1))}\right\}\right), \\ & \left(a_3, \left\{\frac{(c_1,(0.5,0,1,1)_D)}{((0.1,0.8),(0.2,0.8),(0,1),(0,1))}, \frac{(c_2,(0.5,0,1,1)_D)}{((0.1,0.9),(0.3,0.7),(0,1),(0,1))}, \frac{(c_3,(0.5,0,1,1)_D)}{((0.8,0.1),(0.7,0.2),(1,0),(1,0))}, \frac{(c_4,(0.5,0,1,1)_D)}{((0.1,0.8),(0.1,0.9),(0,1),(0,1))}\right\}\right) \Bigg\}.\end{aligned}$$

$$\begin{aligned}(G_P^{IF},B)=\Bigg\{&\Bigg(b_1,\Bigg\{\frac{\left(c_1,(0,0.4,1,1)_D\right)}{((0.5,0.4),(0.6,0.3),(1,0),(1,0))},\frac{\left(c_2,(0,0.4,1,1)_D\right)}{((0.8,0.2),(0.4,0.6),(0,1),(0,1))},\frac{\left(c_3,(0,0.4,1,1)_D\right)}{((0.2,0.7),(0.2,0.8),(0,1),(0,1))},\frac{\left(c_4,(0,0.4,1,1)_D\right)}{((0.7,0.2),(0.5,0.4),(1,0),(1,0))}\Bigg\}\Bigg),\\
&\Bigg(b_2,\Bigg\{\frac{\left(c_1,(0,0.5,0,1)_D\right)}{((0.5,0.5),(0.3,0.7),(0,1),(0,1))},\frac{\left(c_2,(0,0.5,0,1)_D\right)}{((0.7,0.2),(0.7,0.2),(1,0),(1,0))},\frac{\left(c_3,(0,0.5,0,1)_D\right)}{((0.3,0.6),(0.2,0.8),(0,1),(0,1))},\frac{\left(c_4,(0,0.5,0,1)_D\right)}{((0.8,0.1),(0.5,0.5),(0,1),(0,1))}\Bigg\}\Bigg),\\
&\Bigg(b_3,\Bigg\{\frac{\left(c_1,(0.5,0,1,1)_D\right)}{((0.5,0.4),(0.6,0.4),(0,1),(0,1))},\frac{\left(c_2,(0.5,0,1,1)_D\right)}{((0.8,0.1),(0.3,0.7),(0,1),(0,1))},\frac{\left(c_3,(0.5,0,1,1)_D\right)}{((0.2,0.7),(0.7,0.2),(1,0),(1,0))},\frac{\left(c_4,(0.5,0,1,1)_D\right)}{((0.6,0.3),(0.3,0.7),(0,1),(0,1))}\Bigg\}\Bigg)\\
&\Bigg(b_4,\Bigg\{\frac{\left(c_1,(0,0,1,1)_D\right)}{((0.2,0.8),(0.3,0.6),(0,1),(0,1))},\frac{\left(c_2,(0,0,1,1)_D\right)}{((0.1,0.9),(0.3,0.7),(0,1),(0,1))},\frac{\left(c_3,(0,0,1,1)_D\right)}{((0.8,0.2),(0.7,0.3),(1,0),(1,0))},\frac{\left(c_4,(0,0,1,1)_D\right)}{((0.1,0.9),(0.1,0.8),(0,1),(0,1))}\Bigg\}\Bigg)\Bigg\}.\end{aligned}$$

By using basic fuzzy union (maximum) and basic fuzzy intersection (minimum) we have:
$\left(F_P^{IF},A\right)\bigvee_P^{IF}\left(G_P^{IF},B\right)=\left(H_P^{IF},K\right)$ where

$$\begin{aligned}(H_P^{IF},K)=\Bigg\{&\Bigg(k_1,\Bigg\{\frac{\left(c_1,(0,0.4,1,1)_D\right)}{((0.6,0.3),(0.66,0.3),(1,0),(1,0))},\frac{\left(c_2,(0,0.4,1,1)_D\right)}{((0.8,0.2),(0.4,0.54),(0,1),(0,1))},\frac{\left(c_3,(0,0.4,1,1)_D\right)}{((0.2,0.7),(0.26,0.63),(0,1),(0,1))},\frac{\left(c_4,(0,0.4,1,1)_D\right)}{((0.7,0.2),(0.56,0.26),(1,0),(1,0))}\Bigg\}\Bigg),\\
&\Bigg(k_2,\Bigg\{\frac{\left(c_1,(0,0.5,0,1)_D\right)}{((0.6,0.3),(0.25,0.7),(0,1),(0,1))},\frac{\left(c_2,(0,0.5,0,1)_D\right)}{((0.7,0.2),(0.7,0.25),(1,0),(1,0))},\frac{\left(c_3,(0,0.5,0,1)_D\right)}{((0.3,0.6),(0.15,0.85),(0,1),(0,1))},\frac{\left(c_4,(0,0.5,0,1)_D\right)}{((0.8,0.1),(0.45,0.5),(0,1),(0,1))}\Bigg\}\Bigg),\\
&\Bigg(k_3,\Bigg\{\frac{\left(c_1,(0.5,0,1,1)_D\right)}{((0.3,0.6),(0.6,0.4),(0,1),(0,1))},\frac{\left(c_2,(0.5,0,1,1)_D\right)}{((0.45,0.5),(0.3,0.7),(0,1),(0,1))},\frac{\left(c_3,(0.5,0,1,1)_D\right)}{((0.5,0.4),(0.7,0.2),(1,0),(1,0))},\frac{\left(c_4,(0.5,0,1,1)_D\right)}{((0.35,0.55),(0.3,0.7),(0,1),(0,1))}\Bigg\}\Bigg)\\
&\Bigg(k_4,\Bigg\{\frac{\left(c_1,(0,0,1,1)_D\right)}{((0.2,0.8),(0.3,0.6),(0,1),(0,1))},\frac{\left(c_2,(0,0,1,1)_D\right)}{((0.1,0.9),(0.3,0.7),(0,1),(0,1))},\frac{\left(c_3,(0,0,1,1)_D\right)}{((0.8,0.2),(0.7,0.3),(1,0),(1,0))},\frac{\left(c_4,(0,0,1,1)_D\right)}{((0.1,0.9),(0.1,0.8),(0,1),(0,1))}\Bigg\}\Bigg)\Bigg\}.\end{aligned}$$

7.4. *Plithogenic Intuitionistic Fuzzy Soft Intersection*

Example 7.6. Consider Example 7.5 By using basic fuzzy union (maximum) and basic fuzzy intersection (minimum) we have:
$\left(F_P^{IF},A\right)\bigwedge_P^{IF}\left(G_P^{IF},B\right)=\left(H_P^{IF},K\right)$ where

$$\begin{aligned}(H_P^{IF},K)=\Bigg\{&\Bigg(k_1,\Bigg\{\frac{\left(c_1,(0,0.4,1,1)_D\right)}{((0.6,0.4),(0.64,0.3),(1,0),(1,0))},\frac{\left(c_2,(0,0.4,1,1)_D\right)}{((0.7,0.2),(0.34,0.56),(0,1),(0,1))},\frac{\left(c_3,(0,0.4,1,1)_D\right)}{((0.1,0.8),(0.24,0.68),(0,1),(0,1))},\frac{\left(c_4,(0,0.4,1,1)_D\right)}{((0.7,0.2),(0.54,0.32),(1,0),(1,0))}\Bigg\}\Bigg),\\
&\Bigg(k_2,\Bigg\{\frac{\left(c_1,(0,0.5,0,1)_D\right)}{((0.5,0.4),(0.25,0.7),(0,1),(0,1))},\frac{\left(c_2,(0,0.5,0,1)_D\right)}{((0.5,0.4),(0.7,0.25),(1,0),(1,0))},\frac{\left(c_3,(0,0.5,0,1)_D\right)}{((0.1,0.8),(0.15,0.85),(0,1),(0,1))},\frac{\left(c_4,(0,0.5,0,1)_D\right)}{((0.7,0.3),(0.45,0.5),(0,1),(0,1))}\Bigg\}\Bigg),\\
&\Bigg(k_3,\Bigg\{\frac{\left(c_1,(0.5,0,1,1)_D\right)}{((0.3,0.6),(0.2,0.8),(0,1),(0,1))},\frac{\left(c_2,(0.5,0,1,1)_D\right)}{((0.45,0.5),(0.3,0.7),(0,1),(0,1))},\frac{\left(c_3,(0.5,0,1,1)_D\right)}{((0.5,0.4),(0.7,0.2),(1,0),(1,0))},\frac{\left(c_4,(0.5,0,1,1)_D\right)}{((0.35,0.55),(0.1,0.8),(0,1),(0,1))}\Bigg\}\Bigg)\\
&\Bigg(k_4,\Bigg\{\frac{\left(c_1,(0,0,1,1)_D\right)}{((0.2,0.8),(0.3,0.6),(0,1),(0,1))},\frac{\left(c_2,(0,0,1,1)_D\right)}{((0.1,0.9),(0.3,0.7),(0,1),(0,1))},\frac{\left(c_3,(0,0,1,1)_D\right)}{((0.8,0.2),(0.7,0.3),(1,0),(1,0))},\frac{\left(c_4,(0,0,1,1)_D\right)}{((0.1,0.9),(0.1,0.8),(0,1),(0,1))}\Bigg\}\Bigg)\Bigg\}.\end{aligned}$$

7.5. *Plithogenic Neutrosophic Soft Union*

Example 7.7. Consider Example 6.5. Let

$$\begin{aligned}A=\Big\{&a_1=(slow,white,model_1,2015\ and\ befor),a_2=(slow,yellow,model_3,2017),\\
&a_3=(fast,red,model_4,2018)\Big\}and\\
B=\Big\{&b_1=(slow,white,model_1,2015\ and\ befor),b_2=(slow,yellow,model_3,2017),\\
&b_3=(fast,red,model_4,2018)\Big\}\end{aligned}$$

Suppose $\left(F_P^N, A\right)$ and $\left(G_P^N, B\right)$ are two plithogenic neutrosophic soft sets over U such that

$$\begin{aligned}(F_P^N, A) = \Bigg\{ &\left(a_1, \left\{ \frac{(c_1,(0,0.4,1,1)_D)}{((0.6,0.3,0.1),(0.7,0.2,0.1),(1,0,0),(1,0,0))}, \frac{(c_2,(0,0.4,1,1)_D)}{((0.7,0.1,0.2),(0.3,0.5,0.2),(0,0,1),(0,0,1))}, \frac{(c_3,(0,0.4,1,1)_D)}{((0.1,0.1,0.8),(0.3,0.6,0.1),(0,0,1),(0,0,1))} \right\}\right), \\ &\left(a_2, \left\{ \frac{(c_1,(0,0.5,0,1)_D)}{((0.6,0.3,0.1),(0.2,0.6,0.2),(0,0,1),(0,0,1))}, \frac{(c_2,(0,0.5,0,1)_D)}{((0.5,0.2,0.3),(0.7,0.2,0.1),(1,0,0),(1,0,0))}, \frac{(c_3,(0,0.5,0,1)_D)}{((0.1,0.2,0.7),(0.1,0.1,0.8),(0,0,1),(0,0,1))} \right\}\right), \\ &\left(a_3, \left\{ \frac{(c_1,(0.5,0,1,1)_D)}{((0.1,0.2,0.7),(0.2,0.1,0.7),(0,0,1),(0,0,1))}, \frac{(c_2,(0.5,0,1,1)_D)}{((0.1,0.1,0.8),(0.3,0.2,0.5),(0,0,1),(0,0,1))}, \frac{(c_3,(0.5,0,1,1)_D)}{((0.8,0.1,0.1),(0.7,0.1,0.2),(1,0,0),(1,0,0))} \right\}\right) \Bigg\}.\end{aligned}$$

$$\begin{aligned}(G_P^N, B) = \Bigg\{ &\left(b_1, \left\{ \frac{(c_1,(0,0.4,1,1)_D)}{((0.5,0.2,0.3),(0.6,0.3,0.1),(1,0,0),(1,0,0))}, \frac{(c_2,(0,0.4,1,1)_D)}{((0.8,0.1,0.1),(0.2,0.3,0.5),(0,0,1),(0,0,1))}, \frac{(c_3,(0,0.4,1,1)_D)}{((0.3,0.1,0.6),(0.3,0.4,0.3),(0,0,1),(0,0,1))} \right\}\right), \\ &\left(b_2, \left\{ \frac{(c_1,(0,0.5,0,1)_D)}{((0.5,0.3,0.2),(0.4,0.4,0.2),(0,0,1),(0,0,1))}, \frac{(c_2,(0,0.5,0,1)_D)}{((0.4,0.3,0.3),(0.8,0.1,0.1),(1,0,0),(1,0,0))}, \frac{(c_3,(0,0.5,0,1)_D)}{((0.1,0.1,0.8),(0.2,0.2,0.6),(0,0,1),(0,0,1))} \right\}\right), \\ &\left(b_3, \left\{ \frac{(c_1,(0.5,0,1,1)_D)}{((0.3,0.2,0.5),(0.3,0.2,0.5),(0,0,1),(0,0,1))}, \frac{(c_2,(0.5,0,1,1)_D)}{((0.1,0.1,0.8),(0.5,0.1,0.4),(0,0,1),(0,0,1))}, \frac{(c_3,(0.5,0,1,1)_D)}{((0.6,0.2,0.2),(0.6,0.2,0.2),(1,0,0),(1,0,0))} \right\}\right) \Bigg\}.\end{aligned}$$

By using basic fuzzy union (maximum) and basic fuzzy intersection (minimum) we have:
$\left(F_P^N, A\right) \bigvee_P^N \left(G_P^N, B\right) = \left(H_P^N, K\right)$ where

$$\begin{aligned}(H_P^N, K) = \Bigg\{ &\left(k_1, \left\{ \frac{(c_1,(0,0.4,1,1)_D)}{((0.6,0.25,0.1),(0.66,0.25,0.1),(1,0,0),(1,0,0))}, \frac{(c_2,(0,0.4,1,1)_D)}{((0.8,0.1,0.1),(0.26,0.4,0.32),(0,0,1),(0,0,1))}, \frac{(c_3,(0,0.4,1,1)_D)}{((0.3,0.1,0.6),(0.3,0.5,0.18),(0,0,1),(0,0,1))} \right\}\right), \\ &\left(k_2, \left\{ \frac{(c_1,(0,0.5,0,1)_D)}{((0.6,0.3,0.1),(0.3,0.5,0.2),(0,0,1),(0,0,1))}, \frac{(c_2,(0,0.5,0,1)_D)}{((0.5,0.25,0.3),(0.75,0.15,0.1),(1,0,0),(1,0,0))}, \frac{(c_3,(0,0.5,0,1)_D)}{((0.1,0.15,0.7),(0.15,0.15,0.7),(0,0,1),(0,0,1))} \right\}\right), \\ &\left(k_3, \left\{ \frac{(c_1,(0.5,0,1,1)_D)}{((0.2,0.2,0.6),(0.3,0.15,0.5),(0,0,1),(0,0,1))}, \frac{(c_2,(0.5,0,1,1)_D)}{((0.1,0.1,0.8),(0.5,0.15,0.4),(0,0,1),(0,0,1))}, \frac{(c_3,(0.5,0,1,1)_D)}{((0.7,0.15,0.15),(0.7,0.15,0.2),(1,0,0),(1,0,0))} \right\}\right) \Bigg\}.\end{aligned}$$

7.6. *Plithogenic Neutrosophic Soft Intersection*

Example 7.8. Consider Example 7.7 By using basic fuzzy union (maximum) and basic fuzzy intersection (minimum) we have:
$\left(F_P^N, A\right) \bigwedge_P^N \left(G_P^N, B\right) = \left(H_P^N, K\right)$ where

$$\begin{aligned}(H_P^N, K) = \Bigg\{ &\left(k_1, \left\{ \frac{(c_1,(0,0.4,1,1)_D)}{((0.5,0.25,0.3),(0.64,0.25,0.1),(1,0,0),(1,0,0))}, \frac{(c_2,(0,0.4,1,1)_D)}{((0.7,0.1,0.2),(0.24,0.4,0.38),(0,0,1),(0,0,1))}, \frac{(c_3,(0,0.4,1,1)_D)}{((0.1,0.1,0.8),(0.3,0.5,0.22),(0,0,1),(0,0,1))} \right\}\right), \\ &\left(k_2, \left\{ \frac{(c_1,(0,0.5,0,1)_D)}{((0.5,0.3,0.2),(0.3,0.5,0.2),(0,0,1),(0,0,1))}, \frac{(c_2,(0,0.5,0,1)_D)}{((0.4,0.25,0.3),(0.75,0.15,0.1),(1,0,0),(1,0,0))}, \frac{(c_3,(0,0.5,0,1)_D)}{((0.1,0.15,0.8),(0.15,0.15,0.7),(0,0,1),(0,0,1))} \right\}\right), \\ &\left(k_3, \left\{ \frac{(c_1,(0.5,0,1,1)_D)}{((0.2,0.2,0.6),(0.2,0.15,0.7),(0,0,1),(0,0,1))}, \frac{(c_2,(0.5,0,1,1)_D)}{((0.1,0.1,0.8),(0.3,0.15,0.5),(0,0,1),(0,0,1))}, \frac{(c_3,(0.5,0,1,1)_D)}{((0.7,0.15,0.15),(0.6,0.15,0.2),(1,0,0),(1,0,0))} \right\}\right) \Bigg\}.\end{aligned}$$

Proposition 7.9. *Let F_P^z, G_P^z and H_P^z be any three PSSs over U. Then the following results hold:*

(1) $F_P^z \bigvee_P^z G_P^z = G_P^z \bigvee_P^z F_P^z$,
(2) $F_P^z \bigwedge_P^z G_P^z = G_P^z \bigwedge_P^z F_P^z$,
(3) $F_P^z \bigvee_P^z (G_P^z \bigvee_P^z H_P^z) = (F_P^z \bigvee_P^z G_P^z) \bigvee_P^z H_P^z$,
(4) $F_P^z \bigwedge_P^z (G_P^z \bigwedge_P^z H_P^z) = (F_P^z \bigwedge_P^z G_P^z) \bigwedge_P^z H_P^z$.

proof From union and intersection definitions and the fact that fuzzy set, intuitionistic fuzzy and neutrosophic set are commutative and associative, we can get the proof.

Proposition 7.10. *Let (F_P^z, E), (G_P^z, E) and (H_P^z, E) be any three PSSs over U. Then the following results hold:*

(1) $F_P^z \bigvee_P^z (G_P^z \bigwedge_P^z H_P^z) = (F_P^z \bigvee_P^z G_P^z) \bigwedge_P^z (F_P^z \bigvee_P^z H_P^z)$,
(2) $F_P^z \bigwedge_P^z (G_P^z \bigvee_P^z H_P^z) = (F_P^z \bigwedge_P^z G_P^z) \bigvee_P^z (F_P^z \bigwedge_P^z H_P^z)$.

proof. Let $z \equiv Fuzzy$, $\forall x \in E$, and without loss of generality suppose $c = 0$

(1)

$$\begin{aligned}
\lambda_{F_P^F(x) \bigvee_P^F (G_P^F(x) \bigwedge_P^F H_P^F(x))}(x) &= s\left\{\lambda_{F_P^F(x)}(x), \lambda_{(G_P^F(x) \bigwedge_P^F H_P^F(x))}(x)\right\} \\
&= s\left\{\lambda_{F_P^F(x)}(x), t\left(\lambda_{G_P^F(x)}(x), \lambda_{H_P^F(x)}(x)\right)\right\} \\
&= t\{s\left(\lambda_{F_P^F(x)}(x), \lambda_{G_P^F(x)}(x)\right), s\left(\lambda_{F_P^F(x)}(x), \lambda_{H_P^F(x)}(x)\right)\} \\
&= t\left\{\lambda_{(F_P^F(x) \bigvee_P^F G_P^F(x))}(x), \lambda_{(F_P^F(x) \bigvee_P^F H_P^F(x))}(x)\right\} \\
&= \lambda_{(F_P^F(x) \bigvee_P^F G_P^F(x)) \bigwedge_P^F (F_P^F(x) \bigvee_P^F H_P^F(x))}(x)
\end{aligned}$$

For $z \equiv IF$ and $z \equiv N$ use the same method.

(2) We can use the same method in (a).

8. Plithogenic Soft Similarity

In this section we introduce a measure of similarity between two P-FSSs $(z \equiv F)$ and we leave the other similarity when $(z \equiv IF \, and \, N)$ for future research. The set theoretic approach has been taken in this regard because it is popular and very easy for calculation.

Definition 8.1. Let $\left(F_P^F, E\right)$ and $\left(G_P^F, E\right)$ be two P-FSSs over (U, E) as in Definition 6.1. *Similarity* between $\left(F_P^F, E\right)$ and $\left(G_P^F, E\right)$, denoted by $S\left(F_P^F, G_P^F\right)$, is defined as follows:

$S\left(F_P^F, G_P^F\right) = \dfrac{1}{|E|} \sum_{k=1}^{|E|} M_k$ where

$$M_k = 1 - \frac{\sum_{j=1}^{|U|} \sum_{i=1}^{|e|} |F_j(e_{ik}) - G_j(e_{ik})|}{\sum_{j=1}^{|U|} \sum_{i=1}^{|e|} |F_j(e_{ik}) + G_j(e_{ik})|}.$$

Where $e \in E$.

Definition 8.2. Let $\left(F_P^F, E\right)$, $\left(G_P^F, E\right)$ and $\left(H_P^F, E\right)$ be two P-FSSs over (U, E). We say that $\left(F_P^F, E\right)$ and $\left(G_P^F, E\right)$ are *significantly similar* if $S\left(F_P^F, G_P^F\right) \geqslant \frac{1}{2}$.

Proposition 8.3. *Let $\left(F_P^F, E\right)$ and $\left(G_P^F, E\right)$ be any two P-FSSs over (U, E) such that F_P^F or G_P^F a non-zero P-FSS. Then the following holds:*

(1) $S\left(F_P^F, G_P^F\right) = S\left(G_P^F, F_P^F\right)$,
(2) $0 \leqslant S\left(F_P^F, G_P^F\right) \leqslant 1$,
(3) $F_P^F = G_P^F \Rightarrow S\left(F_P^F, G_P^F\right) = 1$,
(4) $F_P^F \subseteq G_P^F \subseteq H_P^F \Rightarrow S\left(F_P^F, H_P^F\right) \leqslant S\left(G_P^F, H_P^F\right)$,

(5) *If* $\left(F_P^F, A\right) \bigwedge_P^F \left(G_P^F, B\right) = \emptyset \Rightarrow S\left(F_P^F, G_P^F\right) = 0.$

proof. Proofs (a)-(d) follows from Definition 8.1, We will give the proof of (e).

For the left hand side we have $\left(F_P^F, A\right) \bigwedge_P^F \left(G_P^F, B\right) = \emptyset$, then

$t\left(F_P^F(e_{ik}), G_P^F(e_{ik})\right) = 0, \forall i, k.$ Now,

by using Definition 8.1 we have

$$M_k = 1 - \frac{\sum_{j=1}^{|U|}\sum_{i=1}^{|e|}|F_j(e_{ik}) - G_j(e_{ik})|}{\sum_{j=1}^{|U|}\sum_{i=1}^{|e|}|F_j(e_{ik}) + G_j(e_{ik})|}.$$

Since $t\left(F_P^F(e_{ik}), G_P^F(e_{ik})\right) = 0$, then $F_j(e_{ik}) - G_j(e_{ik}) = F_j(e_{ik}) + G_j(e_{ik})$ and this gives

$$\frac{\sum_{j=1}^{|U|}\sum_{i=1}^{|e|}|F_j(e_{ik}) - G_j(e_{ik})|}{\sum_{j=1}^{|U|}\sum_{i=1}^{|e|}|F_j(e_{ik}) - G_j(e_{ik})|} = 1. \text{ Then}$$

$M_k = 1 - 1 = 0.$

Example 8.4. Let $U = \{c_1, c_2, c_3, c_4\}$ and $a_1 =$ speed, $a_2 =$ color, $a_3 =$ model, $a_4 =$ manufacturing year. Suppose their attributes values respectively:

Speed $\equiv A_1 = \{slow, fast, veryfast\}$,
Color $\equiv A_2 = \{white, yellow, red, black\}$,
Model $\equiv A_3 = \{model_1, model_2, model_3, model_4\}$,
manufacturing year $\equiv A_4 = \{2015 \; and \; befor, 2016, 2017, 2018, 2019\}$. Suppose

$$E = \Big\{e_1 = (slow, white, model_1, 2015 \; and \; befor), e_2 = (slow, yellow, model_3, 2017),$$
$$e_3 = (fast, red, model_4, 2018)\Big\}.$$

Suppose $\left(F_P^F, E\right)$ and $\left(G_P^F, E\right)$ are two plithogenic fuzzy soft sets over U such that

$$\left(F_P^F, E\right) = \Bigg\{\left(e_1, \left\{\frac{(c_1, (0, 0.4, 1, 1)_D)}{(0.6, 0.7, 1, 1)}, \frac{(c_2, (0, 0.4, 1, 1)_D)}{(0.7, 0.3, 0, 0)}, \frac{(c_3, (0, 0.4, 1, 1)_D)}{(0.1, 0.3, 0, 0)}, \frac{(c_4, (0, 0.4, 1, 1)_D)}{(0.7, 0.6, 1, 1)}\right\}\right),$$
$$\left(e_2, \left\{\frac{(c_1, (0, 0.5, 0, 1)_D)}{(0.6, 0.2, 0, 0)}, \frac{(c_2, (0, 0.5, 0, 1)_D)}{(0.5, 0.7, 1, 1)}, \frac{(c_3, (0, 0.5, 0, 1)_D)}{(0.1, 0.1, 0, 0)}, \frac{(c_4, (0, 0.5, 0, 1)_D)}{(0.7, 0.4, 0, 0)}\right\}\right),$$
$$\left(e_3, \left\{\frac{(c_1, (0.5, 0, 1, 1)_D)}{(0.1, 0.2, 0, 0)}, \frac{(c_2, (0.5, 0, 1, 1)_D)}{(0.1, 0.3, 0, 0)}, \frac{(c_3, (0.5, 0, 1, 1)_D)}{(0.8, 0.7, 1, 1)}, \frac{(c_4, (0.5, 0, 1, 1)_D)}{(0.1, 0.1, 0, 0)}\right\}\right)\Bigg\}.$$

$$
\begin{aligned}
(G_P^F, E) = \Bigg\{ & \left(e_1, \left\{\frac{(c_1,(0,0.4,1,1)_D)}{(0.5,0.6,1,1)}, \frac{(c_2,(0,0.4,1,1)_D)}{(0.8,0.4,0,0)}, \frac{(c_3,(0,0.4,1,1)_D)}{(0.2,0.2,0,0)}, \frac{(c_4,(0,0.4,1,1)_D)}{(0.7,0.5,1,1)}\right\}\right), \\
& \left(e_2, \left\{\frac{(c_1,(0,0.5,0,1)_D)}{(0.5,0.3,0,0)}, \frac{(c_2,(0,0.5,0,1)_D)}{(0.7,0.7,1,1)}, \frac{(c_3,(0,0.5,0,1)_D)}{(0.3,0.2,0,0)}, \frac{(c_4,(0,0.5,0,1)_D)}{(0.8,0.5,0,0)}\right\}\right), \\
& \left(e_3, \left\{\frac{(c_1,(0.5,0,1,1)_D)}{(0.5,0.6,0,0)}, \frac{(c_2,(0.5,0,1,1)_D)}{(0.8,0.3,0,0)}, \frac{(c_3,(0.5,0,1,1)_D)}{(0.2,0.7,1,1)}, \frac{(c_4,(0.5,0,1,1)_D)}{(0.6,0.3,0,0)}\right\}\right) \Bigg\}.
\end{aligned}
$$

Then we can find the similarity between F_P^F and G_P^F as follows:

$$M_1 = 1 - \frac{\sum_{j=1}^{4}\sum_{i=1}^{4} |F_j(e_{i1}) - G_j(e_{i1})|}{\sum_{j=1}^{4}\sum_{i=1}^{4} |F_j(e_{i1}) + G_j(e_{i1})|}$$

Now, for $j = 1$ we have

$$\sum_{i=1}^{4} |F_1(e_{i1}) - G_1(e_{i1})| = |(0.6 - 0.5)| + |(0.7 - 0.6)| + |(1 - 1)| + |(1 - 1)| = 0.2$$

and

$$\sum_{i=1}^{4} |F_1(e_{i1}) + G_1(e_{i1})| = |(0.6 + 0.5)| + |(0.7 + 0.6)| + |(1 + 1)| + |(1 + 1)| = 6.4.$$

For $j = 2$ we have

$$\sum_{i=1}^{4} |F_2(e_{i1}) - G_2(e_{i1})| = |(0.7 - 0.8)| + |(0.3 - 0.4)| + |(0 - 0)| + |(0 - 0)| = 0.2$$

and

$$\sum_{i=1}^{4} |F_2(e_{i1}) + G_2(e_{i1})| = |(0.7 + 0.8)| + |(0.3 + 0.4)| + |(0 + 0)| + |(0 + 0)| = 2.2.$$

For $j = 3$ we have

$$\sum_{i=1}^{4} |F_3(e_{i1}) - G_3(e_{i1})| = |(0.1 - 0.2)| + |(0.3 - 0.2)| + |(0 - 0)| + |(0 - 0)| = 0.2$$

and

$$\sum_{i=1}^{4} |F_3(e_{i1}) + G_3(e_{i1})| = |(0.1 + 0.2)| + |(0.3 + 0.2)| + |(0 + 0)| + |(0 + 0)| = 0.8.$$

For $j = 4$ we have

$$\sum_{i=1}^{4} |F_4(e_{i1}) - G_4(e_{i1})| = |(0.7-0.7)| + |(0.6-0.5)| + |(1-1)| + |(1-1)| = 0.1$$

and

$$\sum_{i=1}^{4} |F_4(e_{i1}) + G_4(e_{i1})| = |(0.7+0.7)| + |(0.6+0.5)| + |(1+1)| + |(1+1)| = 6.5.$$

Then $M_1 = 1 - \frac{\sum_{j=1}^{4}\sum_{i=1}^{4} |F_j(e_{i1}) - G_j(e_{i1})|}{\sum_{j=1}^{4}\sum_{i=1}^{4} |F_j(e_{i1}) + G_j(e_{i1})|} = 1 - \frac{[0.2+0.2+0.2+0.1]}{[6.4+2.2+0.8+6.5]} = 0.96$

Similarly we get $M_2 = 0.92$ and $M_3 = 0.78$. Then the similarity between the two P-FSSs F_P^F and G_P^F is given by

$$S\left(F_P^F, F_P^F\right) = \frac{1}{3}\sum_{k=1}^{3} M_k = \frac{0.96+0.92+0.78}{3} \cong 0.89.$$

9. Conclusions and future research

In this paper we have defined the concept of Plithogenic soft set, and gave some generalizations of this concept. We also gave examples for these concepts. We studied the basic properties of these operations. An important result of such a paper is that new questions can be used as an idea for further research, as such a research always unearths further questions. The work presented in this paper poses interesting new questions to researchers and provides the theoretical framework for further study on plithogenic soft sets. Based on the previous results we may suggest problems in relation to our research that we anticipate to venture elsewhere in the cases of operations in plithogenic soft sets, similarity in plithogenic soft sets and applications on plithogenic soft sets. We can investigate the following topics for further works:

(1) To define and study the "AND" and "OR" operations of plithogenic soft set.
(2) To define and study the application of similarity measure of plithogenic soft set on DM and MD.
(3) To define and study the application of plithogenic soft set operations on DM and MD.
(4) To generalized plithogenic soft sets to plithogenic soft multisets, expert set, possibility set and etc.

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