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Neutrosophic Triplet Partial g - Metric Spaces

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Abstract: In this chapter, neutrosophic triplet partial g - metric spaces are obtained. Then, some definitions and examples are given for neutrosophic triplet partial g - metric space. Based on these definitions, new theorems are given and proved. In addition, it is shown that neutrosophic triplet partial g - metric spaces are different from the classical g - metric spaces, neutrosophic triplet metric spaces. Thus, we add a new structure in neutrosophic triplet theory. Also, thanks to neutrosophic triplet partial g - metric space, researchers can obtain new fixed point theorems for neutrosophic triplet theory.

Keywords: g - metric space, neutrosophic triplet set, neutrosophic triplet metric space, neutrosophic triplet g - metric space, neutrosophic triplet partial g - metric space

1 Introduction

There are many uncertainties in daily life. The logic of classical mathematics is often insufficient to explain these uncertainties. Because it is not always possible to call a situation or event absolutely right or wrong. For example, we cannot always call the weather cold or hot. It can be hot for some, cold for some and cool for others. Similar situations in which we remain indecisive may appear in the professional proficiency assessment. It is often difficult to determine whether a work done or a product produced is always definite good or definite bad. Such a situation reduces the reliability of evaluating professional proficiencies. In order to cope with these uncertainties, Smarandache (1998) defined the concept of neutrosophic logic and neutrosophic set. In the concept of neutrosophic logic and neutrosophic sets, there is T degree of membership, I degree of indeterminacy and F degree of non-membership. These degrees are defined independently of each other. A neutrosophic value is shown by (T, I, F) . In other words, a condition is handled according to both its accuracy and its inaccuracy and its uncertainty. Therefore, neutrosophic logic and neutrosophic set help us to explain many uncertainties in our lives. In addition, many researchers have made studies on this theory [2-27, 50 - 56]. Recently, Baset et al. studied TOPSIS-CRITIC model for sustainable supply chain risk management [51]; Baset et al. obtained resource levelling problem in construction projects under neutrosophic environment [52].

In fact, in the concept of fuzzy logic and fuzzy sets [28] there is only a degree of membership. In addition, the concept of intuitionistic fuzzy logic and intuitionistic fuzzy set [29] includes membership degree, degree of indeterminacy and degree of non-membership. But these degrees are defined dependently of each other. Therefore, neutrosophic set is a generalized state of fuzzy and intuitionistic fuzzy set.

Also, Smarandache and Ali obtained neutrosophic triplet set (NTS) and neutrosophic triplet groups (NTG) [30]. For every element " x " in NTS A , there exist a neutral of " x " and an opposite of " x ". Also, neutral of " x " must different from the classical neutral element. Therefore, the NTS is different from the classical set. Furthermore, a neutrosophic triplet (NT) " x " is showed by $\langle x, \text{neut}(x), \text{anti}(x) \rangle$. Also, many researchers have introduced NT

structures [31-44]. Recently, Şahin, Kargin, Yücel and Özkartepe obtain neutrosophic triplet g – metric spaces [45].

Furthermore, Mustafa and Sims introduced g - metric spaces [46] in 2006. g - metric space is generalized form of metric space. The g - metric spaces have an important role in fixed point theory. Recently, researchers studied g - metric space [46-48]. Also, Salimi and Vetro introduced partial g – metric spaces [49].

In this chapter, we introduce neutrosophic triplet partial g - metric space (NTpgMS). In Section 2, we give definitions and properties for partial g - metric space (pgMS) [49], neutrosophic triplet sets (NTS) [30], neutrosophic triplet metric spaces (NTMS) [32] and neutrosophic triplet g – metric space (NTgMS) [45]. In Section 3, we define NTpgMS and we give some properties for NTpgMS. Also, we show that NTpgMSs are different from the pgMSs, NTMSs and NTgMSs, because the triangle inequality in the NTgMS, NTMS and pgMS differ from the triangle inequality in the NTpgMS. Then, we examine relationship between NTpgMS and NTgMS. In Section 4, we give conclusions.

2 Preliminaries

Definition 2.1: [30] Let $\#$ be a binary operation. A NTS $(X, \#)$ is a set such that for $x \in X$,

- i) There exists neutral of “ x ” such that $x*\text{neut}(x) = \text{neut}(x)*x = x$.
- ii) There exists anti of “ x ” such that $x*\text{anti}(x) = \text{anti}(x)*x = \text{neut}(x)$.

Also, a neutrosophic triplet “ x ” is denoted by $(x, \text{neut}(x), \text{anti}(x))$.

Definition 2.2: [32] Let $(N, *)$ be a NTS and $d_N: N \times N \rightarrow \mathbb{R}^+ \cup \{0\}$ be a function. If $d_N: N \times N \rightarrow \mathbb{R}^+ \cup \{0\}$ and $(N, *)$ satisfies the following conditions, then d_N is called NTM.

- a) $x*y \in N$;
- b) $d_N(x, y) \geq 0$;
- c) If $x = y$, then $d_N(x, y) = 0$;
- d) $d_N(x, y) = d_N(y, x)$;
- e) If there exists at least a $y \in N$ for each $x, z \in N$ such that $d_N(x, z) \leq d_N(x, z*\text{neut}(y))$, then $d_N(x, z*\text{neut}(y)) \leq d_N(x, y) + d_N(y, z)$.

In this case, $((N, *), d_N)$ is called a NTMS.

Definition 2.3: [36] Let $(N, *)$ be a NTS. If $d_p: N \times N \rightarrow \mathbb{R}^+ \cup \{0\}$ function satisfies the following conditions, then d_p is a NTpM. For all $x, y, z \in N$,

- a) $x*y \in N$,
- b) $d_p(x, y) \geq d_p(x, x) \geq 0$,
- c) If $d_p(x, y) = d_p(x, x) = d_p(y, y) = 0$, then there exists at least one pair of elements $x, y \in N$ such that $x = y$,
- d) $d_p(x, y) = d_p(y, x)$,
- e) If for each pair of $x, z \in N$, there exists at least one $y \in N$ such that $d_p(x, z) \leq d_p(x, z*\text{neut}(y))$, then $d_p(x, z*\text{neut}(y)) \leq d_p(x, y) + d_p(y, z) - d_p(y, y)$.

In this case, $((N, *, d_p)$ is called a NTpMS.

Definition 2.4: [45] Let $(X, *)$ be a NTS. If the following conditions hold, then $g: X \times X \times X \rightarrow R^+ \cup \{0\}$ is an NTgM.

- a) $\forall x, y \in X ; x * y \in X$,
- b) If $x = y = z$, then $g(x, y, z) = 0$,
- c) If $x \neq y$, then $g(x, y, z) > 0$,
- d) If $z \neq y$, then $g(x, x, y) \leq g(x, y, z)$,
- e) $g(x, y, z) = g(x, z, y) = g(y, x, z) = g(y, z, x) = g(z, x, y) = g(z, y, x)$, for every $x, y, z \in X$,
- f) If there exists at least an $a \in X$ for each $x, y, z \in X$ such that

$$g(x, y, z) \leq g(x * neut(a), y * neut(a), z * neut(a)),$$
 then

$$g(x * neut(a), y * neut(a), z * neut(a)) \leq G(x, a, a) + G(a, y, z).$$

In this case, $(X, *, g)$ is called NTgMS.

Definition 2.5:[45] Let $(X, *, g)$ be a NTgMS and $\{x_n\}$ be a sequence in this space. A point $x \in X$ is said to be limit of the sequence $\{x_n\}$, if $\lim_{n,m \rightarrow \infty} g(x, x_n, x_m) = 0$ and $\{x_n\}$ is called NT g – convergent to x .

Definition 2.6:[45] Let $(X, *, g)$ be a NTgMS and $\{x_n\}$ be a sequence in this space. $\{x_n\}$ is called NT g – Cauchy sequence if $\lim_{n,m,l \rightarrow \infty} g(x_n, x_m, x_l) = 0$.

Definition 2.7:[45] Let $(X, *, g)$ be a NTgMS. If every $\{x_n\}$ NT g – Cauchy sequence is NT g – convergent, then $(X, *, g)$ is called NT complete NTgMS.

Definition 2.8:[49] Let X be a neutrosophic triplet set. If the following conditions hold, then $g: X \times X \times X \rightarrow R^+ \cup \{0\}$ is a pgM. For all $a, x, y, z \in X$;

- a) If $x = y = z$, then $g(x, y, z) = g(x, x, x) = g(y, y, y) = g(z, z, z)$,
- b) $g(x, x, x) + g(y, y, y) + g(z, z, z) \leq 3 g(x, y, z)$,
- c) If $x \neq y$, then $\frac{1}{3} g(x, x, x) + \frac{2}{3} g(x, x, x) < g(x, y, y)$,
- d) If $y \neq z$, then $g(x, x, y) - \frac{1}{3} g(x, x, x) \leq g(x, y, z) - \frac{1}{3} g(x, x, x)$,
- e) $g(x, y, z) = g(x, z, y) = g(y, x, z) = g(y, z, x) = g(z, x, y) = g(z, y, x)$,
- f) $g(x, y, z) \leq g(x, a, a) + g(a, y, z) - g(a, a, a)$.

Definition 2.9: [49] Let (X, g) be a pgMS and $\{x_n\}$ be a sequence in this space. A point $x \in X$ is said to be limit of the sequence $\{x_n\}$, if $\lim_{n,m \rightarrow \infty} g(x, x_n, x_m) = g(x, x, x)$ and $\{x_n\}$ is called NT p– g – convergent to x .

3 Neutrosophic Triplet Partial g - Metric Space

Definition 3.1: Let $(A, *)$ be a NTS. If the function $d_{NG}: A \times A \times A \rightarrow R^+ \cup \{0\}$ satisfies the below conditions, then p_{NG} is called a NTpgMS. For $\forall x, y, z \in A$;

- a) $x * y \in A$,
- b) $0 \leq p_{NG}(x, x, x) \leq p_{NG}(x, y, z)$,
- c) If $p_{NG}(x, x, x) = p_{NG}(y, y, y) = p_{NG}(z, z, z) = p_{NG}(x, y, z) = 0$, then $x = y = z$,

- d) If $z \neq y$, then $p_{NG}(x, x, y) \leq p_{NG}(x, y, z)$,
- e) $p_{NG}(x, y, z) = p_{NG}(x, z, y) = p_{NG}(y, x, z) = p_{NG}(y, z, x) = p_{NG}(z, x, y) = p_{NG}(z, y, x)$
- f) If there exists at least an $a \in X$ for each $x, y, z \in X$ such that $p_{NG}(x, y, z) \leq p_{NG}(x * neut(a), y * neut(a), z * neut(a))$, then $p_{NG}(x * neut(a), y * neut(a), z * neut(a)) \leq p_{NG}(x, a, a) + p_{NG}(a, y, z) - p_{NG}(a, a, a)$.

In this case, $(A, *, p_{NG})$ is called NTpgMS.

Example 3.2: Let $X = \{0,3,4,6,9\}$ be a set. We show that $(X, *)$ is a NTS on Z_{12} . Also, we obtain that $neut(0) = 0, anti(0) = 0; neut(3) = 9, anti(3) = 6; neut(4) = 4, anti(4) = 4; neut(6) = 6, anti(6) = 6; neut(9) = 9, anti(9) = 9$.

Thus, $(X, .)$ is a NTS and NTs are $(0,0,0), (3,6,9), (4,4,4), (6,6,6)$ and $(9,9,9)$.

Now, we define the function $p_{NG}: X \times X \times X \rightarrow R^+ \cup \{0\}$ such that

$$p_{NG}(x, y, z) = 1 + |4^x - 4^y| + |4^x - 4^z| + |4^y - 4^z|.$$

We show that p_{NG} is a NTpgM.

- a) From Table 1, it is clear that $\forall x, y \in X; x * y \in X$.

*	0	3	4	6	9
0	0	0	0	0	0
3	0	9	0	6	3
4	0	0	4	0	0
6	0	6	0	0	6
9	0	3	0	6	9

Table 1: "*" binary operator under Z_{12}

- b) It is clear that $0 \leq p_{NG}(x, x, x) = 1 \leq p_{NG}(x, y, z)$.
- c) $p_{NG}(x, y, z) = 1 + |4^x - 4^y| + |4^x - 4^z| + |4^y - 4^z| \geq 0$.
- d) If $y \neq z$, it is clear that $p_{NG}(x, x, y) = 1 + |4^x - 4^x| + |4^x - 4^y| + |4^x - 4^y| \leq p_{NG}(x, y, z) = 1 + |4^x - 4^y| + |4^x - 4^z| + |4^y - 4^z|$.
- e) By absolute value function, it is clear that $p_{NG}(x, y, z) = p_{NG}(x, z, y) = p_{NG}(y, x, z) = p_{NG}(y, z, x) = p_{NG}(z, x, y) = p_{NG}(z, y, x)$, for every $x, y, z \in X$.
- f)

For $x = 0, y = 6, z = 3, a = 3, neut(a) = 6$;
 since $p_{NG}(0, 6, 3) \leq p_{NG}(0 * 6, 6 * 6, 3 * 6) = p_{NG}(0, 6, 6)$, we obtain that $p_{NG}(0, 6, 6) \leq p_{NG}(0, 3, 3) + p_{NG}(3, 6, 6) - p_{NG}(3, 3, 3)$.
 For $x = 0, y = 3, z = 9, a = 3, neut(a) = 6$;
 since $p_{NG}(0, 3, 9) \leq p_{NG}(0 * 6, 3 * 6, 9 * 6) = p_{NG}(0, 6, 6)$, we obtain that $p_{NG}(0, 6, 6) \leq p_{NG}(0, 3, 3) + p_{NG}(3, 6, 6) - p_{NG}(3, 3, 3)$.
 For $x = 0, y = 9, z = 3, a = 6, neut(a) = 6$;

since $p_{NG}(0, 9, 3) \leq p_{NG}(0 * 6, 9 * 6, 3 * 6) = p_{NG}(0, 6, 6)$, we obtain that

$$p_{NG}(0, 6, 6) \leq p_{NG}(0, 6, 6) + p_{NG}(6, 6, 6) - p_{NG}(6, 6, 6).$$

For $x = 0, y = 6, z = 9, a = 3, neut(a) = 6$;

since $p_{NG}(0, 6, 9) \leq p_{NG}(0 * 6, 6 * 6, 9 * 6) = p_{NG}(0, 6, 6)$, we obtain that

$$p_{NG}(0, 6, 6) \leq p_{NG}(0, 3, 3) + p_{NG}(3, 6, 6) - p_{NG}(3, 3, 3).$$

For $x = 0, y = 9, z = 6, a = 3, neut(a) = 6$;

since $p_{NG}(0, 9, 6) \leq p_{NG}(0 * 6, 9 * 6, 6 * 6) = p_{NG}(0, 6, 6)$, we obtain that

$$p_{NG}(0, 6, 6) \leq p_{NG}(0, 3, 3) + p_{NG}(3, 9, 6) - p_{NG}(3, 3, 3).$$

For $x = 3, y = 6, z = 9, a = 9, neut(a) = 9$;

since $p_{NG}(3, 6, 9) \leq p_{NG}(3 * 9, 6 * 9, 9 * 9) = p_{NG}(3, 6, 9)$, we obtain that

$$p_{NG}(9, 6, 9) \leq p_{NG}(3, 6, 9) + p_{NG}(9, 6, 9) - p_{NG}(9, 9, 9).$$

For $x = 3, y = 9, z = 6, a = 9, neut(a) = 9$;

since $p_{NG}(3, 9, 6) \leq p_{NG}(3 * 9, 9 * 9, 6 * 9) = p_{NG}(3, 9, 6)$, we obtain that

$$p_{NG}(3, 9, 6) \leq p_{NG}(3, 9, 9) + p_{NG}(9, 9, 6) - p_{NG}(9, 9, 9).$$

For $x = 3, y = 0, z = 6, a = 9, neut(a) = 9$;

since $p_{NG}(3, 0, 6) \leq p_{NG}(3 * 9, 0 * 9, 6 * 9) = p_{NG}(3, 0, 6)$, we obtain that

$$p_{NG}(3, 0, 6) \leq p_{NG}(3, 9, 9) + p_{NG}(9, 0, 6) - p_{NG}(9, 9, 9).$$

For $x = 3, y = 6, z = 0, a = 9, neut(a) = 9$;

since $p_{NG}(3, 6, 0) \leq p_{NG}(3 * 9, 6 * 9, 0 * 9) = p_{NG}(3, 6, 0)$, we obtain that

$$p_{NG}(3, 6, 0) \leq p_{NG}(3, 9, 9) + p_{NG}(9, 6, 0) - p_{NG}(9, 9, 9).$$

For $x = 3, y = 9, z = 0, a = 9, neut(a) = 9$;

since $p_{NG}(3, 9, 0) \leq p_{NG}(3 * 9, 9 * 9, 0 * 9) = p_{NG}(3, 9, 0)$, we obtain that

$$p_{NG}(3, 9, 0) \leq p_{NG}(3, 9, 9) + p_{NG}(9, 9, 0) - p_{NG}(9, 9, 9).$$

For $x = 3, y = 0, z = 9, a = 9, neut(a) = 9$;

since $p_{NG}(3, 0, 9) \leq p_{NG}(3 * 9, 0 * 9, 9 * 9) = p_{NG}(3, 0, 9)$, we obtain that

$$p_{NG}(3, 0, 9) \leq p_{NG}(3, 9, 9) + p_{NG}(9, 0, 9) - p_{NG}(9, 9, 9).$$

For $x = 6, y = 0, z = 3, a = 9, neut(a) = 9$;

since $p_{NG}(6, 0, 3) \leq p_{NG}(6 * 9, 0 * 9, 3 * 9) = p_{NG}(6, 0, 3)$, we obtain that

$$p_{NG}(6, 0, 3) \leq p_{NG}(6, 9, 9) + p_{NG}(9, 0, 3) - h_{p_{NG}}(9, 9, 9).$$

For $x = 6, y = 3, z = 0, a = 9, neut(a) = 9$;

since $p_{NG}(6, 3, 0) \leq p_{NG}(6 * 9, 3 * 9, 0 * 9) = p_{NG}(6, 3, 0)$, we obtain that

$$p_{NG}(6, 3, 0) \leq p_{NG}(6, 9, 9) + p_{NG}(9, 3, 0) - p_{NG}(3, 3, 3).$$

For $x = 6, y = 3, z = 9, a = 9, neut(a) = 9$;

since $p_{NG}(6, 3, 9) \leq p_{NG}(6 * 9, 3 * 9, 9 * 9) = p_{NG}(6, 3, 9)$, we obtain that

$$p_{NG}(6, 3, 9) \leq p_{NG}(6, 9, 9) + p_{NG}(9, 3, 9) - p_{NG}(9, 9, 9).$$

For $x = 6, y = 9, z = 3, a = 9, neut(a) = 9$;

since $p_{NG}(6, 9, 3) \leq p_{NG}(6 * 9, 9 * 9, 3 * 9) = p_{NG}(6, 9, 3)$, we obtain that

$$p_{NG}(6, 9, 3) \leq p_{NG}(6, 9, 9) + p_{NG}(9, 9, 3) - p_{NG}(9, 9, 9).$$

For $x = 6, y = 0, z = 9, a = 9, neut(a) = 9$;

since $p_{NG}(6, 0, 9) \leq p_{NG}(6 * 9, 0 * 9, 9 * 9) = p_{NG}(6, 0, 9)$, we obtain that

$$p_{NG}(6, 0, 9) \leq p_{NG}(6, 9, 9) + p_{NG}(9, 0, 9) - p_{NG}(9, 9, 9).$$

For $x = 6, y = 9, z = 0, a = 9, neut(a) = 9$;

since $p_{NG}(6, 9, 0) \leq p_{NG}(6 * 9, 9 * 9, 0 * 9) = p_{NG}(6, 9, 0)$, we obtain that

$$p_{NG}(6, 9, 0) \leq p_{NG}(6, 9, 9) + p_{NG}(9, 9, 0) - p_{NG}(9, 9, 9).$$

For $x = 9, y = 0, z = 3, a = 9, neut(a) = 9$;

since $p_{NG}(9, 0, 3) \leq p_{NG}(9 * 9, 0 * 9, 3 * 9) = p_{NG}(9, 0, 3)$, we obtain that

$$p_{NG}(9, 0, 3) \leq p_{NG}(9, 9, 9) + p_{NG}(9, 0, 3) - p_{NG}(9, 9, 9).$$

For $x = 9, y = 3, z = 0, a = 9, neut(a) = 9$;

since $p_{NG}(9, 3, 0) \leq p_{NG}(9 * 9, 3 * 9, 0 * 9) = p_{NG}(9, 3, 0)$, we obtain that

$$p_{NG}(9, 3, 0) \leq p_{NG}(9, 9, 9) + p_{NG}(9, 3, 0) - p_{NG}(9, 9, 9).$$

For $x = 9, y = 3, z = 6, a = 9, neut(a) = 9$;

since $p_{NG}(9, 3, 6) \leq p_{NG}(9 * 9, 3 * 9, 6 * 9) = p_{NG}(9, 3, 6)$, we obtain that

$$p_{NG}(9, 3, 6) \leq p_{NG}(9, 9, 9) + p_{NG}(9, 3, 6) - p_{NG}(9, 9, 9).$$

For $x = 9, y = 6, z = 3, a = 9, neut(a) = 9$;

since $p_{NG}(9, 6, 3) \leq p_{NG}(9 * 9, 6 * 9, 3 * 9) = p_{NG}(9, 6, 3)$, we obtain that

$$p_{NG}(9, 6, 3) \leq p_{NG}(9, 9, 9) + p_{NG}(9, 6, 3) - p_{NG}(9, 9, 9).$$

For $x = 9, y = 0, z = 6, a = 9, neut(a) = 9$;

since $p_{NG}(9, 0, 6) \leq p_{NG}(9 * 9, 0 * 9, 6 * 9) = p_{NG}(9, 0, 6)$, we obtain that

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For $x = 9, y = 6, z = 0, a = 9, neut(a) = 9$;

since $p_{NG}(9, 6, 0) \leq p_{NG}(9 * 9, 6 * 9, 0 * 9) = p_{NG}(9, 6, 0)$, we obtain that

$$p_{NG}(9, 6, 0) \leq p_{NG}(9, 9, 9) + p_{NG}(9, 6, 0) - p_{NG}(9, 9, 9).$$

For $x = 0, y = 0, z = 3, a = 6, neut(a) = 6$;

since $p_{NG}(0, 0, 3) \leq p_{NG}(0 * 6, 0 * 6, 3 * 6) = p_{NG}(0, 0, 6)$, we obtain that

$$p_{NG}(0, 0, 6) \leq p_{NG}(0, 6, 6) + p_{NG}(6, 0, 6) - p_{NG}(6, 6, 6).$$

For $x = 0, y = 3, z = 0, a = 6, neut(a) = 6$;

since $p_{NG}(0, 3, 0) \leq p_{NG}(0 * 6, 3 * 6, 0 * 6) = p_{NG}(0, 6, 0)$, we obtain that

$$p_{NG}(0, 6, 0) \leq p_{NG}(0, 6, 6) + p_{NG}(6, 6, 0) - p_{NG}(6, 6, 6).$$

For $x = 3, y = 0, z = 0, a = 6, neut(a) = 6$;

since $p_{NG}(3, 0, 0) \leq p_{NG}(3 * 6, 0 * 6, 0 * 6) = p_{NG}(6, 0, 0)$, we obtain that

$$p_{NG}(6, 0, 0) \leq p_{NG}(6, 6, 6) + p_{NG}(6, 0, 0) - p_{NG}(6, 6, 6).$$

For $x = 0, y = 0, z = 6, a = 9, neut(a) = 9$;

since $p_{NG}(0, 0, 6) \leq p_{NG}(0 * 9, 0 * 9, 6 * 9) = p_{NG}(0, 0, 6)$, we obtain that

$$p_{NG}(0, 0, 6) \leq p_{NG}(0, 9, 9) + p_{NG}(9, 0, 6) - p_{NG}(9, 9, 9).$$

For $x = 0, y = 6, z = 0, a = 9, neut(a) = 9$;

since $p_{NG}(0, 6, 0) \leq p_{NG}(0 * 9, 6 * 9, 0 * 9) = p_{NG}(0, 6, 0)$, we obtain that

$$p_{NG}(0, 6, 0) \leq p_{NG}(0, 9, 9) + p_{NG}(9, 6, 0) - p_{NG}(9, 9, 9).$$

For $x = 6, y = 0, z = 0, a = 9, neut(a) = 9$;

since $p_{NG}(6, 0, 0) \leq p_{NG}(6 * 9, 0 * 9, 0 * 9) = p_{NG}(6, 0, 0)$, we obtain that

$$p_{NG}(6, 0, 0) \leq p_{NG}(6, 9, 9) + p_{NG}(9, 0, 0) - p_{NG}(9, 9, 9).$$

For $x = 0, y = 0, z = 9, a = 9, neut(a) = 9$;

since $p_{NG}(0, 0, 9) \leq p_{NG}(0 * 9, 0 * 9, 9 * 9) = p_{NG}(0, 0, 9)$, we obtain that

$$p_{NG}(0, 0, 9) \leq p_{NG}(0, 9, 9) + p_{NG}(9, 0, 9) - p_{NG}(9, 9, 9).$$

For $x = 0, y = 9, z = 0, a = 9, neut(a) = 9$;

since $p_{NG}(0, 9, 0) \leq p_{NG}(0 * 9, 9 * 9, 0 * 9) = p_{NG}(0, 9, 0)$, we obtain that

$$p_{NG}(0, 9, 0) \leq p_{NG}(0, 9, 9) + p_{NG}(9, 9, 0) - p_{NG}(9, 9, 9).$$

For $x = 9, y = 0, z = 0, a = 9, neut(a) = 9$;

since $p_{NG}(9, 0, 0) \leq p_{NG}(9 * 9, 0 * 9, 0 * 9) = p_{NG}(9, 0, 0)$, we obtain that

$$p_{NG}(9, 0, 0) \leq p_{NG}(9, 9, 9) + p_{NG}(9, 0, 0) - p_{NG}(9, 9, 9).$$

For $x = 3, y = 3, z = 0, a = 6, neut(a) = 6$;

since $p_{NG}(3, 3, 0) \leq p_{NG}(3 * 6, 3 * 6, 0 * 6) = p_{NG}(6, 6, 0)$, we obtain that

$$p_{NG}(6, 6, 0) \leq p_{NG}(6, 6, 6) + p_{NG}(6, 6, 0) - p_{NG}(6, 6, 6).$$

For $x = 3, y = 0, z = 3, a = 6, neut(a) = 6$;

since $p_{NG}(3, 0, 3) \leq p_{NG}(3 * 6, 0 * 6, 3 * 6) = p_{NG}(6, 0, 6)$, we obtain that

$$p_{NG}(6, 0, 6) \leq p_{NG}(6, 6, 6) + p_{NG}(6, 0, 6) - p_{NG}(6, 6, 6).$$

For $x = 0, y = 3, z = 3, a = 6, neut(a) = 6$;

since $p_{NG}(0, 3, 3) \leq p_{NG}(0 * 6, 3 * 6, 3 * 6) = p_{NG}(0, 6, 6)$, we obtain that

$$p_{NG}(0, 6, 6) \leq p_{NG}(0, 6, 6) + p_{NG}(6, 6, 6) - p_{NG}(6, 6, 6).$$

For $x = 3, y = 3, z = 6, a = 9, neut(a) = 9$;

since $p_{NG}(3, 3, 6) \leq p_{NG}(3 * 9, 3 * 9, 6 * 9) = p_{NG}(3, 3, 6)$, we obtain that

$$p_{NG}(3, 3, 6) \leq p_{NG}(3, 9, 9) + p_{NG}(9, 3, 6) - p_{NG}(9, 9, 9).$$

For $x = 3, y = 6, z = 3, a = 9, neut(a) = 9$;

since $p_{NG}(3, 6, 3) \leq p_{NG}(3 * 9, 6 * 9, 3 * 9) = p_{NG}(3, 6, 3)$, we obtain that

$$p_{NG}(3, 6, 3) \leq p_{NG}(3, 9, 9) + p_{NG}(9, 6, 3) - p_{NG}(9, 9, 9).$$

For $x = 6, y = 3, z = 3, a = 9, neut(a) = 9$;

since $p_{NG}(6, 3, 3) \leq p_{NG}(6 * 9, 3 * 9, 3 * 9) = p_{NG}(6, 3, 3)$, we obtain that

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For $x = 3, y = 3, z = 9, a = 9, neut(a) = 9$;

since $p_{NG}(3, 3, 9) \leq p_{NG}(3 * 9, 3 * 9, 9 * 9) = p_{NG}(3, 3, 9)$, we obtain that

$$p_{NG}(3, 3, 9) \leq p_{NG}(3, 9, 9) + p_{NG}(9, 3, 9) - p_{NG}(9, 9, 9).$$

For $x = 3, y = 9, z = 3, a = 9, neut(a) = 9$;

since $p_{NG}(3, 9, 3) \leq p_{NG}(3 * 9, 9 * 9, 3 * 9) = p_{NG}(3, 9, 3)$, we obtain that

$$p_{NG}(3, 9, 3) \leq p_{NG}(3, 9, 9) + p_{NG}(9, 9, 3) - p_{NG}(9, 9, 9).$$

For $x = 9, y = 3, z = 3, a = 9, neut(a) = 9$;

since $p_{NG}(9, 3, 3) \leq p_{NG}(9 * 9, 3 * 9, 3 * 9) = p_{NG}(9, 3, 3)$, we obtain that

$$p_{NG}(9, 3, 3) \leq p_{NG}(9, 9, 9) + p_{NG}(9, 3, 3) - p_{NG}(9, 9, 9).$$

For $x = 6, y = 6, z = 0, a = 9, neut(a) = 9$;

since $p_{NG}(6, 6, 0) \leq p_{NG}(6 * 9, 6 * 9, 0 * 9) = p_{NG}(6, 6, 0)$, we obtain that

$$p_{NG}(6, 6, 0) \leq p_{NG}(6, 9, 9) + p_{NG}(9, 6, 0) - p_{NG}(9, 9, 9).$$

For $x = 6, y = 0, z = 6, a = 9, neut(a) = 9$;

since $p_{NG}(6, 0, 6) \leq p_{NG}(6 * 9, 0 * 9, 6 * 9) = p_{NG}(6, 0, 6)$, we obtain that

$$p_{NG}(6, 0, 6) \leq p_{NG}(6, 9, 9) + p_{NG}(9, 0, 6) - p_{NG}(9, 9, 9).$$

For $x = 0, y = 6, z = 6, a = 9, neut(a) = 9$;

since $p_{NG}(0, 6, 6) \leq p_{NG}(0 * 9, 6 * 9, 6 * 9) = p_{NG}(0, 6, 6)$, we obtain that

$$p_{NG}(0, 6, 6) \leq p_{NG}(0, 9, 9) + p_{NG}(9, 6, 6) - p_{NG}(9, 9, 9).$$

For $x = 6, y = 6, z = 3, a = 9, neut(a) = 9$;

since $p_{NG}(6, 6, 3) \leq p_{NG}(6 * 9, 6 * 9, 3 * 9) = p_{NG}(6, 6, 3)$, we obtain that

$$p_{NG}(6, 6, 3) \leq p_{NG}(6, 9, 9) + p_{NG}(9, 6, 3) - p_{NG}(9, 9, 9).$$

For $x = 6, y = 3, z = 6, a = 9, neut(a) = 9$;

since $p_{NG}(6, 3, 6) \leq p_{NG}(6 * 9, 3 * 9, 6 * 9) = p_{NG}(6, 3, 6)$, we obtain that

$$p_{NG}(6, 3, 6) \leq p_{NG}(6, 9, 9) + p_{NG}(9, 3, 6) - p_{NG}(9, 9, 9).$$

For $x = 3, y = 6, z = 6, a = 9, neut(a) = 9$;

since $p_{NG}(3, 6, 6) \leq p_{NG}(3 * 9, 6 * 9, 6 * 9) = p_{NG}(3, 6, 6)$, we obtain that

$$p_{NG}(3, 6, 6) \leq p_{NG}(9, 9, 9) + p_{NG}(9, 6, 6) - p_{NG}(9, 9, 9).$$

For $x = 6, y = 6, z = 3, a = 9, neut(a) = 9$;

since $p_{NG}(6, 6, 3) \leq p_{NG}(6 * 9, 6 * 9, 3 * 9) = p_{NG}(6, 6, 3)$, we obtain that

$$p_{NG}(6, 6, 3) \leq p_{NG}(6, 9, 9) + p_{NG}(9, 6, 3) - p_{NG}(9, 9, 9).$$

For $x = 6, y = 6, z = 9, a = 9, neut(a) = 9$;

since $p_{NG}(6, 6, 9) \leq p_{NG}(6 * 9, 6 * 9, 9 * 9) = p_{NG}(6, 6, 9)$, we obtain that

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For $x = 6, y = 9, z = 6, a = 9, neut(a) = 9$;

since $p_{NG}(6, 9, 6) \leq p_{NG}(6 * 9, 9 * 9, 6 * 9) = p_{NG}(6, 9, 6)$, we obtain that

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For $x = 3, y = 6, z = 6, a = 9, neut(a) = 9$;

since $p_{NG}(3, 6, 6) \leq p_{NG}(3 * 9, 6 * 9, 6 * 9) = p_{NG}(3, 6, 6)$, we obtain that

$$p_{NG}(3, 6, 6) \leq p_{NG}(3, 9, 9) + p_{NG}(9, 6, 6) - p_{NG}(9, 9, 9).$$

For $x = 9, y = 9, z = 0, a = 9, neut(a) = 9$;

since $p_{NG}(9, 9, 0) \leq p_{NG}(9 * 9, 9 * 9, 0 * 9) = p_{NG}(9, 9, 0)$, we obtain that

$$p_{NG}(9, 9, 0) \leq p_{NG}(9, 9, 9) + p_{NG}(9, 9, 0) - p_{NG}(9, 9, 9).$$

For $x = 9, y = 0, z = 9, a = 9, neut(a) = 9$;

since $p_{NG}(9, 0, 9) \leq p_{NG}(9 * 9, 0 * 9, 9 * 9) = p_{NG}(9, 0, 9)$, we obtain that

$$p_{NG}(9, 0, 9) \leq p_{NG}(9, 9, 9) + p_{NG}(9, 0, 9) - p_{NG}(9, 9, 9).$$

For $x = 0, y = 9, z = 9, a = 9, neut(a) = 9$;

since $p_{NG}(0, 9, 9) \leq p_{NG}(0 * 9, 9 * 9, 9 * 9) = p_{NG}(0, 9, 9)$, we obtain that

$$p_{NG}(0, 9, 9) \leq p_{NG}(0, 9, 9) + p_{NG}(9, 9, 9) - p_{NG}(9, 9, 9).$$

For $x = 9, y = 9, z = 3, a = 9, neut(a) = 9$;

since $p_{NG}(9, 9, 3) \leq p_{NG}(9 * 9, 9 * 9, 3 * 9) = p_{NG}(9, 9, 3)$, we obtain that

$$p_{NG}(9, 9, 3) \leq p_{NG}(9, 9, 9) + p_{NG}(9, 9, 3) - p_{NG}(9, 9, 9).$$

For $x = 9, y = 3, z = 9, a = 9, neut(a) = 9$;

since $p_{NG}(9, 3, 9) \leq p_{NG}(9 * 9, 3 * 9, 9 * 9) = p_{NG}(9, 3, 9)$, we obtain that

$$p_{NG}(9, 3, 9) \leq p_{NG}(9, 9, 9) + p_{NG}(9, 3, 9) - p_{NG}(9, 9, 9).$$

For $x = 3, y = 9, z = 9, a = 9, neut(a) = 9$;

since $p_{NG}(3, 9, 9) \leq p_{NG}(3 * 9, 9 * 9, 9 * 9) = p_{NG}(3, 9, 9)$, we obtain that

$$p_{NG}(3, 9, 9) \leq p_{NG}(3, 9, 9) + p_{NG}(9, 9, 9) - p_{NG}(9, 9, 9).$$

For $x = 9, y = 9, z = 6, a = 9, neut(a) = 9$;

since $p_{NG}(9, 9, 6) \leq p_{NG}(9 * 9, 9 * 9, 6 * 9) = p_{NG}(9, 9, 6)$, we obtain that

$$p_{NG}(9, 9, 6) \leq p_{NG}(9, 9, 9) + p_{NG}(9, 9, 6) - p_{NG}(9, 9, 9).$$

For $x = 9, y = 6, z = 9, a = 9, neut(a) = 9$;

since $p_{NG}(9, 6, 9) \leq p_{NG}(9 * 9, 6 * 9, 9 * 9) = p_{NG}(9, 6, 9)$, we obtain that

$$p_{NG}(9, 6, 9) \leq p_{NG}(9, 9, 9) + p_{NG}(9, 6, 9) - p_{NG}(9, 9, 9).$$

For $x = 6, y = 9, z = 9, a = 9, neut(a) = 9$;

since $p_{NG}(6, 9, 9) \leq p_{NG}(6 * 9, 9 * 9, 9 * 9) = p_{NG}(6, 9, 9)$, we obtain that

$$p_{NG}(6, 9, 9) \leq p_{NG}(6, 9, 9) + p_{NG}(9, 9, 9) - p_{NG}(9, 9, 9).$$

For $x = 0, y = 0, z = 0, a = 9, neut(a) = 9$;

since $p_{NG}(0, 0, 0) \leq p_{NG}(0 * 9, 0 * 9, 0 * 9) = p_{NG}(0, 0, 0)$, we obtain that

$$p_{NG}(0, 0, 0) \leq p_{NG}(0, 9, 9) + p_{NG}(9, 0, 0) - p_{NG}(9, 9, 9).$$

For $x = 3, y = 3, z = 3, a = 6, neut(a) = 6$;

since $p_{NG}(3, 3, 3) \leq p_{NG}(3 * 6, 3 * 6, 3 * 6) = p_{NG}(6, 6, 6)$, we obtain that

$$p_{NG}(6, 6, 6) \leq p_{NG}(6, 6, 6) + p_{NG}(6, 6, 6) - p_{NG}(6, 6, 6).$$

For $x = 6, y = 6, z = 6, a = 9, neut(a) = 9$;

since $p_{NG}(6, 6, 6) \leq p_{NG}(6 * 9, 6 * 9, 6 * 9) = p_{NG}(6, 6, 6)$, we obtain that

$$p_{NG}(6, 6, 6) \leq p_{NG}(6, 9, 9) + p_{NG}(9, 6, 6) - p_{NG}(9, 9, 9).$$

For $x = 9, y = 9, z = 9, a = 9, neut(a) = 9$;

since $p_{NG}(9, 9, 9) \leq p_{NG}(9 * 9, 9 * 9, 9 * 9) = p_{NG}(9, 9, 9)$, we obtain that

$$p_{NG}(9, 9, 9) \leq p_{NG}(9, 9, 9) + p_{NG}(9, 9, 9) - p_{NG}(9, 9, 9).$$

For $x = 4, y = 0, z = 0, a = 4, neut(a) = 4$;

since $p_{NG}(4, 0, 0) \leq p_{NG}(4 * 4, 0 * 4, 0 * 4) = p_{NG}(4, 0, 0)$, we obtain that

$$p_{NG}(4, 0, 0) \leq p_{NG}(4, 4, 4) + p_{NG}(4, 0, 0) - p_{NG}(4, 4, 4).$$

For $x = 4, y = 0, z = 3, a = 6, neut(a) = 6$;

since $p_{NG}(4, 0, 3) \leq p_{NG}h_{dg}(4 * 6, 0 * 6, 3 * 6) = p_{NG}(0, 0, 6)$, we obtain that

$$p_{NG}(4, 0, 3) \leq p_{NG}(4, 6, 6) + p_{NG}(6, 0, 3) - p_{NG}(6, 6, 6).$$

For $x = 4, y = 3, z = 0, a = 6, neut(a) = 6$;

since $p_{NG}(4, 3, 0) \leq p_{NG}(4 * 6, 3 * 6, 0 * 6) = p_{NG}(0, 6, 0)$, we obtain that

$$p_{NG}(0, 6, 0) \leq p_{NG}(0, 6, 6) + p_{NG}(6, 6, 0) - p_{NG}(6, 6, 6).$$

For $x = 3, y = 0, z = 4, a = 6, neut(a) = 6$;

since $p_{NG}(3, 0, 4) \leq p_{NG}(3 * 6, 0 * 6, 4 * 6) = p_{NG}(6, 0, 0)$, we obtain that

$$p_{NG}(6, 0, 0) \leq p_{NG}(6, 6, 6) + p_{NG}(6, 0, 0) - p_{NG}(6, 6, 6).$$

For $x = 3, y = 4, z = 0, a = 6, neut(a) = 6$;

since $p_{NG}(3, 4, 0) \leq p_{NG}(3 * 6, 4 * 6, 0 * 6) = p_{NG}(6, 0, 0)$, we obtain that

$$p_{NG}(6, 0, 0) \leq p_{NG}(6, 6, 6) + p_{NG}(6, 0, 0) - p_{NG}(6, 6, 6).$$

For $x = 4, y = 0, z = 4, a = 4, neut(a) = 4$;

since $p_{NG}(4, 0, 4) \leq p_{NG}(4 * 4, 0 * 4, 4 * 4) = p_{NG}(4, 0, 4)$, we obtain that

$$p_{NG}(4, 0, 4) \leq p_{NG}(4, 4, 4) + p_{NG}(4, 0, 4) - p_{NG}(4, 4, 4).$$

For $x = 4, y = 4, z = 0, a = 4, neut(a) = 4$;

since $p_{NG}(4, 4, 0) \leq p_{NG}(4 * 4, 4 * 4, 0 * 4) = p_{NG}(4, 4, 0)$, we obtain that

$$p_{NG}(4, 4, 0) \leq p_{NG}(4, 4, 4) + p_{NG}(4, 4, 0) - p_{NG}(4, 4, 4).$$

For $x = 0, y = 4, z = 4, a = 4, neut(a) = 4$;

since $p_{NG}(0, 4, 4) \leq p_{NG}(0 * 4, 4 * 4, 4 * 4) = p_{NG}(0, 4, 4)$, we obtain that

$$p_{NG}(0, 4, 4) \leq p_{NG}(0, 4, 4) + p_{NG}(4, 4, 4) - p_{NG}(4, 4, 4).$$

For $x = 4, y = 0, z = 6, a = 9, neut(a) = 9$;

since $p_{NG}(4, 0, 6) \leq p_{NG}(4 * 9, 0 * 9, 6 * 9) = p_{NG}(0, 0, 6)$, we obtain that

$$p_{NG}(0, 0, 6) \leq p_{NG}(0, 9, 9) + p_{NG}(9, 0, 6) - p_{NG}(9, 9, 9).$$

For $x = 4, y = 6, z = 0, a = 9, neut(a) = 9$;

since $p_{NG}(4, 6, 0) \leq p_{NG}(4 * 9, 6 * 9, 0 * 9) = p_{NG}(0, 6, 0)$, we obtain that

$$p_{NG}(0, 6, 0) \leq p_{NG}(0, 9, 9) + p_{NG}(9, 6, 0) - p_{NG}(9, 9, 9).$$

For $x = 6, y = 4, z = 0, a = 9, neut(a) = 9$;

since $p_{NG}(6, 4, 0) \leq p_{NG}(6 * 9, 4 * 9, 0 * 9) = p_{NG}(6, 0, 0)$, we obtain that

$$p_{NG}(6, 0, 0) \leq p_{NG}(6, 9, 9) + p_{NG}(9, 0, 0) - p_{NG}(9, 9, 9).$$

For $x = 6, y = 0, z = 4, a = 9, neut(a) = 9$;

since $p_{NG}(6, 0, 4) \leq p_{NG}(6 * 9, 0 * 9, 4 * 9) = p_{NG}(6, 0, 0)$, we obtain that

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For $x = 4, y = 0, z = 9, a = 9, neut(a) = 9$;

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For $x = 4, y = 9, z = 0, a = 9, neut(a) = 9$;

since $p_{NG}(4, 9, 0) \leq p_{NG}(4 * 9, 9 * 9, 0 * 9) = p_{NG}(0, 9, 0)$, we obtain that

$$p_{NG}(0, 9, 0) \leq p_{NG}(0, 9, 9) + p_{NG}(9, 9, 0) - p_{NG}(9, 9, 9).$$

For $x = 9, y = 0, z = 4, a = 9, neut(a) = 9$;

since $p_{NG}(9, 0, 4) \leq p_{NG}(9 * 9, 0 * 9, 4 * 9) = p_{NG}(9, 0, 0)$, we obtain that

$$p_{NG}(9, 0, 0) \leq p_{NG}(9, 9, 9) + p_{NG}(9, 0, 0) - p_{NG}(9, 9, 9).$$

For $x = 9, y = 4, z = 0, a = 9, neut(a) = 9$;

since $p_{NG}(9, 4, 0) \leq p_{NG}(9 * 9, 4 * 9, 0 * 9) = p_{NG}(9, 0, 0)$, we obtain that

$$p_{NG}(9, 0, 0) \leq p_{NG}(9, 9, 9) + p_{NG}(9, 0, 0) - p_{NG}(9, 9, 9).$$

For $x = 4, y = 3, z = 3, a = 3, neut(a) = 6$;

since $p_{NG}(4, 3, 3) \leq p_{NG}(4 * 6, 3 * 6, 3 * 6) = p_{NG}(0, 6, 6)$, we obtain that

$$p_{NG}(0, 6, 6) \leq p_{NG}(0, 6, 6) + p_{NG}(6, 6, 6) - p_{NG}(6, 6, 6).$$

For $x = 4, y = 3, z = 4, a = 3, neut(a) = 6$;

since $p_{NG}(4, 3, 4) \leq p_{NG}(4 * 6, 3 * 6, 4 * 6) = p_{NG}(0, 6, 0)$, we obtain that

$$p_{NG}(0, 6, 0) \leq p_{NG}(0, 6, 6) + p_{NG}(6, 6, 0) - p_{NG}(6, 6, 6).$$

For $x = 4, y = 4, z = 3, a = 3, neut(a) = 6$;

since $p_{NG}(4, 4, 3) \leq p_{NG}(4 * 6, 4 * 6, 3 * 6) = p_{NG}(0, 0, 6)$, we obtain that

$$p_{NG}(0, 0, 6) \leq p_{NG}(0, 6, 6) + p_{NG}(6, 0, 6) - p_{NG}(6, 6, 6).$$

For $x = 3, y = 4, z = 4, a = 3, neut(a) = 6$;

since $p_{NG}(3, 4, 4) \leq p_{NG}(3 * 6, 4 * 6, 4 * 6) = p_{NG}(6, 0, 0)$, we obtain that

$$p_{NG}(6, 0, 0) \leq p_{NG}(6, 6, 6) + p_{NG}(6, 0, 0) - p_{NG}(6, 6, 6).$$

For $x = 4, y = 3, z = 6, a = 9, neut(a) = 9$;

since $p_{NG}(4, 3, 6) \leq p_{NG}(4 * 9, 3 * 9, 6 * 9) = p_{NG}(0, 3, 6)$, we obtain that

$$p_{NG}(0, 3, 6) \leq p_{NG}(0, 9, 9) + p_{NG}(9, 3, 6) - p_{NG}(9, 9, 9).$$

For $x = 4, y = 6, z = 3, a = 9, neut(a) = 9$;

since $p_{NG}(4, 6, 3) \leq p_{NG}(4 * 9, 6 * 9, 3 * 9) = p_{NG}(0, 6, 3)$, we obtain that

$$p_{NG}(0, 6, 3) \leq p_{NG}(0, 9, 9) + p_{NG}(9, 6, 3) - p_{NG}(9, 9, 9).$$

For $x = 6, y = 4, z = 3, a = 9, neut(a) = 9$;

since $p_{NG}(6, 4, 3) \leq p_{NG}(6 * 9, 4 * 9, 3 * 9) = p_{NG}(6, 0, 3)$, we obtain that

$$p_{NG}(6, 0, 3) \leq p_{NG}(6, 9, 9) + p_{NG}(9, 0, 3) - p_{NG}(9, 9, 9).$$

For $x = 6, y = 3, z = 4, a = 9, neut(a) = 9$;

since $p_{NG}(6, 3, 4) \leq p_{NG}(6 * 9, 3 * 9, 4 * 9) = p_{NG}(6, 3, 0)$, we obtain that

$$p_{NG}(6, 3, 0) \leq p_{NG}(6, 9, 9) + p_{NG}(9, 3, 0) - p_{NG}(9, 9, 9).$$

For $x = 3, y = 6, z = 4, a = 9, neut(a) = 9$;

since $p_{NG}(3, 6, 4) \leq p_{NG}(3 * 9, 6 * 9, 4 * 9) = p_{NG}(3, 6, 0)$, we obtain that

$$p_{NG}(3, 6, 0) \leq p_{NG}(3, 9, 9) + p_{NG}(9, 6, 0) - p_{NG}(9, 9, 9).$$

For $x = 3, y = 4, z = 6, a = 9, neut(a) = 9$;

since $p_{NG}(3, 4, 6) \leq p_{NG}(3 * 9, 4 * 9, 6 * 9) = p_{NG}(3, 0, 6)$, we obtain that

$$p_{NG}(3, 0, 6) \leq p_{NG}(3, 9, 9) + p_{NG}(9, 0, 6) - p_{NG}(9, 9, 9).$$

For $x = 4, y = 3, z = 9, a = 6, neut(a) = 6$;

since $p_{NG}(4, 3, 9) \leq p_{NG}(4 * 6, 3 * 6, 9 * 6) = p_{NG}(0, 6, 6)$, we obtain that

$$p_{NG}(0, 6, 6) \leq p_{NG}(0, 6, 6) + p_{NG}(6, 6, 6) - p_{NG}(6, 6, 6).$$

For $x = 4, y = 9, z = 3, a = 6, neut(a) = 6$;

since $p_{NG}(4, 9, 3) \leq p_{NG}(4 * 6, 9 * 6, 3 * 6) = p_{NG}(0, 6, 6)$, we obtain that

$$p_{NG}(0, 6, 6) \leq p_{NG}(0, 6, 6) + p_{NG}(6, 6, 6) - p_{NG}(6, 6, 6).$$

For $x = 9, y = 4, z = 3, a = 6, neut(a) = 6$;

since $p_{NG}(9, 4, 3) \leq p_{NG}(9 * 6, 4 * 6, 3 * 6) = p_{NG}(6, 0, 6)$, we obtain that

$$p_{NG}(6, 0, 6) \leq p_{NG}(6, 6, 6) + p_{NG}(6, 0, 6) - p_{NG}(6, 6, 6).$$

For $x = 9, y = 3, z = 4, a = 6, neut(a) = 6$;

since $p_{NG}(9, 3, 4) \leq p_{NG}(9 * 6, 3 * 6, 4 * 6) = p_{NG}(6, 6, 0)$, we obtain that

$$p_{NG}(6, 6, 0) \leq p_{NG}(6, 6, 6) + p_{NG}(6, 6, 0) - p_{NG}(6, 6, 6).$$

For $x = 3, y = 9, z = 4, a = 6, neut(a) = 6$;

since $p_{NG}(3, 9, 4) \leq p_{NG}(3 * 6, 9 * 6, 4 * 6) = p_{NG}(6, 6, 0)$, we obtain that

$$p_{NG}(6, 6, 0) \leq p_{NG}(6, 6, 6) + p_{NG}(6, 6, 0) - p_{NG}(6, 6, 6).$$

For $x = 3, y = 4, z = 9, a = 6, neut(a) = 6$;

since $p_{NG}(3, 4, 9) \leq p_{NG}(3 * 6, 4 * 6, 9 * 6) = p_{NG}(6, 0, 6)$, we obtain that

$$p_{NG}(6, 0, 6) \leq p_{NG}(6, 6, 6) + p_{NG}(6, 0, 6) - p_{NG}(6, 6, 6).$$

For $x = 4, y = 4, z = 4, a = 4, neut(a) = 4$;

since $p_{NG}(4, 4, 4) \leq p_{NG}(4 * 4, 4 * 4, 4 * 4) = p_{NG}(4, 4, 4)$, we obtain that

$$p_{NG}(4, 4, 4) \leq p_{NG}(4, 4, 4) + p_{NG}(4, 4, 4) - p_{NG}(4, 4, 4).$$

For $x = 4, y = 4, z = 6, a = 9, neut(a) = 9$;

since $p_{NG}(4, 4, 6) \leq p_{NG}(4 * 9, 4 * 9, 6 * 9) = p_{NG}(0, 0, 6)$, we obtain that

$$p_{NG}(0, 0, 6) \leq p_{NG}(0, 9, 9) + p_{NG}(9, 0, 6) - p_{NG}(9, 9, 9).$$

For $x = 4, y = 6, z = 4, a = 9, neut(a) = 9$;

since $p_{NG}(4, 6, 4) \leq p_{NG}(4 * 9, 6 * 9, 4 * 9) = p_{NG}(0, 6, 0)$, we obtain that

$$p_{NG}(0, 6, 0) \leq p_{NG}(0, 9, 9) + p_{NG}(9, 6, 0) - p_{NG}(9, 9, 9).$$

For $x = 6, y = 4, z = 4, a = 9, neut(a) = 9$;

since $p_{NG}(6, 4, 4) \leq p_{NG}(6 * 9, 4 * 9, 4 * 9) = p_{NG}(6, 0, 0)$, we obtain that

$$p_{NG}(6, 0, 0) \leq p_{NG}(0, 9, 9) + p_{NG}(9, 0, 0) - p_{NG}(9, 9, 9).$$

For $x = 4, y = 4, z = 9, a = 9, neut(a) = 9$;

since $p_{NG}(4, 4, 9) \leq p_{NG}(4 * 9, 4 * 9, 9 * 9) = p_{NG}(0, 0, 9)$, we obtain that

$$p_{NG}(0, 0, 9) \leq p_{NG}(0, 9, 9) + p_{NG}(9, 0, 9) - p_{NG}(9, 9, 9).$$

For $x = 4, y = 9, z = 4, a = 9, neut(a) = 9$;

since $p_{NG}(4, 9, 4) \leq p_{NG}(4 * 9, 9 * 9, 4 * 9) = p_{NG}(0, 9, 0)$, we obtain that

$$p_{NG}(0, 9, 0) \leq p_{NG}(0, 9, 9) + p_{NG}(9, 9, 0) - p_{NG}(9, 9, 9).$$

For $x = 9, y = 4, z = 4, a = 9, neut(a) = 9$;

since $p_{NG}(9, 4, 4) \leq p_{NG}(9 * 9, 4 * 9, 4 * 9) = p_{NG}(9, 0, 0)$, we obtain that

$$p_{NG}(9, 0, 0) \leq p_{NG}(9, 9, 9) + p_{NG}(9, 0, 0) - p_{NG}(9, 9, 9).$$

For $x = 4, y = 6, z = 6, a = 9, neut(a) = 9$;

since $p_{NG}(4, 6, 6) \leq p_{NG}(4 * 9, 6 * 9, 6 * 9) = p_{NG}(0, 6, 6)$, we obtain that

$$p_{NG}(0, 6, 6) \leq p_{NG}(0, 9, 9) + h_{p_{NG}}(9, 6, 6) - p_{NG}(9, 9, 9).$$

For $x = 6, y = 4, z = 6, a = 9, neut(a) = 9$;

since $p_{NG}(6, 4, 6) \leq p_{NG}(6 * 9, 4 * 9, 6 * 9) = p_{NG}(6, 0, 6)$, we obtain that

$$p_{NG}(6, 0, 6) \leq p_{NG}(6, 9, 9) + p_{NG}(9, 0, 6) - p_{NG}(9, 9, 9).$$

For $x = 6, y = 6, z = 4, a = 9, neut(a) = 9$;

since $p_{NG}(6, 6, 4) \leq p_{NG}(6 * 9, 6 * 9, 4 * 9) = p_{NG}(6, 6, 0)$, we obtain that

$$p_{NG}(6, 6, 0) \leq p_{NG}(6, 9, 9) + p_{NG}(9, 6, 0) - p_{NG}(9, 9, 9).$$

For $x = 4, y = 6, z = 9, a = 9, neut(a) = 9$;

since $p_{NG}(4, 6, 9) \leq p_{NG}(4 * 9, 6 * 9, 9 * 9) = p_{NG}(0, 6, 9)$, we obtain that

$$p_{NG}(0, 6, 9) \leq p_{NG}(0, 9, 9) + p_{NG}(9, 6, 9) - p_{NG}(9, 9, 9).$$

For $x = 4, y = 9, z = 6, a = 9, neut(a) = 9$;

since $p_{NG}(4, 9, 6) \leq p_{NG}(4 * 9, 9 * 9, 6 * 9) = p_{NG}(0, 9, 0)$, we obtain that

$$p_{NG}(0, 9, 0) \leq p_{NG}(0, 9, 9) + p_{NG}(9, 9, 0) - p_{NG}(9, 9, 9).$$

For $x = 6, y = 9, z = 4, a = 9, neut(a) = 9$;

since $p_{NG}(6, 9, 4) \leq p_{NG}(6 * 9, 9 * 9, 4 * 9) = p_{NG}(6, 9, 0)$, we obtain that

$$p_{NG}(6, 9, 0) \leq p_{NG}(6, 9, 9) + p_{NG}(9, 9, 0) - p_{NG}(9, 9, 9).$$

For $x = 6, y = 4, z = 9, a = 9, neut(a) = 9$;

since $p_{NG}(6, 4, 9) \leq p_{NG}(6 * 9, 4 * 9, 9 * 9) = p_{NG}(6, 0, 9)$, we obtain that

$$p_{NG}(6, 0, 9) \leq p_{NG}(6, 9, 9) + p_{NG}(9, 0, 9) - p_{NG}(9, 9, 9).$$

Therefore, p_{NG} is a NTpgM.

Corollary 3.3:

1) The NTpgMS differs from the pgMS. Because, there is not a $*$ binary operation in pgMS. Also, triangle inequalities are different in this spaces.

2) The NTpgMS differs from the NTMS due to triangle inequalities.

3) The NTpgMS differs from the NTgMS. Because the triangle inequality in the NTgMS differs from the triangle inequality in the NTpgMS. Also, in a NTpgMS, it can be that $p_{NG}(x, x) \neq 0$.

Theorem 3.4: Let $((X, *), p_{NG})$ be a NTpgMS and $d_p: X \times X \rightarrow R^+ \cup \{0\}$ be a function such that

$$d_p(x, y) = p_{NG}(x, y, y) + p_{NG}(x, x, y). \text{ Then, } d_p \text{ is a NTpM.}$$

Proof:

i) Since $((X, *), p_{NG})$ is a NTpgMS, it is clear that for $\forall x, y \in X; x * y \in X$.

ii) Since p_{NG} is a NTpgMS, $0 \leq d_p(x, x) \leq d_p(x, y)$ implies that

$$0 \leq p_{NG}(x, x, x) + p_{NG}(x, x, x) \leq p_{NG}(x, y, y) + p_{NG}(x, x, y).$$

iii) Since p_{NG} is a NTpgMS, if $d_p(x, x) = d_p(y, y) = d_p(x, y) = 0$, then we obtain $x = y$.

iv) Since p_{NG} is a NTpgMS, we obtain

$$d_p(x, y) = p_{NG}(x, y, y) + p_{NG}(x, x, y) = p_{NG}(y, x, x) + p_{NG}(y, y, x) = d_p(y, x).$$

v) We assume that there exists at least an element $a \in X$ for each x, y and z such that $p_{NG}(x, y, z) \leq p_{NG}(x * \text{neut}(a), y * \text{neut}(a), z * \text{neut}(a))$. Thus, if we assume $a = x$, It is clear that $d_p(x, y) \leq d_p(x, y * \text{neut}(a))$.

Also, since $((X, *), p_{NG})$ is a NTpgMS, it is obvious that

$$p_{NG}(x, y, z) \leq p_{NG}(x * \text{neut}(a), y * \text{neut}(a), z * \text{neut}(a)) \leq p_{NG}(x, a, a) + p_{NG}(a, y, z) - p_{NG}(a, a, a).$$

Hence, we obtain

$$p_{NG}(x, y, y) + p_{NG}(x, x, y) \leq$$

$$p_{NG}(x, a, a) + p_{NG}(x, x, a) + p_{NG}(x, x, a) + p_{NG}(a, y, y) + p_{NG}(a, a, y) - p_{NG}(a, a, a) - p_{NG}(a, a, a).$$

Thus, we have $d_p(x, y) \leq d_p(x, a) + d_p(a, y) - d_p(a, a)$.

Theorem 3.5: Let $((X, *), p_{NG})$ be a NTpgMS. If for all $x \in X, p_{NG}(x, x, x) = 0$, then $((X, *), p_{NG})$ is a NTgMS.

Proof: We suppose that $(X, *)$ is a NTS and $((X, *), p_{NG})$ is a NTpgMS.

i) Since $((X, *), p_{NG})$ is a NTpgMS; then for all $x, y \in X; x * y \in X$.

ii) Since $p_{NG}(x, x, x) = 0$, it is clear that $0 \leq p_{NG}(x, x, x) = 0 \leq p_{NG}(x, y, z)$.

iii) Since $((X, *), p_{NG})$ is a NTpgMS, it is clear that if $x \neq y$, then $p_{NG}(x, y, z) > 0$.

iv) Since $((X, *), p_{NG})$ is a NTpgMS, it is clear that if $y \neq z$, then $p_{NG}(x, x, y) \leq p_{NG}(x, y, z)$.

v) Since $((X, *), p_{NG})$ is a NTpgMS, it is clear that

$$p_{NG}(x, y, z) = p_{NG}(x, z, y) = p_{NG}(y, x, z) = p_{NG}(y, z, x) = p_{NG}(z, x, y) = p_{NG}(z, y, x).$$

vi) We assume that there exists at least an element $a \in X$ for each x, y, z such that

$$p_{NG}(x, y, z) = p_{NG}(x * \text{neut}(a), y * \text{neut}(a), z * \text{neut}(a)), \text{ then}$$

$$p_{NG}(x * \text{neut}(a), y * \text{neut}(a), z * \text{neut}(a)) \leq p_{NG}(x, a, a) + p_{NG}(a, y, z) - p_{NG}(a, a, a).$$

Since $p_{NG}(x, x, x) = 0$, we obtain that

$$p_{NG}(x * \text{neut}(a), y * \text{neut}(a), z * \text{neut}(a)) \leq p_{NG}(x, a, a) + p_{NG}(a, y, z).$$

Thus, $((X, *), p_{NG})$ is a NTpgMS.

Theorem 3.6: Let $((X, *), d_{NG})$ be a NTgM. Then, the function $p_{NG}(x, y, z) = d_{NG}(x, y, z) + k$, $k \in \mathbb{R}^+$ is an NTpgMS.

Proof:

i) Since d_{NG} is a NTgMS, for all $x, y \in X$; $x*y \in X$.

ii) Since d_{NG} is a NTgMS, we obtain $d_{NG}(x, x, x) \leq d_{NG}(x, y, z)$. Thus, it is clear that

$$p_{NG}(x, x, x) = d_{NG}(x, x, x) + k \leq p_{NG}(x, y, z) = d_{NG}(x, y, z) + k.$$

iii) $p_{NG}(x, y, z) = d_{NG}(x, y, z) + k > 0$.

iv) Since d_{NG} is a NTgMS, if $y \neq z$, then $d_{NG}(x, x, y) \leq d_{NG}(x, y, z)$. Thus, it is clear that

$$p_{NG}(x, x, y) = d_{NG}(x, x, y) + k \leq p_{NG}(x, y, z) = d_{NG}(x, y, z) + k$$

v) Since d_{NG} is a NTgMS, we obtain

$$d_{NG}(x, y, z) = d_{NG}(x, z, y) = d_{NG}(y, x, z) = d_{NG}(y, z, x) = d_{NG}(z, x, y) = d_{NG}(z, y, x). \text{ Thus, it is clear that}$$

$$d_{NG}(x, y, z) + k = d_{NG}(x, z, y) + k = d_{NG}(y, x, z) + k = d_{NG}(y, z, x) + k = d_{NG}(z, x, y) + k = d_{NG}(z, y, x) + k.$$

Therefore,

$$p_{NG}(x, y, z) = p_{NG}(x, z, y) = p_{NG}(y, x, z) = p_{NG}(y, z, x) = p_{NG}(z, x, y) = p_{NG}(z, y, x).$$

vi) We assume that there exists at least an $a \in X$ for each $x, y, z \in X$ such that

$$d_{NG}(x, y, z) \leq d_{NG}(x * \text{neut}(a), y * \text{neut}(a), z * \text{neut}(a)). \text{ Thus, we obtain}$$

$$p_{NG}(x, y, z) = d_{NG}(x, y, z) + k \leq$$

$$p_{NG}(x * \text{neut}(a), y * \text{neut}(a), z * \text{neut}(a)) = d_{NG}(x * \text{neut}(a), y * \text{neut}(a), z * \text{neut}(a)) + k. \tag{1}$$

Also, since d_{NG} is a NTgMS, we obtain

$$d_{NG}(x * \text{neut}(a), y * \text{neut}(a), z * \text{neut}(a)) \leq d_{NG}(x, a, a) + d_{NG}(a, y, z). \text{ Therefore, we obtain}$$

$$d_{NG}(x * \text{neut}(a), y * \text{neut}(a), z * \text{neut}(a)) + k \leq d_{NG}(x, a, a) + k + d_{NG}(a, y, z) + k - k. \text{ Thus,}$$

$$p_{NG}(x * \text{neut}(a), y * \text{neut}(a), z * \text{neut}(a)) \leq p_{NG}(x, a, a) + p_{NG}(a, y, z) - k =$$

$$p_{NG}(x * \text{neut}(a), y * \text{neut}(a), z * \text{neut}(a)) \leq p_{NG}(x, a, a) + p_{NG}(a, y, z) - p_{NG}(a, a, a). \tag{2}$$

From (1) and (2), this condition is hold.

In this case, $((X, *), p_{NG})$ is called a NTpgMS.

Corollary 3.7: A NTpgMS can be obtained from a NTgMS.

Definition 3.8: Let $((X, *), p_{NG})$ be a NTpgMS and $\{x_n\}$ be a sequence in this space. A point $x \in X$ is said to be the limit of the sequence $\{x_n\}$, if $\lim_{n,m \rightarrow \infty} p_{NG}(x, x_n, x_m) - p_{NG}(x, x, x) = 0$ and $\{x_n\}$ is called a NT pg – convergent to x .

Definition 3.9: Let $((X, *), p_{NG})$ be a NTpgMS and $\{x_n\}$ be a sequence in this space. $\{x_n\}$ is called a NT pg – Cauchy sequence if there exists at least a $x \in X$ such that $\lim_{n,m,l \rightarrow \infty} p_{NG}(x_n, x_m, x_l) - p_{NG}(x, x, x) = 0$.

Definition 3.10: Let $((X, *), p_{NG})$ be a NTpgMS. If every $\{x_n\}$ NT pg - Cauchy sequence is a NT pg - convergent, then $((X, *), p_{NG})$ is called a NT complete NTpgMS.

Conclusion

In this study we first obtained NTpgMS. We show that NTpgMS is different from pgMS, NTgMS and NTMS. Also, we show that a NTpgMS will provide the properties of a NTgMS under which conditions are met. Thus, we added a new structure to neutrosophic triple structures. Also, thanks to neutrosophic triplet partial g – metric space, researchers can obtain new fixed point theorems for neutrosophic triplet theory and neutrosophic triplet partial g - normed space, neutrosophic triplet partial g – inner product space.

Abbreviations

gM: g - metric

gMS: g - metric space

NT: Neutrosophic triplet

NTS: Neutrosophic triplet set

NTM: Neutrosophic triplet metric

NTMS: Neutrosophic triplet metric space

NTpM: Neutrosophic triplet partial metric

NTpMS: Neutrosophic triplet partial metric space

NTgM: Neutrosophic triplet g - metric

NTgMS: Neutrosophic triplet g - metric space

NTpgM: Neutrosophic triplet partial g - metric

NTpgMS: Neutrosophic triplet partial g - metric space

References

[1] Smarandache F., Neutrosophy: Neutrosophic Probability, Set and Logic, Rehoboth, Amer. Research Press, (1998)

[2] Kandasamy W. B. V. and Smarandache F., Basic neutrosophic algebraic structures and their applications to fuzzy and neutrosophic models, Hexis, Frontigan, (2004) p 219

[3] Kandasamy W. B. V. and Smarandache F., Some neutrosophic algebraic structures and neutrosophic n-algebraic structures, Hexis, Frontigan, (2006) p 219

[4] Smarandache F. and Ali M., Neutrosophic triplet as extension of matter plasma, unmatter plasma and anti-matter plasma, APS Gaseous Electronics Conference, (2016) doi: 10.1103/BAPS.2016.GEC.HT6.110

[5] Smarandache F. and Ali M., The Neutrosophic Triplet Group and its Application to Physics, presented by F. S. to Universidad Nacional de Quilmes, Department of Science and Technology, Bernal, Buenos Aires, Argentina (02 June 2014)

[6] Smarandache F. and Ali M., Neutrosophic triplet group. Neural Computing and Applications, (2016) 1-7.

[7] Smarandache F. and Ali M., Neutrosophic Triplet Field Used in Physical Applications, (Log Number: NWS17-2017-000061), 18th Annual Meeting of the APS Northwest Section, Pacific University, Forest Grove, OR, USA (June 1-3, 2017)

[8] Smarandache F. and Ali M., Neutrosophic Triplet Ring and its Applications, (Log Number: NWS17-2017-000062), 18th Annual Meeting of the APS Northwest Section, Pacific University, Forest Grove, OR, USA (June 1-3, 2017).

[9] Şahin, M., Kargın A., Neutrosophic triplet groups based on set valued neutrosophic quadruple numbers, Neutrosophic Set and Systems, (2019) 30, 122 - 131

[10] Broumi A., Bakali A., Talea M. and Smarandache F., Single Valued Neutrosophic Graphs: Degree, Order and Size. IEEE International Conference on Fuzzy Systems, (2016) pp. 2444-2451.

[11] Broumi A., Bakali A., Talea M. and Smarandache F. Decision-Making Method Based On the Interval Valued Neutrosophic Graph, Future Technologies, IEEE International Conference on Fuzzy Systems, (2016) pp 44-50.

[12] Broumi A., Bakali A., Talea M., Smarandache F. and Vladareanu L., Computation of Shortest Path Problem in a Network with SV-Trapezoidal Neutrosophic Numbers, Proceedings of the 2016 International Conference on Advanced Mechatronic Systems, Melbourne, Australia, (2016) pp.417-422.

[13] Liu P. and Shi L., The Generalized Hybrid Weighted Average Operator Based on Interval Neutrosophic Hesitant Set and Its Application to Multiple Attribute Decision Making, Neural Computing and Applications (2015), 26(2): 457-471

[14] Liu P. and Shi L., Some Neutrosophic Uncertain Linguistic Number Heronian Mean Operators and Their Application to Multi-attribute Group Decision making, Neural Computing and Applications, (2015) doi:10.1007/s00521-015-2122-6

[15] Liu P. and Tang G., Some power generalized aggregation operators based on the interval neutrosophic numbers and their application to decision making, Journal of Intelligent & Fuzzy Systems, (2016) 30, 2517-2528

[16] Liu P. and Tang G., Multi-criteria group decision-making based on interval neutrosophic uncertain linguistic variables and Choquet integral, Cognitive Computation, (2016) 8(6) 1036-1056

[17] Liu P. and Wang Y., Interval neutrosophic prioritized OWA operator and its application to multiple attribute decision making, journal of systems science & complexity, (2016) 29(3): 681-697

[18] Liu P. and Teng F., Multiple attribute decision making method based on normal neutrosophic generalized weighted power averaging operator, Internal journal of machine learning and cybernetics, (2015) 10.1007/s13042-015-0385-y

[19] Liu P., Zhang L., Liu X., and Wang P., Multi-valued Neutrosophic Number Bonferroni mean Operators and Their Application in Multiple Attribute Group Decision Making, Internal journal of information technology & decision making, (2016) 15(5) 1181-1210

[20] Sahin M., Deli I. and Ulucay V., Similarity measure of bipolar neutrosophic sets and their application to multiple criteria decision making, Neural Comput & Applic. (2016) DOI 10. 1007/S00521

[21] Wang H., Smarandache F., Zhang Y. Q., Sunderraman R., Single valued neutrosophic sets. Multispace Multistructure, (2010) 4, 410–413.

[22] Şahin M., Olgun N, Uluçay V., Kargın A. and Smarandache F. A new similarity measure on falsity value between single valued neutrosophic sets based on the centroid points of transformed single valued neutrosophic

numbers with applications to pattern recognition, *Neutrosophic Sets and Systems*, (2017) 15, 31-48, doi: org/10.5281/zenodo570934

[23] Şahin M., Ecemiş O., Uluçay V. and Kargın A., Some new generalized aggregation operators based on centroid single valued triangular neutrosophic numbers and their applications in multi-attribute decision making, *Asian Journal of Mathematics and Computer Research*, (2017) 16(2): 63-84

[24] Chatterjee R., Majumdar P., and Samanta S. K., *Similarity Measures in Neutrosophic Sets-I. Fuzzy Multi-criteria Decision-Making Using Neutrosophic Sets*. Springer, Cham. (2019) 249-294.

[25] Mohana K., and Mohanasundari M. On Some Similarity Measures of Single Valued Neutrosophic Rough Sets. *Neutrosophic Sets and Systems*, (2019) 10

[26] Smarandache F., et al., Word-level neutrosophic sentiment similarity. *Applied Soft Computing*, (2019) 80, 167-176.

[27] Ye J. Similarity measures between interval neutrosophic sets and their applications in multicriteria decision – making. *J. Intell. Fuzzy Syst.* (2014) 26 (1) 165 – 172

[28] Zadeh A. L. Fuzzy sets, *Information and control*, (1965) 8.3 338-353,

[29] Atanassov T. K., Intuitionistic fuzzy sets, *Fuzzy Sets Syst*, (1986) 20:87–96

[30] Şahin M. and Kargın A., Neutrosophic triplet metric topology, *Neutrosophic Set and Systems*, (2019) 27, 154 -162

[31] Ali M., Smarandache F., Khan M., Study on the development of neutrosophic triplet ring and neutrosophic triplet field, *Mathematics-MDPI*, (2018) 6(4), 46

[32] Şahin M. and Kargın A., Neutrosophic triplet normed space, *Open Physics*, (2017) 15, 697-704

[33] Şahin M. and Kargın A., Neutrosophic triplet inner product space, *Neutrosophic Operational Research*, (2017) 2, 193-215,

[34] Smarandache F., Şahin M., Kargın A., Neutrosophic Triplet G- Module, *Mathematics – MDPI*, (2018) 6, 53

[35] Şahin M. and Kargın A., Neutrosophic triplet b – metric space, *Neutrosophic Triplet Research 1*, (2019) 7, 79 -89

[36] Şahin M., Kargın A., Çoban M. A., Fixed point theorem for neutrosophic triplet partial metric space, *Symmetry – MDPI*, (2018) 10, 240

[37] Şahin M., Kargın A., Neutrosophic triplet v – generalized metric space, *Axioms – MDPI*, (2018) 7, 67

[38] Şahin M. and Kargın A., Smarandache F., Neutrosophic triplet topology, *Neutrosophic Triplet Research 1*, (2019), 4, 43 - 53

[39] Şahin M., Kargın A., Neutrosophic triplet normed ring space, *Neutrosophic Set and Systems*, (2018) 21, 20 – 27

[40] Şahin M., Kargın A., Neutrosophic triplet partial inner product space, *Neutrosophic Triplet Research 1*, (2019), 10 - 21

[41] Şahin M., Kargın A., Neutrosophic triplet groups Based on set valued neutrosophic quadruple numbers, *Neutrosophic Set and Systems*, (2019) 30, 122 -131

[42] Şahin M., Kargın A. Neutrosophic triplet partial v – generalized metric space, *Neutrosophic Triplet Research 1*, (2019) 2, 22 - 34

[43] Şahin M., Kargın A., Neutrosophic triplet Lie Algebra, *Neutrosophic Triplet Research 1*, (2019) 6, 68 -78

-
- [44] Şahin M., Kargin A., Isomorphism theorems for Neutrosophic triplet G - module, *Neutrosophic Triplet Research* 1, (2019) 5, 54- 67
- [45] Şahin M., Kargin A., Yücel M. Neutrosophic triplet g - metric space, *Neutrosophic Quadruple Research* 1, (2020) 13, 181 - 202
- [46] Mustafa, Z., & Sims, B. A new approach to generalized metric spaces. *Journal of Nonlinear and convex Analysis*, (2006) 7(2), 289-297.
- [47] Mustafa, Z., Obiedat, H., & Awawdeh, F. Some fixed point theorem for mapping on complete G-metric spaces. *Fixed point theory and Applications*, 2008 (1), 189870.
- [48] Mustafa, Z., Shatanawi, W., & Bataineh, M. Existence of fixed point results in G-metric spaces. *International Journal of Mathematics and Mathematical Sciences*, (2009) doi:10.1155/2009/283028
- [49] Salimi P., Vetro P., A result of Suzuki Type in partial G – metric space, *Acta Mathematica Scientia*, (2014) 34B (2):1–11
- [50] Abdel-Basset, M., Ali M., and Atef A. "Uncertainty assessments of linear time-cost tradeoffs using neutrosophic set." *Computers & Industrial Engineering* 141 (2020): 106286.
- [51] Abdel-Basset, M., & Mohamed, R., A novel plithogenic TOPSIS-CRITIC model for sustainable supply chain risk management. *Journal of Cleaner Production*, (2020) 247, 119586.
- [52] Abdel-Basset, M., Ali M., and Atef A. "Resource levelling problem in construction projects under neutrosophic environment." *The Journal of Supercomputing* (2019) 1-25.
- [53] Abdel-Basset, M., Mohamed, M., Elhoseny, M., Chiclana, F., & Zaied, A. E. N. H. Cosine similarity measures of bipolar neutrosophic set for diagnosis of bipolar disorder diseases. *Artificial Intelligence in Medicine*, (2019) 101, 101735.
- [54] Tey, D. J. Y, et al. "A novel neutrosophic data analytic hierarchy process for multi-criteria decision making method: A case study in kuala lumpur stock exchange." *IEEE Access* (2019) 7: 53687-53697.
- [55] Son, N. T. K, et al. "On the stabilizability for a class of linear time-invariant systems under uncertainty." *Circuits, Systems, and Signal Processing*, (2020) 39.2: 919-960.
- [56] Tanuwijaya, B., et al. "A Novel Single Valued Neutrosophic Hesitant Fuzzy Time Series Model: Applications in Indonesian and Argentinian Stock Index Forecasting." *IEEE Access* (2020)

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