

3-22-2020

Neutrosophic α -Irresolute Multifunction in Neutrosophic Topological Spaces

T. Rajesh Kannan

S. Chandrasekar

Follow this and additional works at: https://digitalrepository.unm.edu/nss_journal

Recommended Citation

Kannan, T. Rajesh and S. Chandrasekar. "Neutrosophic α -Irresolute Multifunction in Neutrosophic Topological Spaces." *Neutrosophic Sets and Systems* 32, 1 (2020). https://digitalrepository.unm.edu/nss_journal/vol32/iss1/25

This Article is brought to you for free and open access by UNM Digital Repository. It has been accepted for inclusion in Neutrosophic Sets and Systems by an authorized editor of UNM Digital Repository. For more information, please contact amywinter@unm.edu, lsloane@salud.unm.edu, sarahrk@unm.edu.



Neutrosophic α -Irresolute Multifunction in Neutrosophic Topological Spaces

T.RajeshKannan¹ and S.Chandrasekar²

^{1,2} PG and Research Department of Mathematics, Arignar Anna Government Arts College, Namakkal(DT),Tamil Nadu, India, rajeshkannan03@yahoo.co.in, chandrumat@gmail.com

* Correspondence: Author (chandrumat@gmail.com)

Abstract: Aim of this present paper is, we define some new type of irresolute multifunction between the two spaces. We obtain some characterization and some properties between such as Lower & Upper α - irresolute multifunction

Keywords: Neutrosophic α -irresolute lower; Neutrosophic α irresolute upper; Neutrosophic α - closed sets; Neutrosophic topological spaces

1. Introduction

C.L. Chang [3] was introduced fuzzy topological space by using .Zadeh’s L.A [18] (uncertain) fuzzy sets. Further Coker [4] was developed the notion of Intuitionistic fuzzy topological spaces by using Atanassov’s[1] Intuitionistic fuzzy set. Neutrality the degree of indeterminacy, as an independent concept was introduced by Smarandache [7]. He also defined the Neutrosophic set of three component Neutrosophic topological spaces (t, f, i) =(Truth, Falsehood, Indeterminacy),The Neutrosophic crisp set concept converted to Neutrosophic topological spaces by A.A.Salama [13]. I.Arokiarani.[2] et al, introduced Neutrosophic α -closed sets. T Rajesh kannan[10] et.al introduced and investigated a new class of continuous multivalued function is called Neutrosophic α - continuous multivalued function in Neutrosophic topological spaces.

Aim of this present paper is, we define some new type of irresolute multifunction between the two spaces. we obtain some characterization and some properties between such as Lower & Upper α - irresolute multifunction.

2. PRELIMINARIES

In this section, we introduce the basic definition for Neutrosophic sets and its operations.

Throughout this presentation, $(R^C_1, \tau_{R^C_1})$ is namely as classical topological spaces on R^C_1 (represent as $CTSR^C_1$), $(R^N_2, \tau_{R^N_2})$ is namely as an Neutrosophic topological spaces on R^N_2 .(represent as $NUTSR^N_2$),The family of all open set in R^C_1 (α –Open in R^C_1 , semi-open in R^C_1 and pre-open in R^C_1 respectively) is denoted by $O(CTSR^C_1)$ ($\alpha O(CTSR^C_1)$, $SO(CTSR^C_1)$ and $PO(CTSR^C_1)$ respectively). The family of all Neutrosophic open set in R^N_2 (α –Open in R^N_2 , semi-open in R^N_2 , and pre-open in R^N_2 , respectively) is denoted by $O(NUTSR^N_2)$.($\alpha O(NUTSR^N_2)$, $SO(NUTSR^N_2)$ and $PO(NUTSR^N_2)$ respectively). The family of all closed set in R^C_1 (α –closed in R^C_1 , semi-closed in R^C_1 and pre-Closed in R^C_1 respectively) is denoted by $C(CTSR^C_1)$.($\alpha C(CTSR^C_1)$, $SC(CTSR^C_1)$ and $PS(CTSR^C_1)$ respectively). The family of all Neutrosophic Closed in R^N_2 (α –closed in R^N_2 , semi-closed in R^N_2 , and pre-closed in R^N_2 , respectively) is denoted by $C(NUTSR^N_2)$.($\alpha C(NUTSR^N_2)$, $SC(NUTSR^N_2)$ and $PC(NUTSR^N_2)$ respectively)

Definition 2.1 [7]

Let R^N_1 be a non-empty fixed set. A Neutrosophic set $A_{R^N_1}$ is an object having the form $A_{R^N_1} = \{ \langle \xi, \mu_{A_{R^N_1}}(\xi), \sigma_{A_{R^N_1}}(\xi), \gamma_{A_{R^N_1}}(\xi) \rangle : \xi \in R^N_1 \}$. Where $\mu_{A_{R^N_1}}(\xi): R^N_1 \rightarrow [0, 1], \sigma_{A_{R^N_1}}(\xi): R^N_1 \rightarrow [0, 1], \gamma_{A_{R^N_1}}(\xi): R^N_1 \rightarrow [0, 1]$, are represent Neutrosophic of the degree of membership function, the degree indeterminacy and the degree of non membership function respectively of each element $\xi \in R^N_1$ to the set $A_{R^N_1}$ with $0 \leq \mu_{A_{R^N_1}}(\xi) + \sigma_{A_{R^N_1}}(\xi) + \gamma_{A_{R^N_1}}(\xi) \leq 3$. This is called standard form generalized fuzzy sets. But also Neutrosophic set may be $0 \leq \mu_{A_{R^N_1}}(\xi) + \sigma_{A_{R^N_1}}(\xi) + \gamma_{A_{R^N_1}}(\xi) \leq 1$

Remark 2.2[7]

we denote $A_{R^N_1} = \{ \langle \xi, \mu_{A_{R^N_1}}(\xi), \sigma_{A_{R^N_1}}(\xi), \gamma_{A_{R^N_1}}(\xi) \rangle \}$ for the Neutrosophic set $A_{R^N_1} = \{ \langle \xi, \mu_{A_{R^N_1}}(\xi), \sigma_{A_{R^N_1}}(\xi), \gamma_{A_{R^N_1}}(\xi) \rangle : \xi \in R^N_1 \}$.

Example 2.3 [7]

Each Intuitionistic fuzzy set $A_{R^N_1}$ is a non-empty set in R^N_1 is obviously on Neutrosophic set having the form $A_{R^N_1} = \{ \langle \xi, \mu_{A_{R^N_1}}(\xi), (1 - (\mu_{A_{R^N_1}}(\xi) + \gamma_{A_{R^N_1}}(\xi))), \gamma_{A_{R^N_1}}(\xi) \rangle : \xi \in R^N_1 \}$

Definition 2.4 [7]

We must introduce the Neutrosophic set 0_N and 1_N in R^N_1 as follows :

$$0_N = \{ \langle \xi, 0, 0, 1 \rangle : \xi \in R^N_1 \} \ \& \ 1_N = \{ \langle \xi, 1, 0, 0 \rangle : \xi \in R^N_1 \}$$

Definition 2.5 [7]

Let R^N_1 be a non-empty set and Neutrosophic sets $A_{R^N_1}$ and $B_{R^N_1}$ in the form NS $A_{R^N_1} = \{ \langle \xi, \mu_{A_{R^N_1}}(\xi), \sigma_{A_{R^N_1}}(\xi), \gamma_{A_{R^N_1}}(\xi) \rangle : \xi \in R^N_1 \}$ & $B_{R^N_1} = \{ \langle \xi, \mu_{B_{R^N_1}}(\xi), \sigma_{B_{R^N_1}}(\xi), \gamma_{B_{R^N_1}}(\xi) \rangle : \xi \in R^N_1 \}$ defined as:

- (1) $A_{R^N_1} \subseteq B_{R^N_1} \Leftrightarrow \mu_{A_{R^N_1}}(\xi) \leq \mu_{B_{R^N_1}}(\xi), \sigma_{A_{R^N_1}}(\xi) \leq \sigma_{B_{R^N_1}}(\xi), \text{ and } \gamma_{A_{R^N_1}}(\xi) \geq \gamma_{B_{R^N_1}}(\xi)$
- (2) $A_{R^N_1}^c = \{ \langle \xi, \gamma_{A_{R^N_1}}(\xi), \sigma_{A_{R^N_1}}(\xi), \mu_{A_{R^N_1}}(\xi) \rangle : \xi \in R^N_1 \}$
- (3) $A_{R^N_1} \cap B_{R^N_1} = \{ \langle \xi, \mu_{A_{R^N_1}}(\xi) \wedge \mu_{B_{R^N_1}}(\xi), \sigma_{A_{R^N_1}}(\xi) \vee \sigma_{B_{R^N_1}}(\xi), \gamma_{A_{R^N_1}}(\xi) \vee \gamma_{B_{R^N_1}}(\xi) \rangle : \xi \in R^N_1 \}$
- (4) $A_{R^N_1} \cup B_{R^N_1} = \{ \langle \xi, \mu_{A_{R^N_1}}(\xi) \vee \mu_{B_{R^N_1}}(\xi), \sigma_{A_{R^N_1}}(\xi) \wedge \sigma_{B_{R^N_1}}(\xi), \gamma_{A_{R^N_1}}(\xi) \wedge \gamma_{B_{R^N_1}}(\xi) \rangle : \xi \in R^N_1 \}$
- (5) $\cap A_{R^N_1} = \{ \langle \xi, \wedge_j \mu_{A_{R^N_1}}(\xi), \wedge_j \sigma_{A_{R^N_1}}(\xi), \vee_j \gamma_{A_{R^N_1}}(\xi) \rangle : \xi \in R^N_1 \}$
- (6) $\cup A_{R^N_1} = \{ \langle \xi, \vee_j \mu_{A_{R^N_1}}(\xi), \vee_j \sigma_{A_{R^N_1}}(\xi), \wedge_j \gamma_{A_{R^N_1}}(\xi) \rangle : \xi \in R^N_1 \}$ for all $\xi \in R^N_1$

Proposition 2.6 [9]

For all $A_{R^N_1}$ and $B_{R^N_1}$ are two Neutrosophic sets then the following condition are true:

- (1) $(A_{R^N_1} \cap B_{R^N_1})^c = (A_{R^N_1})^c \cup (B_{R^N_1})^c$
- (2) $(A_{R^N_1} \cup B_{R^N_1})^c = (A_{R^N_1})^c \cap (B_{R^N_1})^c$

Definition 2.7 [10]

A Neutrosophic topology is a non -empty set R^N_1 is a family $\tau_{N_{R^N_1}}$ of Neutrosophic subsets in R^N_1 satisfying the following axioms:

- (i) $0_N, 1_N \in \tau_{N_{R^N_1}}$
- (ii) $G_{R^N_1} \cap H_{R^N_1} \in \tau_{N_{R^N_1}}$ for any $G_{R^N_1}, H_{R^N_1} \in \tau_{N_{R^N_1}}$
- (iii) $\cup_i G_{R^N_1} \in \tau_{N_{R^N_1}}$ for every $G_{R^N_1} \in \tau_{N_{R^N_1}}, I \in J$

The pair $(R^N_1, \tau_{N_{R^N_1}})$ is called a Neutrosophic topological space.

The element Neutrosophic topological spaces of $\tau_{N_{R^N_1}}$ are called Neutrosophic open sets.

A Neutrosophic set $A_{R^N_1}$ is closed if and only if $A_{R^N_1}^c$ is Neutrosophic open.

Definition 2.8[10]

Let $(R^N_1, \tau_{N_{R^N_1}})$ be Neutrosophic topological spaces.

- $A_{R^N_1} = \{ \langle \xi, \mu_{A_{R^N_1}}(\xi), \sigma_{A_{R^N_1}}(\xi), \gamma_{A_{R^N_1}}(\xi) \rangle : \xi \in R^N_1 \}$ be a Neutrosophic set in R^N_1
1. $\text{Neu-Cl}(A_{R^N_1}) = \cap \{ K_{A_{R^N_1}} : K_{A_{R^N_1}} \text{ is a Neutrosophic closed set in } R^N_1 \text{ and } A_{R^N_1} \subseteq K_{A_{R^N_1}} \}$
 2. $\text{Neu-Int}(A_{R^N_1}) = \cup \{ G_{A_{R^N_1}} : G_{A_{R^N_1}} \text{ is a Neutrosophic open set in } R^N_1 \text{ and } G_{A_{R^N_1}} \subseteq A_{R^N_1} \}$.
 3. Neutrosophic Semi-open if $A_{R^N_1} \subseteq \text{Neu-Cl}(\text{Neu-Int}(A_{R^N_1}))$.
 4. The complement of Neutrosophic Semi-open set is called Neutrosophic semi-closed.
 5. $\text{Neu-sCl}(A_{R^N_1}) = \cap \{ K_{A_{R^N_1}} : K_{A_{R^N_1}} \text{ is a Neutrosophic Semi closed set in } R^N_1 \text{ and } A_{R^N_1} \subseteq K_{A_{R^N_1}} \}$
 6. $\text{Neu-sInt}(A_{R^N_1}) = \cup \{ G_{A_{R^N_1}} : G_{A_{R^N_1}} \text{ is a Neutrosophic Semi open set in } R^C_1 \text{ and } G_{A_{R^N_1}} \subseteq A_{R^N_1} \}$.
 7. Neutrosophic α -open set if $A_{R^N_1} \subseteq \text{Neu-Int}(\text{Neu-Cl}(\text{Neu-Int}(A_{R^N_1})))$.
 8. The complement of Neutrosophic α -open set is called Neutrosophic α -closed.
 9. $\text{Neu}\alpha\text{-Cl}(A_{R^N_1}) = \cap \{ K_{A_{R^N_1}} : K_{A_{R^N_1}} \text{ is a Neutrosophic } \alpha \text{- closed set in } R^N_1 \text{ and } A_{R^N_1} \subseteq K_{A_{R^N_1}} \}$
 10. $\text{Neu}\alpha\text{-Int}(A_{R^N_1}) = \cup \{ G_{A_{R^N_1}} : G_{A_{R^N_1}} \text{ is a Neutrosophic } \alpha \text{- open set in } R^N_1 \text{ and } G_{A_{R^N_1}} \subseteq A_{R^N_1} \}$.
 11. Neutrosophic pre open set if $A_{R^N_1} \subseteq \text{Neu-Int}(\text{Neu-Cl}(A_{R^N_1}))$.
 12. The complement of Neutrosophic Pre-open set is called Neutrosophic pre-closed.
 13. $\text{Neu-pCl}(A_{R^N_1}) = \cap \{ K_{A_{R^N_1}} : K_{A_{R^N_1}} \text{ is a Neutrosophic P- closed set in } R^N_1 \text{ and } A_{R^N_1} \subseteq K_{A_{R^N_1}} \}$
 14. $\text{Neu-pInt}(A_{R^N_1}) = \cup \{ G_{A_{R^N_1}} : G_{A_{R^N_1}} \text{ is a Neutrosophic P - open set in } R^N_1 \text{ and } G_{A_{R^N_1}} \subseteq A_{R^N_1} \}$.

Remark:2.9[11]

Let $A_{R^N_1}$ be an Neutrosophic topological space $(R^N_1, \tau_{NR^C_1})$. Then

- (i) $\text{Neu}\alpha\text{-Cl}(A_{R^N_1}) = A_{R^N_1} \cup \text{Neu-Cl}(\text{Neu-Int}(\text{Neu-Cl}(A_{R^N_1})))$.
- (ii) $\text{Neu}\alpha\text{-Int}(A_{R^N_1}) = A_{R^N_1} \cap \text{Neu-Int}(\text{Neu-Cl}(\text{Neu-Int}(A_{R^N_1})))$.

Definition 2.10[9]

Take ξ_1, ξ_2, ξ_3 are belongs to real numbers 0 to 1 such that $0 \leq \xi_1 + \xi_2 + \xi_3 \leq 1$. An Neutrosophic point $\wp(\xi_1, \xi_2, \xi_3)$ is Neutrosophic set defined by

$$\wp(\xi_1, \xi_2, \xi_3) = \{ (\xi_1, \xi_2, \xi_3) \text{ if } \xi = \wp(0, 0, 1) \text{ if } \xi \neq \wp$$

Take $\wp(\xi_1, \xi_2, \xi_3) = \langle \wp_{\xi_1}, \wp_{\xi_2}, \wp_{\xi_3} \rangle$ Where $\wp_{\xi_1}, \wp_{\xi_2}, \wp_{\xi_3}$ are represent Neutrosophic the degree of membership function, the degree indeterminacy and the degree of non-membership function respectively of each element $\xi \in R^N_1$ to the set $A_{R^N_1}$

Definition:2.11

A Neutrosophic set $A_{R^N_1}$ in R^N_1 is said to be quasi-coincident (q-coincident) with a Neutrosophic set $B_{R^N_1}$ denoted by $A_{R^N_1} q B_{R^N_1}$ if and only if there exists $\xi \in R^N_1$ such that $A_{R^N_1}(\xi) + B_{R^N_1}(\xi) > 1$.

Remark: 2.12

$$A_{R^N_1} q B_{R^N_1} \Leftrightarrow A_{R^N_1} \not\subseteq B_{R^N_1}^C$$

Definition 2.13[9]

Let R^N_1 and R^N_2 be two finite sets. Define $\psi_1: R^N_1 \rightarrow R^N_2$. If $A_{R^N_2} = \{ \langle \theta, \mu_{A_{R^N_2}}(\theta), \sigma_{A_{R^N_2}}(\theta), \gamma_{A_{R^N_2}}(\theta) \rangle : \theta \in R^C_2 \}$ is an NS in R^N_2 , then the inverse image (pre image) $A_{R^N_2}$ under ψ_1 is an NS defined by $\psi_1^{-1}(A_{R^N_2}) = \langle \xi, \psi_1^{-1} \mu_{A_{R^N_2}}(\xi), \psi_1^{-1} \sigma_{A_{R^N_2}}(\xi), \psi_1^{-1} \gamma_{A_{R^N_2}}(\xi) : \xi \in R^N_1 \rangle$. Also define image NS $U = \langle \xi, \mu_U(\xi), \sigma_U(\xi), \gamma_U(\xi) : \xi \in R^N_1 \rangle$ under ψ_1 is an NS defined by $\psi_1(U) = \langle \theta, \psi_1(\mu_{A_{R^N_2}}(\theta)), \psi_1(\sigma_{A_{R^N_2}}(\theta)), \psi_1(\gamma_{A_{R^N_2}}(\theta)) : \theta \in R^N_2 \rangle$

where

$$\psi_1(\mu_{A_{R^N_2}}(\theta)) = \{ \sup \mu_{A_{R^N_2}}(\xi), \text{ if } \psi_1^{-1}(\theta) \neq \emptyset, \xi \in \psi_1^{-1}(\theta) \}$$

$$0, \text{ elsewhere}$$

$$\psi_1(\sigma_{A_{R^N_2}}(\theta)) = \{ \sup \sigma_{A_{R^N_2}}(\xi) \text{ if } \psi_1^{-1}(\theta) \neq \emptyset, \xi \in \psi_1^{-1}(\theta) \}$$

$$\psi_1(\gamma_{A_{R^{N_2}}}(\theta)) = \begin{cases} \inf(\gamma_{A_{R^{N_2}}}(\xi) \text{ if } \psi_1^{-1}(\theta) \neq \emptyset, \xi \in \psi_1^{-1}(\theta) \\ 0, \text{ Elsewhere} \end{cases}$$

Definition 2.14[2]

A mapping $\psi_1 : (R^N_1, \tau_{NR^N_1}) \rightarrow (R^N_2, \tau_{NR^N_2})$ is called a

- (1) Neutrosophic continuous (Neu-continuous) if $\psi_1^{-1}(A_{R^{N_2}}) \in C(CTSR^C_1)$ whenever $A_{R^{N_2}} \in C(NUTSR^N_2)$
- (2) Neutrosophic α -continuous (Neu α - continuous) if $\psi_1^{-1}(A_{R^{N_2}}) \in \alpha C(CTSR^C_1)$ whenever $A_{R^{N_2}} \in C(NUTSR^N_2)$
- (3) Neutrosophic Semi-continuous (Neu Semi - continuous) if $\psi_1^{-1}(A_{R^{N_2}}) \in SC(CTSR^C_1)$ whenever $A_{R^{N_2}} \in C(NUTSR^N_2)$

Definition 2.15.

Let $(R^C_1, \tau_{R^C_1})$ be a topological space in the classical sense and $(R^N_2, \tau_{NR^N_2})$ be an Neutrosophic topological space. $\Psi : (R^C_1, \tau_{R^C_1}) \rightarrow (R^N_2, \tau_{NR^N_2})$ is called a Neutrosophic multifunction if and only if for each $\xi \in R^C_1$, $\Psi(\xi)$ is a Neutrosophic set in R^N_2 .

Definition 2.16

For a Neutrosophic multifunction $\Psi : (R^C_1, \tau_{R^C_1}) \rightarrow (R^N_2, \tau_{NR^N_2})$, the upper inverse $\Psi^+(\Gamma)$ and lower inverse $\Psi^-(\Gamma)$ of a Neutrosophic set $\Gamma_{R^{N_2}}$ in R^N_2 are defined as follows:

$$\begin{aligned} \Psi^+(\Gamma_{R^{N_2}}) &= \{ \xi \in R^C_1 \mid \Psi(\xi) \leq \Gamma_{R^{N_2}} \} \text{ and} \\ \Psi^-(\Gamma_{R^{N_2}}) &= \{ \xi \in R^C_1 \mid \Psi(\xi) \supseteq \Gamma_{R^{N_2}} \}. \end{aligned}$$

Lemma 2.17.

For a Neutrosophic multifunction $\Psi : (R^C_1, \tau_{R^C_1}) \rightarrow (R^N_2, \tau_{NR^N_2})$, we have $\Psi^-(1 - \Gamma_{R^{N_2}}) = R^C_1 - \Psi^+(\Gamma_{R^{N_2}})$, for any Neutrosophic set $\Gamma_{R^{N_2}}$ in R^N_2 .

Lemma:2.18

Let $\Gamma_{R^{N_2}}$ be a subset of Neutrosophic topology $\tau_{NR^N_2}$. then

- 1. $\Gamma_{R^{N_2}}$ is α -closed in R^N_2 iff $\text{Neu-SInt}(\text{Neu-Cl}(\Gamma_{R^{N_2}})) \subset \Gamma_{R^{N_2}}$
- 2. $\text{Neu-SInt}(\text{Neu-Cl}(\Gamma_{R^{N_2}})) = \text{Neu-Cl}(\text{Neu-Int}(\text{Neu-Cl}(\Gamma_{R^{N_2}})))$

Lemma:2.19

Let $\Gamma_{R^{N_2}}$ be a subset of Neutrosophic topology $\tau_{NR^N_2}$. then below are equivalent

- 1. $\Gamma_{R^{N_2}}$ is $\text{Neu}\alpha$ -open in R^N_2
- 2. $U_{R^{N_2}} \subset \Gamma_{R^{N_2}} \subset \text{Neu-Int}(\text{Neu-Cl}(U_{R^{N_2}}))$ for some $U_{R^{N_2}}$ of R^N_2 .
- 3. $U_{R^{N_2}} \subset \Gamma_{R^{N_2}} \subset \text{Neu-S}(\text{Cl}(U_{R^{N_2}}))$ for some $U_{R^{N_2}}$ of R^N_2
- 4. $\Gamma_{R^{N_2}} \subset \text{Neu-SCl}(\text{Neu-Int}(\Gamma_{R^{N_2}}))$

Definition 2.19[6]

A Neutrosophic multifunction $\Psi : (R^C_1, \tau_{R^C_1}) \rightarrow (R^N_2, \tau_{NR^N_2})$ is said to be 1. Neutrosophic upper semi continuous at a point $\xi \in R^C_1$ if for any $\Gamma_{R^{N_2}} \in O(NUTSR^N_2)$, $\Gamma_{R^{N_2}}$ containing $\Psi(\xi)$, there exist $\xi \in U_{R^C_1} \in O(CTSR^C_1)$ such that $\Psi(U_{R^C_1}) \subset \Gamma_{R^{N_2}}$.

2. Neutrosophic lower semi continuous at a point $\xi \in R^C_1$ if for any $\Gamma_{R^{N_2}} \in O(NUTSR^N_2)$, with $\Psi(\xi) \supseteq \Gamma_{R^{N_2}}$, there exist $x \in U_{R^C_1} \in O(CTSR^C_1)$ such that $\Psi(U_{R^C_1}) \supseteq \Gamma_{R^{N_2}}$

3. Neutrosophic upper semi continuous (Neutrosophic lower semi continuous) if it is Neutrosophic upper semi continuous (Neutrosophic lower semi continuous) at each point $\xi \in R^C_1$.

4. Neutrosophic upper pre -continuous at a point $\xi \in R^C_1$ if for any $\Gamma_{R^{N_2}} \in O(NUTSR^N_2)$, Γ containing

- $\Psi(\xi)$, there exist $\xi \in U_{R^c_1} \in PO(CTSR^c_1)$ such that $\Psi(U_{R^c_1}) \subset \Gamma_{R^{N_2}}$
5. Neutrosophic lower pre-continuous at a point $\xi \in R^c_1$ if for any $\Gamma_{R^{N_2}} \in O(NUTSR^{N_2})$, with $\Psi(\xi)q\Gamma_{R^{N_2}}$, there exist $\xi \in U_{R^c_1} \in PO(CTSR^c_1)$ such that $\Psi(U_{R^c_1})q\Gamma_{R^{N_2}}$
 6. Neutrosophic upper pre-continuous (Neutrosophic lower pre-continuous) if it is Neutrosophic upper pre-continuous (Neutrosophic lower pre-continuous) at each point $\xi \in R^c_1$.
 7. Neutrosophic upper α -continuous at a point $\xi \in R^c_1$ if for any $\Gamma_{R^{N_2}} \in O(NUTSR^{N_2})$, Γ containing $\Psi(\xi)$ (that is, $F(\xi) \subset \Gamma$), there exist $\xi \in U_{R^c_1} \in \alpha O(CTSR^c_1)$ such that $\Psi(U_{R^c_1}) \subset \Gamma_{R^{N_2}}$
 8. Neutrosophic lower α -continuous at a point $\xi \in R^c_1$ if for any $\Gamma_{R^{N_2}} \in O(NUTSR^{N_2})$, with $\Psi(\xi)q\Gamma_{R^{N_2}}$, there exist $x \in U_{R^c_1} \in \alpha O(CTSR^c_1)$ such that $\Psi(U_{R^c_1})q\Gamma_{R^{N_2}}$
 9. Neutrosophic upper α -continuous (Neutrosophic lower α -continuous) if it is Neutrosophic upper α -continuous (Neutrosophic lower α -continuous) at each point $\xi \in R^c_1$.
 10. Neutrosophic upper quasi-continuous at a point $\xi \in R^c_1$ if for any $\Gamma_{R^{N_2}} \in O(NUTSR^{N_2})$, $\Gamma_{R^{N_2}}$ containing $\Psi(\xi)$, there exist $\xi \in U_{R^c_1} \in S O(CTSR^c_1)$ such that $\Psi(U_{R^c_1}) \subset \Gamma_{R^{N_2}}$
 11. Neutrosophic lower quasi semi continuous at a point $\xi \in R^c_1$ if for any $\Gamma_{R^{N_2}} \in O(NUTSR^{N_2})$, with $\Psi(\xi)q\Gamma_{R^{N_2}}$, there exist $\xi \in U_{R^c_1} \in S O(CTSR^c_1)$ such that $\Psi(U_{R^c_1})q\Gamma_{R^{N_2}}$
 12. Neutrosophic upper quasi semi continuous (Neutrosophic lower quasi semi continuous) if it is Neutrosophic upper quasi semi continuous (Neutrosophic lower quasi semi continuous) at each point $\xi \in R^c_1$.

III. Lower α -Irresolute Neutrosophic Multifunctions

In this section, we introduce the Definition for Neutrosophic Lower α -irresolute multifunction and its properties

Definition 3.1.

An Neutrosophic multifunction $\Psi : (R^c_1, \tau_{R^c_1}) \rightarrow (R^{N_2}, \tau_{NR^{N_2}})$ is said to be

- (1) Neutrosophic lower α -irresolute at a point $x_0 \in R^c_1$, if for any $\Gamma_{R^{N_2}} \in \alpha O(NUTSR^{N_2})$ such that $\Psi(x_0)q\Gamma_{R^{N_2}}$ there exists $U_{R^c_1} \in \alpha O(CTSR^c_1)$ containing x_0 such that $\Psi(\xi)q\Gamma_{R^{N_2}}, \forall \xi \in U_{R^c_1}$
- (2) Neutrosophic lower α -irresolute if it is Neutrosophic lower α -irresolute at each point of R^c_1 .

Theorem 3.2

Every Neutrosophic lower α -irresolute multifunction is Neutrosophic lower α -continuous multifunction.

Proof:

Letting $x_0 \in R^c_1$, $\Psi : (R^c_1, \tau_{R^c_1}) \rightarrow (R^{N_2}, \tau_{NR^{N_2}})$ and $\Gamma_{R^{N_2}} \in O(NUTSR^{N_2})$

such that $\Psi(x_0)q\Gamma_{R^{N_2}}$. But we know that, Every $\Gamma_{R^{N_2}}, \Gamma_{R^{N_2}} \in O(NUTSR^{N_2})$ is

$\Gamma_{R^{N_2}} \in \alpha O(NUTSR^{N_2})$. Therefore $\Gamma_{R^{N_2}} \in \alpha O(NUTSR^{N_2})$. By our assumption, Neutrosophic lower α -irresolute multifunction, there exists $U_{R^c_1} \in \alpha O(CTSR^c_1)$ containing x_0 such that $\Psi(\xi)q\Gamma_{R^{N_2}}, \forall \xi \in U_{R^c_1}$. Hence Ψ is Neutrosophic lower α -continuous multifunction at x_0 .

Theorem 3.3

Every Neutrosophic lower α -irresolute multifunction is Neutrosophic lower Pre continuous multifunction.

Proof:

Letting $x_0 \in R^c_1$, $\Psi : (R^c_1, \tau_{R^c_1}) \rightarrow (R^{N_2}, \tau_{NR^{N_2}})$ and $\Gamma_{R^{N_2}} \in O(NUTSR^{N_2})$ such that

$\Psi(x_0)q\Gamma_{R^{N_2}}$. But we know that, Every $\Gamma_{R^{N_2}}, \Gamma_{R^{N_2}} \in O(NUTSR^{N_2})$ is

$\Gamma_{R^{N_2}} \in \alpha O(NUTSR^{N_2})$. Therefore $\Gamma_{R^{N_2}} \in \alpha O(NUTSR^{N_2})$. By our assumption, Neutrosophic lower α -irresolute multifunction, there exists $U_{R^c_1} \in \alpha O(CTSR^c_1)$ containing x_0 such that $\Psi(x_0)q\Gamma_{R^{N_2}}, \forall x \in U_{R^c_1}$. every $U_{R^c_1}, U_{R^c_1} \in \alpha O(NUTSR^{N_2})$ is $U_{R^c_1} \in PO(CTSR^c_1)$.

There exists $U_{R^{c_1}} \in PO(CTSR^c_1)$ containing x_0 such that $\Psi(\xi)q\Gamma_{R^{N_2}}, \forall \xi \in U_{R^{c_1}}$. Hence Ψ is Neutrosophic lower Pre-continuous multifunction at x_0 .

Theorem 3.4

Every Neutrosophic lower α -irresolute multifunction is Neutrosophic lower quasi semi continuous multifunction.

Proof:

Letting $x_0 \in R^c_1, \Psi : (R^c_1, \tau_{R^c_1}) \rightarrow (R^{N_2}, \tau_{NR^{N_2}})$ and $\Gamma_{R^{N_2}} \in O(NUTSR^{N_2})$ such that $\Psi(x_0)q\Gamma_{R^{N_2}}$, But we know that, Every $\Gamma_{R^{N_2}}, \Gamma_{R^{N_2}} \in O(NUTSR^{N_2})$ is $\Gamma_{R^{N_2}} \in \alpha O(NUTSR^{N_2})$, Therefore $\Gamma_{R^{N_2}} \in \alpha O(NUTSR^{N_2})$. By our assumption, Neutrosophic lower α -irresolute multifunction, There exists $U_{R^{c_1}} \in \alpha O(CTSR^c_1)$ containing x_0 such that $\Psi(\xi)q\Gamma_{R^{N_2}}, \forall \xi \in U_{R^{c_1}}$. Here every $U_{R^{c_1}}, U_{R^{c_1}} \in \alpha O(NUTSR^{N_2})$ is $U_{R^{c_1}} \in SO(CTTSR^{N_2})$. Finally we get, There exists $U_{R^{c_1}} \in SO(CTSR^c_1)$ containing x_0 such that $\Psi(\xi)q\Gamma_{R^{N_2}}, \forall \xi \in U_{R^{c_1}}$ hence Ψ is Neutrosophic lower quasi semi continuous multifunction at x_0 .

Theorem 3.5

Let $\Psi : (R^c_1, \tau_{R^c_1}) \rightarrow (R^{N_2}, \tau_{NR^{N_2}})$, be an Neutrosophic multifunction and letting $x_0 \in R^c_1$. Then the following statements are equivalent:

- (a) Ψ is Neutrosophic lower α -irresolute at x_0 .
- (b) For any $\Gamma_{R^{N_2}}, \Gamma_{R^{N_2}} \in \alpha O(NUTSR^{N_2})$ with $(x_0)q\Gamma_{R^{N_2}}, \Rightarrow x_0 \in sCl(Int(\Psi^-(\Gamma_{R^{N_2}})))$.
- (c) For any $U_{R^{c_1}}, U_{R^{c_1}} \in SO(CTSR^c_1), x_0 \in U_{R^{c_1}}$ and for each $\Gamma_{R^{N_2}}, \Gamma_{R^{N_2}} \in \alpha O(NUTSR^{N_2})$ with $\Psi(x_0)q\Gamma_{R^{N_2}}$, there exists a $V_{R^{c_1}} \in O(CTSR^c_1), V_{R^{c_1}} \subset U_{R^{c_1}}$ such that $\Psi(\xi)qV_{R^{c_1}}, \forall \xi \in V_{R^{c_1}}$

Proof.

(a) \Rightarrow (b). Let $x_0 \in R^c_1$ and $\Gamma_{R^{N_2}} \in \alpha O(NUTSR^{N_2})$ such that $\Psi(x_0)q\Gamma_{R^{N_2}}$. Then by our assumption (a), we get there exists $U_{R^{c_1}} \in \alpha O(CTSR^c_1)$ such that $x_0 \in U_{R^{c_1}}$ and $\Psi(\xi)q\Gamma_{R^{N_2}}, \forall \xi \in U_{R^{c_1}}$. Thus $x_0 \in U_{R^{c_1}} \subset \Psi^-(\Gamma_{R^{N_2}}) \dots \dots (1)$ Here $U_{R^{c_1}} \in \alpha O(CTSR^c_1)$. we know that for any set $A_{R^{c_1}}, A_{R^{c_1}} \in \alpha O(CTSR^c_1) \Leftrightarrow A_{R^{c_1}} \subset sCl(Int(A_{R^{c_1}}))$. Therefore, $U_{R^{c_1}} \subset sCl(Int(U_{R^{c_1}})) \dots (2)$. from (1) and (2), we get $x_0 \in sCl(Int\Psi^-(\Gamma_{R^{N_2}}))$. Hence (b).

(b) \Rightarrow (c). Let $\Gamma_{R^{N_2}} \in \alpha O(NUTSR^{N_2})$ such that $(x_0)q\Gamma_{R^{N_2}}$, then $x_0 \in sCl(Int\Psi^-(\Gamma_{R^{N_2}}))$. Let $U_{R^{c_1}} \in SO(CTSR^c_1)$ and $x_0 \in U_{R^{c_1}}$. Then $U_{R^{c_1}} \cap Int(\Psi^-(\Gamma_{R^{N_2}})) \neq \emptyset$ and $U_{R^{c_1}} \cap Int(\Psi^-(\Gamma_{R^{N_2}}))$ is semi-open in R^c_1 . Put $V_{R^{c_1}} = Int(U_{R^{c_1}} \cap Int(\Psi^-(\Gamma_{R^{N_2}})))$, Then $V_{R^{c_1}}$ is an open set of $R^c_1, V_{R^{c_1}} \subset U_{R^{c_1}}, V_{R^{c_1}} \neq \emptyset$ and $\Psi(v)q\Gamma_{R^{N_2}}, \forall v \in V_{R^{c_1}}$. (c) \Rightarrow (a). Let $\{U_\xi\}$ be the system of the $SO(CTSR^c_1)$ containing ξ .

Let $U_{R^{c_1}} \in SO(CTSR^c_1)$ and $x_0 \in U_{R^{c_1}}$ and Any $\Gamma_{R^{N_2}} \in \alpha O(NUTSY)$ such that $\Psi(x_0)q\Gamma_{R^{N_2}}$, there exists a nonempty open set $B_U \subset U_{R^{c_1}}$ Such that $\Psi(v)q\Gamma_{R^{N_2}}, \forall v \in B_U$. Let $W_{R^{c_1}} = \cup B_U : U \in \{U_{x_0}\}$, then $W_{R^{c_1}} \in O(CTSR^c_1)$, and $x_0 \in sCl(W_{R^{c_1}})$ and $\Psi(v)q\Gamma_{R^{N_2}}, \forall v \in W_{R^{c_1}}$. Put $S_{R^{c_1}} = W_{R^{c_1}} \cup \{x_0\}$, then $W_{R^{c_1}} \subset S_{R^{c_1}} \subset sCl(W_{R^{c_1}})$. Thus $S_{R^{c_1}} \in \alpha O(CTSR^c_1), x_0 \in S_{R^{c_1}}$ and $\Psi(v)q\Gamma_{R^{N_2}}, \forall v \in S_{R^{c_1}}$. Hence Ψ is Neutrosophic lower α -irresolute at x_0 .

Theorem 3.6

Let $\Psi : (R^c_1, \tau_{R^c_1}) \rightarrow (R^{N_2}, \tau_{NR^{N_2}})$, be an Neutrosophic multifunction. Then the following statements are equivalent:

- (a) Ψ is Neutrosophic lower α -irresolute.
- (b) $\Psi^-(\lambda_{R^{N_2}}) \in \alpha O(CTSR^c_1)$, for every Neutrosophic α -open set $\lambda_{R^{N_2}}$ of R^{N_2} .
- (c) $\Psi^+(\beta_{R^{N_2}}) \in \alpha C(CTSR^c_1)$, for every Neutrosophic α -closed set $\beta_{R^{N_2}}$ of R^{N_2} .

- (d) $sInt(Cl(\Psi^+(\Gamma_{R^{N_2}}))) \subset \Psi^+(Neu - \alpha Cl(\Gamma_{R^{N_2}}))$, for each Neutrosophic set $\Gamma_{R^{N_2}}$ of R^{N_2} .
- (e) $\Psi \left(sInt \left(Cl(V_{R^{C_1}}) \right) \right) \subset Neu - \alpha Cl(\Psi(V_{R^{C_1}}))$, for each subset $V_{R^{C_1}}$ of R^{C_1} .
- (f) $\Psi \left(\alpha Cl(V_{R^{C_1}}) \right) \subset Neu - \alpha Cl(\Psi(V_{R^{C_1}}))$, for each subset $V_{R^{C_1}}$ of R^{C_1} .
- (g) $\alpha Cl(\Psi^+(\Gamma_{R^{N_2}})) \subset \Psi^+(Neu - \alpha Cl(\Gamma_{R^{N_2}}))$, for each Neutrosophic set $\Gamma_{R^{N_2}}$ of R^{N_2} .
- (h) $\Psi \left(Cl \left(Int \left(Cl(A_{R^{C_1}}) \right) \right) \right) \subset Neu - \alpha Cl(\Psi(A_{R^{C_1}}))$, for each subset $A_{R^{C_1}}$ of R^{C_1} .

Proof.

(a) \Rightarrow (b). Let $\lambda_{R^{N_2}} \in \alpha O(NUTSR^{N_2})$ and $x_0 \in \Psi^-(\lambda_{R^{N_2}})$ such that $\Psi(x_0)q\lambda_{R^{N_2}}$, since Ψ is Neutrosophic lower α -irresolute, Applying previous theorem, it follows that $x_0 \in sCl(Int(\Psi^-(\lambda_{R^{N_2}})))$. As x_0 is chosen arbitray in $\Psi^-(\lambda_{R^{N_2}})$, we have $\Psi^-(\lambda_{R^{N_2}}) \subset sCl(Int(\Psi^-(\lambda_{R^{N_2}})))$ and thus $\Psi^-(\lambda_{R^{N_2}}) \in \alpha O(CTSR^{C_1})$. Hence $\Psi^-(\lambda_{R^{N_2}})$ is an α -open in R^{C_1} . (b) \Rightarrow (a). Let $x_0 \in R^{C_1}$ and $\lambda_{R^{N_2}} \in \alpha O(NUTSR^{N_2})$ such that $\Psi(x_0)q\lambda_{R^{N_2}}$, so that $x_0 \in \Psi^-(\lambda_{R^{N_2}})$. By hypothesis $\Psi^-(\lambda_{R^{N_2}}) \in \alpha O(CTSR^{C_1})$. We have $x_0 \in \Psi^-(\lambda_{R^{N_2}}) \subset sCl(Int(\Psi^-(\lambda_{R^{N_2}})))$ and we get Ψ is Neutrosophic lower α -irresolute at x_0 . As x_0 was arbitrarily chosen, Ψ is Neutrosophic lower α -irresolute.

(b) \Leftrightarrow (c). From the definition, both are equivalent.

(c) \Rightarrow (d). Let $\Gamma_{R^{N_2}} \in (NUTS\Gamma_{R^{N_2}})$. taking closure, $Neu - \alpha Cl(\Gamma_{R^{N_2}})$ is Neutrosophic α -closed set in R^{N_2} . By our assumption, $\Psi^+(Neu - \alpha Cl(\Gamma_{R^{N_2}})) \in \alpha C(CTSR^{C_1})$.

We know that $sIntCl(A_{R^{C_1}}) \subset A_{R^{C_1}}$ iff $A_{R^{C_1}} \in \alpha C(CTSR^{C_1})$.

we obtain $\Psi^+(Neu - \alpha Cl(\Gamma_{R^{N_2}})) \supset sInt \left(Cl \left(\Psi^+(Neu - Cl(\Gamma_{R^{N_2}})) \right) \right) \supset sInt \left(Cl \left(\Psi^+(\Gamma_{R^{N_2}}) \right) \right)$.

(d) \Rightarrow (e) Suppose that (d) is satisfied and let $V_{R^{C_1}}$ be an arbitrary subset of R^{C_1} . Let us Take $\Gamma_{R^{N_2}} = \Psi(V_{R^{C_1}})$, Then $V_{R^{C_1}} \subset \Psi^+(\Gamma_{R^{N_2}})$. Therefore, by hypothesis, we have

$$sInt(Cl(V_{R^{C_1}})) \subset sInt(Cl(\Psi^+(\Gamma_{R^{N_2}}))) \subset \Psi^+(Neu - \alpha Cl(\Gamma_{R^{N_2}})).$$

Therefore, $\Psi \left(sInt \left(Cl(V_{R^{C_1}}) \right) \right) \subset \Psi \left(\Psi^+(Neu - \alpha Cl(\Gamma_{R^{N_2}})) \right) \subset Neu - \alpha Cl(\Gamma_{R^{N_2}}) = Neu - \alpha Cl \left(\Psi(V_{R^{C_1}}) \right)$.

(e) \Rightarrow (c). Suppose that (e) is true. and let $\Gamma_{R^{N_2}} \in \alpha C(NUTSR^{N_2})$. Put $V_{R^{C_1}} = \Psi^+(\Gamma)$, Then $\Psi(V_{R^{C_1}}) \subset \Gamma_{R^{N_2}}$. Therefore, by our hypothesis, we have $\Psi \left(sInt \left(Cl(V_{R^{C_1}}) \right) \right) \subset Neu - \alpha Cl \left(\Psi(V_{R^{C_1}}) \right) \subset Neu - \alpha Cl(\Gamma_{R^{N_2}}) = \Gamma_{R^{N_2}}$. And $\Psi^+(\Psi(sInt(Cl(V_{R^{C_1}})))) \subset \Psi^+(\Gamma_{R^{N_2}})$. Since we always have $\Psi^+(\Psi(sInt(Cl(V_{R^{C_1}})))) \supset sInt(Cl(V_{R^{C_1}}))$, Then must verify $\Psi^+(\Gamma_{R^{N_2}}) \supset sInt \left(Cl \left(\Psi^+(\Gamma_{R^{N_2}}) \right) \right)$. We know that $sIntClV_{R^{C_1}} \subset V_{R^{C_1}}$ iff $V_{R^{C_1}} \in \alpha C(CTSR^{C_1})$, Finally we get $F^+(\Gamma_{R^{N_2}}) \in \alpha C(CTSR^{C_1})$.

(c) \Rightarrow (f). Here $V_{R^{C_1}} \subset \Psi^+(\Psi(V_{R^{C_1}}))$, we have $V_{R^{C_1}} \subset \Psi^+(Neu - Cl(\Psi(V_{R^{C_1}})))$. Now $Neu - \alpha Cl(\Psi(V_{R^{C_1}}))$ is an Neutrosophic α -closed set in R^{N_2} and so by our assumption, $\Psi^+(Neu - Cl(\Psi(V_{R^{C_1}}))) \in \alpha C(CTSR^{C_1})$. Thus $\alpha Cl(V_{R^{C_1}}) \subset \Psi\Psi^+(Neu - \alpha Cl(\Psi(V_{R^{C_1}})))$.

Consequently, $\Psi \left(\alpha Cl(V_{R^{C_1}}) \right) \subset \Psi \left(\Psi^+(Neu - \alpha Cl(\Psi(V_{R^{C_1}}))) \right) \subset Neu - \alpha Cl(\Psi(V_{R^{C_1}}))$.

(f) \Rightarrow (c). Let $\Gamma_{R^{N_2}} \in \alpha CO(NUTSR^{N_2})$. Replacing $V_{R^{C_1}}$ by Ψ^+ we get by (f), $\Psi(\alpha Cl(\Psi^+(\Gamma_{R^{N_2}}))) \subset Neu - \alpha Cl(\Psi(\Psi^+(\Gamma_{R^{N_2}}))) \subset Neu - \alpha Cl(\Gamma_{R^{N_2}}) = \Gamma_{R^{N_2}}$. Consequently, $\alpha Cl(\Psi^+(\Gamma_{R^{N_2}})) \subset \Psi^+(\Gamma_{R^{N_2}})$. But $\Psi^+(\Gamma_{R^{N_2}}) \subset \alpha Cl(\Psi^+(\Gamma_{R^{N_2}}))$ and so, $\alpha Cl(\Psi^+(\Gamma_{R^{N_2}})) = \Psi^+(\Gamma_{R^{N_2}})$. Thus $\Psi^+(\Gamma_{R^{N_2}}) \in \alpha C(CTSR^{C_1})$.

(f) ⇒ (g). Let $\Gamma_{R^{N_2}}$ be any Neutrosophic set of R^{N_2} . Replacing $V_{R^{C_1}}$ by $\Psi^+(\Gamma_{R^{N_2}})$ we get by (f), $\Psi(\alpha Cl(\Psi^+(\Gamma_{R^{N_2}}))) \subset NEU - \alpha Cl(\Psi(\Psi^+(\Gamma_{R^{N_2}}))) \subset Neu - \alpha Cl(\Gamma_{R^{N_2}})$. Therefore we get $\alpha Cl(\Psi^+(\Gamma_{R^{N_2}})) \subset \Psi^+(Neu - \alpha Cl(\Gamma_{R^{N_2}}))$.

(g) ⇒ (f). Replacing $\Gamma_{R^{N_2}}$ by $\Psi(V_{R^{C_1}})$, where $V_{R^{C_1}}$ is a subset of R^{C_1} , we get by our result (g), $\alpha Cl(V_{R^{C_1}}) \subset \alpha Cl(\Psi^+(\Psi(V_{R^{C_1}}))) = \alpha Cl(\Psi^+(\Gamma_{R^{N_2}})) = \Psi^+(\alpha Cl(\Gamma_{R^{N_2}})) = \Psi^+(\alpha Cl(\Psi(V_{R^{C_1}})))$. Thus $\Psi(\alpha Cl(V_{R^{C_1}})) \subset \Psi(\Psi^+(\alpha Cl(\Psi(V_{R^{C_1}})))) \subset Neu \alpha Cl(\Psi(V_{R^{C_1}}))$.

(e) ⇒ (h). Clearly is true from the above result.

(h) ⇒ (a). Let $\xi \in R^{C_1}$ and $\Gamma_{R^{N_2}} \in \alpha O(NUTSR^{N_2})$ such that $(\xi)q\Gamma_{R^{N_2}}$. Then $\xi \in \Psi^-(\Gamma_{R^{N_2}})$. We shall show that $\Psi^-(\Gamma_{R^{N_2}}) \in \alpha O(CTSR^{C_1})$. By the hypothesis, We have $\Psi(Cl(Int(Cl(\Psi^+(\Gamma_{R^{N_2}}^c)))))) \subset Neu - \alpha Cl(\Psi(\Psi^+(\Gamma_{R^{N_2}}^c))) \subset (\Gamma_{R^{N_2}}^c)$, Which implies $Cl(Int(Cl(\Psi^+(\Gamma_{R^{N_2}}^c)))) \subset \Psi^+(\Gamma_{R^{N_2}}^c) \subset (\Psi^-(\Gamma_{R^{N_2}}))^c$. Therefore, we obtain $\Psi^-(\Gamma_{R^{N_2}}) \subset Int(Cl(Int(\Psi^-(\Gamma_{R^{N_2}}))))$. Hence $\Psi^-(\Gamma_{R^{N_2}}) \in \alpha O(CTSR^{C_1})$. Put $U_{R^{C_1}} = \Psi^-(\Gamma_{R^{N_2}})$. Then $\xi \in U_{R^{C_1}} \in \alpha O(CTSR^{C_1})$ and $\Psi(u)q\Gamma_{R^{N_2}}$ for every $u \in U_{R^{C_1}}$. Therefore Ψ is Neutrosophic lower α -irresolute.

IV. Upper α -Irresolute Neutrosophic Multifunctions

In this section, we introduce the Definition for Neutrosophic upper α -irresolute multifunction and its properties

Definition 4.1.

An Neutrosophic multifunction $\Psi: (R^{C_1}, \tau_{R^{C_1}}) \rightarrow (R^{N_2}, \tau_{NR^{N_2}})$, is called

- (a) Neutrosophic upper α -irresolute at a point $x_0 \in R^{C_1}$, if for any $\Gamma_{R^{N_2}}, \Gamma_{R^{N_2}} \in \alpha O(NUTSR^{N_2})$ such that $\Psi(x_0) \subset \Gamma_{R^{N_2}}$ there exists $U_{R^{C_1}} \in \alpha O(CTSR^{C_1})$ containing x_0 such that $\Psi(U_{R^{C_1}}) \subset \Gamma_{R^{N_2}}$.
- (b) Neutrosophic upper α -irresolute if it is satisfied that property at each point of R^{C_1} .

Theorem 4.2

Every Neutrosophic upper α -irresolute multifunction is Neutrosophic upper α -continuous multifunction.

Proof:

Letting $x_0 \in R^{C_1}$, $\Psi: (R^{C_1}, \tau_{R^{C_1}}) \rightarrow (R^{N_2}, \tau_{NR^{N_2}})$ and $\Gamma_{R^{N_2}} \in \alpha O(NUTSR^{N_2})$ such that $\Psi(x_0) \subset \Gamma_{R^{N_2}}$. But we know that, every $\Gamma_{R^{N_2}}, \Gamma_{R^{N_2}} \in \alpha O(NUTSR^{N_2})$ is $\Gamma_{R^{N_2}} \in \alpha O(NUTSR^{N_2})$, Therefore $\Gamma_{R^{N_2}} \in \alpha O(NUTSR^{N_2})$. By our assumption, Neutrosophic lower α -irresolute multifunction, There exists $U_{R^{C_1}} \in \alpha O(CTSR^{C_1})$ containing x_0 such that $\Psi(\xi) \subset \Gamma_{R^{N_2}}, \forall \xi \in U_{R^{C_1}}$. Hence Ψ is Neutrosophic lower α -continuous multifunction at x_0 .

Theorem 4.3

Every Neutrosophic upper α -irresolute multifunction is Neutrosophic upper Pre-continuous multifunction.

Proof:

Letting $x_0 \in R^{C_1}$, $\Psi: (R^{C_1}, \tau_{R^{C_1}}) \rightarrow (R^{N_2}, \tau_{NR^{N_2}})$ and $\Gamma_{R^{N_2}} \in \alpha O(NUTSR^{N_2})$ such that $\Psi(x_0) \subset \Gamma_{R^{N_2}}$. But we know that, Every $\Gamma_{R^{N_2}}, \Gamma_{R^{N_2}} \in \alpha O(NUTSR^{N_2})$ is $\Gamma_{R^{N_2}} \in \alpha O(NUTSR^{N_2})$. Therefore $\Gamma_{R^{N_2}} \in \alpha O(NUTSR^{N_2})$. By our assumption, Neutrosophic upper α -irresolute multifunction, There exists $U_{R^{C_1}} \in \alpha O(CTSR^{C_1})$ containing x_0 such that $\Psi(\xi) \subset \Gamma_{R^{N_2}}, \forall \xi \in U_{R^{C_1}}$, every $U_{R^{C_1}}, U_{R^{C_1}} \in \alpha O(NUTSR^{N_2})$ is $U_{R^{C_1}} \in PO(CTSR^{C_1})$. There exists $U_{R^{C_1}} \in PO(CTSR^{C_1})$ containing x_0 such that $\Psi(\xi) \subset \Gamma_{R^{N_2}}, \forall \xi \in U_{R^{C_1}}$ hence Ψ is Neutrosophic upper Pre-continuous multifunction at x_0 .

Theorem 4.4

Every Neutrosophic upper α -irresolute multifunction is Neutrosophic upper quasi semi continuous multifunction.

Proof:

Letting $x_0 \in R^C_1$, $\Psi : (R^C_1, \tau_{R^C_1}) \rightarrow (R^{N_2}, \tau_{NR^{N_2}})$ and $\Gamma_{R^{N_2}} \in O(NUTSR^{N_2})$ such that $\Psi(x_0) \subset \Gamma_{R^{N_2}}$. But we know that, Every $\Gamma_{R^{N_2}}, \Gamma_{R^{N_2}} \in O(NUTSR^{N_2})$ is $\Gamma_{R^{N_2}} \in \alpha O(NUTSR^{N_2})$, Therefore $\Gamma_{R^{N_2}} \in \alpha O(NUTSR^{N_2})$, By our assumption, Neutrosophic upper α -irresolute multifunction, there exists $U_{R^C_1} \in \alpha O(CTSR^C_1)$ containing x_0 such that $\Psi(\xi) \subset \Gamma_{R^{N_2}}, \forall \xi \in U_{R^C_1}$. Every $U_{R^C_1}, U_{R^C_1} \in \alpha O(NUTSR^{N_2})$ is $U_{R^C_1} \in SO(CTTSR^{N_2})$. Their exists $U_{R^C_1} \in SO(CTSR^C_1)$ containing x_0 such that $\Psi(\xi) \subset \Gamma_{R^{N_2}}, \forall \xi \in U_{R^C_1}$. Hence Ψ is Neutrosophic upper quasi semi continuous multifunction at x_0 .

Theorem 4.5

Let $(R^C_1, \tau_{R^C_1}) \rightarrow (R^{N_2}, \tau_{NR^{N_2}})$, be an Neutrosophic multifunction and let $\xi \in R^C_1$. Then the following statements are equivalent:

- (a) Ψ is Neutrosophic Upper α -irresolute at ξ .
- (b) For each $\Gamma_{R^{N_2}} \in \alpha O(NUTSR^{N_2})$ with $(\xi) \subset \Gamma_{R^{N_2}}$, Implies $\xi \in sCl(Int(\Psi^-(\Gamma)))$.
- (c) For any $\xi, \xi \in U_{R^C_1} \in SO(CTSR^C_1)$ and for any $\Gamma_{R^{N_2}} \in \alpha O(NUTSR^{N_2})$ with $(\xi) \subset \Gamma_{R^{N_2}}$, there exists a nonempty open set $V_{R^C_1} \subset U_{R^C_1}$ such that $\Psi(V_{R^C_1}) \subset \Gamma_{R^{N_2}}$.

Proof.

(a) \Rightarrow (b) Let $\xi \in R^C_1$ and $\Gamma_{R^{N_2}} \in \alpha O(NUTSR^{N_2})$ Such that $\Psi(\xi) \subset \Gamma_{R^{N_2}}$. Then by our assumption (a), we get there exists $U_{R^C_1} \in \alpha O(CTSR^C_1)$ such that $\xi \in U_{R^C_1}$ and $F(U_{R^C_1}) \subset \Gamma_{R^{N_2}}$. Thus $\xi \in U_{R^C_1} \subset \Psi^+(\Gamma_{R^{N_2}})$. here $U_{R^C_1} \in \alpha O(CTSR^C_1)$. We know that for any set $A_{R^C_1}, A_{R^C_1} \in \alpha O(CTSR^C_1) \Leftrightarrow A_{R^C_1} \subset sCl(Int(A_{R^C_1}))$. Therefore, $U_{R^C_1} \subset sCl(Int(U_{R^C_1}))$. Finally we get $\xi \in sCl(Int\Psi^+(\Gamma_{R^{N_2}}))$. hence (b).

(b) \Rightarrow (c). Let $\Gamma_{R^{N_2}} \in \alpha O(NUTSR^{N_2})$ such that $\Psi(\xi) \subset \Gamma_{R^{N_2}}$, then $\xi \in sCl(Int\Psi^-(\Gamma_{R^{N_2}}))$. Let $U_{R^C_1} \in SO(CTSR^C_1)$ and $\xi \in U_{R^C_1}$. Then $U_{R^C_1} \cap Int(\Psi^-(\Gamma_{R^{N_2}})) \neq \emptyset$ and $U_{R^C_1} \cap Int(\Psi^-(\Gamma_{R^{N_2}}))$ is semi-open in R^C_1 . Put $V_{R^C_1} = Int(U_{R^C_1} \cap Int(\Psi^-(\Gamma_{R^{N_2}})))$, Then $V_{R^C_1}$ is an open set of R^C_1 , $V_{R^C_1} \subset U_{R^C_1}, V_{R^C_1} \neq \emptyset$ and $\Psi(V_{R^C_1}) \subset \Gamma_{R^{N_2}}$,

(c) \Rightarrow (a). Let $\{U_\xi\}$ be the system of the $SO(CTSR^C_1)$ containing ξ . Let $U_{R^C_1} \in SO(CTSR^C_1)$ and $\xi \in U_{R^C_1}$ and Let $\Gamma_{R^{N_2}} \in \alpha O(NUTSR^{N_2})$ such that $\Psi(\xi) \subset \Gamma_{R^{N_2}}$, there exists a nonempty open set $B_U \subset U_{R^C_1}$ Such that $\Psi(v) \subset \Gamma_{R^{N_2}}, \forall v \in B_U$. Let $W_{R^C_1} = \cup B_U : U_{R^C_1} \in \{U_\xi\}$, then $W_{R^C_1} \in O(CTSR^C_1)$ and $\xi \in sCl(W_{R^C_1})$ and $\Psi(v) \subset \Gamma_{R^{N_2}}, \forall v \in W_{R^C_1}$. Put $S_{R^C_1} = W_{R^C_1} \cup \xi$. Then $W_{R^C_1} \subset S_{R^C_1} \subset sCl(W_{R^C_1})$. Thus $S_{R^C_1} \in \alpha O(CTSR^C_1)$, $\xi \in S_{R^C_1}$ and $\Psi(v) \subset \Gamma_{R^{N_2}}, \forall v \in S$. Hence Ψ is Neutrosophic Upper α -irresolute at ξ .

Theorem 4.6

For an Neutrosophic multifunction $(R^C_1, \tau_{R^C_1}) \rightarrow (R^{N_2}, \tau_{NR^{N_2}})$ the following statements are equivalent:

- (a) Ψ is Neutrosophic upper α -irresolute.
- (b) $\Psi^+(\Gamma_{R^{N_2}}) \in \alpha O(CTSR^C_1)$, for every Neutrosophic α -open set $\Gamma_{R^{N_2}}$ of R^{N_2}
- (c) $\Psi^-(\lambda_{R^{N_2}}) \in \alpha C(CTSR^C_1)$, for each Neutrosophic α -closed set $\lambda_{R^{N_2}}$ of R^{N_2} .
- (d) For each point $\xi \in R^C_1$ and for each α -neighborhood $V_{R^{N_2}}$ of $\Psi(\xi)$ in R^{N_2} , $F^+(V_{R^{N_2}})$ is an α -neighborhood of ξ .
- (e) For each point $\xi \in R^C_1$ and for each α -neighborhood $V_{R^{N_2}}$ of $\Psi(\xi)$ in R^{N_2} , there is an α -neighborhood $U_{R^C_1}$ of ξ such that $\Psi(U_{R^C_1}) \subset V_{R^{N_2}}$.
- (f) $\alpha Cl(\Psi^-(\lambda_{R^{N_2}})) \subset \Psi^-(Neu - \alpha Cl(\lambda_{R^{N_2}}))$ for each Neutrosophic set $\lambda_{R^{N_2}}$ of R^{N_2} .
- (g) $sInt(Cl(\Psi^-(\lambda_{R^{N_2}}))) \subset \Psi^-(Neu - \alpha Cl(\lambda_{R^{N_2}}))$ for any Neutrosophic set λ of R^{N_2} .

Proof.

(a)⇒(b). Let $\Gamma_{R^{N_2}} \in \alpha O(NUTSR^{N_2})$ and $\xi \in \Psi^+(\Gamma_{R^{N_2}})$. Applying previous theorem, we get $\xi \in sCl(Int\Psi^+(\Gamma_{R^{N_2}}))$. Therefore, we obtain $\Psi^+(\Gamma_{R^{N_2}}) \subset sCl(Int\Psi^+(\Gamma_{R^{N_2}}))$. Finally we get $\Psi^+(\Gamma_{R^{N_2}}) \in \alpha O(CTSR^C_1)$.

(b)⇒(a). Let ξ be arbitrarily point in R^C_1 and $\Gamma_{R^{N_2}} \in \alpha O(NUTSR^{N_2})$ such that $\Psi(\xi) \subset \Gamma_{R^{N_2}}$ so $\xi \in \Psi^+(\Gamma_{R^{N_2}})$. By hypothesis $\Psi^+(\Gamma_{R^{N_2}}) \in \alpha O(CTSR^C_1)$, we get $\xi \in \Psi^+(\Gamma_{R^{N_2}}) \subset sCl(Int(\Psi^+(\Gamma_{R^{N_2}})))$ and hence F is Neutrosophic upper α -irresolute at ξ . As ξ is arbitrarily chosen, Ψ is Neutrosophic upper α -irresolute.

(b)⇒(c). This implies easily get from that $[\Psi^-(\Gamma_{R^{N_2}})]^C = [\Psi^+(\Gamma_{R^{N_2}})]^C$, where $\Gamma_{R^{N_2}} \in \alpha O(NUTSY)$

(c)⇒(f). Let $\lambda_{R^{N_2}} \in \alpha O(NUTSR^{N_2})$. Then by our assumption (c), $\Psi^-(Neu - \alpha Cl(\lambda_{R^{N_2}}))$ is an α -closed set in R^C_1 . We have $\Psi^-(Neu - \alpha Cl(\lambda_{R^{N_2}})) \supset sInt(Cl(\Psi^-(Neu - Cl(\lambda_{R^{N_2}})))) \supset sInt(Cl(\Psi^-(\lambda_{R^{N_2}}))) \supset \Psi^-(\lambda) \cup sInt(Cl(\Psi^-(\lambda_{R^{N_2}}))) \supset \alpha Cl(\Psi^-(\lambda_{R^{N_2}}))$. Hence the result.

(f)⇒(g). Let $\lambda_{R^{N_2}} \in \alpha O(NUTSR^{N_2})$. we have $\alpha Cl(\Psi^-(\lambda_{R^{N_2}})) = \Psi^-(\lambda_{R^{N_2}}) \cup sInt(Cl(\Psi^-(\lambda_{R^{N_2}}))) \subset \Psi^-(Neu - \alpha Cl(\lambda_{R^{N_2}}))$. Hence (g).

(g)⇒(c). Let $\lambda_{R^{N_2}}^C \in \alpha C(NUTSR^{N_2})$ Then by (g) we have,

$$sInt(Cl(\Psi^-(\lambda_{R^{N_2}}^C))) \subset \Psi^-(\lambda_{R^{N_2}}^C) \cup sInt(Cl(\Psi^-(\lambda_{R^{N_2}}^C))) \subset \Psi^-(\alpha Cl(\lambda_{R^{N_2}}^C)) = \Psi^-(\lambda_{R^{N_2}}^C).$$

Hence By our result, $\Psi^-(\lambda_{R^{N_2}}^C) \in \alpha C(CTSR^C_1)$.

(b)⇒(d). Let $\xi \in R^C_1$ and $V_{R^{N_2}}$ be an α -neighborhood of $\Psi(\xi)$ in R^{N_2} . Then there is an $\lambda_{R^{N_2}} \in \alpha O(NUTSR^{N_2})$ such that $\Psi(\xi) \subset \lambda_{R^{N_2}} \subset V_{R^{N_2}}$. Hence, $\xi \in \Psi^+(\lambda_{R^{N_2}}) \subset \Psi^+(V_{R^{N_2}})$. Now by hypothesis $\Psi^+(\lambda_{R^{N_2}}) \in \alpha O(CTSR^C_1)$, and Thus $\Psi^+(V_{R^{N_2}})$ is an α -neighborhood of ξ .

(d)⇒(e). Let $\xi \in R^C_1$ and $V_{R^{N_2}}$ be an α -neighborhood of $\Psi(\xi)$ in R^{N_2} . Put $U_{R^C_1} = \Psi^+(V_{R^{N_2}})$. Then $U_{R^C_1}$ is an α -neighborhood of ξ and $\Psi(U) \subset V_{R^{N_2}}$.

(e)⇒(a). Let $\xi \in R^C_1$ and $V_{R^{N_2}}$ be an Neutrosophic set in R^{N_2} such that $\Psi(\xi) \subset V_{R^{N_2}}$. $V_{R^{N_2}}$ being an Neutrosophic α -open set in R^{N_2} , is an α -neighborhood of $\Psi(\xi)$ and according to the hypothesis there is an α -neighborhood $U_{R^C_1}$ of ξ such that $\Psi(U_{R^C_1}) \subset V_{R^{N_2}}$. Therefore $V_{R^{N_2}} \in \alpha O(CTSR^C_1)$ such that $\xi \in U_{R^C_1} \subset U_{R^C_1}$ and hence $\Psi(A) \subset \Psi(U_{R^C_1}) \subset V_{R^{N_2}}$. Hence Ψ is Neutrosophic upper α -irresolute at ξ .

References

1. K. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems 20(1986),87-94.
2. I. Arokiarani, R. Dhavaseelan, S. Jafari, M. Parimala, On Some New Notions and Functions in Neutrosophic Topological Spaces, Neutrosophic Sets and Systems, Vol. 16, 2017, (16-19)
3. C.L. Chang, Fuzzy Topological Spaces, J. Math. Anal. Appl. 24 (1968), 182-190.
4. Dogan Coker, An introduction to Intuitionistic fuzzy topological spaces, Fuzzy Sets and Systems, 88(1997), 81-89
5. R. Dhavaseelan and S. Jafari, Generalized Neutrosophic closed sets, New trends in Neutrosophic theory and applications Volume II- 261-273, (2018)
6. R. Dhavaseelan and S. Jafari, Neutrosophic semi continuous multifunction, New trends in Neutrosophic theory and applications Volume II- 345-354, (2017)
7. Florentin Smarandache, Neutrosophic and Neutrosophic Logic, First International Confer On Neutrosophic, Neutrosophic Logic, Set, Probability and Statistics University of New Mexico, Gallup, NM 87301, USA (2002), smarand@unm.edu
8. Florentin Smarandache, Neutrosophic Set: - A Generalization of Intuitionistic Fuzzy set, Journal of Defense Resources Management. 1(2010), 107-114.
9. P. Iswarya and Dr. K. Bageerathi, On Neutrosophic semi-open sets in Neutrosophic Topological spaces, International Journal of Mathematics Trends and Technology (IJMTT), Vol 37, No. 3, (2016), 24-33.

10. Mani Parimala, Florentin Smarandache, Saeid Jafari and Ramalingam Udhayakumar, Article in Information (Switzerland) October 2018.
11. T Rajesh kannan ,Dr. S.Chandrasekar, Neutrosophic $\omega\alpha$ -closed sets in Neutrosophic topological space, Journal Of Computer And Mathematical Sciences, vol.9(10),1400-1408 Octobe 2018.
12. T Rajesh kannan ,S.Chandrasekar, Neutrosophic α -continuous multifunction in Neutrosophic topological space, The International Journal of Analytical and Experimental Modal Analysis, Volume XI, Issue IX, September 2019, 1360-9367
13. A.A.Salama and S.A.Alblowi, Neutrosophic set and Neutrosophic topological space, ISOR J. Mathematics, Vol.(3), Issue(4), (2012). pp-31-35
14. V.K.Shanthi ,S.Chandrasekar, K.Safina Begam, Neutrosophic Generalized Semi Closed Sets In Neutrosophic Topological Spaces, International Journal of Research in Advent Technology, Vol.6, No.7, July 2018, 1739-1743
15. S S Thakur and Kush Bohre , On Lower and Upper α –Irresolute Intuitionistic Fuzzy Multi Function, Facta Universities, Ser.Math.Inform vol 30, No 4 (2015) Page-361-375
16. Wadei F.AL-omeri and Saeid Jafari , Neutrosophic pre continuous multifunction and almost pre continuous multifunctions, Neutrosophic sets and system, vol 27 2019, 53-69
17. L.A. Zadeh, Fuzzy Sets, Inform and Control 8(1965), 338- 353.
18. C. Maheswari , S. Chandrasekar: Neutrosophic gb-closed Sets and Neutrosophic gb-Continuity, Neutrosophic Sets and Systems, vol. 29, 2019, pp. 89-100, DOI: 10.5281/zenodo.3514409
19. C.Maheswari, M.Sathyabama, S.Chandrasekar., Neutrosophic generalized b-closed Sets In Neutrosophic Topological Spaces, *Journal of physics Conf. Series* 1139 (2018) 012065. doi:10.1088/1742-6596/1139/1/012065
20. V. Banu priya S.Chandrasekar: Neutrosophic α gs Continuity and Neutrosophic α gs Irresolute Maps, *Neutrosophic Sets and Systems*, vol. 28, 2019, pp. 162-170. DOI: 10.5281/zenodo.3382531
21. Abdel-Basset, M., El-hoseny, M., Gamal, A., & Smarandache, F. (2019). A novel model for evaluation Hospital medical care systems based on plithogenic sets. *Artificial intelligence in medicine*, 100, 101710.
22. Abdel-Basset, M., Manogaran, G., Gamal, A., & Chang, V. (2019). A Novel Intelligent Medical Decision Support Model Based on Soft Computing and IoT. *IEEE Internet of Things Journal*.
23. Abdel-Basset, M., Mohamed, R., Zaied, A. E. N. H., & Smarandache, F. (2019). A hybrid plithogenic decision-making approach with quality function deployment for selecting supply chain sustainability metrics. *Symmetry*, 11(7), 903.
24. Abdel-Baset, M., Chang, V., & Gamal, A. (2019). Evaluation of the green supply chain management practices: A novel neutrosophic approach. *Computers in Industry*, 108, 210-220.
25. Abdel-Basset, M., Saleh, M., Gamal, A., & Smarandache, F. (2019). An approach of TOPSIS technique for developing supplier selection with group decision making under type-2 neutrosophic number. *Applied Soft Computing*, 77, 438-452.

Received: 12 Sep, 2019. Accepted: 20 Mar 2020.