6-2020

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IEEE Access
Multidisciplinary | Rapid Review | Open Access Journal

Received June 12, 2020, accepted June 25, 2020, date of publication June 30, 2020, date of current version July 28, 2020.

Digital Object Identifier 10.1109/ACCESS.2020.3006164

Derivable Single Valued Neutrosophic Graphs Based on *KM*-Fuzzy Metric

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ABSTRACT In this paper we consider the concept of *KM*-fuzzy metric spaces and we introduce a novel concept of *KM*-single valued neutrosophic metric graphs based on *KM*-fuzzy metric spaces. Then we investigate the finite *KM*-fuzzy metric spaces with respect to *KM*-fuzzy metrics and we construct the *KM*-fuzzy metric spaces on any given non-empty sets. We try to extend the concept of *KM*-fuzzy metric spaces to a larger class of *KM*-fuzzy metric spaces such as union and product of *KM*-fuzzy metric spaces and in this regard we investigate the class of products of *KM*-single valued neutrosophic metric graphs. In the final, we define some operations such as tensor product, Cartesian product, semi-strong product, strong product, union, semi-ring sum, suspension, and complement of *KM*-single valued neutrosophic metric graphs.

INDEX TERMS (Derivable) *KM*-single valued neutrosophic metric graph, *KM*-fuzzy metric space, triangular-norm (conorm).

I. INTRODUCTION

Classical theory is a pure concept and without quality or criteria, so it is not attractive to use in our world, that's why we use the neutrosophic sets theory as one of a generalizations of set theory in order to deal with uncertainties, which is a key action in the contemporary world introduced by Smarandache for the first time in 1998 [22] and in 2005 [23]. This concept is a new mathematical tool for handling problems involving imprecise, indeterminacy, and inconsistent data. This theory describes an important role in modeling and controlling unsure hypersystems in nature, society and industry. In addition, fuzzy topological spaces as a generalization of topological spaces, have a fundamental role in construction of fuzzy metric spaces as an extension of the concept of metric spaces. The theory of fuzzy metric spaces works on finding the distance between two points as non-negative fuzzy numbers, which have various applications. The structure of fuzzy metric spaces is equipped with mathematical tools such as triangular norms and fuzzy subsets depending on time parameter and on other variables. This theory has been proposed by different researchers with different definitions from several points of views ([3]–[5], [12]), and that this study was applied to the notion of KM-fuzzy metric space introduced in 1975 [4] by Kramosil and Michalek. Fuzzy graphs, introduced by Rosenfeld, are finding an increasing number of applications in modelling real time systems where the level of information inherent in the system varies with respect to different levels of precision. Fuzzy models are becoming useful because of their aim in reducing the difference between the traditional numerical models used in engineering and sciences [19]. The generalization of the concept of a fuzzy graph is noticed by some researchers on more subjects, such as fuzzy graph based on t-norm, intuitionistic fuzzy threshold graphs, m-polar fuzzy graphs and single-valued neutrosophic graphs. Mordeson *et al.* [17] generalized the definition of a fuzzy graph by replacing minimum in the basic definitions with an arbitrary t-norm. They developed a measure on the susceptibility of trafficking in persons for networks by using a t-norm other than minimum [17]. Recently, F. Smarandache, introduced a new concept as a generalization of hypergraphs to n-SuperHypergraph, Plithogenic n-SuperHypergraph {with super-vertices (that are groups of vertices) and hyper-edges {defined on power-set of power-set…} that is the most general form of graph as today}, and n-ary HyperAlgebra, n-ary NeutroHyperAlgebra, n-ary AntiHyperAlgebra respectively, which have several properties and are connected with the real world [24]. Further materials regarding graphs, single-valued neutrosophic metric graphs, hypergraphs, intuitionistic fuzzy set, n-SuperHypergraph and Plithogenic n-SuperHypergraph, and NeutroAlgebras {Smarandache generalized the classical

The associate editor coordinating the review of this manuscript and approving it for publication was Wai Keung Fung.

algebraic structures to neutro algebraic structures (or Neutro Algebras) [whose operations and axioms are partially true, partially indeterminate, and partially false] as extensions of PartialAlgebra, and to AntiAlgebraic structures (or Anti Algebras) [whose operations and axioms are totally false], and in general, he extended any classical structure, in no matter what field of knowledge, to a Neutro structure and an Anti structure}. All these are available in the literature too [2], [7]–[10], [13], [18], [20], [21], [25]–[29].

Regarding these points, we introduce the concept of KM-single valued neutrosophic metric graphs based on the concept of KM-fuzzy metrics. One of the main motivations of KM-single valued neutrosophic metric graphs is obtained from the fuzzy graphs and so we want to use this concept to model many decision making problems in uncertain environment. We need to construct the KM-single valued neutrosophic metric graphs based on finite or infinite sets, so we develop the concept of KM-fuzzy metric on any nonempty set and prove that for every given set with respect to the concept of C-graphable sets one can construct a KM-metric space. It is a natural generalization of the fuzzy graphs to the single-valued neutrosophic metric graphs, so it shows our main motivation for introducing the notion of the KM-single valued neutrosophic metric graphs. This notion is based on one of the fundamental concepts of fuzzy mathematics, which includes tools such as t-norms, t-conorms, and fuzzy subsets. We apply the notation of KM-fuzzy metric spaces to generate the finite KM-single valued neutrosophic metric graphs. We have extended some production operations on the KM-fuzzy metric spaces to the KM-single valued neutrosophic metric graphs.

II. PRELIMINARIES

In this section, we recall some definitions and results, which we use in what follows.

Definition 1 ([11], [14]): Let $G_1 = (V_1, E_1)$, $G_2 = (V_2, E_2)$ be simple graphs, $(x_1, x_2), (y_1, y_2) \in V_1 \times V_2$, where $V_1 \times V_2$ is the vertex set of the following graphs:

(i) categorical(tensor, direct, cardinal, Kronecker) product graph $G_1 \times G_2$:

$E(G_1 \times G_2) = \{(x_1, x_2)(y_1, y_2) \mid x_1y_1 \in E_1 \text{ and } x_2y_2 \in E_2\}$;

(ii) Cartesian product graph $G_1 \otimes G_2$:

$E(G_1 \otimes G_2) = \{(x_1, x_2)(y_1, y_2) \mid (x_1 = y_1 \text{ and } x_2y_2 \in E_2) \text{ or } (x_1y_1 \in E_1 \text{ and } x_2 = y_2)\}$;

(iii) semi-strong product graph $G_1 \cdot G_2$:

$E(G_1 \cdot G_2) = \{(x_1, x_2)(y_1, y_2) \mid (x_1 = y_1 \text{ and } x_2y_2 \in E_2) \text{ or } (x_1y_1 \in E_1 \text{ and } x_2y_2 \in E_2)\}$;

(iv) strong product (symmetric composition) graph $G_1 \odot G_2$:

$E(G_1 \odot G_2) = E(G_1 \otimes G_2) \cup E(G_1 \times G_2)$;

(v) lexicographic product (composition)graph $G_1 \circ G_2(G_1.G_2, G_1[G_2])$:

$E(G_1 \circ G_2) = \{(x_1, x_2)(y_1, y_2) \mid (x_1y_1 \in E_1) \text{ or } (x_1 = y_1 \text{ and } x_2y_2 \in E_2)\}$;

(vi) union graph $G_1 \cup G_2$:

$V(G_1 \cup G_2) = V(G_1) \cup V(G_2)$; and $E(G_1 \cup G_2) = E(G_1) \cup E(G_2)$;

(vii) join product graph $G_1 + G_2$:

$E(G_1 + G_2) = E(G_1) \cup E(G_2) \cup E'$, where E' is the set of all line joining V_1 with V_2.

Definition 2 [16]: A fuzzy graph $G = (V, \sigma, \mu)$ is an algebraic structure of non-empty set V together with a pair of functions $\sigma : V \to [0, 1]$ and $\mu : V \times V \to [0, 1]$ such that for all $x, y \in V$, $\mu(x, y) \leq \sigma(x) \wedge \sigma(y)$. It is called σ as fuzzy vertex set and μ as fuzzy edge set of G.

Definition 3 [1]: A single valued neutrosophic graph (SVN–G) is defined to be a form $G = (V, E, A, B)$ where

(i) $V = \{v_1, v_2, \ldots, v_n\}$, $T_A, I_A, F_A : V \longrightarrow [0, 1]$ denote the degree of membership, degree of indeterminacy and non–membership of the element $v_i \in V$; respectively, and for every $1 \leq i \leq n$, we have $0 \leq T_A(v_i) + I_A(v_i) + F_A(v_i) \leq 3$.

(ii) $E \subseteq V \times V$, $T_B, I_B, F_B : E \longrightarrow [0, 1]$ are called degree of truth–membership, indeterminacy–membership and falsity–membership of the edge $(v_i, v_j) \in E$ respectively, such that for any $1 \leq i, j \leq n$, we have $T_B(v_i, v_j) \leq \min\{T_A(v_i), T_A(v_j)\}$, $I_B(v_i, v_j) \geq \max\{I_A(v_i), I_A(v_j)\}$, $F_B(v_i, v_j) \geq \max\{F_A(v_i), F_A(v_j)\}$ and $0 \leq T_B(v_i, v_j) + I_B(v_i, v_j) + F_B(v_i, v_j) \leq 3$. Also A is called the single valued neutrosophic vertex set of V and B is called the single valued neutrosophic edge set of E.

Definition 4 [15]: A triplet (X, ρ, T) is called a KM-fuzzy metric space, if X is an arbitrary non–empty set, T is a left-continuous t-norm and $\rho : X^2 \times \mathbb{R}^{\geq 0} \to [0, 1]$ is a fuzzy set, such that for each $x, y, z, \in X$ and $t, s \geq 0$, we have:

(i) $\rho(x, y, 0) = 0$,

(ii) $\rho(x, x, t) = 1$ for all $t > 0$,

(iii) $\rho(x, y, t) = \rho(y, x, t)$(commutative property),

(iv) $T(\rho(x, y, t), \rho(y, z, s)) \leq \rho(x, z, t + s)$(triangular inequality),

(vi) $\rho(x, y, -) : \mathbb{R}^{\geq 0} \to [0, 1]$ is a left-continuous map,

(vii) $\lim_{t\to\infty} \rho((x, y, t)) = 1$,

$(viii)$ $\rho(x, y, t) = 1, \forall\, t > 0$ implies that $x = y$.

If (X, ρ, T) satisfies in conditions (i)–(vii), then it is called KM-fuzzy pseudo metric space and ρ is called a KM-fuzzy pseudo metric. a fuzzy version of the triangular inequality. The value $\rho(x, y, t)$ is considered as the degree of nearness from

Theorem 1 [15]: Let (X, ρ, T) be a KM-fuzzy metric space. Then $\rho(x, y, -) : \mathbb{R}^{\geq 0} \to [0, 1]$ is a non-decreasing map.

Proof 1: See [15].

III. FINITE *KM*-FUZZY METRIC SPACE

In this section, we apply the concept of KM-fuzzy metric spaces and construct a new class of KM-fuzzy metric spaces under operation product and union of KM-fuzzy metric spaces. In addition, for any given non-empty set we construct

KM-fuzzy metric space with respect to α-discrete metric, where $\alpha \in \mathbb{R}^+$. From now on, for all $x, y \in [0, 1]$ we consider $T_{min}(x, y) = \min\{x, y\}$, $T_{pr}(x, y) = xy$, $T_{lu}(x, y) = \max(0, x + y - 1)$, $T_{do}(x, y) = \dfrac{xy}{x + y - xy}$ and $\mathcal{C}_T = \{T : [0, 1] \times [0, 1] \to [0, 1] \mid T$ is a left-continuous t-norm$\}$.

Theorem 2: If (X, ρ, T_{min}) is a KM-fuzzy metric space and $T \in \mathcal{C}_T$. Then (X, ρ, T) is a KM-fuzzy metric space.

Proof 2: Let $x, y, z \in X$, $r, s \in \mathbb{R}^{\geq 0}$ and $T \in \mathcal{C}_T$. Since for all $x, y \in [0, 1]$, $T(x, y) \leq T_{min}(x, y)$, we get that $T(\rho(x, y, t), \rho(y, z, s)) \leq T_{min}(\rho(x, y, t), \rho(y, z, s)) \leq \rho(x, z, t + s)$. Hence (X, ρ, T) is a KM-fuzzy metric space.

Let X be an arbitrary set and $\alpha \in \mathbb{R}^+$. For all $x, y \in X$, define $d_\alpha : X \times X \to \mathbb{R}$ by $d_\alpha(x, y) = 0$, where $x = y$ and $d_\alpha(x, y) = \alpha$, where $x \neq y$ as an α-discrete metric. So we have the following theorem.

Theorem 3: Let X be an arbitrary set and $|X| \geq 2$. Then there exists a fuzzy set $\rho : X^2 \times \mathbb{R}^{\geq 0} \to [0, 1]$, such that (X, ρ, T_{min}) is a KM-fuzzy metric space.

Proof 3: Let $|X| \geq 2$ and $\alpha \in \mathbb{R}^+$ be a fixed element. Clearly (X, d_α) is a metric space, now for all $x, y \in X$, $0 \neq m, s, t \in \mathbb{R}^{\geq 0}$, define $\rho : X^2 \times \mathbb{R}^{\geq 0} \to [0, 1]$ by $\rho(x, y, 0) = 0$ and $\rho(x, y, t > 0) = \dfrac{\varphi(t)}{\varphi(t) + md_\alpha(x, y)}$, where $\varphi : \mathbb{R}^{\geq 0} \to \mathbb{R}^{\geq 0}$ is an increasing continuous function and for all $x, y \in X$, we have $\varphi(t) + md_\alpha(x, y) \neq 0$ and $\varphi(t) \to 0$, whence $t \to 0$. Now, we show that (X, ρ, T_{min}) is a KM-fuzzy metric space. We prove only the triangular inequality and for all $x, y, z \in X$, consider the five cases $x = y = z$, $x = y \neq z$, $x = z \neq y$, $x \neq y = z$ and $x \neq y \neq z$. In all cases for $0 \in \{t, s\}$ is clear, now for $0 \notin \{t, s\}$ we investigate it. For $x = y \neq z$, since $\varphi(t + s) \geq \varphi(s)$, we have $\varphi(t + s)(\varphi(s) + m\alpha) - \varphi(s)(\varphi(t + s) + m\alpha) \geq 0$ and so $\dfrac{\varphi(s)}{\varphi(s) + m\alpha} \leq \dfrac{\varphi(t + s)}{\varphi(t + s) + m\alpha}$. If $x \neq y \neq z$, then $d_\alpha(x, y) = d_\alpha(z, y) = d_\alpha(x, z) = \alpha$. Since φ is an increasing map, we get that $m\alpha\varphi(t) \leq m\alpha\varphi(t + s)$ and it implies that $\varphi(t)(\varphi(t+s)+m\alpha) \leq \varphi(t+s)(\varphi(t)+m\alpha)$ and so $\dfrac{\varphi(t)}{\varphi(t) + m\alpha} \leq \dfrac{\varphi(t + s)}{\varphi(t + s) + m\alpha}$, which means that $\rho(x, y, t) \leq \rho(x, z, t + s)$. By a similar way, $\rho(z, y, s) \leq \rho(x, z, t + s)$ and so $T_{min}(\rho(x, y, t), \rho(z, y, s)) \leq \rho(x, z, t + s)$.

The other cases, are proved in a similar way and so (X, ρ, T_{min}) is a KM-fuzzy metric space.

Corollary 1: Let X be an arbitrary set and $|X| \geq 2$. Then there exists a fuzzy set $\rho : X^2 \times \mathbb{R}^{\geq 0} \to [0, 1]$, such that for all $T \in \mathcal{C}_T$, (X, ρ, T) is a KM-fuzzy metric space.

A. FINITE KM-FUZZY METRIC SPACE BASED ON METRIC

In this subsection, we apply the concept of finite metric for constructing of KM-fuzzy metric space on any given non-empty set.

Definition 5: Let X be a finite set. We say that X is a C-graphable set, if $G = (X, E)$ is a connected graph, where $E \subseteq X \times X$ and $G = (X, E)$ is called an X-derived graph. Let $\mathcal{G}_X$ be the set of all connected graphs which are constructed on X as the set of vertices, so we have the following results.

Let $G = (X, E)$ be a connected graph. For all $x, y \in X$, define $d^g(x, y) = \min\{|P_{x,y}|$ where $P_{x,y}$ is a path between $x, y\}$. Obviously, d^g is a metric on X.

Theorem 4: Let X be a finite set and $|X| \geq 2$. Then there exists a non-discrete metric d on X such that (X, d) is a metric space.

Proof 4: Let $|X| \geq 2$. Clearly, X is a C-graphable set and so there exists a graph $G = (X, E) \in \mathcal{G}_X$. For all $x, y \in X$, define $d(x, y) = d^g(x, y)$. Clearly (X, d^g) is a metric space.

Corollary 2: Let $n \in \mathbb{N}$, X be a set and $|X| = n$.

(*i*) If $G = (X, E) \cong K_n$ is the complete graph, then for metric spaces (X, d^g) and (X, d_1), we have $d^g = d_1$.

(*ii*) If $G = (X, E) \cong C_n$ is the cycle graph, then for metric spaces (X, d^g) and (X, d_1), we have $d_1 \leq d^g \leq d_{\frac{\lfloor n \rfloor}{2}}$.

Theorem 5: Let X be a non-empty set. Then there exists a fuzzy subset $\rho : X^2 \times \mathbb{R}^{\geq 0} \to [0, 1]$, such that (X, ρ, T_{pr}) is a KM-fuzzy metric space.

Proof 5: Let $|X| \geq 2$. Then clearly, X is a C-graphable set and by Theorem 4, (X, d^g) is a metric space. For all $x, y \in X$ and for all $0 \neq m, t \in \mathbb{R}^{\geq 0}$, define $\rho(x, y, 0) = 0$ and $\rho(x, y, t > 0) = \dfrac{\varphi(t)}{\varphi(t) + md^g(x, y)}$, where $\varphi : \mathbb{R}^{\geq 0} \to \mathbb{R}^{\geq 0}$ is an increasing continuous function, $\varphi(t) + md^g(x, y) \neq 0$ and $\varphi(t) \to 0$, whence $t \to 0$. Now, we show that (X, ρ, T_{pr}) is a KM-fuzzy metric space and in this regard, only prove triangular inequality property. Let $x, y, z \in X$. For $0 \in \{t, s\}$ is clear, now for $0 \notin \{t, s\}$ we investigate it. Since for all $s, t, m \in \mathbb{R}^+$,

$$\begin{aligned}
&\varphi(t + s)\varphi(s)md^g(x, y) + \varphi(t + s)\varphi(t)md^g(y, z)\\
&\quad \geq \varphi(t)\varphi(s)md^g(x, y) + \varphi(s)\varphi(t)md^g(y, z)\\
&\quad \geq \varphi(s)\varphi(t)md^g(x, z), m^2d^g(y, z)d^g(y, z)\varphi(t + s) > 0,
\end{aligned}$$

we get that $T_{pr}\left(\dfrac{\varphi(t)}{\varphi(t) + md^g(x, y)}, \dfrac{\varphi(s)}{\varphi(s) + md^g(y, z)}\right) \leq \dfrac{\varphi(t + s)}{\varphi(t + s) + md^g(x, z)}$.

It follows that $T_{pr}(\rho(x, y, t), \rho(y, z, s)) \leq \rho(x, z, t + s)$ and so (X, ρ, T_{pr}) is a KM-fuzzy metric space.

Corollary 3: Let X be a non-empty set. Then there exists a fuzzy subset $\rho : X^2 \times \mathbb{R}^{\geq 0} \to [0, 1]$, such that for all left-continuous t-norm $T \leq T_{pr}$, (X, ρ, T) is a KM-fuzzy metric space.

B. OPERATIONS ON KM-FUZZY METRIC SPACES

In this subsection, we extend KM-fuzzy metric spaces to union and product of KM-fuzzy metric spaces. Let (X_1, ρ_1, T) and (X_2, ρ_2, T) be KM-fuzzy metric spaces, $(x_1, y_1), (x_2, y_2) \in X_1 \times X_2$ and $t \in \mathbb{R}^{\geq 0}$. For an arbitrary $T \in \mathcal{C}_T$, define $T(\rho) : (X_1 \times X_2)^2 \times \mathbb{R}^{\geq 0} \to [0, 1]$ by $T(\rho)\big((x_1, y_1), (x_2, y_2), t\big) = T(\rho_1(x_1, x_2, t), \rho_2(y_1, y_2, t))$. So we have the following theorem.

Theorem 6: Let (X_1, ρ_1, T) and (X_2, ρ_2, T) be KM-fuzzy metric spaces. Then $(X_1 \times X_2, T_{min}(\rho), T)$ is a KM-fuzzy metric space.

Proof 6: Let $(x_1, y_1), (x_2, y_2), (x_3, y_3) \in X_1 \times X_2$ and $t, s \in \mathbb{R}^{\geq 0}$.

(i) Since for all $x_1, x_2 \in X_1, y_1, y_2 \in X_2, \rho_1(x_1, x_2, 0) = 0$ and $\rho_2(y_1, y_2, 0) = 0$, have $T_{min}(\rho)\big((x_1, y_1), (x_2, y_2), 0\big) = 0$.

(ii) $T_{min}(\rho)\big((x_1, y_1), (x_2, y_2), t\big) = 1$ if and only if $T_{min}(\rho_1(x_1, x_2, t), \rho_2(y_1, y_2, t)) = 1$ if and only if $\rho_1(x_1, x_2, t) = \rho_2(y_1, y_2, t) = 1$ if and only if $(x_1, y_1) = (x_2, y_2)$.

(iii) It is clear that $T_{min}(\rho)$ is a commutative map.

(iv)

$$\begin{aligned}
&T\big(T_{min}(\rho)((x_1, y_1), (x_2, y_2), t), T_{min}(\rho)((x_2, y_2), (x_3, y_3), s)\big)\\
&= T\big(T_{min}(\rho_1(x_1, x_2, t), \rho_2(y_1, y_2, t)),\\
&\quad T_{min}(\rho_1(x_2, x_3, s), \rho_2(y_2, y_3, s))\big) \leq T_{min}(T\big(\rho_1(x_1,\\
&\quad x_2, t), \rho_1(x_2, x_3, s)\big), T\big(\rho_2(y_1, y_2, t), \rho_2(y_2, y_3, s)\big)))\\
&\leq T_{min}(\rho_1(x_1, x_3, t+s), \rho_2(y_1, y_3, t+s))\\
&= T_{min}(\rho)((x_1, y_1), (x_3, y_3), t+s).
\end{aligned}$$

(v) Since ρ_1, ρ_2 are left-continuous maps, we get that ρ is a left-continuous map.

(vi) Clearly $\lim\limits_{t\to\infty} T_{min}(\rho_1(x_1, x_2, t), \rho_2(y_1, y_2, t)) = T_{min}(\lim\limits_{t\to\infty} \rho_1(x_1, x_2, t), \lim\limits_{t\to\infty} \rho_2(y_1, y_2, t)) = T_{min}(1, 1) = 1$. Thus $(X_1 \times X_2, T_{min}(\rho), T)$ is a KM-fuzzy metric space.

is easy to check that (X_1, ρ_1, T_{lu}) and (X_2, ρ_2, T_{lu}) are KM-fuzzy metric spaces and by Theorem 6, $(X_1 \times X_2, T_{min}(\rho), T_{lu})$ is a KM-fuzzy metric space.

where $\rho_1(x, y, t) = \dfrac{\min(x, y) + t}{\max(x, y) + t}$ and $\rho_2(x, y, t) = \dfrac{\min(x, y)}{\max(x, y)}$. Applying Theorem 6, $(\mathbb{R}^{\geq 0} \times \mathbb{N}, \rho, T_{pr})$ is a KM-fuzzy metric space, where $\rho((x_1, y_1), (x_2, y_2), t) = \min\{\dfrac{\min(x_1, x_2) + t}{\max(x_1, x_2) + t}, \dfrac{\min(y_1, y_2)}{\max(y_1, y_2)}\}$.

Let $X_1 \cap X_2 = \emptyset$, (X_1, ρ_1, T) and (X_2, ρ_2, T) be KM-fuzzy metric spaces, $x, y \in X_1 \cup X_2$ and $t \in \mathbb{R}^{\geq 0}$. Consider $\epsilon(x, y, t) = \bigwedge\limits_{x,u\in X_1\ y,v\in X_2} (\rho_1(x, u, t) \wedge \rho_2(y, v, t)))$, define $\rho_1 \cup \rho_2 : (X_1 \cup X_2)^2 \times \mathbb{R}^{\geq 0} \to [0, 1]$ by

$$(\rho_1 \cup \rho_2)(x, y, t) = \begin{cases} \rho_1(x, y, t) & \text{if } x, y \in X_1,\\ \rho_2(x, y, t) & \text{if } x, y \in X_2,\\ \epsilon(x, y, t) & \text{if } x \in X_1, y \in X_2, . \end{cases}$$

So we have the following theorem.

Theorem 7: Let (X_1, ρ_1, T) and (X_2, ρ_2, T) be KM-fuzzy metric spaces. Then $(X_1 \cup X_2, \rho_1 \cup \rho_2, T)$ is a KM-fuzzy metric space, where $X_1 \cap X_2 = \emptyset$.

Proof 7: Let $x, y, z \in X_1 \cup X_2$ and $t, s \in \mathbb{R}^{\geq 0}$. We only prove the triangular inequality property and other cases are immediate. Let $x, y \in X_1$(for $x, y \in X_2$, one can prove in a similar way), then $T\big((\rho_1 \cup \rho_2)(x, y, t), (\rho_1 \cup \rho_2)(y, z, s)\big) = T\big(\rho_1(x, y, t), (\rho_1 \cup \rho_2)(y, z, s)\big)$. If $z \in X_1$, then $T\big((\rho_1 \cup \rho_2)(x, y, t), (\rho_1 \cup \rho_2)(y, z, s)\big) = T\big(\rho_1(x, y, t), \rho_1(y, z, s)\big) \leq \rho_1(x, z, t+s) = (\rho_1 \cup \rho_2)(x, z, t+s)$. If $z \in X_2$, then $T\big((\rho_1 \cup \rho_2)(x, y, t), (\rho_1 \cup \rho_2)(y, z, s)\big) = T\big(\rho_1(x, y, t), \epsilon\big) \leq \epsilon = (\rho_1 \cup \rho_2)(x, z, t+s)$. Let $x \in X_1, y \in X_2$. Then $T\big((\rho_1 \cup \rho_2)(x, y, t), (\rho_1 \cup \rho_2)(y, z, s)\big) = T\big(\epsilon, (\rho_1 \cup \rho_2)(y, z, s)\big)$. If $z \in X_2$, since $x \in X_1$ and $y \in X_2$, we get that $(\rho_1 \cup \rho_2)(x, z, t+s) = \epsilon$ and so $T\big(\epsilon, (\rho_1 \cup \rho_2)(y, z, s)\big) = T\big(\epsilon, \rho_2(y, z, s)\big) \leq \epsilon = (\rho_1 \cup \rho_2)(x, z, t+s)$. If $z \in X_1$, since $x \in X_1$ and $y \in X_2$, we get that $(\rho_1 \cup \rho_2)(x, z, t+s) \neq \epsilon$ and so $T\big(\epsilon, (\rho_1 \cup \rho_2)(y, z, s)\big) = T\big(\epsilon, \epsilon\big) \leq \epsilon \leq \rho_1(x, z, t+s)\big) = (\rho_1 \cup \rho_2)(x, z, t+s)$. It follows that $(X_1 \cup X_2, \rho_1 \cup \rho_2, T)$ is a KM-fuzzy metric space.

IV. *KM*-SINGLE VALUED NEUTROSOPHIC METRIC GRAPH

In this section, we introduce a novel concept as KM-single valued neutrosophic metric graphs and analyse some their properties.

Definition 6: Let (V, ρ, T) be a fuzzy metric space and $G^* = (V, E)$ be a simple graph. Then $G = \big(X = (T_V, I_V, F_V), Y = (T_E, I_E, F_E), \rho, T, S\big)$ is called a KM-single valued neutrosophic metric graph (a strong KM-single valued neutrosophic metric graph) on G^*, if there exists some time $t \in \mathbb{R}^{\geq 0}$ (for $t = 0$, we call starting time) such that for all $xy \in E$, we have

(i) the functions $T_V : V \to [0, 1], I_V : V \to [0, 1]$ and $F_V : V \to [0, 1]$ represent the degree of truth-membership, indeterminacy-membership and falsity-membership of the element $x \in V$, respectively. There is no restriction on the sum of $T_V(x), I_V(x)$ and $F_V(x)$, therefore $0 \leq T_V(x) + I_V(x) + F_V(x) \leq 3$ for all $x \in V$.

(ii) the functions $T_E : E \subseteq V \times V \to [0, 1], I_E : E \subseteq V \times V \to [0, 1]$ and $F_E : E \subseteq V \times V \to [0, 1]$ are defined by $T\big(T_E(xy), T(T_V(x), T_V(y))\big) \leq \rho(x, y, t)$ $\big(T(T_E(xy), T(T_V(x), T_V(y)) = \rho(x, y, t)\big)$), $S\big(I_E(xy), S(I_V(x), I_V(y))\big) \geq \rho(x, y, t)$ $\big(S\big(I_E(xy), S(I_V(x), I_V(y))\big) = \rho(x, y, t)\big)$ and $S\big(F_E(xy), S(F_V(x), F_V(y))\big) \geq \rho(x, y, t)$ $\big(S\big(F_E(xy), S(F_V(x), F_V(y))\big) = \rho(x, y, t)\big)$, where S is a triangular conorm as a dual of triangular norm T, via a negation η.

We call X as a KM-single valued neutrosophic metric vertex set of G and Y is KM-single valued neutrosophic edge set of G.

In definition of KM-single valued neutrosophic metric graph, if $t \to \infty$, then for all $x, y \in V$, $\rho(x, y, t) \to 1$ and so it follows that $F_E(xy) = S(F_V(x), F_V(y)) = I_E(xy) = S(I_V(x), I_V(y)) = \rho(x, y, t)$ and $T_E(xy), T(T_V(x), T_V(y))$ can be any given fuzzy values. The concept of KM-single valued neutrosophic metric graph is a generalization of KM-fuzzy metric graph, where is introduced by M. Hamidi et.al [6].

Theorem 8: Let (V, ρ, T) be a fuzzy metric space and $G = (X, Y, \rho, T, S)$ be a KM-single valued neutrosophic metric graph on $G^* = (V, E)$. Then for starting time:

(i) for all $xy \in E$, $T_E(xy) = 0$ or $T_V(x) = 0$ or $T_V(y) = 0$.

(ii) $|Range(I_E))| = |Range(I_V))| = |Range(F_E))| = |Range(F_V))| = |[0, 1]|$.

Proof 8: (i) Let $xy \in E$. Since $G = (X, Y, \rho, T, S)$ is a KM-single valued neutrosophic metric graph $G^* = (V, E)$, we get that $T\big(T_E(xy), T(T_V(x), T_V(y))\big) \le \rho(x, y, 0)$. Hence $T\big(T_E(xy), T(T_V(x), T_V(y))\big) = 0$ and so $T_E(xy) = 0$ or $T_V(x) = 0$ or $T_V(y) = 0$.

(*ii*) It is immediate by Definition.

Example 1: Let $V = \{1, 2, 3, 4\}$ and $x, y \in X$. Consider a fuzzy subset $\rho(x, y, 0) = 0$ and $\rho(x, y, t > 0) = \dfrac{\min\{x, y\} + t}{\max\{x, y\} + t}$. We take the negation $\eta(m) = 1 - m(m \in [0, 1])$ and obtain a KM-single valued neutrosophic metric graph $G = (V, \big(X = (T_V, I_V, F_V), Y = (T_E, I_E, F_E), \rho, T_{min}, S_{max}\big))$ on the cycle graph C_4 for $t = 1$, in Figure 1.

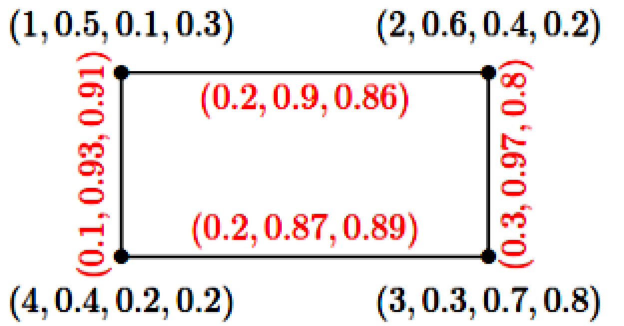

FIGURE 1. ***KM*-single valued neutrosophic metric graph $G = (X, Y, \rho, T_{min}, S_{max})$.**

Let (V, ρ, T) be a fuzzy metric space and $G = (X, Y, \rho, T, S)$ be a KM-single valued neutrosophic metric graph on $G^* = (V, E)$ and $\alpha, \beta, \gamma \in [0, 1]$. Define $T_V^\alpha = \{x \in V \mid T_V(x) \ge \alpha\}$, $I_V^\beta = \{x \in V \mid I_V(x) \le \beta\}$, $F_V^\gamma = \{x \in V \mid F_V(x) \le \gamma\}$, $T_E^\alpha = \{xy \in E \mid T_E(x) \ge \alpha\}$, $I_E^\beta = \{xy \in E \mid I_E(x) \le \beta\}$, $F_E^\gamma = \{xy \in E \mid F_E(x) \le \gamma\}$, $X^{(\alpha,\beta,\gamma)} = \{x \in V \mid T_V(x) \ge \alpha, I_V(x) \le \beta, F_V(x) \le \gamma\}$ and $Y^{(\alpha,\beta,\gamma)} = \{xy \in E \mid T_E(x) \ge \alpha, I_E(x) \le \beta, F_E(x) \le \gamma\}$.

Theorem 9: Let (V, ρ, T) be a fuzzy metric space and $G = (X, Y, \rho, T, S)$ be a KM-single valued neutrosophic metric graph on $G^* = (V, E)$ and $\alpha, \beta, \gamma \in [0, 1]$. Then $X^{(\alpha,\beta,\gamma)} = T_V^\alpha \cap I_V^\beta \cap F_V^\gamma$ and $Y^{(\alpha,\beta,\gamma)} = T_E^\alpha \cap I_E^\beta \cap F_E^\gamma$.

Proof 9: Let $x \in X^{(\alpha,\beta,\gamma)}$. Then $T_V(x) \ge \alpha, I_V(x) \le \beta$ and $F_V(x) \le \gamma$ implies that $x \in T_V^\alpha \cap I_V^\beta \cap F_V^\gamma$ and conversely. In similar a way, one can see that $Y^{(\alpha,\beta,\gamma)} = T_E^\alpha \cap I_E^\beta \cap F_E^\gamma$.

Let $G = (X, Y, \rho, T, S)$ be a KM-single valued neutrosophic metric graph on $G^* = (V, E)$. Consider $\alpha_{min} = \bigwedge_{xy \in E} T\big(T_E(xy), T(T_V(x), T_V(y))\big)$, $\beta_{max} = \bigvee_{xy \in E} S\big(I_E(xy), S(I_V(x), I_V(y))\big)$, $\gamma_{max} = \bigvee_{xy \in E} S\big(F_E(xy), S(F_V(x), F_V(y))\big)$. Thus we have the following theorem.

Theorem 10: Let (V, ρ, T) be a fuzzy metric space and $G = (X, Y, \rho, T, S)$ be a KM-single valued neutrosophic metric graph on $G^* = (V, E)$. Then For any $\alpha \le \alpha_{min}, \beta \ge \beta_{max}, \gamma \ge \gamma_{max}$, $G^{(\alpha,\beta,\gamma)} = \big(X^{(\alpha,\beta,\gamma)}, Y^{(\alpha,\beta,\gamma)}\big)$ is a subgraph of $G^* = (X, Y)$. parameters of $\mathbb{R}^+$.

Proof 10: Let $xy \in E$. Since $T\big(T_E(xy), T(T_V(x), T_V(y))\big) \le T_{min}\big(T_E(xy), T(T_V(x), T_V(y))\big)$, we get that $T_E(xy) \ge \alpha_{min} \ge \alpha$. So for any $\alpha \le \alpha_{min}$, ${T_E}^\alpha \subseteq E$. Also since $S\big(I_E(xy), S(I_V(x), I_V(y))\big) \ge S_{max}\big(I_E(xy), S(I_V(x), I_V(y))\big)$, we get that $I_E(xy) \le \beta_{max} \le \beta$. So for any $\beta \ge \beta_{max}$, ${I_E}^\beta \subseteq E$. In a similar way, can see that ${F_E}^\beta \subseteq E$. Using Theorem 9, $Y^{(\alpha,\beta,\gamma)} \subseteq E$ and so $G^{(\alpha,\beta,\gamma)} = \big(X^{(\alpha,\beta,\gamma)}, Y^{(\alpha,\beta,\gamma)}\big)$ is a subgraph of $G^* = (X, Y)$.

Theorem 11: Let (V, ρ, T) be a KM-fuzzy metric space and $G^* = (V, E)$ be a simple graph.

(*i*) If $T_E \le \rho, I_E \ge \rho$ and $F_E \ge \rho$ then $G = (X, Y, \rho, T)$ is a KM-single valued neutrosophic metric graph on G^*.

(*ii*) If $G = (X, Y, \rho, T_{min}, S_{max})$ is a KM-single valued neutrosophic metric graph on G^* and $T_E > \rho, I_E < \rho$ and $F_E < \rho$, then $G = (X, Y)$ is not a single valued neutrosophic graph on G^*.

(*iii*) If $G = (X, Y, \rho, T_{min}, S_{max})$ is a strong KM-single valued neutrosophic metric graph on G^*, then $G = (X, Y)$ is a KM-single valued neutrosophic graph on G^* if and only if $\rho(x, y, t) \ge T_E(xy), \rho(x, y, t) \le I_E(xy)$ and $\rho(x, y, t) \le F_E(xy)$.

Proof 11: Let $x, y \in V$. Then for some $t \in \mathbb{R}^{\ge 0}$:

(*i*) Since $T\big(T_E(xy), T(T_V(x), T_V(y))\big) \le T_E(xy)$, $S\big(I_E(xy), S(I_V(x), I_V(y))\big) \ge I_E(xy)$ and $S\big(F_E(xy), S(F_V(x), F_V(y))\big) \ge F_E(xy)$ then $T_E \le \rho, I_E \ge \rho$ and $F_E \ge \rho$ imply that $T\big(T_E(xy), T(T_V(x), T_V(y))\big) \le \rho(x, y, t)$, $S\big(I_E(xy), S(I_V(x), I_V(y))\big) \ge \rho(x, y, t)$ and $S\big(F_E(xy), S(F_V(x), F_V(y))\big) \ge \rho(x, y, t)$. So $G = (X, Y, \rho, T)$ is a KM-single valued neutrosophic graph metric graph on G^*.

(*ii*) Let $G = (X, Y)$ be a single valued neutrosophic graph on G^*. For all $xy \in E$, since $G = (X, Y, \rho, T_{min}, S_{max})$ is a KM-single valued neutrosophic metric graph on G^*, using $T_E(xy) \le T_{min}(T_V(x), T_V(y)), I_E(xy) \ge S_{max}(I_V(x), I_V(y))$ and $F_E(xy) \ge S_{max}(F_V(x), F_V(y))$, we get that $T_E(xy) = T_{min}\big(T_E(xy), T_{min}(T_V(x), T_V(y))\big) \le \rho(x, y, t)$, $I_E(xy) = S_{max}\big(I_E(xy), S_{max}(I_V(x), I_V(y))\big) \ge \rho(x, y, t)$ and $F_E(xy) = S_{max}\big(F_E(xy), S_{max}(F_V(x), F_V(y))\big) \ge \rho(x, y, t)$ which it is a contradiction.

(*iii*) $G = (X, Y)$ is a single valued neutrosophic graph on G^* if and only if for all $xy \in E$, $T_E(xy) \le T_{min}(T_V(x), T_V y))$, $I_E(xy) \ge S_{max}(I_V(x), I_V(y))$ and $F_E(xy) \ge S_{max}(F_V(x), F_V(y))$. Then $G = (X, Y)$ is a KM-single valued neutrosophic graph on G^* if and only if $T_{min}(T_E(xy)(xy), T_{min}(T_V(x), T_V(y))) = T_E(xy)$, $S_{max}(I_E(xy)(xy), S_{max}(I_V(x), I_V(y))) = I_E(xy)$ and $S_{max}(F_E(xy)(xy), S_{max}(F_V(x), F_V(y))) = F_E(xy)$ if and only if $\rho(x, y, t) \ge T_E(xy)$, $\rho(x, y, t) \le I_E(xy)$ and $\rho(x, y, t) \le F_E(xy)$.

Corollary 4: Let $G = (X, Y, \rho, T, S)$ be a KM-fuzzy metric connected graph on $G^* = (V, E)$. Then for starting time $G = (X, Y)$ is not a single valued neutrosophic graph on G^*.

Theorem 12: Let (V, ρ, T) be a KM-fuzzy metric space, $G^* = (V, E)$ be a simple graph and $xy \in E$. Then for $T_V, I_V, F_V : V \to [0, 1]$ and $T_E, I_E, F_E : E \to [0, 1]$, (*i*) If $T_V(x)+T_V(y)\leq 1, I_V(x)+I_V(y)=1$ and $F_V(x)+F_V(y)=1$ then $G=(X,Y,\rho,T_{lu},S_{lu})$ is a KM-single valued neutrosophic metric graph on G^*.

(*ii*) If $T_E(xy)+1\leq T_V(xy)+T_V(x)+T_V(y)\leq 2, I_V(x)+I_V(y)=F_V(x)+F_V(y)=1$, then $G=(X,Y,\rho,T_{lu},S_{lu})$ is a KM-single valued neutrosophic metric graph on G^*.

Proof 12: Let $x,y\in V$. Then for some $t\in\mathbb{R}^{\geq 0}$:

(*i*) W have

$$\begin{aligned}&T_{lu}(T_E(xy),T_{lu}(T_V(x),T_V(y))\\&\quad=\max\big(0,T_E(xy)+T_{lu}(T_V(x),T_V(y))-1\big)\\&\quad=\max\big(0,T_E(xy)+\max(0,T_V(x)+T_V(y)-1)-1\big).\end{aligned}$$

If $T_V(x)+T_V(y)\leq 1$, then $T_{lu}(T_E(xy),T_{lu}(T_V(x),T_V(y))=\max\big(0,T_E(xy)-1\big)=0$, since for all $x,y\in V$ we have $T_E(xy)\leq 1$. It concludes that for any time $t\in\mathbb{R}^{\geq 0}$ get that $T_{lu}(T_E(xy),T_{lu}(T_V(x),T_V(y))\leq\rho(x,y,t)$. In addition, $I_V(x)+I_V(y)=1$, implies that

$$\begin{aligned}&S_{lu}(I_E(xy),S_{lu}(I_V(x),I_V(y))\\&\quad=\min\big(1,I_E(xy)+S_{lu}(I_V(x),I_V(y))\big)\\&\quad=\min\big(1,I_E(xy)+\min(1,I_V(x)+I_V(y))\big)\\&\quad=\min\big(1,I_E(xy)+1\big)=1\geq\rho(x,y,t).\end{aligned}$$

In a similar way, one can prove that $S_{lu}(F_E(xy),S_{lu}(F_V(x),F_V(y))\geq\rho(x,y,t)$ and so $G=(X,Y,\rho,T_{lu},S_{lu})$ is a KM-single valued neutrosophic metric graph on G^*.

(*ii*) Because $T_E(xy)+1\leq T_E(xy)+T_V(x)+T_V(y)\leq 2$, we get that $T_V(x)+T_V(y)\geq 1$ and by item (*i*), have $T_{lu}(T_E(xy),T_{lu}(T_V(x),T_V(y))=T_{lu}(0,T_E(xy)+T_V(x)+T_V(y)-2)=0$. Moreover, $I_V(x)+I_V(y)=F_V(x)+F_V(y)=1$, implies that

$$\begin{aligned}&S_{lu}(F_E(xy),S_{lu}(F_V(x),F_V(y))\\&\quad=\min\big(1,F_E(xy)+S_{lu}(F_V(x),F_V(y))\big)\\&\quad=\min\big(1,F_E(xy)+\min(1,F_V(x)+F_V(y))\big)\\&\quad=\min\big(1,F_E(xy)+1\big)=1\geq\rho(x,y,t).\end{aligned}$$

KM-fuzzy metric graph on G^*. It follows that $G=(X,Y,\rho,T_{lu},S_{lu})$ is a KM-single valued neutrosophic metric graph on G^*.

A. OPERATIONS ON KM-FUZZY METRIC GRAPHS

In this section, for any given two KM-single valued neutrosophic metric graphs, define some product operations and show that the product of KM-single valued neutrosophic metric graphs is a KM-fuzzy metric graph. From now on, we consider $G_1=(X_1=(T_V^{(1)},I_V^{(1)},F_V^{(1)}),Y_1=(T_E^{(1)},I_E^{(1)},F_E^{(1)}),\rho_1,T,S)$, $G_2=(X_2=(T_V^{(2)},I_V^{(2)},F_V^{(2)}),Y_2=(T_E^{(2)},I_E^{(2)},F_E^{(2)}),\rho_2,T,S)$ as KM-single valued neutrosophic metric graphs on simple graphs $G_1^*=(V_1,E_1)$ and $G_2^*=(V_2,E_2)$, respectively.

Definition 7: Let G_1, G_2 be KM-single valued neutrosophic metric graphs on simple graphs G_1^* and G_2^*, respectively. Define the categorical product (tensor product) of fuzzy subsets $X_1\times X_2=(T_V^{(1)}\times T_V^{(2)},I_V^{(1)}\times I_V^{(2)},F_V^{(1)}\times F_V^{(2)})$, $Y_1\times Y_2=(T_E^{(1)}\times T_E^{(2)},I_E^{(1)}\times I_E^{(2)},F_E^{(1)}\times F_E^{(2)})$, where $T_V^{(1)}\times T_V^{(2)},I_V^{(1)}\times I_V^{(2)},F_V^{(1)}\times F_V^{(2)}:V(G_1^*\times G_2^*)\to[0,1]$ by

$$\begin{aligned}(T_V^{(1)}\times T_V^{(2)})(x_1,x_2)&=T_{min}(T_V^{(1)}(x_1),T_V^{(2)}(x_2)),\\(I_V^{(1)}\times I_V^{(2)})(x_1,x_2)&=S_{max}(I_V^{(1)}(x_1),I_V^{(2)}(x_2)),\\(F_V^{(1)}\times F_V^{(2)})(x_1,x_2)&=S_{max}(F_V^{(1)}(x_1),F_V^{(2)}(x_2)),\end{aligned}$$

and $T_E^{(1)}\times T_E^{(2)},I_E^{(1)}\times I_E^{(2)},F_E^{(1)}\times F_E^{(2)}:E(G_1^*\times G_2^*)\to[0,1]$ by

$$\begin{aligned}&(T_E^{(1)}\times T_E^{(2)})((x_1,x_2)(y_1,y_2))=T_{min}(T_E^{(1)}(x_1y_1),\\&\quad T_E^{(2)}(x_2y_2)),(I_E^{(1)}\times I_E^{(2)})((x_1,x_2)(y_1,y_2))=S_{max}\\&\quad(I_E^{(1)}(x_1y_1),I_E^{(2)}(x_2y_2),(F_E^{(1)}\times F_E^{(2)})((x_1,x_2)(y_1,y_2))\\&=S_{max}(F_E^{(1)}(x_1y_1),F_E^{(2)}(x_2y_2).\end{aligned}$$

Theorem 13: Let G_1 and G_2 be KM-single valued neutrosophic metric graphs on simple graphs G_1^* and G_2^*, respectively. Then $G_1\times G_2=(X_1\times X_2,Y_1\times Y_2,T_{min}(\rho),T,S)$ is a KM-single valued neutrosophic metric graph on $G_1^*\times G_2^*$.

Proof 13: Firstly, by Theorem 6, $(V_1\times V_2,T_{min}(\rho),T)$ is a KM-fuzzy metric space. Let $(x_1,x_2)(y_1,y_2)\in E(G_1^*\times G_2^*)$. Since G_1 is a KM-single valued neutrosophic metric graph on G_1^* and G_2 is a KM-single valued neutrosophic metric graph on G_2^*, for some $t_1,t_2\in\mathbb{R}^{\geq 0}$, we get that

$$\begin{aligned}&T\big((T_E^{(1)}\times T_E^{(2)})((x_1,x_2)(y_1,y_2)),T\big((T_V^{(1)}\times T_V^{(2)})\\&\quad(x_1,x_2),(T_V^{(1)}\times T_V^{(2)})(y_1,y_2)\big)=T\big(T_{min}(T_E^{(1)}(x_1y_1),\\&\quad T_E^{(2)}(x_2y_2)),T\big((T_{min}(T_V^{(1)}(x_1),T_V^{(1)}(x_2)),(T_{min}\\&\quad(T_V^{(1)}(y_1),T_V^{(2)}(y_2)))\big)\leq T\big(T_E^{(1)}(x_1\,y_1),T\big(T_V^{(1)}(x_1),\\&\quad T_V^{(1)}(y_1))\big)\leq\rho_1(x_1,y_1,t_1)\text{ and}\\&\quad T\big((T_E^{(1)}\times T_E^{(2)})((x_1,x_2)(y_1,y_2)),T\big((T_V^{(1)}\times T_V^{(2)})\\&\quad(x_1,x_2),(T_V^{(1)}\times T_V^{(2)})(y_1,y_2)\big)=T\big(T_{min}(T_E^{(1)}(x_1y_1),\\&\quad T_E^{(2)}(x_2y_2)),T\big((T_{min}(T_V^{(1)}(x_1),T_V^{(2)}(x_2)),(T_{min}(T_V^{(1)}\\&\quad(y_1),T_V^{(2)}(y_2)))\big)\leq T\big(T_E^{(2)}(x_2\,y_2),T\big(T_V^{(2)}(x_2),\\&\quad T_V^{(2)}(y_2))\big)\leq\rho_2(x_2,y_2,t_2).\end{aligned}$$

Consider $t=\max\{t_1,t_2\}$, so by Theorem 1, we obtain

$$\begin{aligned}&T\big((T_E^{(1)}\times T_E^{(2)}((x_1,x_2)(y_1,y_2)),\\&\qquad T\big((T_V^{(1)}\times T_V^{(2)})(x_1,x_2),(T_V^{(1)}\times T_V^{(2)})(y_1,y_2)\big)\\&\quad\leq T_{min}\big(\rho_1(x_1,y_1,t_1),\rho_2(x_2,y_2,t_2)\big)\\&\quad\leq T_{min}(\rho)((x_1,x_2),(y_1,y_2),t).\end{aligned}$$

I addition,

$$\begin{aligned}&S\big((I_E^{(1)}\times I_E^{(2)})((x_1,x_2)(y_1,y_2)),\\&\qquad S\big((I_V^{(1)}\times I_V^{(2)})(x_1,x_2),(I_V^{(1)}\times I_V^{(2)})(y_1,y_2)\big)\\&\quad=S\big(S_{max}(I_E^{(1)}(x_1y_1),I_E^{(2)}(x_2y_2)),S\big((S_{max}(I_V^{(1)}(x_1),\\&\quad I_V^{(1)}(x_2)),(S_{max}(I_V^{(1)}(y_1),I_V^{(2)}(y_2)))\big)\\&\quad\geq S\big(I_E^{(1)}(x_1\,y_1),S\big(I_V^{(1)}(x_1),I_V^{(1)}(y_1)\big)\big)\geq\rho_1(x_1,y_1,t_1)\end{aligned}$$

and $S\big((I_E^{(1)} \times I_E^{(2)})((x_1,x_2)(y_1,y_2)), S\big((I_V^{(1)} \times I_V^{(2)})(x_1,x_2), (I_V^{(1)} \times I_V^{(2)})(y_1,y_2)\big) = S\big(S_{max}(I_E^{(1)}(x_1y_1), I_E^{(2)}(x_2y_2)), S\big((S_{max}(I_V^{(1)}(x_1), I_V^{(2)}(x_2)), (S_{max}(I_V^{(1)}(y_1), I_V^{(2)}(y_2))\big)\big) \geq S\big(I_E^{(2)}(x_2\,y_2), S\big(I_V^{(2)}(x_2), I_V^{(2)}(y_2)\big)\big) \geq \rho_2(x_2,y_2,t_2)$.

Consider $t = \min\{t_1, t_2\}$, so by Theorem 1, we obtain

$$S\big((I_E^{(1)} \times I_E^{(2)}((x_1,x_2)(y_1,y_2)), S\big((I_V^{(1)} \times I_V^{(2)})(x_1,x_2), (I_V^{(1)} \times I_V^{(2)})(y_1,y_2)\big) \geq S_{max}\big(\rho_1(x_1,y_1,t_1), \rho_2(x_2,y_2,t_2)\big) \geq S_{max}(\rho)((x_1,x_2),(y_1,y_2),t).$$

In a similar way, can see that $S\big((F_E^{(1)} \times F_E^{(2)}((x_1,x_2)(y_1,y_2)), S\big((F_V^{(1)} \times F_V^{(2)})(x_1,x_2), (F_V^{(1)} \times F_V^{(2)})(y_1,y_2)\big) \geq S_{max}(\rho)((x_1,x_2),(y_1,y_2),t)$. Thus $G_1 \times G_2 = (X_1 \times X_2, Y_1 \times Y_2, T_{min}(\rho), T, S)$ is a KM-single valued neutrosophic metric graph on $G_1^* \times G_2^*$.

Definition 8: Let G_1, G_2 be KM-single valued neutrosophic metric graphs on simple graphs G_1^* and G_2^*, respectively. Define the Cartesian product (or product) fuzzy subsets $X_1 \otimes X_2 = (T_V^{(1)} \otimes T_V^{(2)}, I_V^{(1)} \otimes I_V^{(2)}, F_V^{(1)} \otimes F_V^{(2)})$, $Y_1 \otimes Y_2 = (T_E^{(1)} \otimes T_E^{(2)}, I_E^{(1)} \otimes I_E^{(2)}, F_E^{(1)} \otimes F_E^{(2)})$, where $T_V^{(1)} \otimes T_V^{(2)}, I_V^{(1)} \otimes I_V^{(2)}, F_V^{(1)} \otimes F_V^{(2)} : V(G_1^* \times G_2^*) \to [0,1]$ by $(T_V^{(1)} \otimes T_V^{(2)})(x_1,x_2) = T_{min}(T_V^{(1)}(x_1), T_V^{(2)}(x_2))$, $(I_V^{(1)} \otimes I_V^{(2)})(x_1,x_2) = S_{max}(I_V^{(1)}(x_1), I_V^{(2)}(x_2))$, $(F_V^{(1)} \otimes F_V^{(2)})(x_1,x_2) = S_{max}(F_V^{(1)}(x_1), F_V^{(2)}(x_2))$, and $T_E^{(1)} \otimes T_E^{(2)}, I_E^{(1)} \otimes I_E^{(2)}, F_E^{(1)} \otimes F_E^{(2)} : E(G_1^* \times G_2^*) \to [0,1]$ by $(T_E^{(1)} \otimes T_E^{(2)})((x,x_2)(x,y_2)) = T_{min}(T_V^{(1)}(x), T_E^{(2)}(x_2y_2))$, $(T_E^{(1)} \otimes T_E^{(2)})((x_1,y)(y_1,y)) = T_{min}(T_V^{(2)}(y), T_E^{(1)}(x_1y_1))$, $(I_E^{(1)} \otimes I_E^{(2)})((x,x_2)(x,y_2)) = S_{max}(I_V^{(1)}(x), I_E^{(2)}(x_2y_2))$, $(I_E^{(1)} \otimes I_E^{(2)})((x_1,y)(y_1,y)) = S_{max}(I_V^{(2)}(y), I_E^{(1)}(x_1y_1))$, $(F_E^{(1)} \otimes F_E^{(2)})((x,x_2)(x,y_2)) = S_{max}(F_V^{(1)}(x), F_E^{(2)}(x_2y_2))$, $(F_E^{(1)} \otimes F_E^{(2)})((x_1,y)(y_1,y)) = S_{max}(F_V^{(2)}(y), F_E^{(1)}(x_1y_1))$.

Example 2: Consider the KM-fuzzy metric spaces $(V_1 = \{1,2\}, \rho_1, T_{min})$, $(V_2 = \{3,4,5\}, \rho_2, T_{min})$, where $\rho_1(1,1,t>0) = 1$, $\rho_1(2,2,t>0) = 1$, $\rho_1(1,2,t>0) = \dfrac{1+t}{2+t}$, $\rho_1(x,y,0) = 0$, $x,y \in V_1$ and for all $x,y \in \{3,4,5\}$,

$$\rho_2(x,y,t) = \begin{cases} \dfrac{\min\{x,y\}+t}{\max\{x,y\}+t} & \text{if } t>0 \\ 0 & \text{if } t=0 \end{cases}$$

. We take the negation $\eta(m) = 1-m (m \in [0,1])$ and obtain the KM-single valued neutrosophic metric graphs $G_1 = (V_1, \big(X = (T_V, I_V, F_V), Y = (T_E, I_E, F_E), \rho_1, T_{min}, S_{max}\big))$ in unit time $t_1 = 1$ and $G_2 = (V_2, \big(X = (T_V, I_V, F_V), Y = (T_E, I_E, F_E), \rho_2, T_{min}, S_{max}\big))$ in unit time $t_2 = 1$ on G_1^* and G_2^* in Figure 2, where $A = (0.6, 0.4, 0.2)$, $B = (0.5, 0.1, 0.3)$, $C = (0.3, 0.5, 0.7)$, $D = (0.5, 0.6, 0.2)$, $E = (0.1, 0.2, 0.5)$, $AB = (0.5, 0.97, 0.95)$, $CE = (0.1, 0.96, 0.91)$, $ED = (0.5, 0.98, 0.99)$ and $DC = (0.3, 0.93, 0.96)$. Now, we obtain the KM-fuzzy metric graph $G_1 \times G_2$ in

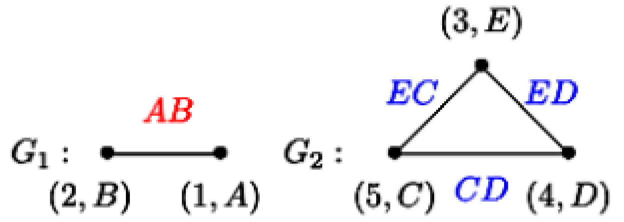

FIGURE 2. ***KM*-single valued neutrosophic metric graphs G_1 and G_2 for $t = 1$.**

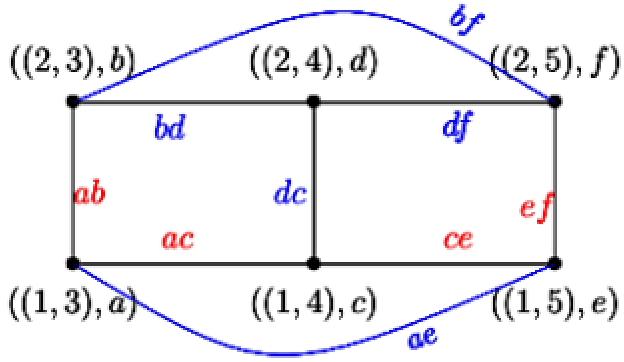

FIGURE 3. ***KM*-single valued neutrosophic metric graph $G_1 \otimes G_2$ for $t = 1$.**

Figure 3, where $a = (0.1, 0.4, 0.5)$, $b = (0.1, 0.4, 0.5)$, $c = (0.5, 0.6, 0.2)$, $d = (0.5, 0.6, 0.3)$, $e = (0.3, 0.5, 0.7)$, $f = (0.3, 0.5, 0.7)$, $ab = (0.1, 0.97, 0.95)$, $dc = (0.5, 0.97, 0.95)$, $ef = (0.3, 0.97, 0.95)$, $bd = (0.5, 0.98, 0.99)$, $df = (0.3, 0.93, 0.96)$, $ac = (0.5, 0.98, 0.99)$, $ce = (0.3, 0.93, 0.96)$, $bf = (0.1, 0.96, 0.961)$ and $ae = (0.1, 0.96, 0.91)$.

Theorem 14: Let G_1 and G_2 be KM-single valued neutrosophic metric graphs on simple graphs G_1^* and G_2^*, respectively.

(*i*) If $(G_1^* \otimes G_2^*, T(\rho), T)$ is a KM-fuzzy metric space, then $T(\rho) = \rho_1$ or $T(\rho) = \rho_2$, where $T \in \mathcal{C}_T$.

(*ii*) $G_1 \otimes G_2 = (X_1 \otimes X_2, Y_1 \otimes Y_2, T_{min}(\rho), T, S)$ is a KM-single valued neutrosophic metric graph on $G_1^* \otimes G_2^*$.

Proof 14: (*i*) Let $(x_1,x_2),(y_1,y_2) \in E(G_1^* \otimes G_2^*)$. Then $x_1 = y_1$ and $x_2y_2 \in E(G_2^*)$ or $x_2 = y_2$ and $x_1y_1 \in E(G_1^*)$. If $x_1 = y_1$ and $x_2y_2 \in E(G_2^*)$, then $T(\rho)\big((x_1,x_2),(x_1,y_2),t\big) = T(\rho_1(x_1,x_1,t), \rho_2(x_2,y_2,t)) = T(1, \rho_2(x_2,y_2,t)) = \rho_2(x_2,y_2,t)$. If $x_2 = y_2$ and $x_1y_1 \in E(G_1^*)$, then $T(\rho)\big((x_1,x_2),(y_1,x_2),t\big) = T(\rho_1(x_1,y_1,t), \rho_2(x_2,x_2,t)) = T(\rho_1(x_1,y_1,t),1) = \rho_1(x_1,y_1,t)$.

(*ii*) Firstly, by Theorem 6, $(V_1 \times V_2, T_{min}(\rho), T)$ is a KM-fuzzy metric space. Let $(x_1,x_2),(y_1,y_2) \in E(G_1^* \otimes G_2^*)$. Since G_1 is a KM-single valued neutrosophic metric graph on G_1^* and G_2 is a KM-single valued neutrosophic metric graph on G_2^*, for some $t_1, t_2 \in \mathbb{R}^{\geq 0}$, give $t = \max\{t_1,t_2\}$, so by item (*i*) and Theorem 1, we get that

$$T\big((T_E^{(1)} \otimes T_E^{(2)})((x,x_2)(x,y_2)), T\big((T_V^{(1)} \otimes T_V^{(2)})(x,x_2), (T_V^{(1)} \otimes T_V^{(2)})(x,y_2)\big)$$

$$= T\big(T_{min}(T_V^{(1)}(x), T_E^{(2)}(x_2y_2)), T\big(T_{min}(T_V^{(1)}(x), T_V^{(2)}(x_2)), T_{min}(T_V^{(1)}(x), T_V^{(2)}(y_2))\big)\big)$$
$$\le T\big(T_E^{(2)}(x_2\, y_2), T\big(T_V^{(2)}(x), T_V^{(2)}(y_2)\big)\big) \le \rho_2(x_2, y_2, t_2)$$
$$\le T_{min}(\rho)((x, x_2)(x, y_2), t) \text{ and}$$
$$T\big((T_E^{(1)} \otimes T_E^{(2)})((x_1, y)(y_1, y)), T\big((T_V^{(1)} \otimes T_V^{(2)})(x_1, y), (T_V^{(1)} \otimes T_V^{(2)})(y_1, y)\big)\big)$$
$$= T\big(T_{min}(T_V^{(2)}(y), T_E^{(1)}(x_1y_1)), T\big(T_{min}(T_V^{(1)}(x_1), T_V^{(2)}(y)), T_{min}(T_V^{(1)}(y_1), T_V^{(2)}(y))\big)\big)$$
$$\le T\big(T_E^{(1)}(x_1y_1), T\big(T_V^{(1)}(x_1), T_V^{(1)}(y_1)\big)\big) \le \rho_1(x_1, y_1, t_1)$$
$$\le T_{min}(\rho)((x_1, y)(y_1, y), t)$$

Now, give $t = \min\{t_1, t_2\}$, so by item (i) and Theorem 1, we get that

$$S\big((I_E^{(1)} \otimes I_E^{(2)})((x, x_2)(x, y_2)), S\big((I_V^{(1)} \otimes I_V^{(2)})(x, x_2), (I_V^{(1)} \otimes I_V^{(2)})(x, y_2)\big)$$
$$= S\big(S_{max}(I_V^{(1)}(x), I_E^{(2)}(x_2y_2)), S\big(S_{max}(I_V^{(1)}(x), I_V^{(2)}(x_2)), S_{max}(I_V^{(1)}(x), I_V^{(2)}(y_2))\big)\big)$$
$$\ge S\big(I_E^{(2)}(x_2\, y_2), S\big(I_V^{(2)}(x), I_V^{(2)}(y_2)\big)\big) \ge \rho_2(x_2, y_2, t_2)$$
$$\ge S_{max}(\rho)((x, x_2)(x, y_2), t) \text{ and } S\big((I_E^{(1)} \otimes I_E^{(2)})((x_1, y)(y_1, y)), S\big((T_V^{(1)} \otimes I_V^{(2)})(x_1, y), (T_V^{(1)} \otimes I_V^{(2)})(y_1, y)\big)$$
$$= S\big(S_{max}(I_V^{(2)}(y), I_E^{(1)}(x_1y_1)), T\big(S_{max}(I_V^{(1)}(x_1), I_V^{(2)}(y)), S_{max}(I_V^{(1)}(y_1), I_V^{(2)}(y))\big)\big)$$
$$\ge S\big(I_E^{(1)}(x_1y_1), S\big(I_V^{(1)}(x_1), I_V^{(1)}(y_1)\big)\big) \ge \rho_1(x_1, y_1, t_1)$$
$$\ge S_{max}(\rho)((x_1, y)(y_1, y), t).$$

In a similar way, can see that $S\big((F_E^{(1)} \otimes F_E^{(2)}((x_1, x_2)(y_1, y_2)), S\big((F_V^{(1)} \otimes F_V^{(2)})(x_1, x_2), (F_V^{(1)} \otimes F_V^{(2)})(y_1, y_2)\big) \ge S_{max}(\rho)((x_1, x_2), (y_1, y_2), t)$. Thus $G_1 \otimes G_2 = (X_1 \otimes X_2, Y_1 \otimes Y_2, T_{min}(\rho), T, S)$ is a KM-single valued neutrosophic metric graph on $G_1^* \otimes G_2^*$.

Definition 9: Let G_1, G_2 be KM-single valued neutrosophic metric graphs on simple graphs G_1^* and G_2^*, respectively.

Define the semi-strong product of fuzzy subsets $X_1 \cdot X_2 = (T_V^{(1)} \cdot T_V^{(2)}, I_V^{(1)} \cdot I_V^{(2)}, F_V^{(1)} \cdot F_V^{(2)}), Y_1 \cdot Y_2 = (T_E^{(1)} \cdot T_E^{(2)}, I_E^{(1)} \cdot I_E^{(2)}, F_E^{(1)} \cdot F_E^{(2)})$, where $T_V^{(1)} \cdot T_V^{(2)}, I_V^{(1)} \cdot I_V^{(2)}, F_V^{(1)} \cdot F_V^{(2)} : V(G_1^* \times G_2^*) \to [0, 1]$ by $(T_V^{(1)} \cdot T_V^{(2)})(x_1, x_2) = T_{min}(T_V^{(1)}(x_1), T_V^{(2)}(x_2)), (I_V^{(1)} \cdot I_V^{(2)})(x_1, x_2) = S_{max}(I_V^{(1)}(x_1), I_V^{(2)}(x_2)), (F_V^{(1)} \cdot F_V^{(2)})(x_1, x_2) = S_{max}(F_V^{(1)}(x_1), F_V^{(2)}(x_2))$, and $T_E^{(1)} \cdot T_E^{(2)}, I_E^{(1)} \cdot I_E^{(2)}, F_E^{(1)} \cdot F_E^{(2)} : E(G_1^* \times G_2^*) \to [0, 1]$ by $(T_E^{(1)} \cdot T_E^{(2)})((x, x_2)(x, y_2)) = T_{min}(T_V^{(1)}(x), T_E^{(2)}(x_2y_2)), (T_E^{(1)} \cdot T_E^{(2)})((x_1, x_2)(y_1, y_2)) = T_{min}(T_E^{(1)}(x_1y_1)), T_E^{(2)}(x_2y_2)), (I_E^{(1)} \cdot I_E^{(2)})((x, x_2)(x, y_2)) = S_{max}(I_V^{(1)}(x), I_E^{(2)}(x_2y_2)), (I_E^{(1)} \cdot I_E^{(2)})(x_1, x_2)(y_1, y_2)) = S_{max}(I_E^{(1)}(x_1y_1)), I_E^{(2)}(x_2y_2)), (F_E^{(1)} \cdot F_E^{(2)})((x, x_2)(x, y_2)) = S_{max}(F_V^{(1)}(x), F_E^{(2)}(x_2y_2)), (F_E^{(1)} \cdot F_E^{(2)})((x_1, x_2)(y_1, y_2)) = S_{max}(F_E^{(1)}(x_1y_1), F_E^{(2)}(x_2y_2))$.

Example 3: Consider the KM-single valued neutrosophic metric graphs G_1 and G_2 in Example 2. So we obtain the KM-fuzzy metric graph $G_1 \cdot G_2$ in Figure 4, where $a = (0.1, 0.4, 0.5), b = (0.1, 0.4, 0.5), c = (0.5, 0.6, 0.2), d = (0.5, 0.6, 0.3), e = (0.3, 0.5, 0.7), f = (0.3, 0.5, 0.7), be = (0.1, 0.97, 0.95), bc = (0.5, 0.98, 0.99), af = (0.1, 0.97, 0.95), bd = (0.5, 0.98, 0.99), df = (0.3, 0.93, 0.96), ac = (0.5, 0.98, 0.99), ce = (0.3, 0.93, 0.96), bf = (0.1, 0.96, 0.961), cf = (0.3, 0.97, 0.96), de = (0.3, 0.97, 0.96), ad = (0.5, 0.98, 0.99)$ and $ae = (0.1, 0.96, 0.91)$.

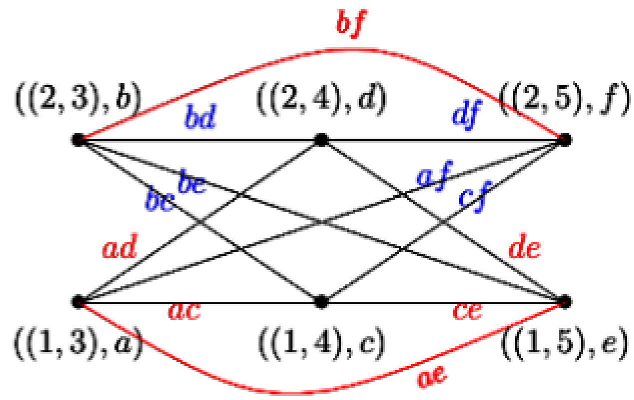

FIGURE 4. ***KM*-single valued neutrosophic metric graph $G_1 \cdot G_2$ for $t = 1$.**

Theorem 15: Let G_1 and G_2 be KM-single valued neutrosophic metric graphs on simple graphs G_1^* and G_2^*, respectively. Then $G_1 \cdot G_2 = (X_1 \cdot X_2, Y_1 \cdot Y_2, T_{min}(\rho), T, S)$ is a KM-single valued neutrosophic metric graph on $G_1^* \cdot G_2^*$.

Proof 15: It is similar to Theorems 13 and 14.

Definition 10: Let G_1, G_2 be KM-single valued neutrosophic metric graphs on simple graphs G_1^* and G_2^*, respectively. Define the strong product of fuzzy subsets $X_1 \odot X_2 = (T_V^{(1)} \odot T_V^{(2)}, I_V^{(1)} \odot I_V^{(2)}, F_V^{(1)} \odot F_V^{(2)}), Y_1 \odot Y_2 = (T_E^{(1)} \odot T_E^{(2)}, I_E^{(1)} \odot I_E^{(2)}, F_E^{(1)} \odot F_E^{(2)})$, where $T_V^{(1)} \odot T_V^{(2)}, I_V^{(1)} \odot I_V^{(2)}, F_V^{(1)} \odot F_V^{(2)} : V(G_1^* \times G_2^*) \to [0, 1]$ by $(T_V^{(1)} \odot T_V^{(2)})(x_1, x_2) = T_{min}(T_V^{(1)}(x_1), T_V^{(2)}(x_2)), (I_V^{(1)} \odot I_V^{(2)})(x_1, x_2) = S_{max}(I_V^{(1)}(x_1), I_V^{(2)}(x_2)), (F_V^{(1)} \odot F_V^{(2)})(x_1, x_2) = S_{max}(F_V^{(1)}(x_1), F_V^{(2)}(x_2))$, and $T_E^{(1)} \odot T_E^{(2)}, I_E^{(1)} \odot I_E^{(2)}, F_E^{(1)} \odot F_E^{(2)} : E(G_1^* \times G_2^*) \to [0, 1]$ by $(T_E^{(1)} \odot T_E^{(2)})((x, x_2)(x, y_2)) = T_{min}(T_V^{(1)}(x), T_E^{(2)}(x_2y_2)), (T_E^{(1)} \odot T_E^{(2)})((x_1, y)(x_2, y)) = T_{min}(T_V^{(2)}(y), T_E^{(1)}(x_1y_1)), (T_E^{(1)} \odot T_E^{(2)})((x_1, x_2)(y_1, y_2)) = T_{min}(T_E^{(1)}(x_1y_1)), T_E^{(2)}(x_2y_2)), (I_E^{(1)} \odot I_E^{(2)})((x, x_2)(x, y_2)) = S_{max}(I_V^{(1)}(x), I_E^{(2)}(x_2y_2)), (I_E^{(1)} \odot I_E^{(2)})((x_1, y)(x_2, y)) = T_{min}(I_V^{(2)}(y), I_E^{(1)}(x_1y_1)), (I_E^{(1)} \odot I_E^{(2)})(x_1, x_2)(y_1, y_2)) = S_{max}(I_E^{(1)}(x_1y_1)), I_E^{(2)}(x_2y_2)), (F_E^{(1)} \odot F_E^{(2)})((x, x_2)(x, y_2)) = S_{max}(F_V^{(1)}(x), F_E^{(2)}(x_2y_2)), (F_E^{(1)} \odot F_E^{(2)})((x_1, y)(x_2, y)) = T_{min}(F_V^{(2)}(y), F_E^{(1)}(x_1y_1))(F_E^{(1)} \odot F_E^{(2)})((x_1, x_2)(y_1, y_2)) = S_{max}(F_E^{(1)}(x_1y_1), F_E^{(2)}(x_2y_2))$.

Example 4: Consider the KM-fuzzy metric spaces $(V_1 = \{1, 2\}, \rho_1, T_{min})$, $(V_2 = \{3, 4, 5\}, \rho_2, T_{min})$, where for all

$x, y \in \{1,2\}$, $\rho_1(x,y,0) = 0, \rho_1(x,y,t>0) = \dfrac{\min\{x,y\}+t}{\max\{x,y\}+t}$ and for all $x, y \in \{3,4,5\}$,

$$\rho_2(x,y,0)=0,\ \rho_2(x,y,t>0)=\begin{cases}1 & \text{if } x=y\\ \dfrac{5+t}{10+t} & \text{if } x\neq y.\end{cases}$$

We take the negation $\eta(m) = 1-m(m \in [0,1])$ and obtain the KM-single valued neutrosophic metric graphs $G_1 = (V_1, \big(X = (T_V, I_V, F_V), Y = (T_E, I_E, F_E), \rho_1, T_{min}, S_{max}\big))$ in unit time $t_1 = 2$ and $G_2 = (V_2, \big(X = (T_V, I_V, F_V), Y = (T_E, I_E, F_E), \rho_2, T_{min}, S_{max}\big))$ in unit time $t_2 = 1$ on G_1^* and G_2^* in Figure 5, where $A = (0.1, 0.5, 0.4), B = (0.2, 0.3, 0.3), C = (0.3, 0.4, 0.5), D = (0.4, 0.6, 0.5), E = (0.5, 0.2, 0.1), AB = (0.5, 0.97, 0.95), DE = (0.5, 0.98, 0.99)$ and $DC = (0.3, 0.93, 0.96)$. Now, we obtain the

FIGURE 5. ***KM*-single valued neutrosophic metric graphs G_1, G_2 for $t_1 = 2, t_2 = 1$.**

KM-single valued neutrosophic metric graph $G_1 \odot G_2$ in Figure 6, where $a = (0.1, 0.5, 0.5)$, $b = (0.1, 0.6, 0.5)$, $c = (0.1, 0.5, 0.4)$, $d = (0.2, 0.4, 0.5)$, $e = (0.2, 0.6, 0.5)$, $f = (0.2, 0.3, 0.3)$, $ab = (0.1, 0.93, 0.96)$, $bc = (0.1, 0.98, 0.99)$, $de = (0.2, 0.93, 0.96)$, $ef = (0.2, 0.98, 0.99)$, $ad = (0.3, 0.97, 0.95)$, $be = (0.4, 0.97, 0.95)$, $cf = (0.5, 0.97, 0.95)$, $ae = (0.3, 0.97, 0.96)$, $bf = (0.5, 0.98, 0.99)$, $bd = (0.3, 0.97, 0.96)$, $ce = (0.5, 0.98, 0.99)$.

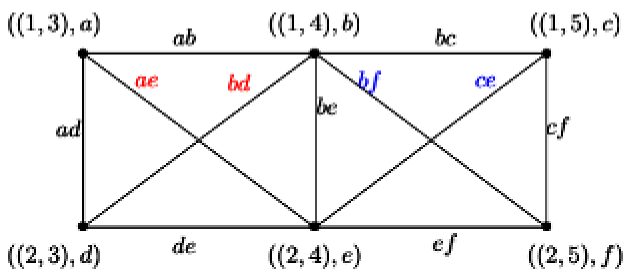

FIGURE 6. ***KM*-single valued neutrosophic $G_1 \odot G_2$ for $t = 2$.**

Theorem 16: Let G_1 and G_2 be KM-single valued neutrosophic metric graphs on simple graphs G_1^* and G_2^*, respectively. Then $G_1 \odot G_2 = (X_1 \odot X_2, Y_1 \odot Y_2, T_{min}(\rho), T, S)$ is a KM-single valued neutrosophic metric graph on $G_1^* \odot G_2^*$.

Proof 16: It is similar to Theorems 13 and 14.

Definition 11: Let G_1, G_2 be KM-single valued neutrosophic metric graphs on simple graphs G_1^* and G_2^*, respectively. Define the union of fuzzy subsets $X_1 \cup X_2 = (T_V^{(1)} \cup T_V^{(2)}, I_V^{(1)} \cup I_V^{(2)}, F_V^{(1)} \cup F_V^{(2)})$, $Y_1 \cup Y_2 = (T_E^{(1)} \cup T_E^{(2)}, I_E^{(1)} \cup I_E^{(2)}, F_E^{(1)} \cup F_E^{(2)})$, where $T_V^{(1)} \cup T_V^{(2)}, I_V^{(1)} \cup I_V^{(2)}, F_V^{(1)} \cup F_V^{(2)} : (V_1 \cup V_2) \to [0,1]$ by

$$(T_V^{(1)} \cup T_V^{(2)})(x_1,x_2) = \begin{cases} T_V^{(1)}(x) & \text{if } x \in V_1 \setminus V_2\\ T_V^{(2)}(x) & \text{if } x \in V_2 \setminus V_1\\ T_{min}(T_V^{(1)}(x), T_V^{(2)}(x)) & \text{if } x \in V_2 \cap V_1,\end{cases}$$

$$(I_V^{(1)} \cup I_V^{(2)})(x_1,x_2) = \begin{cases} I_V^{(1)}(x) & \text{if } x \in V_1 \setminus V_2\\ I_V^{(2)}(x) & \text{if } x \in V_2 \setminus V_1\\ S_{max}(I_V^{(1)}(x), I_V^{(2)}(x)) & \text{if } x \in V_2 \cap V_1,\end{cases}$$

$$(F_V^{(1)} \cup F_V^{(2)})(x_1,x_2) = \begin{cases} F_V^{(1)}(x) & \text{if } x \in V_1 \setminus V_2\\ F_V^{(2)}(x) & \text{if } x \in V_2 \setminus V_1\\ S_{max}(F_V^{(1)}(x), F_V^{(2)}(x)) & \text{if } x \in V_2 \cap V_1\end{cases}$$

and $T_E^{(1)} \cup T_E^{(2)}, I_E^{(1)} \cup I_E^{(2)}, F_E^{(1)} \cup F_E^{(2)} : (E_1 \cup E_2) \to [0,1]$, by

$$(T_E^{(1)} \cup T_E^{(2)})(xy) = \begin{cases} T_E^{(1)}(xy) & \text{if } xy \in E_1 \setminus E_2\\ T_E^{(2)}(xy) & \text{if } xy \in E_2 \setminus E_1\\ T_{min}(T_E^{(1)}(xy), T_E^{(2)}(xy)) & \text{if } xy \in E_2 \cap E_1,\end{cases}$$

$$(I_E^{(1)} \cup I_E^{(2)})(xy) = \begin{cases} I_E^{(1)}(xy) & \text{if } xy \in E_1 \setminus E_2\\ I_E^{(2)}(xy) & \text{if } xy \in E_2 \setminus E_1\\ S_{max}(I_E^{(1)}(xy), I_E^{(2)}(xy)) & \text{if } xy \in E_2 \cap E_1,\end{cases}$$

$$(F_E^{(1)} \cup F_E^{(2)})(xy) = \begin{cases} F_E^{(1)}(xy) & \text{if } xy \in E_1 \setminus E_2\\ F_E^{(2)}(xy) & \text{if } xy \in E_2 \setminus E_1\\ S_{max}(F_E^{(1)}(xy), F_E^{(2)}(xy)) & \text{if } xy \in E_2 \cap E_1.\end{cases}$$

Example 5: Consider the KM-single valued neutrosophic metric graphs G_1 and G_2 in Example 4. It is easy to see that KM-single valued neutrosophic metric metric graph $G_1 \cup G_2$ with $t = 2$ in Figure 4.

Theorem 17: Let $G_1 = (X_1, Y_1, \rho_1, T, S)$, $G_2 = (X_2, Y_2, \rho_2, T, S)$ be KM-single valued neutrosophic metric graphs on simple graphs $G_1^* = (V_1, E_1)$ and $G_2^* = (V_2, E_2)$, respectively. If $G_1^* = (V_1, E_1)$ and $G_2^* = (V_2, E_2)$ are two simple graphs, where $V_1 \cap V_2 = \emptyset$, then $G_1 \cup G_2 = (X_1 \cup X_2, Y_1 \cup Y_2, \rho_1 \cup \rho_2, T, S)$ is a KM-single valued neutrosophic metric graph on $G_1^* \cup G_2^*$.

Proof 17: Firstly, by Theorem 7, $(V_1 \cup V_2, T_{min}(\rho), T)$ is a KM-fuzzy metric space. Let $xy \in E(G_1^* \cup G_2^*)$. Since $G_1^* = (V_1, E_1)$ and $G_2^* = (V_2, E_2)$ are two simple graphs, $xy \in E_1 \setminus E_2$ implies that $(x, y \in V_1 \setminus V_2)$ and $xy \in E_2 \setminus E_1$ implies that $(x, y \in V_2 \setminus V_1)$. Since G_1 is a KM-single valued neutrosophic metric graph on G_1^* and G_2 is a KM-single valued neutrosophic metric graph on G_2^*, for some $t_1, t_2 \in \mathbb{R}^{\geq 0}$, take $t = \max\{t_1, t_2\}$ so by Theorems 1 and 7, if $xy \in E_1 \setminus E_2$, we have

$$\begin{aligned}&T\big((T_E^{(1)} \cup T_E^{(2)})(xy), T\big((T_V^{(1)} \cup T_V^{(2)})(x), (T_V^{(1)} \cup T_V^{(2)})(y)\big)\\&\quad= T\big(T_V^{(1)}(xy), T\big(T_V^{(1)}(x), T_V^{(1)}(y)\big)\\&\quad\leq \rho_1(x, y, t) \leq (\rho_1 \cup \rho_2)(x, y, t).\end{aligned}$$

In a similar way, if $xy \in E_2 \setminus E_1$, one can see that $T_{min}\big((T_E^{(1)} \cup T_E^{(2)}(xy), T_{min}\big((T_V^{(1)} \cup T_V^{(2)})(x), (T_V^{(1)} \cup T_V^{(2)})(y)\big) \leq \rho_2(x, y, t) = (\rho_1 \cup \rho_2)(x, y, t)$. Other cases is similar to.

Now consider $t = \min\{t_1, t_2\}$ so by Theorems 1 and 7, if $xy \in E_1 \setminus E_2$, we have

$$\begin{aligned}&S\big((I_E^{(1)} \cup I_E^{(2)})(xy), S\big((I_V^{(1)} \cup I_V^{(2)})(x), (I_V^{(1)} \cup I_V^{(2)})(y)\big)\\&\quad= S\big(I_V^{(1)}(xy), T\big(I_V^{(1)}(x), I_V^{(1)}(y)\big)\\&\quad\geq \rho_1(x, y, t) \geq (\rho_1 \cup \rho_2)(x, y, t).\end{aligned}$$

In a similar way, if $xy \in E_2 \setminus E_1$, one can see that $S\big((I_E^{(1)} \cup T_E^{(2)}(xy), S\big((I_V^{(1)} \cup I_V^{(2)})(x), (I_V^{(1)} \cup I_V^{(2)})(y)\big) \geq \rho_2(x, y, t) = (\rho_1 \cup \rho_2)(x, y, t)$. Other cases is similar to and in a similar way, we can prove that $S\big((F_E^{(1)} \cup F_E^{(2)})(xy), S\big((F_V^{(1)} \cup F_V^{(2)})(x), (F_V^{(1)} \cup F_V^{(2)})(y)\big) \geq (\rho_1 \cup \rho_2)(x, y, t)$. Thus $G_1 \cup G_2 = (X_1 \cup X_2, Y_1 \cup Y_2, \rho_1 \cup \rho_2, T, S)$ is a KM-single valued neutrosophic metric graph on $G_1^* \cup G_2^*$.

Definition 12: Let G_1, G_2 be KM-single valued neutrosophic metric graphs on simple graphs G_1^* and G_2^*, respectively. Define the semi-ring sum of fuzzy subsets $X_1 \oslash X_2 = (T_V^{(1)} \oslash T_V^{(2)}, I_V^{(1)} + I_V^{(2)}, F_V^{(1)} \oslash F_V^{(2)})$, $Y_1 \oslash Y_2 = (T_E^{(1)} \oslash T_E^{(2)}, I_E^{(1)} + I_E^{(2)}, F_E^{(1)} + F_E^{(2)})$, where $T_V^{(1)} \oslash T_V^{(2)}, I_V^{(1)} \oslash I_V^{(2)}, F_V^{(1)} \oslash F_V^{(2)} : (V_1 \oslash V_2) \to [0, 1]$ by $(T_V^{(1)} \oslash T_V^{(2)})(x_1, x_2) = (T_V^{(1)} \cup T_V^{(2)})(x_1, x_2)$, $(I_V^{(1)} \oslash I_V^{(2)})(x_1, x_2) = (I_V^{(1)} \cup I_V^{(2)})(x_1, x_2)$, $(F_V^{(1)} \oslash F_V^{(2)})(x_1, x_2) = (F_V^{(1)} \cup F_V^{(2)})(x_1, x_2)$ and $T_E^{(1)} \oslash T_E^{(2)}, I_E^{(1)} \oslash I_E^{(2)}, F_E^{(1)} \oslash F_E^{(2)} : (E_1 \oslash E_2) \to [0, 1]$, by

$$(T_E^{(1)} \oslash T_E^{(2)})(xy) = \begin{cases} T_E^{(1)}(xy) & \text{if } xy \in E_1 \setminus E_2\\ T_E^{(2)}(xy) & \text{if } xy \in E_2 \setminus E_1\\ 0 & \text{if } xy \in E_2 \cap E_1,\end{cases}$$

$$(I_E^{(1)} \oslash I_E^{(2)})(xy) = \begin{cases} I_E^{(1)}(xy) & \text{if } xy \in E_1 \setminus E_2\\ I_E^{(2)}(xy) & \text{if } xy \in E_2 \setminus E_1\\ 1 & \text{if } xy \in E_2 \cap E_1,\end{cases}$$

$$(F_E^{(1)} \oslash F_E^{(2)})(xy) = \begin{cases} F_E^{(1)}(xy) & \text{if } xy \in E_1 \setminus E_2\\ F_E^{(2)}(xy) & \text{if } xy \in E_2 \setminus E_1\\ 1 & \text{if } xy \in E_2 \cap E_1.\end{cases}$$

Theorem 18: Let G_1 and G_2 be KM-single valued neutrosophic metric graphs on simple graphs G_1^* and G_2^*, respectively. If G_1^* and G_2^* are two simple graphs, where $V_1 \cap V_2 = \emptyset$, then $G_1 \oslash G_2 = (X_1 \oslash X_2, Y_1 \oslash Y_2, \rho_1 \cup \rho_2, T, S)$ is a KM-single valued neutrosophic metric graph on $G_1^* \cup G_2^*$.

Proof 18: It is similar to Theorem 17.

Definition 13: Let G_1, G_2 be KM-single valued neutrosophic metric graphs on simple graphs G_1^* and G_2^*, respectively. Define the join(or suspension) of fuzzy subsets $X_1 + X_2 = (T_V^{(1)} + T_V^{(2)}, I_V^{(1)} + I_V^{(2)}, F_V^{(1)} + F_V^{(2)})$, $Y_1 + Y_2 = (T_E^{(1)} + T_E^{(2)}, I_E^{(1)} + I_E^{(2)}, F_E^{(1)} + F_E^{(2)})$, where $T_V^{(1)} + T_V^{(2)}, I_V^{(1)} + I_V^{(2)}, F_V^{(1)} + F_V^{(2)} : (V_1 \oslash V_2) \to [0, 1]$ by $(T_V^{(1)} + T_V^{(2)})(x_1, x_2) = (T_V^{(1)} \cup T_V^{(2)})(x_1, x_2)$, $(I_V^{(1)} \oslash I_V^{(2)})(x_1, x_2) = (I_V^{(1)} \cup I_V^{(2)})(x_1, x_2)$, $(F_V^{(1)} \oslash F_V^{(2)})(x_1, x_2) = (F_V^{(1)} \cup F_V^{(2)})(x_1, x_2)$ and $T_E^{(1)} + T_E^{(2)}, I_E^{(1)} + I_E^{(2)}, F_E^{(1)} + F_E^{(2)} : (E_1 \oslash E_2) \to [0, 1]$, by

$$(T_E^{(1)} + T_E^{(2)})(xy) = \begin{cases} T_E^{(1)}(xy) \cup T_E^{(2)}(xy) & \text{if } xy \in E_1 \cup E_2\\ (\rho_1 \cup \rho_2)(x, y, t) & \text{if } xy \in E'(x \in V_1, y \in V_2),\end{cases}$$

$$(I_E^{(1)} + I_E^{(2)})(xy) = \begin{cases} I_E^{(1)}(xy) \cup I_E^{(2)}(xy) & \text{if } xy \in E_1 \cup E_2\\ (\rho_1 \cup \rho_2)(x, y, t) & \text{if } xy \in E'(x \in V_1, y \in V_2),\end{cases}$$

$$(F_E^{(1)} + F_E^{(2)})(xy) = \begin{cases} F_E^{(1)}(xy) \cup F_E^{(2)}(xy) & \text{if } xy \in E_1 \cup E_2\\ (\rho_1 \cup \rho_2)(x, y, t) & \text{if } xy \in E'(x \in V_1, y \in V_2),\end{cases}$$

where E' is the set of all edges joining the vertices of V_1 and V_2 and $t \in R^{\geq 0}$.

Theorem 19: Let G_1 and G_2 be KM-single valued neutrosophic metric graphs on simple graphs G_1^* and G_2^*, respectively. If G_1^* and G_2^* are two simple graphs, where $V_1 \cap V_2 = \emptyset$, then $G_1 + G_2 = (X_1 + X_2, Y_1 + Y_2, \rho_1 \cup \rho_2, T, S)$ is a KM-single valued neutrosophic metric graph on $G_1^* + G_2^*$.

Proof 19: Let $xy \in E(G_1^* + G_2^*)$. Then $xy \in E_1 \setminus E_2, xy \in E_2 \setminus E_1$ or $xy \in E'$. We only consider $xy \in E'$ and other cases are similar to Theorem 17. Since $xy \in E'$, we get that $(x \in V_1 \setminus V_2, y \in V_2 \setminus V_1)$ or $(y \in V_1 \setminus V_2, x \in V_2 \setminus V_1)$. If $x \in V_1 \setminus V_2, y \in V_2 \setminus V_1(y \in V_1 \setminus V_2, x \in V_2 \setminus V_1$ is proved in a similar way), for some $t_1, t_2 \in \mathbb{R}^{\geq 0}$, take $t = \max\{t_1, t_2\}$ so by Theorem 1, we have $T\big((T_E^{(1)} + T_E^{(2)})(xy), T\big((T_V^{(1)} + T_V^{(2)})(x), (T_V^{(1)} + T_V^{(2)})(y)\big) \leq T\big((\rho_1 \cup \rho_2)(x, y, t), T\big(T_V^{(1)}(x), T_V^{(1)}(y)\big) \leq (\rho_1 \cup \rho_2)(x, y, t)$. Now, consider $t = \min\{t_1, t_2\}$ so by Theorem 1, we have $S\big((I_E^{(1)} + I_E^{(2)})(xy), S\big((I_V^{(1)} + I_V^{(2)})(x), (I_V^{(1)} + I_V^{(2)})(y)\big) \geq S\big((\rho_1 \cup \rho_2)(x, y, t), S\big(I_V^{(1)}(x), I_V^{(1)}(y)\big) \geq (\rho_1 \cup \rho_2)(x, y, t)$ and $S\big((F_E^{(1)} + F_E^{(2)})(xy), S\big((F_V^{(1)} + I_V^{(2)})(x), (F_V^{(1)} + F_V^{(2)})(y)\big) \geq S\big((\rho_1 \cup \rho_2)(x, y, t), S\big(F_V^{(1)}(x), F_V^{(1)}(y)\big) \geq (\rho_1 \cup \rho_2)(x, y, t)$. It follows that $G_1 + G_2 = (X_1 + X_2, Y_1 + Y_2, \rho_1 \cup \rho_2, T, S)$ is a KM-single valued neutrosophic metric graph on $G_1^* + G_2^*$.

Definition 14: Let (V, ρ, T) be a KM-single valued neutrosophic metric space and $G^* = (V, E)$ be a simple graph. If $G = (X, Y, \rho, T, S)$ is a KM-fuzzy metric graph on G^*, then define the complement of fuzzy subsets $\overline{X} = (\overline{T_V}, \overline{I_V}, \overline{F_V})$, $\overline{Y} = (\overline{T_E}, \overline{I_E}, \overline{F_E})$, where $\overline{T_V}, \overline{I_V}, \overline{F_V} : V \to [0, 1]$ and $\overline{T_E}, \overline{I_E}, \overline{F_E} : E \to [0, 1]$ by $\overline{T_V}(x) = T_V(x), \overline{I_V}(x) = I_V(x), \overline{F_V}(x) = F_V(x)$ and $\overline{T_E}(xy) = \rho(x, y, t) - T\big(T_E(xy), T(T_V(x), T_V(y))\big), \overline{I_E}(xy) = S\big(I_E(xy), S(I_V(x), I_V(y))\big), \overline{F_E}(xy) = S\big(F_E(xy), S(F_V(x), F_V(y))\big)$, where $x, y \in V$. We will denote the complement of a KM-single valued neutrosophic metric graph $G = (X, Y, \rho, T, S)$, by $\overline{G} = (\overline{X}, \overline{Y}, \rho, T, S)$.

Theorem 20: Let (V, ρ, T) be a KM-fuzzy metric space and $G^* = (V, E)$ be a simple graph. If $G = (X, Y, \rho, T, S)$ is a KM-single valued neutrosophic metric graph on G^*, then $\overline{G} = (\overline{X}, \overline{Y}, \rho, T, S)$ is a KM-single valued neutrosophic metric graph.

Proof 20: Let $x, y \in V$. Since G is a KM-single valued neutrosophic metric graph on G^*, for some $t \in \mathbb{R}^{\geq 0}$,

$$\begin{aligned} &T\big(\overline{T_E}(xy), T\big(\overline{T_V}(x), \overline{T_V}(y)\big)\big) \\ &= T\big(\rho(x, y, t) - T\big(T_E(xy), T(T_V(x), T_V(y))\big), T\big(T_V(x), \\ &\quad T_V(y)\big)\big) \leq \rho(x, y, t) - T\big(T_E(xy), T(T_V(x), T_V(y))\big) \\ &\quad \leq \rho(x, y, t). \end{aligned}$$

In addition,

$$\begin{aligned} &S\big(\overline{I_E}(xy), S\big(\overline{I_V}(x), \overline{I_V}(y)\big)\big) \\ &\quad = S\big(S\big(I_E(xy), S(I_V(x), I_V(y))\big), S\big(I_V(x), I_V(y)\big)\big) \\ &\quad \geq S\big(I_E(xy), S(I_V(x), I_V(y))\big) \geq \rho(x, y, t). \end{aligned}$$

In a similar way, it is easy to see that $S\big(\overline{F_E}(xy), S\big(\overline{F_V}(x), \overline{F_V}(y)\big)\big) \geq \rho(x, y, t)$. It follows that $\overline{G} = (\overline{X}, \overline{Y}, \rho, T, S)$ is a KM-single valued neutrosophic metric graph.

Example 6: Consider the KM-single valued neutrosophic-metric graph G in Example 1. So obtain a KM-single valued neutrosophic metric graph $\overline{G}$ on the cycle graph C_4 for $t = 1$, in Figure 7.

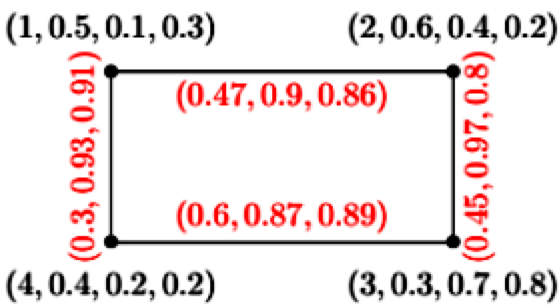

FIGURE 7. ***KM*-single valued neutrosophic metric graph $\overline{G}$.**

V. CONCLUSION

The current paper has introduced a novel concept fuzzy algebra as KM-single valued neutrosophic metric graph and a new generalization of graphs based on KM-fuzzy metric spaces. This work extended and obtained some properties in KM-fuzzy metric spaces. Also it showed that every non empty set converted to a KM-fuzzy metric space, the product and union of KM-fuzzy metric spaces is a KM-fuzzy metric space, the extended KM-fuzzy metric spaces are constructed using the some algebraic operations on KM-fuzzy metric spaces, the concept of complement of KM-single valued neutrosophic metric graph is defined and investigated some its properties. We hope that these results are helpful for further studies in theory of graphs. In our future studies, we hope to obtain more results regarding intuitionistic metric graphs, neutrosophic metric graphs, KM-single valued neutrosophic metric hypergraphs, bipolar KM-single valued neutrosophic metric graphs, automorphism KM-single valued neutrosophic metric graphs and their applications.

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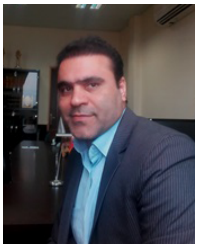

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