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Single valued neutrosophic mappings defined by single valued neutrosophic relations with applications

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Abstract: In this paper, we introduce the notion of single valued neutrosophic mapping defined by single valued neutrosophic relation which is considered as a generalization of fuzzy mapping defined by fuzzy relation and several properties related to this notion are studied. Moreover, we generalize the notion of fuzzy topology on fuzzy sets introduced by Kandil et al. to the setting of single valued neutrosophic sets. As applications, we establish the property of continuity in single valued neutrosophic topological space and investigate relationships among various types of single valued neutrosophic continuous mapping.

Keywords: Single valued neutrosophic set; Binary relation; Mapping; Topology; Continuous mapping.

1 Introduction

It is a well-known fact by now that mappings in crisp set theory are among the oldest acquaintances of modern mathematics and, play an important role in many mathematical branches (both pure and applied), as well as in topology and its analysis approaches. The uses of mappings appear also in formal logic [13], category theory [35], graph theory [11], group theory [6] and in computer science [31]. In general, it was and still more common.

In fuzzy setting, the concept of fuzzy mapping has received far attention. It has appeared in many papers, for instance, S. Heilpern [12] introduced this concept and proved a fixed point theorem for fuzzy contraction mappings. In [17], S. Lou and L. Cheng proved that fuzzy controllers can be regarded as a fuzzy mapping from the set of linguistic variables describing the observed object to that of linguistic variables describing the controlled objects. Thereafter, Lim et al. [18] investigated the equivalence relations and mappings for fuzzy sets and relationship between them. Ismail and Massa'deh [9] defined L-fuzzy mappings and studied their operations, also they developed many properties of classical mappings into L-fuzzy case. For the study of fuzzy continuous mappings in fuzzy topological space, an extended approaches are proposed, R.N. Bhaumik and M.N Mukherjee [5] investigated some properties of fuzzy completely continuous mapping. Mukherjee and B. Ghosh [27] pay attention to the introduction and studying of the concepts of certain classes of mappings between fuzzy topological spaces. Each of these mappings presents a stronger form of the fuzzy continuous mappings. In this regard, we find that other authors also contributed a lot to this field, like M. K. Single and A. R. Single [36], B. Ahmed [1] and M. K. Mishra et al. [26].

In [3], Attanassov introduced the concept of intuitionistic fuzzy set which is an extension of fuzzy set, characterized by a membership (truth-membership) function and a non-membership (falsity-membership) function for the elements of a universe X. Moreover, there is a restriction that the sum of both values is less and equal to one. Recently, F. Smarandache [32] generalized the Atanassov's intuitionistic fuzzy sets and other types of sets to the notion of neutrosophic sets. He introduced this concept to deal with imprecise and indeterminate data. Neutrosophic sets are characterized by truth membership function (T), indeterminacy membership function (I) and falsity membership function (F). Many researchers have studied and applied in different fields the neutrosophic sets and its various extensions such as decision making problems (e.g. [39, 41]), image processing (e.g. [8, 44]), educational problem (e.g. [25]), conflict resolution (e.g. [28]), social problems (e.g. [29, 24]), medical diagnosis (e.g. [22, 40, 42]), supply chain management (e.g. [20]), construction projects (e.g. [21]) and to address the conditions of uncertainty and inconsistency (e.g. [23]) and others. In particular, to exercise neutrosophic sets in real life applications suitably, Wang et al. [37] introduced the concept of single valued neutrosophic set as a subclass of a neutrosophic set, and investigated some of its properties. Very recently, Kim et al. [15] studied a single valued neutrosophic (relation/ transitive closure/ equivalence relation class/ partition). The studies, whether theoretical or applied on single valued neutrosophic set have been progressing rapidly. For instance, [2, 7, 14] and more others.

Motivated by recent developments relating to this framework, in this paper, we introduce the notion of single valued neutrosophic mapping defined by single valued neutrosophic relation as a generalization of fuzzy mappings introduced by Ismail and Massa'deh [9] and many properties related to this notion are studied. Also, we generalize the notion of fuzzy topology on fuzzy sets introduced by A. Kandil et al. [16] to the setting of single valued neutrosophic sets to establish the continuity property of single valued neutrosophic mapping. To that end, we investigate relation among various types of single valued neutrosophic continuous mappings.

The contents of the paper are organized as follows. In Section 2, we recall the necessary basic concepts and properties of single valued neutrosophic sets, single valued neutrosophic relations and some related notions that will be needed throughout this paper. In Section 3, the notion of single valued neutrosophic mapping defined by single valued neutrosophic relation is introduced and some properties related to this notion are studied. In Section 4, we establish as an application the single valued neutrosophic continuous mapping in single valued neutrosophic topological space and relationships between various types of single valued neutrosophic continuous mapping are explained. Finally, we present some conclusions and discuss future research in Section 5.

2 Preliminaries

This section contains the basic definitions and properties of single valued neutrosophic sets and some related notions that will be needed throughout this paper.

2.1 Single valued neutrosophic sets

The notion of fuzzy sets was first introduced by Zadeh [43].

Definition 2.1. [43] Let X be a nonempty set. A fuzzy set $A = \{\langle x, \mu_A(x)\rangle \mid x \in X\}$ is characterized by a membership function $\mu_A : X \to [0, 1]$, where $\mu_A(x)$ is interpreted as the degree of membership of the element x in the fuzzy subset A for any $x \in X$.

In 1983, Atanassov [3] proposed a generalization of Zadeh membership degree and introduced the notion of the intuitionistic fuzzy set.

Definition 2.2. [3] Let X be a nonempty set. An intuitionistic fuzzy set (IFS, for short) A on X is an object of the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\}$ characterized by a membership function $\mu_A : X \to [0, 1]$ and a non-membership function $\nu_A : X \to [0, 1]$ which satisfy the condition:

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1, \text{ for any } x \in X.$$

In 1998, Smarandache [32] defined the concept of a neutrosophic set as a generalization of Atanassov's intuitionistic fuzzy set. Also, he introduced neutrosophic logic, neutrosophic set and its applications in [33, 34]. In particular, Wang et al. [37] introduced the notion of a single valued neutrosophic set.

Definition 2.3. [33] Let X be a nonempty set. A neutrosophic set (NS, for short) A on X is an object of the form $A = \{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x)\rangle \mid x \in X\}$ characterized by a membership function $\mu_A : X \to]^-0, 1^+[$ and an indeterminacy function $\sigma_A : X \to]^-0, 1^+[$ and a non-membership function $\nu_A : X \to]^-0, 1^+[$ which satisfy the condition:

$$^-0 \leq \mu_A(x) + \sigma_A(x) + \nu_A(x) \leq 3^+, \text{ for any } x \in X.$$

Certainly, intuitionistic fuzzy sets are neutrosophic sets by setting $\sigma_A(x) = 1 - \mu_A(x) - \nu_A(x)$.

Next, we show the notion of single valued neutrosophic set as an instance of neutrosophic set which can be used in real scientific and engineering applications.

Definition 2.4. [37] Let X be a nonempty set. A single valued neutrosophic set (SVNS, for short) A on X is an object of the form $A = \{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x)\rangle \mid x \in X\}$ characterized by a truth-membership function $\mu_A : X \to [0, 1]$, an indeterminacy-membership function $\sigma_A : X \to [0, 1]$ and a falsity-membership function $\nu_A : X \to [0, 1]$.

The class of single valued neutrosophic sets on X is denoted by $SVN(X)$.

For any two SVNSs A and B on a set X, several operations are defined (see, e.g., [37, 38]). Here we will present only those which are related to the present paper.

(i) $A \subseteq B$ if $\mu_A(x) \leq \mu_B(x)$ and $\sigma_A(x) \leq \sigma_B(x)$ and $\nu_A(x) \geq \nu_B(x)$, for all $x \in X$,

(ii) $A = B$ if $\mu_A(x) = \mu_B(x)$ and $\sigma_A(x) = \sigma_B(x)$ and $\nu_A(x) = \nu_B(x)$, for all $x \in X$,

(iii) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \wedge \sigma_B(x), \nu_A(x) \vee \nu_B(x)\rangle \mid x \in X\}$,

(iv) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \vee \sigma_B(x), \nu_A(x) \wedge \nu_B(x)\rangle \mid x \in X\}$,

(v) $\overline{A} = \{\langle x, 1 - \nu_A(x), 1 - \sigma_A(x), 1 - \mu_A(x)\rangle \mid x \in X\}$,

(vi) $[A] = \{\langle x, \mu_A(x), \sigma_A(x), 1 - \mu_A(x)\rangle \mid x \in X\}$,

(vii) $\langle A\rangle = \{\langle x, 1 - \nu_A(x), \sigma_A(x), \nu_A(x)\rangle \mid x \in X\}$.

In the sequel, we need the following definition of level sets (which is also often called (α, β, γ)-cuts) of a single valued neutrosophic set.

Definition 2.5. [2] Let A be a single valued neutrosophic set on a set X. The (α, β, γ)-cut of A is a crisp subset

$$A_{\alpha,\beta,\gamma} = \{x \in X \mid \mu_A(x) \geq \alpha \text{ and } \sigma_A(x) \geq \beta \text{ and } \nu_A(x) \leq \gamma\},$$

where $\alpha, \beta, \gamma \in]0, 1]$.

Definition 2.6. [2] Let A be a single valued neutrosophic set on a set X. The support of A is the crisp subset on X given by

$$Supp(A) = \{x \in X \mid \mu_A(x) \neq 0 \text{ and } \sigma_A(x) \neq 0 \text{ and } \nu_A(x) \neq 0\}.$$

2.2 Single valued neutrosophic relations

Kim et al. [15] introduced the concept of single valued neutrosophic relation as a natural generalization of fuzzy and intuitionistic fuzzy relation.

Definition 2.7. [15] A single valued neutrosophic binary relation (A single valued neutrosophic relation, for short) from a universe X to a universe Y is a single valued neutrosophic subset in $X \times Y$, i.e., is an expression R given by

$$R = \{\langle (x, y), \mu_R(x, y), \sigma_R(x, y), \nu_R(x, y)\rangle \mid (x, y) \in X \times Y\},$$

where $\mu_R : X \times Y \to [0, 1]$, and $\sigma_R : X \times Y \to [0, 1]$, and $\nu_A : X \times Y \to [0, 1]$.
For any $(x, y) \in X \times Y$. The value $\mu_R(x, y)$ is called the degree of a membership of (x, y) in R, $\sigma_R(x, y)$ is called the degree of indeterminacy of (x, y) in R and $\nu_R(x, y)$ is called the degree of non-membership of (x, y) in R.

Example 2.8. Let $X = \{a, b, c, d, e\}$. Then the single valued neutrosophic relation R defined on X by

$$R = \{\langle (x, y), \mu_R(x, y), \sigma_R(x, y), \nu_R(x, y)\rangle \mid x, y \in X\},$$

where μ_R, σ_R and ν_R are given by the following tables:

$\mu_R(.,.)$	a	b	c	d	e
a	0.35	0	0	0.35	0.30
b	0	0.40	0	0.35	0.45
c	0.20	0	0.65	0	0.70
d	0	0	0	1	0
e	0.25	0.35	0	0	0.60

$\sigma_R(.,.)$	a	b	c	d	e
a	0.5	0.5	0.42	0.2	0
b	0.60	0.12	0.40	0.80	0.10
c	0	1	0.02	0.75	0.15
d	0.33	1	0.88	0	0.10
e	0.20	0.55	1	0.55	0.30

$\nu_R(.,.)$	a	b	c	d	e
a	0	1	0.40	0.25	0.25
b	0.30	0.35	0.20	0.35	0.10
c	0.80	1	0	0.85	0.15
d	1	1	1	0	1
e	0.70	0.55	1	0.90	0.30

Next, the following definitions is needed to recall.

Definition 2.9. [30] Let R and P be two single valued neutrosophic relations from a universe X to a universe Y.

(i) The transpose (inverse) R^t of R is the single valued neutrosophic relation from the universe Y to the universe X defined by

$$R^t = \{\langle (x,y), \mu_{R^t}(x,y), \sigma_{R^t}(x,y), \nu_{R^t}(x,y)\rangle \mid (x,y) \in X \times Y\},$$

where

$$\begin{cases} \mu_{R^t}(x,y) = \mu_R(y,x) \\ \quad \text{and} \\ \sigma_{R^t}(x,y) = \sigma_R(y,x) \\ \quad \text{and} \\ \nu_{R^t}(x,y) = \nu_R(y,x)\,, \end{cases}$$

for any $(x,y) \in X \times Y$.

(ii) R is said to be contained in P or we say that P contains R, denoted by $R \subseteq P$, if for all $(x,y) \in X \times Y$ it holds that $\mu_R(x,y) \leq \mu_P(x,y)$, $\sigma_R(x,y) \leq \sigma_P(x,y)$ and $\nu_R(x,y) \geq \nu_P(x,y)$.

(iii) The intersection (resp. *the union*) of two single valued neutrosophic relations R and P from a universe X to a universe Y is a single valued neutrosophic relation defined as

$$\begin{aligned} R \cap P \;=\; & \{\langle (x,y), \min(\mu_R(x,y), \mu_P(x,y)), \min(\sigma_R(x,y), \sigma_P(x,y)), \max(\nu_R(x,y), \nu_P(x,y))\rangle \mid (x,y) \\ & \in X \times Y\} \end{aligned}$$

and

$$R \cup P = \{\langle (x,y), \max(\mu_R(x,y), \max(\sigma_R(x,y), \sigma_P(x,y)), \min(\nu_R(x,y), \nu_P(x,y)))\rangle \mid (x,y) \in X \times Y\}\,.$$

Definition 2.10. [30, 38] Let R be a single valued neutrosophic relation from a universe X into itself.

(i) *Reflexivity*: $\mu_R(x,x) = \sigma_R(x,x) = 1$ and $\nu_R(x,x) = 0$, for any $x \in X$.

(ii) *Symmetry*: for any $x, y \in X$ then

$$\begin{cases} \mu_R(x,y) = \mu_R(y,x) \\ \sigma_R(x,y) = \sigma_R(y,x) \\ \nu_R(x,y) = \nu_R(y,x) \end{cases},$$

(iii) *Antisymmetry*: for any $x, y \in X$, $x \neq y$ then

$$\begin{cases} \mu_R(x,y) \neq \mu_R(y,x) \\ \sigma_R(x,y) \neq \sigma_R(y,x) \\ \nu_R(x,y) \neq \nu_R(y,x) \end{cases},$$

(iv) *Transitivity*: $R \circ R \subset R$ i.e., $R^2 \subset R$.

3 Single valued neutrosophic mappings defined by single valued neutrosophic relations

In this section, we generalize the notion of fuzzy mapping defined by fuzzy relation introduced by Ismail and Massa'deh [9] to the setting of single valued neutrosophic sets. Also, the main properties related to single valued neutrosophic mapping are studied.

Definition 3.1. Let A be a single valued neutrosophic set on X and B be a single valued neutrosophic set on Y, let $f : Supp\ A \to Supp\ B$ be an ordinary mapping and R be a single valued neutrosophic relation on $X \times Y$. Then f_R is called a single valued neutrosophic mapping if for all $(x, y) \in Supp\ A \times Supp\ B$ the following condition is satisfied:

$$\mu_R(x,y) = \begin{cases} min(\mu_A(x), \mu_B(f(x)) \ , \ if\ y = f(x) \\ 0\ , \ \ Otherwise\, , \end{cases}$$

and

$$\sigma_R(x,y) = \begin{cases} min(\sigma_A(x), \sigma_B(f(x)) \ , \ if\ y = f(x) \\ 0\ , \ \ Otherwise\, , \end{cases}$$

and

$$\nu_R(x,y) = \begin{cases} max(\nu_A(x), \nu_B(f(x)) \ , \ if\ y = f(x) \\ 1\ , \ \ Otherwise\, , \end{cases}$$

Example 3.2. Let $X = \{\alpha, \beta, \gamma\}$, $Y = \{a, b, c\}$, $A \in SVNS(X)$ and $B \in SVNS(Y)$ given by

$$A = \{\langle \alpha, 0.5, 0.2, 0.8\rangle, \langle \beta, 0.1, 0.7, 0.3\rangle, \langle \gamma, 0, 0.9, 1\rangle\}$$

$$B = \{\langle a, 0, 1, 0.3\rangle, \langle b, 0.1, 0.5, 0.2\rangle, \langle c, 0.7, 0.2, 0.4\rangle\}.$$

We will construct the single valued neutrosophic mapping f_R by :

(i) an ordinary mapping $f : \{\alpha, \beta\} \to \{b, c\}$ such that $f(\alpha) = b$ and $f(\beta) = c$,

(ii) a single valued neutrosophic relation R defined by :

$\mu_R(\alpha, f(\alpha)) = \mu_R(\alpha, b) = \mu_A(\alpha) \wedge \mu_B(b) = 0.1$

$\mu_R(\beta, f(\beta)) = \mu_R(\beta, c) = \mu_A(\beta) \wedge \mu_B(c) = 0.1$

$\mu_R(\alpha, a) = \mu_R(\alpha, c) = \mu_R(\beta, a) = \mu_R(\beta, b) = \mu_R(\gamma, a) = \mu_R(\gamma, b) = \mu_R(\gamma, c) = 0$

In similar way, it holds that

$\sigma_R(\alpha, f(\alpha)) = \sigma_R(\alpha, b) = \sigma_A(\alpha) \wedge \sigma_B(b) = 0.2$

$\sigma_R(\beta, f(\beta)) = \sigma_R(\beta, c) = \sigma_A(\beta) \wedge \sigma_B(c) = 0.2$

$\sigma_R(\alpha, a) = \sigma_R(\alpha, c) = \sigma_R(\beta, a) = \sigma_R(\beta, b) = \sigma_R(\gamma, a) = \sigma_R(\gamma, b) = \sigma_R(\gamma, c) = 0$

and

$\nu_R(\alpha, f(\alpha)) = \nu_R(\alpha, b) = \nu_A(\alpha) \vee \nu_B(b) = 0.8$

$\nu_R(\beta, f(\beta)) = \nu_R(\beta, c) = \nu_A(\beta) \vee \nu_B(c) = 0.4$

$\nu_R(\alpha, a) = \nu_R(\alpha, c) = \nu_{R_I}(\beta, a) = \nu_{R_I}(\beta, b) = \sigma_R(\gamma, a) = \sigma_R(\gamma, b) = \sigma_R(\gamma, c) = 1.$

Hence, $\mu_R(x, y) = \{\langle(\alpha, f(\alpha)), 0.1, 0.2, 0.8\rangle, \langle(\beta, f(\beta)), 0.1, 0.2, 0.4\rangle, \langle(\alpha, a), 0, 0, 1\rangle,$
$\langle(\alpha, c), 0, 0, 1\rangle, \langle(\beta, a), 0, 0, 1\rangle, \langle(\beta, b), 0, 0, 1\rangle, \langle(\gamma, a), 0, 0, 1\rangle, \langle(\gamma, b), 0, 0, 1\rangle, \langle(\gamma, c), 0, 0, 1\rangle\}.$

Thus, f_R is a single valued neutrosophic mapping.

Example 3.3. Let $X = \mathbb{Q}$, $Y = \mathbb{R}$, $A \in SVNS(X)$ and $B \in SVNS(Y)$ given by:
$\mu_A(x) = 0.3$, $\sigma_A(x) = 0.25$ and $\nu_A(x) = 0.5$, for any $x \in \mathbb{Q}$.
$\mu_B(x) = \sigma_B(x) = \nu_B(x) = 0.5$, for any $x \in \mathbb{R}$.

We will construct the single valued neutrosophic mapping f_R by :

(i) an ordinary mapping $f : \mathbb{Q} \to \mathbb{R}$ such that $f(x) = x^2$,

(ii) a single valued neutrosophic relation R defined by :

$\mu_R(x, f(x)) = \mu_R(x, x^2) = \mu_A(x) \wedge \mu_B(x^2) = 0.3$
$\sigma_R(x, f(x)) = \sigma_R(x, x^2) = \sigma_A(x) \wedge \mu_B(x^2) = 0.25$
$\nu_R(x, f(x)) = \nu_R(x, x^2) = \nu_A(x) \vee \nu_B(x^2) = 0.5$

Thus, f_R is a single valued neutrosophic mapping.

Remark 3.4. From the above definition, we can construct the single valued neutrosophic mapping by this method

(i) We determine the $Supp\ A$ and $Supp\ B$.

(ii) We determine the ordinary mapping from $Supp\ A$ to $Supp\ B$.

(iii) We determine the single valued neutrosophic relation by its membership function, indeterminacy function and non-membership function.

(iv) Finally, we conclude the construction of the single valued neutrosophic mapping.

Definition 3.5. Let f_R, g_S be two single valued neutrosophic mappings, then f_R and g_S are equal if and only if $f = g$ and $R = S$ i.e., $(\mu_R(x, f(x)) = \mu_S(x, g(x)),\ \sigma_R(x, f(x)) = \sigma_S(x, g(x))$, and $\nu_R(x, f(x)) = \nu_S(x, g(x)))$.

Definition 3.6. Let A be a single valued neutrosophic set on X, let $f : Supp\ A \to Supp\ A$ be an ordinary mapping such that $f(x) = x$ and R be a single valued neutrosophic relation on $X \times X$. Then f_R is called a single valued neutrosophic identity mapping if for all $x, y \in Supp\ A$ the following conditions are satisfied:

$$\mu_R(x,y) = \begin{cases} \mu_A(x)\ ,\ if\ x=y \\ 0\ ,\ Otherwise\ , \end{cases}$$

and

$$\sigma_R(x,y) = \begin{cases} \sigma_A(x)\ ,\ if\ x=y \\ 0\ ,\ Otherwise\ , \end{cases}$$

and

$$\nu_R(x,y) = \begin{cases} \nu_A(x)\ ,\ if\ x=y \\ 1\ ,\ Otherwise\ , \end{cases}$$

Definition 3.7. Let A, B and C are a single valued neutrosophic sets on X, Y and Z respectively, let f : $Supp\ A \to Supp\ B$ and g : $Supp\ B \to Supp\ C$ are an ordinary mappings and R, S are a single valued neutrosophic relations on $X \times Y$ and $Y \times Z$ respectively. Then $(g \circ f)_T$ is called the composition of single valued neutrosophic mappings f_R and g_R such that $g \circ f$: $Supp\ A \to Supp\ C$ and the single valued neutrosophic relation T is defined by

$$\begin{cases} \mu_T(x,z) = sup_y(min(\mu_R(x,y), \mu_S(y,z))) \\ \quad \text{and} \\ \sigma_T(x,z) = sup_y(min(\sigma_R(x,y), \sigma_S(y,z))) \\ \quad \text{and} \\ \nu_T(x,z) = inf_y(max(\nu_R(x,y), \nu_S(y,z)))\ , \end{cases}$$

for any $(x,z) \in Supp\ A \times Supp\ C$.

Example 3.8. Let $X = \mathbb{N}$, $Y = \mathbb{R}$ and $Z = \mathbb{R}$, and let $A \in SVNS(X)$, $B \in SVNS(Y)$ and $C \in SVNS(Z)$, defined as follows :

$\mu_A(n) = \sigma_A(n) = \frac{1}{1+n}$ and $\nu_A(n) = \frac{n}{2+2n}$, for any $n \in \mathbb{N}$.

$\mu_B(x) = \sigma_B(x) = \begin{cases} 0.25\ ,\ if\ x \in [-1,1] \\ 0\ ,\ Otherwise\ , \end{cases}$ and $\nu_B(x) = \begin{cases} 0.5\ ,\ if\ x \in [-1,1] \\ 1\ ,\ Otherwise\ , \end{cases}$

$\mu_C(x) = \sigma_C(x) = \frac{|cos(x)|}{3}$ and $\nu_C(x) = \frac{|sin(x)|}{3}$, for any $x \in \mathbb{R}$.

We define a single valued neutrosophic mappings $f_R : A \to B$ and $g_S : B \to C$ by :

(i) an ordinary mappings $f : Supp\ A \longrightarrow Supp\ B$, defined for any $n \in Supp\ A$ by :

$f(n) = \begin{cases} 1\ ,\ if\ n\ is\ an\ even\ number, \\ -1\ , if\ n\ is\ an\ odd\ number\ , \end{cases}$

and $g : Supp\ B \longrightarrow Supp\ C$ defined by $g(x) = 2x$, for any $x \in [-1,1]$.

(ii) a single valued neutrosophic relations R and S defined by :

$\mu_R(n, f(n)) = \sigma_R(n, f(n)) = \wedge\{\mu_A(n),\ \mu_B(f(n))\} = \wedge\{\frac{1}{1+n},\ 0.25\}$,

$\nu_R(n, f(n)) = \vee\{\nu_A(n),\ \nu_B(f(n))\} = \vee\{\frac{n}{2+2n},\ 0.5\}$ and

$\mu_S(x, g(x)) = \sigma_S(x, g(x)) = \wedge\{\mu_B(x),\ \mu_C(g(x))\} = \begin{cases} \wedge\{0.25,\ \frac{|cos(2x)|}{3}\}\ ,\ x \in [-1,1], \\ 0\ ,\ otherwise\ , \end{cases}$

and $\nu_S(x, g(x)) = \vee\{\nu_B(x),\ \nu_C(g(x))\} = \begin{cases} \vee\{0.5,\ \frac{|sin(2x)|}{3}\}\ ,\ x \in [-1,1], \\ 1\ ,\ otherwise. \end{cases}$

Then, the composition $g_S \circ f_R = (g \circ f)_T$ is defined by :

(i) an ordinary mapping $f : Supp\ A \longrightarrow Supp\ C$, defined for any $n \in Supp\ A$ by :

$$(g \circ f)(n) = \begin{cases} 2\ ,\ if\ n\, is\ an\ even\ number, \\ -2\ ,\ if\ n\, is\ an\ odd\ number\ , \end{cases}$$

(ii) a single valued neutrosophic relation T defined by :

$$\begin{aligned} \mu_T(n,(g \circ f)(n)) = \sigma_T(n,(g \circ f)(n)) &= \begin{cases} \wedge\{\frac{1}{1+n}, 0.25,\ \frac{|cos(2)|}{3}\}\ ,\ if\ n\, is\ an\ even\ number \\ \wedge\{\frac{1}{1+n}, 0.25,\ \frac{|cos(-2)|}{3}\}\ ,\ if\ n\, is\ an\ odd\ number \end{cases} \\ &= \wedge\{\frac{1}{1+n}, 0.25,\ \frac{|\ cos(2)\ |}{3}\} \\ &= \wedge\{\frac{1}{1+n}, 0.25\}, \end{aligned}$$

$$\begin{aligned} \nu_T(n,(g \circ f)(n)) &= \begin{cases} \vee\{\frac{n}{2+2n}, 0.25,\ \frac{|sin(2)|}{3}\}\ ,\ if\ n\, is\ an\ even\ number \\ \vee\{\frac{n}{2+2n}, 0.25,\ \frac{|sin(-2)|}{3}\}\ ,\ if\ n\, is\ an\ odd\ number \end{cases} \\ &= \vee\{\frac{n}{2+2n}, 0.25,\ \frac{|\ sin(2)\ |}{3}\} \\ &= \vee\{\frac{2}{2+2n}, 0.25\}. \end{aligned}$$

Remark 3.9. The single valued neutrosophic identity mapping Id_R is neutral for the composition of single valued neutrosophic mappings.

In the sequel, we need to introduce the notion of the direct image and the inverse image of a single valued neutrosophic set by a single valued neutrosophic mapping.

Definition 3.10. Let $f_R : A \to B$ be a single valued neutrosophic mapping from a single valued neutrosophic set A to another single valued neutrosophic set B and $C \subseteq A$. The direct image of C by f_R is defined by $f_R(C) = \{\langle y, \mu_{f_R(C)}(y), \sigma_{f_R(C)}(y), \nu_{f_R(C)}(y)\rangle \mid y \in Y\}$, where

$$\mu_{f_R(C)}(y) = \begin{cases} \mu_B(y)\ ,\ if\ y \in f(supp(C)) \\ 0\ ,\ \ Otherwise\, , \end{cases}$$

and

$$\sigma_{f_R(C)}(y) = \begin{cases} \sigma_B(y)\ ,\ if\ y \in f(supp(C)) \\ 0\ ,\ \ Otherwise\, , \end{cases}$$

and

$$\nu_{f_R(C)}(y) = \begin{cases} \nu_B(y)\ ,\ if\ y \in f(supp(C)) \\ 1\ ,\ \ Otherwise. \end{cases}$$

Similarly, if $C' \subseteq B$. The inverse image of C' by f is defined by

$$f_R^{-1}(C') = \{\langle x, \mu_{f_R^{-1}(C')}(x), \sigma_{f_R^{-1}(C')}(x), \nu_{f_R^{-1}(C')}(x)\rangle \mid x \in X\},$$

where

$$\mu_{f_R^{-1}(C')}(x) = \begin{cases} \mu_A(x)\ ,\ if\ x \in f^{-1}(supp(C')) \\ 0\ ,\ Otherwise\ , \end{cases}$$

and

$$\sigma_{f_R^{-1}(C')}(x) = \begin{cases} \sigma_A(x)\ ,\ if\ x \in f^{-1}(supp(C')) \\ 0\ ,\ Otherwise\ , \end{cases}$$

and

$$\nu_{f_R^{-1}(C')}(x) = \begin{cases} \nu_A(x)\ ,\ if\ x \in f^{-1}(supp(C')) \\ 1\ ,\ Otherwise. \end{cases}$$

Example 3.11. Let $X = [0, +\infty[$, $Y = \mathbb{R}$ and $A \in SVNS(X)$ defined for any $x \in X$ by :

$\mu_A(x) = \sigma_A(x) = \begin{cases} cos(x)\ ,\ if\ x \in [0, \frac{\pi}{2}] \\ 0\ ,\ Otherwise\ , \end{cases}$ $\quad \nu_A(x) = \begin{cases} 0.9\ ,\ if\ x \in [0, \frac{\pi}{2}] \\ 1\ ,\ Otherwise. \end{cases}$

Also, let $B \in SVNS(Y)$ given by :

$\mu_B(y) = \sigma_B(y) = \begin{cases} y\ ,\ if\ y \in [0, 1] \\ 0\ ,\ Otherwise\ , \end{cases}$ $\quad \nu_B(y) = \begin{cases} 0.2\ ,\ if\ y \in [0, 1] \\ 1\ ,\ Otherwise. \end{cases}$

We define the single valued neutrosophic mapping $f_R : A \to B$ by:

(i) an ordinary mapping $f : Supp\ A \longrightarrow Supp\ B$, defined for any $x \in [0, \frac{\pi}{2}]$ by $f(x) = \frac{x}{4}$.

(ii) a single valued neutrosophic relation R defined by $\mu_R(x, f(x)) = \sigma_R(x, f(x)) = \mu_A(x) \wedge \mu_B(f(x)) = cos(x) \wedge \frac{1}{4}x$ and $\nu_R(x, f(x)) = \nu_A(x) \vee \nu_B(f(x)) = 0.9$

Now, if we take C an SVNS on X, where $C \subseteq A$ given by :

$\mu_C(x) = \sigma_C(x) = \begin{cases} -x+1\ ,\ if\ x \in [0, \frac{1}{2}] \\ 0\ ,\ Otherwise\ , \end{cases}$ $\quad \nu_C(x) = \begin{cases} 0.99\ ,\ if\ y \in [0, \frac{1}{2}] \\ 1\ ,\ Otherwise\ , \end{cases}$

Then, the direct image of C by f_R is defined by :

$\mu_{f_R(C)}(y) = \begin{cases} \mu_B(y)\ ,\ if\ y \in f(supp(C)) \\ 0\ ,\ Otherwise\ , \end{cases} = \begin{cases} y\ ,\ if\ y \in [0, \frac{1}{8}] \\ 0\ ,\ Otherwise\ , \end{cases}$

$\sigma_{f_R(C)}(y) = \begin{cases} \sigma_B(y)\ ,\ if\ y \in f(supp(C)) \\ 0\ ,\ Otherwise\ , \end{cases} = \begin{cases} y\ ,\ if\ y \in [0, \frac{1}{8}] \\ 0\ ,\ Otherwise\ , \end{cases}$

and

$\nu_{f_R(C)}(y) = \begin{cases} \nu_B(y)\ ,\ if\ y \in f(supp(C)) \\ 0\ ,\ Otherwise\ , \end{cases} = \begin{cases} 0.2\ ,\ if\ y \in [0, \frac{1}{8}] \\ 1\ ,\ Otherwise. \end{cases}$

Moreover, it is easy to show that $f_R(C) \subseteq B$.

Next, if we take C' an SVNS on Y, where $C' \subseteq B$ given by :

$\mu_{C'}(y) = \sigma_{C'}(y) = \begin{cases} sin(y)\ ,\ if\ y \in [0, \frac{1}{3}] \\ 0\ ,\ Otherwise\ , \end{cases}$ $\quad \nu_{C'}(y) = \begin{cases} 0.4\ ,\ if\ y \in [0, \frac{1}{3}] \\ 1\ ,\ Otherwise\ , \end{cases}$

Then, the inverse image of C' by f is defined by :

$\mu_{f_R^{-1}(C')}(x) = \begin{cases} \mu_A(x)\ ,\ if\ x \in f^{-1}(supp(C')) \\ 0\ ,\ Otherwise\ , \end{cases} = \begin{cases} cos(x)\ ,\ if\ x \in [0, \frac{4}{3}] \\ 0\ ,\ Otherwise\ , \end{cases}$

$\sigma_{f_R^{-1}(C')}(x) = \begin{cases} \sigma_A(x)\ ,\ if\ x \in f^{-1}(supp(C')) \\ 0\ ,\ Otherwise\ , \end{cases} = \begin{cases} cos(x)\ ,\ if\ x \in [0, \frac{4}{3}] \\ 0\ ,\ Otherwise\ , \end{cases}$

and $\nu_{f_R^{-1}(C')}(x) = \begin{cases} \nu_A(x)\ ,\ if\ x \in f^{-1}(supp(C')) \\ 1\ ,\ Otherwise\ , \end{cases} = \begin{cases} 0.9\ ,\ if\ x \in [0, \frac{4}{3}] \\ 1\ ,\ Otherwise. \end{cases}$

Moreover, it is easy to show that $f_R^{-1}(C') \subseteq A$.

Now, we introduce the product of single valued neutrosophic sets and single valued neutrosophic projection mappings.

Definition 3.12. Let A be a single valued neutrosophic set on X and B be a single valued neutrosophic set on Y. The product of A and B, denoted by $A \times B$ is a single valued neutrosophic set on $X \times Y$ defined by :
$\mu_{X\times Y}(x,y) = min\{\mu_A(x),\, \mu_B(y)\}$, $\sigma_{X\times Y}(x,y) = min\{\sigma_A(x),\, \sigma_B(y)\}$, $\nu_{X\times Y}(x,y) = max\{\nu_A(x),\, \nu_B(y)\}$.
Also, we introduce the first single valued neutrosophic projection mapping $(P_1)_R : A \times B \longrightarrow A$ by:

(i) an ordinary mapping $P_1 : Supp(A\times B) \longrightarrow Supp(A)$ such that $P_1(x,y) = x$ for any $(x,y) \in Supp(A\times B)$,

(i) a single valued neutrosophic relation R defined by :

$$\begin{aligned}\mu_R((x,y),P_1(x,y)) &= min\{\mu_{A\times B}(x,y),\ \mu_A(P_1(x,y))\}\}\\ &= min\{\mu_A(x),\, \mu_B(y),\ \mu_A(x)\}\}\\ &= min\{\mu_A(x),\ \mu_B(y)\}\end{aligned}$$

and

$$\begin{aligned}\sigma_R((x,y),P_1(x,y)) &= min\{\sigma_{A\times B}(x,y),\ \sigma_A(P_1(x,y))\}\}\\ &= min\{\sigma_A(x),\, \sigma_B(y),\ \sigma_A(x)\}\}\\ &= min\{\sigma_A(x),\ \sigma_B(y)\}\end{aligned}$$

and

$$\begin{aligned}\nu_R((x,y),P_1(x,y)) &= max\{\nu_{A\times B}(x,y),\ \nu_A(P_1(x,y))\}\}\\ &= max\{\nu_A(x),\, \nu_B(y),\ \nu_A(x)\}\}\\ &= max\{\nu_A(x),\ \nu_B(y)\}\end{aligned}$$

The second single valued neutrosophic projection mapping is defined analogously.

4 Continuity property in single valued neutrosophic topological space

The aim of the present section, is to introduce and study the notion of single valued neutrosophic continuous mapping in single valued neutrosophic topological spaces. The basic properties, and relationships with some types of continuity are also obtained.

4.1 Single valued neutrosophic topology

In this subsection, we generalize the notion of fuzzy topology on fuzzy sets introduced by Kandil et al. [16] to the setting of single valued neutrosophic sets to establish the continuity property of single valued neutrosophic mapping.

Definition 4.1. Let A be a single valued neutrosophic set on the set X and $O_A = \{U$ is an SVNS on X : $U \subseteq A\}$. We define a single valued neutrosophic topology on single valued neutrosophic set A by the family $T \subseteq O_A$ which satisfies the following conditions :

(i) $A,\ 0_\sim \in T$;

(ii) if $U_1,\, U_2\, \in T$, then $U_1 \cap U_2 \in T$;

(iii) if $U_i \in T$ for all $i \in I$, then $\cup_I U_i \in T$.

T is called a single valued neutrosophic topology of A and the pair (A, T) is a single valued neutrosophic topological space (SVN-TOP, for short). Every element of T is called a single valued neutrosophic open set (SVNOS, for short).

Example 4.2. Let $X = \mathcal{P}(\mathbb{R}^2)$ and $\alpha \in]0, 1[$, A be a single valued neutrosophic set on X given by :

$$\mu_A(\theta) = \begin{cases} 1\,,\; if\ \theta = \emptyset \\ \alpha\,,\;\; 0 < |\theta| < \infty\,, \\ 0\,,\;\; Otherwise\,, \end{cases} \qquad \sigma_A(\theta) = \begin{cases} 1\,,\; if\ \theta = \emptyset \\ \frac{\alpha}{2}\,,\;\; 0 < |\theta| < \infty\,, \\ 0\,,\;\; Otherwise\,, \end{cases} \qquad \nu_A(\theta) = \begin{cases} 0\,,\; if\ \theta = \emptyset \\ 1-\alpha\,,\;\; 0 < |\theta| < \infty\,, \\ 0.5\,,\;\; Otherwise. \end{cases}$$

Then, the family $T = \{A, 0_\sim, U\}$ where:

$$\mu_U(\theta) = \begin{cases} \frac{\alpha}{3}\,,\;\; |\theta| < \infty\,, \\ 0\,,\;\; Otherwise\,, \end{cases} \qquad \sigma_U(\theta) = \begin{cases} \frac{\alpha}{4}\,,\;\; |\theta| < \infty\,, \\ 0\,,\;\; Otherwise\,, \end{cases} \qquad \nu_U(\theta) = \begin{cases} 1\,,\;\; |\theta| < \infty\,, \\ 0.8\,,\;\; Otherwise\,, \end{cases}$$

is a single valued neutrosophic topology on A.

Inspired by the notion of interior (resp. closure) on intuitionistic fuzzy topological space on a set introduced by Atanassov [4], we generalize these notions in single valued neutrosophic topology on a single valued neutrosophic set.

Definition 4.3. Let (A, T) be a single valued neutrosophic topological space, for every single valued neutrosophic subset G of X we define the interior and closure of G by:

$int(G) = \{\langle x, \max\limits_{x\in X} \mu_U(x), \max\limits_{x\in X} \sigma_U(x), \min\limits_{x\in X} \nu_U(x)\rangle \mid x \in U \subseteq G\}$ and

$cl(G) = \{\langle x, \min\limits_{x\in X} \mu_K(x), \min\limits_{x\in X} \sigma_K(x), \max\limits_{x\in X} \nu_K(x)\rangle \mid x \in A$ and $G \subseteq K\}$

Example 4.4. Let $X = \{a, b, c\}$ and $A, B, C, D \in SVNS(X)$ such that

$A = \{< a, 0.5, 0.7, 0.1 >, < b, 0.7, 0.9, 0.2 >, < c, 0.6, 0.8, 0 >\}$

$B = \{< a, 0.5, 0.6, 0.2 >, < b, 0.5, 0.6, 0.4 >, < c, 0.4, 0.5, 0.4 >\}$

$C = \{< a, 0.4, 0.5, 0.5 >, < b, 0.6, 0.7, 0.3 >, < c, 0.2, 0.3, 0.3 >\}$

$D = \{< a, 0.5, 0.6, 0.2 >, < b, 0.6, 0.7, 0.3 >, < c, 0.4, 0.5, 0.3 >\}$

$E = \{< a, 0.4, 0.5, 0.5 >, < b, 0.5, 0.6, 0.4 >, < c, 0.2, 0.3, 0.4 >\}$

Then the family $T = \{A,\ 0_\sim,\ B, C, D, E\}$ is an SVN-TOP of A.

Now, we suppose that $G \in SVNS(X)$ given by $G = \{< a, 0.41, 0.5, 0.6), < b, 0.3, 0.2, 0.6 >, < c, 0.2, 0.3, 0.7 >\}$. Then, $int(G) = 0_\sim$ and $cl(G) = \overline{E} \cap 1_\sim = \overline{E}$.

Definition 4.5. Let (A, T) be a single valued neutrosophic topological space and $U \in SVNS(A, T)$. Then U is called :

1. a single valued neutrosophic semiopen set (SVNSOS) if $U \subseteq cl(int(U))$;
2. a single valued neutrosophic α-open set (SVNαOS) if $U \subseteq int(cl(int(U)))$;
3. a single valued neutrosophic preopen set (SVNPOS) if $U \subseteq int(cl(U))$;
4. a single valued neutrosophic regular open set (SVNROS) if $U = int(cl(U))$.

4.2 Single valued neutrosophic continuous mappings

In this subsection, we will study some interesting properties of single valued neutrosophic continuous mappings in single valued neutrosophic topological space and relations between various types of single valued neutrosophic continuous mapping. First, we introduce the notion of single valued neutrosophic continuous mapping.

Definition 4.6. Let (A, T) (B, L) be two single valued neutrosophic topological spaces. The mapping f_R : $(A, T) \to (B, L)$ is a single valued neutrosophic continuous if and only if the inverse of each L-open single valued neutrosophic set is T-open single valued neutrosophic set.

Example 4.7. Let (A, T) and (B, T') be two single valued neutrosophic topological spaces, where $\mu_A(x) = 0.8$, $\sigma_A(x) = 0.88$ and $\nu_A(x) = 0.1$, for any $x \in \mathbb{R}_+$ and

$$\mu_B(y) = \begin{cases} 0.5\ ,\ if\ y \geq 0 \\ 0.8\ ,\ Otherwise\,, \end{cases} \quad \sigma_B(y) = \begin{cases} 0.88\ ,\ if\ y \geq 0 \\ 0\ ,\ Otherwise\,, \end{cases} \quad \nu_B(y) = \begin{cases} 0.1\ ,\ if\ y \geq 0 \\ 0.3\ ,\ Otherwise\,, \end{cases}$$

We suppose that $T = \{A, 0_\sim, U_1\}$, where

$$\mu_{U_1}(x) = \begin{cases} 0.8\ ,\ if\ x \in [0, \sqrt{2}] \\ 0\ ,\ Otherwise\,, \end{cases} \quad \sigma_{U_1}(x) = \begin{cases} 0.88\ ,\ if\ x \in [0, \sqrt{2}] \\ 0\ ,\ Otherwise\,, \end{cases} \quad \nu_{U_1}(x) = \begin{cases} 0.1\ ,\ if\ x \in [0, \sqrt{2}] \\ 1\ ,\ Otherwise\,, \end{cases}$$

Also, we suppose that $T' = \{B, 0_\sim, U_1'\}$, where

$$\mu_{U_1'}(y) = \begin{cases} 0.5\ ,\ if\ y \in [0, 2] \\ 0\ ,\ Otherwise\,, \end{cases} \quad \sigma_{U_1'}(y) = \begin{cases} 0.8\ ,\ if\ y \in [0, 2] \\ 0\ ,\ Otherwise\,, \end{cases} \quad \nu_{U_1'}(y) = \begin{cases} 0.2\ ,\ if\ y \in [0, 2] \\ 0.4\ ,\ Otherwise. \end{cases}$$

Then, the single valued neutrosophic mapping $f_R : A \to B$ define by :

(i) an ordinary mapping $f : \mathbb{R}_+ \longrightarrow \mathbb{R}_+$ such that $f(x) = x^2$, for any $x \in \mathbb{R}_+$,

(ii) a single valued neutrosophic relation R defined by :

$\mu_R(x, f(x)) = 0.5$ $\sigma_R(x, f(x)) = 0.88$ and $\nu_R(x, f(x)) = 0.1$.

is a single valued neutrosophic continuous mapping. Indeed, it is easy to show that $f_R^{-1}(B) = A$ and $f_R^{-1}(0_\sim) = 0_\sim$ and we have,

$$\begin{aligned} \mu_{f_R^{-1}(U_1')}(x) &= \begin{cases} \mu_A(x)\ ,\ if\ x \in f^{-1}(supp(U_1')) \\ 0\ ,\ Otherwise\,, \end{cases} \\ &= \begin{cases} 0.8\ ,\ if\ x \in [0, \sqrt{2}] \\ 0\ ,\ Otherwise\,, \end{cases} \\ &= \mu_{U_1}(x), \end{aligned}$$

$$\begin{aligned} \sigma_{f_R^{-1}(U_1')}(x) &= \begin{cases} \sigma_A(x)\ ,\ if\ x \in f^{-1}(supp(U_1')) \\ 0\ ,\ Otherwise\,, \end{cases} \\ &= \begin{cases} 0.88\ ,\ if\ x \in [0, \sqrt{2}] \\ 0\ ,\ Otherwise\,, \end{cases} \\ &= \sigma_{U_1}(x), \end{aligned}$$

and

$$\begin{aligned}\nu_{f_R^{-1}(U_1')}(x) &= \begin{cases} \nu_A(x)\,,\ if\ x \in f^{-1}(supp(U_1')) \\ 1\,,\ Otherwise\,, \end{cases} \\ &= \begin{cases} \nu_A(x)\,,\ if\ x \in [0, \sqrt{2}] \\ 1\,,\ Otherwise\,, \end{cases} \\ &= \begin{cases} 0.1\,,\ if\ x \in [0, \sqrt{2}] \\ 1\,,\ Otherwise\,, \end{cases} \\ &= \nu_{U_1}(x). \end{aligned}$$

Hence, $f_R^{-1}(U_1') = U_1 \in T$. Thus, f_R is a single valued neutrosophic continuous mapping.

Remark 4.8. Let (A, T) be a single valued neutrosophic topological space. Then the single valued neutrosophic identity mapping $Id_R : (A, T) \to (A, T)$ is a single valued neutrosophic continuous mapping.

Next, we provide the relationships between various types of single valued neutrosophic continuous mapping. First, we generalize the notions of precontinuous mapping, α-continuous mapping introduced by Güray et al. [10] to the setting of single valued neutrosophic sets.

Definition 4.9. Let $f_R : (A, T) \to (B, T')$ be a single valued neutrosophic mapping. Then f_R is called :

1. a single valued neutrosophic precontinuous mapping if $f_R^{-1}(U')$ is a SVNPOS on A for every SVNOS U' on B;
2. a single valued neutrosophic α-continuous mapping if $f_R^{-1}(U')$ is a SVNαOS on A for every SVNOS U' on B.

The following proposition shows the relationship between single valued neutrosophic continuous mapping and single valued neutrosophic α-continuous mapping.

Proposition 4.10. *Let $f_R : (A, T) \to (B, T')$ be a single valued neutrosophic mapping. If f_R is a single valued neutrosophic continuous mapping, then f_R is a single valued neutrosophic α-continuous mapping.*

Proof. Let U' be a SVNOS in B and we need to show that $f_R^{-1}(U')$ is an SVNαOS in A. The fact that f_R is a single valued neutrosophic continuous mapping implies that $f_R^{-1}(U')$ is a SVNOS in A. From Definition 3.10, it follows that

$$\mu_{f_R^{-1}(U')}(x) = \begin{cases} \mu_A(x)\,,\ if\ x \in f^{-1}(supp(U')) \\ 0\,,\ Otherwise\,, \end{cases} \qquad \sigma_{f_R^{-1}(U')}(x) = \begin{cases} \sigma_A(x)\,,\ if\ x \in f^{-1}(supp(U')) \\ 0\,,\ Otherwise\,, \end{cases}$$

and $\nu_{f_R^{-1}(U')}(x) = \begin{cases} \nu_A(x)\,,\ if\ x \in f^{-1}(supp(U')) \\ 1\,,\ Otherwise. \end{cases}$

We conclude that, $f_R^{-1}(U')$ is a SVNαOS in A. Hence, f_R is a single valued neutrosophic α-continuous mapping. □

Remark 4.11. The converse of the above implication is not necessarily holds. Indeed, let us consider the single valued neutrosophic mapping f_R given in Example 4.7 and T be a SVN-topology given by $T = \{0_\sim, A, U_1\}$, where: $\mu_A(x) = 1,\ \sigma_A(x) = 0.99,\ \nu_A(x) = 0.001$ and

$$\mu_{U_1}(x) = \begin{cases} 1\,,\ if\ x \in [0, 1] \\ 0\,,\ Otherwise, \end{cases} \quad \sigma_{U_1}(x) = \begin{cases} 0.99\,,\ if\ x \in [0, 1] \\ 0\,,\ Otherwise, \end{cases} \quad \nu_{U_1}(x) = \begin{cases} 0.001\,,\ if\ x \in [0, 1] \\ 1\,,\ Otherwise. \end{cases}$$

Hence, $int(f_R^{-1}(U_1')) = U_1$, $cl(U_1) = 1_\sim$ and $int(1_\sim) = A$. Thus, $f_R^{-1}(U_1') \subseteq int(cl(int(f_R^{-1}(U_1'))))$. We conclude that $f_R^{-1}(U_1')$ is an SVNαS but not SVNOS and f_R is a single valued neutrosophic α-continuous mapping but not a single valued neutrosophic continuous mapping.

The following proposition shows the relationship between single valued neutrosophic α-continuous mapping and single valued neutrosophic pre-continuous mapping.

Proposition 4.12. *Let $f_R : (A,T) \to (B,T')$ be a single valued neutrosophic mapping. If f_R is a single valued neutrosophic α-continuous mapping, then f_R is a single valued neutrosophic pre-continuous mapping.*

Proof. Let U' be an SVNOS in B and we need to show that $f_R^{-1}(U')$ is a SVNPOS in A. The fact that f_R is a single valued neutrosophic α-continuous mapping implies that $f_R^{-1}(U')$ is a SVNαOS in A. From Definition 3.10, it follows that

$\mu_{f_R^{-1}(U')}(x) = \begin{cases} \mu_A(x)\,,\ if\ x \in f^{-1}(supp(U')) \\ 0\,,\ Otherwise\,, \end{cases}$ $\sigma_{f_R^{-1}(U')}(x) = \begin{cases} \sigma_A(x)\,,\ if\ x \in f^{-1}(supp(U')) \\ 0\,,\ Otherwise\,, \end{cases}$

and $\nu_{f_R^{-1}(U')}(x) = \begin{cases} \nu_A(x)\,,\ if\ x \in f^{-1}(supp(U')) \\ 1\,,\ Otherwise. \end{cases}$

We conclude that, $f_R^{-1}(U')$ is an SVNPOS in A. Hence, f_R is a single valued neutrosophic pre-continuous mapping. □

Remark 4.13. The converse of the above implication is not necessarily holds. Indeed, let (A,T) and (B,T') be two single valued neutrosophic topological spaces, where $\mu_A(x) = 1$, $\sigma_A(x) = 1$ and $\nu_A(x) = 0.005$, for any $x \in \mathbb{R}_+$ and

$\mu_B(y) = \begin{cases} 0.7\,,\ if\ y \geq 0 \\ 0\,,\ Otherwise\,, \end{cases}$ $\sigma_B(y) = \begin{cases} 0.9\,,\ if\ y \geq 0 \\ 0.8\,,\ Otherwise\,, \end{cases}$ $\nu_B(y) = \begin{cases} 0.01\,,\ if\ y \geq 0 \\ 0.03\,,\ Otherwise\,, \end{cases}$

We suppose that $T = \{A, 0_\sim, U_1\}$, where
$\mu_{U_1}(x) = 0$ $\sigma_{U_1}(x) = 1$ and $\nu_{U_1}(x) = 1$.
Also, we suppose that $T' = \{B, 0_\sim, U_1'\}$, where

$\mu_{U_1'}(y) = \begin{cases} 0.7\,,\ if\ y \in [0,4] \\ 0\,,\ Otherwise\,, \end{cases}$ $\sigma_{U_1'}(y) = \begin{cases} 0.5\,,\ if\ y \in [0,4] \\ 0\,,\ Otherwise\,, \end{cases}$ $\nu_{U_1'}(y) = \begin{cases} 0.12\,,\ if\ y \in [0,4] \\ 0.32\,,\ Otherwise. \end{cases}$

Then, the single valued neutrosophic mapping $f_R : A \to B$ define by :

(i) an ordinary mapping $f : \mathbb{R}_+ \longrightarrow \mathbb{R}_+$ such that $f(x) = \sqrt{x}$, for any $x \in \mathbb{R}_+$,

(ii) a single valued neutrosophic relation R defined by :
$\mu_R(x, f(x)) = 0.7$ $\sigma_R(x, f(x)) = 0.9$ and $\nu_R(x, f(x)) = 0.01$.

$$\begin{aligned} \mu_{f_R^{-1}(U_1')}(x) &= \begin{cases} \mu_A(x)\,,\ if\ x \in f^{-1}(supp(U_1')) \\ 0\,,\ Otherwise\,, \end{cases} \\ &= \begin{cases} 1\,,\ if\ x \in [0,16] \\ 0\,,\ Otherwise\,, \end{cases} \end{aligned}$$

$$\begin{aligned} \sigma_{f_R^{-1}(U_1')}(x) &= \begin{cases} \sigma_A(x)\,,\ if\ x \in f^{-1}(supp(U_1')) \\ 0\,,\ Otherwise\,, \end{cases} \\ &= \begin{cases} 1\,,\ if\ x \in [0,16] \\ 0\,,\ Otherwise\,, \end{cases} \end{aligned}$$

$$\begin{aligned}
\nu_{f_R^{-1}(U_1')}(x) &= \begin{cases} \nu_A(x)\ ,\ if\ x \in f^{-1}(supp(U_1')) \\ 1\ ,\ Otherwise\ , \end{cases} \\
&= \begin{cases} \nu_A(x)\ ,\ if\ x \in [0,16] \\ 1\ ,\ Otherwise\ , \end{cases} \\
&= \begin{cases} 0.01\ ,\ if\ x \in [0,16] \\ 1\ ,\ Otherwise\ , \end{cases}
\end{aligned}$$

Hence, $cl(f_R^{-1}(U_1')) = \overline{0_\sim} = 1_\sim$ and $int(1_\sim) = A$. Thus, $f_R^{-1}(U_1') \subseteq int(cl(f_R^{-1}(U_1')))$. We conclude that $f_R^{-1}(U_1')$ is an SVNPOS and f_R is a single valued neutrosophic pre-continuous but not a single valued neutrosophic continuous.

5 Conclusion

In this work, we have generalized the notion of fuzzy mapping defined by fuzzy relation introduced by Ismail and Massa'deh to the setting of single valued neutrosophic sets. Also, the main properties related to the single valued neutrosophic mapping have been studied. Next, as an application we have established the single valued neutrosophic continuous mapping in the single valued neutrosophic topological spaces. Future work will be directed to study the notion of the single valued neutrosophic mapping for other types of topologies based on the single valued neutrosophic sets.

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