Neutrosophic N-Soft Sets with TOPSIS method for Multiple Attribute Decision Making

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Abstract: The objective of this article is to introduce a new hybrid model of neutrosophic N-soft set which is combination of neutrosophic set and N-soft set. We introduce some basic operations on neutrosophic N-soft sets along with their fundamental properties. For multi-attribute decision-making (MADM) problems with neutrosophic N-soft sets, we propose an extended TOPSIS (technique based on order preference by similarity to ideal solution) method. In this method, we first propose a weighted decision matrix based comparison method to identify the positive and the negative ideal solutions. Afterwards, we define a separation measurement of these solutions. Finally, we calculate relative closeness to identify the optimal alternative. At length, a numerical example is rendered to illustrate the developed scheme in medical diagnosis via hypothetical case study.

Keywords: Neutosophic N-soft set, operations on neutosophic N-soft sets, MADM, TOPSIS, medical diagnosis.

1. Introduction

In contemporary decision-making science, multi-attribute decision-making (MADM) phenomenon plays a significant role in solving many real world problems. To deal with uncertainties, researchers have introduced different theories including, Fuzzy set (FS) [54] that comprises a mapping communicating the degree of association and intuitionistic fuzzy set (IFS) [10, 11] that comprises a pair of mappings communicating the degree of association and the degree of non-association of members of the universe to the unit closed interval with the restriction that sum of degree of association and degree of non-association should not exceed one. Smarandache [46, 47] introduced neutrosophic sets as an extension of IFSs. A neutrosophic object comprises three degrees, namely, degree of association, indeterminacy, and the degree of non-association to each alternative. Smarandache’s Neutrosophic Set [50] is a generalization of Intuitionistic Fuzzy Set, Inconsistent Intuitionistic Fuzzy Set (Picture Fuzzy Set, Ternary Fuzzy Set), Pythagorean Fuzzy Set (Atanassov’s Intuitionistic Fuzzy Set of second type), $q$-Rung Orthopair Fuzzy Set, Spherical Fuzzy Set, and $n$-Hyper-Spherical Fuzzy Set; while Neutrosoplication is a generalization of Regret Theory, Grey System Theory, and Three-Ways Decision. In 1999, Molodtsov [32] presented the notion of soft set as an important mathematical tool to deal with uncertainties. In 2007, Aktas and Cagman [6] extended the idea of soft sets to soft groups. In 2010, Feng et al. [18, 19] presented several results on soft sets, fuzzy soft sets and rough sets. In 2009 and 2011, Ali et al. [7, 8] introduced various properties of soft sets, fuzzy soft sets and rough sets. In 2011, Cagman et al. [12], and Shabir and Naz [51] independently presented soft topological spaces. Arockiarani et al. [9], in 2013, introduced the notion of fuzzy...


Soft sets provide binary evaluation of the objects and other mathematical models like fuzzy sets, intuitionistic fuzzy sets and neutrosophic sets associate values in the interval $[0,1]$. These models fail to deal with the situation when modeling on real world problems associate non-binary evaluations. Non-binary evaluations are also expected in rating or ranking positions. The ranking can be expressed in multinary values in the form of number of stars, dots, grades or any generalized notation. Motivated by these concerns, in 2017, Fatimah et al. [17] floated the idea of N-soft set as an extended model of soft set, in order to describe the importance of grades in real life. In 2018 and 2019, Akram et al. [1]-[3] introduced group decision-making methods based on hesitant N-soft sets and intuitionistic fuzzy N-soft rough set.

The technique for the order of preference by similarity to ideal solution (TOPSIS) was initially developed by Hwang and Yoon [26] in 1981. The core idea in the TOPSIS method is that selected alternative should have least geometric distance from positive ideal solution and maximum geometric distance from negative ideal solution. Positive ideal solution represents the condition for

The goal of this paper is to present a new hybrid model "neutrosophic N-soft set" and their applications to the decision making (DM). Neutrosophic N-soft set is the generalization of N-soft set, fuzzy N-soft set and intuitionistic fuzzy N-soft.

The comparison analysis of the proposed model with some existing models is given in Table 1.

<table>
<thead>
<tr>
<th>Sets</th>
<th>Parametrization</th>
<th>Non Binary Evaluation</th>
<th>Truth Membership</th>
<th>Falsity Membership</th>
<th>Indeterminacy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fuzzy set [54]</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
</tr>
<tr>
<td>Intuitionistic fuzzy set [10]</td>
<td>×</td>
<td>×</td>
<td>✓</td>
<td></td>
<td>×</td>
</tr>
<tr>
<td>Neutrosophic set [46]</td>
<td>×</td>
<td>×</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Soft Set [12]</td>
<td>✓</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
</tr>
<tr>
<td>N-soft Set [17]</td>
<td>✓</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
</tr>
<tr>
<td>Fuzzy N-soft Set[1]</td>
<td>✓</td>
<td></td>
<td>✓</td>
<td></td>
<td>×</td>
</tr>
<tr>
<td>Intuitionistic N-soft Set [3]</td>
<td>✓</td>
<td></td>
<td>✓</td>
<td>✓</td>
<td>×</td>
</tr>
<tr>
<td>Neutrosophic N-soft Set (Proposed)</td>
<td>✓</td>
<td></td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>

Table 1: Comparison with other existing theories

The rest of paper is organized as follows. In Section 2, we recall some fundamental concepts of N-soft set, fuzzy neutrosophic set and fuzzy neutrosophic soft set. In Section 3, we propose our new hybrid model fuzzy neutrosophic N-soft set along with their examples. We also present some basic operations on fuzzy neutrosophic N-soft set with illustrations. We also investigate fundamental
properties of the proposed model by using defined operations. In Section 4, we construct relations by using fuzzy neutrosophic N-soft set and define composition of fuzzy neutrosophic N-soft sets using relations. We also define some new choice functions and score functions in connection with fuzzy neutrosophic N-soft sets. In Section 5, we proposed DM method for medical diagnosis by the model. In Section 6, we give a numerical example of this diagnosis method via conjectural case study. In Section 7, we conclude with some future directions and give suggestions for future work.

2. Preliminaries

In this segment, we review some essential definitions and a few aftereffects of N-soft and neutrosophic sets that would be accommodating in the following segments.

**Definition 2.1** [54] A fuzzy set \( \vartheta \) in \( \mathbb{X} \) is assessed up by a mapping with \( \mathbb{X} \) as domain and membership degree in \([0,1]\). The accumulation of all fuzzy sets (FSs) in the universal set \( \mathbb{X} \) is signified by \( \vartheta(\mathbb{X}) \).

**Definition 2.2** [46, 47] A neutrosophic set (NS) \( \mathbb{P} \) over the universe of discourse \( \mathbb{X} \) is defined as
\[
\mathbb{P} = \{ \langle \varphi, (\mathbb{T}_{\mathbb{P}}(\varphi), \mathbb{I}_{\mathbb{P}}(\varphi), \mathbb{F}_{\mathbb{P}}(\varphi)) \rangle : \varphi \in \mathbb{X} \}
\]
where \( \mathbb{T}_{\mathbb{P}}, \mathbb{I}_{\mathbb{P}}, \mathbb{F}_{\mathbb{P}} : \mathbb{X} \rightarrow [-0,1+] \) and \(-0 \leq \mathbb{T}_{\mathbb{P}}(\varphi) + \mathbb{I}_{\mathbb{P}}(\varphi) + \mathbb{F}_{\mathbb{P}}(\varphi) \leq 3\). The mapping \( \mathbb{T}_{\mathbb{P}} \) stands for degree of membership, \( \mathbb{I}_{\mathbb{P}} \) is the degree of indeterminacy and \( \mathbb{F}_{\mathbb{P}} \) is the degree of falsity of points of the given set. From philosophical perspective, the neutrosophic set takes the entries from some subset of \([-0,1+] \). But it many actual applications, it is inconvenient to utilize neutrosophic set with entries from such subsets. Therefore, we consider the neutrosophic set which takes the entries from some subset of \([0,1]\).

**Definition 2.3** [9] Let \( \mathbb{X} \) be a space of objects (points). A fuzzy neutrosophic set (FNS) \( \mathbb{P} \) in \( \mathbb{X} \) is despirit by a truth-membership function \( \mathbb{T}_{\mathbb{P}} \), an indeterminacy membership-function \( \mathbb{I}_{\mathbb{P}} \) and a falsity-membership function \( \mathbb{F}_{\mathbb{P}} \). In mathematical form, this collection is expressed as
\[
\mathbb{P} = \{ \langle \varphi, (\mathbb{T}_{\mathbb{P}}(\varphi), \mathbb{I}_{\mathbb{P}}(\varphi), \mathbb{F}_{\mathbb{P}}(\varphi)) \rangle : \varphi \in \mathbb{X}, \mathbb{T}_{\mathbb{P}}, \mathbb{I}_{\mathbb{P}}, \mathbb{F}_{\mathbb{P}} \in [0,1] \}
\]
with the constraint that sum of \( \mathbb{T}_{\mathbb{P}}(\varphi), \mathbb{I}_{\mathbb{P}}(\varphi) \) and \( \mathbb{F}_{\mathbb{P}}(\varphi) \) should fall in \([0,3]\) i.e.
\[0 \leq \mathbb{T}_{\mathbb{P}}(\varphi) + \mathbb{I}_{\mathbb{P}}(\varphi) + \mathbb{F}_{\mathbb{P}}(\varphi) \leq 3\]

**Definition 2.4** [32] Let \( \mathbb{X} \) be the set of points and \( \mathbb{E} \) be the set of attributes with \( \mathbb{L} \) in \( \mathbb{E} \). Let \( \mathcal{P}(\mathbb{X}) \) denotes collection of subsets of \( \mathbb{X} \). The pair \( (\zeta, \mathcal{L}) \) is said to be a soft set (SS) over \( \mathbb{X} \), where \( \zeta \) is a function given by
\[\zeta : \mathcal{L} \rightarrow \mathcal{P}(\mathbb{X})\]
Thus, an SS is expressed in mathematical form as
\[(\zeta, \mathcal{L}) = \{ (\xi, \zeta(\xi)) : \xi \in \mathcal{L} \}.

**Definition 2.5** [9] Let \( \mathbb{X} \) be the initial universal set and \( \mathbb{E} \) be the set of parameters. We consider the non-empty set \( \mathcal{L} \subseteq \mathbb{E} \). Let \( \mathbb{P}(\mathbb{X}) \) signifies the set of all NSs of \( \mathbb{X} \). The accretion \( \Omega_{\mathcal{L}} \) is called the neutrosophic soft set (NSS) over \( \mathbb{X} \), where \( \Omega_{\mathcal{L}} \) is a function given by \( \Omega_{\mathcal{L}} : \mathcal{L} \rightarrow \mathbb{P}(\mathbb{X}) \). We can write it as
\[\Omega_{\mathcal{L}} = \{ (\xi, (\varphi, \mathbb{T}_{\mathcal{L}}(\varphi), \mathbb{I}_{\mathcal{L}}(\varphi), \mathbb{F}_{\mathcal{L}}(\varphi)) : \varphi \in \mathbb{X}) : \xi \in \mathcal{L} \}
\]
Notice that if \( \Omega_{\mathcal{L}}(\xi) = \{ (\varphi, 0,1,1) : \varphi \in \mathbb{X} \} \), then NS-element \( (\xi, \Omega_{\mathcal{L}}(\xi)) \) does not seem to appear in the NSS \( \Omega_{\mathcal{L}} \). The set of all NSSs over \( \mathbb{X} \) is symbolized by \( \mathcal{N}(\mathbb{X}_\mathcal{E}) \).

**Definition 2.6** [17] Let \( \mathbb{X} \) be a set of points and \( \mathbb{E} \) be a set of attributes with \( \mathcal{L} \) in \( \mathbb{E} \). Let \( \mathcal{G} = \{0,1,2,\ldots,N-1\} \) be the set of ordered grades where \( N \in \{2,3,\ldots\} \). The N-soft set (NSS) on \( \mathbb{X} \) is denoted by \( (\zeta, \mathcal{L}, N) \) where \( \zeta : \mathcal{L} \rightarrow 2^{\mathbb{X} \times \mathcal{G}} \) is a map characterized by
\[
(\zeta(\xi)) = (\varphi, \pi(\xi))
\]

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\[ \forall \varphi \in \mathbb{X}, \xi \in \mathcal{L}, r_{L(\xi)} \in \mathcal{G}. \]

**Definition 2.7** [17] A weak complement of N-soft set \((\zeta, \mathcal{L}, N)\) is another N-soft set \((\zeta^\prime, \mathcal{L}, N)\) gratifying \(\zeta^\prime \cap \zeta = \varphi, \forall \xi \in \mathbb{X}.\)

**Definition 2.8** [17] A top weak complement of N-soft set \((\zeta, \mathcal{L}, N)\) is an N-soft set \((\zeta^\star, \mathcal{L}, N)\), where

\[
(\zeta^\star, \mathcal{L}, N) = \begin{cases} 
    (\varphi, N - 1), & \text{if } r_{L(\xi)}(\varphi) < N - 1 \\
    (\varphi, 0), & \text{if } r_{L(\xi)}(\varphi) = N - 1
\end{cases}, \quad \forall \xi \in \mathbb{X}.
\]

**Definition 2.9** [17] A bottom weak complement of N-soft set \((\zeta, \mathcal{L}, N)\) is one more N-soft set \((\zeta^\ast, \mathcal{L}, N)\), where

\[
(\zeta^\ast, \mathcal{L}, N) = \begin{cases}
    (\varphi, 0), & \text{if } r_{L(\xi)}(\varphi) > 0, \\
    (\varphi, N - 1), & \text{if } r_{L(\xi)}(\varphi) = 0.
\end{cases}
\]

### 3 Neutrosophic N-soft Set

In this section, we propose a novel structure neutrosophic N-soft set (NNSS), which is blend of NS and NSS. We present some definitions and operations on NNSS too. Some properties of NNSS associated with these operations also have been set up.

**Definition 3.1** Let \(\mathbb{X}\) be the initial universe set, \(\mathcal{E}\) the set of attributes and \(\mathcal{G}\) the aggregate of ordered grades. We consider non-empty subset \(\mathcal{L}\) of \(\mathcal{E}\). Let \(\hat{P}(\mathbb{X} \times \mathcal{G})\) be the collection of all NSSs of \(\mathbb{X} \times \mathcal{G}\). A neutrosophic N-soft set (NNSS) is signified by \((\lambda, \Omega, N)\), where \(\Omega = (\zeta, \mathcal{L}, N)\) is an NSS. If there is no ambiguity, we can abbreviate it as \(\lambda_{\mathcal{L}}\) represented by the mapping \(\lambda_{\mathcal{L}}: \mathcal{L} \rightarrow \hat{P}(\mathbb{X} \times \mathcal{G})\). Mathematically,

\[ \lambda_{\mathcal{L}} = \{(\xi, \Gamma_{\mathcal{L}}(\xi)): \Gamma_{\mathcal{L}}(\xi) = \{(\varphi, T_{L(\xi)}(\varphi), I_{L(\xi)}(\varphi), F_{L(\xi)}(\varphi)), r_{L}(\xi)(\varphi)\}, r_{L} \in \mathcal{G}, \varphi \in \mathbb{X}, T_{L}, I_{L}, F_{L} \in [0,1], \xi \in \mathcal{E}\} \]

In short form, we may write

\[ \lambda_{\mathcal{L}} = \{(\xi, \Gamma_{\mathcal{L}}(\xi)): \xi \in \mathcal{E}\} \]

where

\[ \Gamma_{\mathcal{L}}(\xi) = \{(\varphi, T_{L(\xi)}(\varphi), I_{L(\xi)}(\varphi), F_{L(\xi)}(\varphi)), r_{L}(\xi)(\varphi)): r_{L} \in \mathcal{G}, \varphi \in \mathbb{X}, T_{L}, I_{L}, F_{L} \in [0,1]\} \]

The accretion of all NNSSs is denoted by \(\text{NNS}(\mathbb{X})\).

Our proposed structure is more generalized then other existing models. The existing models are special cases of our proposed model, as shown in Table 2

<table>
<thead>
<tr>
<th>Neutrosophic N-soft Set (Proposed)</th>
<th>((\xi, ((\varphi, T_{L(\xi)}(\varphi), I_{L(\xi)}(\varphi), F_{L(\xi)}(\varphi)), r_{L(\xi)}(\varphi))))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intuitionistic N-soft Set [3]</td>
<td>((\xi, ((\varphi, T_{L(\xi)}(\varphi), 0, F_{L(\xi)}(\varphi)), r_{L(\xi)}(\varphi))))</td>
</tr>
<tr>
<td>Fuzzy N-soft Set [1]</td>
<td>((\xi, ((\varphi, T_{L(\xi)}(\varphi), 0, 0), r_{L(\xi)}(\varphi))))</td>
</tr>
<tr>
<td>N-soft Set [17]</td>
<td>((\xi, r_{L(\xi)}(\varphi))))</td>
</tr>
</tbody>
</table>

Table 2: Comparison with N-soft set and it's other existing generalization

**Example 3.2** Let \(\mathbb{X} = \{\varphi_1, \varphi_2\}\) and \(\mathcal{E} = \{\xi_1, \xi_2, \xi_3\}\). Consider \(\mathcal{E} \supseteq \mathcal{L} = \{\xi_1, \xi_2\}\). Define NNSS as \(\lambda_{\mathcal{L}} = \{(\xi_i, \Gamma_{\mathcal{L}}(\xi_i)): \xi_i \in \mathcal{L}, i = 1,2\}\), where 8SS is given in Table 3 below:

<table>
<thead>
<tr>
<th>((\xi, L, B))</th>
<th>(\xi_1)</th>
<th>(\xi_2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\varphi_1)</td>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td>(\varphi_2)</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

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Now, we define N8SS as
\[ \Gamma_L(\xi_1) = \{(\phi_1, 0.8, 0.5, 0.1), 6\}, \{(\phi_2, 0.6, 0.2, 0.9), 4\}\]
\[ \Gamma_L(\xi_2) = \{(\phi_1, 0.5, 0.7, 0.3), 3\}, \{(\phi_2, 0.7, 0.4, 0.8), 5\}\]
The tabular representation of N8SS is given in Table 4.

<table>
<thead>
<tr>
<th>(\lambda)</th>
<th>(\xi_1)</th>
<th>(\xi_2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\phi_1)</td>
<td>((0.8, 0.5, 0.1), 6)</td>
<td>((0.5, 0.7, 0.3), 3)</td>
</tr>
<tr>
<td>(\phi_2)</td>
<td>((0.6, 0.2, 0.9), 4)</td>
<td>((0.7, 0.4, 0.8), 5)</td>
</tr>
</tbody>
</table>

Table 4: Tabular representation of N8SS

Remarks:
1. Every N2SS \((\lambda, \Omega, 2)\) is generally equal to NSS.
2. Any arbitrary NNSS over the universe \(\mathbb{X}\) can also be thought of as N\((N + 1)\)-soft set. For example, an N8SS can also be treated as an N9SS for the grade 8 is never used as can be seen in Table 4.
This observation may be extended on the parallel track.

Now, we head towards presenting some arithmetical notions related to NNSS.

**Definition 3.3** Let \(\lambda_L, \lambda_M \in \text{NSS}(\mathbb{X})\). \(\lambda_L\) is said to be NNS- subset of \(\lambda_M\), if
\[ L \subseteq M, \]
\[ T_{L(\xi)}(\psi) \leq T_{M(\xi)}(\psi), \]
\[ I_{L(\xi)}(\psi) \geq I_{M(\xi)}(\psi), \]
\[ F_{L(\xi)}(\psi) \geq F_{M(\xi)}(\psi), \]
\[ r_{L(\xi)}(\psi) \leq r_{M(\xi)}(\psi) \]
\[ \forall \xi \in E, \psi \in X, r_L \in \mathbb{G}. \] We demonstrate it by \(\lambda_L \subseteq \lambda_M\). \(\lambda_M\) is said to be NNS- superset of \(\lambda_L\).

**Example 3.4** Let \(\mathbb{X} = \{\phi_1, \phi_2\}\) and \(E = \{\xi_1, \xi_2, \xi_3\}\). Consider \(E \supseteq L = \{\xi_1, \xi_2\}\). Consider N8SS \(\lambda_L\) as given in Example 3.2. Let \(M = E\). Define N8SS \(\lambda_M\) as
\[ \lambda_M = \{(\xi_i, \Gamma_M(\xi_i)); \xi_i \in M, i = 1, 2, 3\}\]
where 8SS is given in Table 5 below.

<table>
<thead>
<tr>
<th>((\xi, M, \mathbb{N}))</th>
<th>(\xi_1)</th>
<th>(\xi_2)</th>
<th>(\xi_3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\phi_1)</td>
<td>7</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>(\phi_2)</td>
<td>5</td>
<td>7</td>
<td>3</td>
</tr>
</tbody>
</table>

Table 5: Tabular representation of 8SS

Now, we define N8SS
\[ \Gamma_M(\xi_1) = \{(\phi_1, 0.9, 0.4, 0.0), 7\}, \{(\phi_2, 0.7, 0.1, 0.8), 5\}\]
\[ \Gamma_M(\xi_2) = \{(\phi_1, 0.6, 0.5, 0.2), 4\}, \{(\phi_2, 0.9, 0.3, 0.8), 7\}\]
\[ \Gamma_M(\xi_3) = \{(\phi_1, 0.8, 0.5, 0.1), 6\}, \{(\phi_3, 0.5, 0.7, 0.3), 3\}\]
having tabular form
Table 6: Tabular representation of N8SS

<table>
<thead>
<tr>
<th>( \lambda_M )</th>
<th>( \xi_1 )</th>
<th>( \xi_2 )</th>
<th>( \xi_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \varphi_1 )</td>
<td>(0.9,0.4,0,0),7</td>
<td>(0.6,0.5,0,2),4</td>
<td>(0.8,0.5,0,1),6</td>
</tr>
<tr>
<td>( \varphi_2 )</td>
<td>(0.7,0.1,0,8),5</td>
<td>(0.9,0.3,0,8),7</td>
<td>(0.5,0.7,0,3),3</td>
</tr>
</tbody>
</table>

Table 7: Tabular representation of null N8SS

<table>
<thead>
<tr>
<th>( \lambda_{L\phi} )</th>
<th>( \xi_1 )</th>
<th>( \xi_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \varphi_1 )</td>
<td>(0,1,1),0</td>
<td>(0,1,1),0</td>
</tr>
<tr>
<td>( \varphi_2 )</td>
<td>(0,1,1),0</td>
<td>(0,1,1),0</td>
</tr>
</tbody>
</table>

Table 8: Tabular representation of absolute N8SS

<table>
<thead>
<tr>
<th>( \lambda_\hat{L} )</th>
<th>( \xi_1 )</th>
<th>( \xi_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \varphi_1 )</td>
<td>(1,0,0),7</td>
<td>(1,0,0),7</td>
</tr>
<tr>
<td>( \varphi_2 )</td>
<td>(1,0,0),7</td>
<td>(1,0,0),7</td>
</tr>
</tbody>
</table>
1. \( \lambda_L \sqsubseteq \lambda_L \).
2. \( \lambda_L \phi \sqsubseteq \lambda_L \).
3. \( \lambda_L \sqsubseteq \lambda_L \).
4. \( \lambda_K \sqsubseteq \lambda_L \) and \( \lambda_L \sqsubseteq \lambda_M \Rightarrow \lambda_K \sqsubseteq \lambda_M \).

**Proof.** The proof follows directly from definitions of related terms.

**Proposition 3.11** Let \( \lambda_K, \lambda_L, \lambda_M \in \text{NNS}(\mathfrak{X}) \). Then,

1. \( \lambda_K = \lambda_L \) and \( \lambda_L = \lambda_M \Rightarrow \lambda_K = \lambda_M \).
2. \( \lambda_L \sqsubseteq \lambda_M \) and \( \lambda_M \sqsubseteq \lambda_L \Rightarrow \lambda_L = \lambda_M \).

**Proof.** Straight forward.

**Definition 3.12** Let \( \lambda_L \in \text{NNS}(\mathfrak{X}) \). Then weak complement of NNSS \( \lambda_L \) is symbolized by \( \lambda_L^C \) and defined as

\[
\lambda_L^C = \{(\xi, \Gamma_L^C) : \xi \in E \}
\]

where

\[
\Gamma_L^C = \{(\varphi, F_L(\xi)(\varphi), 1 - I_L(\xi)(\varphi), T_L(\xi)(\varphi)), r^*_{L(\xi)}(\varphi)) : \varphi \in X \}
\]

Here \( r^*_{L(\xi)}(\varphi) \) denotes weak complement defined in Definition 2.7.

**Example 3.13** Let \( \mathfrak{X} = \{\varphi_1, \varphi_2\} \) and \( E = \{\xi_1, \xi_2, \xi_3\} \). Consider \( E \supseteq \mathbb{L} = \{\xi_1, \xi_2\} \). Define complement of N8SS \( \lambda_L \) given in Example 3.2 as \( \lambda_L^L = \{(\xi_i, \Gamma_L^L(\xi_i)) : \xi_i \in \mathbb{L}, i = 1, 2\} \) i.e.

\[
\begin{align*}
\Gamma_L^L(\xi_1) &= \{(\varphi_1, 0.1, 0.5, 0.8), 5), (\varphi_2, 0.9, 0.8, 0.6, 7)\} \\
\Gamma_L^L(\xi_2) &= \{(\varphi_1, 0.3, 0.3, 0.5), 4), (\varphi_2, 0.8, 0.6, 0.7, 2)\}
\end{align*}
\]

The tabular form is given in Table 9.

<table>
<thead>
<tr>
<th>( \lambda_L^C )</th>
<th>( \xi_1 )</th>
<th>( \xi_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \varphi_1 )</td>
<td>((0.8,0.5,0.1),5)</td>
<td>((0.5,0.7,0.3),4)</td>
</tr>
<tr>
<td>( \varphi_2 )</td>
<td>((0.6,0.2,0.9),7)</td>
<td>((0.7,0.4,0.8),2)</td>
</tr>
</tbody>
</table>

**Table 9:** Tabular representation of weak complement of N8SS

**Proposition 3.14** Let \( \lambda_L \in \text{NNS}(\mathfrak{X}) \), then

1. \( (\lambda_L^C)^C \neq \lambda_L \).
2. \( \lambda_L^C \neq \lambda_L \).
3. \( \lambda_L^C \neq \lambda_L^C \).

**Proof.** Straight forward.

**Definition 3.15** Let \( \lambda_L \in \text{NNS}(\mathfrak{X}) \). Then top weak complement of NNSS \( \lambda_L \) is symbolized by \( \lambda_L^* \) and defined as

\[
\lambda_L^* = \{(\xi, \Gamma_L^*) : \xi \in E \}
\]

Where,

\[
\Gamma_L^* = \{(\varphi, F_L(\xi)(\varphi), 1 - I_L(\xi)(\varphi), T_L(\xi)(\varphi)), r^*_{L(\xi)}(\varphi)) : \varphi \in X \}
\]

where, \( r^*_{L(\xi)}(\varphi) \) denotes top weak complement defined in Definition 2.8.

**Example 3.16** Let \( \mathfrak{X} = \{\varphi_1, \varphi_2\} \) and \( E = \{\xi_1, \xi_2, \xi_3\} \). Consider \( E \supseteq \mathbb{L} = \{\xi_1, \xi_2\} \). Define complement of N8SS \( \lambda_L \) given in Example 3.2 as \( \lambda_L^L = \{(\xi_i, \Gamma_L^L(\xi_i)) : \xi_i \in \mathbb{L}, i = 1, 2\} \) i.e.

\[
\Gamma_L^L(\xi_1) = \{(\varphi_1, 0.1, 0.5, 0.8, 7), (\varphi_2, 0.9, 0.8, 0.6, 7)\}
\]
\[ \Gamma_L^*(\xi_i) = \{((\varphi_1, 0.3,0.3,0.5),7),((\varphi_2, 0.8,0.6,0.7),7)\} \]

In tabular form given in Table 10.

<table>
<thead>
<tr>
<th>( \lambda^*_L )</th>
<th>( \xi_1 )</th>
<th>( \xi_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \varphi_1 )</td>
<td>(0.8,0.5,0.1,7)</td>
<td>(0.5,0.7,0.3,7)</td>
</tr>
<tr>
<td>( \varphi_2 )</td>
<td>(0.6,0.2,0.9,7)</td>
<td>(0.7,0.4,0.8,7)</td>
</tr>
</tbody>
</table>

Table 10: Tabular representation of top weak complement of N8SS

**Proposition 3.17** Let \( \lambda_L \in \text{NNS}(X) \). Then,
1. \((\lambda_L^*)^* \neq \lambda_L^*\).
2. \(\lambda^*_L \circ = \lambda^*_L \).
3. \(\lambda^*_L = \lambda^*_L \).

**Proof.** The proof follows quickly from definitions of relevant terms.

**Definition 3.18** Let \( \lambda_L \in \text{NNS}(X) \). Then the bottom weak complement of \( \text{NNS} \) \( \lambda_L \) is symbolized by \( \lambda_L^* \) and defined as follows

\[ \lambda_L^* = \{(\xi, \Gamma_L^*): \xi \in E\} \]

where

\[ \Gamma_L^* = \{(\varphi, F_{L}(\xi)(\varphi), 1 - I_{L}(\xi)(\varphi), T_{L}(\xi)(\varphi)): \varphi \in X\} \]

Here \( r_{L}(\xi)(\varphi) \) denotes top weak complement defined in Definition 2.9.

**Example 3.19** Let \( X = \{\varphi_1, \varphi_2\} \) and \( E = \{\xi_1, \xi_2, \xi_3\} \). Consider \( E \supseteq L = \{\xi_1, \xi_2\} \). Bottom weak complement of N8SS \( \lambda_L \) defined in Example 3.2 as \( \lambda_L = \{(\xi_i, \Gamma_L(\xi_i)): \xi_i \in L, i = 1,2\} \) where

\[ \Gamma_L(\xi_1) = \{((\varphi_1, 0.1,0.5,0.8),7),((\varphi_2, 0.9,0.8,0.6),7)\} \]

\[ \Gamma_L(\xi_2) = \{((\varphi_1, 0.3,0.3,0.5),7),((\varphi_2, 0.8,0.6,0.7),7)\} \]

In tabular form the bottom weak complement of N8SS is given in Table 11.

<table>
<thead>
<tr>
<th>( \lambda_L^* )</th>
<th>( \xi_1 )</th>
<th>( \xi_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \varphi_1 )</td>
<td>(0.8,0.5,0.1,0)</td>
<td>(0.5,0.7,0.3,0)</td>
</tr>
<tr>
<td>( \varphi_2 )</td>
<td>(0.6,0.2,0.9,0)</td>
<td>(0.7,0.4,0.8,0)</td>
</tr>
</tbody>
</table>

Table 11: Tabular representation of bottom weak complement of N8SS

**Proposition 3.20** Let \( \lambda_L \in \text{NNS}(X) \). Then,
1. \((\lambda_L^*)^* \neq \lambda_L^*\).
2. \(\lambda^*_L \circ = \lambda_L^* \).
3. \(\lambda^*_L = \lambda^*_L \).

**Proof.** Straight forward.

**Definition 3.21** Let \( \lambda_L, \lambda_M \in \text{NNS}(X) \). Then the difference of \( \lambda_L \) and \( \lambda_M \) is symbolized by \( \lambda_L \setminus \lambda_M \) and is defined as

\[ \lambda_L \setminus \lambda_M = \{(\xi, (T_{L}(\xi)(M(\xi)(\varphi)), 1 - I_{L}(\xi)(M(\xi)(\varphi)), F_{L}(\xi)(M(\xi)(\varphi)), r_{L}(\xi)(M(\xi)(\varphi))): \varphi \in \mathbb{X})\}: \xi \in E\} \]

where \( T_{L}(\xi)(M(\xi)(\varphi)), 1 - I_{L}(\xi)(M(\xi)(\varphi)), F_{L}(\xi)(M(\xi)(\varphi)) \) and \( r_{L}(\xi)(M(\xi)(\varphi)) \) are defined as

\[ T_{L}(\xi)(M(\xi)(\varphi)) = \min \{T_{L}(\xi)(\varphi), F_{M(\xi)(\varphi)}\} \]

\[ 1 - I_{L}(\xi)(M(\xi)(\varphi)) = \max \{1 - I_{L}(\xi)(\varphi), 1 - F_{M(\xi)(\varphi)}\} \]

\[ F_{L}(\xi)(M(\xi)(\varphi)) = \max \{F_{L}(\xi)(\varphi), T_{M(\xi)(\varphi)}\} \]
\[ r_{(L\cup M)}(\varphi) = \begin{cases} r_{L}(\varphi) - r_{M}(\varphi), & \text{if } r_{L}(\varphi) > r_{M}(\varphi), \\ 0, & \text{otherwise} \end{cases} \]

**Definition 3.22** Let \( \lambda_L, \lambda_M \in \text{NNS}(\mathbb{X}) \). Then addition of \( \lambda_L \) and \( \lambda_M \) is symbolized by \( \lambda_L \oplus \lambda_M \) and is defined as

\[
\lambda_L \oplus \lambda_M = \{(\xi, ((\varphi, T_{L}(\xi) \oplus M_{\xi}(\varphi), I_{L}(\xi) \oplus M_{\xi}(\varphi), F_{L}(\xi) \oplus M_{\xi}(\varphi)), r_{L}(\xi)(\varphi)): \varphi \in \mathbb{X}) \}: \xi \in L \}
\]

where \( T_{L}(\xi) \oplus M_{\xi}(\varphi) \) and \( F_{L}(\xi) \oplus M_{\xi}(\varphi) \) are given as

\[
T_{L}(\xi) \oplus M_{\xi}(\varphi) = \min(T_{L}(\varphi) + T_{M}(\varphi),1)
\]

\[
F_{L}(\xi) \oplus M_{\xi}(\varphi) = \min(F_{L}(\varphi) + F_{M}(\varphi),1)
\]

**Definition 3.23** Let \( \lambda_L, \lambda_M \in \text{NNS}(\mathbb{X}) \) be expressed as \( \lambda_L = (\lambda_1, \Omega_1, N) \) and \( \lambda_M = (\lambda_2, \Omega_2, N_1) \) where \( \Omega_1 = (\xi_1, L, N_2) \) and \( \Omega_2 = (\xi_2, M, N_1) \) are NSSs. Then their restricted union is symbolized by \( \lambda_L \sqcup_R \lambda_M \) and defined as \( (\varphi, \Omega_1 \sqcup_R \Omega_2, \max(N_1, N_2)) \) where \( \Omega_1 \sqcup_R \Omega_2 = (\varphi, \Xi \sqcup_R \varphi, \max(N_1, N_2)) \) i.e.

\[
\lambda_L \sqcup_R \lambda_M = \{(\xi, ((\varphi, T_{L}(\xi) \sqcup M_{\xi}(\varphi), I_{L}(\xi) \sqcup M_{\xi}(\varphi), F_{L}(\xi) \sqcup M_{\xi}(\varphi), r_{L}(\xi)(\varphi), r_{M}(\xi)(\varphi))): \xi \in L \}\}
\]

**Example 3.24** Consider again \( \lambda_L, \lambda_M \) as given in Examples 3.2 and 3.4 respectively. The restricted union \( \lambda_L \sqcup_R \lambda_M \) is given in Table 12.

<table>
<thead>
<tr>
<th>( \lambda_L \sqcup_R \lambda_M )</th>
<th>( \xi_1 )</th>
<th>( \xi_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \varphi_1 )</td>
<td>(0.9,0.4,0.0,7)</td>
<td>(0.6,0.5,0.2,4)</td>
</tr>
<tr>
<td>( \varphi_2 )</td>
<td>(0.7,0.1,0.8,5)</td>
<td>(0.9,0.3,0.8,7)</td>
</tr>
</tbody>
</table>

**Example 3.26** Consider again \( \lambda_L, \lambda_M \) as given in Examples 3.2 and 3.4 respectively. The extended union \( \lambda_L \sqcup_E \lambda_M \) is given in Table 13.

<table>
<thead>
<tr>
<th>( \lambda_L \sqcup_E \lambda_M )</th>
<th>( \xi_1 )</th>
<th>( \xi_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \varphi_1 )</td>
<td>(0.9,0.4,0.0,7)</td>
<td>(0.6,0.5,0.2,4)</td>
</tr>
<tr>
<td>( \varphi_2 )</td>
<td>(0.7,0.1,0.8,5)</td>
<td>(0.9,0.3,0.8,7)</td>
</tr>
</tbody>
</table>

**Definition 3.25** Let \( \lambda_L, \lambda_M \in \text{NNS}(\mathbb{X}) \) be expressed as \( \lambda_L = (\lambda_1, \Omega_1, N) \) and \( \lambda_M = (\lambda_2, \Omega_2, N_1) \) where \( \Omega_1 = (\xi_1, L, N_2) \) and \( \Omega_2 = (\xi_2, M, N_1) \) are NSSs. Then their extended union is symbolized by \( \lambda_L \sqcup_E \lambda_M \) and defined as \( (\varphi, \Omega_1 \sqcup_E \Omega_2, \max(N_1, N_2)) \) where \( \Omega_1 \sqcup_E \Omega_2 = (\varphi, \Xi \sqcup_E \varphi, \max(N_1, N_2)) \) i.e.

\[
\lambda_L \sqcup_E \lambda_M = \{(\xi, ((\varphi, T_{L}(\xi) \sqcup E M_{\xi}(\varphi), I_{L}(\xi) \sqcup E M_{\xi}(\varphi), F_{L}(\xi) \sqcup E M_{\xi}(\varphi), r_{L}(\xi)(\varphi), r_{M}(\xi)(\varphi)): \xi \in L \}\}
\]
Theorem 3.27 Let $\lambda_L, \lambda_M \in \text{NNS}(\mathbb{X})$. Then their extended-union $\lambda_L \sqcup_{\mathcal{E}} \lambda_M$ is the smallest NNSS containing both $\lambda_L$ and $\lambda_M$.

Proof. Straight forward.

Definition 3.28 Let $\lambda_L, \lambda_M \in \text{NNS}(\mathbb{X})$ be expressed as $\lambda_L = (\lambda_1, \Omega_1, N)$ and $\lambda_M = (\lambda_2, \Omega_2, N_2)$ where $\Omega_1 = (\zeta_1, \mathcal{L}, N_2)$ and $\Omega_2 = (\zeta_2, \mathcal{M}, N_1)$ are NSSs. Then their restricted intersection is symbolized by $(\lambda_L, \Omega_1, N_2) \cap_{\mathcal{R}} (\lambda_2, \Omega_2, N_1)$ and is defined as $(y, \Omega_1 \cup_{\mathcal{R}} \Omega_2, \min(N_1, N_2))$ where $\Omega_1 \cap_{\mathcal{R}} \Omega_2 = (Y, \mathcal{L} \cap \mathcal{M}, \min(N_1, N_2))$ i.e.

$(\lambda_L, \Omega_1, N_2) \cap_{\mathcal{R}} (\lambda_2, \Omega_2, N_1) = \{(\xi, ((\varphi, T_{\mathcal{E}}(\xi)(\varphi) \land T_{\mathcal{M}}(\xi)(\varphi), I_{\mathcal{L}}(\xi)(\varphi) \lor I_{\mathcal{M}}(\xi)(\varphi), F_{\mathcal{L}}(\xi)(\varphi) \lor F_{\mathcal{M}}(\xi)(\varphi)), r_{\mathcal{E}}(\xi)(\varphi) \land r_{\mathcal{M}}(\xi)(\varphi)) : \varphi \in \mathbb{X}) : \xi \in \mathcal{L} \cap \mathcal{M}\}$

Example 3.29 Consider again $\lambda_L, \lambda_M$ as given in Examples 3.2, 3.4 respectively. The restricted intersection $\lambda_L \cap_{\mathcal{R}} \lambda_M$ is given in Table 14.

<table>
<thead>
<tr>
<th>$\lambda_L \cap_{\mathcal{R}} \lambda_M$</th>
<th>$\xi_1$</th>
<th>$\xi_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varphi_1$</td>
<td>(0.8,0.5,0.1),6</td>
<td>(0.5,0.7,0.3),3</td>
</tr>
<tr>
<td>$\varphi_2$</td>
<td>(0.6,0.2,0.9),4</td>
<td>(0.7,0.4,0.8),5</td>
</tr>
</tbody>
</table>

Table 14: Tabular representation of restricted intersection of two NBSs

Theorem 3.30 Let $\lambda_L, \lambda_M \in \text{NNS}(\mathbb{X})$. Then their restricted-intersection $\lambda_L \cap_{\mathcal{E}} \lambda_M$ is the largest NNSS contained in both $\lambda_L$ and $\lambda_M$.

Proof. Straight forward.

Definition 3.31 Let $\lambda_L, \lambda_M \in \text{NNS}(\mathbb{X})$ be expressed as $\lambda_L = (\lambda_1, \Omega_1, N)$ and $\lambda_M = (\lambda_2, \Omega_2, N_2)$ where $\Omega_1 = (\zeta_1, \mathcal{L}, N_2)$ and $\Omega_2 = (\zeta_2, \mathcal{M}, N_1)$ are NSSs. Then their restricted intersection is symbolized by $(\lambda_L, \Omega_1, N_2) \cap_{\mathcal{E}} (\lambda_2, \Omega_2, N_1)$ and is defined as $(y, \Omega_1 \cup_{\mathcal{E}} \Omega_2, \min(N_1, N_2))$, where $\Omega_1 \cap_{\mathcal{E}} \Omega_2 = (Y, \mathcal{L} \cup \mathcal{M}, \min(N_1, N_2))$ i.e.

$(\lambda_L, \Omega_1, N_2) \cap_{\mathcal{E}} (\lambda_2, \Omega_2, N_1) = \{(\xi, ((\varphi, T_{\mathcal{E}}(\xi)(\varphi) \land T_{\mathcal{M}}(\xi)(\varphi), I_{\mathcal{L}}(\xi)(\varphi) \lor I_{\mathcal{M}}(\xi)(\varphi), F_{\mathcal{L}}(\xi)(\varphi) \lor F_{\mathcal{M}}(\xi)(\varphi)), r_{\mathcal{E}}(\xi)(\varphi) \land r_{\mathcal{M}}(\xi)(\varphi)) : \varphi \in \mathbb{X}) : \xi \in \mathcal{L} \cup \mathcal{M}\}$

Example 3.32 Consider again $\lambda_L, \lambda_M$ as given in Examples 3.2, 3.4 respectively. The extended intersection $\lambda_L \cap_{\mathcal{E}} \lambda_M$ is given in Table 15.

<table>
<thead>
<tr>
<th>$\lambda_L \cap_{\mathcal{E}} \lambda_M$</th>
<th>$\xi_1$</th>
<th>$\xi_2$</th>
<th>$\xi_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varphi_1$</td>
<td>(0.8,0.5,0.1),6</td>
<td>(0.5,0.7,0.3),3</td>
<td>(0.8,0.5,0.1),6</td>
</tr>
<tr>
<td>$\varphi_2$</td>
<td>(0.6,0.2,0.9),4</td>
<td>(0.7,0.4,0.8),5</td>
<td>(0.5,0.7,0.3),3</td>
</tr>
</tbody>
</table>

Table 15: Tabular representation of extended intersection of two NBSs

For any two NNSS $\lambda_L$ and $\lambda_M$ over same set of points $\mathbb{X}$ and using the operations defined above, we conclude the following proposition:

Proposition 3.33 Let $\lambda_L$ and $\lambda_M$ be two NNSS

1. $\lambda_L \sqcup_{\mathcal{E}} \lambda_L = \lambda_L$
2. $\lambda_L \sqcup_{\mathcal{E}} \lambda_M = \lambda_M \sqcup_{\mathcal{E}} \lambda_L$
3. $\lambda_L \cap_{\mathcal{E}} \lambda_L = \lambda_L$
For any three NNSS $\lambda_L$, $\lambda_M$ and $\lambda_N$ over same set of points $X$ and using the operations defined above, we conclude the following proposition:

**Proposition 3.34** Let $\lambda_L$, $\lambda_M$ and $\lambda_N$ be three NNSS

1. $\lambda_L \cup_X (\lambda_M \cup_X \lambda_N) = (\lambda_L \cup_X \lambda_M) \cup_X \lambda_N$
2. $\lambda_L \cap_R (\lambda_M \cap_R \lambda_N) = (\lambda_L \cap_R \lambda_M) \cap_R \lambda_N$
3. $\lambda_L \cup_X (\lambda_M \cap_R \lambda_N) = (\lambda_L \cup_X \lambda_M) \cap_R (\lambda_L \cup_X \lambda_N)$
4. $\lambda_L \cap_R (\lambda_M \cup_X \lambda_N) = (\lambda_L \cap_R \lambda_M) \cup_X (\lambda_L \cap_R \lambda_N)$

**Definition 3.35** Let $\lambda_L, \lambda_M \in \text{NNS}(X)$ be expressed as $\lambda_L = (\lambda_1, \Omega_1, N)$ and $\lambda_M = (\lambda_2, \Omega_2, N_1)$ where $\Omega_1 = (\xi_1, L, N_2)$ and $\Omega_2 = (\xi_2, M, N_1)$ are NSSs. Then AND Operation is symbolized by $(\lambda_1, \Omega_1, N_2) \wedge (\lambda_2, \Omega_2, N_1)$ or shortly $\lambda_L \wedge \lambda_M$ and is defined as $(\lambda_1, \Omega_1, N_2) \wedge (\lambda_2, \Omega_2, N_1) = (\lambda_L, L \times M, \min(N_1, N_2))$, where degree of membership, indeterminacy and non-membership are given as follows:

$$T_{X(\xi_1, \xi_2)}(\phi) = \min\{T_L(\xi_1)(\phi), T_M(\xi_2)(\phi)\},$$

$$I_{X(\xi_1, \xi_2)}(\phi) = \frac{I_L(\xi_1)(\phi) + I_M(\xi_2)(\phi)}{2},$$

$$F_{X(\xi_1, \xi_2)}(\phi) = \max\{F_L(\xi_1)(\phi), F_M(\xi_2)(\phi)\},$$

$$R_{X(\xi_1, \xi_2)}(\phi) = \min\{R_L(\xi_1)(\phi), R_M(\xi_2)(\phi)\},$$

for all $\phi \in X$.

**Definition 3.36** Let $\lambda_L, \lambda_M \in \text{NNS}(\mathbb{X})$ be two NNS be expressed as $\lambda_L = (\lambda_1, \Omega_1, N)$ and $\lambda_M = (\lambda_2, \Omega_2, N_1)$ where $\Omega_1 = (\xi_1, L, N_2)$ and $\Omega_2 = (\xi_2, M, N_1)$ are NSSs. Then OR operation is symbolized by $(\lambda_1, \Omega_1, N_2) \lor (\lambda_2, \Omega_2, N_1)$ or shortly $\lambda_L \lor \lambda_M$ and is defined as $(\lambda_1, \Omega_1, N_2) \lor (\lambda_2, \Omega_2, N_1) = (\lambda_L, L \times M, \min(N_1, N_2))$, where degree of membership, indeterminacy and non-membership are given as follows:

$$T_{X(\xi_1, \xi_2)}(\phi) = \max\{T_L(\xi_1)(\phi), T_M(\xi_2)(\phi)\},$$

$$I_{X(\xi_1, \xi_2)}(\phi) = \frac{I_L(\xi_1)(\phi) + I_M(\xi_2)(\phi)}{2},$$

$$F_{X(\xi_1, \xi_2)}(\phi) = \min\{F_L(\xi_1)(\phi), F_M(\xi_2)(\phi)\},$$

$$R_{X(\xi_1, \xi_2)}(\phi) = \min\{R_L(\xi_1)(\phi), R_M(\xi_2)(\phi)\},$$

for all $\phi \in X$.

**Definition 3.37** The Truth-favorite of an NNSS $\lambda_L$ is denoted by $\lambda_M = \lambda_L$ and is defined by

$$T_L(\xi)(\phi) = \min\{T_L(\xi)(\phi) + I_L(\xi)(\phi) + 1\},$$

$$I_L(\xi)(\phi) = 0,$$

$$F_L(\xi)(\phi) = F_M(\xi)(\phi),$$

$$R_L(\xi)(\phi) = R_M(\xi)(\phi),$$

for all $\phi \in X, \xi \in L$. 

M. Riaz, K. Naeem, I. Zareef and D. Afzal, Neutrosophic N-Soft Sets with TOPSIS method
Definition 3.38 The Falsity-favorite of an NNSS $\lambda_\xi$ is denoted by $\lambda_\xi = \bar{\overline{\nabla}} \lambda_\xi$ and is defined by

$$
\begin{align*}
T_{\xi}(\varphi) &= T_{\mathcal{M}}(\varphi) \\
I_{\xi}(\varphi) &= 0 \\
F_{\xi}(\varphi) &= \min\{F_{\xi}(\varphi) + I_{\xi}(\varphi), 1\} \\
r_{\xi}(\varphi) &= r_{\mathcal{M}}(\varphi)
\end{align*}
$$

for all $\varphi \in \mathcal{X}, \xi \in \mathcal{L}$.

Proposition 3.39 Let $\lambda_\xi$ be an NNSS, then

1. $\overline{\bar{\nabla}} \overline{\nabla} \lambda_\xi = \nabla \lambda_\xi$.
2. $\bar{\overline{\nabla}} \lambda_\xi = \nabla \lambda_\xi$.

Proof. Follows immediately from definitions.

Definition 3.40 Let $\lambda_\xi \in \text{NNS}(\mathcal{X})$. Then scalar multiplication of $\lambda_\xi$ with $\alpha$ is symbolized by $\lambda_\xi \otimes \alpha$ and is defined as

$$
\lambda_\xi \otimes \alpha = \{ (\xi, ((\varphi, T_{\xi}(\varphi) \otimes \alpha, I_{\xi}(\varphi) \otimes \alpha, F_{\xi}(\varphi) \otimes \alpha, r_{\xi}(\varphi) \otimes \alpha) : \varphi \in \mathcal{X}) : \xi \in \mathcal{E} \}
$$

where $T_{\xi}(\varphi) \otimes \alpha$, $I_{\xi}(\varphi) \otimes \alpha$, $F_{\xi}(\varphi) \otimes \alpha$ and $r_{\xi}(\varphi) \otimes \alpha$ are defined by

$$
\begin{align*}
T_{\xi}(\varphi) \otimes \alpha &= \min\{T_{\xi}(\varphi) \times \alpha, 1\} \\
I_{\xi}(\varphi) \otimes \alpha &= \min\{I_{\xi}(\varphi) \times \alpha, 1\} \\
F_{\xi}(\varphi) \otimes \alpha &= \min\{F_{\xi}(\varphi) \times \alpha, 1\} \\
r_{\xi}(\varphi) \otimes \alpha &= (r_{\xi}(\varphi) \times \alpha, \text{if } 0 \leq r_{\xi}(\varphi) \times \alpha < N - 1, N - 1, \text{otherwise})
\end{align*}
$$

4 Relations On Neutrosophic N-Soft Sets

Definition 4.1 Let $\lambda_\xi$ and $\lambda_\mathcal{M}$ be two NNSSs defined over the universe $(\mathcal{X}, \mathcal{L})$ and $(\mathcal{X}, \mathcal{M})$ respectively. Neutrosophic N-soft relation $\overline{\mathcal{R}}$ is defined as $\overline{\mathcal{R}}(\xi_i, \xi_j) = \lambda_\xi(\xi_i) \cap \mathcal{R} \lambda_\mathcal{M}(\xi_j)$, $\forall \xi_i \in \mathcal{L}$ and $\forall \xi_j \in \mathcal{M}$.
\( \mathfrak{R} : \mathcal{N} \to P(\mathcal{X}) \)

is an NNSS over \((\mathcal{X}, \mathcal{N})\), where \(\mathcal{N} \subseteq \mathcal{L} \times \mathcal{M}\).

**Definition 4.2** The composition \(\circ\) of two neutrosophic N-soft relations \(\mathfrak{R}_1\) and \(\mathfrak{R}_2\) is defined by

\[
(\mathfrak{R}_1 \circ \mathfrak{R}_2)(l, n) = \mathfrak{R}_1(l, m) \cap \mathfrak{R}_2(m, n)
\]

where \(\mathfrak{R}_1\) is neutrosophic N-soft relations from \(\lambda_L\) to \(\lambda_M\) over the universe \((\mathcal{X}, \mathcal{L})\) and \((\mathcal{X}, \mathcal{M})\) respectively and \(\mathfrak{R}_2\) is neutrosophic N-soft relations from \(\lambda_M\) to \(\lambda_N\) over the universe \((\mathcal{X}, \mathcal{M})\) and \((\mathcal{X}, \mathcal{N})\) respectively.

**Definition 4.3** Let \(\mathfrak{R}_1\) is neutrosophic N-soft relation over the universe \((\mathcal{X}, \mathcal{L})\) and \(\mathfrak{R}_2\) is neutrosophic N-soft relation over the universe \((\mathcal{X}, \mathcal{M})\). The union and intersection of \(\mathfrak{R}_1\) and \(\mathfrak{R}_2\) defined as below

\[
(\mathfrak{R}_1 \cup \mathfrak{R}_2)(l, m) = \max\{\mathfrak{R}_1(l, m), \mathfrak{R}_2(l, m)\}
\]

\[
(\mathfrak{R}_1 \cap \mathfrak{R}_2)(l, m) = \min\{\mathfrak{R}_1(l, m), \mathfrak{R}_2(l, m)\}
\]

where \(\mathfrak{R}_1 : \mathcal{L} \times \mathcal{M} \to P(\mathcal{X})\) and \(\mathfrak{R}_2 : \mathcal{L} \times \mathcal{M} \to P(\mathcal{X})\).

**Definition 4.4** Let \(\lambda_L\) in \((\mathcal{X}, \mathcal{L})\) be a neutrosophic N-soft set. Let \(\mathfrak{R}\) for \(\lambda_L\) to \(\lambda_M\). Then max-min-max composition of neutrosophic N-soft set with \(\lambda_L\) is another neutrosophic N-soft set \(\lambda_M\) of \((\mathcal{X}, \mathcal{M})\) which is denoted by \(\mathfrak{R} \circ \lambda_L\). The membership function, indeterminate function, non-membership function and grading function of \(\lambda_M\) are defined, respectively, as

\[
T_{\mathfrak{R} \circ \lambda_L}(m) = \max_l\{\min(T_L(l), T_L(l, m))\},
\]

\[
I_{\mathfrak{R} \circ \lambda_L}(m) = \min_l\{\max(I_L(l), I_L(l, m))\},
\]

\[
F_{\mathfrak{R} \circ \lambda_L}(m) = \min_l\{\max(F_L(l), F_L(l, m))\},
\]

\[
r_{\mathfrak{R} \circ \lambda_L}(m) = \max_l\{\min(r_L(l), r_L(l, m))\},
\]

\[\forall l \in \mathcal{L}, m \in \mathcal{M}, r_L \in \mathcal{G} \).

**Definition 4.5** Let \(\lambda_L\) be a neutrosophic N-soft set. Then the choice function of \(\lambda_L\) is defined as

\[
C(\lambda_L) = r_L + T_L - I_L - F_L.
\]

**Definition 4.6** Let \(\lambda_L\) and \(\lambda_M\) be two neutrosophic N-soft sets. Then the score function of \(\lambda_L\) and \(\lambda_M\) is defined as

\[
S_{LM} = C(\lambda_L) - C(\lambda_M).
\]

**Definition 4.7** Let \(\lambda_L\) be a neutrosophic N-soft set. We define score function for \(\lambda_L\) as

\[
S_L = r_i + T_i - I_i - F_i.
\]

5 Application of Neutrosophic N-Soft Set to Medical Diagnosis

In this Section, we discuss the execution of N-soft set and neutrosophic set in medical diagnosis. In some previous studies of the neutrosophic set and neutrosophic soft set, there are many examples of medical diagnosis but all of them have lack of parameterized evaluation characterization. First we propose Algorithm 1 as given below.

**Algorithm 1**

**Step 1:** Input a set \(\mathfrak{B}\) of patients, a set \(\mathfrak{S}\) of symptoms as parameter set and a set \(\mathfrak{D}\) of diseases.

**Step 2:** Construct a relation \(\mathfrak{L}(\mathfrak{B} \leftrightarrow \mathfrak{S})\) between the patients and symptoms.

**Step 3:** Construct a relation a relation \(\mathfrak{R}(\mathfrak{S} \leftrightarrow \mathfrak{D})\) between the symptoms and the diseases.

**Step 4:** Compute the composition relation \(\mathfrak{R}(\mathfrak{B} \leftrightarrow \mathfrak{D})\) the relation of patients and diseases by using Definition 4.4.
Step 5: Obtain the choice function of $\mathcal{R}$ by using Definition 4.5.

Step 6: Choose the highest choice value of patient corresponding to disease gives the higher possibility of the patient affected with the respective disease.

Flow chart portrayal of Algorithm 1 is given in Figure 1:

![Flow chart representation of Algorithm 1](image_url)

Now we demonstrate how neutrosophic N-soft set (NNSS) can be efficiently employed in multi-criteria group decision making (MCGDM). First of all, we propose an extension of TOPSIS to NNSS. In this study, we choose TOPSIS because our goal is to solve a medical diagnosis decision making problem. Since medical diagnosis involves similarities (in symptoms) and TOPSIS method is most appropriate method for handling such problems. A detailed study of TOPSIS may be found in [26]. The procedural steps of Neutrosophic N-soft set TOPSIS Method to examine critical situation of each patient is given in Algorithm 2.

Algorithm 2 (Neutrosophic N-soft set TOPSIS Method)

Step 1: Constructing weighed parameter matrix $\mathcal{H}$ by using ranking values obtained in Step 4 of Algorithm 1 composition relation $\mathcal{R}(\mathfrak{F} \rightarrow \mathfrak{D})$ and relates it with linguistic ratings from Table 26.

$$\mathcal{H} = \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1n} \\ r_{21} & r_{12} & \cdots & r_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ r_{i1} & r_{i2} & \cdots & r_{in} \\ \vdots & \vdots & \ddots & \vdots \\ r_{m1} & r_{m2} & \cdots & r_{mn} \end{bmatrix} = [r_{ij}]_{m \times n}$$

Step 2: Creating normalized decision matrix $\mathcal{B}$. Throughout from now, we shall use $L_n = \{1,2,3,\ldots, n\}$ $\forall n \in N$

$$b_{ij} = \frac{r_{ij}}{\sqrt{\sum_{k=1}^{m} r_{kj}^2}} \quad (1)$$

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Step 3: Creating weighted vector $\mathbf{W} = \{W_1, W_2, W_3, \ldots, W_n\}$ by using the expression

$$W_j = \frac{w_j}{\sum_{k=1}^{m} w_k}, \quad w_k = \frac{1}{m} \sum_{i=1}^{m} b_{ij}$$

(2)

Step 4: Constructing weighted decision matrix $\mu$.

$$\mu = \begin{bmatrix} \mu_{11} & \mu_{12} & \cdots & \mu_{1n} \\ \mu_{21} & \mu_{22} & \cdots & \mu_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \mu_{m1} & \mu_{m2} & \cdots & \mu_{mn} \end{bmatrix} = [\mu_{ij}]_{m \times n}$$

where $\mu_{ij} = W_j b_{ij}$

(3)

Step 5: Finding positive ideal solution (PIS) and negative ideal solution (NIS) by using the Equations

$$PIS = \{\mu_1^+, \mu_2^+, \mu_3^+, \ldots, \mu_j^+, \ldots, \mu_n^+\} = \{\max(\mu_{ij}) : i \in L_n\}$$

(4)

$$NIS = \{\mu_1^-, \mu_2^-, \mu_3^-, \ldots, \mu_j^-, \ldots, \mu_n^-\} = \{\min(\mu_{ij}) : i \in L_n\}$$

(5)

Step 6: Calculate separation measurements of PIS ($S_i^+$) and NIS ($S_i^-$) for each parameter by using the equations

$$S_i^+ = \sqrt{\sum_{j=1}^{n} (\mu_{ij} - \mu_{ij}^+)^2}, \quad \forall i \in L_n$$

(6)

and

$$S_i^- = \sqrt{\sum_{j=1}^{n} (\mu_{ij} - \mu_{ij}^-)^2}, \quad \forall i \in L_n$$

(7)

Step 7: Calculating of relative closeness of alternative to the ideal solution by using the equation

$$C_i^+ = \frac{S_i^-}{S_i^+ + S_i^-}, \quad 0 \leq C_i^+ \leq 1, \quad \forall i \in L_n$$

(8)

Step 8: Ranking the preference order.

Flow chart portrayal of neutrosophic N-soft set TOPSIS method is shown in Figure 2.
5.1 Numerical Example

Now we employ the above Algorithm 1 to find the decision factor about the following top four deadliest diseases in the world. Due to the following risk factors, these diseases progress slowly. Here is some detail about these diseases:

D₁: Coronary artery disease (CAD)
CAD occurs when the vessels that transfer blood towards heart become narrowed. CAD leads to heart failure, arrhythmias and chest pain. Risk factors for CAD are

<table>
<thead>
<tr>
<th>High blood pressure</th>
<th>High cholesterol</th>
<th>Smoking</th>
</tr>
</thead>
<tbody>
<tr>
<td>Family history of CAD</td>
<td>Diabetes</td>
<td>Obesity</td>
</tr>
</tbody>
</table>

Table 16: Risk factors for CAD

D₂: Stroke
This fatal disease occurs when some artery in the brain is blocked or leaks. The risk factors for Stroke are:

<table>
<thead>
<tr>
<th>High blood pressure</th>
<th>Being female</th>
<th>Smoking</th>
</tr>
</thead>
<tbody>
<tr>
<td>Family history of stroke</td>
<td>Being American</td>
<td>Being African</td>
</tr>
</tbody>
</table>

Table 17: Risk factors for Stroke

D₃: Lower respiratory infections (LRI)
This disease occurs due to tuberculosis, pneumonia, influenza, flu, or bronchitis. Risk factors for LRI contain
Poor air quality | Asthma | Smoking  
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Weak immune system</td>
<td>HIV</td>
<td>Crowded child-care settings</td>
</tr>
</tbody>
</table>

Table 18: Risk factors for LRI

**D₄: Chronic obstructive pulmonary disease (COPD)**
This disease is a long-term, progressive lung disease that makes breathing difficult. Risk factors for COPD are

<table>
<thead>
<tr>
<th>Family history</th>
<th>Lungs irritation</th>
</tr>
</thead>
<tbody>
<tr>
<td>History of respiratory infections</td>
<td>Smoking</td>
</tr>
</tbody>
</table>

Table 19: Risk factors for COPD

**D₅: Trachea, bronchus and lungs cancers**
Respiratory cancers incorporate diseases of the bronchus, larynx, lungs and trachea. The risk factors for Trachea, bronchus and lungs cancers involve

<table>
<thead>
<tr>
<th>Use of coal for cooking</th>
<th>Tobacco usage</th>
<th>Poor air quality</th>
</tr>
</thead>
<tbody>
<tr>
<td>Family history of disease</td>
<td>Smoking</td>
<td>Diesel fumes</td>
</tr>
</tbody>
</table>

Table 20: Risk factors for Trachea, bronchus and lungs cancers

Core in certain sense is the most basic part occurring in the considered knowledge. Core can be translated as the arrangement of most trademark some portion of knowledge, which cannot be abstained from when decreasing the data. The core risk factor of all diseases discussed above is “smoking”. For computational purpose, let's decide the grading values depending upon the degree of membership function as in Table 21:

<table>
<thead>
<tr>
<th>Degree of membership function</th>
<th>Grading values</th>
</tr>
</thead>
<tbody>
<tr>
<td>T = 0</td>
<td>0</td>
</tr>
<tr>
<td>0 &lt; T ≤ 0.2</td>
<td>1</td>
</tr>
<tr>
<td>0.2 &lt; T ≤ 0.4</td>
<td>2</td>
</tr>
<tr>
<td>0.4 &lt; T ≤ 0.6</td>
<td>3</td>
</tr>
<tr>
<td>0.6 &lt; T ≤ 0.8</td>
<td>4</td>
</tr>
<tr>
<td>0.8 &lt; T ≤ 1.0</td>
<td>5</td>
</tr>
</tbody>
</table>

Table 21: Ranking scale

Table 22 yields relation between symptoms and patients:
The relation between the symptoms and the diseases is given in Table 23:

<table>
<thead>
<tr>
<th>𝓡</th>
<th>Headache(𝔰₁)</th>
<th>Shortness of breath(𝔰₂)</th>
<th>Angina(𝔰₃)</th>
</tr>
</thead>
<tbody>
<tr>
<td>𝓡₁</td>
<td>((0.7,0.2,0.5),4)</td>
<td>((0.6,0.3,0.4),3)</td>
<td>((0.4,0.6,0.5),2)</td>
</tr>
<tr>
<td>𝓡₂</td>
<td>((0.9,0.3,0.1),5)</td>
<td>((0.7,0.4,0.3),4)</td>
<td>((0.8,0.5,0.2),4)</td>
</tr>
<tr>
<td>𝓡₃</td>
<td>((0.6,0.6,0.4),3)</td>
<td>((0.5,0.5,0.8),3)</td>
<td>((0.2,0.4,0.8),1)</td>
</tr>
<tr>
<td>𝓡₄</td>
<td>((0.2,0.5,0.8),1)</td>
<td>((0.3,0.1,0.7),2)</td>
<td>((0.7,0.1,0.3),4)</td>
</tr>
</tbody>
</table>

Table 22: Relation between symptoms and patients

The composition relation of patients and diseases in Table 24:

<table>
<thead>
<tr>
<th>𝒫</th>
<th>𝒳₁</th>
<th>𝒳₂</th>
<th>𝒳₃</th>
<th>𝒳₄</th>
</tr>
</thead>
<tbody>
<tr>
<td>𝒫₁</td>
<td>(3.8)</td>
<td>4</td>
<td>2.8</td>
<td>3.7</td>
</tr>
<tr>
<td>𝒫₂</td>
<td>4.2</td>
<td>5.5</td>
<td>2.9</td>
<td>4</td>
</tr>
<tr>
<td>𝒫₃</td>
<td>2.6</td>
<td>2.6</td>
<td>2.6</td>
<td>2.8</td>
</tr>
<tr>
<td>𝒫₄</td>
<td>2.5</td>
<td>1.3</td>
<td>1.1</td>
<td>3.9</td>
</tr>
</tbody>
</table>

Table 23: Relation between the symptoms and the diseases

The composition relation of patients and diseases in Table 24:

<table>
<thead>
<tr>
<th>𝒫</th>
<th>𝒳₁</th>
<th>𝒳₂</th>
<th>𝒳₃</th>
<th>𝒳₄</th>
</tr>
</thead>
<tbody>
<tr>
<td>𝒫₁</td>
<td>(0.7,0.4,0.5),4</td>
<td>(0.7,0.2,0.5),4</td>
<td>(0.6,0.3,0.5),3</td>
<td>(0.7,0.5,0.5),4</td>
</tr>
<tr>
<td>𝒫₂</td>
<td>(0.8,0.4,0.2),4</td>
<td>(0.9,0.3,0.1),5</td>
<td>(0.6,0.3,0.4),3</td>
<td>(0.7,0.5,0.2),4</td>
</tr>
<tr>
<td>𝒫₃</td>
<td>(0.6,0.6,0.4),3</td>
<td>(0.6,0.6,0.4),3</td>
<td>(0.6,0.6,0.4),3</td>
<td>(0.6,0.4,0.4),3</td>
</tr>
<tr>
<td>𝒫₄</td>
<td>(0.5,0.5,0.5),3</td>
<td>(0.4,0.5,0.6),2</td>
<td>(0.3,0.5,0.7),2</td>
<td>(0.7,0.5,0.3),4</td>
</tr>
</tbody>
</table>

Table 24: Composition relation of patients and diseases

Table 25 gives choice values of the relation 𝒫:

<table>
<thead>
<tr>
<th>𝒫</th>
<th>𝒳₁</th>
<th>𝒳₂</th>
<th>𝒳₃</th>
<th>𝒳₄</th>
</tr>
</thead>
<tbody>
<tr>
<td>𝒫₁</td>
<td>3.8</td>
<td>4</td>
<td>2.8</td>
<td>3.7</td>
</tr>
<tr>
<td>𝒫₂</td>
<td>4.2</td>
<td>5.5</td>
<td>2.9</td>
<td>4</td>
</tr>
<tr>
<td>𝒫₃</td>
<td>2.6</td>
<td>2.6</td>
<td>2.6</td>
<td>2.8</td>
</tr>
<tr>
<td>𝒫₄</td>
<td>2.5</td>
<td>1.3</td>
<td>1.1</td>
<td>3.9</td>
</tr>
</tbody>
</table>

Table 25: Choice values of relation 𝒫

From Table 25, we conclude that the patients 𝒫₁ and 𝒫₂ are likely to be suffering from 𝒳₂ whereas 𝒫₃ and 𝒫₄ are suffering from 𝒳₄.

In order to examine the intensity level of the disease of the patients, we use neutrosophic N-soft TOPSIS method which is demonstrated in Algorithm 2. First, we decide the grading values as a function of linguistic terms as Table 26:
Now we construct weighted parameter matrix by using Step 9 and Table 26 as

$$\mathcal{H} = \begin{bmatrix} 4 & 4 & 3 \\ 4 & 5 & 3 \\ 3 & 3 & 3 \\ 3 & 2 & 2 \end{bmatrix} = \begin{bmatrix} C & C & G \\ C & VC & G \\ G & G & G \\ G & S & S \end{bmatrix}$$

Creating normalized decision matrix $\mathcal{B}$ by using Equation 1

$$\mathcal{B} = \begin{bmatrix} 0.57 & 0.54 & 0.54 & 0.53 \\ 0.57 & 0.68 & 0.54 & 0.53 \\ 0.43 & 0.41 & 0.54 & 0.40 \\ 0.43 & 0.27 & 0.36 & 0.53 \end{bmatrix}$$

Now by using Equation 2 construct weight vector $\mathcal{W} = \{W_1, W_2, W_3, W_4\} = \{0.58, 0.14, 0.14, 0.14\}$

By using Equation 3 the weighted decision matrix $\mu$ is

$$\mu = \begin{bmatrix} 0.33 & 0.07 & 0.07 & 0.07 \\ 0.33 & 0.09 & 0.07 & 0.07 \\ 0.25 & 0.06 & 0.07 & 0.07 \\ 0.25 & 0.04 & 0.05 & 0.07 \end{bmatrix}$$

The positive ideal solution (PIS) and negative ideal solution (NIS) by using the Equations 4 and 5 as

$$PIS = \{0.33, 0.09, 0.07, 0.07\}$$

$$NIS = \{0.25, 0.04, 0.05, 0.06\}$$

The separation measurements of PIS and NIS for each parameter by using the Equations 6 and 7 are

$$S_1^+ = 0.11$$

$$S_2^+ = 0.06$$

$$S_3^+ = 0.02$$

$$S_4^+ = 0.01$$

$$S_1^- = 0.11$$

$$S_2^- = 0.06$$

$$S_3^- = 0.03$$

$$S_4^- = 0.02$$

The relative closeness of alternatives to the ideal solution by using Equation 8 are

$$C_1^+ = 0.5$$

$$C_2^+ = 0.5$$

<table>
<thead>
<tr>
<th>Linguistic Terms</th>
<th>Grading Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Undetermined (U)</td>
<td>0</td>
</tr>
<tr>
<td>Very Stable (VS)</td>
<td>1</td>
</tr>
<tr>
<td>Stable (S)</td>
<td>2</td>
</tr>
<tr>
<td>Grave (G)</td>
<td>3</td>
</tr>
<tr>
<td>Critical (C)</td>
<td>4</td>
</tr>
<tr>
<td>Very Critical (VC)</td>
<td>5</td>
</tr>
</tbody>
</table>

Table 26: Linguistic terms for evaluation of parameters
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\[ C_3^+ = 0.6 \]
\[ C_4^+ = 0.7 \]

Ranking the preference order is
\[ C_4^+ \geq C_3^+ \geq C_2^+ \geq C_1^+ \]
which indicates that condition of patient \( p_4 \) is most critical. The pictorial representation of the rankings of the patients is demonstrated with the assistance of a chart as given in Figure 3.

Figure 3: Ranking of patients w.r.t. intensity level of disease

5. Conclusion

The purpose of this work is to lay the foundation of theory of neutrosophic N-soft set as a hybrid model of neutrosophic sets and N-soft sets. We established some basic operations on neutrosophic N-soft sets along with their fundamental properties. We introduced the notions of NNS-subset, null-NNS, absolute-NNS, complements of NNS, truth-favorite, falsity-favorite, relations on NNS, composition of NNSS and score function of NNS. We explained these concepts with the help of illustrations. We presented a novel application of multi-attribute decision-making (MADM) based on neutrosophic N-soft set by using Algorithm 1. We proposed neutrosophic N-soft sets TOPSIS method as demonstrated in Algorithm 2 for MADM in medical diagnosis. We defined separation measurements of positive ideal solution and negative ideal solution to compute a relative closeness to identify the optimal alternative. Lastly, a numerical example is given to illustrate the developed method for medical diagnosis.

This may be the starting point for neutrosophic N-soft set mathematical concepts and information structures that are based on neutrosophic set and N-soft set theoretic operations. We have studied a few concepts only, it will be necessary to carry out more theoretical research to recognize a general framework for the practical applications. The proposed model of neutrosophic N-soft set can be elaborated with new research topics such as image processing, expert systems, soft computing techniques, fusion rules, cognitive maps, graph theory and decision-making of real world problems.
We hope that this study will prove a ground-breaking and will open new doors for the vibrant researchers in this field.

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Conflicts of Interest

The authors declare that they have no conflict of interest.

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