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Abstract. In this paper we have introduced the concept of score and accuracy function of the Quadripartitioned Single valued Neutrosophic Numbers (QSVNN) and also defined ranking methods between two QSVNNs which is based on its score function. Dombi operators are used in solving many Multicriteria Attribute Group decision making (MAGDM) problems because of its very good flexibility with a general parameter.Here Dombi T-norm and T-conorm operations of two QSVNNs are defined. Based on this Dombi operations, we introduced two Dombi weighted aggregation operators QSVNDWAA and QSVNDWGA under Quadripartitioned Single valued Neutrosophic environment and also studied its properties. Finally, we discussed about Multicriteria Attribute Decision making method (MADM) using QSVNDWAA or QSVNDWGA operator and also an illustrative example is given for the proposed method which gives a detailed results to select the best alternative based upon the ranking orders.

Keywords: Quadripartitioned single valued neutrosophic sets, Score and Accuracy functions, Dombi Weighted Aggregation Operators .

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1. Introduction

Fuzzy sets which allows the elements to have a degrees of membership in the set and it was introduced by Zadeh [31] in 1965. The degrees of membership lies in the real unit interval $[0, 1]$. Intuitionstic fuzzy set (IFS) allows both membership and non membership to the elements and this was introduced by Atnassov [1] in 1983. By introducing one more component in IFS set neutrosophic set was introduced by Smarandache [19] in 1998. Neutrosophic set has three components truth membership function, indeterminacy membership function and falsity membership function respectively. This neutrosophic set helps to handle the indeterminate and inconsistent information effectively. Later Wang [21] (2010) introduced the concept of Single valued Neutrosophic set (SVNS) which is a generalization of classic set, fuzzy set, interval valued fuzzy set and intuitionstic fuzzy set.

In 1982 Pawlak [17] defined the standard version of rough set theory which is given in terms of a pair of sets that is lower and upper approximation sets. It provided a new approach to vagueness which is defined by a boundary region of a set. Later Yang(2017) [23] defined a new hybrid model of single valued neutrosophic rough set model and it has many applications in medical diagnosis, decision making problems, image processing etc., Neutrosophic set helps to solve many real life world problems [2–6] because of its uncertainty analysis in data sets. K.Mohana, M.Mohanasundari [15] studied On Some Similarity Measures of Single Valued Neutrosophic Rough Sets and applied the concept in Medical Diagnosis problem. When indeterminacy component in neutrosophic set is divided into two parts namely 'Contradiction' (both true and false) 'Unknown' (neither true nor false) we get four components that is T,C,U,F which define a new set called 'Quadripartitioned Single valued neutrosophic set' (QSVNS)introduced by Rajashi Chatterjee., et al. [18] And this is completely based on Belnap's four valued logic and Smarandache's 'Four Numerical valued neutrosophic logic'. By combining the concept of rough set and QSVNS a new hybrid model of 'Quadripartitioned Single valued neutrosophic Rough set' (QSVNRS) was introduced by K.Mohana and M.Mohanasundari. [16]

Many mathematical operations like average, aggregate, sum, count, max, min are performed with the help of aggregation operations.Multicriteria Attribute decision making (MADM) is an approach which is used to select a best one when several alternatives are included under consideration of many attributes. So many researchers [8, 11, 24–27, 29] pay attention to solve the Multicriteria Attribute decision making problems using the concept of various correlation coefficients of the different sets like fuzzy set, IFS, SVNS, QSVNS. And also many researchers [12–14, 20, 22, 28, 30, 33] used aggregation operators as one of the tool to solve a Multicriteria decision making problem and also studied its properties. Dombi Bonferroni mean operators were introduced by Dombi [10] in 1982 which is used in many Multicriteria Attribute Group decision making (MAGDM) problems because of its very good flexibility with a general parameter. J.Chen and J.Ye [7] studied Some Single-valued Neutrosophic Dombi Weighted Aggregation Operators for Multiple Attribute Decision-Making problem.

In this paper Section 2 deals about the basic definitions of Quadripartitioned Single valued neutrosophic sets, Score and accuracy function of single valued neutrosophic number, Dombi T norm and T conorm operations of two single valued neutrosophic numbers(SVNN) and its properties. We have defined Score and accuracy function of quadripartitioned single valued neutrosophic number, Dombi T norm and T conorm operations of two quadripartitioned single valued neutrosophic numbers(QSVNN) in Section 3. Based on the operations of Dombi T norm and T conorm on two QSVNNs we have defined two aggregation operators QSVNDWAA and QSVNDWGA and also studied its properties. Section 4 deals about Multicriteria Attribute Decision making (MADM) method using the above proposed operators QSVNDWAA and QSVNDWGA. Finally an illustrative example is given in the method which we have discussed in Section 4.

2. **Preliminaries**

2.1 Quadripartitioned single valued neutrosophic sets

Definition 2.1. [19]
Neutrosophic set is defined over the non-standard unit interval $]^{-}0,\ 1^{+}[$ whereas single valued neutrosophic set is defined over standard unit interval $[0,1]$.It means a single valued neutrosophic set A is defined by

$$A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle : x \in X\}$$

where $T_A(x), I_A(x), F_A(x) : X \to [0,1]$ such that $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$

Definition 2.2. [18]
Let X be a non-empty set. A quadripartitioned single valued neutrosophic set (QSVNS) A over X characterizes each element in X by a truth-membership function $T_A(x)$, a contradiction membership function $C_A(x)$, an ignorance membership function $U_A(x)$ and a falsity membership function $F_A(x)$ such that for each $x \in X$, $T_A, C_A, U_A, F_A \in [0,1]$ and $0 \leq T_A(x) + C_A(x) + U_A(x) + F_A(x) \leq 4$ when X is discrete, A is represented as $A = \sum_{i=1}^{n} \langle T_A(x_i), C_A(x_i), U_A(x_i), F_A(x_i)\rangle / x_i, x_i \in X$.

Definition 2.3. [18]
The complement of a QSVNS A is denoted by A^C and is defined as,
$A^C = \sum_{i=1}^{n} \langle F_A(x_i), U_A(x_i), C_A(x_i), T_A(x_i)\rangle / x_i, x_i \in X$ i.e., $T_{A^C}(x_i) = F_A(x_i)$,
$C_{A^C}(x_i) = U_A(x_i), U_{A^C}(x_i) = C_A(x_i), F_{A^C}(x_i) = T_A(x_i), x_i \in X$

Definition 2.4. [18]
Consider two QSVNS A and B, over X. A is said to be contained in B, denoted by $A \subseteq B$ iff $T_A(x) \leq T_B(x), C_A(x) \leq C_B(x), U_A(x) \geq U_B(x)$, and $F_A(x) \geq F_B(x)$

Definition 2.5. [18]
The union of two QSVNS A and B is denoted by $A \cup B$ and is defined as,

$A \cup B = \sum_{i=1}^{n} \langle T_A(x_i) \vee T_B(x_i), C_A(x_i) \vee C_B(x_i), U_A(x_i) \wedge U_B(x_i), F_A(x_i) \wedge F_B(x_i) \rangle /$
$x_i, x_i \in X$

Definition 2.6. [18]
The intersection of two QSVNS A and B is denoted by $A \cap B$ and is defined as,

$A \cap B = \sum_{i=1}^{n} \langle T_A(x_i) \wedge T_B(x_i), C_A(x_i) \wedge C_B(x_i), U_A(x_i) \vee U_B(x_i), F_A(x_i) \vee F_B(x_i) \rangle /$
$x_i, x_i \in X$

Definition 2.7. [21]
Let X be a universal set. A $SVNS$ N in X is described by a truth-membership function $t_N(x)$, an indeterminacy-membership function $u_N(x)$, and a falsity-membership function $v_N(x)$. Then a $SVNS$ N can be denoted as the following form:

$$N = \{\langle x, t_N(x), u_N(x), v_N(x)\rangle \, | x \in X\}$$

where the functions $t_N(x), u_N(x), v_N(x) \in [0,1]$ satisfy the condition $0 \leq t_N(x) + u_N(x) + v_N(x) \leq 3$ for $x \in X$. For convenient expression, a basic element $\langle x, t_N(x), u_N(x), v_N(x)\rangle$ in N is denoted by $s = \langle t, u, v, \rangle$ which is called a SVNN. For any SVNN $s = \langle t, u, v, \rangle$, its score and accuracy functions can be introduced, respectively as follows:

$$E(s) = (2 + t - u - v)/3, \quad E(s) \in [0,1],$$
$$H(s) = t - v, \quad H(s) \in [-1,1]$$

According to the two functions $E(s)$ and $H(s)$, the comparison and ranking of two SVNNs are introduced by the following definition.

Definition 2.8. [32] Let $s_1 = \langle t_1, u_1, v_1\rangle$ and $s_2 = \langle t_2, u_2, v_2\rangle$ be two SVNNs. Then the ranking method for s_1 and s_2 is defined as follows:

(1) If $E(s_1) > E(s_2)$ then $s_1 \succ s_2$,

(2) If $E(s_1) = E(s_2)$ and $H(s_1) > H(s_2)$ then $s_1 \succ s_2$,

(3) If $E(s_1) = E(s_2)$ and $H(s_1) = H(s_2)$ then $s_1 = s_2$.

Definition 2.9. [10] Let p and q be any two real numbers. Then, the Dombi T-norm and T-conorm between p and q are defined as follows:

$$O_D(p,q) = \frac{1}{1+\left\{\left(\frac{1-p}{p}\right)^{\rho}+\left(\frac{1-q}{q}\right)^{\rho}\right\}^{1/\rho}},$$
$$O_D^c(p,q) = 1 - \frac{1}{1+\left\{\left(\frac{p}{1-p}\right)^{\rho}+\left(\frac{q}{1-q}\right)^{\rho}\right\}^{1/\rho}},$$

where $\rho \geq 1$ and $(p,q) \in [0,1] \times [0,1]$.
According to the Dombi T-norm and T-conorm, we define the Dombi operations of SVNNs.

Definition 2.10. [7] Let $s_1 = \langle t_1, u_1, v_1 \rangle$ and $s_2 = \langle t_2, u_2, v_2 \rangle$ be two SVNNs, $\rho \geq 1$, and $\lambda > 0$. Then, the Dombi T-norm and T-conorm operations of SVNNs are defined below:

(1) $s_1 \oplus s_2 = \left\langle 1 - \frac{1}{1+\left\{\left(\frac{t_1}{1-t_1}\right)^\rho + \left(\frac{t_2}{1-t_2}\right)^\rho\right\}^{1/\rho}}, \frac{1}{1+\left\{\left(\frac{1-u_1}{u_1}\right)^\rho + \left(\frac{1-u_2}{u_2}\right)^\rho\right\}^{1/\rho}}, \frac{1}{1+\left\{\left(\frac{1-v_1}{v_1}\right)^\rho + \left(\frac{1-v_2}{v_2}\right)^\rho\right\}^{1/\rho}} \right\rangle$

(2) $s_1 \otimes s_2 = \left\langle \frac{1}{1+\left\{\left(\frac{1-t_1}{t_1}\right)^\rho + \left(\frac{1-t_2}{t_2}\right)^\rho\right\}^{1/\rho}}, 1 - \frac{1}{1+\left\{\left(\frac{u_1}{1-u_1}\right)^\rho + \left(\frac{u_2}{1-u_2}\right)^\rho\right\}^{1/\rho}}, 1 - \frac{1}{1+\left\{\left(\frac{v_1}{1-v_1}\right)^\rho + \left(\frac{v_2}{1-v_2}\right)^\rho\right\}^{1/\rho}} \right\rangle$

(3) $\lambda s_1 = \left\langle 1 - \frac{1}{1+\left\{\lambda\left(\frac{t_1}{1-t_1}\right)^\rho\right\}^{1/\rho}}, \frac{1}{1+\left\{\lambda\left(\frac{1-u_1}{u_1}\right)^\rho\right\}^{1/\rho}}, \frac{1}{1+\left\{\lambda\left(\frac{1-v_1}{v_1}\right)^\rho\right\}^{1/\rho}} \right\rangle$

(4) $s_1^\lambda = \left\langle \frac{1}{1+\left\{\lambda\left(\frac{1-t_1}{t_1}\right)^\rho\right\}^{1/\rho}}, 1 - \frac{1}{1+\left\{\lambda\left(\frac{u_1}{1-u_1}\right)^\rho\right\}^{1/\rho}}, 1 - \frac{1}{1+\left\{\lambda\left(\frac{v_1}{1-v_1}\right)^\rho\right\}^{1/\rho}} \right\rangle$

Definition 2.11. [7] Let $s_j = \langle t_j, u_j, v_j \rangle \, (j = 1, 2, ..., n)$ be a collection of SVNNs and $w = (w_1, w_2, ..., w_n)$ be the weight vector for s_j with $w_j \in [0,1]$ and $\sum_{j=1}^n w_j = 1$. Then, the SVNDWAA and SVNDWGA operators are defined respectively as follows:

$$\text{SVNDWAA}\,(s_1, s_2, ..., s_n) = \bigoplus_{j=1}^n w_j s_j$$

$$\text{SVNDWGA}\,(s_1, s_2, ..., s_n) = \bigotimes_{j=1}^n s_j^{w_j}$$

3. Quadripartitioned single valued Neutrosophhic Dombi Operations

Definition 3.1. For an QSVNNs $q = \langle t, c, u, f \rangle$ its score and accuracy functions are defined by,

$$E(q) = (3 + t - c - u - f)/4, \quad E(q) \in [0,1], \tag{1}$$

$$H(q) = t - f, \quad H(q) \in [-1,1] \tag{2}$$

The following definition defined the comparison and ranking of any two QSVNNs based on the two functions E(s) and H(s).

Definition 3.2. Let $q_1 = \langle t_1, c_1, u_1, f_1 \rangle$ and $q_2 = \langle t_2, c_2, u_2, f_2 \rangle$ be two QSVNNs. Then the ranking method for q_1 and q_2 is defined as follows:

(1) If $E(q_1) > E(q_2)$ then $q_1 \succ q_2$,

(2) If $E(q_1) = E(q_2)$ and $H(q_1) > H(q_2)$ then $q_1 \succ q_2$,

(3) If $E(q_1) = E(q_2)$ and $H(q_1) = H(q_2)$ then $q_1 = q_2$.

Definition 3.3. The Dombi T-norm and T-conorm operations of any two QSVNNs $q_1 = \langle t_1, c_1, u_1, f_1\rangle$ and $q_2 = \langle t_2, c_2, u_2, f_2\rangle$ are defined as follows:

(1) $$q_1 \oplus q_2 = \left\langle 1 - \frac{1}{1+\left\{\left(\frac{t_1}{1-t_1}\right)^\rho + \left(\frac{t_2}{1-t_2}\right)^\rho\right\}^{1/\rho}}, 1 - \frac{1}{1+\left\{\left(\frac{c_1}{1-c_1}\right)^\rho + \left(\frac{c_2}{1-c_2}\right)^\rho\right\}^{1/\rho}}, \frac{1}{1+\left\{\left(\frac{1-u_1}{u_1}\right)^\rho + \left(\frac{1-u_2}{u_2}\right)^\rho\right\}^{1/\rho}}, \frac{1}{1+\left\{\left(\frac{1-f_1}{f_1}\right)^\rho + \left(\frac{1-f_2}{f_2}\right)^\rho\right\}^{1/\rho}} \right\rangle$$

(2) $$q_1 \otimes q_2 = \left\langle \frac{1}{1+\left\{\left(\frac{1-t_1}{t_1}\right)^\rho + \left(\frac{1-t_2}{t_2}\right)^\rho\right\}^{1/\rho}}, \frac{1}{1+\left\{\left(\frac{1-c_1}{c_1}\right)^\rho + \left(\frac{1-c_2}{c_2}\right)^\rho\right\}^{1/\rho}}, 1 - \frac{1}{1+\left\{\left(\frac{u_1}{1-u_1}\right)^\rho + \left(\frac{u_2}{1-u_2}\right)^\rho\right\}^{1/\rho}}, 1 - \frac{1}{1+\left\{\left(\frac{f_1}{1-f_1}\right)^\rho + \left(\frac{f_2}{1-f_2}\right)^\rho\right\}^{1/\rho}} \right\rangle$$

(3) $$\lambda q_1 = \left\langle 1 - \frac{1}{1+\left\{\lambda\left(\frac{t_1}{1-t_1}\right)^\rho\right\}^{1/\rho}}, 1 - \frac{1}{1+\left\{\lambda\left(\frac{c_1}{1-c_1}\right)^\rho\right\}^{1/\rho}}, \frac{1}{1+\left\{\lambda\left(\frac{1-u_1}{u_1}\right)^\rho\right\}^{1/\rho}}, \frac{1}{1+\left\{\lambda\left(\frac{1-f_1}{f_1}\right)^\rho\right\}^{1/\rho}} \right\rangle$$

(4) $$q_1^\lambda = \left\langle \frac{1}{1+\left\{\lambda\left(\frac{1-t_1}{t_1}\right)^\rho\right\}^{1/\rho}}, \frac{1}{1+\left\{\lambda\left(\frac{1-c_1}{c_1}\right)^\rho\right\}^{1/\rho}}, 1 - \frac{1}{1+\left\{\lambda\left(\frac{u_1}{1-u_1}\right)^\rho\right\}^{1/\rho}}, 1 - \frac{1}{1+\left\{\lambda\left(\frac{f_1}{1-f_1}\right)^\rho\right\}^{1/\rho}} \right\rangle$$

4. Dombi Weighted Aggregation Operators of QSVNNs

In this section we introduce two Dombi weighted aggregation operators QSVNDWAA and QSVNDWGA which is based on the Dombi operations of QSVNNs in Definition 3.3 and also studied its properties.

Definition 4.1. A collection of QSVNNs is denoted by $q_j = \langle t_j, c_j, u_j, f_j\rangle\,(j = 1, 2, ..., n)$ and $w = (w_1, w_2, ..., w_n)$ be the weight vector for q_j with $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$. Then the QSVNDWAA and QSVNDWGA operators are defined as follows.

$$\text{QSVNDWAA}\ (q_1, q_2, ..., q_n) = \bigoplus_{j=1}^{n} w_j q_j$$

$$\text{QSVNDWGA}\ (q_1, q_2, ..., q_n) = \bigotimes_{j=1}^{n} q_j^{w_j}$$

Theorem 3.1 A collection of QSVNNs is denoted by $q_j = \langle t_j, c_j, u_j, f_j\rangle\,(j = 1, 2, ..., n)$ and $w = (w_1, w_2, ..., w_n)$ be the weight vector for q_j with $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$. Then the aggregated value of the QSVNDWAA operator is still a QSVNN and is calculated by the following formula,

$$QSVNDWAA(q_1, q_2, ..., q_n) = \left\langle 1 - \frac{1}{1 + \left\{ \sum_{j=1}^{n} w_j \left(\frac{t_j}{1-t_j} \right)^{\rho} \right\}^{1/\rho}}, 1 - \frac{1}{1 + \left\{ \sum_{j=1}^{n} w_j \left(\frac{c_j}{1-c_j} \right)^{\rho} \right\}^{1/\rho}}, \frac{1}{1 + \left\{ \sum_{j=1}^{n} w_j \left(\frac{1-u_j}{u_j} \right)^{\rho} \right\}^{1/\rho}}, \frac{1}{1 + \left\{ \sum_{j=1}^{n} w_j \left(\frac{1-f_j}{f_j} \right)^{\rho} \right\}^{1/\rho}} \right\rangle \quad (3)$$

We can prove this theorem using mathematical induction.

Proof: When $n = 2$ by using the Dombi operations of QSVNNs in Definition (3.3) we can have the following result

$$QSVNDWAA(q_1, q_2) = q_1 \oplus q_2$$

$$= \left\langle 1 - \frac{1}{1 + \left\{ w_1 \left(\frac{t_1}{1-t_1} \right)^{\rho} + w_2 \left(\frac{t_2}{1-t_2} \right)^{\rho} \right\}^{1/\rho}}, 1 - \frac{1}{1 + \left\{ w_1 \left(\frac{c_1}{1-c_1} \right)^{\rho} + w_2 \left(\frac{c_2}{1-c_2} \right)^{\rho} \right\}^{1/\rho}}, \frac{1}{1 + \left\{ w_1 \left(\frac{1-u_1}{u_1} \right)^{\rho} + w_2 \left(\frac{1-u_2}{u_2} \right)^{\rho} \right\}^{1/\rho}}, \frac{1}{1 + \left\{ w_1 \left(\frac{1-f_1}{f_1} \right)^{\rho} + w_2 \left(\frac{1-f_2}{f_2} \right)^{\rho} \right\}^{1/\rho}} \right\rangle$$

$$= \left\langle 1 - \frac{1}{1 + \left\{ \sum_{j=1}^{2} w_j \left(\frac{t_j}{1-t_j} \right)^{\rho} \right\}^{1/\rho}}, 1 - \frac{1}{1 + \left\{ \sum_{j=1}^{2} w_j \left(\frac{c_j}{1-c_j} \right)^{\rho} \right\}^{1/\rho}}, \frac{1}{1 + \left\{ \sum_{j=1}^{2} w_j \left(\frac{1-u_j}{u_j} \right)^{\rho} \right\}^{1/\rho}}, \frac{1}{1 + \left\{ \sum_{j=1}^{2} w_j \left(\frac{1-f_j}{f_j} \right)^{\rho} \right\}^{1/\rho}} \right\rangle$$

when $n = k$, Equation (1) becomes,

$$QSVNDWAA(q_1, q_2, ..., q_k) = \left\langle 1 - \frac{1}{1 + \left\{ \sum_{j=1}^{k} w_j \left(\frac{t_j}{1-t_j} \right)^{\rho} \right\}^{1/\rho}}, 1 - \frac{1}{1 + \left\{ \sum_{j=1}^{k} w_j \left(\frac{c_j}{1-c_j} \right)^{\rho} \right\}^{1/\rho}}, \frac{1}{1 + \left\{ \sum_{j=1}^{k} w_j \left(\frac{1-u_j}{u_j} \right)^{\rho} \right\}^{1/\rho}}, \frac{1}{1 + \left\{ \sum_{j=1}^{k} w_j \left(\frac{1-f_j}{f_j} \right)^{\rho} \right\}^{1/\rho}} \right\rangle$$

When $n = k+1$ we have the following result

$$QSVNDWAA(q_1, q_2, ..., q_k, q_{k+1}) = \left\langle 1 - \frac{1}{1+\left\{\sum_{j=1}^{k} w_j \left(\frac{t_j}{1-t_j}\right)^{\rho}\right\}^{1/\rho}}, 1 - \frac{1}{1+\left\{\sum_{j=1}^{k} w_j \left(\frac{c_j}{1-c_j}\right)^{\rho}\right\}^{1/\rho}}, \frac{1}{1+\left\{\sum_{j=1}^{k} w_j \left(\frac{1-u_j}{u_j}\right)^{\rho}\right\}^{1/\rho}}, \frac{1}{1+\left\{\sum_{j=1}^{k} w_j \left(\frac{1-f_j}{f_j}\right)^{\rho}\right\}^{1/\rho}} \right\rangle \oplus w_{k+1} q_{k+1}$$

$$= \left\langle 1 - \frac{1}{1+\left\{\sum_{j=1}^{k+1} w_j \left(\frac{t_j}{1-t_j}\right)^{\rho}\right\}^{1/\rho}}, 1 - \frac{1}{1+\left\{\sum_{j=1}^{k+1} w_j \left(\frac{c_j}{1-c_j}\right)^{\rho}\right\}^{1/\rho}}, \frac{1}{1+\left\{\sum_{j=1}^{k+1} w_j \left(\frac{1-u_j}{u_j}\right)^{\rho}\right\}^{1/\rho}}, \frac{1}{1+\left\{\sum_{j=1}^{k+1} w_j \left(\frac{1-f_j}{f_j}\right)^{\rho}\right\}^{1/\rho}} \right\rangle$$

Hence we proved that Theorem 3.1 is true for $n = k+1$. Thus Equation (1) is true for all n.

The operator QSVNDWAA satisfies the following properties.

(1) Reducibility : If $w = (1/n, 1/n, ..., 1/n)$, then it is obvious that there exits,

$$QSVNDWAA(q_1, q_2, ..., q_n) = \left\langle 1 - \frac{1}{1+\left\{\sum_{j=1}^{n} \frac{1}{n} \left(\frac{t_j}{1-t_j}\right)^{\rho}\right\}^{1/\rho}}, 1 - \frac{1}{1+\left\{\sum_{j=1}^{n} \frac{1}{n} \left(\frac{c_j}{1-c_j}\right)^{\rho}\right\}^{1/\rho}}, \frac{1}{1+\left\{\sum_{j=1}^{n} \frac{1}{n} \left(\frac{1-u_j}{u_j}\right)^{\rho}\right\}^{1/\rho}}, \frac{1}{1+\left\{\sum_{j=1}^{n} \frac{1}{n} \left(\frac{1-f_j}{f_j}\right)^{\rho}\right\}^{1/\rho}} \right\rangle$$

(2) Idempotency : Let all the QSVNNs be denoted by $q_j = \langle t_j, c_j, u_j, f_j \rangle = q (j = 1, 2, ..., n)$. Then QSVNDWAA $(q_1, q_2, ..., q_n) = q$.

(3) Commutativity: Let any QSVNS $(q_1^{'}, q_2^{'}, ..., q_n^{'})$ be any permutation of $(q_1, q_2, ..., q_n)$. Then there is QSVNDWAA $(q_1^{'}, q_2^{'}, ..., q_n^{'})$ = QSVNDWAA $(q_1, q_2, ..., q_n)$.

(4) Boundedness: Let $q_{min} = min(s_1, s_2, ..., s_n)$ and $q_{max} = max(s_1, s_2, ..., s_n)$. Then $q_{min} \leq QSVNDWAA(q_1, q_2, ..., q_n) \leq q_{max}$

Proof: (1) Given $q_j = \langle t_j, c_j, u_j, f_j \rangle = q (j = 1, 2, ..., n)$ Property (1) is trivially true based on equation (3)

(2) The following result is derived from the equation (3) and we get,

$$QSVNDWAA(q_1,q_2,...,q_n)=\left\langle 1-\frac{1}{1+\left\{\sum_{j=1}^{n}w_j\left(\frac{t_j}{1-t_j}\right)^{\rho}\right\}^{1/\rho}},1-\frac{1}{1+\left\{\sum_{j=1}^{n}w_j\left(\frac{c_j}{1-c_j}\right)^{\rho}\right\}^{1/\rho}},\frac{1}{1+\left\{\sum_{j=1}^{n}w_j\left(\frac{1-u_j}{u_j}\right)^{\rho}\right\}^{1/\rho}},\frac{1}{1+\left\{\sum_{j=1}^{n}w_j\left(\frac{1-f_j}{f_j}\right)^{\rho}\right\}^{1/\rho}}\right\rangle$$

$$=\left\langle 1-\frac{1}{1+\left\{\left(\frac{t}{1-t}\right)^{\rho}\right\}^{1/\rho}},1-\frac{1}{1+\left\{\left(\frac{c}{1-c}\right)^{\rho}\right\}^{1/\rho}},\frac{1}{1+\{(\frac{1-u}{u})^{\rho}\}^{1/\rho}},\frac{1}{1+\left\{\left(\frac{1-f}{f}\right)^{\rho}\right\}^{1/\rho}}\right\rangle$$

$$=\left\langle 1-\frac{1}{1+\frac{t}{1-t}},1-\frac{1}{1+\frac{c}{1-c}},1-\frac{1}{1+\frac{1-u}{u}},1-\frac{1}{1+\frac{1-f}{f}}\right\rangle=\langle t,c,u,f\rangle=q$$

QSVNDWAA $(q_1,q_2,...,q_n)=q$ holds.

(3) This property is obvious.

(4) Consider $q_{min}=min(q_1,q_2,...,q_n)=\langle t^-,c^-,u^-,f^-\rangle$ and $q_{max}=max(q_1,q_2,...,q_n)=\langle t^+,c^+,u^+,f^+\rangle$ Then,

$t^-=\min_j(t_j),c^-=\min_j(c_j),u^-=\max_j(u_j),f^-=\max_j(f_j)$

$t^+=\max_j(t_j),c^+=\max_j(c_j),u^+=\min_j(u_j),f^+=\min_j(f_j)$

Therefore we get the following inequalities.

$$1-\frac{1}{1+\left\{\sum_{j=1}^{n}w_j\left(\frac{t^-}{1-t^-}\right)^{\rho}\right\}^{1/\rho}}\leq 1-\frac{1}{1+\left\{\sum_{j=1}^{n}w_j\left(\frac{t_j}{1-t_j}\right)^{\rho}\right\}^{1/\rho}}\leq 1-\frac{1}{1+\left\{\sum_{j=1}^{n}w_j\left(\frac{t^+}{1-t^+}\right)^{\rho}\right\}^{1/\rho}}$$

$$1-\frac{1}{1+\left\{\sum_{j=1}^{n}w_j\left(\frac{c^-}{1-c^-}\right)^{\rho}\right\}^{1/\rho}}\leq 1-\frac{1}{1+\left\{\sum_{j=1}^{n}w_j\left(\frac{c_j}{1-c_j}\right)^{\rho}\right\}^{1/\rho}}\leq 1-\frac{1}{1+\left\{\sum_{j=1}^{n}w_j\left(\frac{c^+}{1-c^+}\right)^{\rho}\right\}^{1/\rho}}$$

$$\frac{1}{1+\left\{\sum_{j=1}^{n}w_j\left(\frac{1-u^+}{u^+}\right)^{\rho}\right\}^{1/\rho}}\leq\frac{1}{1+\left\{\sum_{j=1}^{n}w_j\left(\frac{1-u_j}{u_j}\right)^{\rho}\right\}^{1/\rho}}\leq\frac{1}{1+\left\{\sum_{j=1}^{n}w_j\left(\frac{1-u^-}{u^-}\right)^{\rho}\right\}^{1/\rho}}$$

$$\frac{1}{1+\left\{\sum_{j=1}^{n}w_j\left(\frac{1-f^+}{f^+}\right)^{\rho}\right\}^{1/\rho}}\leq\frac{1}{1+\left\{\sum_{j=1}^{n}w_j\left(\frac{1-f_j}{f_j}\right)^{\rho}\right\}^{1/\rho}}\leq\frac{1}{1+\left\{\sum_{j=1}^{n}w_j\left(\frac{1-f^-}{f^-}\right)^{\rho}\right\}^{1/\rho}}$$

Hence $q_{min}\leq QSVNDWAA(q_1,q_2,...,q_n)\leq q_{max}$ holds.

Theorem 3.2 A collection of QSVNNs is denoted by $q_j=\langle t_j,c_j,u_j,f_j\rangle\,(j=1,2,...,n)$ and $w=(w_1,w_2,...,w_n)$ be the weight vector for q_j with $w_j\in[0,1]$ and $\sum_{j=1}^{n}w_j=1$. Then the aggregated value of the QSVNDWGA operator is still a QSVNN and is calculated by the following formula:

$$QSVNDWGA(q_1, q_2, ..., q_n) = \left\langle \frac{1}{1+\left\{\sum_{j=1}^{n} w_j \left(\frac{1-t_j}{t_j}\right)^{\rho}\right\}^{1/\rho}}, \frac{1}{1+\left\{\sum_{j=1}^{n} w_j \left(\frac{1-c_j}{c_j}\right)^{\rho}\right\}^{1/\rho}}, 1-\frac{1}{1+\left\{\sum_{j=1}^{n} w_j \left(\frac{u_j}{1-u_j}\right)^{\rho}\right\}^{1/\rho}}, 1-\frac{1}{1+\left\{\sum_{j=1}^{n} w_j \left(\frac{f_j}{1-f_j}\right)^{\rho}\right\}^{1/\rho}} \right\rangle \quad (4)$$

The proof is similar to the proof of Theorem (3.1).

This QSVNDWGA operator also satisfies the following properties.

(1) Reducibility : If $w = (1/n, 1/n, ..., 1/n)$, then it is obvious that there exits,

$$QSVNDWGA(q_1, q_2, ..., q_n) = \left\langle \frac{1}{1+\left\{\sum_{j=1}^{n} \frac{1}{n} \left(\frac{1-t_j}{t_j}\right)^{\rho}\right\}^{1/\rho}}, \frac{1}{1+\left\{\sum_{j=1}^{n} \frac{1}{n} \left(\frac{1-c_j}{c_j}\right)^{\rho}\right\}^{1/\rho}}, 1-\frac{1}{1+\left\{\sum_{j=1}^{n} \frac{1}{n} \left(\frac{u_j}{1-u_j}\right)^{\rho}\right\}^{1/\rho}}, 1-\frac{1}{1+\left\{\sum_{j=1}^{n} \frac{1}{n} \left(\frac{f_j}{1-f_j}\right)^{\rho}\right\}^{1/\rho}} \right\rangle$$

(2) Idempotency : Let all the QSVNNs be denoted by $q_j = \langle t_j, c_j, u_j, f_j \rangle = q (j = 1, 2, ..., n)$. Then QSVNDWGA $(q_1, q_2, ..., q_n) = q$.
(3) Commutativity: Let any QSVNS $(q_1^{'}, q_2^{'}, ..., q_n^{'})$ be any permutation of $(q_1, q_2, ..., q_n)$. Then there is QSVNDWGA $(q_1^{'}, q_2^{'}, ..., q_n^{'})$ = QSVNDWGA $(q_1, q_2, ..., q_n)$.
(4) Boundedness: Let $q_{min} = min(q_1, q_2, ..., q_n)$ and $q_{max} = max(q_1, q_2, ..., q_n)$. Then $q_{min} \leq QSVNDWGA(q_1, q_2, ..., q_n) \leq q_{max}$

To prove the above properties it is similar to the operator properties of QSVNDWAA. Hence it is not repeated here.

5. MADM method using QSVNDWAA operator or QSVNDWGA operator

This section deals about the MADM method to handle the MADM problems effectively with QSVNN information by using the QSVNDWAA operator or QSVNDWGA operator.
Let $A = \{A_1, A_2, ..., A_m\}$ and $C = \{C_1, C_2, ..., C_n\}$ be a discrete set of alternatives and attributes respectively. The weight vector of the above attributes is given by $w = \{w_1, w_2, ..., w_n\}$ such that $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$.

To make a better decision to choose the alternative $A_i(i=1,2,...,m)$, a decision maker needs to analyse the attributes $C_j(j=1,2,...,n)$ by the QSVNN $q_{ij}=\langle t_{ij},c_{ij},u_{ij},f_{ij}\rangle\,(i=1,2,...,m;j=1,2,...,n)$ then we get a QSVNN decision matrix $D=(d_{ij})_{m\times n}$

The following decision steps are needed to handle the MADM problems under QSVNN information by using the operator QSVNDWAA or QSVNDWGA.

Step 1 : Collect the QSVNN $q_i(i=1,2,...,m)$ for the given alternative $A_i(i=1,2,...,m)$ by using the operator QSVNDWAA

$$q_i=QSVNDWAA(q_{i1},q_{i2},...,q_{in})$$
$$=\left\langle 1-\frac{1}{1+\left\{\sum_{j=1}^{n}w_j\left(\frac{t_{ij}}{1-t_{ij}}\right)^{\rho}\right\}^{1/\rho}},1-\frac{1}{1+\left\{\sum_{j=1}^{n}w_j\left(\frac{c_{ij}}{1-c_{ij}}\right)^{\rho}\right\}^{1/\rho}},\frac{1}{1+\left\{\sum_{j=1}^{n}w_j\left(\frac{1-u_{ij}}{u_{ij}}\right)^{\rho}\right\}^{1/\rho}},\frac{1}{1+\left\{\sum_{j=1}^{n}w_j\left(\frac{1-f_{ij}}{f_{ij}}\right)^{\rho}\right\}^{1/\rho}}\right\rangle$$

or by using QSVNDWGA operator

$$q_i=QSVNDWGA(q_{i1},q_{i2},...,q_{in})$$
$$=\left\langle \frac{1}{1+\left\{\sum_{j=1}^{n}w_j\left(\frac{1-t_{ij}}{t_{ij}}\right)^{\rho}\right\}^{1/\rho}},\frac{1}{1+\left\{\sum_{j=1}^{n}w_j\left(\frac{1-c_{ij}}{c_{ij}}\right)^{\rho}\right\}^{1/\rho}},1-\frac{1}{1+\left\{\sum_{j=1}^{n}w_j\left(\frac{u_{ij}}{1-u_{ij}}\right)^{\rho}\right\}^{1/\rho}},1-\frac{1}{1+\left\{\sum_{j=1}^{n}w_j\left(\frac{f_{ij}}{1-f_{ij}}\right)^{\rho}\right\}^{1/\rho}}\right\rangle$$

where $w=(w_1,w_2,...,w_n)$ is the weight vector such that $w_j\in[0,1]$ and $\sum_{j=1}^{n}w_j=1$

Step 2: Score values $E(q_i)$ can be calculated by using Equation (1) with the collective QSVNN $q_i(i=1,2,...,m)$

Step 3: Select the best one according to rank given to the alternatives.

6. **Illustrative Example**

This section illustrates an example for a MADM problem about investment alternatives under a QSVNN environment. An investment company chooses three possible alternatives for investing their money by considering the four attributes. Let A_1, A_2, A_3 be three alternatives which represent food, car and computer company respectively. Let C_1, C_2, C_3, C_4 be the four attributes which denotes i) Knowledge (or) Expertise ii) Start up costs iii) Market or Demand iv) Competition respectively. Here the alternatives under given attributes are expressed by the form of QSVNNs. When three alternatives under four attributes are evaluated we get a quadripartitioned single valued neutrosophic decision matrix $D = (q_{ij})_{m\times n}$ where $q_{ij} = \langle t_{ij}, c_{ij}, u_{ij}, f_{ij}\rangle$ $(i = 1,2,3; j = 1,2,3,4)$ which is given below.

$$D = \begin{bmatrix} \langle 0.5,0.6,0.2,0.1\rangle & \langle 0.4,0.2,0.3,0.1\rangle & \langle 0.4,0.2,0.3,0.1\rangle & \langle 0.6,0.7,0.1,0.5\rangle \\ \langle 0.5,0.1,0.8,0.7\rangle & \langle 0.2,0.1,0.8,0.7\rangle & \langle 0.5,0.4,0.7,0.3\rangle & \langle 0.5,0.4,0.7,0.3\rangle \\ \langle 0.1,0.2,0.5,0.7\rangle & \langle 0.1,0.5,0.3,0.4\rangle & \langle 0.3,0.2,0.7,0.8\rangle & \langle 0.9,0.8,0.4,0.1\rangle \end{bmatrix}$$

The weight vector for the above four attributes is given as $w = (0.35, 0.25, 0.25, 0.15)$. Hence the proposed operator of QSVNDWAA (or) QSVNDWGA are used here to solve MADM problem under QSVNN information.

The following steps are needed to solve MADM problem when we use the operator QSVND-WAA. Step 1 : By using Equation(1) for $\rho = 1$ derive the collective QSVNNs of q_i for the alternative $A_i (i = 1,2,3)$ which is given below.

$q_1 = \langle 0.4760, 0.6667, 0.2034, 0.1136\rangle$,

$q_2 = \langle 0.4483, 0.25, 0.7568, 0.4565\rangle$,

$q_3 = \langle 0.6038, 0.5, 0.4414, 0.3404\rangle$

Step 2 : Score values $E(q_i)$ can be calculated by using Equation (1) of the collective QSVNN $q_i (i = 1,2,3)$ for the alternatives $A_i (i = 1,2,3)$ gives the following results.

$E(q_1) = 0.6231, E(q_2) = 0.4962, E(q_3) = 0.5805$

Step 3: The ranking order is given according to the obtained score values

$q_1 > q_3 > q_2$ and the best one is q_1

The same MADM problem can also be solved by using the another proposed operator that is QSVNDWGA. The following steps are needed to solve the MADM problem.

Step 1 : By using Equation (4) for $\rho = 1$ derive the collective QSVNNs of q_i for the alternative $A_i (i = 1,2,3)$ which is given below.

$q_1 = \langle 0.4545, 0.3033, 0.2416, 0.1965\rangle$,

$q_2 = \langle 0.3636, 0.1429, 0.7692, 0.6111\rangle$,

$q_3 = \langle 0.1429, 0.2712, 0.5328, 0.6667\rangle$

Step 2 : Score values $E(q_i)$ can be calculated by using Equation (1) of the collective QSVNN $q_i(i = 1, 2, 3)$ for the alternatives $A_i(i = 1, 2, 3)$ gives the following results.

$E(q_1) = 0.6783, E(q_2) = 0.4601, E(q_3) = 0.4181$

Step 3: The ranking order is given according to the obtained score values
$q_1 > q_2 > q_3$ and the best one is q_1

The following Table 1 and 2 shows the ranking results for the parameters of $\rho \in [1, 10]$ of the quadripartitioned single valued neutrosophic Dombi weighted arithmetic average (QSVND-WAA) operator and quadripartitioned single valued neutrosophic Dombi weighted geometric average (QSVNDWGA) operator respectively.

We can observe the following results from Tables 1 and 2.

1) Different aggregation operators that is QSVNDWAA and QSVNDWGA shows different ranking orders. But the ranking orders due to different operational parameters are same according to the one operator. This results that the operational parameter ρ is not sensitive in this decision making problem since we get the same ranking orders corresponding to the QSVNDWAA and QSVNDWGA operator.

Table 1. Ranking results of the operator QSVNDWAA for different operational parameters.

ρ	$E(q_1), E(q_2), E(q_3)$	**Ranking Order**
1	0.6231,0.4962,0.5805	$q_1 > q_3 > q_2$
2	0.6596,0.5044,0.6323	$q_1 > q_3 > q_2$
3	0.6601,0.5089,0.6468	$q_1 > q_3 > q_2$
4	0.6619,0.5118,0.6535	$q_1 > q_3 > q_2$
5	0.6637,0.5139,0.6577	$q_1 > q_3 > q_2$
6	0.6652,0.5154,0.6605	$q_1 > q_3 > q_2$
7	0.6665,0.5166,0.6626	$q_1 > q_3 > q_2$
8	0.6675,0.5181,0.6641	$q_1 > q_3 > q_2$
9	0.6683,0.5184,0.6654	$q_1 > q_3 > q_2$
10	0.6689,0.5191,0.6664	$q_1 > q_3 > q_2$

TABLE 2. Ranking results of the operator QSVNDWGA for different operational parameters.

ρ	$E(q_1), E(q_2), E(q_3)$	**Ranking Order**
1	0.6783,0.4601,0.4181	$q_1 > q_2 > q_3$
2	0.6619,0.4424,0.3994	$q_1 > q_2 > q_3$
3	0.6475,0.4308,0.3877	$q_1 > q_2 > q_3$
4	0.6380,0.4236,0.3799	$q_1 > q_2 > q_3$
5	0.6315,0.4196,0.3745	$q_1 > q_2 > q_3$
6	0.6269,0.4158,0.3706	$q_1 > q_2 > q_3$
7	0.6236,0.4136,0.3678	$q_1 > q_2 > q_3$
8	0.6204,0.4119,0.3656	$q_1 > q_2 > q_3$
9	0.6185,0.4105,0.3638	$q_1 > q_2 > q_3$
10	0.6167,0.4094,0.3624	$q_1 > q_2 > q_3$

1) The ranking orders according to the operators QSVNDWAA and QSVNDWGA are different
2) Ranking orders are not affected by different operational parameters of $\rho \in [0, 1]$ in both the operators which shows that ρ is not sensitive in this decision making problem.
3) These aggregation methods of the operators QSVNDWAA and QSVNDWGA provides new method to solve MADM problems under an QSVNN environment.

7. Conclusion

In this paper we have studied the Dombi operations of QSVNN based on the Dombi T-norm and T-conorm operations and also we have proposed the two weighted aggregation operators QSVNDWAA , QSVNDWGA and investigate their properties. Multiple Attribute Decision making is one of the effective approach which helps us to the problems involving a selection from a finite number of alternatives are included under finite number of attributes. To solve these type of MADM problems ranking orders are used to select the best one among the given alternatives. This paper also deals about MADM method by using the proposed QSVNDWAA and QSVNDWGA operator under a QSVNN environment. Using these aggregation operators we calculate the score function of the alternatives with respect to the given attributes and this score function helps us to rank the alternatives and choose the best one. Finally we illustrated an example of a MADM problem for the proposed aggregation operators.

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