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Abstract: Within this paper, we present and research the definition of interval valued neutrosophic topological space along with interval valued neutrosophic finer and interval valued neutrosophic coarser topologies. We also describe interval valued neutrosophic interior and closer of an interval valued neutrosophic set. Interval valued neutrosophic subspace topology also studied. Some examples and theorems are presented concerning this concept.

Keywords: Interval valued neutrosophic topology, Interval valued neutrosophic subspace topology

1. Introduction

The notion of fuzzy set has invaded almost all branches of mathematics since its introduction by Zadeh[20]. Fuzzy sets and fuzzy logic has been applied in many real applications to handle uncertaintely fuzzy set theory is very successful in handling uncertainties arising from vagueness or partial belongingness of an element in a set, it cannot model all type of uncertainties pre – veiling in different real physical problems such as problems involving incomplete information. Turksen [18] introducted the idea of interval valued fuzzy sets.

Later, Atanassov[10] introduced the concept generalization of fuzzy set, which is known as intuitionistic fuzzy sets. Intuitionistic fuzzy sets take into account both the degree of membership and non – membership. Further, intuitionistic fuzzy sets were extended to the interval valued intuitionistic fuzzy sets[11]. The interval valued intuitionistic fuzzy set uses a pair of interval $[t^-, t^+], 0 \leq t^- \leq t^+ \leq 1$ and $[f^-, f^+]$, $0 \leq f^- \leq f^+ \leq 1$ with $t^+ + f^+ \leq 1$, to describe the degree of true belief and false belief. Because of the restriction that $t^+ + f^+ \leq 1$, intuitionistic fuzzy sets and interval valued intuitionistic fuzzy sets can only handle incomplete information not the indeterminate information and inconsistent information which exists commonly in belief systems.

As a generalization of fuzzy set and intuitionistic fuzzy set, neutrosophic set have been introduced and developed by F. Smaramdache[15,16 & 17]. It is a logic in which each proposition is calculated to have degree of truth(T), a degree of indeterminacy(I) and a degree of falsity(F). Smarandache's neutrosophic concept have wide range of real applications for many fields of

[1,2,3,4,5,6,7 & 8] information system, computer science, artificial intelligence, applied mathematics, decision making, mechanics, electrical and electronics, medicine and management science etc.

Salama, Albloe[14] proposed the concept of neutrosophic topological space. Later, Wang, Smarandache, Zhang and Sunderraman introduced the notion of interval valued neutrosophic set[19]. An interval valued neutrosophic set A defined on X, $x = x(T, I, F) \in A$ with T, I and F being the subinterval of [0,1]. Lupianez discusses the relation between interval value neutrosophic sets and topology [12]

The purpose of this article is to propose the idea of interval valued neutrosophic topological space and discuss the some of the basic properties.

2. Preliminaries

Definition 2.1[19] Let X be a space of points (objects), with a generic element in X denoted by x. An interval valued neutrosophic set(INS) A in X is characterized by truth – membership function T_A, indeterminacy – membership function I_A and falsity – membership function F_A. For each point x in X, $T_A(x), I_A(x), F_A(x) \subseteq [0,1]$.

Example 2.2[19] Suppose, $X = \{x_1, x_2, x_3\}$. The strength is x_1, the trust is x_2 and the price is x_3. The x_1, x_2 and x_3 values are given in $[0,1]$. They're obtained from some domain experts ' questionnaire, their choice could be degree of goodness, degree of indeterminacy, and degree of poorness. A and B are the interval neutrosophic sets of X define by $A =< \frac{[0.2,0.4],[0.3,0.5],[0.3,0.5]}{x_1}, \frac{[0.5,0.7],[0,0.2],[0.2,0.3]}{x_2}, \frac{[0.6,0.8],[0.2,0.3],[0.2,0.3]}{x_3} >$ $B =< \frac{[0.5,0.7],[0.1,0.3],[0.1,0.3]}{x_1}, \frac{[0.2,0.3],[0.2,0.4],[0.5,0.8]}{x_2}, \frac{[0.4,0.6],[0,0.1],[0.3,0.4]}{x_3} >$

Definition 2.3[19] An interval neutrosophic set A is empty $if\ and\ only\ if$ its $\inf T_A(x) = \sup T_A(x) = 0$, $\inf I_A(x) = \sup I_A(x) = 1$ and $\inf F_A(x) = \sup F_A(x) = 0$, for all x in X.

Definition 2.4(Containment) [19] An interval neutrosophic set A is contained in the other interval neutrosophic set B, $A \subseteq B$, $if\ and\ only\ if$

$$\inf T_A(x) \le \inf T_B(x)\,, \sup T_A(x) \le \sup T_B(x)$$
$$\inf I_A(x) \ge \inf I_B(x)\,, \sup I_A(x) \ge \sup T_B(x)$$
$$\inf F_A(x) \ge \inf F_B(x)\,, \sup F_A(x) \ge \sup F_B(x)$$

for all x in X.

Definition 2.5[19] Two interval neutrosophic sets A and B are equal, written as $A = B, if\ and\ only\ if\ A \subseteq B$ and $B \subseteq A$. Let $0_N =< 0,1,1 >$ and $1_N =< 1,0,0 >$.

Definition 2.6[19] The complement of an interval neutrosophic set A is denoted by $\bar{A}$ and is defined by $T_{\bar{A}}(x) = F_A(x)$; $\inf I_{\bar{A}}(x) = 1 - \sup I_A(x)$;$\sup I_{\bar{A}}(x) = 1 - \inf I_A(x)$; $F_{\bar{A}}(x) = T_A(x)$ for all x in X.

Example 2.7[19] Let A be the interval neutrosophic set defined in Example 2.3, then $\bar{A} =< \frac{[0.3,0.5],[0.5,0.7],[0.3,0.4]}{x_1}, \frac{[0.2,0.3],[0.8,0],[0.5,0.7]}{x_2}, \frac{[0.2,0.3],[0.7,0.8],[0.6,0.8]}{x_3} >$

Definition 2.8 (Intersection) [19] The intersection of two interval neutrosophic sets A and B is an interval neutrosophic set $C = A \cap B$, whose truth-membership, indeterminacy – membership and false – membership are related to those of A and B by

$$\inf T_C(x) = \min(\inf T_A(x), \inf T_B(x)), \quad \sup T_C(x) = \min(\sup T_A(x), \sup T_B(x))$$
$$\inf I_C(x) = \max(\inf I_A(x), \inf I_B(x)), \quad \sup T_C(x) = \max(\sup I_A(x), \sup I_B(x))$$
$$\inf F_C(x) = \max(\inf F_A(x), \inf F_B(x)), \quad \sup T_C(x) = \max(\sup F_A(x), \sup F_B(x))$$

for all x in X.

Example 2.9[19] Let A and B be the interval neutrosophic sets defined in Example 2.3, then $A \cap B = < \frac{[0.2,0.4],[0.3,0.5],[0.3,0.5]}{x_1}, \frac{[0.2,0.3],[0.2,0.4],[0.5,0.8]}{x_2}, \frac{[0.4,0.6],[0.2,0.3],[0.3,0.4]}{x_3} >$.

Theorem 2.10[19] $A \cap B$ is the largest interval neutrosophic set contained in both A and B.

Definition 2.11(Union) [19] The union of two interval neutrosophic sets A and B is an interval neutrosophic set C, written as $C = A \cup B$, whose truth – membership, indeterminacy – membership and false membership are related to those of A and B by

$$\inf T_C(x) = \max(\inf T_A(x), \inf T_B(x)), \quad \sup T_C(x) = \max(\sup T_A(x), \sup T_B(x))$$
$$\inf I_C(x) = \min(\inf I_A(x), \inf I_B(x)), \quad \sup T_C(x) = \min(\sup I_A(x), \sup I_B(x))$$
$$\inf F_C(x) = \min(\inf F_A(x), \inf F_B(x)), \quad \sup T_C(x) = \min(\sup F_A(x), \sup F_B(x))$$

for all x in X.

Example 2.12[19] Let A and B be the interval neutrosophic sets defined in Example 2.3, then $A \cup B =< \frac{[0.5,0.7],[0.1,0.3],[0.1,0.3]}{x_1}, \frac{[0.5,0.7],[0,0.2],[0.2,0.3]}{x_2}, \frac{[0.6,0.8],[0,0.1],[0.2,0.3]}{x_3} >$.

Theorem 2.13[19] $A \cup B$ is the smallest interval neutrosophic set containing both A and B.

3. Interval Valued Neutrosophic Topological Spaces

With some examples and results, we give the concept of interval valued neutrosophic topologi cal spaces.

Definition 3.1 An interval valued neutrosophic topological space of interval valued neutrosophic set (In short IVN topological space) is a pair (X, τ_N) where X is a nonempty set and τ_N is a family of IVN sets on X satisfying the following axioms:

1. $0_N, 1_N \in \tau_N$
2. $A, B \in \tau_N \Rightarrow A \cap B \in \tau_N$
3. $A_i \in \tau_N, i \in I \Rightarrow \cup_{i \in I} A_i \in \tau_N$

τ_N is called an interval valued neutrosophic topology on X. τ_N members are called interval valued neutrosophic open sets (In Short IVN open sets).

Example 3.2 Assume that $X = \{a, b\}$. Here a is denoted by quality of Computers, b is denoted by Price of Computers. The value of a and b are in [0,1]. These are collected from some domain expects questionnaire; their choices could be degree of excellence, degree of indeterminacy, degree of poorness. The IVN set are

$0_N = \langle[0,0],[1,1],[1,1]\rangle$, $1_N = \langle[1,1],[0,0],[0,0]\rangle$, $A = \langle\frac{([0.1,0.4],[0.2,0.7],[0.4,0.6])}{a},\frac{([0.6,0.8],[0.2,0.3],[0.2,0.3])}{b}\rangle$, $B = \langle\frac{([0.1,0.3],[0.3,0.8],[0.5,0.8])}{a},\frac{([0.2,0.7],[0.4,0.8],[0.3,0.7])}{b}\rangle$,. $\tau_N = \{0_N, 1_N, A, B\}$ is called an IVN topology on X. (X, τ_N) is called an $IVNTS$.

Example3.3 Let $X = \{a, b\}$ and the IVN sets are

$C = \langle\frac{([0.4,0.7],[0.5,0.7],[0.4,0.9])}{a},\frac{([0.2,0.3],[0.4,0.5],[0.7,0.9])}{b}\rangle$, $D = \langle\frac{([0.5,0.8],[0.3,0.5],[0.2,0.7])}{a},\frac{([0.5,0.7],[0.1,0.5],[0.3,0.7])}{b}\rangle$.

$\tau_N = \{0_N, 1_N, C, D\}$ is called an IVN topology on X. (X, τ_N) is called an IVN topological space.

Theorem 3.4 Let $\{\tau_{N_i}: i \in I\}$ be a family of IVN topologies of IVN sets on X. Then $\cap_i \{\tau_{N_i}: i \in I\}$ is also an IVN topology of IVN sets on X.

Proof: (i) $0_N, 1_N \in \tau_{N_i}$ for each $i \in I$, Hence $0_N, 1_N \in \bigcap_{i\in I} \tau_{N_i}$.(ii) Let $\{A_i: i \in I\}$ be a arbitrary family of IVN sets where $A_i \in \bigcap_{i\in I} \tau_{N_i}$ for each $i \in I$.Then for each $i \in I$. $A_i \in \tau_{N_i}$ for $i \in I$ and since for each $i \in I$, τ_{N_i} is a IVN topology, Therefore $\bigcup_{i\in I} A_i \in \tau_{N_i}$ for each $i \in I$. Hence $\bigcup_{i\in I} A_i \in \bigcap_{i\in I} \tau_{N_i}$

But union of IVN topologies as seen in the following example need not be an IVN topology.

Example: 3.5 In example 3.2 and 3.3 the families $\tau_{N_1} = \{0_N, 1_N, A, B\}$ and $\tau_{N_2} = \{0_N, 1_N, C, D\}$ are IVN topologies in X. For X, however their union $\tau_{N_1} \cup \tau_{N_2} = \{0_N, 1_N, A, B, C, D\}$ is not a IVN topology.

Definition 3.6 Let (X, τ_N) be an IVN topological space. An IVN set A of X is called an interval valued neutrosophic closed set (in short IVN -closed set) if its complement A^c is an IVN open set in τ_N.

Example 3.7 Let us consider the Example 3.2, the IVN closed sets in (X, τ_N) are $A^c = \langle\frac{([0.4,0.6],[0.3,0.8],[0.1,0.4])}{a},\frac{([0.2,0.3],[0.7,0.8],[0.6,0.8])}{b}\rangle$, $B^c = \langle\frac{([0.5,0.8],[0.2,0.7],[0.1,0.3])}{a},\frac{([0.3,0.7],[0.2,0.6],[0.2,0.7])}{b}\rangle$, $0_N^c = 1_N$ and $1_N^c = 0_N$ are the IVN – closed sets in (X, τ_N).

Theorem 3.8 Let (X, τ_N) be an IVN topological space. Then (i) $0_N, 1_N$ are IVN – closed sets. (ii) Arbitrary intersection of IVN – closed sets is IVN – closed set. (iii) Finite union of IVN – closed sets is IVN – closed set.

Proof: (i) since $0_N, 1_N \in \tau_N$, $0_N^c = 1_N$ and $1_N^c = 0_N$, therefore 0_N^c and 1_N^c are IVN – closed sets. (ii) Let $\{A_i: i \in I\}$be an arbitrary family of IVN – closed sets in (X, τ_N) and let $A = \bigcap_{i\in I} A_i$ Now

$A^c = \left(\bigcap_{i\in I} A_i\right)^c = \bigcup_{i\in I} (A_i)^c$ and $A^c \in \tau_N$ for each $i \in I$, hence $\bigcup_{i\in I} (A_i)^c \in \tau_N$, therefore $A^c \in \tau_N$.

Thus A is an IVN– closed set. (iii) Let $\{A_k: k = 1,2, \dots \dots \dots nn\}$ be a family of IVN – closed set in

(X,τ_N) and let $G=\bigcup_{k=1}^{n}A_k$. Now $(G)^c=\left(\bigcup_{k=1}^{n}A_k\right)^c=\bigcap_{k=1}^{n}A_k^c$ and $(A_k)^c\in\tau_N$ for $k=1,2,.........n$

, so $\bigcap_{k=1}^{n}A_k^c\in\tau_N$. Hence $G^c\in\tau_N$, thus G is IVN – closed set.

Definition 3.9 Let both (X,τ_{N_1}) and (X,τ_{N_2}) be two $IVNTS$. If each $A\in\tau_{N_2}$ implies $A\in\tau_{N_1}$, then $\tau_{N_1}$is called interval valued neutrosophic finer topology than τ_{N_2} and τ_{N_2} is called interval valued neutrosophic coarser topology than τ_{N_1}

Example 3.10 Let $X=\{a,b\}$ and IVN sets are $A=\langle\frac{([0.5,0.7],[0.3,0.6],[0.2,0.8])}{a},\frac{([0.4,0.6],[0.3,0.5],[0.4,0.7])}{b}\rangle$, $B=\langle\frac{([0.3,0.7],[0.4,0.6],[0.3,0.8])}{a},\frac{([0.1,0.7],[0.3,0.8],[0.2,0.6])}{b}\rangle$, $C=\langle\frac{([0.5,0.7],[0.3,0.6],[0.2,0.8])}{a},\frac{([0.4,0.7],[0.3,0.5],[0.2,0.6])}{b}\rangle$, $D=\langle\frac{([0.3,0.7],[0.4,0.6],[0.3,0.8])}{a},\frac{([0.1,0.7],[0.3,0.8],[0.4,0.7])}{b}\rangle$. Let $\tau_{N_1}=\{0_N,1_N,A,B,C,D\}$ and $\tau_{N_2}=\{0_N,1_N,A,C\}$ be an IVN topologies on X and let (X,τ_{N_1}) and (X,τ_{N_2})be a IVN topological spaces. If τ_{N_1} is IVN finer topology than τ_{N_2} andτ_{N_2} is IVN coarser topology than τ_{N_1}

Definition 3.11 Let (X,τ_N) be a IVN topological space. A subcollection $\mathfrak{B}$ of τ_N is said to be base of τ_N if every element of τ_N can be expressed as the arbitray IVN union of some elements of $\mathfrak{B}$, then $\mathfrak{B}$ is called an IVN basis for the IVN topology τ_N.

Example 3.12 In Example 3.10, for the IVN topology $\tau_{N_1}=\{0_N,1_N,A,B,C,D\}$. The sub collection $\mathfrak{B}=\{0_N,1_N,A,B,C\}$ of $P(X)$ is a IVN basis for the IVN topology τ_{N_1}.

Definition 3.13 Let (X,τ_N) be a IVN topological space and $A\in IVNs(X)$, the interior and closure of A is denoted by $IVN\ Int(A)$ and $IVN\ Cl(A)$ are defined as
$IVN\ Int(A)=\cup\{G\in\tau_N: G\subseteq A\}$, $IVN\ Cl(A)=\cap\{K\in\tau_N^c: A\subseteq K\}$

Example 3.14 Let us take an Example 3.3 and consider an IVN set
$E=\langle\frac{([0.4,0.6],[0.4,0.7],[0.2,0.7])}{a},\frac{([0.3,0.5],[0.3,0.6],[0.3,0.5])}{b}\rangle$. Now $IVN\ Int(E)=0_N$ and $IVN\ Cl(E)=1_N$.

Theorem 3.15 Let (X,τ_N) be a IVN topological space and $A,B\in IVNs(X)$ then the following properties holds:

(i) $IVN\ Int(A)\subseteq A$

(ii) $A\subseteq B\Rightarrow IVN\ Int(A)\subseteq IVN\ Int(B)$

(iii) $IVN\ Int(A)\in\tau_N$

(iv) $A\in\tau_N\ iff\ IVN\ Int(A)=A$

(v) $IVN\ Int(IVN\ Int(A))=IVN\ Int(A)$

(vi) $IVN\ Int(0_N)=0_N$, $IVN\ Int(1_N)=1_N$

Proof:

(i) Straight forward.

(ii) $A \subseteq B \Rightarrow$ All of the IVN open sets in A that are also in B. Both IVN open sets included in A also included in B. $ie.,\{K \in \tau_N : K \subseteq A\} \subseteq \{G \in \tau_N : G \subseteq B\}$. $ie.,\cup\{K \in \tau_N : K \subseteq A\} \subseteq \cup \{G \in \tau_N : G \subseteq B\}$. $ie., IVN\ Int(A) \subseteq IVNInt(B)$.

(iii) $IVN\ Int(A) = \cup\{K \in \tau_N : K \subseteq A\}$. It is clear that $\cup\{K \in \tau_N : K \subseteq A\} \in \tau_N$. So, $IVN\ Int(A) \in \tau_N$.

(iv) Let $A \in \tau_N$, then by(i), $IVN\ Int(A) \subseteq A$. Now since $A \in \tau_N$ and $IVN\ Int(A) \subseteq A$. Therefore $A \subseteq \cup\{G \in \tau_N : G \subseteq A\} = IVN\ Int(A)$, $A \subseteq INV\ Int(A)$. Thus $IVN\ Int(A) = A$. Conversely, let $IVN\ Int(A) = A$. Since by (iii), $IVN\ Int(A) \in \tau_N$. Therefore $A \in \tau_N$.

(v) By (iii), $IVN\ Int(A) \in \tau_N$. Therefore by (iv), $IVN\ Int(IVN\ Int(A)) = IVN\ Int(A)$.

(vi) We know that $0_N, 1_N \in \tau_N$, by (iv), $IVN\ Int(0_N) = 0_N$, $IVN\ Int(1_N) = 1_N$.

Theorem 3.16 Let (X, τ_N) be a $IVNTS$ and $A, B \in IVNs(X)$ then possess the following properties:

(i) $A \subseteq IVN\ Cl(A)$

(ii) $A \subseteq B \Rightarrow IVN\ Cl(A) \subseteq IVN\ Cl(B)$

(iii) $(IVN\ Cl(A))^c \in \tau_N$

(iv) $A^c \in \tau_N\ iff\ IVN\ Cl(A) = A$

(v) $IVN\ Cl(IVN\ Cl(A)) = IVN\ Cl(A)$

(vi) $IVN\ Cl(0_N) = 0_N$, $IVN\ Cl(1_N) = 1_N$

Proof:

Straight forward.

Theorem 3.17 Let (X, τ_N) be a IVN topological space and $A, B \in IVNs(X)$then hold the following properties:

(i) $IVN\ Int(A \cap B) = IVN\ Int(A) \cap IVN\ Int(B)$

(ii) $IVN\ Int(A \cup B) \supseteq IVNInt(A) \cup IVNInt(B)$

(iii) $IVN\ Cl(A \cup B) = IVN\ Cl(A) \cup IVNInt(B)$

(iv) $IVN\ Cl(A \cap B) \subseteq IVN\ Cl(A) \cap IVN\ Int(B)$

(v) $(IVN\ Int(A))^c = IVN\ Cl\ (A^c)$

(vi) $(IVN\ Cl(A))^c = IVN\ Int(A^c)$

Proof:

(i) By Theorem 3.15(i), $IVN\ Int\ (A) \subseteq A$ and $IVN\ Int(B) \subseteq B$. Thus $IVN\ Int(A) \cap IVN\ Int(B) \subseteq A \cap B$. Hence $IVN\ Int(A) \cap IVN\ Int(B) \subseteq IVNInt(A \cap B)$ -----------(1)

Again since $A \cap B \subseteq A$, by Theorem 3.15(ii). $IVN\ Int(A \cap B) \subseteq IVNInt(A)$. Similarly $IVN\ Int(A \cap B) \subseteq IVNInt(B)$.

Hence $IVN\ Int(A \cap B) \subseteq IVNInt(A) \cap IVNInt(B)$ --------(2) from (1) and (2) we get, $IVN\ Int(A \cap B) = IVNInt(A) \cap IVN\ Int(B)$.

(ii) Since $A \subseteq A \cup B$. $IVN\ Int(A) \subseteq IVNInt(A \cup B)$ by Theorem 3.15(ii). Similarly $IVN\ Int(B) \subseteq IVNInt(A \cup B)$. Hence $IVN\ Int(A) \cup IVN\ Int(B) \subseteq IVNInt(A \cup B)$.

(iii) By Theorem 3.16(i), $A \subseteq IVN\ Cl(A)$ and $B \subseteq IVN\ Cl(B)$. Thus $A \cup B \subseteq IVN\ Cl(A) \cup IVNCl(B)$, $IVN\ Cl(A \cup B) \subseteq IVN\ Cl(A) \cup IVNCl(B)$-----------(1)

Again since $A \subseteq A \cup B$, by Theorem 3.16(ii). $IVN\ Cl(A) \subseteq IVNCl(A \cup B)$. Similarly $IVN\ Cl(B) \subseteq IVNCl(A \cup B)$. Hence $IVN\ Cl(A) \cup IVN\ Cl(B) \subseteq IVNCl(A \cup B)$------(2) from (1) and (2) we get $IVN\ Cl(A) \cup IVN\ Cl(B) = IVNCl(A \cup B)$.

(iv) Since $A \cap B \subseteq A$, $IVN\ Cl(A \cap B) \subseteq IVN\ Cl(A)$ by Theorem 3.16(ii), Similarly, $IVN\ Cl(A \cap B) \subseteq IVN\ Cl(B)$. Hence $IVN\ Cl(A \cap B) \subseteq IVN\ Cl(A) \cap IVNCl(B)$.

(v) $\{IVN\ Int(A)\}^c = [\cup \{G \in \tau_N : G \subseteq A\}]^c = \cap \{G \in \tau_N^c : A^c \subseteq G\}$,
$\{IVN\ Int(A)\}^c = IVN\ Cl(A)^c$.

(vi) $\{IVN\ Cl(A)\}^c = [\cap \{G \in \tau_N^c : A^c \subseteq G\}]^c = \cup \{G \in \tau_N : G \subseteq A\}$,
$\{IVN\ Cl(A)\}^c = IVN\ Int(A)^c$.

In theorem 3.17((ii) and (iv)), the equality does not hold. Let us display this by an example below

Example 3.18 Let $X = \{a, b\}$ and the IVN sets are $0_N = < \frac{[0,0],[0,0],[1,1]}{a}, \frac{[0,0],[0,0],[1,1]}{b} >$;

$1_N = < \frac{[1,1],[0,0],[0,0]}{a}, \frac{[1,1],[0,0],[0,0]}{b} >$; $A = < \frac{[0.1,0.4],[0.2,0.7],[0.4,0.6]}{a}, \frac{[0.6,0.8],[0.2,0.3],[0.2,0.3]}{b} >$;

$B = < \frac{[0.1,0.3],[0.3,0.8],[0.5,0.8]}{a}, \frac{[0.2,0.7],[0.4,0.8],[0.3,0.7]}{b} >$, $\tau_N = \{0_N, 1_N, A, B\}$ is an IVN topology on X. Let us consider two IVN sets $C = < \frac{[0.1,0.4],[0.3,0.7],[0.5,0.6]}{a}, \frac{[0.4,0.8],[0.2,0.3],[0.2,0.3]}{b} >$ and $D = <$ $\frac{[0,0.3],[0.2,0.8],[0.4,0.9]}{a}, \frac{[0.6,0.7],[0.3,0.6],[0.2,0.5]}{b} >$; Now $C \cup D = < \frac{[0.1,0.4],[0.2,0.7],[0.4,0.6]}{a}, \frac{[0.6,0.8],[0.2,0.3],[0.2,0.3]}{b} > =$ A; $IVN\ Int(C \cup D) = IVN\ Int(A) = A$; $IVN\ Int(C) = 0_N$, $IVN\ Int(D) = 0_N$, $IVN\ Int(C) \cup IVN\ Int(D) = 0_N$;

Therefore $IVN\ Int(C \cup D) \neq IVN\ Int(C) \cup IVN\ Int(D)$.

By Theorem 3.17(v), $IVN\ Cl(C)^c = (IVN\ Int(C))^c = (0_N)^c = 1_N$, $IVN\ Cl(D)^c = (IVN\ Int(D))^c = (0_N)^c = 1_N$, $IVN\ Int(C) \cap IVN\ Int(D) = 1_N$; $IVN\ Cl(C^c \cap D^c) = IVN\ Cl((C \cup D)^c) = (IVN\ Int(C \cup D))^c = (IVN\ Int(A))^c = A^c$; $IVN\ Cl(C^c \cap D^c) \neq IVN\ Cl(C^c) \cup IVN\ Cl(D^c)$.

4. Interval Valued Neutrosophic Subspace Topology

In this section we present, along with some examples and findings, the definition of interval valued neutrosophic subspace topology.

Theorem 4.1 Let (X, τ_N) be a IVN topological space on X and $Y \in P(X)$. Then the collection $\tau_{NY} = \{Y \cap G : G \in \tau_N\}$ is a IVN topology on X.

Proof:

(i) Since $0_N, 1_N \in \tau_N$, therefore $Y \cap 0_N = 0_N \in \tau_{NY}$ and $Y \cap 1_N = Y \in \tau_{NY}$.

(ii) Let $Y_k \in \tau_{NY}, \forall\, k \in I$, then $Y_k = Y \cap G_k$ where $G_k \in \tau_N$ for each $k \in I$. Now $\bigcup_{k \in I} Y_k = \bigcup_{k \in I} (Y \cap G_k) = Y \cap \left(\bigcup_{k \in I} G_k \right) \in \tau_{NY}$. Since $\bigcup_{k \in I} G_k \in \tau_N$ as each $G_k \in \tau_N$.

(iii) Let $Y_1, Y_2 \in \tau_{NY}$, $Y_1 = Y \cap G_1$ and $Y_2 = Y \cap G_2$ where $G_1, G_2 \in \tau_N$ Now $Y_1 \cap Y_2 = (Y \cap G_1) \cap (Y \cap G_2) = Y \cap (G_1 \cap G_2) \in \tau_{NY}$, since $G_1 \cap G_2 \in \tau_N$ as $G_1, G_2 \in \tau_N$.

Definition 4.2 Let (X, τ_N) be an IVN topological space on X and Y is a interval values neutrosophic subset (In short IVN subset) of X, the collection $\tau_{NY} = \{Y \cap G: G \in \tau_N\}$ is called interval valued neutrosophic subspace (In short IVN subspace) of Y. Y is called IVN subspace of X.

Example 4.3 Let us consider the IVN topology $\tau_{N_1} = \{0_N, 1_N, A, B, C, D\}$ as in Example 3.10 and an IVN set $Y =< \frac{[0.4,0.6],[0.3,0.7],[0.1,0.5]}{a}, \frac{[0.5,0.9],[0.4,1],[0.2,0.6]}{b} >$, $0_N = Y \cap 0_N = 0_N$;

$G_1 = Y \cap A$, $G_1 =< \frac{[0.4,0.6],[0.3,0.7],[0.2,0.8]}{a}, \frac{[0.4,0.6],[0.4,1],[0.4,0.7]}{b} >$;

$G_2 = Y \cap B$, $G_2 =< \frac{[0.3,0.6],[0.4,0.7],[0.3,0.8]}{a}, \frac{[0.1,0.7],[0.4,1],[0.2,0.6]}{b} >$;

$G_3 = Y \cap C$, $G_3 =< \frac{[0.4,0.6],[0.3,0.7],[0.2,0.8]}{a}, \frac{[0.4,0.7],[0.4,1],[0.2,0.6]}{b} >$;

$G_4 = Y \cap D$, $G_4 =< \frac{[0.3,0.6],[0.4,0.7],[0.3,0.8]}{a}, \frac{[0.1,0.7],[0.4,1],[0.4,0.7]}{b} >$; Then $\tau_{NY} = \{0_N, 1_N, G_1, G_2, G_3\}$ is an IVN subspace topology for τ_{N_1} and τ_{NY} is called IVN subspace of (X, τ_{N_1}).

Theorem 4.4 Let (X, τ_N) be an IVN topological space, $\mathfrak{B}$ be an IVN basis for τ_N and Y is an IVN subset of X. Then the family $\mathfrak{B}_Y = \{Y \cap G: G \in \mathfrak{B}\}$ is an IVN basis for IVN subspace topology τ_{NY}.

Proof:

Let $U \in \tau_{NY}$ be arbitrary, then there exists an IVN set $G \in \tau_N$ such that $U = Y \cap G$. Since $\mathfrak{B}$ is an IVN basis for τ_N, therefore there exists a sub collection $\{\chi_i: i \in I\}$ of $\mathfrak{B}$ such that $G = \bigcup_{i \in I} \chi_i$. Now, $U = Y \cap G = \bigcup_{i \in I} \chi_i = \bigcup_{i \in I} (Y \cap \chi_i)$. Since $Y \cap \chi_i \in B_Y$, therefore B_Y is a IVN basis for an IVN subspace topology τ_{NY}.

5. Conclusion

The concept of interval valued neutrosophic topological space, interval valued neutrosophic interior and interval valued neutrosophic closure of an interval valued neutrosophic sets were introduced. An interval valued neutrosophic subspace topology of interval valued neutrosophic sets are also introduced. The newly introduced' Interval Valued Neutrophic Topological Spaces' is a stronger version of 'Neutrosophic Topological Spaces'.

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