Generalized Neutrosophic Separation Axioms in Neutrosophic Soft Topological Spaces

A. Mehmood

F. Nadeem

G. Nordo

M. Zamir

C. Park

See next page for additional authors

Generalized Neutrosophic Separation Axioms in Neutrosophic Soft Topological Spaces

Authors

A. Mehmood, F. Nadeem, G. Nordo, M. Zamir, C. Park, H. Kalsoom, S. Jabeen, and M. I. KHAN

NSS

Neutrosophic Sets and Systems, Vol. 32, 2020

University of New Mexico

Generalized Neutrosophic Separation Axioms in Neutrosophic Soft Topological Spaces

A. Mehmood [1,*], F. Nadeem [2], G. Nordo [3], M. Zamir [4], C. Park [5], H. Kalsoom [6], S. Jabeen [7] and M. I. KHAN [8]

[1]Department of Mathematics and Statistics, Riphah International University, Sector I 14, Islamabad, Pakistan; mehdaniyal@gmail.com

[2]Department of Mathematics, University of Science and Technology, Bannu, Khyber Pakhtunkhwa, Pakistan; fawadnadeem2@gmail.com

[3]MIFT – Dipartimento di Scienze Matematiche e Informatiche, scienze Fisiche e scienze della Terra, Messina University, Messina, Italy; giorgio.nordo@unime.it

[4]Department of Mathematics, University of Science and Technology, Bannu, Khyber Pakhtunkhwa, Pakistan; zamirburqi@ustb.edu.pk

[5]Department of Mathematics, Research Institute for Natural Sciences, Hanyang University; baak@hanyang.ac.kr

[6]School of Mathematical Sciences, Zhejiang University, Hangzhou 310027, China; humaira87@zju.edu.cn

[7]School of Mathematics and system Sciences Beihang University Beijing, Beijing, China; shamoona011@buaa.edu.cn

[8]Department of Epidemiology and Biostatistics, Anhui Medical University, China; ddqec@ustb.edu.pk

*Correspondence: mehdaniyal@gmail.com

Abstract. The idea of neutrosophic set was floated by Smarandache by considering a truth membership, an indeterminacy membership and a falsehood or falsity membership functions. The engagement between neutrosophic set and soft set was done by Maji. More over it was used effectively to model uncertainty in different areas of application, such as control, reasoning, pattern recognition and computer vision. The first aim of this paper leaks out the notion of neutrosophic soft p-open set,neutrosophic soft p-closed sets and their important characteristics. Also the notion of neutrosophic soft p-neighborhood and neutrosophic soft p-separation axioms in neutrosophic soft topological spaces are developed. Important results are examed marrying to these newly defined notion relative to soft points. The notion of neutrosophic soft p-separation axioms of neutrosophic soft topological spaces is diffused in different results concerning soft points. Furthermore, properties of neutrosophic soft -P^i-space ($i = 0, 1, 2, 3, 4$) and linkage between them is built up.

Keywords: neutrosophic soft set; neutrosophic soft point; neutrosophic soft p-open set; neutrosophic soft p-neighborhood; neutrosophic soft p-separation axioms.)

1. Introduction

The outdated fuzzy sets is behaviorized by the membership worth or the grade of membership worth. Some times it may be very difficult to assign the membership worth for a fuzzy sets.This gap was bridged with the introduction of interval valued fuzzy sets. In some real life problems in expert system, belief system and so forth,we must take in account the truth-membership and the falsity-membership simultaneously for appropriate narration of an object in uncertain,ambiguous atmosphere. Fuzzy sets and interval valued fuzzy sets are badly failed to handle this situation. The importance of intuitionistic fuzzy sets is automatically come in play in such a hazardous situation.The intuitionistic fuzzy sets can only handle the imperfect information supposing both the truth-membership or association(or simply membership)and falsity-membership(or non-membership)values. It fails to switch the indeterminate and inconsistent information which exists in belief system. Smarandache [14] bounced up conception of neutrosophic set which is a mathematical technique for facing problems involving imprecise, indeterminacy and inconsistent data.The words neutrosophy and neutrosophic were introduced by Smarandache. Neutrosophy (noun) means knowledge of neutral thought, while neutrosophic (adjective), means having the nature of or having the behavior of neutrosophy. This theory is nothing but just generalization of ordinary sets, fuzzy set theory [15], intuitionistic fuzzy set theory [1] etc. Some work have been supposed on neutrosophic sets by some mathematicians in many area of mathematics [4,12]. Many practical problems in economics, engineering, environment,medical science social science etc.can not be treated by conventional methods, because conventional methods have genetic complexities. These complexities may be taking birth due to the insufficiency of the theories of paramertrization tools. Each of these theories has its transmissible difficulties, as was exposed by Molodtsov [11]. Molodtsov developed an absolutely modern approach to cope with uncertainty and vagueness and applied it more and more in different directions such as smoothness of functions, game theory, operations research, Riemann integration, perron integration, and so forth. Meticulously,theory of soft set is free from the parameterization meagerness condition of fuzzy set theory, rough set theory. probability theory for facing with uncertainty Shabir and Naz [13] floated the conception of soft topological spaces, which are defined over an initial universe of discourse with a fixed set of parameters, and showed that a soft topological space produces a parameterized family of topological spaces. Theoretical studies of soft topological spaces were also done by some authors in [2, 3, 6, 8]. Kattak et al. [9] leaked out the notion of some basic result in soft bi topological spaces with respect to soft points.These results supposed the engagement of soft limit point, soft interior point, soft neighborhood, the relation between soft weak structures and soft weak closures. Moreover the authors also addressed soft sequences uniqueness of limit in soft weak-Hausdorff spaces, the product of soft Hausdorff spaces with respect to soft points in different soft weak open set and the marriage between soft Hausdorff space and the diagonal. The combination of Neutrosophic set with soft sets was first introduced by Maji [10]. This combination makes entirely a new mathematical model Neutrosophic Soft Set and later this notion was improved by Deli and Broumi [7]. Work was progressively continue,later on mathematician came in action and defined a new mathematical structure known as neutrosophic soft topological spaces. Neutrosophic soft topological spaces were presented by Bera in [5].

M. Abdel-Basset et al. [16] proposed some novel similarity measures for bipolar and interval-valued bipolar neutrosophic set such as the cosine similarity measures and weighted cosine similarity measures. The propositions of these similarity measures are examined, and two multi-attribute decision making techniques are presented based on proposed measures. For verifying the feasibility of proposed measures, two numerical examples are presented in comparison with the related methods for demonstrating the practicality of the proposed method. Finally, the authors applied the proposed measures of similarity for diagnosing bipolar disorder diseases significantly.

M. Abdel-Basset et al. [17] supposed the objective function of scheduling problem to minimize the costs of daily resource fluctuations using the precedence relationships during the project completion time. The authors designed a resource leveling model based on neutrosophic set to overcome the ambiguity caused by the real-world problems. In this model, trapezoidal neutrosophic numbers are used to estimate the activities durations. The crisp model for activities time is obtained by applying score and accuracy functions. The authors produced a numerical example to illustrate the validation of the proposed model in this study.

Arif et al. [18] introduced the notion of most generalized neutrosophic soft open sets in neutrosophic soft topological structures relative to neutrosophic soft points. The authors leaked out the concept of most generalized separation axioms in neutrosophic soft topological spaces with respect to soft points. Gradually the study is extended to deliberate important results related to these newly defined concepts with respect to soft points. Several related properties, structural characteristics have been investigated. The convergence of sequence in neutrosophic soft topological space is defined and its uniqueness in neutrosophic soft most generalized-Hausdorff space relative to soft points is reflected. The authors further studied and switched over neutrosophic monotonous soft function and its characteristics to multifarious results. The authors lastly addressed neutrosophic soft product spaces under most generalized neutrosophic soft open set with respect to crisp points.

The first aim of this article bounces the notion of neutrosophic soft p-open set,neutrosophic soft p-neighborhood and neutrosophic soft p-separation axioms in neutrosophic soft topology which is defined on neutrosophic soft sets. Later on the important results are discussed related to these newly defined concepts with respect to soft points. Finally, the concept of p-separation axioms of neutrosophic soft topological spaces is diffused in different results with respect to soft points. Furthermore, properties of neutrosophic soft-P^i-space ($i = 0, 1, 2, 3, 4$) and some switch between them are discussed. We hope that these results will best fit for future study on neutrosophic soft topology to carry out a general framework for practical applications.

2. Preliminaries

In this phase we now state certain useful definitions, theorems, and several existing results for neutrosophic soft sets that we require in the next sections.

Definition 2.1. [14] A neutrosophic set A on the universe set X is defined as:
A $= \{\langle x, T^A(x), I^A(x), F^A(x)\rangle : x \in X\}$,
where T, I, $F : X \to]^-0, 1^+[$ and $^-0 \leqq T^A(x) + I^A(x) + F^A(x) \leqq 3^+$.

Definition 2.2. [11] Let X be an initial universe, E be a set of all parameters, and P(x) denote the power set of X. A pair (F, E) is called a soft set over X, where F is a mapping given by $F : E \to P(X)$. In other words, the soft set is a parameterized family of subsets of the set X. For $\lambda \in E$, $F(\lambda)$ may be considered as the set of λ-elements of the soft set (F, E), or as the set of λ-approximate element of the set, i.e.
$(F, E) = \{(\lambda, F(\lambda)) : \lambda \in E, F : E \to P(X)\}$.

After the neutrosophic soft set was defined by Maji [10], this concept was modified by Deli and Broumi [7] as given below:

Definition 2.3. [7] Let X be an initial universe set and E be a set of parameters. Let P(X) denote the set of all neutrosophic sets of X. Then a neutrosophic soft set $(\widetilde{F}, E)$ over X is a set defined by a set valued function $\widetilde{F}$ representing a mapping $\widetilde{F} : E \to P(X)$, where $\widetilde{F}$ is called the approximate function of the neutrosophic soft set $(\widetilde{F}, E)$. In other words, the neutrosophic soft set is a parameterized family of some elements of the set P(X) and therefore it can be written as a set of ordered pairs:
$(\widetilde{F}, E) = \{(\lambda, \langle x, T^{\widetilde{F}(\lambda)}(x), I^{\widetilde{F}(\lambda)}(x), F^{\widetilde{F}(\lambda)}(x)\rangle : x \in X) : \lambda \in E\}$,
where $T^{\widetilde{F}(\lambda)}(x)$, $I^{\widetilde{F}(\lambda)}(x)$, $F^{\widetilde{F}(\lambda)}(x) \in [0, 1]$ are respectively called the truth-membership, indeterminacy-membership, and falsity-membership function of $\widetilde{F}(\lambda)$. Since the supremum of each T, I, F is 1, the inequality $0 \leqq T^{\widetilde{F}(\lambda)}(x) + I^{\widetilde{F}(\lambda)}(x) + F^{\widetilde{F}(\lambda)}(x) \leqq 3$ is obvious.

Definition 2.4. [5] Let $(\widetilde{F}, E)$ be a neutrosophic soft set over the universe set X. The complement of $(\widetilde{F}, E)$ is denoted by $(\widetilde{F}, E)^c$ and is defined by:
$(\widetilde{F}, E)^c = \{(\lambda, \langle x, F^{\widetilde{F}(\lambda)}(x), 1 - I^{\widetilde{F}(\lambda)}(x), T^{\widetilde{F}(\lambda)}(x)\rangle : x \in X) : \lambda \in E\}$.
It is obvious that $((\widetilde{F}, E)^c)^c = (\widetilde{F}, E)$.

Definition 2.5. [10] Let $(\widetilde{F}, E)$ and $(\widetilde{G}, E)$ be two neutrosophic soft sets over the universe set X. $(\widetilde{F}, E)$ is said to be a neutrosophic soft subset of $(\widetilde{G}, E)$ if $T^{\widetilde{F}(\lambda)}(x) \leqq T^{\widetilde{G}(\lambda)}(x)$, $I^{\widetilde{F}(\lambda)}(x)$

$\leqq I^{\widetilde{G}(\lambda)}(x)$, $F^{\widetilde{F}(\lambda)}(x) \geqq F^{\widetilde{G}(\lambda)}(x)$,$\forall\lambda \in E$, $\forall x \in X$. It is denoted by $(\widetilde{F}, E) \subseteqq (\widetilde{G}, E)$. $(\widetilde{F}, E)$ is said to be neutrosophic soft equal to $(\widetilde{G}, E)$ if $(\widetilde{F}, E)$ is a neutrosophic soft subset of $(\widetilde{G}, E)$ and $(\widetilde{G}, E)$ is a neutrosophic soft subset of $(\widetilde{F}, E)$. It is denoted by $(\widetilde{F}, E) = (\widetilde{G}, E)$.

3. Applications of Neutrosophic Soft Point and its Characteristics

Definition 3.1. Let $(\widetilde{F}^1, E)$ and $(\widetilde{F}^2, E)$ be two neutrosophic soft sets over universe set X. Then their union is denoted by $(\widetilde{F}^1, E) \sqcup (\widetilde{F}^2, E) = (\widetilde{F}^3, E)$ and is defined by:

$(\widetilde{F}^3, E) = \{(\lambda, \langle x, T^{\widetilde{F}^3(\lambda)}(x), I^{\widetilde{F}^3(\lambda)}(x), F^{\widetilde{F}^3(\lambda)}(x)\rangle : x \in X) : \lambda \in E\}$,

where $T^{\widetilde{F}^3(\lambda)}(x) = \max\ \{T^{\widetilde{F}^1(\lambda)}(x), T^{\widetilde{F}^2(\lambda)}(x)\}$,

$I^{\widetilde{F}^3(\lambda)}(x) = \max\ \{I^{\widetilde{F}^1(\lambda)}(x), I^{\widetilde{F}^2(\lambda)}(x)\}$,

$F^{\widetilde{F}^3(\lambda)}(x) = \max\ \{F^{\widetilde{F}^1(\lambda)}(x), F^{\widetilde{F}^2(\lambda)}(x)\}$.

Definition 3.2. Let $(\widetilde{F}^1, E)$ and $(\widetilde{F}^2, E)$ be two neutrosophic soft sets over the universe set X. Then their intersection is denoted by $(\widetilde{F}^1, E) \sqcap (\widetilde{F}^2, E) = (\widetilde{F}^3, E)$ and is defined by:

where

$T^(\widetilde{F}^3, E) = \min\ \{T^{\widetilde{F}^1(\lambda)}(x), T^{\widetilde{F}^2(\lambda)}(x)\}$

$I^{\widetilde{F}^3(\lambda)}(x) = \max\ \{I^{\widetilde{F}^1(\lambda)}(x), I^{\widetilde{F}^2(\lambda)}(x)\}$,

$F^{\widetilde{F}^3(\lambda)}(x) = \max\ \{F^{\widetilde{F}^1(\lambda)}(x), F^{\widetilde{F}^2(\lambda)}(x)\}$.

Definition 3.3. A neutrosophic soft set $(\widetilde{F}, E)$ over the universe set X is said to be a null neutrosophic soft set if $T^{\widetilde{F}(\lambda)}(x) = 0$, $I^{\widetilde{F}(\lambda)}(x) = 0$, $F^{\widetilde{F}(\lambda)}(x) = 1$; $\forall\lambda \in E$, $\forall x \in X$. It is denoted by $0^{(X,E)}$.

Definition 3.4. A neutrosophic soft set $(\widetilde{F}, E)$ over the universe set X is said to be an absolute neutrosophic soft set if $T^{\widetilde{F}(\lambda)}(x) = 1$, $I^{\widetilde{F}(\lambda)}(x) = 1$, $F^{\widetilde{F}(\lambda)}(x) = 0$; $\forall\lambda \in E$, $\forall x \in X$. It is denoted by $1^{(X,E)}$.

Clearly, $0^{c}(X, E) = 1^{(X,E)}$ and $1^{c}(X, E) = 0^{(X,E)}$.

Definition 3.5. Let NSS(X, E) be the family of all neutrosophic soft sets over the universe set X and $\Im \sqsubset$ NSS(X, E). Then $\Im$ is said to be a neutrosophic soft topology on X if:

1. $0^{(X,E)}$ and $1^{(X,E)}$ belong to $\Im$,
2. the union of any number of neutrosophic soft sets in $\Im$ belongs to $\Im$,
3. the intersection of a finite number of neutrosophic soft sets in $\Im$ belongs to $\Im$.

Then $(X, \Im, E)$ is said to be a neutrosophic soft topological space over X. Each member of $\Im$ is said to be a neutrosophic soft open set.

Definition 3.6. Let $(X, \Im, E)$ be a neutrosophic soft topological space over X and $(\widetilde{F}, E)$ be a subset of neutrosophic soft topological space over X. Then $(\widetilde{F}, E)$ is said to be a neutrosophic soft closed set iff its complement is a neutrosophic soft open set.

Definition 3.7. Let $(X, \Im, E)$ be a neutrosophic soft topological space over X and $(\widetilde{F}, E)$ be a subset of neutrosophic soft topological space over X. Then $(\widetilde{F}, E)$ is said to be a neutrosophic soft p-open (NSPO) set if $(\widetilde{F}, E) \subseteq NSint(NScl((\widetilde{F}, E)))$

Definition 3.8. Let $(X, \Im, E)$ be a neutrosophic soft topological space over X and $(\widetilde{F}, E)$ be a subset of neutrosophic soft topological space over X. Then $(\widetilde{F}, E)$ is said to be a neutrosophic soft p-closed (NSPC) set if $(\widetilde{F}, E) \supseteq NScl(NSint((\widetilde{F}, E)))$

Definition 3.9. Let NS be the family of all neutrosophic sets over the universe set X and x $\in$ X. The neutrosophic set $x^{(\alpha,\beta,\gamma)}$ is called a neutrosophic point, for $0 < \alpha, \beta, \gamma \leqq 1$, and is defined as follow:

$$x^{(\alpha,\beta,\gamma)}(y) = \begin{cases} (\alpha, \beta, \gamma), \ if \ y = x \\ (0, 0, 1), \ if \ y \neq x. \end{cases} \tag{1}$$

It is clear that every neutrosophic set is the union of its neutrosophic points.

Definition 3.10. Suppose that X $= \{x^1, x^2\}$. Then neutrosophic set

A $= \{\langle x^1, 0.1, 0.3, 0.5\rangle, \langle x^2, 0.5, 0.4, 0.7\rangle\}$

is the union of neutrosophic points $x^1(0.1, 0.3, 0.5) and\ x^2(0.5, 0.4, 0.7)$.

Now we define the concept of neutrosophic soft points for neutrosophic soft sets.

Definition 3.11. Let NSS(X, E) be the family of all neutrosophic soft sets over the universe set X. Then neutrosophic soft set $x^\lambda(\alpha, \beta, \gamma)$ is called a neutrosophic soft point, for every x $\in$ X, $0 < \alpha, \beta, \gamma \leqq 1$, $\lambda \in$ E, and is defined as follows:

$$x^{\lambda(\alpha,\beta,\gamma)}(\lambda')(y) = \begin{cases} (\alpha, \beta, \gamma) \ if \ \lambda' = \lambda \ and \ y = x \\ (0, 0, 1), \ if \ \lambda' \neq \lambda \ or \ y \neq x. \end{cases} \tag{2}$$

Definition 3.12. Suppose that the universe set X is given by X $= \{x^1, x^2\}$ and the set of parameters by E $= \{\lambda^1, \lambda^2\}$. Let us consider neutrosophic soft sets $(\widetilde{F}, E)$ over the universe X as follows:

$$(\widetilde{F}, E) = \left\{ \begin{array}{c} \lambda^1 = \{\langle x^1, 0.3, 0.7, 0.6\rangle, \langle x^2, 0.4, 0.3, 0.8\rangle\} \\ \lambda^2 = \{\langle x^1, 0.4, 0.6, 0.8\rangle, \langle x^2, 0.3, 0.7, 0.2\rangle\}. \end{array} \right\} \tag{3}$$

It is clear that$(\widetilde{F}, E)$ is union of its neutrosophic soft point $x^1\lambda^{1(0.3,0.7,0.6)}$,$x^1\lambda^{2(0.4,0.6,0.8)}$,$x^2\lambda^1$, and $x^2\lambda^{2(0.3,0.7,0.6)}$. Here

$$x^1\lambda^{1(0.3,0.7,0.6)} = \left\{ \begin{array}{c} \lambda^1 = \{\langle x^1, 0.3, 0.7, 0.6\rangle, \langle x^2, 0, 0, 1\rangle\} \\ \lambda^2 = \{\langle x^1, 0, 0, 1\rangle, \langle x^2, 0, 0, 1\rangle\}. \end{array} \right\} \tag{4}$$

$$x^1\lambda^{2(0.4,0.6,0.8)} = \left\{ \begin{array}{c} \lambda^1 = \{\langle x^1, 0.3, 0.7, 0.6\rangle, \langle x^2, 0, 0, 1\rangle\} \\ \lambda^2 = \{\langle x^1, 0.4, 0.6, 0.8\rangle, \langle x^2, 0, 0, 1\rangle\}. \end{array} \right\}. \tag{5}$$

$$x^2\lambda^{1(0.4,0.3,0.8)} = \left\{ \begin{array}{c} \lambda^1 = \{\langle x^1, 0, 0, 1\rangle, \langle x^2, 0.4, 0.3, 0.8\rangle\} \\ \lambda^2 = \{\langle x^1, 0, 0, 1\rangle, \langle x^2, 0, 0, 1\rangle\}. \end{array} \right\}. \tag{6}$$

$$x^2\lambda^{2(0.3,0.7,0.2)} = \left\{ \begin{array}{c} \lambda^1 = \{\langle x^1, 0, 0, 1\rangle, \langle x^2, 0, 0, 1\rangle\} \\ \lambda^2 = \{\langle x^1, 0, 0, 1\rangle, \langle x^1, 0.3, 0.7, 0.2\rangle\}. \end{array} \right\}. \tag{7}$$

Definition 3.13. Let $(\widetilde{F}, E)$ be a neutrosophic soft set over the universe set X. We say that $x^{\lambda^{(\alpha,\beta,\gamma)}} \in (\widetilde{F}, E)$ read as belonging to the neutrosophic soft set $(\widetilde{F}, E)$ whenever $\alpha \leqq T^{\widetilde{F}(\lambda)}(x)$,$\beta \leqq I^{\widetilde{F}(\lambda)}(x)$ and $\gamma \geqq F^{\widetilde{F}(\lambda)}(x)$.

Definition 3.14. Let $(X, \Im, E)$ be a neutrosophic soft topological space over X. A neutrosophic soft set $(\widetilde{F}, E)$ in $(X, \Im, E)$ is called a neutrosophic soft p-neighborhood of the neutrosophic soft point $x^{\lambda^{(\alpha,\beta,\gamma)}} \in (\widetilde{F}, E)$, if there exists a neutrosophic soft p-open set $(\widetilde{G}, E)$ such that $x^{\lambda^{(\alpha,\beta,\gamma)}} \in (\widetilde{G}, E) \sqsubset (\widetilde{F}, E)$.

Theorem 3.15. *Let $(X, \Im, E)$ be a neutrosophic soft topological space and $(\widetilde{F}, E)$ be a neutrosophic soft set over X. Then $(\widetilde{F}, E)$ is a neutrosophic soft p-open set if and only if $(\widetilde{F}, E)$ is a neutrosophic soft p-neighborhood of its neutrosophic soft points.*

Proof. Let $(\widetilde{F}, E)$ be a neutrosophic soft p-open set and $x^{\lambda^{(\alpha,\beta,\gamma)}} \in (\widetilde{F}, E)$. Then $x^{\lambda^{(\alpha,\beta,\gamma)}} \in (\widetilde{F}, E) \sqsubset (\widetilde{F}, E)$.

Therefore, $(\widetilde{F}, E)$ is a neutrosophic soft p-neighborhood of $x^{\lambda^{(\alpha,\beta,\gamma)}}$.

Conversely, let $(\widetilde{F}, E)$ be a neutrosophic soft p-neighborhood of its neutrosophic soft points. Let $x^{\lambda^{(\alpha,\beta,\gamma)}} \in (\widetilde{F}, E)$. Since $(\widetilde{F}, E)$ is a neutrosophic soft p-neighborhood of the neutrosophic soft point $x^{\lambda^{(\alpha,\beta,\gamma)}}$, there exists $(\widetilde{G}, E) \in \Im$ such that $x^{\lambda^{(\alpha,\beta,\gamma)}} \in (\widetilde{G}, E) \sqsubset (\widetilde{F}, E)$. Since $(\widetilde{F}, E)$ $= \sqcup \{x^{\lambda^{(\alpha,\beta,\gamma)}} : x^{\lambda^{(\alpha,\beta,\gamma)}} \in (\widetilde{F}, E)\}$, it follows that $(\widetilde{F}, E)$ is a union of neutrosophic soft p-open sets and hence $(\widetilde{F}, E)$ is a neutrosophic soft p-open set.

The p-neighborhood system of a neutrosophic soft point $x^{\lambda^{(\alpha,\beta,\gamma)}}$, denoted by U $(x^{\lambda^{(\alpha,\beta,\gamma)}}, E)$, is the family of all its p-neighborhoods. □

Theorem 3.16. *The neighborhood system U $(x^{\lambda^{(\alpha,\beta,\gamma)}}, E)$ at $x^{\lambda^{(\alpha,\beta,\gamma)}}$ in a neutrosophic soft topological space $(X, \Im, E)$ has the following properties.*

1) If $(\widetilde{F}, E) \in U\ (x^{\lambda^{(\alpha,\beta,\gamma)}}, E)$, then $x^{\lambda^{(\alpha,\beta,\gamma)}} \in (\widetilde{F}, E)$;

2) If $(\widetilde{F}, E) \in U\ (x^{\lambda^{(\alpha,\beta,\gamma)}}, E)$ and $(\widetilde{F}, E) \sqsubset (\widetilde{H}, E)$, then $(\widetilde{H}, E)$, then $(\widetilde{H}, E) \in U\ (x^{\lambda^{(\alpha,\beta,\gamma)}}, E)$;

3) If $(\widetilde{F}, E)$, If $(\widetilde{G}, E) \in U\ (x^{\lambda^{(\alpha,\beta,\gamma)}}, E)$, then $(\widetilde{F}, E) \sqcap (\widetilde{G}, E) \in U\ (x^{\lambda^{(\alpha,\beta,\gamma)}}, E)$;

4) If $(\widetilde{F}, E) \in U\ (x^{\lambda^{(\alpha,\beta,\gamma)}}, E)$, then there exists a $(\widetilde{G}, E) \in U\ (x^{\lambda^{(\alpha,\beta,\gamma)}}, E)$ such that $(\widetilde{G}, E) \in U\ (y^{\lambda'^{(\alpha',\beta',\gamma')}}, E)$, for each $y^{\lambda'^{(\alpha',\beta',\gamma')}} \in (\widetilde{G}, E)$.

Proof. The proof of 1), 2), and 3) is obvious from definition 3.12.

4) If $(\widetilde{F}, E) \in$ U $(x^{\lambda(\alpha,\beta,\gamma)}, E)$, then there exists a neutrosophic soft p-open set $(\widetilde{G}, E)$ such that $x^{\lambda(\alpha,\beta,\gamma)} \in (\widetilde{G}, E) \sqsubset (\widetilde{F}, E)$. From Proposition 3.1, $(\widetilde{G}, E) \in$ U $(x^{\lambda(\alpha,\beta,\gamma)}, E)$, so for each $y^{\lambda'(\alpha',\beta',\gamma')} \in (\widetilde{G}, E)$, $(\widetilde{G}, E) \in$ U $(y^{\lambda'(\alpha',\beta',\gamma')}, E)$ is obtained. □

Definition 3.17. Let $x^{\lambda(\alpha,\beta,\gamma)}$ and $y^{\lambda'(\alpha',\beta',\gamma')}$ be two neutrosophic soft points. For the neutrosophic soft points $x^{\lambda(\alpha,\beta,\gamma)}$ and $y^{\lambda'(\alpha',\beta',\gamma')}$ over a common universe X, we say that neutrosophic soft points are distinct points if $x^{\lambda(\alpha,\beta,\gamma)} \sqcap y^{\lambda'(\alpha',\beta',\gamma')} = 0^{(X,E)}$.

It is clear that $x^{\lambda(\alpha,\beta,\gamma)}$ and $y^{\lambda'(\alpha',\beta',\gamma')}$ are distinct neutrosophic soft points if and only if x $\neq$ y or $\lambda' \neq \lambda$.

4. **Neutrosophic Soft p-Separation Structures**

In this phase, we suppose neutrosophic soft p-separation axioms and neutrosophic soft topological subspace consisting of distinct neutrosophic soft points of neutrosophic soft topological space over X.

Definition 4.1. a) Let $(X, \Im, E)$ be a neutrosophic soft topological space over the crisp set X , and $x^{\lambda(\alpha,\beta,\gamma)} > y^{\lambda'(\alpha',\beta',\gamma')}$ are neutrosophic soft points. If there exist neutrosophic soft p-open sets $(\widetilde{F}, E)$ and $(\widetilde{G}, E)$ such that

$x^{\lambda(\alpha,\beta,\gamma)} \in (\widetilde{F}, E)$ and $x^{\lambda(\alpha,\beta,\gamma)} \sqcap (\widetilde{G}, E) = 0^{(X,E)}$ or

$y^{\lambda'(\alpha',\beta',\gamma')} \in (\widetilde{G}, E)$ and $y^{\lambda'(\alpha',\beta',\gamma')} \sqcap (\widetilde{F}, E) = 0^{(X,E)}$,

then $(X, \Im, E)$ is called a neutrosophic soft-P^o-space.

b) Let $(X, \Im, E)$ be a neutrosophic soft topological space over the crisp set X and $x^{\lambda(\alpha,\beta,\gamma)} > y^{\lambda'(\alpha',\beta',\gamma')}$ are neutrosophic soft points. If there exist neutrosophic soft p-open sets $(\widetilde{F}, E)$ and $(\widetilde{G}, E)$ such that

$x^{\lambda(\alpha,\beta,\gamma)} \in (\widetilde{F}, E)$, $x^{\lambda(\alpha,\beta,\gamma)} \sqcap (\widetilde{G}, E) = 0^{(X,E)}$ or

$y^{\lambda'(\alpha',\beta',\gamma')} \in (\widetilde{G}, E)$, $y^{\lambda'(\alpha',\beta',\gamma')} \sqcap (\widetilde{F}, E) = 0^{(X,E)}$,

then $(X, \Im, E)$ is called a neutrosophic soft-P^1-space.

c) Let $(X, \Im, E)$ be a neutrosophic soft topological space over the crisp set X, and $x^{\lambda(\alpha,\beta,\gamma)} > y^{\lambda'(\alpha',\beta',\gamma')}$ are neutrosophic soft points. If there exist neutrosophic soft p-open sets $(\widetilde{F}, E)$ and $(\widetilde{G}, E)$ such that

$x^{\lambda(\alpha,\beta,\gamma)} \in (\widetilde{F}, E)$, $y^{\lambda'(\alpha',\beta',\gamma')} \in (\widetilde{G}, E)$ and $(\widetilde{F}, E) \sqcap (\widetilde{G}, E) = 0^{(X,E)}$,

then $(X, \Im, E)$ is called a neutrosophic soft-P^2-space.

Example 4.2. Let X = $\{x^1, x^2\}$ be a universe set, E= $\{\lambda^1, \lambda^2\}$ be a parameters set, and $(x^1)\lambda^{1(0.1,0.4,0.7)}$, $(x^1)\lambda^{2(0.2,0.5,0.6)}$, $(x^2)\lambda^{1(0.3,0.3,0.5)}$, and $(x^2)\lambda^{2(0.4,0.4,0.4)}$ be neutrosophic soft points. Then the family $\Im = \{0^{(X,E)}, 1^{(X,E)}, (\widetilde{F}^1, E), (\widetilde{F}^2, E), (\widetilde{F}^3, E), (\widetilde{F}^4, E),$
$(\widetilde{F}^5, E), (\widetilde{F}^6, E), (\widetilde{F}^7, E), (\widetilde{F}^8, E)\}$, where

$(\widetilde{F}^1, E) = \{x^{1\lambda^{1(0.1,0.4,0.7)}}\}$,
$(\widetilde{F}^2, E) = \{(x^1)\lambda^{2(0.2,0.5,0.6)}\}$,
$(\widetilde{F}^3, E) = \{(x^2)\lambda^1(0.3, 0.3, 0.5)\}$,
$(\widetilde{F}^4, E) = (\widetilde{F}^1, E) \sqcup (\widetilde{F}^2, E)$,
$(\widetilde{F}^5, E) = (\widetilde{F}^1, E) \sqcup (\widetilde{F}^3, E)$,
$(\widetilde{F}^6, E) = (\widetilde{F}^2, E) \sqcup (\widetilde{F}^3, E)$,
$(\widetilde{F}^7, E) = (\widetilde{F}^1, E) \sqcup (\widetilde{F}^2, E) \sqcup (\widetilde{F}^3, E)$,
$(\widetilde{F}^8, E) = \{(x^1)\lambda^{1(0.1,0.4,0.7)}, (x^1)\lambda^{2(0.2,0.5,0.6)}, (x^2)\lambda^{2(0.3,0.3,0.5)}, (x^2)\lambda^2(0.4, 0.4, 0.4)\}$,

is a neutrosophic soft topology over X. Hence, $(X, \Im, E)$ is a neutrosophic soft topological space over X. Also, $(X, \Im, E)$ is a neutrosophic soft- P^0-space but not a neutrosophic soft-P^1-space because for neutrosophic soft points $(x^1)\lambda^1(0.1, 0.4, 0.7)$ and $(x^2)\lambda^2(0.4, 0.4, 0.4)$,$(X, \Im, E)$ is not a neutrosophic soft-P^1-space.

Example 4.3. Let X = $\mathbb{N}$ be a natural numbers set and E = $\{\lambda\}$ be a parameters set. Here $n^{\lambda(\alpha n,\beta n,\gamma n)}$ are neutrosophic soft points. Here we can give $(\alpha n, \beta n, \gamma n)$ appropriate values and the neutrosophic soft points $n^{\lambda(\alpha n,\beta n,\gamma n)}$,$m^{\lambda(\alpha n,\beta n,\gamma n)}$ are distinct neutrosophic soft points if and only if n $\neq$ m. It is clear that there is one-to-one compatibitily between the set of natural numbers and the set of neutrosophic soft points $N^\lambda = \{n^{\lambda(\alpha n,\beta n,\gamma n)}\}$.

Then we give cofinite topology on this set. Then neutrosophic soft set $(\widetilde{F}, E)$ is a neutrosophic soft p-open set if and only if the finite neutrosophic soft point is discarded from N^λ. Hence, $(X, \Im, E)$ is a neutrosophic soft-P^1-space but not a neutrosophic soft-P^2-space.

Example 4.4. Let X = $\{x^1, x^2\}$ be a universe set, E = $\{\lambda^1, \lambda^2\}$ be a parameters set, and $x^1\lambda^{1(0.1,0.4,0.7)}$,$x^1\lambda^{2(0.2,0.5,0.6)}$,
$x^2\lambda^{1(0.3,0.3,0.5)}$, and $x^2\lambda^{2(0.4,0.4,0.4)}$, be neutrosophic soft points. Then the family
$\Im = \{0^{(X,E)}, 1^{(X,E)}, (\widetilde{F}^1, E), (\widetilde{F}^2, E), ..., (\widetilde{F}^{15}, E)\}$, where
$(\widetilde{F}^1, E) = \{x^1\lambda^{1(0.1,0.4,0.7)}\}$,
$(\widetilde{F}^2, E) = \{(x^1)\lambda^{2(0.2,0.5,0.6)}\}$,
$(\widetilde{F}^3, E) = \{(x^2)\lambda^{1(0.3,0.3,0.5)}\}$,
$(\widetilde{F}^4, E) = \{(x^2)\lambda^{2(0.4,0.4,0.4)}\}$,
$(\widetilde{F}^5, E) = (\widetilde{F}^1, E) \sqcup (\widetilde{F}^2, E)$,
$(\widetilde{F}^6, E) = (\widetilde{F}^1, E) \sqcup (\widetilde{F}^3, E)$,
$(\widetilde{F}^7, E) = (\widetilde{F}^1, E) \sqcup (\widetilde{F}^4, E)$,
$(\widetilde{F}^8, E) = (\widetilde{F}^2, E) \sqcup (\widetilde{F}^3, E)$,
$(\widetilde{F}^9, E) = (\widetilde{F}^2, E) \sqcup (\widetilde{F}^4, E)$,
$(\widetilde{F}^{10}, E) = (\widetilde{F}^3, E) \sqcup (\widetilde{F}^4, E)$,
$(\widetilde{F}^{11}, E) = (\widetilde{F}^1, E) \sqcup (\widetilde{F}^2, E) \sqcup (\widetilde{F}^3, E)$,

$(\widetilde{F}^{12}, E) = (\widetilde{F}^1, E) \sqcup (\widetilde{F}^2, E) \sqcup (\widetilde{F}^4, E)$,
$(\widetilde{F}^{13}, E) = (\widetilde{F}^2, E) \sqcup (\widetilde{F}^3, E) \sqcup (\widetilde{F}^4, E)$,
$(\widetilde{F}^{14}, E) = (\widetilde{F}^1, E) \sqcup (\widetilde{F}^3, E) \sqcup (\widetilde{F}^4, E)$,
$(\widetilde{F}^{15}, E) = \{(x^1)\lambda^{1(0.1,0.4,0.7)}, (x^1)\lambda^{2(0.2,0.5,0.6)}, (x^2)\lambda^{2(0.3,0.3,0.5)}, (x^2)\lambda^{2(0.4,0.4,0.4)}\}$,

is a neutrosophic soft topology over X. Hence, $(X, \Im, E)$ is a neutrosophic soft topological space over X. Also, $(X, \Im, E)$ is a neutrosophic soft -P^2-space.

Theorem 4.5. *Let $(X, \Im, E)$ be a neutrosophic soft topological space over X. Then $(X, \Im, E)$ is a neutrosophic soft-P^1-space if and only if each neutrosophic soft point is a neutrosophic soft p-closed set.*

Proof. Let$(X, \Im, E)$ be a neutrosophic soft-P^1-space and $x^{\lambda(\alpha,\beta,\gamma)}$ be an arbitrary neutrosophic soft point. We show that $(x^{\lambda(\alpha,\beta,\gamma)})^{\lambda}$ is a neutrosophic soft p-open set. Let $y^{\lambda'(\alpha',\beta',\gamma')} \in (x^{\lambda(\alpha,\beta,\gamma)})^{\lambda}$; then $x^{\lambda(\alpha,\beta,\gamma)}$ and $y^{\lambda'(\alpha',\beta',\gamma')}$ are distinct neutrosophic soft points. Hence, x $\neq$ y or $\lambda' \neq \lambda$.

Since $(X, \Im, E)$ is a neutrosophic soft-P^1-space, there exists a neutrosophic soft p-open set $(\widetilde{G}, E)$ such that

$y^{\lambda'(\alpha',\beta',\gamma')} \in (\widetilde{G}, E)$ and $x^{\lambda(\alpha,\beta,\gamma)} \sqcap (\widetilde{G}, E) = 0^{(X,E)}$.

Then, since $x^{\lambda(\alpha,\beta,\gamma)} \sqcap (\widetilde{G}, E) = 0^{(X,E)}$, we have $y^{\lambda'(\alpha',\beta',\gamma')} \in (\widetilde{G}, E) \sqsubset (x^{\lambda(\alpha,\beta,\gamma)})^{\lambda}$. This implies that $(x^{\lambda(\alpha,\beta,\gamma)})^{\lambda}$ is a neutrosophic soft p-open set, i.e. $x^{\lambda(\alpha,\beta,\gamma)}$ is a neutrosophic soft p-closed set.

Suppose that each neutrosophic soft point $x^{\lambda(\alpha,\beta,\gamma)}$ is a neutrosophic soft p-closed set. Then $(x^{\lambda(\alpha,\beta,\gamma)})^{\lambda}$ is a neutrosophic soft p-open set. Let $x^{\lambda(\alpha,\beta,\gamma)} \sqcap y^{\lambda'(\alpha',\beta',\gamma')} = 0^{(X,E)}$. Thus $y^{\lambda'(\alpha',\beta',\gamma')} \in (x^{\lambda(\alpha,\beta,\gamma)})^{\lambda}$ and $x^{\lambda(\alpha,\beta,\gamma)} \sqcap (x^{\lambda(\alpha,\beta,\gamma)})^{\lambda} = 0^{(X,E)}$. Therefore, $(X, \Im, E)$ is a neutrosophic soft-P^1-space over X. □

Theorem 4.6. *Let $(X, \Im, E)$ be a neutrosophic soft topological space over X. Then $(X, \Im, E)$ is a neutrosophic soft-P^2-space iff for distinct neutrosophic soft points $x^{\lambda(\alpha,\beta,\gamma)}$ and $y^{\lambda'(\alpha',\beta',\gamma')}$, there exists a neutrosophic soft p-open set $(\widetilde{F}, E)$ containing $x^{\lambda}(\alpha, \beta, \gamma)$ but not $y^{\lambda'}(\alpha', \beta', \gamma')$ such that $y^{\lambda'(\alpha',\beta',\gamma')}$ does not belong to $\overline{(\widetilde{F}, E)}$.*

Proof. Let $x^{\lambda(\alpha,\beta,\gamma)}$ and $y^{\lambda'(\alpha',\beta',\gamma')}$ be two neutrosophic soft points in neutrosophic soft -P^2-space $(X, \Im, E)$.

Then there exist disjoint neutrosophic soft p-open set $(\widetilde{F}, E)$, $(\widetilde{G}, E)$ such that

$x^{\lambda(\alpha,\beta,\gamma)} \in (\widetilde{F}, E)$, $y^{\lambda'(\alpha',\beta',\gamma')} \in (\widetilde{G}, E)$.

Since $x^{\lambda(\alpha,\beta,\gamma)} \sqcap y^{\lambda'(\alpha',\beta',\gamma')} = 0^{(X,E)}$ and $(\widetilde{F}, E) \sqcap (\widetilde{G}, E) = 0^{(X,E)}$, $y^{\lambda'(\alpha',\beta',\gamma')}$ does not belong to $(\widetilde{F}, E)$. It implies that $y^{\lambda'(\alpha',\beta',\gamma')}$ does not belong to $\overline{(\widetilde{F}, E)}$.

Next suppose that, for distinct neutrosophic soft points $x^{\lambda(\alpha,\beta,\gamma)}$, $y^{\lambda'(\alpha',\beta',\gamma')}$, there exists a neutrosophic soft p-open set $(\widetilde{F},E)$ containing $x^{\lambda(\alpha,\beta,\gamma)}$ but not $y^{\lambda'(\alpha',\beta',\gamma')}$ such that $y^{\lambda'(\alpha',\beta',\gamma')}$ does not belong to $(\overline{\widetilde{F},E})$. Then $y^{\lambda'(\alpha',\beta',\gamma')} \in ((\overline{\widetilde{F},E}))^c$, i.e. $(\widetilde{F},E)$ and $((\overline{\widetilde{F},E}))^c$ are disjoint neutrosophic soft p-open sets containing $x^{\lambda(\alpha,\beta,\gamma)}$, $y^{\lambda'(\alpha',\beta',\gamma')}$ respectively. □

Theorem 4.7. *Let $(X,\Im,E)$ be a neutrosophic soft-P^1-space for every neutrosophic soft point $x^{\lambda(\alpha,\beta,\gamma)} \in (\widetilde{F},E) \in \Im$. If there exists a neutrosophic soft p-open set $(\widetilde{G},E)$ such that*

$x^{\lambda(\alpha,\beta,\gamma)} \in (\widetilde{G},E) \sqsubset (\overline{\widetilde{G},E}) \sqsubset (\widetilde{F},E)$,

then $(X,\Im,E)$ is a neutrosophic soft-P^2-space.

Proof. Suppose that $x^{\lambda(\alpha,\beta,\gamma)} \sqcap y^{\lambda'(\alpha',\beta',\gamma')} = 0^{(X,E)}$. Since $(X,\Im,E)$ is a neutrosophic soft T^1-space, $x^{\lambda(\alpha,\beta,\gamma)}$ and $y^{\lambda'(\alpha',\beta',\gamma')}$ are neutrosophic soft p-closed sets in $\Im$. Thus $x^{\lambda(\alpha,\beta,\gamma)} \in (y^{\lambda'}(\alpha',\beta',\gamma'))^c \in \Im$. Then there exists a neutrosophic soft p-open set $(\widetilde{G},E)$ in $\Im$ such that

$x^{\lambda(\alpha,\beta,\gamma)} \in (\widetilde{G},E) \sqsubset (\overline{\widetilde{G},E}) \sqsubset (y^{\lambda'}(\alpha',\beta',\gamma'))^c$.

Hence, we have $y^{\lambda'(\alpha',\beta',\gamma')} \in ((\overline{\widetilde{G},E}))^c$, $x^{\lambda(\alpha,\beta,\gamma)} \in (\widetilde{G},E)$, and $(\widetilde{G},E) \sqcap ((\overline{\widetilde{G},E}))^c = 0^{(X,E)}$, i,e. $(X,\Im,E)$ is a neutrosophic soft P^2-space. □

Remark 4.8. Let $(X,\Im,E)$ be a neutrosophic soft-P^1-space for i = 0, 1, 2. For each x $\neq$ y, neutrosophic points $x^(\alpha,\beta,\gamma)$ and $y^(\alpha',\beta',\gamma')$ have neighborhoods satisfying conditions of-P^i-space in neutrosophic topological space $(X,\Im^\lambda)$ for each $\lambda \in E$ because $x^{\lambda(\alpha,\beta,\gamma)}$ and $y^{\lambda'(\alpha',\beta',\gamma')}$ are distinct neutrosophic soft points.

Definition 4.9. Let $(X,\Im,E)$ be a neutrosophic soft topological space over X, $(\widetilde{F},E)$ be a neutrosophic soft p-closed set , and $x^{\lambda(\alpha,\beta,\gamma)} \sqcap (\widetilde{F},E) = 0^{(X,E)}$. If there exist neutrosophic soft p-open open sets $(^1,E)$ and $(\widetilde{G}^2,E)$ such that $x^{\lambda(\alpha,\beta,\gamma)} \in (\widetilde{G}^1,E)$, $(\widetilde{F},E) \sqsubset (\widetilde{G}^2,E)$, and $(\widetilde{G}^1,E) \sqcap (\widetilde{G}^2,E) = 0^{(X,E)}$, then $(X,\Im,E)$ is called a neutrosophic soft b-regular space. $(X,\Im,E)$ is said to be a neutrosophic soft-P^3-space if is both a neutrosophic soft p-regular and neutrosophic soft-P^1-space.

Theorem 4.10. *Let $(X,\Im,E)$ be a neutrosophic soft topological space over X, $(X,\Im,E)$ is a neutrosophic soft-P^3-space if and only if for every $x^{\lambda(\alpha,\beta,\gamma)} \in (\widetilde{F},E) \in \Im$, there exists $(\widetilde{G},E)$ $\in \Im$ such that $x^{\lambda(\alpha,\beta,\gamma)} \in (\widetilde{G},E) \sqsubset (\overline{\widetilde{G},E}) \sqsubset (\widetilde{F},E)$.*

Proof. Let $(X,\Im,E)$ be a neutrosophic soft P^3-space and $x^{\lambda(\alpha,\beta,\gamma)} \in (\widetilde{F},E) \in \Im$. Since $(X,\Im,E)$ is a neutrosophic soft-P^3-space for the neutrosophic soft point $x^{\lambda(\alpha,\beta,\gamma)}$ and neutrosophic soft p-closed set $(\widetilde{F},E)^c$, there exist $(\widetilde{G}^1,E)$, $(\widetilde{G}^2,E) \in \Im$ such that $x^{\lambda(\alpha,\beta,\gamma)} \in (\widetilde{G}^1,E)$, $(\widetilde{F},E)^c \sqsubset (\widetilde{G}^2,E)$, and $(\widetilde{G}^1,E) \sqcap (\widetilde{G}^2,E) = 0^{(X,E)}$. thus, we have $x^{\lambda(\alpha,\beta,\gamma)} \in (\widetilde{G}^1,E)$ $\sqsubset (\widetilde{G}^2,E)^c \sqsubset (\widetilde{F},E)$. Since $(\widetilde{G}^2,E)^c$ is a neutrosophic soft p-closed set, $(\overline{\widetilde{G}^1,E}) \sqsubset (\widetilde{G}^2,E)^c$.

Conversely , let $x^{\lambda(\alpha,\beta,\gamma)} \sqcap (\widetilde{H}, E) = 0^{(X,E)}$ and $(\widetilde{H}, E)$ be a neutrosophic soft p-closed set. Thus, $x^{\lambda(\alpha,\beta,\gamma)} \in (\widetilde{H}, E)^c$ and from the condition of the theorem, we have $x^{\lambda(\alpha,\beta,\gamma)} \in (\widetilde{G}, E)$ $\sqsubset \overline{(\widetilde{G}, E)} \sqsubset (\widetilde{H} \cdot E)^c$.

Then $x^{\lambda(\alpha,\beta,\gamma)} \in (\widetilde{G}, E)$, $(\widetilde{H}, E) \sqsubset (\overline{(\widetilde{G}, E)})^c$, and $(\widetilde{G}, E) \sqcap \overline{(\widetilde{G}, E)}^c = 0^{(X,E)}$ are satisfied, i.e. $(X, \Im, E)$ is a neutrosophic soft-P^3-space. □

Definition 4.11. A neutrosophic soft topological space $(X, \Im, E)$ over X is called a neutrosophic soft p-normal space if for every pair of disjoint neutrosophic soft b-closed set $(\widetilde{F}^1, E)$, $(\widetilde{F}^2, E)$, there exists disjoint neutrosophic soft p-open sets $(\widetilde{G}^1, E)$, $(\widetilde{G}^2, E)$ such that $(\widetilde{F}^1, E)$ $\sqsubset (\widetilde{G}^1, E)$ and $(\widetilde{F}^2, E) \sqsubset (\widetilde{G}^2, E)$.

$(X, \Im, E)$ is said to be a neutrosophic soft b-T^4-space if it is both a neutrosophic soft p-normal and neutrosophic soft-P^1-space.

Theorem 4.12. *Let $(X, \Im, E)$ be a neutrosophic soft topological space over X . Then $(X, \Im, E)$ is a neutrosophic soft-P^4-space if and only if, for each neutrosophic soft p-closed set $(\widetilde{F}, E)$ and neutrosophic soft p-open set $(\widetilde{G}, E)$ with $(\widetilde{F}, E) \sqsubset (\widetilde{G}, E)$, there exists a neutrosophic soft p-open set $(\widetilde{D}, E)$ such that*

$(\widetilde{F}, E) \sqsubset (\widetilde{D}, E) \sqsubset \overline{(\widetilde{D}, E)} \sqsubset (\widetilde{G}, E).$

Proof. Let $(X, \Im, E)$ be a neutrosophic soft-P^4-space, $(\widetilde{F}, E)$ be a neutrosophic soft p-closed set and $(\widetilde{F}, E) \sqsubset (\widetilde{G}, E) \in \Im$. Then $(\widetilde{G}, E)^c$ is a neutrosophic soft p-closed set and $(\widetilde{F}, E)$ $\sqcap (\widetilde{G}, E)^c = 0^{(X,E)}$. Since $(X, \Im, E)$ is a neutrosophic soft-P^4-space, there exist neutrosophic soft p-open sets $(\widetilde{D}^1, E)$ and $(\widetilde{D}^2, E)$ such that $(\widetilde{F}, E) \sqsubset (\widetilde{D}^1, E)$, $(\widetilde{G}, E)^c \sqsubset (\widetilde{D}^2, E)$, and $(\widetilde{D}^1, E) \sqcap (\widetilde{D}^2, E) = 0^{(X,E)}$. This implies that

$(\widetilde{F}, E) \sqsubset (\widetilde{D}^1, E) \sqsubset (\widetilde{D}^2, E)^c \sqsubset (\widetilde{G}, E).$

$(\widetilde{D}^2, E)^c$ is a neutrosophic soft p-closed set and $\overline{(\widetilde{D}^1, E)} \sqsubset (\widetilde{D}^2, E)^c$ is satisfied. Thus,

$(\widetilde{F}, E) \sqsubset (\widetilde{D}^1, E) \sqsubset \overline{(\widetilde{D}^1, E)} \sqsubset (\widetilde{G}, E)$

is obtained.

Conversely, let $(\widetilde{F}^1, E)$, $(\widetilde{F}^2, E)$ be two disjoint neutrosophic soft p-closed sets. Then $(\widetilde{F}^1, E) \sqsubset (\widetilde{F}^2, E)^c$. From the condition of theorem, there exists a neutrosophic soft p-open set $(\widetilde{D}, E)$ such that

$(\widetilde{F}^1, E) \sqsubset (\widetilde{D}, E) \sqsubset \overline{(\widetilde{D}^1, E)} \sqsubset (\widetilde{F}^2, E)^c$.

Thus, $(\widetilde{D}, E)$, $(\overline{(\widetilde{D}, E)})^c$ are neutrosophic soft p-open sets and $(\widetilde{F}^1, E) \sqsubset (\widetilde{D}, E)$, $(\widetilde{F}^2, E) \sqsubset$ $(\overline{(\widetilde{D}, E)})^c$, and $(\widetilde{D}, E) \sqcap (\overline{(\widetilde{D}, E)})^c = 0^{(X,E)}$ are obtained. Hence, $(X, \Im, E)$ is a neutrosophic soft-P^4-space. □

Definition 4.13. Let $(X, \Im, E)$ be a neutrosophic soft topological space over X and $(\widetilde{F}, E)$ be an arbitrary neutrosophic soft set. Then $\Im^{(\widetilde{F},E)} = \{(\widetilde{F}, E) \sqcap (\widetilde{H}, E) : (\widetilde{H}, E) \in \Im\}$ is said to be neutrosophic soft topology on $(\widetilde{F}, E)$ and $((\widetilde{F}, E), \Im^{(\widetilde{F},E)}, E)$ is called a neutrosophic soft topological subspace of $(X, \Im, E)$.

Theorem 4.14. *Let $(X, \Im, E)$ be a neutrosophic soft topological space over X. If $(X, \Im, E)$ is a neutrosophic soft-P^i-space, then the neutrosophic soft topological subspace $((\widetilde{F}, E), \Im^{(\widetilde{F},E)}, E)$ is a neutrosophic soft-P^i-space for i = 0, 1, 2, 3.*

Proof. Let $x^{\lambda(\alpha,\beta,\gamma)}$, $y^{\lambda'(\alpha',\beta',\gamma')} \in ((\widetilde{F}, E), \Im^{(\widetilde{F},E)}, E)$ such that $x^{\lambda(\alpha,\beta,\gamma)} \sqcap y^{\lambda'(\alpha',\beta',\gamma')} = 0^{(X,E)}$. Thus , there exist neutrosophic soft p-open set $(\widetilde{F}^1, E)$ and $(\widetilde{F}^2, E)$ satisfying the conditions of neutrosophic soft -P^i-space such that $x^{\lambda(\alpha,\beta,\gamma)} \in (\widetilde{F}^1, E)$, $y^{\lambda'(\alpha',\beta',\gamma')} \in (\widetilde{F}^2, E)$. Then $x^{\lambda(\alpha,\beta,\gamma)} \in (\widetilde{F}^1, E) \sqcap (\widetilde{F}, E)$ and $y^{\lambda'(\alpha',\beta',\gamma')} \in (\widetilde{F}^2, E) \sqcap (\widetilde{F}, E)$. Also, the neutrosophic soft p-open set $(\widetilde{F}^1, E) \sqcap (\widetilde{F}, E)$, $(\widetilde{F}^2, E) \sqcap (\widetilde{F}, E)$ in $\Im^{(\widetilde{F},E)}$ satisfy the conditions of neutrosophic soft-P^i-space for i = 0, 1, 2, 3. □

Theorem 4.15. *Let $(X, \Im, E)$ be a neutrosophic soft topological space over X. If $(X, \Im, E)$ is a neutrosophic soft-P^4-space and $(\widetilde{F}, E)$ is a neutrosophic soft p-closed set in $(X, \Im, E)$, then $((\widetilde{F}, E), \Im^{(\widetilde{F},E)}, E)$ is a neutrosophic soft -P^4-space.*

Proof. Let $(X, \Im, E)$ be a neutrosophic soft P^4-space and $(\widetilde{F}, E)$ be a neutrosophic soft p-closed set in $(X, \Im, E)$. Let $(\widetilde{F}^1, E)$ and $(\widetilde{F}^2, E)$ be two neutrosophic soft p-closed sets in $((\widetilde{F}, E), \Im^{(\widetilde{F},E)}, E)$ such that $(\widetilde{F}^1, E) \sqcap (\widetilde{F}^2, E) = 0^{(X,E)}$. When $(\widetilde{F}, E)$ is a neutrosophic soft p-closed set in $(X, \Im, E)$, $(\widetilde{F}^1, E)$ and $(\widetilde{F}^2, E)$ are neutrosophic soft p-closed sets in $(X, \Im, E)$. Since $(X, \Im, E)$ is a neutrosophic soft-P^4-space, there exist neutrosophic soft p-open sets $(\widetilde{G}^1, E)$ and $(\widetilde{G}^2, E)$ such that $(\widetilde{F}^1, E) \sqsubset (\widetilde{G}^1, E)$, $(\widetilde{F}^2, E) \sqsubset (\widetilde{G}^2, E)$ and $(\widetilde{G}^1, E)$ $\sqcap$ $(\widetilde{G}^2, E) = 0^{(X,E)}$. Then $(\widetilde{F}^1, E) = (\widetilde{G}^1, E) \sqcap (\widetilde{F}, E)$, $(\widetilde{F}^2, E) = (\widetilde{G}^2, E) \sqcap (\widetilde{F}, E)$ and $((\widetilde{G}^1, E) \sqcap (\widetilde{F}, E)) \sqcap ((\widetilde{G}^2, E) \sqcap (\widetilde{F}, E)) = 0^{(X,E)}$. This implies that $((\widetilde{F}, E), \Im^{(\widetilde{F},E)}, E)$ is a neutrosophic soft-P^4-space. □

5. **Conclusion**

Neutrosophic soft p-separation structures are the most imperative and fascinating notions in neutrosophic soft topology.We have introduced neutrosophic soft p-separation axioms in neutrosophic soft topological structures with respect to soft points, which are defined over an initial universe of discourse with a fixed set of parameters(data,decision variables). We further investigated and scrutinized some essential features of the initiated neutrosophic soft p-separation structures. It is supposed that these results will be very very useful for future studies on neutrosophic soft topology to carry out a general framework for practical applications. Applications of neutrosophic soft p-separation structures in neutrosophic soft topological spaces can be traced out in decision making problems.

References

1. K. Atanassov, Intuitionistic fuzzy sets, Fuzzy sets Syst 1986, 20, 87-96.
2. S. Bayramov; C. Gunduz, On intuitionistic fuzzy soft topological spaces, TWMS J. Pure Appl. Math. 2014, 5, 66-79.
3. S. Bayramov; C. Gunduz, A new approach to separability and compactness in soft topological spaces, TWMS J. Pure Appl. Math. 2018, 9, 82-93.
4. T. Bera; N.K. Mahapatra, On neutrosophic soft function, Ann. Fuzzy Math. Inform. 2016, 12, 101-199.
5. T. Bera; N.K. Mahapatra, Introduction to neutrosophic soft topological space, Opsearch 2017, 54, 841-867.
6. N. Cagman; S. Karatas; S. Enginoglu, Soft topology, Comput. Math. Appl. 2011, 62, 351-358.
7. I. Deli; S. Broumi, Neutrosophic soft relations and some properties, Ann. Fuzzy Math. Inform. 2015, 9, 169-182.
8. C. Gunduz Aras; S. Bayramov, On the Tietze extension theorem in soft topological spaces, Proceedings of the Institute of Mathematics and Mechanics of the National Academy of Sciences of Azerbaijan 2017, 43, 105-115.
9. A. M. Khattak; M. Zamir; M. Alam; F. Nadeem; S. Jabeen; A. Zaighum,'Weak Soft Open Sets in Soft Bi Topological Spaces', Journal of Mathematical Extension, 2020, 14, 85-116.
10. P.K. Maji, Neutrosophic soft set, Ann. Fuzzy Math. Inform. 2013, 5, 157-168.
11. D. Molodtsov, Soft set theory-first results, Comput. Math. Appl. 1999, 37, 19-31.
12. A.A Salma; S.A. Alblowi, Neutrosophic set and neutrosophic topological spaces, IOSR J. Math. 2012, 3, 31-35.
13. M. Shabir; M. Naz, On soft topological spaces, Comput. Math. Appl. 2011, 61, 1786-1799.
14. F. Smarandache, Neutrosophic set, a generalisation of the intuitionistic fuzzy sets, Int. J. Pure Appl. Math. 2005, 24, 287- 297.
15. L.A. Zadeh, Fuzzy sets, Inform. Control 1965, 8, 338-353.
16. M. Abdel-Basset; M. Mohamed; M. Elhoseny; F. Chiclana; A.E.H. Zaied, Cosine similarity measures of bipolar neutrosophic set for diagnosis of bipolar disorder diseases, Artificial Intelligence in Medicine 2019, 101 101735.
17. M. Abdel-Basset; A. Mumtaz; A. Asma, Resource levelling problem in construction projects under neutrosophic environment, The Journal of Supercomputing 2019, 1-25.
18. M. Arif; F. Nadeem; C. Park; G. Nordo; S. Abdullah, Neutrosophic soft topological structure relative to most generalized neutrosophic soft open set, Communication in Mathematics and Applications, [In press]

Received: Oct 21, 2019. Accepted: Mar 20, 2020