Neutrosophic Inventory Backorder Problem Using Triangular Neutrosophic Numbers

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Abstract: A company may have backorders if they run out of the stock in their stores, in which case, it can just place a new order to restock its shelves. A customer who is willing to wait for some time until the company has restocked the products would have to place a backorder. A backorder only exists if customers are willing to wait for the order. In this paper, a neutrosophic inventory backorder problem using a triangular neutrosophic numbers is introduced. First, we fuzzify the carrying cost and shortage cost as triangular neutrosophic numbers and the signed distance method is used to defuzzify them. From these, we can obtain the neutrosophic optimal shortage quantity and the neutrosophic total cost. A numerical example is provided to illustrate the proposed model in neutrosophic environment.

Keywords: Neutrosophic EOQ; Neutrosophic set; Signed distance method; Triangular neutrosophic numbers.

1. Introduction

Backorders represents any quantity of inventory an enterprise customer have ordered but have not yet received as it presently isn't to be had in stock. An enterprise's backorders are an essential factor in its inventory control evaluation. The quantity of items on backorder and how long it takes to fulfill these customer orders can offer perception into how properly the company manages its stock.

Sen and Malakar [13] considered an EOQ model with shortage, considering the various parameters as triangular, trapezoidal fuzzy number and parabolic fuzzy number. Intuitionistic fuzzy set - a generalization of fuzzy set was introduced by Atanassov [1]. Yao and Lee [15] developed a fuzzy inventory with or without backorder for fuzzy order quantity with trapezoidal fuzzy number. Bulancak and Kirkavak [3] applied trapezoidal fuzzy number for EOQ with backorder.

Fuzzy inventory model without shortages was proposed by Dutta and Kumar[4]. Carrying cost and set up cost are expressed as fuzzy trapezoidal numbers and for defuzzification signed distance method is used by them. Mahuya Deb and Prabjot Kaur[6] developed an intuitionistic fuzzy inventory backorder problem using triangular intuitionistic fuzzy numbers. D. Banerjee and S. Pramanik[2] developed a single-objective linear goal programming problem with neutrosophic numbers. F.

Smarandache[14] introduced neutrosophic set and neutrosophic logic by considering the non-standard analysis. F. Smarandache[16] introduced the plithogenic set -as generalization of crisp, fuzzy, intuitionistic fuzzy, and neutrosophic sets

Neutrosophic set is the take a look at of neutralities origin, nature and scope and additionally their interactions with exceptional ideational spectra. To deal with unsure information processing, the brand new emerging tool known as neutrosophic set is used. Neutrosophic set is a powerful and popular formal framework that has the potential to address uncertainty analysis in information sets. However, the neutrosophic set desires to be specified detail. So that, we define an example of neutrosophic set called as single-valued neutrosophic set (SVNS). Single valued neutrosophic set is an instance of neutrosophic set. The SVNS is a set of generalization of a classic set, fuzzy set, interval value fuzzy set, intuitionistic fuzzy set and para consistent set. The single-valued neutrosophic set is used in lots of locations like professional machine, information fusion gadget, query answering device, bioinformatics and scientific informatics and many others.

Pranab Biswas, Surapati Pramanik, Bibhas C. Giri [12] introduced multi-attribute group decision making based on expected value of neutrosophic trapezoidal numbers. An exact formula of expected value for neutrosophic trapezoidal number is established. Irfan Deli and Yusuf subas[5] discussed two special forms of single valued neutrosophic numbers such as single valued trapezoidal neutrosophic numbers and single valued triangular neutrosophic numbers. M.Mullai and S.Broumi[7] proposed neutrosophic inventory model without shortages. Also neutrosophic inventory model with price break for finding the optimal solution of the model for the optimal order quantity was established by M.Mullai and R. Surya[8].

In this paper, neutrosophic inventory backorder model is established by taking the parameters as triangular neutrosophic numbers. The neutrosophic optimal shortage quantity and the neutrosophic optimal total cost are derived in this model and signed distance method is used for defuzzification. A neutrosophic set may help in solving membership function when it is not defined accurately. Without difficulty, the work can also manage the inventory system of any company in neutrosophic backorder model. The novelty of this model is to give more accurate results than existing methods whenever uncertain and unexpected situations arise in back order inventory system. To illustrate the results of this model, sensitivity analysis is presented for crisp, fuzzy, intuitionistic fuzzy and neutrosophic sets and the results are discussed briefly.

2. Preliminaries

The basic definitions involving neutrosophic set, single valued neutrosophic sets and triangular neutrosophic numbers which are very useful for the proposed model are outlined here.

Definition 2.1 (Irfan Deli and Yusuf Subas., 2014) (Neutrosophic set)

Let E be a universe. A neutrosophic set A in E is characterized by a truth-membership function T_A, an indeterminacy-membership function I_A and a falsity-membership function F_A. $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard elements of [0,1]. It can be written as

$$A=\{\langle\ x, T_A(x),\ I_A(x),\ F_A(x)\ \rangle : x \in E, T_A(x),\ I_A(x),\ F_A(x) \in\ ]0^-, 1^+[\ \}.$$

There is no restriction on the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$, so $0^- \leq T_A(x) + I_A(x) + F_A(x) \leq 3+$.

Definition 2.2 (Irfan Deli and Yusuf Subas., 2014) (Single-valued neutrosophic set)

Let E be a universe. A single valued neutrosophic set A, which can be used in real scientific and engineering applications, in E is characterized by a truth-membership function T_A, an indeterminacy-membership function I_A and a falsity-membership function F_A. $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard elements of [0,1]. It can be written as

$$A=\{\langle\ x, T_A(x),\ I_A(x),\ F_A(x)\ \rangle : x\ \epsilon\ E, T_A(x),\ I_A(x), F_A(x)\ \epsilon\ [0,\ 1]\ \}.$$

There is no restriction on the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$, so $0\ \leq\ T_A(x)+I_A(x)\ +\ F_A(x)\ \leq\ 3$.

Definition 2.3 (Irfan Deli and Yusuf Subas., 2014) (Triangular neutrosophic numbers)

Let the triangular neutrosophic number $\tilde{a} = \langle(a_1, b_1, c_1); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}}\rangle$ is a special neutrosophic set on the real line set R, whose truth-membership, indeterminacy-membership, and falsity-membership functions are defined as follows:

$$\mu\tilde{a}(x) = \begin{cases} (x-a_1)w_{\tilde{a}}/(b_1-a_1) & \text{if } a_1 \leq x \leq b_1 \\ w_{\tilde{a}} & \text{if } x = b_1 \\ (c_1-x)w_{\tilde{a}}/(c_1-b_1) & \text{if } b_1 \leq x \leq c_1 \\ 0 & \text{if } otherwise \end{cases}$$

$$\nu\tilde{a}(x) = \begin{cases} (b_1-x+(x-a_1)u_{\tilde{a}})/(b_1-a_1) & \text{if } a_1 \leq x \leq b_1 \\ u_{\tilde{a}} & \text{if } x = b_1 \\ (x-b_1+(c_1-x)u_{\tilde{a}})/(c_1-b_1) & \text{if } b_1 \leq x \leq c_1 \\ 1 & \text{if } otherwise \end{cases}$$

$$\lambda\tilde{a}(x) = \begin{cases} (b_1-x+(x-a_1)y_{\tilde{a}})/(b_1-a_1) & \text{if } a_1 \leq x \leq b_1 \\ y_{\tilde{a}} & \text{if } x = b_1 \\ (x-b_1+(c_1-x)y_{\tilde{a}})/(c_1-b_1) & \text{if } b_1 \leq x \leq c_1 \\ 1 & \text{if } otherwise \end{cases}$$

respectively.

If $a_1 \geq 0$ and at least $c_1 > 0$ then $\tilde{a} = \langle(a_1, b_1, c_1); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}}\rangle$ is called a positive triangular neutrosophic number, denoted by $\tilde{a} > 0$. Likewise, if $c_1 \leq 0$ and at least $a_1 < 0$, then $\tilde{a} = \langle(a_1, b_1, c_1); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}}\rangle$ is called a negative triangular neutrosophic number, denoted by $\tilde{a} < 0$. A triangular neutrosophic number $\tilde{a} = \langle(a_1, b_1, c_1); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}}\rangle$ may express an ill-known quantity about a, which is approximately equal to a.

Definition 2.4 (Sushil Kumar. U and Rajput .S., 2006)(Signed distance method)

Let $\tilde{D} \epsilon F$. We define the signed distance of $\tilde{D}$ measured from $\tilde{0}$ as

$$d(\tilde{D}, \tilde{0}) = \frac{1}{2}\int_0^1 [D_L(\alpha) + D_R(\alpha)]d\alpha$$

Definition 2.5 (Mahuya Deb and Prabjot Kaur., 2016) (Defuzzification)

(i) Defuzzification for Triangular Fuzzy Number

The defuzzification value for a triangular fuzzy number(a_1, a_2, a_3) is given by

$$A = \frac{a_1 + 2a_2 + a_3}{4}$$

(ii) Defuzzification for Triangular Intuitionistic Fuzzy Number

Let $\hat{A} = (a_1, a_2, a_3)(a'_1, a_2, a'_3)$ be a triangular intuitionistic fuzzy number. Then the signed distance of $\hat{A}$ can be calculated as follows

$$D^s(\widehat{A}, \widehat{0}) = \frac{1}{4}[\int_0^1 L_\mu(\alpha) + \int_0^1 L_\mu(\alpha) + \int_0^1 L_\mu(\alpha) + \int_0^1 L_\mu(\alpha)]$$

$$= \frac{1}{4}[\int_0^1 \{a_1 - \alpha(a_2 - a_1)\}\delta\alpha + \int_0^1 \{a_3 - \alpha(a_3 - a_2)\}\delta\alpha + \int_0^1 \{a_2 - (1-\alpha)(a_2 - a'_1)\}\delta\alpha$$

$$+ \int_0^1 \{a_2 + (1-\alpha)(a'_3 - a_2)\}\delta\alpha]$$

$$= \frac{a_1 + 2a_2 + a_3 + a'_1 + 2a_2 + a'_3}{8}$$

3. Notations

C_h^N - Neutrosophic carrying cost per unit quantity per unit time

C_s^N - Neutrosophic shortage cost per unit quantity per unit time

D^N - Neutrosophic total demand

$(TC)^N$ - Neutrosophic total cost

Q^N - Neutrosophic order quantity

Q^{*N} - Neutrosophic optimal order quantity

$F(q)^N$ - Defuzzified total neutrosophic cost

4. Assumptions

• At the opening of every cycle, only a single order is produced and the entire lot is delivered in one batch.

• Q^N is the neutrosophic lot-size per cycle whereas S_1^N is the neutrosophic initial inventory level after fulfilling the back-logged quantity of previous cycle and $Q^N - S_1^N$ is the maximum shortage level.

• T^N is the cycle length where t_1^N is the period with no shortage.

5. Neutrosophic model with shortages

This section describes the inventory model with backorder in neutrosophic environment. Since the inventory carrying cost and shortage cost are in neutrosophic numbers, we represent them by triangular neutrosophic numbers as follows:

Let $C_h^N = (C_{h_1}^N, C_{h_2}^N, C_{h_3}^N)(C_{h_1}'^N, C_{h_2}^N, C_{h_3}'^N)(C_{h_1}''^N, C_{h_2}^N, C_{h_3}''^N)$

$C_s^N = (C_{s_1}^N, C_{s_2}^N, C_{s_3}^N)(C_{s_1}'^N, C_{s_2}^N, C_{s_3}'^N)(C_{s_1}''^N, C_{s_2}^N, C_{s_3}''^N)$

To defuzzify the triangular neutrosophic numbers, the signed distance method is defined as follows:

Let $A^N = (a_1, a_2, a_3)(a'_1, a_2, a'_3)(a''_1, a_2, a''_3)$ be a triangular neutrosophic number. Then the signed distance of A^N is written as

$$D^s(A^N, 0) = \frac{a_1 + 2a_2 + a_3 + a''_1 + 2a_2 + a''_3}{8}$$

The neutrosophic total cost is given by

$$(TC)^N = \frac{1}{T}[\frac{C_h^N s_1^{2N}}{2D^N} + \frac{1}{2D^N} C_s^N (Q^N - s_1^N)^2]$$

$$= (C_{h_1}^N, C_{h_2}^N, C_{h_3}^N)(C_{h_1}''^N, C_{h_2}^N, C_{h_3}''^N)\frac{s_1^{2N}}{2D^N} + \frac{(Q^N - s_1^N)^2}{2D^N}(C_{s_1}^N, C_{s_2}^N, C_{s_3}^N)(C_{s_1}''^N, C_{s_2}^N, C_{s_3}''^N)$$

$$= (C_{h_1}^N \frac{s_1^{2^N}}{2D^N} + \frac{(Q^N - s_1^N)^2}{2D^N} C_{s_1}^N, C_{h_2}^N \frac{s_1^{2^N}}{2D^N} + \frac{(Q^N - s_1^N)^2}{2D^N} C_{s_2}^N, C_{h_3}^N \frac{s_1^{2^N}}{2D^N} + \frac{(Q^N - s_1^N)^2}{2D^N} C_{s_3}^N)(C_{h_1}{}''^N \frac{s_1^{2^N}}{2D^N} +$$

$$\frac{(Q^N - s_1^N)^2}{2D^N} C_{s_1}{}''^N, C_{h_2}^N \frac{s_1^{2^N}}{2D^N} + \frac{(Q^N - s_1^N)^2}{2D^N} C_{s_2}^N, C_{h_3}{}''^N \frac{s_1^{2^N}}{2D^N} + \frac{(Q^N - s_1^N)^2}{2D^N} C_{s_3}{}''^N)$$

The defuzzified neutrosophic total cost using above signed distance method is given by

$$F(q)^N = \frac{1}{8}[(C_{h_1}^N \frac{s_1^{2^N}}{2D^N} + \frac{(Q^N - s_1^N)^2}{2D^N} C_{s_1}^N) + 2(C_{h_2}^N \frac{s_1^{2^N}}{2D^N} + \frac{(Q^N - s_1^N)^2}{2D^N} C_{s_2}^N) + (C_{h_3}^N \frac{s_1^{2^N}}{2D^N} + \frac{(Q^N - s_1^N)^2}{2D^N} C_{s_3}^N)$$

$$+ (C_{h_1}{}''^N \frac{s_1^{2^N}}{2D^N} + \frac{(Q^N - s_1^N)^2}{2D^N} C_{s_1}{}''^N) + 2(C_{h_2}^N \frac{s_1^{2^N}}{2D^N} + \frac{(Q^N - s_1^N)^2}{2D^N} C_{s_2}^N) + (C_{h_3}{}''^N \frac{s_1^{2^N}}{2D^N}$$

$$+ \frac{(Q^N - s_1^N)^2}{2D^N} C_{s_3}{}''^N)]$$

To find the minimum of $D(F(q)^N)$ by taking the derivative $D(F(q)^N)$ and equating it to zero,

(i.e) $\frac{1}{8}\{\frac{s_1^N}{D^N}[(C_{h_1}^N + C_{s_1}^N) + 4(C_{h_2}^N + C_{s_2}^N) + (C_{h_3}^N + C_{s_3}^N) + (C_{h_1}{}''^N + C_{s_1}{}''^N) + (C_{h_3}{}''^N + C_{s_3}{}''^N)] - \frac{Q^N}{D^N}[C_{s_1}^N +$

$4C_{s_2}^N + C_{s_3}^N + C_{s_1}{}''^N + C_{s_3}{}''^N]\} = 0$, we get

$$s_1^N = \frac{C_{s_1}^N + 4C_{s_2}^N + C_{s_3}^N + C_{s_1}{}''^N + C_{s_3}{}''^N}{(C_{h_1}^N + C_{s_1}^N) + 4(C_{h_2}^N + C_{s_2}^N) + (C_{h_3}^N + C_{s_3}^N) + (C_{h_1}{}''^N + C_{s_1}{}''^N) + (C_{h_3}{}''^N + C_{s_3}{}''^N)} Q^N$$

$$s_1^N = \frac{C_{s_1}^N + 4C_{s_2}^N + C_{s_3}^N + C_{s_1}{}''^N + C_{s_3}{}''^N}{(C_{h_1}^N + C_{s_1}^N) + 4(C_{h_2}^N + C_{s_2}^N) + (C_{h_3}^N + C_{s_3}^N) + (C_{h_1}{}''^N + C_{s_1}{}''^N) + (C_{h_3}{}''^N + C_{s_3}{}''^N)} D^N T^N \quad \dots\dots\dots\dots (1)$$

Also at $s_1^N = s_1^{N^*}$, we get $D^2(F(s_1^N)) > 0$

Hence, the minimum neutrosophic total cost is given by

$$F(q^N)^* = \frac{1}{8}\left[\left(C_{h_1}^N \frac{s_1^{2^{N^*}}}{2D^N} + \frac{(Q^N - s_1^{N^*})^2}{2D^N} C_{s_1}^N\right) + 2\left(C_{h_2}^N \frac{s_1^{2^{N^*}}}{2D^N} + \frac{(Q^N - s_1^{N^*})^2}{2D^N} C_{s_2}^N\right) + \left(C_{h_3}^N \frac{s_1^{2^{N^*}}}{2D^N} + \frac{(Q^N - s_1^{N^*})^2}{2D^N} C_{s_3}^N\right) +\right.$$

$$\left.\left(C_{h_1}{}''^N \frac{s_1^{2^{N^*}}}{2D^N} + \frac{(Q^N - s_1^{N^*})^2}{2D^N} C_{s_1}{}''^N\right) + 2\left(C_{h_2}^N \frac{s_1^{2^{N^*}}}{2D^N} + \frac{(Q^N - s_1^{N^*})^2}{2D^N} C_{s_2}^N\right) + \left(C_{h_3}{}''^N \frac{s_1^{2^{N^*}}}{2D^N} + \frac{(Q^N - s_1^{N^*})^2}{2D^N} C_{s_3}{}''^N\right)\right] \quad \dots\dots\dots (2)$$

6. Numerical Example

A commodity is to be furnished at a constant rate of 20 units per day. A penalty cost will be charged at a rate of Rs 8 per day, if it is past due for missing the scheduled shipping date. The cost of carrying the commodity in inventory is Rs 14 per unit per month. The production process is such that each month (30 days) a batch of items is started and is available for delivery any time after the end of the month. Find the optimal level of inventory at the beginning of each month. Find the optimal level of inventory at the beginning of each month.

Solution:

Given D = 20, T = 30 , C_h = 14/30 = 0.47 and C_s = 8

Using [4], the shortage quantity and minimum total cost for crisp set, fuzzy set and intuitionistic fuzzy sets are calculated. Also, they are compared with neutrosophic optimal shortage quantity and minimum neutrosophic total cost [by equation (1) and (2)] and tabulated as follows:

	Crisp Set	Fuzzy Set	Intuitionistic Fuzzy Set	Neutrosophic Set
D	20	20	20	20
T	30	30	30	30
C_h	14/30 = 0.47	(0.46,0.49,0.51)	(0.44,0.47,0.49) (0.42, 0.47, 0.51)	(0.44, 0.47, 0.49) (0.42, 0.47, 0.51) (0.4, 0.47, 0.53)
C_s	8	(6, 7, 9)	(6, 7, 9) (4, 7, 10)	(6, 7, 9) (4, 7, 10) (5, 7, 9)
Shortage quantity	567.376	563.654	563	563.06
Minimum total cost	260.993	264.917	266.612	266.513

7. Analytical Observations

In this section, the analysis of shortage quantity and minimum total cost for crisp set, fuzzy set, intuitionistic fuzzy set and neutrosophic set for table:1 is shown graphically.
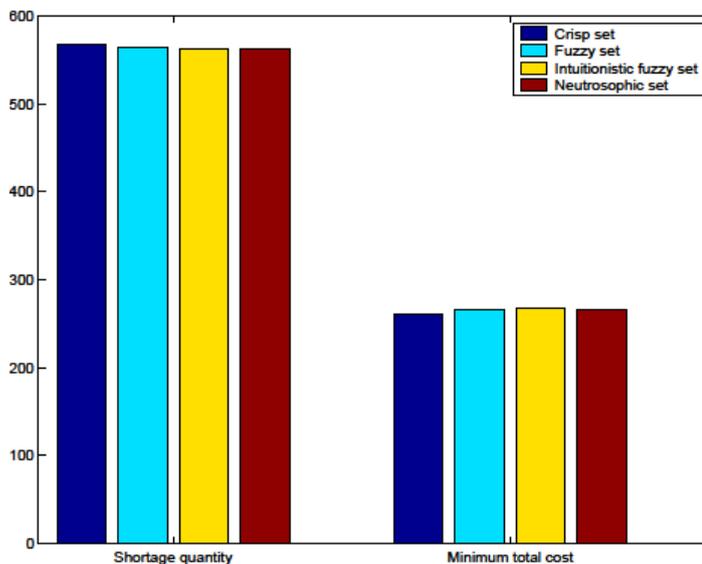

Figure 1: Neutrosophic backorder problem

Also, from the above analytical observations, we conclude that,

• The analysis of the problem under the optimal shortage quantity in neutrosophic environment is closer to crisp, fuzzy and intuitionistic fuzzy environments.

• The optimal shortage quantity in neutrosophic set increases when the optimal shortage quantity in intuitionistic fuzzy set decreases.

• The minimum total cost in neutrosophic set decreases when the minimum total cost in intuitionistic fuzzy set increases.

8. Conclusions

In this proposed model, the neutrosophic total cost and neutrosophic optimal shortage quantity in triangular neutrosophic numbers are obtained. In neutrosophic environment, the shortage quantity is as close to the inuitionistic fuzzy set. The benefit of the neutrosophic inventory model gives better result than fuzzy and intuitionistic fuzzy inventory models. A comprehensive sensitivity analysis has been performed to illustrate the impact of demand on the ordering policy comparing with existing methods. The present proposed work is helpful for business organizations where customer's demands are not fulfilled instantly. In future, the various neutrosophic inventory models will be developed with various limitations such as lead time, backlogging and deteriorating items, etc.

Conflicts of Interest
The authors declare no conflict of interest.

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