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Abstract: Neutrosophic set is a unique concept endowed with unconnected degree of indeterminacy excluded in the non-classical sets it generalizes. This paper communicates shortly on the notions of α - lower level and α - upper level sets of a neutrosophic set and investigates some basic properties.

Keywords: Neutrosophic set; α - lower level and α - upper level sets of a neutrosophic set

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1. Introduction

Uncertainty is unavoidable in real life situations as classical structure cannot handle them. Dealing with vague, uncertain or imperfect information was a huge task for many years. Many models were proposed in order to suitably integrate uncertainty into the system description. Zadeh [12] noticed typically that the collections of objects encountered in real world do not have exactly sharp boundaries of membership as described by a German mathematician, George Cantor (1845-1918). Consequently, he introduced fuzzy set concept and delineated it as a collection of objects with graded membership. However, Atanassov [6] initiated an extension of fuzzy set called intuitionistic fuzzy set. Intuitionistic fuzzy set accommodates additional degrees of freedom (non-membership and hesitation margin) into set description and is broadly used as a tool of intensive research by scholars and scientists.

One of the motivating generalizations of fuzzy set theory and intuitionistic fuzzy set theory is neutrosophic set theory introduced by Smarandache [11]. A neutrosophic set theory is independently characterized by a truth membership function, an indeterminate membership function and a falsity membership function. Therefore, the neutrosophic set theory has become a popular subject of research in problems associated with uncertainty.

Very recently, the scholarly world has witnessed growing research interests in the theory of neutrosophic sets such as medical diagnosis [1, 4, 5], database [7], topology [10], image processing [8], and decision-making problem [2, 3, 9].

The paper attempts to develop the concepts of α - lower level and α - upper level sets of a neutrosophic set and investigates some basic properties based on the related research of fuzzy sets and intuitionistic fuzzy sets with the aim to create a paradigm shift in the aspects of algebra.

2. Preliminaries

In this section, we will give some preliminary information that will be useful in the sequel of the paper

Definition 2.1 [11] A neutrosophic set (NS) A in a non-empty set X is a structure of the form

$A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle \mid x \in X\}$, where $T_A, I_A, F_A : X \longrightarrow]^-0, 1^+[$ define respectively the degree of membership (or Truth), the degree of indeterminacy, and the degree of non-membership (or Falsehood) of the element $x \in X$ to the set $x \in A$ with the condition $^-0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+$.
Here, $1^+ = 1 + c$, where 1 is its standard part and c its non-standard part. Analogously, $^-0 = 0 - c$ is expressed in turn.
The above definition has been used by several authors in literature with sizable number of publications. On the contrary, the results presented in this paper are devoid of non-standard and restricted to the interval $[0, 1]$ for practical techniques.

As an illustration, let us consider the following example.
Example 2.1 Assume that $X = \{a, b, c\}$, where a characterizes the competence, b characterizes the reliability and c indicates the costs of the objects. It may be further assumed that the values of a, b and c are in $[0, 1]$ and they are obtained from some surveys of some connoisseurs. The connoisseurs may impose their view in three components viz. the degree of goodness, the degree of indeterminacy and that of poorness to describe the characteristics of the objects. Suppose A is a neutrosophic set in X, such that,

$A = \{(a, \langle 0.3, 0.4, 0.5\rangle), (b, \langle 0.5, 0.2, 0.3\rangle), (c, \langle 0.7, 0.2, 0.2\rangle)\}$, where the degree of goodness of capability is 0.3, degree of indeterminacy of capability is 0.4 and degree of falsity of capability is 0.5 implying $T_A(a) = 0.3$, $I_A(b) = 0.4$, $F_A(c) = 0.5$ etc.

For simplicity, $A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle \mid x \in X\}$, can be expressed as $A(x) = \left(T_A(x), I_A(x), F_A(x)\right)$ since the membership functions T_A, I_A, F_A are defined from X into the unit interval $[0, 1]$.

Definition 2.2 [11] Let A and B be two neutrosophic sets in a non-empty set X. Then

$(\boldsymbol{i})$ $\boldsymbol{A \subseteq B \Leftrightarrow T_A(x) \leq T_B(x), I_A(x) \leq I_B(x), F_A(x) \geq F_B(x).}$
$(\boldsymbol{ii})$ $\boldsymbol{A = B \Leftrightarrow T_A(x) = T_B(x), I_A(x) = I_B(x), F_A(x) = F_B(x).}$
$(\boldsymbol{iii})$ $\boldsymbol{A \cap B} = \{\langle x, \wedge(T_A(x), T_B(x)), \wedge(I_A(x), I_B(x)), \vee(F_A(x), F_B(x))\rangle \mid x \in X\}$.
(iv) $\boldsymbol{A \cup B} = \{\langle x, \vee(T_A(x), T_B(x)), \vee(I_A(x), I_B(x)), \wedge(F_A(x), F_B(x))\rangle \mid x \in X\}$, where $\wedge$ and $\vee$ are minimum and maximum operations.
(v) $A^c = \{\langle x, F_A(x), 1 - I_A(x), T_A(x))\rangle \mid x \in X\}$.
(vi) $A \setminus B = \{\langle x, T_A \wedge F_B(x), I_A(x) \wedge 1 - I_B(x), F_A(x) \vee T_B(x))\rangle \mid x \in X\}$.

With reference to Definition 2.2 (v), $(A^c)^c = A$.

Remark 2.1 If $\{A_i \mid i \in J\}$ is a family of neutrosophic sets, then $\left(\cup_{i \in J} A_i\right)^c = \cap_{i \in J} A_i^c$ and $\left(\cap_{i \in J} A_i\right)^c = \cup_{i \in J} A_i^c$.

Proposition 2.1 Let A, B, C, D be any neutrosophic sets in a non-empty set X, we have

(i) if $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$.
(ii) if $A \subseteq B$, then $A^c \subseteq B^c$.
(iii) if $A \subseteq B$ and $A \subseteq C$, then $A \subseteq B \cap C$.
(iv) if $A \subseteq B$ and $C \subseteq B$, then $A \cup C \subseteq B$.
(v) if $A \subseteq B$ and $C \subseteq D$, then $A \cup C \subseteq B \cup D$ and $A \cap C \subseteq B \cap D$.

Proof. Immediate from definitions.

Definition 2.3 [11] A neutrosophic set A in a non-empty set X is said to be universe neutrosophic set if $T_A(x) = I_A(x) = 1$, $F_A(x) = 0$, $\forall x \in X$. It is denoted by 1_N.

A neutrosophic set A in a non-empty set X is said to be null neutrosophic set if $T_A(x) = I_A(x) = 0$, $F_A(x) = 1$, $\forall\, x \in X$. It is denoted by 0_N.

3. Main Results

Definition 3.1 Let A be any neutrosophic set in a non-empty set X. Then for any $\alpha \in [0,1]$, the α – lower level and the α – upper level sets of A denoted by $L(A,\alpha)$ and $U(A,\alpha)$ are respectively defined as follows:

$$L(A,\alpha) = \{x \in X \mid T_A(x) \geq \alpha,\ I_A(x) \geq \alpha,\ F_A(x) \leq \alpha\} \text{ and}$$

$$U(A,\alpha) = \{x \in X \mid T_A(x) \leq \alpha,\ I_A(x) \leq \alpha,\ F_A(x) \geq \alpha\}.$$

Example 3.1 Let $A = \{(a,\langle 0.4, 0.3, 0.5\rangle), (b,\langle 0.5, 0.3, 0.1\rangle), (c,\langle 0.2, 0.5, 0.9\rangle)\}$ and $\alpha \in [0,1]$. Then $L(A, 0.1) = L(A, 0.2) = L(A, 0.3) = \{b\}$, $L(A, 0.4) = \{\emptyset\}$, $\alpha \geq 0.4$. However, $U(A,\alpha) = \{\emptyset\}$, $0.1 \leq \alpha \leq 0.3$, $U(A, 0.4) = \{a\}$, $U(A, 0.5) = \{a, c\}$, $U(A, 0.6) = \{c\}$, $\alpha \geq 0.6$.

If A, B, C are neutrosophic sets in a non-empty X and $\alpha, \beta \in [0,1]$, then the results in the following proposition are not difficult to verify from definitions.

Proposition 3.1

(i) $A \subseteq B \Longrightarrow L(A,\alpha) \subseteq L(B,\alpha)$.
(ii) $\alpha \geq \beta \Longrightarrow L(A,\alpha) \supseteq L(A,\beta)$.
(iii) $L\left(\bigcap_{i\in J} A_i, \alpha\right) = \bigcap_{i\in J} L(A_i,\alpha)$.
(iv) $U(A,\alpha) \subseteq L(A,\alpha)$.

Proposition 3.2

$(\boldsymbol{i})$ $L(A \cup B,\alpha) = L(A,\alpha) \cup L(B,\alpha)$.
(ii) $L(A \cap B,\alpha) = L(A,\alpha) \cap L(B,\alpha)$.
(iii) $A = B \Leftrightarrow L(A,\alpha) = L(B,\alpha)$, $\forall\, \alpha \in [0,1]$.

Proof.

(i) $L(A \cup B,\alpha) = \{x \in X \mid T_{A\cup B}(x) \geq \alpha,\ I_{A\cup B}(x) \geq \alpha,\ F_{A\cup B}(x) \leq \alpha\}$
$= \{x \in X \mid T_A(x) \vee T_B(x) \geq \alpha,\ I_A(x) \vee I_B(x) \geq \alpha,\ F_A(x) \wedge F_B(x) \leq \alpha\}$
$= \{x \in X \mid T_A(x) \geq \alpha \cup T_B(x) \geq \alpha,\ I_A(x) \geq \alpha \cup I_B(x) \geq \alpha,\ F_A(x) \leq \alpha \cup F_B \leq \alpha\}$
$=$
$\{x \in X \mid T_A(x) \geq \alpha,\ I_A(x) \geq \alpha,\ F_A(x) \leq \alpha\} \cup \{x \in X \mid T_B(x) \geq \alpha,\ I_B(x) \geq \alpha,\ F_B(x) \leq \alpha\}$
$= L(A,\alpha) \cup L(B,\alpha)$

Hence, $L(A \cup B,\alpha) = L(A,\alpha) \cup L(B,\alpha)$.

(ii) Similar to the proof of (i).

(iii) Clearly, $A = B \Longrightarrow T_A(x) = T_B(x),\ I_A(x) = I_B(x),\ F_A(x) = F_B(x)\ \forall\, x \in X$.
Undoubtedly, $L(A,\alpha) = \{x \in X \mid T_A(x) \geq \alpha,\ I_A(x) \geq \alpha,\ F_A(x) \leq \alpha\}$ and
$L(B,\alpha) = \{x \in X \mid T_B(x) \geq \alpha,\ I_B(x) \geq \alpha,\ F_B(x) \leq \alpha\}$.
But $A = B\ \forall\, x \in X$. Hence, $L(A,\alpha) = L(B,\alpha)$, $\forall\, \alpha \in [0,1]$.
Conversely, suppose that $\forall\, \alpha \in [0,1]$, $L(A,\alpha) = L(B,\alpha)$ but $A \neq B$. Moreover, $A \neq B$ if and only if
there exists some $y \in X$ such that $T_A(y) \neq T_B(y)$, $I_A(y) \neq I_B(y)$, $F_A(y) \neq F_B(y)$. Without loss of generality, assume that $T_A(y) \leq T_B(y)$, $I_A(y) \leq I_B(y)$, $F_A(y) \leq F_B(y)$ and let $\gamma = T_B(y) = I_B(y) =$

$F_B(y)$. It must be that $y \notin L(A,\gamma)$ but $y \in L(B,\gamma)$. Then $L(A,\alpha)$ and $L(B,\alpha)$ are identical, and this
is a contradiction.

The distributive laws are satisfied for α – lower level sets of a neutrosophic set.

Proposition 3.3
(i) $L(A \cup (B \cap C), \alpha) = L(A \cup B, \alpha) \cap L(A \cup C, \alpha)$.
(ii) $L(A \cap (B \cup C), \alpha) = L(A \cap B, \alpha) \cup L(A \cap C, \alpha)$.

Proof. Similar to the proof of Proposition 3.2.

Theorem 3.1 Let A be a neutrosophic set in a non-empty set X and $\alpha, \beta \in [0,1]$. If α comprises all finite values in $[0,1]$ and $\alpha \leq \beta$, then $\cap L(A,\alpha) = L(A,\beta)$.

Proof.
Let $x \in \cap L(A,\alpha)$. Then $x \in L(A,\alpha)\ \forall\, \alpha \in [0,1]$.
$\Rightarrow T_A(x) \geq \alpha,\ I_A(x) \geq \alpha,\ F_A(x) \leq \alpha\ \ \forall\, \alpha \in [0,1],\ x \in X$.
Since $\alpha \leq \beta$, then $T_A(x) \geq \alpha \leq \beta,\ I_A(x) \geq \alpha \leq \beta,\ F_A(x) \leq \alpha \leq \beta\ \ \forall\, \alpha \in [0,1]$.
$\Rightarrow \cap L(A,\alpha) \subseteq L(A,\beta)$.
Conversely, let $x \in L(A,\beta)$, then $T_A(x) \geq \beta,\ I_A(x) \geq \beta,\ F_A(x) \leq \beta,\ \forall\ x \in X$.
$\Rightarrow T_A(x) \geq \beta \geq \alpha,\ I_A(x) \geq \beta \geq \alpha,\ F_A(x) \leq \beta \leq \alpha,\ \forall\, \alpha \in [0,1]$.
$\Rightarrow T_A(x) \geq \alpha,\ I_A(x) \geq \alpha,\ F_A(x) \leq \alpha,\ \forall\, \alpha \in [0,1]$.
$\Rightarrow L(A,\beta) \subseteq \cap L(A,\alpha)$.
Hence, $\cap L(A,\alpha) = L(A,\beta)$.

Proposition 3.4 Let A be a universal neutrosophic set in a non-empty set X and $\alpha \in [0,1]$. Then $L(A,0) = X$.

Proof. Straightforward.

Remark 3.1 If A is a universal neutrosophic set in a non-empty set X and $\alpha \in [0,1]$, then $L(A,0) = L(A,1)$.

Theorem 3.2 If $L(A,\alpha),\ \alpha \in [0,1]$ be the α – lower level sets of a neutrosophic set in a non-empty set X such that $\cap\, \alpha U(F_A,\alpha)$ is restricted to non-zero values, then $A = \cup_{\alpha \in [0,1]} \alpha L(A,\alpha)$.

Proof.
$A(x) = (T_A(x),\ I_A(x),\ F_A(x)) = (a,b,c)$ and for each $\alpha \in (a,1],\ \alpha \in (b,1],\ \alpha \in (0,c)$, we have $T_A(x) = a < \alpha,\ I_A(x) = b < \alpha$ and $F_A(x) = c > \alpha$. Thus, $L(A,\alpha) = (0,0,0)$.
However, for each $\alpha \in (0,a],\ \alpha \in (0,b],\ \alpha \in [c,1)$, we have $T_A(x) = a \geq \alpha,\ I_A(x) = b \geq \alpha$ and $F_A(x) = c \leq \alpha$. Thus, $L(A,\alpha) = (1,1,1)$.
Hence, $\cup_{\alpha \in [0,1]} \alpha L(A,\alpha) = (\vee_{\alpha \in (0,a]} \alpha = a = T_A(x),\ \vee_{\alpha \in (0,b]} \alpha = b = I_A(x),\ \wedge_{\alpha \in [c,1)} \alpha = c = F_A(x))$ with the restriction on $\cap\, \alpha U(F_A,\alpha)$ to be considered non-zero values. This completes the proof.

Example 3.2 Let A be any neutrosophic set in a non-empty set X, given by
$A = \{(a, \langle 0.4, 0.3, 0.5\rangle), (b, \langle 0.5, 0.3, 0.1\rangle), (c, \langle 0.2, 0.5, 0.9\rangle)\}$.

For expediency, let us denote A as

$A = \{(0.4, 0.3, 0.5)/a, (0.5, 0.2, 0.3)/b, (0.7, 0.2, 0.2)/c\}$.

Then

$$L(A, 0.1) = \{(1,1,0)/a, (1,1,1)/b, (1,1,0)/c\}$$
$$L(A, 0.2) = \{(1,1,0)/a, (1,1,1)/b, (1,1,0)/c\}$$
$$L(A, 0.3) = \{(1,1,0)/a, (1,1,1)/b, (0,1,0)/c\}$$
$$L(A, 0.4) = \{(1,0,0)/a, (1,0,1)/b, (0,1,0)/c\}$$
$$L(A, 0.5) = \{(0,0,1)/a, (1,0,1)/b, (0,1,0)/c\}$$
$$L(A, 0.9) = \{(0,0,1)/a, (0,0,1)/b, (0,0,1)/c\}$$

It is not difficult to see that

$$A = 0.1L(A, 0.1) \cup 0.2L(A, 0.2) \cup 0.3L(A, 0.3) \cup 0.4L(A, 0.4) \cup 0.5\, L(A, 0.5) \cup 0.9L(A, 0.9).$$

The following results presented below are for α – upper level sets of a neutrosophic set.

Proposition 3.5

(i) $A \subseteq B \Longrightarrow U(B, \alpha) \subseteq U(A, \alpha)$.
(ii) $\alpha \le \beta \Longrightarrow U(A, \alpha) \subseteq U(A, \beta)$.
(iii) $\bigcap_{i\in J} U(A_i, \alpha) \subseteq U\left(\bigcap_{i\in J} A_i, \alpha\right)$.

Proof. Straightforward.

Proposition 3.6 If A and B are two neutrosophic sets in a non-empty set X and $\alpha \in [0, 1]$, then

$(\boldsymbol{i})$ $U(A\cap B, \alpha) \supseteq U(A, \alpha) \cap U(B, \alpha)$.
(ii) $U(A\cup B, \alpha) = U(A, \alpha) \cup U(B, \alpha)$.
(iii) $A = B \Leftrightarrow U(A, \alpha) = U(B, \alpha), \ \forall\, \alpha \in [0, 1]$.

Proof.

(i) $U(A\cap B, \alpha) = \{\, x \in X \mid T_{A\cap B}(x) \le \alpha,\ I_{A\cap B}(x) \le \alpha,\ F_{A\cap B}(x) \ge \alpha \,\}$

$= \{\, x \in X \mid T_A(x) \wedge T_B(x) \le \alpha,\ I_A(x) \wedge I_B(x) \le \alpha,\ F_A(x) \vee F_B(x) \ge \alpha \,\}$

$\ge \{\, x \in X \mid T_A(x) \le \alpha \cap T_B(x) \le \alpha,\ I_A(x) \le \alpha \cap I_B(x) \le \alpha,\ F_A(x) \ge \alpha \cup F_B \ge \alpha \,\}$

$=$

$\{\, x \in X \mid T_A(x) \le \alpha,\ I_A(x) \le \alpha,\ F_A(x) \ge \alpha \,\} \cap \{\, x \in X \mid T_B(x) \le \alpha,\ I_B(x) \le \alpha,\ F_B(x) \ge \alpha \,\}$

$= U(A, \alpha) \cap U(B, \alpha)$

Hence, $U(A\cap B, \alpha) \supseteq U(A, \alpha) \cap U(B, \alpha)$.

(ii) It is obtained in a similar way.

(iii) The proof is similar to the proof of Proposition 3.2(iii).

Proposition 3.7

(i) $U(A\cup(B\cap C), \alpha) \subseteq U(A\cup B, \alpha) \cap U(A\cup C, \alpha)$.
(ii) $U(A\cap(B\cup C), \alpha) \subseteq U(A\cap B, \alpha) \cup U(A\cap C, \alpha)$.

Proof. Similar to the proof of Proposition 3.6(i).

Proposition 3.8 Let A be a null neutrosophic set in a non-empty set X and $\alpha \in [0, 1]$. Then $U(A, 0) = X$.

Proof. Straightforward.

Remark 3.2 If A is a null neutrosophic set in a non-empty set X and $\alpha \in [0,1]$, then $U(A,0) = U(A,1)$.

Theorem 3.3 If $U(A,\alpha)$, $\alpha \in [0,1]$ be the α – upper level sets of a neutrosophic set in a non-empty set X such that $\cap\, \alpha U(T_A,\alpha)$ and $\cap\, \alpha U(I_A,\alpha)$ are restricted to non-zero values, then $A = \cap_{\alpha\in[0,1]} \alpha U(A,\alpha)$.

Proof.
The proof is analogous to the proof of Theorem 3.2.
Let $A(x) = (T_A(x),\ I_A(x),\ F_A(x)) = (a,b,c)$. Then $T_A(x) = a > \alpha$, $I_A(x) = b > \alpha$ and $F_A(x) = c < \alpha$, $\forall\ \alpha \in [0,a)$, $\alpha \in [0,b)$, $\alpha \in (c,1]$. Thus, $U(A,\alpha) = (0,0,0)$.
On the other hand, $T_A(x) = a \leq \alpha$, $I_A(x) = b \leq \alpha$ and $F_A(x) = c \geq \alpha$, $\forall\ \alpha \in [a,1)\ \alpha \in [b,1)\ \alpha \in (0,c]$. Thus, $U(A,\alpha) = (1,1,1)$.
Hence, $\cap_{\alpha\in[0,1]} \alpha U(A,\alpha) = (\wedge_{\alpha\in[a,1)}\alpha = a = T_A(x),\ \wedge_{\alpha\in[b,1)}\alpha = b = I_A(x),\ \vee_{\alpha\in(0,c]}\alpha = c = F_A(x))$ with the restriction on $\cap\, \alpha U(T_A,\alpha)$ and $\cap\, \alpha U(I_A,\alpha)$ to be considered non-zero values. Hence the proof.

5. Conclusions (authors also should add some future directions points related to her/his research)

The concepts of α – lower level and α – upper level sets and their properties in neutrosophic sets are described. This study is worthy of level sets extension in the hybrid set structures such as neutrosophic multisets, neutrosophic soft sets and rough neutrosophic sets.

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