Neutrosophic Sets and Systems

Volume 31 Article 7

5-2-2020

Neutrosophic Bipolar Fuzzy Set and its Application in Medicines Preparations

Raja Muhammad Hashim

Muhammad Gulistan

Inayatur Rehman

Nasruddin Hassan

Abdul Muhaimin Nasruddin

Neutrosophic Bipolar Fuzzy Set and its Application in Medicines Preparations

Raja Muhammad Hashim[1], Muhammad Gulistan[1], Inayatur Rehman[2], Nasruddin Hassan[3,]* and Abdul Muhaimin Nasruddin[4]

[1] Department of Mathematics and Statistic Hazara University, Mansehra Pakistan; hashimmaths@hu.edu.pk, gulistanmath@hu.edu.pk ,

[2] Department of Mathematics & Sciences, College of Arts and Applied Sciences, Dhofar University Salalah, Oman; irehman@du.edu.om,

[3] School of Mathematical Sciences, Faculty of Science and Technology, University Kebangsaan Malaysia, Bangi 43600, Selangor Malaysia; nas@ukm.edu.my

[4] Department of Management and Marketing, Faculty of Economics and Management, Universiti Putra Malaysia, Serdang 43400, Selangor Malaysia; abdulmuhaimin085@gmail.com

* Correspondence: nas@ukm.edu.my; Tel.: (+60 192145750)

Abstract: To tackle the real life problems we come across, in various fields like computer sciences, medical sciences, social sciences and engineering works where we are facing many ambiguities and imprecisions. Here we bring an idea of neutrosophic bipolar fuzzy decision making where hybridized multi-attributes are involved, which is a very helpful tool to tackle the ambiguities and imprecisions. We present the neutrosophic bipolar fuzzy transformation techniques. The different types of attributes are transformed into unified neutrosophic bipolar fuzzy values. It includes the group decision making mode based on hybrid decision making problems with exact values, interval values and linguistic variables. Calculations of weights by decision makers, composition of aggregated weighted neutrosophic bipolar fuzzy decision matrices, determination of entropy weights, finding positive ideal solution(PIS),and negative ideal solution(NIS), calculation of grey relational coefficient ,calculation of degree of weighted grey relational coefficient of each alternative, determination of relative relational degree of each alternative from the positive ideal solution (PIS) and negative ideal solution (NIS) and ranking of the alternatives are the concepts which are introduced in the case of neutrosophic bipolar fuzzy hybrid multi-attribute group decision making. Eventually, we apply these concepts and techniques upon hybrid multi-attributes decision making problem of selecting the best medicine to cure some particular diseases and develop an algorithm for neutrosophic bipolar fuzzy hybrid multi-attribute group decision making.

Keywords: Neutrosophic bipolar fuzzy sets; multi-attribute group decision making; neutrosophic bipolar fuzzy transformation techniques; interval values and linguistic variables.

1. Introduction

The concept of fuzzy set theory was basically given by Zadeh [1]. The idea of fuzzy set theory has been extended to vague fuzzy set [2-5], interval-valued fuzzy set, intuitionistic fuzzy set [6], L-fuzzy set, Q-fuzzy set [7-11], probabilistic fuzzy set and so on, [12-19]. All these versions had limitations in different situations. Smarandache [20], gave the idea of neutrosophic set which is the generalization of all previous versions of fuzzy sets. Unfortunately, these, models were handling the problems involving only positive preferences and opinions, whereas human mind tends to work in both directions, positive and negative, in order to come up with a decision. Therefore, to bridge up this deficiency Zhang [21], introduced the notion of bipolar fuzzy sets. The features of bipolar fuzzy sets were considered and discussed in detail by Naveed at al. [22-24], Dubois et al. [25] and Silva et al. [26]. The applications of neutrosophic set theory are found in various fields of life, like computer sciences, physical sciences, medical sciences, social sciences, engineering and multi-criteria group decision making problems. The uses of neutrosophic theory for sets in decision making problems (DMP) have been considered by Basset et al. [27-31]. Qun et al. [32] and many others in many [33-36], they gave the idea of linguistic multiple attribute group decision making (LMAGDM). Chen [37] and Hung [38], introduced the idea of manipulation of multiple attribute decision making problems depends upon fuzzy sets. Later on Zhan et al. [39] applied the neutrosophic cubic sets in multi-criteria decision-making issues. Gulistan et al. [40] discussed the notion of neutrosophic cubic graphs and gave the real-life applications in industrial areas. Applications of neutrosophic sets in different directions can be seen in [41-44] and [45-52].

Neutrosophic sets are more general versions to handle the uncertain data problems when compared to the different versions of fuzzy sets. When handling uncertain issues where both positive and negative characteristics are involved, the bipolar fuzzy sets are found to be helpful. In propensity to take decisions considering both positive and negative preferences, we [45], recently defined the concept of neutrosophic bipolar fuzzy sets. We also defined neutrosophic bipolar fuzzy weighted averaging and neutrosophic bipolar fuzzy ordered weighted averaging operators.

In this paper, we will extend the neutrosophic bipolar fuzzy set by introducing the idea of neutrosophic bipolar fuzzy hybrid multi-attribute group decision making where we use the different neutrosophic bipolar fuzzy transformation techniques. We give the new conversion techniques between the exact values and neutrosophic bipolar fuzzy numbers. The conversion techniques between interval values and neutrosophic bipolar fuzzy numbers have also been considered and likewise we also discuss the transformations techniques between linguistic variables and neutrosophic bipolar fuzzy numbers. Graphical representations of the notions in this paper have been considered as well. Finally, numerical example related to a medicine company which intends to prepare three different types of medicines for a certain type of disease.

2. Preliminaries

In this section we provide some of the precursors in developing our new concept.

Definition 2.1. [1] A fuzzy set maps the elements of a universe X to the unit interval $[0,1]$.

Definition 2.2. [13] Let X be a universe of discourse. An intuitionistic fuzzy set, A in X is an object having the following form $A = \{\langle x, \mu(x), \nu(x)\rangle : x \in X\}$

where $\mu_A(x)$ is known as a degree of membership and $\nu_A(x)$ is known as a degree of non-membership of the element X to the IFS A with the condition, $0 \le \mu(x) \le 1$, $0 \le \nu(x) \le 1$, $0 \le \mu(x) + \nu(x) \le 1$. For each IFS A in X. The hesitancy indeterminacy degree measure as follows, $\pi_A(x) = 1 - \mu(x) - \nu(x)$. Then $\pi_A(x)$ is known as degree of indeterminacy membership of x to the set A and $\forall\ x \in X$.

Definition 2.3. [21] Let X be a non-empty set. Then a bipolar fuzzy set, is an object of the form $B = \langle x, \langle \mu^+(x),\ \mu^-(x)\rangle : x \in X\rangle$, where $\mu^+(x): X \to [0,1]$ and $\mu^-(x): X \to [-1,0]$, $\mu^+(x)$ is a positive material and $\mu^-(x)$ is a negative material of $x \in X$. For simplicity, we write the bipolar fuzzy set as $B = \langle \mu^+,\ \mu^-\rangle$ instead of $B = \langle x, \langle \mu^+(x),\ \mu^-(x)\rangle : x \in X\rangle$.

Definition 2.4. [32, 34, 41] A single valued neutrosophic set, is defined as;

$$A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle : x \in X\},$$

where X be the universe of discourse and A is characterized by a t-membership function $T_A: X \to [0,1]$, an i-membership function $I_A: X \to [0,1]$ and a f-membership function $F_A : X \to [0,1]$, where $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$.

Definition 2.5. [6] A neutrosophic set, is defined as:

$$A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle : x \in X\}$$

and X is a universe of discourse and A is characterized by a t-membership function $T_A : X \to]0^-, 1^+[$, an i-membership function $I_A : X \to]0^-, 1^+[$ and a f-membership function $F_A : X \to]0^-, 1^+[$. There is no condition on the sum of $T_A(x), I_A(x), F_A(x)$, so $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$.

Definition 2.6. [45] Let X be a non-vacuous set. Then a neutrosophic bipolar fuzzy set, is an object of the form $NB = (NB^+, NB^-)$where

$NB^+ = \langle y, \langle T_{NB^+}, I_{NB^+}, F_{NB^+}\rangle : x \in X\rangle$, $, NB^- = \langle y, \langle T_{NB^-}, I_{NB^-}, F_{NB^-}\rangle : x \in X\rangle$ such that $T_{NB^+}, I_{NB^+}, F_{NB^+} : X \to [0,1]$ and $T_{NB^-}, I_{NB^-}, F_{NB^-} : X \to [-1,0]$.

Definition 2.7. [45] Let $NB_j = (NB_j^+, NB_j^-)$ be the collection of neutrosophic bipolar fuzzy values. Then a mapping $NBFWA_\omega : \Omega^n \to \Omega$ defined by

$$NBFWA_\omega(NB_1, NB_2, \ldots, NB_n) = \omega_1 NB_1 \oplus \omega_2 NB_2 \oplus, \ldots, \oplus \omega_n NB_n$$

is called a neutrosophic bipolar fuzzy weighted averaging $(NBFWA)$ operator of dimension n, where $w = (w_1, w_2, \ldots, w_n)^T$ is the weight vector of $NB_j(j = 1,2, \ldots, n)$, with $\omega_j \in [0,1]$ and $\Sigma_{j=1}^n w_j = 1$.

Especially, if $\omega = \left(\frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n}\right)^T$, then the $NBFWA$ operator is reduced to a neutrosophic bipolar fuzzy averaging $(NBFA)$ operator of dimension n, which is defined as follows:

$$NBFA(NB_1, NB_2, \ldots, NB_n) = \frac{1}{n}(NB_1 \oplus NB_2 \oplus, \ldots, \oplus NB_n).$$

Definition 2.8. [45] Let $NB_j = (NB_j^+, NB_j^-)$ be a collection of neutrosophic bipolar fuzzy values. A neutrosophic bipolar fuzzy ordered weighted averaging $(NBFOWA)$operator of n dimension is a mapping $NBFOWA : \Omega^n \to \Omega$, that has an associated vector:

$\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ such that $\omega_j \in [0,1]$ and $\Sigma_{j=1}^n \omega_j = 1$. Furthermore

$$NBFOWA_\omega(NB_1^+, NB_2^+, \ldots, NB_n^+) = \omega_1 NB_{\sigma(1)}^+ \oplus \omega_2 NB_{\sigma(2)}^+ \oplus, \ldots, \oplus \omega_n NB_{\sigma(n)}^+$$

$$NBFOWA_\omega(NB_1^-, NB_2^-, \ldots, NB_n^-) = \omega_1 NB_{\sigma(1)}^- \oplus \omega_2 NB_{\sigma(2)}^- \oplus, \ldots, \oplus \omega_n NB_{\sigma(n)}^-$$

where $(\sigma(1), \sigma(2), \ldots, \sigma(n))$ is a permutation of $(1,2, \ldots, n)$ such that $NB_{\sigma(j-1)} \ge NB_{\sigma(j)}$ for all j .

Especially, if $\omega = \left(\frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n}\right)^T$, then the $NBFOWA$ operator is reduced to a bipolar fuzzy averaging

(NBFA) operator of dimension n .

Definition 2.9. [17] A linguistic variable, is a variable whose values are words or sentences in natural or artificial language.

3. Neutrosophic Bipolar Fuzzy Transformations Techniques

In this section we develop the neutrosophic bipolar fuzzy hybrid (MADM) with different types of data values. The neutrosophic bipolar fuzzy hybrid (MADM) problem based on four different data types, exact values, intervals, NBFNs and linguistic terms. Let $NB = \{NB_1, NB_2, \dots, NB_n\}$ be a finite set of alternatives, and let $C = \{c_1, c_2, \dots c_n\}$ be a set of attributes with weight vector $w = (w_1, w_2, \dots, w_m)$, where $w \geq 0$ $(j = 1,2,\dots,m)$ and

$$\sum_{j=1}^{m} w_j = 1.$$

Let $R^k = (a_{ij}^{(k)})_{n\times m}$ be a neutrosophic bipolar fuzzy hybrid decision matrix, where $(a_{ij}^{(k)})$ will be the exact values, intervals, NBFNs, and linguistic terms. We need to transform three other types of attributed values in R^k into unified NBFNs. In the following discussion, we will explore the transformation techniques for each of the data types.

3.1. Conversion between exact values and NBFNs

The values of different attributes have different dimensions. Thus, the real numbers in the hybrid decision making need to be standardized in order to eliminate interference in the results. Generally, there are two kinds of attributes, the benefit type and the cost. The higher the benefit type value is, the better it is. While in the cost type, it is the opposite. For the benefit type, formula is

$$b_{ij}^{(k)} = \frac{a_{ij}^{(k)}}{\sqrt{\sum_{i=1}^{m} (a_{ij}^{(k)})^2}}. \tag{1}$$

The cost type formula is;

$$b_{ij}^{(k)} = \frac{\left(\frac{1}{a_{ij}^{(k)}}\right)}{\sqrt{\sum_{i=1}^{m}\left(\frac{1}{(a_{ij}^{(k)})}\right)^2}}. \tag{2}$$

Standardized precise number can be transformed into neutrosophic bipolar fuzzy numbers as

$$a_{ij}^{(k)} = ((\mu_{ij}^{+(k)}, I_{ij}^{+(k)}, F_{ij}^{+(k)}), (\mu_{ij}^{-(k)}, I_{ij}^{-(k)}, F_{ij}^{-(k)}))$$

$$\mu_{ij}^{+(k)} = b_{ij}^{(k)}, F_{ij}^{(k)} = \frac{\mu_{ij}^{(k)}}{2}, I_{ij}^{(k)} = \frac{\mu_{ij}^{(k)}}{3}, \mu_{ij}^{-(k)} = -1 + b_{ij}^{(k)},$$

$$F_{ij}^{-(k)} = \frac{\mu_{ij}^{-(k)}}{2}, I_{ij}^{-(k)} = \frac{\mu_{ij}^{-(k)}}{3} \tag{3}$$

For intervals and NBFNs, for the benefit type formula is,

$$b_{ij}^{L(k)} = \frac{a_{ij}^{L(k)}}{\sqrt{\sum_{i=1}^{m} (a_{ij}^{U(k)})^2}}, \quad b_{ij}^{U(k)} = \frac{a_{ij}^{U(k)}}{\sqrt{\sum_{i=1}^{m} (a_{ij}^{L(k)})^2}}. \quad (4)$$

For the cost type formula is;

$$b_{ij}^{L(k)} = \frac{\left(\frac{1}{a_{ij}^{U(k)}}\right)}{\sqrt{\sum_{i=1}^{m} \left(\frac{1}{(a_{ij}^{L(k)})}\right)^2}}, \quad b_{ij}^{U(k)} = \frac{a_{ij}^{L(k)}}{\sqrt{\sum_{i=1}^{m} \left(\frac{1}{(a_{ij}^{U(k)})}\right)^2}}. \quad (5)$$

Standardized interval numbers can be transformed into neutrosophic bipolar fuzzy numbers as follows;

$$a_{ij}^{(k)} = \left(\left(\mu_{ij}^{+(k)}, I_{ij}^{+(k)}, F_{ij}^{+(k)}\right), \left(\mu_{ij}^{-(k)}, I_{ij}^{-(k)}, F_{ij}^{-(k)}\right)\right), \quad \mu_{ij}^{(k)} = b_{ij}^{L(k)}, F_{ij}^{(k)} = \frac{\mu_{ij}^{(k)}}{3}, I_{ij}^{(k)} = \frac{\mu_{ij}^{(k)}}{2}$$

$$\mu_{ij}^{-(k)} = -1 + b_{ij}^{U(k)}, F_{ij}^{-(k)} = \frac{\mu_{ij}^{-(k)}}{3}, I_{ij}^{-(k)} = \frac{\mu_{ij}^{-(k)}}{2} \quad (6)$$

Note: The indeterminacy $I \neq 1 - \mu - F$. We have defined functions F and I as in [3,6] to be used in this paper.

3.2. Conversion between linguistic variables and NBFNs

Linguistic variables are used usually when situations are complex or not well defined. The words or sentences given by the decision makers for rating or ranking like very good, good, fine, poor, very poor etc., can be converted into, and expressed as a quantities (NBFNs). The linguistic variables for the position of the decision makers can be expressed in NBFNs in Table 1 and shown as in Figure 1.

Table 1. Linguistic variable for the important of decision makers

Linguistic variable	NBFNs
Very important	((0.85, 0.42, 0.28), (-0.10, -0.05, -0.03))
Important	((0.70, 0.35, 0.23), (-0.2, -0.10, -0.06))
Medium	((0.55, 0.27,0.18), (-0.30, -0.15, -0.10))
Unimportant	((0.30, 0.15, 0.10), (-0.60, -0.30, -0.20))
Very unimportant	((0.10, 0.05, 0.03), (-0.90, -0.45, -0.30))

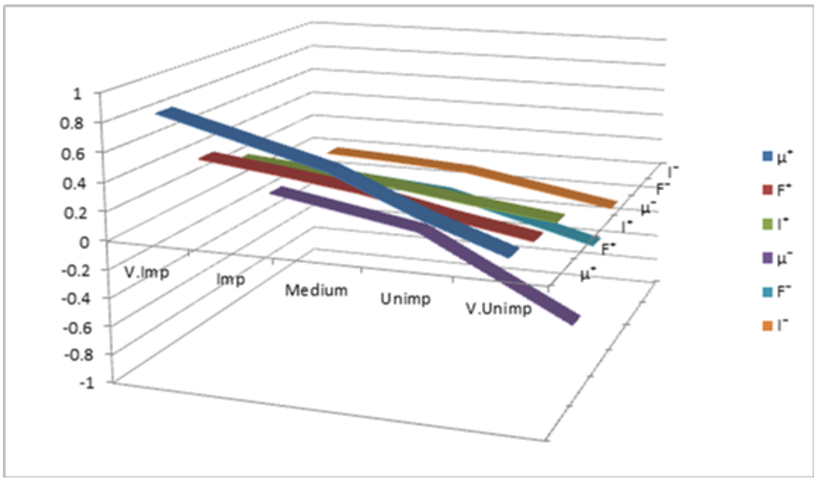

Figure 1. Graphical representation of importance of linguistic variables

Table 2. Conversion of linguistic variable into NBFNs

Linguistic variable	NBFNs
Extremely high (EH)	((0.95,0.47,0.31), (-0.03,-0.015,-0.01))
Very very high (VVH)	((0.83,0.41,0.27), (-0.10,-0.05,-0.03))
Very high (VH)	((0.77,0.38,0.25), (-0.12,-0.06,-0.04))
High (H)	((0.65,0.32,0.21), (-0.21,-0.10,-0.07))
Medium high (MH)	((0.55,0.27,0.18), (-0.32,-0.16,-0.10))
Medium (M)	((0.50,0.25,0.16), (-0.38,-0.19,-0.12))
Medium low (ML)	((0.35,0.17,0.11), (-0.45,-0.22,-0.15))
Low (L)	((0.22,0.11,0.07), (-0.3,-0.15,-0.1))
Very low (VL)	((0.12,0.06,0.04), (-0.87,-0.43,-0.29))
Very very low (VVL)	((0.06,0.03,0.02), (-0.93,-0.46,-0.31))

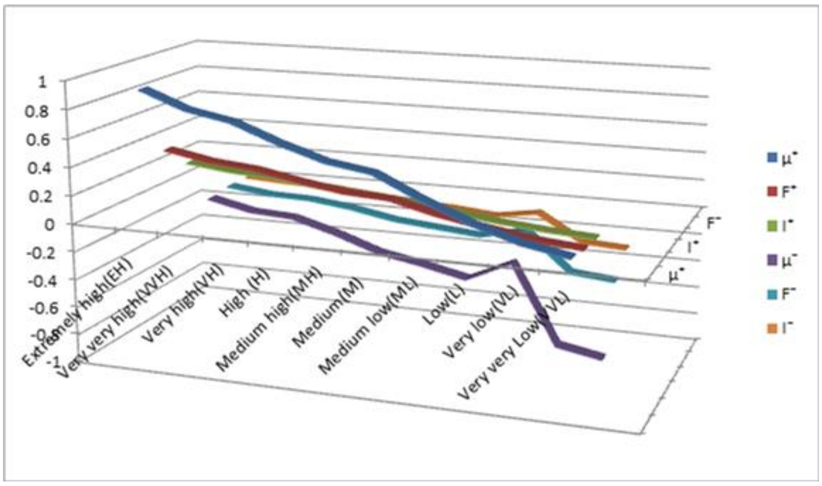

Figure 2. The rating of alternatives

The ratings of alternatives with respect to qualitative criteria can be converted into NBFNs as shown in Table 2 and shown as in Figure 2.

4. Neutrosophic Bipolar Fuzzy Hybrid Multi-Attribute Decision-Making

Neutrosophic bipolar fuzzy hybrid multi-attribute decision making problems are defined on a set of alternatives, from which the decision makers must select the best alternative according to some criteria. Suppose that there exists an alternative set $NB = \{NB_{1,}NB_{2,}, \dots, NB_{n,}\}$ which consists of n alternatives, the decision makers will choose the best one from NB according to an attribute set $C = \{c_1, c_2, \dots, c_m\}$ in which m attributes are there. For convenience, we denote the weight vector of attribute by $w = \{w_1, w_2, \dots, w_m\}^T$, where $w_j \geq 0\ (j = 1,2, \dots, m)$ and

$$\sum_{j=1}^{m} w_j = 1.$$

We develop an algorithm for neutrosophic bipolar fuzzy hybrid MADM as follows:

Step 1. Consider the neutrosophic bipolar fuzzy hybrid decision matrix of each decision maker. The neutrosophic bipolar fuzzy hybrid decision matrix involves four different data types: exact values, intervals, NBFNs, and linguistic terms.

Step 2. In this step we use the transformation techniques to transform exact values, interval values, and linguistic variables, into neutrosophic bipolar fuzzy information. Assume that the rating of alternative $A_i (j = 1,2, \dots, n)$ with respect to attribute c_j given by the kth experts e_k can be expressed in $a_{ij}^{(k)} = ((\mu_{ij}^{+(k)}, I_{ij}^{+(k)}, F_{ij}^{+(k)}), (\mu_{ij}^{-(k)}, I_{ij}^{-(k)}, F_{ij}^{-(k)}))$. Hence a hybrid multiattribute group decision-making problem can be concisely expressed in a matrix format as:

$$R^{(k)} = (\alpha_{ij}^{(k)})_{n\times m} = \begin{bmatrix} \alpha_{11}^{(k)} & \alpha_{11}^{(k)} & .. & .. & \alpha_{1m}^{(k)} \\ \alpha_{21}^{(k)} & \alpha_{22}^{(k)} & .. & .. & \alpha_{2m}^{(k)} \\ . & . & & & . \\ . & . & .. & .. & . \\ . & . & & & . \\ \alpha_{n1}^{(k)} & \alpha_{n2}^{(k)} & .. & .. & \alpha_{nm}^{(k)} \end{bmatrix} \quad (7)$$

where $a_{ij}^{(k)} = ((\mu_{ij}^{+(k)}, I_{ij}^{+(k)}, F_{ij}^{+(k)}), (\mu_{ij}^{-(k)}, I_{ij}^{-(k)}, F_{ij}^{-(k)}))$.

Step 3. In this step we calculate the weight of each decision maker. Calculate the weight with respect to the Kth decision maker e_k . Determine the weights of decision makers, let $D_k = ((\mu_{ij}^{+(k)}, I_{ij}^{+(k)}, F_{ij}^{+(k)}), (\mu_{ij}^{-(k)}, I_{ij}^{-(k)}, F_{ij}^{-(k)}))$ be a neutrosophic bipolar fuzzy number for rating of the Kth decision maker. Then the weight of the Kth decision maker can be obtained as follows:

$$\lambda_k = \frac{(\mu_k^+ + I_k^+(\mu_k^+/(\mu_k^+ + F_k^+))) + |(\mu_k^- + I_k^-(\mu_k^-/(\mu_k^- + F_k^-)))|}{\sum_{k=1}^{t} (\mu_k^+ + I_k^+(\mu_k^+/(\mu_k^+ + F_k^+))) + |(\mu_k^- + I_k^-(\mu_k^-/(\mu_k^- + F_k^-)))|} \quad \text{where} \quad \sum_{k=1}^{t} \lambda_k = 1 \quad (8)$$

Step 4. Compose the aggregated weighted neutrosophic bipolar fuzzy decision matrix. In this step, aggregated weighted neutrosophic bipolar fuzzy decision matrix R is formed by considering the aggregated neutrosophic bipolar fuzzy decision matrix and weights vector of decision maker. The aggregated neutrosophic bipolar fuzzy decision matrix (ANBFDM) was formed by applying the neutrosophic bipolar fuzzy weighted averaging operator (NBFWAO). By considering weights $\lambda_k(k = 1,2,\ldots,t)$ of decision makers, elements β_{ij} of (ANBFDM) can be calculated by using (NBFWA) as follows:

$$\beta_{ij} = [(\mu_{ij}^{+'} = 1 - \prod_{k=1}^{t}(1-\mu_{ij}^{+(k)})^{\lambda_k}, I_{ij}^{+'} = \frac{1-\prod_{k=1}^{t}(1-\mu_{ij}^{+(k)})^{\lambda_k}}{2},$$

$$F_{ij}^{+'} = \frac{1-\prod_{k=1}^{t}(1-\mu_{ij}^{+(k)})^{\lambda_k}}{3})^{\lambda k}), (\mu_{ij}^{-'} = -\prod_{k=1}^{t}(1-\mu_{ij}^{-(k)})^{\lambda_k},$$

$$I_{ij}^{-'} = \frac{-\prod_{k=1}^{t}(1-\mu_{ij}^{-(k)})^{\lambda_k}}{2}, F_{ij}^{-'} = \frac{-\prod_{k=1}^{t}(1-\mu_{ij}^{-(k)})^{\lambda_k}}{3})]. \quad (9)$$

where

$$R = (\beta_{ij})_{n\times m} = ((\mu_{ij}^{+'}, I_{ij}^{+'}, F_{ij}^{+'}), (\mu_{ij}^{-'}, I_{ij}^{-'}, F_{ij}^{-'}))_{n\times m}$$

Step 5. Determine the entropy weights of the selection criteria. In this step, all criteria may not be assumed to be of equal importance. w represents a set of grades of importance. Let w_j be the weights of the criteria, the neutrosophic bipolar fuzzy entropy H_j is calculated by equations;

$$H_j = \frac{1}{n}\sum_{i=1}^{n}\frac{\min((\mu_{ij}^{+'}, I_{ij}^{+'}, F_{ij}^{+'}),(|\mu_{ij}^{-'}|,|I_{ij}^{-'}|,|F_{ij}^{-'}|))}{\max((\mu_{ij}^{+'}, I_{ij}^{+'}, F_{ij}^{+'}),(|\mu_{ij}^{-'}|,|I_{ij}^{-'}|,|F_{ij}^{-'}|))}. \quad (10)$$

The entropy weights of the jth criteria can be calculated as follows:

$$w_j = \frac{1-H_j}{m-\sum_{j=1}^{m} H_j} \quad (11)$$

Step 6. Determine the positive ideal solution (PIS) and the negative ideal solution (NIS) based on neutrosophic bipolar fuzzy numbers. Both solutions are vectors of NBFN elements, and they are resulting AWNBFDM matrix as follows:

$$r^{+} = ((\mu_1^{+'}, I_1^{+'}, F_1^{+'}), (\mu_1^{-'}, I_1^{-'}, F_1^{-'}))^{+}, ((\mu_2^{+'}, I_2^{+'}, F_2^{+'}), (\mu_2^{-'}, I_2^{-'}, F_2^{-'}))^{+}, \ldots$$
$$,\ldots,((\mu_m^{+'}, I_m^{+'}, F_m^{+'}), (\mu_m^{-'}, I_m^{-'}, F_m^{-'}))^{+}.$$
$$r^{-} = ((\mu_1^{+'}, I_1^{+'}, F_1^{+'}), (\mu_1^{-'}, I_1^{-'}, F_1^{-'}))^{-}, ((\mu_2^{+'}, I_2^{+'}, F_2^{+'}), (\mu_2^{-'}, I_2^{-'}, F_2^{-'}))^{-}, \ldots$$
$$,\ldots,((\mu_m^{+'}, I_m^{+'}, F_m^{+'}), (\mu_m^{-'}, I_m^{-'}, F_m^{-'}))^{-}. \quad (12)$$

where

$$((\mu_j^{+'}, I_j^{+'}, F_j^{+'}), (\mu_j^{-'}, I_j^{-'}, F_j^{-'}))^{+}$$
$$= (\max_i(\mu_{ij}^{+'}), \min_i(I_{ij}^{+'}), \min_i(F_{ij}^{+'}))\min_i(\mu_{ij}^{-'}), \max_i(I_{ij}^{-'}), \max_i(F_{ij}^{-'}))),\ j = 1,2,\ldots,m,$$
$$((\mu_j^{+'}, I_j^{+'}, F_j^{+'}), (\mu_j^{-'}, I_j^{-'}, F_j^{-'}))^{-}$$
$$= (\min_i(\mu_{ij}^{+'}), \max_i(I_{ij}^{+'}), \max_i(F_{ij}^{+'})), \max_i(\mu_{ij}^{-'}), \min_i(I_{ij}^{-'}), \min_i(F_{ij}^{-'}))),\ j = 1,2,\ldots,m. \quad (13)$$

Step 7. Find the grey relational coefficient of each evaluation value from positive ideal solution

(PIS) and negative ideal solution (NIS) by using the following equations, respectively. The grey relational coefficients of each evaluation value from PIS and NIS are defined as:

$$\xi_{ij}^{+}=\frac{\min\limits_{1\le i\le n}\min\limits_{1\le j\le m} d\left(\gamma_{ij},r_j^{+}\right)+\tau\max\limits_{1\le i\le n}\max\limits_{1\le j\le m} d\left(\gamma_{ij},r_j^{+}\right)}{d\left(\gamma_{ij},r_j^{+}\right)+\tau\max\limits_{1\le i\le n}\max\limits_{1\le j\le m} d\left(\gamma_{ij},r_j^{+}\right)},$$
$$i=1,2,\ldots,n,\ j=1,2,\ldots,m,$$
$$\xi_{ij}^{-}=\frac{\min\limits_{1\le i\le n}\min\limits_{1\le j\le m} d(\gamma_{ij},r_j^{-})+\tau\max\limits_{1\le i\le n}\max\limits_{1\le j\le m} d(\gamma_{ij},r_j^{-})}{d(\gamma_{ij},r_j^{-})+\tau\max\limits_{1\le i\le n}\max\limits_{1\le j\le m} d(\gamma_{ij},r_j^{-})}, \tag{14}$$
$$i=1,2,\ldots,n,\ j=1,2,\ldots,m,$$

where $\tau\in[0,1]$. Generally, $\tau=0.5$ is used.

Step 8. Find out the degree of weighted grey relational coefficient of each alternative as follows:

$$\xi_i^{+}=\sum_{j=1}^{m} w_j\xi_{ij}^{+},\qquad \xi_i^{-}=\sum_{j=1}^{m} w\xi_{ij}^{-},\ \text{ where } i=1,2,\ldots,n. \tag{15}$$

Step 9. Find out the relative relational degree of each alternative from the positive ideal solution (PIS) and negative ideal solution (NIS) by using the formula as follows:

$$\xi_i=\frac{\xi_i^{+}}{\xi_i^{+}+\xi_i^{-}},\ i=1,2,\ldots,n. \tag{16}$$

Step 10. Rank of alternatives. We rank the alternatives according to the $\xi_i, i=1,2,\ldots,n$, in descending order and choose the alternative with the maximum ξ_i.

5. Numerical Applications

A medicine company intends to prepare three different types of medicines A_1, A_2 and A_3 (Alternatives) depending upon different compositions, to cure some ailment. Three attributes are involved to select the best medicine for the treatment,

(i). Effectiveness (c_1), (ii). Economy (c_2), (iii). Timings (c_3) .

The positive effects of the medicines on the person who needs medical care, are taken as a positive truth membership functions while negative effects of adverse reactions, are the negative truth membership functions, less time consumption to cure the ailment is taken as a positive indeterminacy function whereas more time consumption is taken as negative indeterminacy functions. likewise, positive and negative economic factors are placed as a positive and negative falsity functions.

This is a hybrid MADM problem involving three different data types: exact values, intervals and linguistic terms. To resolve this matter, we apply the developed method for the ranking and selection of the more effective, fast acting and more economic medicine (alternative). Three experts (e_1, e_2, e_3) are involved in the selection process. Each expert expresses his/her preferences depending upon the worth of the alternatives and upon his/her own knowledge over them. The hybrid decision matrices R^1, R^2 and R^3 given by the experts e_1, e_2 and e_3 are shown in Tables 3, 4 and 5.

Step 1. Consider the neutrosophic bipolar fuzzy hybrid decision matrix of each decision maker. The neutrosophic bipolar fuzzy hybrid decision matrix involves four different data types: exact values, intervals, NBFNs, and linguistic terms.

Step 2. Transform the hybrid decision matrix of each decision maker into neutrosophic bipolar fuzzy decision matrix. The exact values and intervals in the hybrid decision matrices given by the decision makers shown in Tables $3-6$ are standardized and then transformed into a neutrosophic bipolar fuzzy number. The linguistic evaluations shown in Tables $3-6$ are converted into NBFNs by using Table 1. Then, the neutrosophic bipolar fuzzy decision matrix $R^{(k)}(k=1,2,3,4)$ of each decision maker shown in Tables 6, 7, 8 and 9 .

Step 3. Determine the weights of decision makers. The importance of the decision makers in the group decision making process is shown in Table 9. These linguistic variables used can be converted into NBFNs by utilizing Table 2. In order to obtain the weights $\lambda_k(k=1,2,3,4)$ of the decision makers, and formula (11) is used:

Table 3. HDM R^1 by e_1

	C_1	C_2	C_3
A_1	VH	2	[20,30]
A_2	H	3	[15,25]
A_3	M	4	[18,24]

Table 4. HDM R^2 by e_2

	C_1	C_2	C_3
A_1	VH	5	[12,24]
A_2	H	3	[18,26]
A_3	M	4	[16,22]

Table 5. HDM R^3 by e_3

	C_1	C_2	C_3
A_1	VH	6	[20,22]
A_2	H	4	[15,18]
A_3	M	3	[12,20]

Table 6. Neutrosophic bipolar fuzzy decision matrix R^1 given by the expert e_1

	C_1	C_2	C_3
A_1	(0.85,0.42,0.28,−0.1,−0.05,−0.03)	(0.78,0.39,0.26,−0.22,−0.11,−0.07)	(0.44,0.22,0.15,−0.03,−0.02,−0.01)
A_2	(0.70,0.35,0.23,−0.20,−0.10,−0.06)	(0.51,0.26,0.17,−0.49,−0.24,−0.16)	(0.33,0.16,0.11,−0.19,−0.10,−0.06)
A_3	(0.45,0.22,0.15,−0.30,−0.15,−0.10)	(0.39,0.2,0.13,−0.61,−0.30,−0.20)	(0.40,0.2,0.13,−0.12,−0.06,−0.04)

Table 7. Neutrosophic bipolar fuzzy decision matrix R^2 given by the expert e_2

	C_1	C_2	C_3
A_1	(0.85,0.42,0.28,−0.1,−0.05,−0.03)	(0.43,0.22,0.14,−0.57,−0.28,−0.19)	(0.29,0.14,0.10,−0.11,−0.06,−0.04)
A_2	(0.70,0.35,0.23,−0.20,−0.10,−0.06)	(0.71,0.36,0.24,−0.29,−0.14,−0.10)	(0.43,0.22,0.14,−0.03,−0.02,−0.01)
A_3	(0.55,0.27,0.18,−0.30,−0.15,−0.10)	(0.54,0.27,0.18,−0.46,−0.23,−0.15)	(0.38,0.19,0.13,−0.18,−0.09,−0.06)

Table 8. Neutrosophic bipolar fuzzy decision matrix R^3 given by the expert e_3

	C_1	C_2	C_3
A_1	(0.85,0.42,0.28,−0.1,−0.05,−0.03)	(0.37,0.18,0.12,−0.63,−0,32,−0.21)	(0.57,0.28,0.19,−0.21,−0.10,−0.07)
A_2	(0.70,0.35,0.23,−0.02,−0.10,−0.06)	(0.55,0.28,0.18,−0.45,−0.22,−0.15)	(0.43,0.22,0.14,−0.35,−0.18,−0.12)
A_3	(0.55,0.27,0.18,−0.30,−0.15,−0.10)	(0.73,0.36,0.24,−0.27,−0.14,−0.09)	(0.34,0.17,0.11,−0.28,−0.14,−0.10)

Table 9. The importance of decision makers

	Linguistic variable	
d_1	Very important	$k=1$
d_2	Important	$k=2$
d_3	Medium	$k=3$

Using (8) we calculate the λ_k which are $\lambda_1 = 0.353, \lambda_2 = 0.334, \lambda_3 = 0.312$ as shown in Figure 3.:

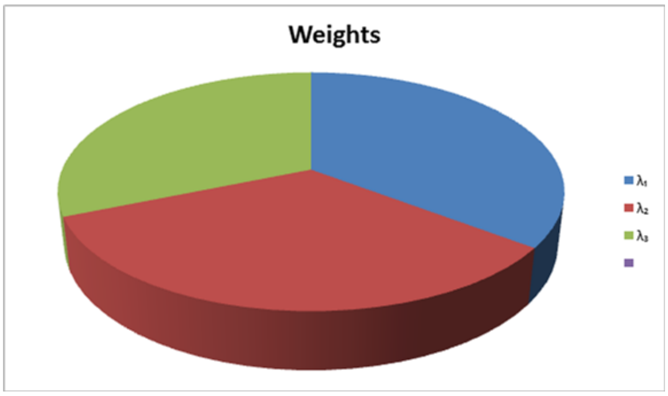

Figure 3. The weight vector

Step 4. Construct the aggregated neutrosophic bipolar fuzzy decision matrix based on the ideas of decision makers. By formula (9), we get the bipolar fuzzy decision matrix R by aggregating all the neutrosophic bipolar fuzzy decision matrices $R^{(K)}(K = 1,2,3)$. The neutrosophic bipolar fuzzy decision matrix R is shown in Table 10.

Table 10. Neutrosophic bipolar fuzzy decision matrix R,

	C_1	C_2	C_3
A_1	$(0.85, 0.42, 0.28, -0.1, -0.05, -0.03)$	$(0.58, 0.29, 0.19, -0.41, -0.20, -0.14)$	$(0.44, 0.22, 0.15, -0.08, -0.04, -0.03)$
A_2	$(0.70, 0.35, 0.23, -0.20, -0.10, -0.06)$	$(0.60, 0.30, 0.20, -0.40, -0.20, -0.13)$	$(0.39, 0.02, 0.13, -0.12, -0.06, -0.04)$
A_3	$(0.45, 0.22, 0.15, -0.30, -0.15, -0.10)$	$(0.57, 0.28, 0.19, -0.43, -0.22, -0.14)$	$(0.37, 0.18, 0.12, -0.18, -0.09, -0.06)$

Step 5. Calculate the entropy weights of the criteria. Use formula (10) to calculate the neutrosophic bipolar fuzzy entropy H_j $(j = 1,2,3)$,

$$H_1 = 0.72, H_2 = 0.78, H_3 = 0.86.$$

Then, use formula (11) to obtain the entropy weights below which are shown in Figure 4.

$$w_1 = 0.44, w_2 = 0.34, w_3 = 0.22.$$

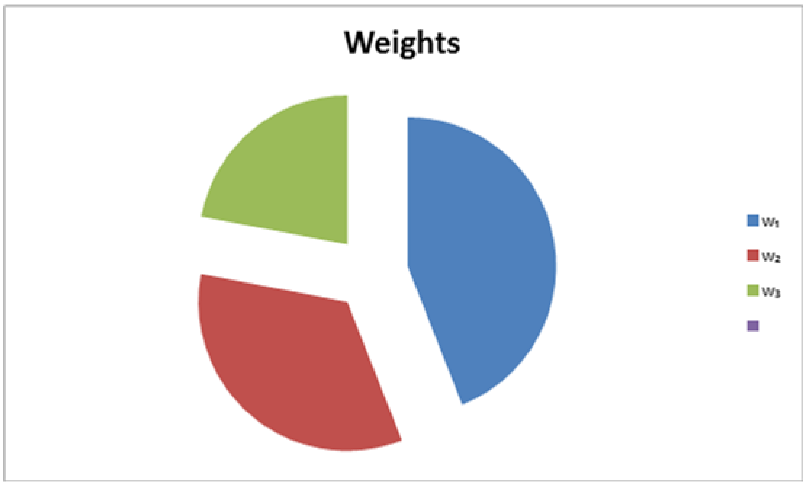

Figure 4. The entropy weight vector

Step 6. The neutrosophic bipolar fuzzy positive ideal solution (PIS) and neutrosophic bipolar fuzzy negative ideal solution (NIS) were obtained as;

$$r^+ = ((0.85,0.22.0.15,-0.30,-0.05,-0.03)),$$
$$((0.60,0.28,0.19,-0.43,-0.20,-0.13)),(0.44,0.18,0.12,-0.18,-0.04,-0.03).$$

$$r^- = ((0.45,0.42.0.28,-0.10,-0.15,-0.10)),$$
$$((0.57,0.30,0.20,-0.40,-0.22,-0.14)),(0.37,0.22,0.15,-0.08,-0.09,-0.05).$$

Step 7. Find out the grey relational coefficient of each alternative from PIS and NIS respectively as in the positive ideal solution ξ^+ and the negative ideal solution ξ^-.

$$\text{Positive ideal solution} \quad \xi^+ = (\xi_{ij}^+)_{3\times3} = \begin{bmatrix} 0.47 & 0.85 & 0.77 \\ 0.40 & 1.00 & 0.71 \\ 0.40 & 1.00 & 0.77 \end{bmatrix}$$

$$\text{Negative ideal solution} \quad \xi^- = (\xi_{ij}^-)_{3\times3} = \begin{bmatrix} 0.40 & 0.89 & 0.77 \\ 0.40 & 1.00 & 0.77 \\ 0.42 & 1.00 & 0.77 \end{bmatrix}$$

Step 8. According to the above step, the attributes weight vector is:

$$w = (0.44,0.34,0.22)$$

then the degree of grey relational coefficient of each alternative from positive ideal solution (PIS) and negative ideal solution (NIS) can be calculated and are;

$$\xi_1^+ = 0.67, \xi_2^+ = 0.68, \xi_3^+ = 0.69.$$
$$\xi_1^- = 0.65, \xi_2^- = 0.69, \xi_3^- = 0.70.$$

Step 9. Calculate the relative relational degree of each alternative below and shown in Figure 5.

$$\xi_1 = 0.507, \xi_2 = 0.496, \xi_3 = 0.500$$

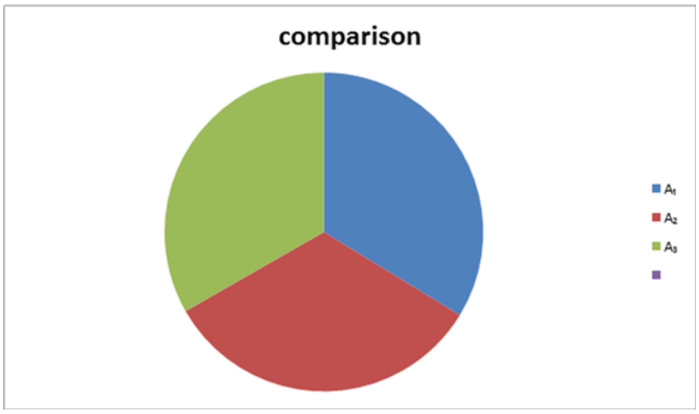

Figure 5. The relative relational degree of alternatives

Step 10. Rank the alternatives. The relative relational degree of alternatives is determined, and then six alternatives are ranked as; $A_1 > A_3 > A_2$. So the alternative A_1 is selected as an appropriate alternative.

6. Comparison Analysis

There is no doubt about that fuzzy sets and all models of fuzzy sets, are helping us out in variety of fields. Amidst of other applications, the decision-making problems are rendered to all versions of fuzzy sets for resolution and can be seen in [27, 29, 30, 34, 45, 47]. Similarity measures have been studied in [16, 45, 49]. Bipolarity in human reasoning and affective decision making studied in [26]. Hybrid multi-attribute group decision making based on intuitionistic fuzzy information and GRA method, discussed in [33]. Recently, [45] defined neutrosophic bipolar fuzzy set and neutrosophic bipolar fuzzy weighted averaging (NBFWA) and neutrosophic bipolar fuzzy ordered weighted averaging (NBFOWA) operators, similarity measures and gave an algorithm and application of neutrosophic bipolar fuzzy sets in decision making in case of multi-attributes.

7. Conclusions

Continuing the work on neutrosophic bipolar fuzzy sets we discussed hybrid multi-attributes group decision making based on neutrosophic bipolar fuzzy sets with different neutrosophic bipolar fuzzy transformation techniques. We apply these concepts and techniques upon hybrid multi-attributes decision making problem of selecting the best medicine to cure some diseases and develop an algorithm for neutrosophic bipolar fuzzy hybrid multi-attribute group decision making. In future the developed technique and procedure can be used in different decision-making problems, like numerical analysis for root convergence [53-58], signature theory, signal processing and operations management [59].

Funding: This research received no external funding

Conflicts of Interest: The authors declare no conflict of interest

REFERENCES

1. Zadeh, L. A. Fuzzy sets. Inf. Control, 1965; **8**, pp.338-353.
2. Alhazaymeh, K.; Hassan, N. Vague soft set relations and functions. J. Intell. Fuzzy Systems, 2015; **28**(3), pp. 1205-1212.
3. Alhazaymeh, K.; Hassan, N. Mapping on generalized vague soft expert set, Int. J. Pure Appl. Math., 2014; **93**(3), pp. 369-376.
4. Alhazaymeh, K.; Hassan, N. Generalized vague soft expert set, Int. J. Pure Appl. Math., 2014; **93**(3), 351-360.
5. Alhazaymeh, K.; Hassan, N. Application of generalized vague soft expert set in decision making, Int. J. Pure Appl. Math., 2014; **93**(3), 361-367.
6. Alhazaymeh, K.; Halim, S.A.; Salleh, A.R.; Hassan, N. Soft intuitionistic fuzzy sets, Appl. Math. Sci., 2012; **6**(54), pp. 2669-2680.
7. Adam, F.; Hassan, N. Q-fuzzy soft set, Appl. Math. Sci., 2014; **8**(174), pp. 8689-8695.
8. Adam, F.; Hassan, N. Operations on Q-fuzzy soft set, Appl. Math. Sci., 2014; **8**(175), pp. 8697-8701.
9. Adam, F.; Hassan, N. Q-fuzzy soft matrix and its application, AIP Conf. Proc., 2014; **1602**, pp. 772-778.
10. Adam, F.; Hassan, N. Properties on the multi Q-fuzzy soft matrix, AIP Conf. Proc., 2014; **1614**, pp. 834-839.
11. Adam, F.; Hassan, N. Multi Q-fuzzy soft set and its application, Far East J. Math. Sci., 2015; **97**(7), pp. 871-881.
12. Lee, K.M. Comparison of interval-valued fuzzy sets, intuitionistic fuzzy sets, and bipolar-valued fuzzy sets. J. Fuzzy Logic Intelligent Systems, 2004; **14**, pp.125-129
13. Atanassov, K. Intuitionistic fuzzy sets, Fuzzy Sets Syst.,1986; **20**, pp.87-96.
14. Kandel, A.; Byatt,W. Fuzzy sets, fuzzy algebra and fuzzy statistics. Proc. of the IEEE, 1978; **66**, pp. 1619-1639.

15. Meghdadi, A. H.; Akbarzadeh, M. Probabilistic fuzzy logic and probabilistic fuzzy systems. The 10th IEEE International Conference on Fuzzy Systems, 2001; **3**, pp.1127-1130.
16. Hung,W. L.; Yang, M. S. Similarity measures of intuitionistic fuzzy sets based on Hausdorff distance. Pattern Recognit. Lett., 2004; **25**, pp.1603-1611.
17. Delgado. M.; Verdegay,J. L.; Vila, M. A. Linguistic decision making models. Int. J. Intell. Syst., 1992; **7**, pp.479-492.
18. Truck, I. Comparison and links between two 2-tuple linguistic models for decision making. Knowledge-Based Systems, 2015; **87**, pp.61-68.
19. Varnamkhasti, M.J.; Hassan, N. Neurogenetic algorithm for solving combinatorial engineering problems, J. Appl. Math., 2012, 253714.
20. Smarandache, F. A Unifying Field in Logics: Neutrosophic Logic. Neutrosophy, Neutrosophic Set, Neutrosophic Probability. American Research Press, Rehoboth, NM, 1999.
21. Zhang, W. R. "Bipolar fuzzy sets and relations: a computational framework for cognitive modeling and multiagent decision analysis," in Proceedings of the Industrial Fuzzy Control and Intelligent Systems Conference, and the NASA Joint Technology Workshop on Neural Networks and Fuzzy Logic and Fuzzy Information Processing Society Biannual Conference, pp. 305–309, San Antonio, Tex, USA, December 1994.
22. Yaqoob, N.; Ansari, M. A. Bipolar, -fuzzy ideals in ternary semigroups. Int. J. Math. Anal., 2013; **7**, pp.1775-1782.
23. Yaqoob, N.; Aslam, M.; Davvaz, B.; Ghareeb, A. Structures of bipolar fuzzy -hyperideals in -semihypergroups. J. Intell. Fuzzy Systems, 2014; **27**, pp.3015-3032.
24. Yaqoob, N.; Aslam, M.; Rehman, I.; Khalaf, M.M. New types of bipolar fuzzy sets in -semihypergroups, Songklanakarin J. Sci. Technol., 2016; **38**, pp.119-127.
25. Dubois, D.; Kaci, S.; Prade, H. Bipolarity in reasoning and decision, an Introduction. International Conference on Inf. Pro. Man. Unc. IPMU, 2004; 04, pp.959-966.
26. Silva Da, R.; Livet, P. Bipolarity in human reasoning and affective decision making. Int. J. Intell. Syst., 2008; **23**, pp.898-922.
27. Abdel-Basset, M.; Mohamed, M. A novel and powerful framework based on neutrosophic sets to aid patients with cancer. Future Generation Computer Systems, 2019; **98**, pp. 144-153.
28. Abdel-Basset, M.; Manogaran, G.; Gamal, A.; Smarandache, F. A hybrid approach of neutrosophic sets and DEMATEL method for developing supplier selection criteria. Design Automation for Embedded Systems, 2018; **22**, pp. 257-278.
29. Abdel-Basset, M.; Manogaran, G.; Gamal, A.; Smarandache, F. A group decision making framework based on neutrosophic TOPSIS approach for smart medical device selection. Journal of Medical Systems, 2019; **43**(38), pp. 1-13. doi:10.1007/s10916019-1156-1
30. Abdel-Basset, M.; Saleh, M.; Gamal, A.; Smarandache, F. An approach of TOPSIS technique for developing supplier selection with group decision making under type-2 neutrosophic number. Appl. Soft Comput., 2019; **77**, pp. 438-452.
31. Abdel-Basset, Mohamed, and Rehab Mohamed. "A novel plithogenic TOPSIS-CRITIC model for sustainable supply chain risk management." Journal of Cleaner Production **247** (2020): 119586.
32. Qun, W.; Peng, Wu.; Ligang, Z.; Huayou, C.; Xianjun, G. Some new Hamacher aggregation operators under single-valued neutrosophic 2-tuple linguistic environment and their applications to multi-attribute group decision making, Computers & Industrial Engineering, 2018; **116**, pp. 144-162.
33. Guo, J. Hybrid multiattribute group decision making based on intuitionistic fuzzy information and GRA Method. ISRN Applied Mathematics, 2013; **2013**, 146026.
34. Liu, P.; Wang, Y. Multiple attribute decision-making method based on single-valued neutrosophic normalized weighted Bonferroni mean. Neural Comput. Appl., 2014; **25**, pp.2001-2010.
35. Liu, P.; Shi, L. The generalized hybrid weighted average operator based on interval neutrosophic hesitant set and its application to multiple attribute decision making. Neural Comput. Appl. 2015; **26**, pp.457-471.
36. Peng, J. J. J.; Wang, Q.; Wang, J.; Zhang, H.Y.; Chen, X.H. Simplified neutrosophic sets and their applications in multi-criteria group decision-making problems. Int. J. System Sciences, 2015; **8**, pp. 2342-2358.
37. Chen, S. M. A new approach to handling fuzzy decision-making problems. IEEE Trans. Syst. Man Cybern.,1988; **18**, pp.1012-1016.
38. Hung, W. L; Yang, M. S. Similarity measures of intuitionistic fuzzy sets based on Hausdorff distance. Pattern Recognit. Lett., 2004; **25**, pp.1603-1611.
39. Zhan, J.; Khan, M.; Gulistan, M.; Ali, A. Applications of neutrosophic cubic sets in multi-criteria decision making. Int. J. for Uncertainty Quantification, 2017; **7**, pp.377-394.

40. Gulistan, M.; Yaqoob, N.; Rashid, Z.; Smarandache, F.; Wahab, H. A. A Study on Neutrosophic Cubic Graphs with Real Life Applications in Industries. Symmetry 2018; **10**, 203.
41. Sahin,R. ; Kucuk, A. Subsethood measure for single valued neutrosophic sets. J. Intell. Fuzzy Systems, 2015; **29**, pp.525-530.
42. Zavadskas, E. K; Bausys, R.; Kaklauskas, A.; Ubarte, I.; Kuzminske, A.; Gudiene, N. Sustainable market valuation of buildings by the single-valued neutrosophic MAMVA method. Appl. Soft Comput., 2017; **57**, pp.74-87.
43. Zhan, J.; Khan, M.; Gulistan, M.; Ali, A. Applications of neutrosophic cubic sets in multi-criteria decision making. Int. J. for Uncertainty Quantification, 2017; **7**, pp.377-394.
44. Zhang, H. Y.; Wang, J.Q.; Chen, X.H. Interval neutrosophic sets and their application in multicriteria decision making problems. The Scientific World Journal, 2014; **2014**, 645953.
45. Hashim, R. M.; Gulistan, M.; Smarandache, F. Applications of Neutrosophic Bipolar fuzzy Sets in HOPE Foundation for Planning to Build a children Hospital with Different Types of similarity measures. Symmetry 2018; **10**, 331.
46. Zhang, W. R. Bipolar fuzzy sets, Proc. of FUZZ-IEEE, 1998, pp. 835-840.
47. Biswas, P.; Pramanik, S.; Giri, B. C. Entropy based grey relational analysis method for multi-attribute decision making under single valued neutrosophic assessments. Neutrosophic Sets and Systems, 2014; **2**, pp. 102–110.
48. Majumdar, P.; Samanta, S.K. On similarity and entropy of neutrosophic sets. J. Intell. Fuzzy Systems, 2014; **26**, pp.1245-1252.
49. Abu Qamar, M.; Hassan, N. Entropy, measures of distance and similarity of Q-neutrosophic soft sets and some applications. Entropy 2018; **20**, 672.
50. Abu Qamar, M.; Hassan, N. Characterizations of group theory under Q-neutrosophic soft environment. Neutrosophic Sets and Systems, 2019; **27**, pp. 114–130.
51. Biswas, P.; Pramanik, S.; Giri, B. C. Aggregation of triangular fuzzy neutrosophic set information and its application to multi-attribute decision making. Neutrosophic Sets and Systems, 2016; **12**, pp. 20-40.
52. Biswas, P.; Pramanik, S.; Giri, B. C. Entropy based grey relational analysis method for multi-attribute decision making under single valued neutrosophic assessments. Neutrosophic Sets and Systems, 2014; **2**, pp. 102–110.
53. Jamaludin, N.; Monsi, M.; Hassan, N.; Suleiman, M. Modification on interval symmetric single-step procedure ISS-5δ for bounding polynomial zeros simultaneously, AIP Conf. Proc., 2013; **1522**, pp. 750-756.
54. Jamaludin, N.; Monsi, M.; Hassan, N.; Kartini, S. On modified interval symmetric single-step procedure ISS2-5D for the simultaneous inclusion of polynomial zeros, Int. J. Math. Anal., 2013; **7**(20), pp. 983-988.
55. Monsi, M.; Hassan, N.; Rusli, S.F. The point zoro symmetric single-step procedure for simultaneous estimation of polynomial zeros, J. Appl. Math., 2012; **2012**, 709832.
56. Sham, A.W.M.; Monsi, M.; Hassan, N.; Suleiman, M. A modified interval symmetric single step procedure ISS-5D for simultaneous inclusion of polynomial zeros, AIP Conf. Proc., 2013; **1522**, pp. 61-67.
57. Sham, A.W.M.; Monsi, M.; Hassan, N. An efficient interval symmetric single step procedure ISS1-5D for simultaneous bounding of real polynomial zeros, Int. J. Math. Anal., 2013; **7**(20), pp. 977-981.
58. Abu Bakar, N.; Monsi, M.; Hassan, N. An improved parameter regula falsi method for enclosing a zero of a function, Appl. Math. Sci., 2012; **6**(28), pp. 1347-1361.
59. Biswas, P.; Pramanik, S.; Giri, B. C. TOPSIS method for multi-attribute group decision-making under single valued neutrosophic environment. Neural Comput. Appl., 2016; **27** (3), pp. 727-737.

Received: Oct 28, 2019. Accepted: Jan 27, 2020