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Abstract: We introduced the theory of Single valued neutrosophic hypergroup as the initial theory of single valued neutrosophic hyper algebra and also developed some results on single valued neutrosophic hypergroup.

Keywords: Hypergroup; Level sets; Single valued neutrosophic sets; Single valued neutrosophic hypergroup.

1. Introduction

Florentin Smarandache introduced Neutrosophic sets in 1998 [16], which is the generalization of the intuitionistic fuzzy sets. In some real time situations, decision makers faced some difficulties with uncertainty and inconsistency values. Neutrosophic sets helped the decision makers to deal with uncertainty values. Abdel-Basset et.al. used neutrosophic concept in real life decision-making problems [1-7]. The concept of single valued neutrosophic set was introduced by Wang. et. al [17].

As a generalization of classical algebraic structure, Algebraic hyper structure was introduced by F. Marty [11]. Corsini and Leoreanu-Fotea developed the applications of hyper structure [9]. Algebraic hyperstructures has many applications in fuzzy sets, lattices, artificial intelligence, automation, combinatorics. Corsini introduced hypergroup theory [8]. After while the hyperstructure theory has seen broader applications in many fields. Some of the recent works on hyperstructures related to vague soft groups, vague soft rings and vague soft ideals can be found in [12, 13].

In this paper we develop the theory of single valued neutrosophic hypergroup and also established some results on single valued neutrosophic hypergroup.

2. Preliminaries

Definition 2.1 [17] Let X be a space of points (objects), with a generic element in X denoted by x. A neutrosophic set A in X is characterized by a truth-membership function T_A, an indeterminancy-membership function I_A and a falsity-membership function F_A. $T_A(x), I_A(x)$ and $F_A(x)$ are real standard or non-standard subsets of $]0^-, 1^+[$.

$T_A: X \to]0^-, 1^+[$

$I_A: X \to]0^-, 1^+[$

$F_A: X \to]0^-, 1^+[$

There is no restriction on the sum of $T_A(x), I_A(x)$ and $F_A(x)$, so $0^- \leq supT_A(x) + supI_A(x) + supF_A(x) \leq 3^+$.

Definition 2.2 [17] Let X be a space of points (objects),with a generic element of X denoted by x. A single valued neutrosophic set (SVNS) A in X is characterized by T_A, I_A and F_A. For each point x in X, $T_A, I_A, F_A \in [0,1]$.

Definition 2.3 [17] The complement of a SVNS A is denoted by c(A) and is defined by

$T_{c(A)}(x) = F_A(x)$

$I_{c(A)}(x) = 1 - I_A(x)$

$F_{c(A)}(x) = T_A(x)$, for all x in X.

Definition 2.4 [17] A SVNS A is contained in the other SVNS B, $A \subseteq B$, if and only if,

$T_A(x) \leq T_B(x)$

$I_A(x) \geq I_B(x)$

$F_A(x) \geq F_B(x)$, for all x in X.

Definition 2.5 [17] The union of two SVNS s A and B is a SVNS C, written as $C = A \cup B$, whose truth, indeterminancy and falsity-membership functions are defined by,

$T_C(x) = \max(T_A(x), T_B(x))$

$I_C(x) = \min(I_A(x), I_B(x))$

$F_C(x) = \min(F_A(x), F_B(x))$, for all x in X.

Definition 2.6 [17] The intersection of two SVNS s A and B is a SVNS C, written as $C = A \cap B$, whose truth, indeterminancy and falsity-membership functions are defined by,

$T_C(x) = \min(T_A(x), T_B(x))$

$I_C(x) = \max(I_A(x), I_B(x))$

$F_C(x) = \max(F_A(x), F_B(x))$, for all x in X.

Definition 2.7 [17] The falsity-favorite of a SVNS B, written as $B\nabla A$, whose truth and falsity-membership functions are defined by

$T_B(x) = T_A(x)$

$I_B(x) = 0$

$F_B(x) = \min\{F_A(x) + I_A(x), 1\}$, for all x in X.

Definition 2.8 [13] A hypergroup $\langle H, \circ \rangle$ is a set H equipped with an associative hyperoperation $(\circ): H \times H \to P(H)$ which satisfies $x \circ H = H \circ x = H$ for all $x \in H$ (Reproduction axiom)

Definition 2.9 [13] A hyperstructure $\langle H, \circ \rangle$ is called an H_v-group if the following axioms hold:

(i) $x \circ (y \circ z) \cap (x \circ y) \circ z \neq \emptyset$ for all $x, y, z \in H$,

(ii) $x \circ H = H \circ x = H$ for all $x \in H$.

If $\langle H, \circ \rangle$ only satisfies (i), then $\langle H, \circ \rangle$ is called a H_v- semigroup.

Definition 2.10 [13] A subset K of H is called a subhypergroup if $\langle K, \circ \rangle$ is a hypergroup of $\langle H, \circ \rangle$.

3. Single Valued Neutrosophic Hypergroup.

Throughout this section H denotes the hypergroup $< H, \circ >$

Definition 3.1 *Let $\mathcal{A}$ be a single valued neutrosophic set over H. Then $\mathcal{A}$ is called a single valued neutrosophic hypergroup over H, if the following conditions are satisfied* (i) $\forall\ p, q \in H$, $min\{T_{\mathcal{A}}(p), T_{\mathcal{A}}(q)\} \leq inf\{T_{\mathcal{A}}(r):\ r \in p \circ q\}$,

$$max\{I_{\mathcal{A}}(p), I_{\mathcal{A}}(q)\} \geq sup\{I_{\mathcal{A}}(r):\ r \in p \circ q\}\ and$$

$$max\{F_{\mathcal{A}}(p), F_{\mathcal{A}}(q)\} \geq sup\{F_{\mathcal{A}}(r):\ r \in p \circ q\}$$

$(ii)\ \forall\ l, p \in H, there\ exists\ q \in H\ such\ that\ p \in l \circ q\ and$

$$min\{T_{\mathcal{A}}(l), T_{\mathcal{A}}(p)\} \leq T_{\mathcal{A}}(q),$$
$$max\{I_{\mathcal{A}}(l), I_{\mathcal{A}}(p)\} \geq I_{\mathcal{A}}(q)\ and$$
$$max\{F_{\mathcal{A}}(l), F_{\mathcal{A}}(p)\} \geq F_{\mathcal{A}}(q)$$

$(iii)\ \forall\ l, p \in H, there\ exists\ r \in H\ such\ that\ p \in r \circ l\ and$

$$min\{T_{\mathcal{A}}(l), T_{\mathcal{A}}(p)\} \leq T_{\mathcal{A}}(r),$$
$$max\{I_{\mathcal{A}}(l), I_{\mathcal{A}}(p)\} \geq I_{\mathcal{A}}(r)\ and$$
$$max\{F_{\mathcal{A}}(l), F_{\mathcal{A}}(p)\} \geq F_{\mathcal{A}}(r)$$

If $\mathcal{A}$ satisfies condition (i) then $\mathcal{A}$ is a single valued neutrosophic semihypergroup over H. Condition (ii) and (iii) represent the left and right reproduction axioms respectively. Then $\mathcal{A}$ is a single valued neutrosophic subhypergroup of H.

Example 3.2 If the family of t-level sets of SVNS $\mathcal{A}$ over H

$\mathcal{A}_{\mathrm{t}} = \{\mathrm{p} \in \mathrm{H}\ |\ \mathrm{T}_{\mathcal{A}}(\mathrm{p}) \geq \mathrm{t}, \mathrm{I}_{\mathcal{A}}(\mathrm{p}) \leq \mathrm{t}\ \text{and}\ \mathrm{F}_{\mathcal{A}}(\mathrm{p}) \leq \mathrm{t}\}$ is a subhypergroup of H then,

$\mathcal{A}$ is a single valued neutrosophic hypergroup over H.

Theorem 3.3 Let $\mathcal{A}$ be a SVNS over H. Then $\mathcal{A}$ is a single valued neutrosophic hypergroup over H iff $\mathcal{A}$ is a single valued neutrosophic semihypergroup over H and also $\mathcal{A}$ satisfies the left and right reproduction axioms.

Proof. The proof is obvious from Definition: 3.1

Theorem 3.4 Let $\mathcal{A}$ be a SVNS over H. If $\mathcal{A}$ is a single valued neutrosophic hypergroup over H ,then $\forall\ \mathrm{t} \in [0,1]\ \mathcal{A}_{\mathrm{t}} \neq \emptyset$ is a subhypergroup of H.

Proof. Let $\mathcal{A}$ be a single valued neutrosophic hypergroup over H and let $\mathrm{p}, \mathrm{q} \in \mathcal{A}_{\mathrm{t}}$, then

$\mathrm{T}_{\mathcal{A}}(\mathrm{p}), \mathrm{T}_{\mathcal{A}}(\mathrm{q}) \geq \mathrm{t}, \mathrm{I}_{\mathcal{A}}(\mathrm{p}), \mathrm{I}_{\mathcal{A}}(\mathrm{q}) \leq \mathrm{t}$ and $\mathrm{F}_{\mathcal{A}}(\mathrm{p}), \mathrm{F}_{\mathcal{A}}(\mathrm{q}) \leq \mathrm{t}$.

Then we have,

$$\inf\{\mathrm{T}_{\mathcal{A}}(\mathrm{r}): \mathrm{r} \in \mathrm{p} \circ \mathrm{q}\} \geq \min\{\mathrm{T}_{\mathcal{A}}(\mathrm{p}), \mathrm{T}_{\mathcal{A}}(\mathrm{q})\} \geq \min\{\mathrm{t}, \mathrm{t}\} = \mathrm{t}$$
$$\sup\{\mathrm{I}_{\mathcal{A}}(\mathrm{r}): \mathrm{r} \in \mathrm{p} \circ \mathrm{q}\} \leq \mathrm{t}\ \text{and}$$
$$\sup\{\mathrm{F}_{\mathcal{A}}(\mathrm{r}): \mathrm{r} \in \mathrm{p} \circ \mathrm{q}\} \leq \mathrm{t}$$

This implies $\mathrm{r} \in \mathcal{A}_{\mathrm{t}}$. Then $\forall\ \mathrm{r} \in \mathrm{p} \circ \mathrm{q}\ , \mathrm{p} \circ \mathrm{q} \subseteq \mathcal{A}_{\mathrm{t}}$.

Thus $\forall \mathrm{r} \in \mathcal{A}_{\mathrm{t}}$, we obtain $\mathrm{r} \circ \mathcal{A}_{\mathrm{t}} \subseteq \mathcal{A}_{\mathrm{t}}$

Now, Let $\mathrm{l}, \mathrm{p} \in \mathcal{A}_{\mathrm{t}}$, then there exist $\mathrm{q} \in \mathrm{H}$ such that $\mathrm{p} \in \mathrm{l} \circ \mathrm{q}$ and

$$\{\mathrm{T}_{\mathcal{A}}(\mathrm{q})\} \geq \min\{\mathrm{T}_{\mathcal{A}}(\mathrm{l}), \mathrm{T}_{\mathcal{A}}(\mathrm{p})\} \geq \min\{\mathrm{t}, \mathrm{t}\} = \mathrm{t}$$
$$\{\mathrm{I}_{\mathcal{A}}(\mathrm{q})\} \leq \mathrm{t}\ \text{and}$$

$\{\mathrm{F}_{\mathcal{A}}(\mathrm{q})\} \leq \mathrm{t}$. This implies $\mathrm{q} \in \mathcal{A}_{\mathrm{t}}$

This proves that $\mathcal{A}_{\mathrm{t}} \subseteq \mathrm{r} \circ \mathcal{A}_{\mathrm{t}}$. As such $\mathcal{A}_{\mathrm{t}} = \mathrm{r} \circ \mathcal{A}_{\mathrm{t}}$

Which proves that $\mathcal{A}_{\mathrm{t}}$ is a subhypergroup of H.

Theorem 3.5 Let $\mathcal{A}$ be a SVNS over H. Then the following are equivalent,

(i) $\mathcal{A}$ is a single valued neutrosophic hypergroup over H

(ii) $\forall\ \mathrm{t} \in [0,1]\ \mathcal{A}_{\mathrm{t}} \neq \emptyset$ is a subhypergroup of H.

Proof. (i) $\Rightarrow$ (ii) The proof is obvious from Theorem : 3.4.

(ii) ⇒ (i) Now assume that $\mathcal{A}_t$ is a subhypergroup of H.

Let $p, q \in \mathcal{A}_{t_0}$ and let $\min\{T_{\mathcal{A}}(p), T_{\mathcal{A}}(q)\} = \max\{I_{\mathcal{A}}(p), I_{\mathcal{A}}(q)\} = \max\{F_{\mathcal{A}}(p), F_{\mathcal{A}}(q)\} = t_0$

Since $p \circ q \subseteq \mathcal{A}_{t_0}$, then for every $r \in p \circ q$, $T_{\mathcal{A}}(r) \geq t_0$, $I_{\mathcal{A}}(r) \leq t_0$, $F_{\mathcal{A}}(r) \leq t_0$

$$\min\{T_{\mathcal{A}}(p), T_{\mathcal{A}}(q)\} \leq \inf\{T_{\mathcal{A}}(r) : r \in p \circ q\},$$
$$\max\{I_{\mathcal{A}}(p), I_{\mathcal{A}}(q)\} \geq \sup\{I_{\mathcal{A}}(r) : r \in p \circ q\} \text{ and}$$
$$\max\{F_{\mathcal{A}}(p), F_{\mathcal{A}}(q)\} \geq \sup\{F_{\mathcal{A}}(r) : r \in p \circ q\}$$

Condition (i) is verified.

Next, let $l, p \in \mathcal{A}_{t_1}$, for every $t_1 \in [0,1]$ and

let $\min\{T_{\mathcal{A}}(l), T_{\mathcal{A}}(q)\} = \max\{I_{\mathcal{A}}(l), I_{\mathcal{A}}(p)\} = \max\{F_{\mathcal{A}}(l), F_{\mathcal{A}}(q)\} = t_1$

Then there exist $q \in \mathcal{A}_{t_1}$ such that $p \in l \circ q \subseteq \mathcal{A}_{t_1}$. Since $q \in \mathcal{A}_{t_1}$,

$$T_{\mathcal{A}}(q) \geq t_1 = \min\{T_{\mathcal{A}}(l), T_{\mathcal{A}}(q)\}$$
$$I_{\mathcal{A}}(q) \leq t_1 = \max\{I_{\mathcal{A}}(l), I_{\mathcal{A}}(q)\}$$
$$F_{\mathcal{A}}(q) \leq t_1 = \max\{F_{\mathcal{A}}(l), F_{\mathcal{A}}(q)\}$$

Condition (ii) is verified. Similarly, (iii) .

Theorem 3.6 Let $\mathcal{A}$ be a SVNS over H. Then $\mathcal{A}$ be a single valued neutrosophic hypergroup over H iff $\forall\, \alpha, \beta, \gamma \in [0,1]$, $\mathcal{A}_{(\alpha,\beta,\gamma)}$ is a subhypergroup of H.

Proof. The proof is straight forward.

Theorem 3.7 Let $\mathcal{A}$ be a single valued neutrosophic hypergroup over H and $\forall\, t_1, t_2 \in [0,1]$ $\mathcal{A}_{t_1}$ and $\mathcal{A}_{t_2}$ be the t-level sets of $\mathcal{A}$ with $t_1 \geq t_2$, then $\mathcal{A}_{t_1}$ is a subhypergroup of $\mathcal{A}_{t_2}$.

Proof. $\forall t_1, t_2 \in [0,1]$, $\mathcal{A}_{t_1}$ and $\mathcal{A}_{t_2}$ be the t-level sets of $\mathcal{A}$ with $t_1 \geq t_2$

This implies that $\mathcal{A}_{t_1} \subseteq \mathcal{A}_{t_2}$

By Theorem 3.4. $\mathcal{A}_{t_1}$ is a subhypergroup of $\mathcal{A}_{t_2}$.

Theorem 3.8 Let $\mathcal{A}$ and $\mathcal{B}$ be single valued neutrosophic hypergroups over H. Then $\mathcal{A} \cap \mathcal{B}$ is a single valued neutrosophic hypergroup over H if it is non-null.

Proof. Suppose $\mathcal{A}$ and $\mathcal{B}$ be single valued neutrosophic hypergroups over H.

By Definition: 2.6. $\mathcal{A} \cap \mathcal{B} = \{< p, T_{\mathcal{A}\cap\mathcal{B}}(p), I_{\mathcal{A}\cap\mathcal{B}}(p), F_{\mathcal{A}\cap\mathcal{B}}(p) > : p \in H\}$

where $T_{\mathcal{A}\cap\mathcal{B}}(p) = T_{\mathcal{A}}(p) \wedge T_{\mathcal{B}}(p)$, $I_{\mathcal{A}\cap\mathcal{B}}(p) = I_{\mathcal{A}}(p) \vee I_{\mathcal{B}}(p)$ and $F_{\mathcal{A}\cap\mathcal{B}}(p) = F_{\mathcal{A}}(p) \vee F_{\mathcal{B}}(p)$

For all $p, q \in H$

$$\text{(i) } \min\{T_{\mathcal{A}\cap\mathcal{B}}(p), T_{\mathcal{A}\cap\mathcal{B}}(q)\} = \min\{T_{\mathcal{A}}(p) \wedge T_{\mathcal{B}}(p), T_{\mathcal{A}}(q) \wedge T_{\mathcal{B}}(q)\}$$
$$\leq \min\{T_{\mathcal{A}}(p), T_{\mathcal{A}}(q)\} \wedge \min\{T_{\mathcal{B}}(p), T_{\mathcal{B}}(q)\}$$
$$\leq \inf\{T_{\mathcal{A}}(r) : r \in p \circ q\} \wedge \inf\{T_{\mathcal{B}}(r) : r \in p \circ q\}$$
$$\leq \inf\{T_{\mathcal{A}}(r) \wedge T_{\mathcal{B}}(r) : r \in p \circ q\}$$
$$= \inf\{T_{\mathcal{A}\cap\mathcal{B}}(r) : r \in p \circ q\}$$

Similarly, we can prove that $\max\{I_{\mathcal{A}\cap\mathcal{B}}(p), I_{\mathcal{A}\cap\mathcal{B}}(q)\} \geq \sup\{I_{\mathcal{A}\cap\mathcal{B}}(r) : r \in p \circ q\}$

$$\max\{F_{\mathcal{A}\cap\mathcal{B}}(p), F_{A\cap B}(q)\} \geq \sup\{F_{\mathcal{A}\cap\mathcal{B}}(r) : r \in p \circ q\}$$

(ii) $\forall\, l, p \in H$, there exists $q \in H$ such that $p \in l \circ q$,

$$\min\{T_{\mathcal{A}\cap\mathcal{B}}(l), T_{\mathcal{A}\cap\mathcal{B}}(p)\} = \min\{T_{\mathcal{A}}(l) \wedge T_{\mathcal{B}}(l)\}, \{T_{\mathcal{A}}(p) \wedge T_{\mathcal{B}}(p)\}$$
$$= \min\{T_{\mathcal{A}}(l), T_{\mathcal{A}}(p)\} \wedge \min\{T_{\mathcal{B}}(l), T_{B}(p)\}$$
$$\leq T_{\mathcal{A}}(q) \wedge T_{\mathcal{B}}(q) = T_{\mathcal{A}\cap\mathcal{B}}(q)$$

Therefore, $\mathcal{A} \cap \mathcal{B}$ is a single valued neutrosophic hypergroup over H.

Theorem 3.9 Let $\mathcal{A}$ and $\mathcal{B}$ be single valued neutrosophic hypergroups over H. Then $\mathcal{A} \cup \mathcal{B}$ is a single valued neutrosophic hypergroup over H.

Proof. By Definition: 2.5.

$$\mathcal{A} \cup \mathcal{B} = \{< \mathrm{p}, \mathrm{T}_{\mathcal{A}\cup\mathcal{B}}(\mathrm{p}), \mathrm{I}_{\mathcal{A}\cup\mathcal{B}}(\mathrm{p}), \mathrm{F}_{\mathcal{A}\cup\mathcal{B}}(\mathrm{p}) > : \mathrm{p} \in \mathrm{H}\}$$

where $\mathrm{T}_{\mathcal{A}\cup\mathcal{B}}(\mathrm{p}) = \mathrm{T}_{\mathcal{A}}(\mathrm{p}) \vee \mathrm{T}_{\mathcal{B}}(\mathrm{p})$, $\mathrm{I}_{\mathcal{A}\cup\mathcal{B}}(\mathrm{p}) = \mathrm{I}_{\mathcal{A}}(\mathrm{p}) \wedge \mathrm{I}_{\mathcal{B}}(\mathrm{p})$ and $\mathrm{F}_{\mathcal{A}\cup\mathcal{B}}(\mathrm{p}) = \mathrm{F}_{\mathcal{A}}(\mathrm{p}) \wedge \mathrm{F}_{\mathcal{B}}(\mathrm{p})$

For all $\mathrm{p}, \mathrm{q} \in \mathrm{H}$,

$$\begin{aligned}
\min\{\mathrm{T}_{\mathcal{A}\cup\mathcal{B}}(\mathrm{p}), \mathrm{T}_{\mathcal{A}\cup\mathcal{B}}(\mathrm{q})\} &= \min\{\mathrm{T}_{\mathcal{A}}(\mathrm{p}) \vee \mathrm{T}_{\mathcal{B}}(\mathrm{p}), \mathrm{T}_{\mathcal{A}}(\mathrm{q}) \vee \mathrm{T}_{\mathcal{B}}(\mathrm{q})\} \\
&\leq \min\{\mathrm{T}_{\mathcal{A}}(\mathrm{p}), \mathrm{T}_{\mathcal{A}}(\mathrm{q})\} \vee \min\{\mathrm{T}_{\mathcal{B}}(\mathrm{p}), \mathrm{T}_{\mathcal{B}}(\mathrm{q})\} \\
&\leq \inf\{\mathrm{T}_{\mathcal{A}}(\mathrm{r}) : \mathrm{r} \in \mathrm{p} \circ \mathrm{q}\} \vee \inf\{\mathrm{T}_{\mathcal{B}}(\mathrm{r}) : \mathrm{r} \in \mathrm{p} \circ \mathrm{q}\} \\
&\leq \inf\{\mathrm{T}_{\mathcal{A}}(\mathrm{r}) \vee \mathrm{T}_{\mathcal{B}}(\mathrm{r}) : \mathrm{r} \in \mathrm{p} \circ \mathrm{q}\} \\
&= \inf\{\mathrm{T}_{\mathcal{A}\cup\mathcal{B}}(\mathrm{r}) : \mathrm{r} \in \mathrm{p} \circ \mathrm{q}\}
\end{aligned}$$

Similarly, the other holds.

Theorem 3.10 Let $\mathcal{A}$ be a single valued neutrosophic hypergroup over H. Then the falsity- favorite of $\mathcal{A}$ (ie., $\nabla\mathcal{A}$) is also a single valued neutrosophic hypergroup over H.

Proof. By Definition: 2.7. $\mathcal{B} = \nabla\mathcal{A}$, where the membership values are $\mathrm{T}_{\mathcal{B}}(\mathrm{x}) = \mathrm{T}_{\mathcal{A}}(\mathrm{x})$, $\mathrm{I}_{\mathcal{B}}(\mathrm{x}) = 0$ and $\mathrm{F}_{\mathcal{B}}(\mathrm{x}) = \min\{\mathrm{F}_{\mathcal{A}}(\mathrm{x}) + \mathrm{I}_{\mathcal{A}}(\mathrm{x}), 1\}$

Then we have to prove for $\mathrm{F}_{\mathcal{B}}$, $\forall \mathrm{p}, \mathrm{q} \in \mathrm{H}$

$$\begin{aligned}
\max\{\mathrm{F}_{\mathcal{B}}(\mathrm{p}), \mathrm{F}_{\mathcal{B}}(\mathrm{q})\} &= \max\{\mathrm{F}_{\mathcal{A}}(\mathrm{p}) + \mathrm{I}_{\mathcal{A}}(\mathrm{p}) \wedge 1, \mathrm{F}_{\mathcal{A}}(\mathrm{q}) + \mathrm{I}_{\mathcal{A}}(\mathrm{q}) \wedge 1\} \\
&= \max\{\mathrm{F}_{\mathcal{A}}(\mathrm{p}) + \mathrm{I}_{\mathcal{A}}(\mathrm{p}), \mathrm{F}_{\mathcal{A}}(\mathrm{q}) + \mathrm{I}_{\mathcal{A}}(\mathrm{q})\} \wedge 1 \\
&\geq (\max\{\mathrm{F}_{\mathcal{A}}(\mathrm{p}), \mathrm{F}_{\mathcal{A}}(\mathrm{q})\} + \max\{\mathrm{I}_{\mathcal{A}}(\mathrm{p}), \mathrm{I}_{\mathcal{A}}(\mathrm{q})\}) \wedge 1 \\
&\geq (\sup\{\mathrm{F}_{\mathcal{A}}(\mathrm{r}) : \mathrm{r} \in \mathrm{p} \circ \mathrm{q}\} + \sup\{\mathrm{I}_{\mathcal{A}}(\mathrm{r}) : \mathrm{r} \in \mathrm{p} \circ \mathrm{q}\}) \wedge 1 \\
&= \sup\{\mathrm{F}_{\mathcal{A}}(\mathrm{r}) + \mathrm{I}_{\mathcal{A}}(\mathrm{r}) \wedge 1 : \mathrm{r} \in \mathrm{p} \circ \mathrm{q}\} \\
&= \sup\{\mathrm{F}_{\mathcal{B}}(\mathrm{r}) : \mathrm{r} \in \mathrm{p} \circ \mathrm{q}\})
\end{aligned}$$

In similar manner the other conditions holds.

4. Conclusions

In this paper, we have developed the theory of hypergroup for the single-valued neutrosophic set by introducing several hyperalgebraic structures and some results were verified. The future research related to this work involve the development of other hyperalgebraic theory for the single-valued neutrosophic sets and interval-valued neutrosophic sets.

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Conflicts of Interest

The authors declare no conflict of interest.

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