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Linear Side-Loaded Transmission-Line Antennas

Robert W. Kulterman

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NAME AND ADDRESS

LINEAR SIDE-LOADED TRANSMISSION-LINE ANTENNAS

By

Robert W. Kulterman

A Thesis

Submitted in Partial Fulfillment of the
Requirements for the Degree of
Doctor of Science in Electrical Engineering

The University of New Mexico

1962

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ROBERT W. HOLZNER

A Thesis

Submitted in Partial Fulfillment of the

Requirements for the Degree of

Doctor of Science in Electrical Engineering

The University of New Mexico

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DOCTOR OF SCIENCE

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1934

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4.7 The final answer is $x = 4$.

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and...

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and...

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1. INTRODUCTION

1.1 A Low Frequency Trailing-Wire Antenna

A need has arisen in recent years for a mobile transmitting antenna capable of radiating large amounts of power as efficiently as possible in the frequency range from 10 to 40 kc. Several mechanical and electrical problems are encountered in building and operating such an antenna.

First of all, the antenna must be physically long if it is going to have any appreciable electrical length. For example, consider an antenna operating at 40 kc. In order for this antenna to have an electrical length of 0.5 radians, it would have to be 1950 feet long. Since 40 kc is the high end of the frequency band and 0.5 radians is electrically fairly short, we must deal with antennas in the range from 1000 to 10,000 feet. Obviously, antennas of this length are going to be difficult to drag behind an airplane or even to hang from a helicopter. If we further require that the antenna be excited at a point of symmetry, the dragging or hanging problem becomes impossible for all practical purposes. Thus, it would seem that the physical size of the antenna and the need for mobility restrict the antenna to some type of end-fed structure. One other possibility is to use the dragging vehicle as one arm of the antenna. The physical size of the antenna limits the effectiveness of this plan for most practical

dragging vehicles which would be relatively small compared to an antenna one thousand feet long, or longer.

Impedance matching is another problem which becomes important in the operation of an electrically short antenna ($\beta h < 1$) that is to handle large amounts of power. For an electrically short antenna, the input impedance is much larger than the input resistance, and the input resistance is of the order of 20 ohms or less. It is not an easy task to match efficiently a 10 to 40 kc transmitter to a load of this type. For example, at $\beta h = 0.5$, the input impedance of a long thin linear center-fed antenna is [1, page 176] $Z_0 = 5 - j 1800$ ohms. In order to obtain a 50% transfer of power from the transmitter to the antenna, the 1800 ohms of reactance must be tuned out, and the transmitter matched to 5 ohms. At high power levels, these things are not easy to do especially since all the equipment used must satisfy the space and weight limitations of the dragging vehicle.

This paper proposes a side-loaded transmission-line antenna as a possible solution to the problem discussed above. It will be shown that this antenna has certain characteristics that make it very desirable as a physically long, electrically short, mobile antenna. Before these characteristics are discussed in detail, the history of the antenna will be presented.

1.2 The History of the Side-Loaded Transmission-Line Antenna

The antenna of Figure 1.1 was first analyzed by Harrison and King [2] under the title of a Hybrid Transmission-Line Antenna. They derived expressions for the input impedance of

Figure 1.1--Hybrid transmission-line antenna

Figure 1-1-Hybrid transmission-line antenna

this antenna subject to the restriction that Z_1 be located at the center of the antenna. They pointed out that this antenna was not practically significant, since it has no point of symmetry. In most situations it is thus impossible to approximate the voltage, V_0 , by coupling to a tank coil or connecting a transmission line across the terminals without thereby completely altering the circuit. This objection is overcome, however, by the physically long trailing wire antenna being considered here. For example, consider an airplane one hundred feet long dragging an antenna two thousand feet long. In the analysis of this situation, it is reasonable to approximate the dragging vehicle and the transmitter it carries by a point source. Thus it is possible with the physically long trailing wire antenna to approximate the voltage, V_0 , without completely altering the circuit.

Sparks and Williams [3] were the first to discuss the advantages of using this hybrid transmission line as a trailing wire antenna. They pointed out the fact that an end-fed structure greatly simplifies the problem of dragging an antenna from an airplane or hanging it from a helicopter. From the theory they presented, it can be shown that the hybrid transmission-line antenna has the same radiation pattern as a linear center-fed antenna, and its input impedance and current distribution can be controlled by adjusting the side-load impedance, the end-load impedance and the separation of the conductors. They did not consider the possibility of relocating the side load or of using more than one side load since their work was based

on Harrison and King's solution [2] which is only good for one side load located at the center of the antenna.

1.3 A General Side-Loaded Transmission-Line Antenna and Its Advantages

The hybrid transmission-line antenna does have advantages as a trailing wire antenna as pointed out above. To gain further knowledge of the advantages of this antenna, the author made a numerical investigation of the equations for the input impedance. This investigation revealed that the input impedance function of a hybrid transmission-line antenna presented the possibility of a higher input resistance than the equivalent linear center-fed antenna. For electrically short antennas, however, the higher input resistance is always associated with a higher input reactance, which is a disadvantage. Further investigation seemed to indicate that the situation might be improved by varying the location of the side load on the transmission line, and also by inserting more than one side load.

In order to investigate the last two possibilities, the theory of Harrison and King had to be extended. In Chapter 2, the theory is extended to account for one side load located at any point on the antenna. Using the newly developed theory, it is shown that it is possible to obtain input impedance functions that have an operating frequency with the following simultaneous advantages:

- a) The input resistance is higher than the input resistance of any other known linear antenna configuration.

- b) The input reactance is zero or very small.
- c) Both the input resistance and the input reactance are slowly varying functions of frequency.

In this paper, the antenna investigated in Chapter 2 is called a single side-loaded transmission-line antenna. It is the same as the hybrid transmission-line antenna except that the side load can be located at any point on the transmission line.

Another result of the theoretical work performed in Chapter 2 is an equivalent circuit. It is shown that the input impedance and antenna efficiency¹ calculated from the equivalent circuit are identical with those calculated from the antenna equations. The equivalent circuit gives, without the necessity of detailed calculations, a clear picture of the input impedance function of any given side-loaded transmission-line antenna.

In Chapter 3, the theory is extended to account for more than one side load. The antenna analyzed is called a general side-loaded transmission-line antenna and includes m side loads arbitrarily located on the two wire transmission lines. In Chapter 4, these general equations are used to explore the possibilities of a double side-loaded transmission-line antenna. The investigation included both approximate methods and more precise computer calculations. It is shown that the advantages listed above can be obtained using double side-loaded transmission-line antennas with short circuits for end loads. From the calculations presented in Chapter 4, the double side-loaded

¹By antenna efficiency, we mean here the ratio of the power radiated to the sum of the power radiated plus the power lost in the side load and end load.

transmission-line antennas appear to have more desirable input impedance functions and better efficiencies than the single side-loaded transmission-line antennas. The calculations in Chapter 4, however, are only a small sample of the many configurations possible with single and double side-loaded transmission-line antennas. Furthermore, antennas with more than two side loads are not investigated numerically at all.

Chapter 6 presents a comparison of theory and experimental data. Because of limited facilities, the experimental models measured are scaled down in physical dimensions and up in frequency from the trailing wire antennas discussed above. This scaling introduced effects not accounted for in the original theory which made the comparison of theory and experiment difficult. Once these effects are accounted for, however, the comparison of theory and experiment does verify the general theory very well. The experimental measurements were performed on models of two single side-loaded transmission-line antennas and two double side-loaded transmission line antennas.

1.4 The Radiation Characteristics of the Side-Loaded Transmission-Line Antenna Used as a Trailing Wire Antenna

In Appendix B, the radiation properties of a general side-loaded transmission-line antenna are investigated. It is shown that they are equivalent to the superposition of the appropriate set of asymmetrically-driven linear antennas; we consider the symmetrically-driven antenna to be only a special case of the asymmetrically-driven antenna. Thus to investigate the radiation properties of a general side-loaded transmission-line

antenna, all that is needed is to investigate the radiation properties of asymmetrically-driven linear antennas and then to superimpose the results from the appropriate set.

The far fields and the equivalent electric dipole moments of an asymmetrically-driven linear antenna are calculated in Chapter 5. The effects of curvature and slope are included in the calculations. The curvature and slope are caused by trailing the antenna behind some dragging vehicle, and are illustrated in Figure 1.2. The asymmetrically-driven linear antenna of Figure 1.2 could not be fed from the dragging vehicle without using feed lines which would completely alter the circuit. This, however, is not necessary. In practice the antenna of Figure 1.2 would be an end-fed side-loaded transmission-line antenna. As explained above, we merely use the asymmetrically-driven linear antenna theoretically to investigate the radiation characteristics of the side-loaded transmission-line antenna.

The curves considered in the calculations of Chapter 5 are only those that would be reasonable for a trailing wire antenna. The calculations show that neither the far fields nor the equivalent electric dipole moments are very dependent on the curvature of the antenna. Both, however, are very dependent on the slope of the antenna.

Figure 1.2--Asymmetrically-driven linear antenna used as a trailing wire antenna

1954

1954

1954

2. THE SINGLE SIDE LOADED TRANSMISSION LINE ANTENNA

2.1 Introduction

The purpose of this chapter is to obtain theoretical expressions for the input impedance and the efficiency of a single side-loaded transmission-line antenna. Once these expressions have been obtained it will be shown that the antenna can be replaced by an equivalent circuit. It will then be shown that the equivalent circuit makes it possible to apply the methods of circuit analysis to the problem of determining the effect of the various antenna parameters on the input impedance and the efficiency of the antenna.

The mathematical formulation of the problem is that presented by C. W. Harrison [2, page 172] and is based on a separation of the currents into those which can be treated by conventional transmission-line theory and a radiating current which must be treated by antenna theory. The radiating current is the algebraic sum of the currents in the two wires of the transmission line as long as the separation of the wires is small compared to a wavelength.

Harrison considered only the case of a side-loaded antenna containing one side load located at the middle of the antenna. The theory in this chapter extends the solution so that it is good for one side load located at any point on the antenna.

2.1

The purpose of this report is to provide a comprehensive overview of the current state of the art in the field of artificial intelligence. This report is intended for a general audience and is not intended to be a technical treatise. The report is organized into several sections, each of which will be discussed in detail. The first section, "Introduction", will provide a brief overview of the field of artificial intelligence and its history. The second section, "Current State of the Art", will discuss the most recent developments in the field. The third section, "Future Directions", will discuss the challenges and opportunities that lie ahead. The fourth section, "Conclusion", will provide a summary of the key findings of the report. The report is written in a clear and concise style, and is intended to be a valuable resource for anyone interested in the field of artificial intelligence.

Another extension of Harrison's work presented in this chapter is the derivation of an equivalent circuit. It is shown that the input impedance and efficiency calculated from the equivalent circuit are identical with those calculated from the antenna equations. The equivalent circuit gives a clear picture of what the input impedance function of any given side-loaded transmission-line antenna will be without carrying out detailed calculations.

2.2 The Input Impedance

Referring to Figure 2.1, we make the following assumptions:

a) The ratios $\frac{h}{a}$ and $\frac{h}{b}$ are very large. In the experiments to be described in Chapter 6 these ratios were of the order of ten or greater. In actual practice they would probably be of the order of one hundred or greater.

b) The structure radiates an electromagnetic field characterized by the scalar and vector potentials $\{\phi, \vec{A}\}$ such that the only component of $\vec{A}$ is oriented in the z-direction. In this regard, we neglect any radiation from the horizontal members (which contain Z_2 and V_0); and we neglect the physical length of Z_1 .

c) All conductors are assumed to be perfect so that the tangential component of the electric field on the surface of each wire is zero.

King [1, Section II.12] shows that the z component of $\vec{A}$, A_z , is the only component of $\vec{A}$, and that σ is infinite, implying that A_z is approximated by the solution of

Figure 2.1--Single side-loaded transmission-line antenna

$$A_z'' + \beta^2 A_z = 0 \quad (2.1)$$

where

$$\beta = \omega \sqrt{\mu_0 \epsilon_0} . \quad (2.2)$$

Here the prime represents differentiation with respect to z and (2.2) implies that we are operating in the atmosphere. Actually, this differential equation was derived under the assumption that A_z has complete radial symmetry about the conductors. This is not true for the Side-Loaded Transmission-Line Antenna of Figure 2.1. However, we assume that for our purposes, where we only consider A_z on the surface of each cylindrical member, the radial asymmetries for each member are negligible. In the following we drop the subscript "z." We now write the assumed complete solution for the vector potential on each member:

$$A_{11}(z) = -\frac{j}{v} [C_{11} \sin \beta z + C_{12} \cos \beta z] \quad (2.3)$$

$$A_{12}(z) = -\frac{j}{v} [C_{21} \sin \beta z + C_{22} \cos \beta z] \quad (2.4)$$

$$A_2(z) = -\frac{j}{v} [C_1 \sin \beta z + C_2 \cos \beta z] , \quad (2.5)$$

where v is the velocity of light and is given by $v = 1/\sqrt{\mu_0 \epsilon_0}$. The Lorentz condition relating the vector and scalar potentials for this situation is

$$\phi = j \frac{\omega}{\beta^2} \frac{\partial A}{\partial z} . \quad (2.6)$$

Applying the boundary conditions at the side load, z_1 , we have for the potential ϕ

$$V_1 = \phi_{11}(\ell_1) - \phi_{12}(\ell_1) . \quad (2.7)$$

Combining (2.3, 2.4, 2.6, and 2.7), we find

$$A_1 + A_2 = A_3 + A_4 + A_5 + A_6 + A_7 + A_8 + A_9 + A_{10} + A_{11} + A_{12} + A_{13} + A_{14} + A_{15} + A_{16} + A_{17} + A_{18} + A_{19} + A_{20} + A_{21} + A_{22} + A_{23} + A_{24} + A_{25} + A_{26} + A_{27} + A_{28} + A_{29} + A_{30} + A_{31} + A_{32} + A_{33} + A_{34} + A_{35} + A_{36} + A_{37} + A_{38} + A_{39} + A_{40} + A_{41} + A_{42} + A_{43} + A_{44} + A_{45} + A_{46} + A_{47} + A_{48} + A_{49} + A_{50} + A_{51} + A_{52} + A_{53} + A_{54} + A_{55} + A_{56} + A_{57} + A_{58} + A_{59} + A_{60} + A_{61} + A_{62} + A_{63} + A_{64} + A_{65} + A_{66} + A_{67} + A_{68} + A_{69} + A_{70} + A_{71} + A_{72} + A_{73} + A_{74} + A_{75} + A_{76} + A_{77} + A_{78} + A_{79} + A_{80} + A_{81} + A_{82} + A_{83} + A_{84} + A_{85} + A_{86} + A_{87} + A_{88} + A_{89} + A_{90} + A_{91} + A_{92} + A_{93} + A_{94} + A_{95} + A_{96} + A_{97} + A_{98} + A_{99} + A_{100}$$

where ϵ_0 is the permittivity of free space, ϵ is the permittivity of the medium, ρ is the charge density, and ∇^2 is the Laplacian operator.

Since the wave equation is a differential equation with respect to r and θ , it is necessary to assume that we are operating in the atmosphere.

Actually, this differential equation was derived under the assumption that A has complete radial symmetry about the con-

ductor. This is not true for the side-loaded transmission

line antenna of Figure 1. However, we assume that for our

purpose, which we will consider A on the surface of each

cylindrical member, the radial asymmetry for each member

is negligible. In the following we drop the subscript r and

we now write the boundary condition for the vector

potential on each member:

$$A_1(z) = \frac{1}{\sqrt{2}} C_1 \sin \theta + C_2 \cos \theta \quad (2.3)$$

$$A_2(z) = \frac{1}{\sqrt{2}} C_3 \sin \theta + C_4 \cos \theta \quad (2.4)$$

$$A_3(z) = \frac{1}{\sqrt{2}} C_5 \sin \theta + C_6 \cos \theta \quad (2.5)$$

where v is the velocity of light and is given by $v = 1/\sqrt{\epsilon_0 \epsilon}$.

The boundary condition relating the vector and scalar potentials

for this situation is

$$\nabla^2 A = -\frac{1}{v^2} \frac{\partial V}{\partial t} \quad (2.6)$$

Applying the boundary conditions at the side lead, $\theta = \pi/2$,

we have for the potential A

$$A = \frac{1}{\sqrt{2}} C_1 \sin \theta + C_2 \cos \theta \quad (2.7)$$

Over the top lead, $\theta = 0$ and $\theta = \pi$, we have

$$V_1 = (C_{11} - C_{21}) \cos \beta \ell_1 - (C_{12} - C_{22}) \sin \beta \ell_1 . \quad (2.8)$$

From the continuity of the vector potential

$$A_{11}(\ell_1) = A_{12}(\ell_1) . \quad (2.9)$$

Combining (2.3, 2.4, and 2.9), we find

$$(C_{11} - C_{21}) \sin \beta \ell_1 = - (C_{12} - C_{22}) \cos \beta \ell_1 . \quad (2.10)$$

Solving (2.8 and 2.10) simultaneously for C_{11} and C_{12} , we obtain

$$C_{11} = C_{21} + V_1 \cos \beta \ell_1 \quad (2.11)$$

$$C_{12} = C_{22} - V_1 \sin \beta \ell_1 . \quad (2.12)$$

Substituting (2.11 and 2.12) in (2.3) and repeating (2.4 and 2.5), we obtain the following set of equations for the vector potential:

$$A_{11}(z) = -\frac{j}{v} \left[\{C_{21} + V_1 \cos \beta \ell_1\} \sin \beta z + \{C_{22} - V_1 \sin \beta \ell_1\} \cos \beta z \right] \quad (2.13)$$

$$A_{12}(z) = -\frac{j}{v} [C_{21} \sin \beta z + C_{22} \cos \beta z] \quad (2.14)$$

$$A_2(z) = -\frac{j}{v} [C_1 \sin \beta z + C_2 \cos \beta z] . \quad (2.15)$$

For convenience we define

$$\begin{aligned} A_1(z) &= A_{1n}(z) \\ \phi_1(z) &= \phi_{1n}(z) \quad \text{for } \ell_{n-1} \leq z \leq \ell_n , \\ I_1(z) &= I_{1n}(z) \end{aligned} \quad (2.16)$$

where $\ell_0 = 0$, $\ell_2 = 2h$, and $n = 1, 2$. Next we consider the boundary conditions at $z = 2h$. From Figure 2.1 we observe that

$$V_2 = \phi_1(2h) - \phi_2(2h) . \quad (2.17)$$

from the continuity of the vector potential

$$A_{11}(z) = A_{12}(z) \quad (2.10)$$

Combining (2.8), (2.9), and (2.10), we find

$$C_{11} \cos \beta_1 z + C_{21} \sin \beta_1 z = C_{12} \cos \beta_2 z + C_{22} \sin \beta_2 z \quad (2.11)$$

Solving (2.8) and (2.10) simultaneously for C_{11} and C_{12} , we obtain

$$C_{11} = C_{21} + V_1 \cos \beta_1 z \quad (2.12)$$

$$C_{12} = C_{22} - V_1 \sin \beta_1 z \quad (2.13)$$

Substituting (2.11) and (2.12) in (2.9) and repeating (2.8) and

(2.9), we obtain the following set of equations for the vector

potential:

$$A_{11}(z) = \frac{1}{V_1} [C_{21} \cos \beta_1 z + C_{22} \sin \beta_1 z] \quad (2.14)$$

$$A_{12}(z) = -\frac{1}{V_1} [C_{21} \sin \beta_1 z + C_{22} \cos \beta_1 z] \quad (2.15)$$

$$A_2(z) = \frac{1}{V_1} [C_{21} \sin \beta_2 z + C_{22} \cos \beta_2 z] \quad (2.16)$$

For convenience we define

$$A_1(z) = A_{11}(z) \quad (2.17)$$

$$A_2(z) = A_{12}(z) \quad (2.18)$$

$$I_1(z) = I_{11}(z) \quad (2.19)$$

where $\beta_1 = 0$, $\beta_2 = 2\beta$, and $\beta = \beta_1/2$. Next we consider the

boundary conditions at $z = 0$. From Figure 1 we observe

that

$$V_1 = 0 \quad (2.20)$$

Combining (2.6, 2.14, 2.15, 2.16, and 2.17), we find

$$V_2 = C_3 \cos 2\beta h - C_4 \sin 2\beta h, \quad (2.18)$$

where

$$C_3 = C_{21} - C_1 \quad (2.19)$$

$$C_4 = C_{22} - C_2. \quad (2.20)$$

Referring again to Figure 2.1, we observe that at $z = 0$

$$V_0 = \phi_1(0) - \phi_2(0). \quad (2.21)$$

Combining (2.6, 2.13, 2.15, 2.16, and 2.21), we obtain

$$V_0 = C_3 + V_1 \cos \beta \ell_1. \quad (2.22)$$

It has been shown by King [1, Section II.12] that the vector potential on the surface of a very long thin cylinder (length = $2h$, radius = r_0 ; $\frac{h}{r_0}$ large) oriented along the z axis and carrying a current I is closely approximated by

$$A(z, r_0) = \frac{\mu_0}{4\pi} \int_0^{2h} I(z') K_{r_0}(z, z') dz', \quad (2.23a)$$

where

$$K_{r_0} = \frac{e^{-j\beta \left[r_0^2 + (z-z')^2 \right]^{1/2}}}{\left[r_0^2 + (z-z')^2 \right]^{1/2}}. \quad (2.23b)$$

In the derivation of (2.23), it is assumed that the current, I , is in a filament rather than in a cylinder with finite cross section. Some limitations of this approximation are discussed by Duncan and Hinchey [8]. We will, however, use (2.23) because it has been successful in predicting experimental measurements, and because it is relatively easy to use as compared to more exact formulas.

Combining (2.1)

$$V = C_1 \nabla \phi$$

where

$$C_1 = \frac{1}{\sqrt{2}}$$

$$C_1 = C_2 = C_3$$

Referring again to (2.1)

$$V = C_1 \nabla \phi$$

Combining (2.1)

$$V = C_1 \nabla \phi$$

It has been assumed that

vector potential

(length = $1/\sqrt{2}$)

and carrying a charge

$$A(x, y)$$

where

$$A(x, y) = \frac{1}{\sqrt{2}}$$

$$A(x, y) = \frac{1}{\sqrt{2}}$$

In the derivation of

is in a similar manner

section. Some attention

by Duncan and Hickey

cause it has been

essential

compared to more

Another approximation we will need was derived by C. W. Harrison [2, page 173, 174] in his original solution of the problem. Assume that there are two very long, thin cylinders oriented parallel to each other; see Figure 2.2. We then say that the z component of $\vec{A}$ on the surface of one of the cylinders, cylinder (1) for example, is the sum of two contributions; one is from I_1 , and the other is from I_2 . Using (2.23), we then find for $\vec{A}$ on the surface of cylinder (1)

$$A_1(z) = \frac{\mu_0}{4\pi} \int_0^{2h} [I_1(z') K_a(z, z') - I_2(z') K_b(z, z')] dz' . \quad (2.24)$$

In a similar manner we find for $\vec{A}$ on the surface of cylinder (2)

$$A_2(z) = \frac{\mu_0}{4\pi} \int_0^{2h} [I_1(z') K_b(z, z') - I_2(z') K_a(z, z')] dz' . \quad (2.25)$$

In order to separate the radiating antenna currents from the transmission line currents, Harrison defined

$$A_d(z) = \frac{\mu_0}{4\pi} \int_0^{2h} [I_1(z') - I_2(z')] K_d(z, z') dz' . \quad (2.26)$$

If (2.26) is added and subtracted from the right-hand side of (2.24 and 2.25), the result is

$$A_1(z) = A_d(z) + \frac{\mu_0}{4\pi} \int_0^{2h} \{ I_1(z') [K_a(z, z') - K_d(z, z')] - I_2(z') [K_b(z, z') - K_d(z, z')] \} dz' \quad (2.27)$$

Figure 2.2--Two long thin cylinders oriented parallel

AS = 2

Figure 1. Two long thin cylinders oriented parallel

$$A_2(z) = A_d(z) + \frac{\mu_0}{4\pi} \int_0^{2h} \left\{ I_1(z') [K_b(z, z') - K_d(z, z')] - I_2(z') [K_a(z, z') - K_d(z, z')] \right\} dz' . \quad (2.28)$$

It is shown by Harrison [2, page 174]¹ that if $\frac{h}{a}$ and $\frac{h}{b}$ are very large and I_1 and I_2 sufficiently smooth, then we can write

$$A_1(z) \cong A_d(z) + \frac{\mu_0}{2\pi} \left[I_1(z) \ln \frac{d}{a} - I_2(z) \ln \frac{d}{b} \right] \quad (2.29)$$

$$A_2(z) \cong A_d(z) + \frac{\mu_0}{2\pi} \left[I_1(z) \ln \frac{d}{b} - I_2(z) \ln \frac{d}{a} \right] . \quad (2.30)$$

An expression for the input impedance of the antenna can now be obtained by combining (2.13, 2.14, 2.15, 2.19, and 2.20) with (2.29 and 2.30) and using the boundary conditions as expressed in (2.18 and 2.22). The process is carried out in the following manner. Subtracting (2.30) from (2.29), we obtain (dropping the approximation sign for convenience)

$$A_1(z) - A_2(z) = \frac{\mu_0}{2\pi} \ln \frac{b}{a} [I_1(z) + I_2(z)] . \quad (2.31)$$

Combining (2.13, 2.14, 2.15, 2.19, 2.20, and 2.31), we find

$$I_1(z) + I_2(z) = -j \frac{2}{R_c} \left[\{C_3 + V_1 \cos \beta \ell_1\} \sin \beta z + \{C_4 - V_1 \sin \beta \ell_1\} \cos \beta z \right] \quad (2.32)$$

for $\ell_0 \leq z \leq \ell_1$, and

$$\begin{aligned} \frac{\mu_0}{4\pi} \int_0^{2h} I_1(z') [K_a(z, z') - K_d(z, z')] dz' &\cong \frac{\mu_0}{4\pi} I_1(z) \int_{-\infty}^{\infty} [K_a(z, z') - K_d(z, z')] dz' \\ &\cong \frac{\mu_0}{2\pi} I_1(z) \ln \frac{d}{a} \end{aligned}$$

$$I_1(z) + I_2(z) = -j \frac{2}{R_c} [C_3 \sin \beta z + C_4 \cos \beta z] \quad (2.33)$$

for $\ell_1 \leq z \leq \ell_2$, where

$$R_c = \frac{\eta}{\pi} \ln \frac{b}{a} \quad (2.34)$$

and

$$\eta = \sqrt{\frac{\mu_o}{\epsilon_o}} \quad (2.35)$$

From Figure 2.1 we observe that $I_1(0) = I_2(0)$ and $I_1(2h) = I_2(2h)$. Using this information in (2.32 and 2.33), we obtain

$$I_1(0) = -\frac{j}{R_c} [C_4 - V_1 \sin \beta \ell_1] \quad (2.36)$$

$$I_1(2h) = -\frac{j}{R_c} [C_3 \sin 2\beta h + C_4 \cos 2\beta h] \quad (2.37)$$

From (2.22 and 2.36) we find

$$Z_o = \frac{V_o}{I_1(0)} = jR_c \left[\frac{C_3 + V_1 \cos \beta \ell_1}{C_4 - V_1 \sin \beta \ell_1} \right] \quad (2.38)$$

If we make the definitions

$$\gamma_1 Z_1 = \frac{V_1}{C_4} \quad (2.39)$$

and

$$\gamma_2 = \frac{C_3}{C_4} \quad (2.40)$$

and substitute (2.39 and 2.40) in (2.38), we obtain

$$Z_o = jR_c \left[\frac{\gamma_2 + \gamma_1 Z_1 \cos \beta \ell_1}{1 - \gamma_1 Z_1 \sin \beta \ell_1} \right] \quad (2.41)$$

We now have obtained an expression for the input impedance of the antenna. This expression, however, contains the parameters γ_1 and γ_2 , defined above, which we have not evaluated

$$I_1(z) = I_2(z) = \frac{1}{2} [I_1(z) + I_2(z)] = \frac{1}{2} [I_1(z) + I_2(z)] \quad (2.35)$$

for $z = \frac{1}{2} (z_1 + z_2)$

$$I_1(z) = \frac{1}{2} [I_1(z) + I_2(z)] = \frac{1}{2} [I_1(z) + I_2(z)] \quad (2.36)$$

and

$$I_2(z) = \frac{1}{2} [I_1(z) + I_2(z)] = \frac{1}{2} [I_1(z) + I_2(z)] \quad (2.37)$$

From Equations (2.35) and (2.36) we obtain

Using this information in (2.35) and (2.36) we obtain

$$I_1(z) = \frac{1}{2} [I_1(z) + I_2(z)] = \frac{1}{2} [I_1(z) + I_2(z)] \quad (2.38)$$

$$I_2(z) = \frac{1}{2} [I_1(z) + I_2(z)] = \frac{1}{2} [I_1(z) + I_2(z)] \quad (2.39)$$

From (2.38) and (2.39) we find

$$I_1(z) = \frac{1}{2} [I_1(z) + I_2(z)] = \frac{1}{2} [I_1(z) + I_2(z)] \quad (2.40)$$

if we make the substitutions

$$I_1(z) = \frac{1}{2} [I_1(z) + I_2(z)] = \frac{1}{2} [I_1(z) + I_2(z)] \quad (2.41)$$

and

$$I_2(z) = \frac{1}{2} [I_1(z) + I_2(z)] = \frac{1}{2} [I_1(z) + I_2(z)] \quad (2.42)$$

and substitute (2.41) and (2.42) in (2.38) we obtain

$$I_1(z) = \frac{1}{2} [I_1(z) + I_2(z)] = \frac{1}{2} [I_1(z) + I_2(z)] \quad (2.43)$$

We now have obtained an expression for the input impedance

of the antenna. This expression, however, contains the term

which we have not yet evaluated.

yet. Because it is the simpler of the two to evaluate, we start first with γ_2 . From Figure 2.1 we observe that

$$V_2 = Z_2 I_1(2h) . \quad (2.42)$$

Combining (2.18, 2.37, 2.40 and 2.42), we obtain

$$\gamma_2 = \left[\frac{R_c \sin 2\beta h - jZ_2 \cos 2\beta h}{R_c \cos 2\beta h + jZ_2 \sin 2\beta h} \right] . \quad (2.43)$$

Next we seek a method for evaluating γ_1 . To do this we return to (2.29 and 2.30). Adding these two expressions, we find

$$A_1(z) + A_2(z) = 2A_d(z) + \frac{\mu_0}{2\pi} I_T(z) \left[\ln \frac{d}{b} + \ln \frac{d}{a} \right] , \quad (2.44)$$

where

$$I_T = I_1 - I_2 . \quad (2.45)$$

Referring to (2.26), we see that as yet "d" is an arbitrary constant. Because it turns out to be convenient in evaluating γ_1 , we choose "d" such that

$$\ln \frac{d}{b} + \ln \frac{d}{a} = 0 , \quad (2.46)$$

which is equivalent to

$$d = \sqrt{ab} . \quad (2.47)$$

Combining (2.13, 2.14, 2.15, 2.26, 2.44, 2.45, and 2.46), we obtain

$$\int_0^{2h} I_T(z') K_d(z, z') dz' = j \frac{4\pi}{\eta} \left[- \left\{ \frac{C_{21} + C_1}{2} + \frac{V_1}{2} \cos \beta \ell_1 \right\} \sin \beta z \right. \\ \left. - \left\{ \frac{C_{22} + C_2}{2} - \frac{V_1}{2} \sin \beta \ell_1 \right\} \cos \beta z \right] \quad (2.48)$$

for $\ell_0 \leq z \leq \ell_1$, and

yes. Because it is the simpler of the two to evaluate we

start with P_1 . From (1.1) we have that

$$V_2 = \frac{1}{2}I_1(2\pi)$$

Combining (2.18), (2.19), (2.20) and (2.21) we obtain

$$V_2 = \frac{1}{2} \left[\frac{1}{2} \cos 2\theta + \frac{1}{2} \sin 2\theta \right]$$

Next we seek a method for evaluating V_2 to this

extent to (2.22 and 2.23). Adding these two expressions we

find

$$A_1(z) + A_2(z) = 2A_2(z) + \frac{1}{2} \ln \frac{d}{b} + \ln \frac{d}{b}$$

where

$$I_1 = I_2 = 1$$

Referring to (2.25) we see that as yet V_2 is an arbitrary

constant. Because it turns out to be convenient to evaluate

V_1 we choose V_2 such that

$$\ln \frac{d}{b} + \ln \frac{d}{b} = 0$$

which is equivalent to

$$d = \sqrt{b}$$

Combining (2.17), (2.18), (2.19), (2.20) and (2.21) we

obtain

$$V_1 = \frac{1}{2} \left[\frac{1}{2} \cos 2\theta + \frac{1}{2} \sin 2\theta \right]$$

for V_1 , V_2 , V_3 and

$$\int_0^{2h} I_T(z') K_d(z, z') dz' = j \frac{4\pi}{\eta} \left[- \left\{ \frac{C_{21} + C_1}{2} \right\} \sin \beta z - \left\{ \frac{C_{21} + C_2}{2} \right\} \cos \beta z \right] \quad (2.49)$$

for $\ell_1 \leq z \leq \ell_2$. If the integral equations in (2.48 and 2.49) could be solved for the ratio

$$Z_d = - \frac{V_1}{2I_T(\ell_1)} , \quad (2.50)$$

it would be possible to evaluate γ_1 , and (2.41) would be a means of evaluating the input impedance of the antenna.

From Appendix A, equations (A.10 and A.11), we observe that (2.48 and 2.49) are the integral equations used to determine the current and impedance of the asymmetrically-driven linear antenna of Figure 2.3. In the same Appendix a method is presented for solving (2.48 and 2.49) for the ratio (2.50) so we shall assume here that Z_d can be evaluated.

Using (2.50), it is quite simple to evaluate γ_1 . First we set $z = \ell_1$ in (2.33) and find

$$I_1(\ell_1) + I_2(\ell_1) = - j \frac{2}{R_c} [C_3 \sin \beta \ell_1 + C_4 \cos \beta \ell_1] . \quad (2.51)$$

Substituting (2.45) in (2.51), we obtain

$$2I_1(\ell_1) - I_T(\ell_1) = - j \frac{2}{R_c} [C_3 \sin \beta \ell_1 + C_4 \cos \beta \ell_1] . \quad (2.52)$$

From Figure 2.1 we observe that

$$I_1(\ell_1) = \frac{V_1}{Z_1} . \quad (2.53)$$

Combining (2.39, 2.50, 2.52, and 2.53), we finally obtain

$$\gamma_1 Z_1 = - \frac{j}{R_c} \left[\frac{4Z_d Z_1}{4Z_d + Z_1} \right] [\cos \beta \ell_1 + \gamma_2 \sin \beta \ell_1] . \quad (2.54)$$

Figure 2.3--Asymmetrically-driven linear antenna

At this point we combine (2.41, 2.43, and 2.54) in order to obtain a single expression for the input impedance of the antenna. The result is

$$Z_o = jR_c \left[\frac{\left\{ \frac{R_c \sin 2\beta h - jZ_2 \cos 2\beta h}{R_c \cos \beta(2h - \ell_1) + jZ_2 \sin \beta(2h - \ell_1)} \right\} - j \frac{Z_{o1}}{R_c} \cos \beta \ell_1}{\left\{ \frac{R_c \cos 2\beta h + jZ_2 \sin 2\beta h}{R_c \cos \beta(2h - \ell_1) + jZ_2 \sin \beta(2h - \ell_1)} \right\} + j \frac{Z_{o1}}{R_c} \sin \beta \ell_1} \right], \quad (2.55)$$

where

$$Z_{o1} = \frac{4Z_d Z_1}{4Z_d + Z_1} \quad (2.56)$$

Except for Z_d , (2.55 and 2.56) are completely determined in terms of physical parameters of the single side-loaded transmission-line antenna of Figure 2.1 which can be measured quite simply. Z_d is the input impedance of the antenna of Figure 2.3 which we shall call the equivalent radiating dipole antenna. We choose this name since the radiating currents for the antenna of Figure 2.1 are equivalent to the currents on the antenna of Figure 2.3.

Looking ahead to the possibility of an equivalent circuit for the antenna, we rearrange (2.55) to the form

$$Z_o = R_c \left[\frac{Z'_{T1} \cos \beta \ell_1 + jR_c \sin \beta \ell_1}{R_c \cos \beta \ell_1 + jZ'_{T1} \sin \beta \ell_1} \right], \quad (2.57)$$

where

$$Z'_{T1} = Z_{T1} + Z_{o1} \quad (2.58)$$

$$Z_{T1} = R_c \left[\frac{Z_2 \cos \beta(2h - \ell_1) + jR_c \sin \beta(2h - \ell_1)}{R_c \cos \beta(2h - \ell_1) + jZ_2 \sin \beta(2h - \ell_1)} \right] \quad (2.59)$$

The equivalence of (2.57, 2.58 and 2.59) with (2.55 and 2.56) is shown in Appendix C.

At this point we combine (2.41), (2.42), and (2.43) in order

to obtain a single expression for the input impedance of the

antenna. The result is

$$Z_o = jR_o \left[\frac{R_c \cos(kL) + jZ_o \sin(kL)}{R_c \cos(kL) - jZ_o \sin(kL)} \right] + jZ_o \sin(kL)$$

where

$$R_o = \frac{1}{2} \left(\frac{Z_{11} + Z_{22}}{Z_{11} - Z_{22}} \right)$$

Except for Z_o , (2.44) and (2.45) are completely determined in

terms of physical parameters of the single side-loaded beam

mission-line antenna of Figure 2.1 which can be measured. With

equation (2.44) the input impedance of the antenna at length L

which we shall call the equivalent radiating dipole antenna

We choose this name since the radiating currents for the an

tern of Figure 2.1 are equivalent to the currents on the

antenna of Figure 2.2.

Looking ahead to the possibility of an equivalent circuit

for the antenna, we rearrange (2.44) to the form

$$Z_o = R_o \left[\frac{Z_{11} \cos(kL) + jZ_o \sin(kL)}{Z_{11} \cos(kL) - jZ_o \sin(kL)} \right] + jZ_o \sin(kL)$$

where

$$Z_{11} = Z_{11} + Z_{22}$$

$$Z_{11} = \frac{1}{2} \left(\frac{Z_{11} + Z_{22}}{Z_{11} - Z_{22}} \right)$$

The equivalence of (2.44) and (2.45) with (2.46) and (2.47)

is shown in Appendix C.

It is worth mentioning at this point that the rearrangement of (2.55 and (2.56) to the form (2.57, 2.58, and 2.59) would be a rather hopeless search if one did not already know what the equivalent circuit was. It is also quite obvious that (2.55) doesn't in any direct way suggest any simple equivalent circuit. Thus, it would seem that we need the equivalent circuit to rearrange the equations, and we need the rearranged equations to see clearly what the equivalent circuit should be. Actually the situation is not this bad. If we had started the problem by setting the origin of the z axis at the side load, Z_1 , rather than at the generator, the resulting equation for the input impedance, Z_0 , would have been more suggestive of both the equivalent circuit and the rearrangement leading to (2.57, 2.58, and 2.59). The system used in the above solution with the origin of the z axis at the driving point was chosen because it leads more naturally into the solution of the general side-loaded antenna with m loads which is presented in Chapter 3.

2.3 Efficiency of the Single Side-Loaded Transmission-Line Antenna

In considering the efficiency of the single side-loaded transmission-line antenna, we will first consider losses in the side load and end load. After we have developed the equivalent circuit, a method for handling any losses due to the finite conductivity of the wires will then be presented. From Figure 2.1 we observe that the total power input and the power lost in the side load and end load are given by the following equations.

$$\text{Total Power Input} = |V_o|^2 G_o, \quad (2.60)$$

$$\text{where } G_o = \frac{R_o}{|Z_o|^2}. \quad (2.61)$$

$$\text{Power lost in the side load and end load} = |V_1|^2 G_1 + |V_2|^2 G_2, \quad (2.62)$$

$$\text{where } G_1 = \frac{R_1}{|Z_1|^2}, \quad G_2 = \frac{R_2}{|Z_2|^2}. \quad (2.63)$$

Thus the expression for efficiency can be written

$$\text{Efficiency} = 100 \left[1 - \frac{|V_1|^2 G_1}{|V_o|^2 G_o} - \frac{|V_2|^2 G_2}{|V_o|^2 G_o} \right]. \quad (2.64)$$

Using (2.22, 2.39, and 2.40), we obtain

$$\frac{V_o}{V_1} = \frac{\gamma_2}{\gamma_1 Z_1} + \cos \beta \ell_1. \quad (2.65)$$

Substituting (2.43 and 2.54) in (2.65), we find after some rearrangement

$$\frac{V_o}{V_1} = -\frac{R_c(4Z_d + Z_1)}{j4Z_d Z_1} \left[\sin \beta \ell_1 - j \cos \beta \ell_1 \left\{ \frac{Z_2 \cos \beta(2h - \ell_1) + jR_c \sin \beta(2h - \ell_1)}{R_c \cos \beta(2h - \ell_1) + jZ_2 \sin \beta(2h - \ell_1)} \right\} \right] + \cos \beta \ell_1. \quad (2.66)$$

Referring to (2.57, 2.58, and 2.59), we see that (2.66) can be written in the form

$$\frac{V_o}{V_1} = \frac{R_c \sin \beta \ell_1 - jZ_{T1} \cos \beta \ell_1}{-jZ_{o1}} + \cos \beta \ell_1 = \frac{R_c \sin \beta \ell_1 - jZ'_{T1} \cos \beta \ell_1}{-jZ_{o1}}. \quad (2.67)$$

Taking the square of the magnitude of (2.67), we obtain

$$\frac{V_o^2}{V_1^2} = \frac{[R_c \sin \beta \ell_1 + X'_{T1} \cos \beta \ell_1]^2 + R'^2_{T1} \cos^2 \beta \ell_1}{|Z_{o1}|^2} = \frac{R'^2_{T1}}{G_o |Z_{o1}|^2} \quad (2.68)$$

where

$$(2.60) \quad \text{Total Power Input} = |V|^2 \frac{1}{Z_0}$$

$$(2.61) \quad \text{where } Z_0 = \frac{R_0}{1 + \frac{X_0^2}{R_0^2}}$$

$$(2.62) \quad \text{Power loss in the side load and end load} = |V|^2 \frac{1}{Z_1} + |V|^2 \frac{1}{Z_2}$$

$$(2.63) \quad \text{where } Z_1 = \frac{R_1}{1 + \frac{X_1^2}{R_1^2}} \quad \text{and} \quad Z_2 = \frac{R_2}{1 + \frac{X_2^2}{R_2^2}}$$

Thus the expression for efficiency can be written

$$(2.64) \quad \text{Efficiency} = 100 \left[\frac{|V|^2 \frac{1}{Z_0} - |V|^2 \frac{1}{Z_1} - |V|^2 \frac{1}{Z_2}}{|V|^2 \frac{1}{Z_0}} \right]$$

Using (2.52), (2.53), and (2.60), we obtain

$$(2.65) \quad \frac{V_0}{V_1} = \frac{1}{1 + \frac{R_1}{R_0} \cos \theta_1}$$

Substituting (2.47) and (2.54) in (2.65), we find after some

rearrangement

$$(2.66) \quad \frac{V_0}{V_1} = \frac{1}{1 + \frac{R_1}{R_0} \cos \theta_1} \left[\frac{1 + \frac{R_1}{R_0} \cos \theta_1}{1 + \frac{R_1}{R_0} \cos \theta_1} + \frac{1 + \frac{R_1}{R_0} \cos \theta_1}{1 + \frac{R_1}{R_0} \cos \theta_1} \right]$$

Referring to (2.51), (2.52), and (2.53), we see that (2.66) can be

written in the form

$$(2.67) \quad \frac{V_0}{V_1} = \frac{1 + \frac{R_1}{R_0} \cos \theta_1}{1 + \frac{R_1}{R_0} \cos \theta_1} + \frac{1 + \frac{R_1}{R_0} \cos \theta_1}{1 + \frac{R_1}{R_0} \cos \theta_1}$$

Taking the square of the magnitude of (2.67), we obtain

$$(2.68) \quad \left| \frac{V_0}{V_1} \right|^2 = \left[\frac{1 + \frac{R_1}{R_0} \cos \theta_1}{1 + \frac{R_1}{R_0} \cos \theta_1} \right]^2 + \left[\frac{1 + \frac{R_1}{R_0} \cos \theta_1}{1 + \frac{R_1}{R_0} \cos \theta_1} \right]^2$$

$$G_o = \frac{R_{T1}^o}{\left[R_c \sin \beta \ell_1 + X_{T1}^o \cos \beta \ell_1 \right]^2 + R_{T1}^o{}^2 \cos^2 \beta \ell_1}, \quad (2.69)$$

and (2.69) can be found by separating the inverse of (2.57) into its real and imaginary parts.

Next we combine (2.18, 2.39, and 2.40) and find

$$V_2 = \frac{V_1}{Z_1} \left[\gamma_2 \cos 2\beta h - \sin 2\beta h \right]. \quad (2.70)$$

Substituting (2.43 and 2.54) in (2.70), we find after some rearrangement

$$\frac{V_2}{V_1} = \frac{4Z_d + Z_1}{4Z_d Z_1} \left[\frac{R_c}{R_c \cos \beta(2h - \ell_1) + jZ_2 \sin \beta(2h - \ell_1)} \right] Z_2. \quad (2.71)$$

Taking the square of the magnitude of (2.71), we obtain

$$\left| \frac{V_2}{V_1} \right|^2 = |Z_2|^2 \left| \frac{4Z_d + Z_1}{4Z_d Z_1} \right|^2 \times \frac{R_c^2}{\left[R_c \cos \beta(2h - \ell_1) - X_2 \sin \beta(2h - \ell_1) \right]^2 + R_2^2 \sin^2 \beta(2h - \ell_1)}. \quad (2.72)$$

Referring to (2.57, 2.58, and 2.59), we see that (2.72) can be rewritten in the form

$$\left| \frac{V_2}{V_1} \right|^2 = \frac{|Z_2|^2}{|Z_{o1}|^2} \frac{R_{T1}}{R_2} = \frac{R_{T1}}{G_2 |Z_{o1}|^2}. \quad (2.73)$$

Combining (2.68 and 2.73), we obtain

$$\left| \frac{V_2}{V_o} \right|^2 = \frac{G_o R_{T1}}{G_2 R_{T1}^o}. \quad (2.74)$$

Substituting (2.68 and 2.74) in (2.64), we find for the efficiency

$$V_1 = \frac{1}{\sqrt{2}} \begin{bmatrix} \cos \theta_1 \\ \sin \theta_1 \end{bmatrix}, \quad V_2 = \frac{1}{\sqrt{2}} \begin{bmatrix} -\sin \theta_1 \\ \cos \theta_1 \end{bmatrix}$$

and (λ_1, λ_2) can be found by separating the variables of (2.10)

into its real and imaginary parts.

Next we combine (2.10), (2.11), and (2.12) to obtain

$$V_1^* \left[\frac{1}{2} (A + B) + \frac{1}{2} (A - B) \cos 2\theta_1 \right] V_1 = \lambda_1 V_1$$

Substituting (2.10) and (2.11) in (2.10), we find after some

rearranging

$$\frac{1}{2} (A + B) + \frac{1}{2} (A - B) \cos 2\theta_1 = \lambda_1$$

Taking the square of the magnitude of (2.11), we obtain

$$\left| \frac{V_2}{V_1} \right|^2 = \frac{1}{2} \left[\frac{A + B}{A - B} + \frac{A - B}{A + B} \cos 2\theta_1 \right]$$

$$R_1$$

$$R_1 = \frac{1}{2} \left[\frac{A + B}{A - B} + \frac{A - B}{A + B} \cos 2\theta_1 \right]$$

Referring to (2.10), (2.11), and (2.12), we see that (2.11) can be

rewritten in the form

$$\left| \frac{V_2}{V_1} \right|^2 = \frac{1}{2} \left[\frac{R_1}{R_2} + \frac{R_2}{R_1} \cos 2\theta_1 \right]$$

Combining (2.08) and (2.13), we obtain

$$\left| \frac{V_2}{V_1} \right|^2 = \frac{1}{2} \left[\frac{R_1}{R_2} + \frac{R_2}{R_1} \cos 2\theta_1 \right]$$

Substituting (2.08) and (2.13) in (2.13), we find for the

relation

$$\text{Efficiency} = 100 \left[1 - \frac{|Z_{O1}|^2 G_1}{R_{T1}^i} - \frac{R_{T1}}{R_{T1}^i} \right] . \quad (2.75)$$

2.4 The Equivalent Circuit

Having the equations for the input impedance arranged in the form of (2.56, 2.57, 2.58 and 2.59), the equivalent circuit is quite simple to construct. Z_O , given by (2.57), is just the input impedance of a lossless transmission line terminated in the impedance, Z_{T1}^i , with a characteristic impedance, R_C , and length, ℓ_1 . Z_{O1} , given by (2.56), is the parallel combination of the $4Z_d$ and Z_1 ; Z_{T1}^i , given by (2.58), is the series combination of Z_{T1} and Z_{O1} ; and Z_{T1} , given by (2.59), is the input impedance of a lossless transmission line terminated in the impedance, Z_2 , with a characteristic impedance, R_C , and length $2h - \ell_1$. Combining these facts leads to the equivalent circuit of Figure 2.4 whose input impedance, Z_O , is given by (2.57). Thus we have an equivalent circuit which can be used to calculate the input impedance of the single side-loaded transmission-line antenna.

It would be convenient to use the equivalent circuit of Figure 2.4 for efficiency calculations as well as for calculations of input impedance. In order to check this possibility we observe from Figure 2.4 that

$$\text{Total Power Input} = |I_{T1}^i|^2 R_{T1}^i \quad (2.76)$$

and

$$\begin{aligned} \text{Power Lost in } Z_1 \text{ and } Z_2 &= |V_1|^2 G_1 + |V_2|^2 G_2 \\ &= |I_{T1}^i|^2 |Z_{O1}|^2 G_1 + |I_{T1}^i|^2 R_{T1} . \end{aligned} \quad (2.77)$$

$$Z_{in} = \frac{Z_1 + jZ_2}{1 + jZ_1 Z_2}$$

2.4 The Equivalent Circuit

Having the equations for the input impedance

in the form of (2.3), (2.4) and (2.5) the equivalent

circuit is quite simple to construct. Z_{in} given by (2.3)

is just the input impedance of a lossless transmission line

terminated in the impedance Z_L with a characteristic

impedance Z_0 and length l , given by (2.4) and (2.5)

parallel combination of the Z_1 and Z_2 given by (2.5)

is the series combination of Z_1 and Z_2 given by (2.4)

(2.5) is the input impedance of a lossless transmission

line terminated in the impedance Z_L with a characteristic

impedance Z_0 and length l . Combining these facts

leads to the equivalent circuit of Figure 2.4 where Z_{in}

impedance Z_{in} is given by (2.3). Thus we have an equivalent

circuit which can be used to calculate the input

impedance of the single side-loaded transmission line antenna.

It would be convenient to use the equivalent circuit of

Figure 2.4 for efficiency calculations as well as for calculating

calculations of input impedances. In order to check this

possibility we observe from Figure 2.4 that

$$Z_{in} = \frac{Z_1 + jZ_2}{1 + jZ_1 Z_2}$$

and

$$Z_{in} = \frac{Z_1 + jZ_2}{1 + jZ_1 Z_2}$$

$$Z_{in} = \frac{Z_1 + jZ_2}{1 + jZ_1 Z_2}$$

Figure 2.4--The equivalent circuit for a single side-loaded transmission-line antenna

Figure 2. The equivalent circuit for a single antenna

Using (2.76 and 2.77) to calculate efficiency, we find

$$\text{Efficiency} = 100 \left[1 - \frac{|Z_{o1}|^2 G_1}{R_{T1}} - \frac{R_{T1}}{R_{T1}} \right], \quad (2.78)$$

which is identical with (2.75). This proves that the equivalent circuit of Figure 2.4 can be used for efficiency calculations as well as input impedance calculations.

Now that we have an equivalent circuit for the antenna, all the methods that have been developed for analyzing circuits and transmission lines can be used to analyze it. As an example, we might consider the problem of accounting for the losses due to the finite conductivity of the antenna conductors. From a knowledge of transmission line theory we simply rewrite (2.57 and 2.59) in the form

$$Z_o = Z_c \left[\frac{Z_{T1} \cosh \gamma \ell_1 + j Z_c \sinh \gamma \ell_1}{Z_c \cosh \gamma \ell_1 + j Z_{T1} \sinh \gamma \ell_1} \right] \quad (2.79)$$

$$Z_{T1} = Z_c \left[\frac{Z_2 \cosh \gamma (2h - \ell_1) + j Z_c \sinh \gamma (2h - \ell_1)}{Z_c \cosh \gamma (2h - \ell_1) + j Z_2 \sinh \gamma (2h - \ell_1)} \right], \quad (2.80)$$

where

$$Z_c = \sqrt{\frac{R + j\omega L}{G + j\omega C}}, \quad (2.81)$$

and

$$\gamma = \alpha + j\beta_\ell = \sqrt{(R + j\omega L)(G + j\omega C)}. \quad (2.82)$$

L, C, R, and G are, respectively, the inductance, capacitance, resistance and conductance per unit length of the transmission line given by [9, Table 9.01, page 364]

$$L = \frac{\mu_o}{\pi} \cosh^{-1} \left(\frac{b}{2a} \right) \quad (2.83)$$

Using (2.75) and (2.77) to calculate efficiency we find

$$\text{Efficiency} = 100 \left[\frac{R_{TL}}{R_{TL} + R_{in}} \right] \left[\frac{R_{in}}{R_{in} + R_{TL}} \right] \quad (2.78)$$

which is identical with (2.77). This proves that the equivalent circuit of Figure 2.5 can be used for efficiency calculations as well as input impedance calculations.

Now that we have an equivalent circuit for the antenna, all the methods that have been developed for analyzing circuits and transmission lines can be used to analyze it. As an example, we might consider the problem of accounting for the losses due to the finite conductivity of the antenna conductors. From a knowledge of transmission line theory we can simply rewrite (2.77) and (2.79) in the form

$$Z_o = Z_c \left[\frac{Z_{TL} \cosh \gamma L + Z_c \sinh \gamma L}{Z_c \cosh \gamma L + Z_{TL} \sinh \gamma L} \right] \quad (2.79)$$

$$Z_{TL} = Z_c \left[\frac{Z_o \cosh \gamma L + Z_c \sinh \gamma L}{Z_o \sinh \gamma L + Z_c \cosh \gamma L} \right] \quad (2.80)$$

where

$$Z_c = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \quad (2.81)$$

and

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)} \quad (2.82)$$

L, C, R, and G are, respectively, the inductance, capacitance, resistance and conductance per unit length of the transmission line given by (2.7) Table 2.01, page 364.

$$\gamma = \frac{1}{2} \cosh^{-1} \left(\frac{R + j\omega L}{Z_c} \right) \quad (2.83)$$

$$C = \frac{\pi \epsilon_0}{\cosh^{-1} \left(\frac{b}{2a} \right)} \quad (2.84)$$

$$R = \frac{R_s}{\pi a} \left[\frac{b/2a}{\sqrt{(b/2a)^2 - 1}} \right] \quad (2.85)$$

$$G = \frac{\pi \sigma_d}{\cosh^{-1} \left(\frac{b}{2a} \right)} \quad (2.86)$$

for the structure under consideration here. σ_d is the conductivity of the dielectric and for the atmosphere is practically zero. Thus we assume $G = 0$. R_s is the skin effect surface resistivity of the conductors given by

$$R_s = \sqrt{\frac{\pi f \mu}{\sigma_l}} \quad (2.87)$$

For the frequencies under consideration here (10 kc to 50 kc) and assuming low loss lines (σ_l very large), it is reasonable to assume $\frac{R}{\omega L} \ll 1$. Also $b \gg a$. Using these facts, we can write

$$Z_c \cong \sqrt{\frac{L}{C}} = \frac{1}{\pi} \sqrt{\frac{\mu_0}{\epsilon_0}} \cosh^{-1} \frac{b}{2a} \cong \frac{1}{\pi} \sqrt{\frac{\mu_0}{\epsilon_0}} \ln \frac{b}{a} = R_c \quad (2.88)$$

and

$$d \cong \frac{R}{2R_c} \quad (2.89)$$

$$\beta_l = \omega \sqrt{\mu_0 \epsilon_0} = \beta \quad (2.90)$$

or

$$\gamma \cong \frac{R}{2R_c} + j\beta \quad (2.91)$$

Substituting (2.91) into (2.58 and 2.80) gives new values for Z_{T1} , Z_{T1}^* , R_{T1} , and R_{T1}^* . With these new values (2.78) now

$$C = \frac{1}{\cosh \left(\frac{p}{\lambda} \right)}$$

$$R = \frac{R_s}{\sqrt{1 + \left(\frac{p}{\lambda} \right)^2}}$$

$$C = \frac{1}{\cosh \left(\frac{p}{\lambda} \right)}$$

for the structure under consideration here is a constant
 derivative of the distribution and for the structure is
 practically zero. Thus we assume $C = 0$ and the effect
 effect surface resistivity of the conductors given by

$$R_s = \sqrt{\frac{\mu}{\epsilon}}$$

For the frequencies under consideration here, 10 to 30 kc,
 and assuming low loss lines (ϵ very large), it is reasonable

to assume $\frac{R}{Z_0} \ll 1$. Also p is a small constant and we can

write

$$Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{1}{\epsilon}} \cosh \left(\frac{p}{\lambda} \right)$$

and

$$dZ = \frac{R}{Z_0}$$

$$dZ = \frac{R}{Z_0} = \frac{R}{\sqrt{\frac{L}{C}}} = \frac{R}{\sqrt{\frac{1}{\epsilon}}} = R \sqrt{\epsilon}$$

or

$$Z = \frac{R}{\epsilon} + \lambda$$

Substituting (2.91) into (2.88 and 2.89) gives new values

for Z_0 , Z , C , and R .

includes the effects of losses due to the finite conductivity of the antenna conductors.

This completes the theoretical treatment of the single side-loaded transmission-line antenna. In Chapter 4 the expressions obtained for input impedance and efficiency will be used to investigate numerically some specific examples. Then, in Chapter 6, we will compare some of the numerical results of Chapter 4 with measurements made on some experimental models of the single side-loaded transmission-line antenna.

Before closing Chapter 2 it might be well to show the advantage of the equivalent circuit in obtaining a rough picture of how the input impedance varies without having to carry out detailed calculations. This discussion will also serve to motivate the insertion of additional side loads leading quite naturally to Chapter 3. In Chapter 3, we will discuss the general theory of side-loaded transmission-line antennas concerned with transmission-line antennas containing m arbitrarily located side loads.

Referring to Figure 2.5 and equations (2.56, 2.57, 2.58, and 2.59), consider the following reasoning. If we are dealing with an electrically short antenna ($\beta h < 1$), then Z_d is capacitive and by choosing Z_1 such that

$$4X_d = -X_1 [\beta_o h]^1 \quad (2.92)$$

¹It is true that (2.92) is not the strict definition of antiresonance. We are dealing here with electrically short antennas and for best efficiencies we will want high Q side and end loads. Thus the real parts of all impedances will in general be much smaller than the imaginary parts. For this

includes the effects of losses due to the finite conductivity

of the antenna conductors.

This includes the ohmic resistance of the conductors.

side-loaded transmission-line antenna. In Chapter 5, the

experimental conditions for input impedance and efficiency will

be used to investigate numerically some specific examples.

Then in Chapter 6, we will compare some of the numerical results

with those of Chapter 4 with measurements made on some experimental

models of the simple side-loaded transmission-line antenna.

Before starting Chapter 7, it might be well to mention the

advantage of the equivalent circuit in obtaining a rough

picture of how the input impedance varies with frequency.

carry out detailed calculations. This discussion will also

serve to motivate the treatment of additional side loads.

leading quite naturally to Chapter 7. In Chapter 7, we will

discuss the general theory of a side-loaded transmission-line

antenna compared with transmission-line antenna consisting

of a series of side loads.

Referring to Figure 1.5 and equations 1.25, 1.26, 1.27,

and 1.28, consider the following reasoning. If we are dealing

with an electrically short antenna ($h \ll \lambda$), then βh is approx-

imately and by choosing βh such that

$$\beta h \approx \frac{\pi}{2} \quad (1.29)$$

it is true that (1.29) is not the strict definition of

antenna resonance. We are dealing here with electrically short

antennas and the input impedance will vary with βh as

shown in Figure 1.6. The input impedance will be

purely resistive when $\beta h \approx \frac{\pi}{2}$ and the input power will be

maximum.

Z_{O1} will be the impedance of an antiresonant circuit antiresonant at the frequency

$$f_o = \frac{\beta_o v}{2\pi} \quad (2.93)$$

Assuming lossless conductors and again remembering the antenna is electrically short, Z_{T1} will be just an inductive reactance if we set $Z_2 = 0$. Z_{T1} then, which is just the series combination of Z_{O1} and Z_{T1} , will still be essentially the impedance of an antiresonant circuit with the antiresonant point still at $f = f_o$. The effect of the lossless transmission line between Z_{T1} and Z_o is to shift the antiresonant point to a lower frequency otherwise leaving the general shape of the curve unchanged. Figure 2.6 illustrates the above discussion graphically. The interesting result is a comparison of the plots of R_o and R_d . It can be seen from Figure 2.6 that over a small range of frequency or βh the real part of the input impedance of the single side loaded transmission line antenna, R_o , is many times greater than the real part of the input impedance of the equivalent center-fed linear antenna, R_d , defined above and illustrated in Figure 2.5. Near the peak of R_o the imaginary part of Z_o , X_o , will pass through zero. At the particular frequency where $X_o = 0$ and $R_o \gg R_d$ the problem of impedance matching a generator to the antenna for maximum power transfer would be much simpler for the single side-loaded transmission-line antenna than it would be for the equivalent center-

l(con't) situation the frequency defined by (2.92) and the frequency defined by

$$16X_1(R_d^2 + X_d^2) + 4X_d(R_1^2 + X_1^2) = 0$$

(which is equivalent to $X_{O1} = 0$) are almost identical.

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Figure 2.5--The equivalent circuit and the equivalent center-fed linear antenna for a single side-loaded transmission-line antenna with the end load shorted

Figure 3.5 - The equivalent circuit and the equivalent
 circuit for a single side-loaded
 transmission line antenna with the load antenna

Figure 2.6--The impedance curves for a single side-loaded transmission-line antenna with the end load shorted

Figure 1.0 - The reflection coefficient R and the transmission coefficient T for a transmission line with a load impedance Z_L .

fed linear antenna where for electrically short antennas $R_d \ll |X_d|$. The advantage is somewhat academic in this case, however, since the peak of R_o for electrically short antennas is very sharp and it would be impossible in practice to operate the antenna at this advantageous point.

To gain some of the advantages discussed above without introducing the need for operating at an exact frequency, consider the situation illustrated in Figure 2.7. Now rather than setting $Z_2 = 0$ we have made Z_2 a large inductance such that Z_{T1} is now the impedance of an antiresonant circuit with the antiresonant point occurring at a frequency slightly higher than the antiresonant frequency of the impedance Z_{o1} . Thus the impedance of Z'_{T1} now contains two antiresonant points slightly separated in frequency and again Z_o will be very similar to Z'_{T1} except that the whole curve is shifted slightly in the direction of lower frequencies. Figure 2.8 illustrates the above discussion graphically. We now have a point of operation, $\beta_3 h$, in Figure 2.8, at which $R_o > R_d$, $X_o = 0$ giving us the impedance matching advantage discussed in the previous example. At the same time, however, R_o is a slowly varying function of frequency in this region and dX_o/df , although, not necessarily small is at least a minimum. Thus a single side-loaded transmission-line antenna operating as described above would be much easier to match to a driving generator than its equivalent center-fed linear antenna also illustrated in Figure 2.7. This advantage again is for the case of electrically short antennas. This last structure does have one practical disadvantage, however. For antennas

fed linear antenna where for electrically short antennas

$R \ll X$. The advantage is somewhat academic in this case,

however, since the peak of R for electrically short antennas

is very sharp and it would be impossible in practice to oper-

ate the antenna at this advantageous point.

To gain some of the advantages discussed above without

introducing the need for operating at an exact frequency,

consider the situation illustrated in Figure 1. Now rather

than setting $X_0 = 0$ we have made X_0 a large inductance such

that X_0 is now the impedance of an antiresonant circuit with

the antiresonant point occurring at a frequency slightly higher

than the antiresonant frequency of the impedance Z_0 . Thus

the impedance of X_0 now contains two antiresonant points

slightly separated in frequency and again X_0 will be very

similar to Z_0 except that the whole curve is shifted slightly

in the direction of lower frequencies. Figure 2 illustrates

the above discussion graphically. We now have a point of

operation, X_0 , in Figure 1, at which $R_0 > R$, $X_0 = 0$, giving us the impedance matching advantage discussed in the

previous example. At the same time, however, R_0 is a slowly

varying function of frequency in this region and $dX_0/d\omega$

although not necessarily small is at least a minimum. Thus

a single side-loaded transmission-line antenna operating as

described above would be much easier to match to a driving

generator than its equivalent center-fed linear antenna also

illustrated in Figure 1. This advantage again is for the

case of electrically short antennas. This last advantage

does have one practical disadvantage, however. For antennas

.....

Figure 2.7--The equivalent circuit and the equivalent center-fed linear antenna for a single side-loaded transmission-line antenna with an inductive end load

Figure 1. The equivalent circuit of the
center-fed dipole antenna with a
transmission line

Figure 1. The equivalent circuit of the
center-fed dipole antenna with a
transmission line

Figure 2.8--The impedance curves for a single side-loaded transmission-line antenna with an inductive end load

R_0
 X_0
 R_0

X_0
 R_0

X_0
 R_0

which are quite short electrically, Z_2 must be an extremely large inductance to actually obtain the behavior described above. To obtain the necessary inductance without introducing large losses might be difficult in a practical design situation and so there is still considerable room for improvement.

In order to obtain the characteristics we are looking for, another possibility is to vary the location of the loads on the antenna or to insert more side loads. Our present theory will allow us to locate the one side load arbitrarily but the end load must remain fixed. It turns out that moving the one side load along the length of the antenna does not produce any significant changes in the way the input impedance behaves, and so we are faced with extending the theory to include more side loads if we are going to make any significant improvements over what has been described above. This leads then to Chapter 3 where, as mentioned above, we will develop a general theory for m side loads allowing us more freedom in design possibilities.

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and so we are faced with extending the theory to include more

side loads if we are going to make any significant improve-

ments over what has been described above. This leads then to

Chapter 5 where, as mentioned above, we will develop a general

theory for n side loads allowing us more freedom in design

possibilities.

3. THE GENERAL THEORY OF SIDE LOADED TRANSMISSION LINE ANTENNAS

3.1 Introduction

In this chapter, we extend the concepts developed in Chapter 2 to cover the general case of m side loads arbitrarily located on the transmission line antenna. After developing all the necessary theory, the appropriate equivalent circuit is developed for the general case. In Chapter 4 the analysis of specific cases is again carried out in an approximate manner using the equivalent circuits to gain an intuitive picture of the variation of the input impedance with frequency. Numerical calculations of the specific cases are also presented in Chapter 4 and compared with experimental data in Chapter 6.

3.2 The General Solution for m Side Loads on One Side

The antenna to be analyzed is illustrated in Figure 3.1 and is a side-loaded transmission-line antenna containing m side loads and one end load with side load k located a distance ℓ_k from the generator. Using the same approximations that were used to arrive at (2.3, 2.4, and 2.5) of Chapter 2, we obtain the following equations for the vector potential on each member.

$$A_{11}(z) = -\frac{j}{V}[C_{11}\sin \beta z + C_{12}\cos \beta z]$$

3.1 Introduction

In this chapter

Chapter 3 is devoted to

clearly located

developing off the

plant circuit is

the analysis of

approximate manner

inductive effects

with frequency

are also presented

data in Chapter

3.2 The general case

The antenna is

and is a

side loads and

from the generator

were used to arrive

obtain the following

each member

$A_{ij}(z)$

Figure 3.1--The general side-loaded transmission-line antenna with all side loads on the same side of the transmission line

Figure 1. A general side-loaded transmission line circuit with shunt loads at each end of the transmission line.

$$\begin{aligned}
A_{12}(z) &= -\frac{j}{v} [C_{21} \sin \beta z + C_{22} \cos \beta z] \\
&\vdots \\
&\vdots \\
&\vdots
\end{aligned} \tag{3.1}$$

$$\begin{aligned}
A_{1,m+1} &= -\frac{j}{v} [C_{m+1,1} \sin \beta z + C_{m+1,2} \cos \beta z] \\
A_2(z) &= -\frac{j}{v} [C_1 \sin \beta z + C_2 \cos \beta z] .
\end{aligned}$$

These equations can also be written in the more compact form

$$A_{1n}(z) = -\frac{j}{v} [C_{n1} \sin \beta z + C_{n2} \cos \beta z] \quad n = 1, 2, 3 \dots m+1 \tag{3.2}$$

$$A_2(z) = -\frac{j}{v} [C_1 \sin \beta z + C_2 \cos \beta z] . \tag{3.3}$$

Using (2.6) and applying the boundary conditions on ϕ at each of the m side loads, we find

$$V_n = \phi_{1n}[\ell_n] - \phi_{1,n+1}[\ell_n] = (C_{n1} - C_{n+1,1}) \cos \beta \ell_n - (C_{n2} - C_{n+1,2}) \sin \beta \ell_n \tag{3.4}$$

for $n = 1, 2, 3 \dots m$. From the continuity of the vector potential we obtain

$$A_{1n}[\ell_n] = A_{1,n+1}[\ell_n] \tag{3.5}$$

for $n = 1, 2, 3 \dots m$. Combining (3.2) and (3.5), we find

$$(C_{n1} - C_{n+1,1}) \sin \beta \ell_n = - (C_{n2} - C_{n+1,2}) \cos \beta \ell_n \tag{3.6}$$

for $n = 1, 2, 3 \dots m$. Solving (3.4) and (3.6) for C_{n1} and C_{n2} in terms of $C_{n+1,1}$ and $C_{n+1,2}$, the result is

$$C_{n1} = C_{n+1,1} + V_n \cos \beta \ell_n \tag{3.7}$$

$$C_{n2} = C_{n+1,2} - V_n \sin \beta \ell_n \tag{3.8}$$

for $n = 1, 2, 3 \dots m$. Substituting (3.7) and (3.8) in (3.2), we can rewrite (3.2) and (3.3) in the form

$$A_{12}(z) = -\frac{1}{V} [C_{21} \sin \beta z + C_{22} \cos \beta z]$$

(3.1)

$$A_{1,m+1}(z) = -\frac{1}{V} [C_{m+1,1} \sin \beta z + C_{m+1,2} \cos \beta z]$$

$$A_2(z) = -\frac{1}{V} [C_1 \sin \beta z + C_2 \cos \beta z]$$

These equations can also be written in the more compact form

$$A_{1n}(z) = -\frac{1}{V} [C_{n1} \sin \beta z + C_{n2} \cos \beta z] \quad n = 1, 2, 3, \dots, m+1 \quad (3.2)$$

$$A_2(z) = -\frac{1}{V} [C_1 \sin \beta z + C_2 \cos \beta z] \quad (3.3)$$

Using (3.6) and applying the boundary conditions on Φ at each

of the m side loads, we find

$$V_n = A_{1n} \left[\frac{1}{V} (C_{n1} \sin \beta z + C_{n2} \cos \beta z) \right] = (C_{n1} \sin \beta z + C_{n2} \cos \beta z) \sin \beta z \quad (3.4)$$

for $n = 1, 2, 3, \dots, m$. From the continuity of the vector

potential we obtain

$$A_{1n} \left[\frac{1}{V} (C_{n1} \sin \beta z + C_{n2} \cos \beta z) \right] = A_{1,n+1} \left[\frac{1}{V} (C_{n+1,1} \sin \beta z + C_{n+1,2} \cos \beta z) \right] \quad (3.5)$$

for $n = 1, 2, 3, \dots, m$. Combining (3.2) and (3.5), we find

$$(C_{n1} \sin \beta z + C_{n2} \cos \beta z) = (C_{n+1,1} \sin \beta z + C_{n+1,2} \cos \beta z) \quad (3.6)$$

for $n = 1, 2, 3, \dots, m$. Solving (3.4) and (3.6) for C_{n1} and C_{n2}

in terms of $C_{n+1,1}$ and $C_{n+1,2}$, the result is

$$C_{n1} = C_{n+1,1} + V \cos \beta z \quad (3.7)$$

$$C_{n2} = C_{n+1,2} - V \sin \beta z \quad (3.8)$$

for $n = 1, 2, 3, \dots, m$. Substituting (3.7) and (3.8) in (3.2),

we can rewrite (3.2) and (3.3) in the form

$$A_{1n}(z) = -\frac{j}{v} \left[\left\{ C_{m+1,1} + \sum_{p=n}^m V_p \cos \beta \ell_p \right\} \sin \beta z + \left\{ C_{m+1,2} - \sum_{p=n}^m V_p \sin \beta \ell_p \right\} \cos \beta z \right] \quad (3.9)$$

$$A_2(z) = -\frac{j}{v} \left[C_1 \sin \beta z + C_2 \cos \beta z \right] \quad (3.10)$$

for $n = 1, 2, 3 \dots m+1$, where $\sum_{p=n}^m V_p \cos \beta \ell_p = \sum_{p=n}^m V_p \sin \beta \ell_p = 0$ for $n = m+1$. For convenience, we define

$$A_1(z) = A_{1n}(z)$$

$$\phi_1(z) = \phi_{1n}(z) \quad \text{for } \ell_{n-1} \leq z \leq \ell_n \quad \text{where } \ell_0 = 0, \ell_{m+1} = 2h$$

and $n = 1, 2, 3 \dots m+1$. (3.11)

$$I_1(z) = I_{1n}(z)$$

At $z = 2h$, we find from the boundary conditions on ϕ

$$V_{m+1} = \phi_1(2h) - \phi_2(2h) . \quad (3.12)$$

Combining (2.6, 3.9, 3.10, 3.11 and 3.12), we obtain

$$V_{m+1} = C_3 \cos 2\beta h - C_4 \sin 2\beta h , \quad (3.13)$$

where

$$C_3 = C_{m+1,1} - C_1 \quad (3.14)$$

$$C_4 = C_{m+1,2} - C_2 . \quad (3.15)$$

At $z = 0$, we find from the boundary conditions on ϕ

$$V_0 = \phi_1(0) - \phi_2(0) . \quad (3.16)$$

Combining (2.6, 3.9, 3.10, 3.11, and 3.16), the result is

$$V_0 = C_3 + \sum_{p=1}^m V_p \cos \beta \ell_p . \quad (3.17)$$

Referring to the discussion in Chapter 2, we write for the integral representation of the vector potential on the surface of the conductors

of the conductors

integral representation of the vector potential on the surface

Referring to the discussion in Chapter 2, we write for the

$$V_0 = C_0 + \sum_{p=1}^{\infty} C_p \cos \beta_p z$$

Combining (2.6), (2.9), (2.10), (2.11), and (2.12), the result is

$$V_0 = A_1(0) - A_2(0)$$

At $z = 0$, we find from the boundary conditions on ψ

$$C_0 = C_{m+1,2} - C_2$$

$$C_1 = C_{m+1,1} - C_1$$

where

$$V_{m+1} = 0 \cos 2\beta_m z + C_{m+1,1} \sin 2\beta_m z$$

Combining (2.6), (2.9), (2.10), (2.11), and (2.12), we obtain

$$V_{m+1} = A_1(2h) - A_2(2h)$$

At $z = 2h$, we find from the boundary conditions on ψ

$$I_1(z) = I_2(z)$$

$$A_1(z) = A_2(z) \quad \text{for } 2h - 2 \leq z \leq 2h$$

$$A_1(z) = A_2(z)$$

for $n = m+1$. For convenience, we define

$$\text{for } n = 1, 2, \dots, m+1, \text{ where } \sum_{p=1}^n C_p \cos \beta_p z = 0$$

$$A_2(z) = -\frac{1}{2} C_1 \sin \beta_1 z + C_2 \cos \beta_1 z$$

$$A_1(z) = -\frac{1}{2} C_{m+1,1} \sin \beta_{m+1} z + \sum_{p=1}^m C_p \cos \beta_p z$$

$$A_1(z) = \frac{\mu_0}{4\pi} \int_0^{2h} [I_1(z') K_a(z, z') - I_2(z') K_b(z, z')] dz' \quad (3.18)$$

$$A_2(z) = \frac{\mu_0}{4\pi} \int_0^{2h} [I_1(z') K_b(z, z') - I_2(z') K_a(z, z')] dz' . \quad (3.19)$$

Using the same system we used in Chapter 2 to separate the radiating antenna currents from the transmission line currents, we define

$$A_d(z) = \frac{\mu_0}{4\pi} \int_0^{2h} [I_1(z') - I_2(z')] K_d(z, z') dz' , \quad (3.20)$$

where again $d = \sqrt{ab}$.

If we add and subtract (3.20) in the right hand side of (3.18) and (3.19), we obtain

$$A_1(z) = A_d(z) + \frac{\mu_0}{4\pi} \int_0^{2h} \left\{ I_1(z') [K_a(z, z') - K_d(z, z')] - I_2(z') [K_b(z, z') - K_d(z, z')] \right\} dz' \quad (3.21)$$

$$A_2(z) = A_d(z) + \frac{\mu_0}{4\pi} \int_0^{2h} \left\{ I_1(z') [K_b(z, z') - K_d(z, z')] - I_2(z') [K_a(z, z') - K_d(z, z')] \right\} dz' . \quad (3.22)$$

Using Harrison's approximation on (3.21) and (3.22), the result is

$$A_1(z) = A_d(z) + \frac{\mu_0}{2\pi} \left[I_1(z) \ln \frac{d}{a} - I_2(z) \ln \frac{d}{b} \right] \quad (3.23)$$

$$A_2(z) = A_d(z) + \frac{\mu_0}{2\pi} \left[I_1(z) \ln \frac{d}{b} - I_2(z) \ln \frac{d}{a} \right] . \quad (3.24)$$

$$A_1(z) = \frac{1}{2\pi} \int_{-\pi}^{\pi} [1 - K_1(z, z') - 1 - K_2(z, z')] dz' \quad (5.18)$$

$$A_2(z) = \frac{1}{2\pi} \int_{-\pi}^{\pi} [1 - K_1(z, z') - 1 - K_2(z, z')] dz' \quad (5.19)$$

Using the same system we used in Chapter 3 to separate the radiating antenna currents from the transmission line currents

we obtain

$$A_3(z) = \frac{1}{2\pi} \int_{-\pi}^{\pi} [1 - K_1(z, z') - 1 - K_2(z, z')] dz' \quad (5.20)$$

where $\sigma = \sqrt{1 - \epsilon}$

If we add and subtract (5.20) in the right hand side of (5.18)

and (5.19), we obtain

$$A_1(z) = A_3(z) + \frac{1}{2\pi} \int_{-\pi}^{\pi} [1 - K_1(z, z') - 1 - K_2(z, z')] dz' \quad (5.21)$$

$$A_2(z) = A_3(z) + \frac{1}{2\pi} \int_{-\pi}^{\pi} [1 - K_1(z, z') - 1 - K_2(z, z')] dz' \quad (5.22)$$

Using Harrison's approximation on (5.21) and (5.22), the result

$$A_1(z) = A_3(z) + \frac{1}{2\pi} \int_{-\pi}^{\pi} [1 - K_1(z, z') - 1 - K_2(z, z')] dz' \quad (5.23)$$

$$A_2(z) = A_3(z) + \frac{1}{2\pi} \int_{-\pi}^{\pi} [1 - K_1(z, z') - 1 - K_2(z, z')] dz' \quad (5.24)$$

Subtracting (3.24) from (3.23), we find

$$A_1(z) - A_2(z) = \frac{\mu_0}{2\pi} \ln \frac{b}{a} \left[I_1(z) + I_2(z) \right] \quad (3.25)$$

Combining (3.9, 3.10, 3.11, 3.14, 3.15, and 3.25), we obtain

$$I_1(z) + I_2(z) = -j \frac{2}{R_c} \left[\left\{ C_3 + \sum_{p=n}^m V_p \cos \beta \ell_p \right\} \sin \beta z + \left\{ C_4 - \sum_{p=n}^m V_p \sin \beta \ell_p \right\} \cos \beta z \right] \quad (3.26)$$

for $\ell_{n-1} \leq z \leq \ell_n$, where $\ell_0 = 0$, $\ell_{m+1} = 2h$, $n = 1, 2, 3 \dots m+1$, and $R_c = \eta/\pi \ln b/a$. We observe that $I_1(0) = I_2(0)$ and $I_1(2h) = I_2(2h)$. Using this information in (3.26), we find

$$I_1(0) = -\frac{j}{R_c} \left[C_4 - \sum_{p=1}^m V_p \sin \beta \ell_p \right] \quad (3.27)$$

$$I_1(2h) = -\frac{j}{R_c} \left[C_3 \sin 2\beta h + C_4 \cos 2\beta h \right] \quad (3.28)$$

Combining (3.17) and (3.28), we find for the input impedance

$$Z_0 = \frac{V_0}{I_1(0)} = jR_c \frac{\left[C_3 + \sum_{p=1}^m V_p \cos \beta \ell_p \right]}{\left[C_4 - \sum_{p=1}^m V_p \sin \beta \ell_p \right]} \quad (3.29)$$

We make the following definitions:

$$\gamma_n Z_n = \frac{V_n}{C_4} \quad (3.30)$$

for $n = 1, 2, 3 \dots m$,

$$\gamma_{m+1} = \frac{C_3}{C_4} \quad (3.31)$$

Substituting (3.30) and (3.31) in (3.29), we obtain

$$Z_0 = jR_c \left[\frac{\gamma_{m+1} + \sum_{p=1}^m \gamma_p Z_p \cos \beta \ell_p}{1 - \sum_{p=1}^m \gamma_p Z_p \sin \beta \ell_p} \right]. \quad (3.32)$$

From Figure 3.1, we observe that

$$V_{m+1} = Z_{m+1} I_1(2h). \quad (3.33)$$

Combining (3.13, 3.28, 3.31, and 3.33), we find

$$\gamma_{m+1} = \frac{R_c \sin \beta \ell_{m+1} - j Z_{m+1} \cos \beta \ell_{m+1}}{R_c \cos \beta \ell_{m+1} + j Z_{m+1} \sin \beta \ell_{m+1}}. \quad (3.34)$$

To evaluate γ_n for $n = 1, 2, 3 \dots m$, we add (3.23) to (3.24) and obtain the result

$$A_1(z) + A_2(z) = 2A_d(z) + \frac{\mu_0}{2\pi} I_T(z) \left[\ln \frac{d}{b} + \ln \frac{d}{a} \right], \quad (3.35)$$

where

$$I_T = I_1 - I_2. \quad (3.36)$$

Defining "d" as we did in Chapter 2 and combining (3.9, 3.10, 3.11, 3.20, 3.35, and 3.36), we obtain

$$\begin{aligned} \int_0^{2h} I_T(z') K_d(z, z') dz' = j \frac{4\pi}{\eta} \left[- \left\{ \frac{C_{m+1,1} + C_1}{2} + \frac{1}{2} \sum_{p=n}^m V_p \cos \beta \ell_p \right\} \sin \beta z \right. \\ \left. - \left\{ \frac{C_{m+1,2} + C_2}{2} - \frac{1}{2} \sum_{p=n}^m V_p \sin \beta \ell_p \right\} \cos \beta z \right] \quad (3.37) \end{aligned}$$

for $\ell_{n-1} \leq z \leq \ell_n$, where $\ell_0 = 0$, $\ell_{m+1} = 2h$ and $n = 1, 2, 3 \dots m+1$.

If the integral equations in (3.37) could be solved for the ratios

$$Z_{dn} = - \frac{V_n}{2I_T(\ell_n)} \quad (3.38)$$

$$\begin{aligned}
 & \text{From Figure 1, we observe that} \\
 & \text{the function } f(x) \text{ is defined by} \\
 & f(x) = \begin{cases} 0 & \text{if } x < 0 \\ x & \text{if } 0 \leq x \leq 1 \\ 1 & \text{if } x > 1 \end{cases}
 \end{aligned}$$

Combining (2.13), (2.14), and (2.15), we find that

$$\begin{aligned}
 & \text{To evaluate } I_2 \text{ for } n = 1, 2, \dots, m, \text{ we add (2.13) to (2.14)} \\
 & \text{and obtain the identity}
 \end{aligned}$$

$$\begin{aligned}
 & \text{where} \\
 & \text{and}
 \end{aligned}$$

$$\begin{aligned}
 & \text{where} \\
 & \text{and}
 \end{aligned}$$

Defining ϕ_n as we did in Chapter 2 and combining (2.13), (2.14),

(2.15), and (2.16), we obtain

$$\begin{aligned}
 & \text{where} \\
 & \text{and}
 \end{aligned}$$

for $n = 1, 2, \dots, m$, where $\phi_n = 0$ and $n = 1, 2, \dots, m-1$.

If the integral equations in (2.17) could be solved for the

$$\begin{aligned}
 & \text{values} \\
 & \text{then}
 \end{aligned}$$

for $n = 1, 2, 3 \dots m$, then it would be possible to evaluate γ_n for $n = 1, 2, 3 \dots m$, and (3.32) would be a means of evaluating the input impedance of the multiply side-loaded antenna theoretically. In Appendix B, it is shown that the integral equations (3.37) are identical with those that describe the multiply-fed linear antenna of Figure 3.2. A method is also presented in Appendix B for determining the input impedances of such a multiply-fed linear antenna and these impedances are just the ratios of (3.38). Thus we assume here that the Z_{dn} for $n = 1, 2, 3 \dots m$ can be evaluated and proceed to evaluate γ_n .

To do this we start by evaluating (3.26) successively at $z = \ell_n$ for $n = 1, 2, 3 \dots m$. The result is

$$I_1(\ell_n) + I_2(\ell_n) = -j \frac{2}{R_c} \left[\left\{ C_3 + \sum_{p=n+1}^m V_p \cos \beta \ell_p \right\} \sin \beta \ell_n + \left\{ C_4 - \sum_{p=n+1}^m V_p \sin \beta \ell_p \right\} \cos \beta \ell_n \right] \quad (3.39)$$

for $n = 1, 2, 3 \dots m$. Rearranging (3.39), we find

$$I_1(\ell_n) + I_2(\ell_n) = -j \frac{2}{R_c} \left[C_3 \sin \beta \ell_n + C_4 \cos \beta \ell_n - \sum_{p=n+1}^m V_p \sin \beta (\ell_p - \ell_n) \right] \quad (3.40)$$

for $n = 1, 2, 3 \dots m$. Substituting (3.36) in (3.40), we obtain

$$2I_1(\ell_n) - I_T(\ell_n) = -j \frac{2}{R_c} \left[C_3 \sin \beta \ell_n + C_4 \cos \beta \ell_n - \sum_{p=n+1}^m V_p \sin \beta (\ell_p - \ell_n) \right] \quad (3.41)$$

for $n = 1, 2, 3 \dots m$. From Figure 3.1 we observe that

$$I_1(\ell_n) = \frac{V_n}{Z_n} \quad (3.42)$$

Combining (3.30, 3.38, 3.41, and 3.42), we finally obtain

for $n = 1, 2, 3, \dots$, then it would be possible to evaluate

γ_n for $n = 1, 2, 3, \dots$ and (3.11) would be satisfied for all n .

evaluating the input impedance of the multiply-fed antenna

antenna characteristically. In Appendix B, it is shown that

the integral equations (3.7) are identical with those

that describe the multiply-fed linear antenna of Figure 3.1.

A method is also presented in Appendix A for determining

the input impedances of each multiply-fed linear antenna

and these impedances are used in the calculation of (3.11). Thus

we assume here that the Z_{in} for $n = 1, 2, 3, \dots$ can be

evaluated and proceed to evaluate γ_n .

From (3.11) we see that we must evaluate (3.11) successively

for $n = 1, 2, 3, \dots$. The result is

$$I_n(x) = \frac{1}{\gamma_n} \left[\frac{1}{2} \left(\frac{1}{\gamma_n} + \frac{1}{\gamma_{n-1}} \right) \int_{-L/2}^{L/2} I_{n-1}(x') dx' + \frac{1}{2} \left(\frac{1}{\gamma_n} - \frac{1}{\gamma_{n-1}} \right) \int_{-L/2}^{L/2} I_{n-1}(x') dx' \right]$$

for $n = 1, 2, 3, \dots$. For $n = 1$, $I_1(x)$ is given by (3.7).

for $n = 1, 2, 3, \dots$, rearranging (3.7) we find

$$I_n(x) = \frac{1}{\gamma_n} \left[\frac{1}{2} \left(\frac{1}{\gamma_n} + \frac{1}{\gamma_{n-1}} \right) \int_{-L/2}^{L/2} I_{n-1}(x') dx' + \frac{1}{2} \left(\frac{1}{\gamma_n} - \frac{1}{\gamma_{n-1}} \right) \int_{-L/2}^{L/2} I_{n-1}(x') dx' \right]$$

for $n = 1, 2, 3, \dots$. Substituting (3.7) in (3.10), we obtain

$$I_n(x) = \frac{1}{\gamma_n} \left[\frac{1}{2} \left(\frac{1}{\gamma_n} + \frac{1}{\gamma_{n-1}} \right) \int_{-L/2}^{L/2} I_{n-1}(x') dx' + \frac{1}{2} \left(\frac{1}{\gamma_n} - \frac{1}{\gamma_{n-1}} \right) \int_{-L/2}^{L/2} I_{n-1}(x') dx' \right]$$

for $n = 1, 2, 3, \dots$. From Figure 3.1 we observe that

$$I_n(x) = \frac{1}{\gamma_n} \left[\frac{1}{2} \left(\frac{1}{\gamma_n} + \frac{1}{\gamma_{n-1}} \right) \int_{-L/2}^{L/2} I_{n-1}(x') dx' + \frac{1}{2} \left(\frac{1}{\gamma_n} - \frac{1}{\gamma_{n-1}} \right) \int_{-L/2}^{L/2} I_{n-1}(x') dx' \right]$$

Comparing (3.10) and (3.11) we see that

Figure 3.2--Multiply-fed linear antenna

R_c - characteristic
impedance of
two wire
transmission line

$$l_{m+1} = 2h$$

$$Z'_{Tk} = Z_{ok} + Z_{Tk}$$

$$Z_{Tk} = R_c \left[\frac{Z'_{Tk+1} \cos \beta(l_{k+1} - l_k) + jR_c \sin \beta(l_{k+1} - l_k)}{R_c \cos \beta(l_{k+1} - l_k) + jZ'_{Tk+1} \sin \beta(l_{k+1} - l_k)} \right]$$

Figure 3.3--The equivalent circuit of the general side-loaded transmission-line antenna

R-characteristic
impedance of
two wire
transmission line

$$Z_{TK} = R_C \left[\frac{Z_{L1} \cos \beta l + j Z_0 \sin \beta l}{Z_0 \cos \beta l + j Z_{L1} \sin \beta l} \right]$$

Figure 5.5- The equivalent circuit of the general end-loaded transmission-line antenna

$$\gamma_n Z_n = -j \frac{4Z_{dn} Z_n}{R_c (4Z_{dn} + Z_n)} \left[\gamma_{m+1} \sin \beta \ell_n + \cos \beta \ell_n - \sum_{p=n+1}^m \gamma_p Z_p \sin \beta (\ell_p - \ell_n) \right] \quad (3.43)$$

for $n = 1, 2, 3 \dots m$. Using (3.43), we can evaluate γ_m in terms of γ_{m+1} , γ_{m-1} in terms of γ_{m+1} and γ_m , γ_{m-2} in terms of γ_{m+1} , γ_m and γ_{m-1} , etc.

If we now combine (3.32, 3.34, and 3.43), we obtain an expression for the input impedance of the multiply side-loaded transmission-line antenna in terms of its measurable physical parameters plus the Z_{dn} which are shown in Appendix B to be the input impedances of the equivalent multiply-fed linear antenna of Figure 3.2. For convenience we also define

$$Z_{on} = \frac{4Z_n Z_{dn}}{4Z_{dn} + Z_n} \quad (3.44)$$

In Appendix C, it is shown that (3.32, 3.34, and 3.43) are identical with the equations for the input impedance of the circuit of Figure 3.3. This circuit therefore serves as an equivalent circuit for the general side-loaded transmission-line antenna. The origin of the circuit was just to extend in an analogous way the equivalent circuit that was already developed for the single side-loaded transmission-line antenna and then to prove the equivalence algebraically having the circuit to work with in carrying out the proof.

3.3 The General Solution for m Side Loads Located on Both Sides of the Transmission Line

The theory for the side-loaded transmission-line antenna considered up to this point is not completely general since

$$Y_{m+1} = \frac{1}{Z_{m+1}} = \frac{1}{Z_m + j\omega L_{m+1}} = \frac{1}{Z_m} \frac{1}{1 + j\omega L_{m+1} Y_m} = Y_m \frac{1}{1 + j\omega L_{m+1} Y_m} \quad (5.45)$$

for $n = 1, 2, \dots, m$. Using (5.45), we can evaluate Y_m in terms of Y_{m+1} and Y_{m-1} in terms of Y_m and Y_{m-1} , etc. If we continue (5.35), (5.36), and (5.37), we obtain an expression for the input impedance of the multiply side-loaded transmission-line antenna in terms of the measurable physical parameters plus the Z_m which are shown in Appendix B to be the input impedance of the equivalent multiply-fed linear antenna of Figure 5.3. For convenience we also define

$$Z_{on} = \frac{Z_n}{1 + j\omega L_n Y_n} \quad (5.46)$$

In Appendix C, it is shown that (5.35), (5.36), and (5.46) are identical with the equations for the input impedance of the circuit of Figure 5.3. This circuit therefore serves as an equivalent circuit for the general side-loaded transmission-line antenna. The origin of the circuit was found to extend in an analogous way the equivalent circuit that was already developed for the single side-loaded transmission-line antenna and then to prove the equivalence algebraically having the circuit to work with in carrying out the proof.

5.3 The General Solution for a Side Loaded Antenna on Both Sides of the Transmission Line

The theory for the side-loaded transmission-line antenna considered up to this point is not completely general since

all the side loads are located on the same side or in the same wire of the two wire transmission line. To see what effect shifting some of the side loads to the opposite side of the transmission line antenna will have on the equations already developed, we first consider the case of shifting just one side load. This antenna is illustrated in Figure 3.4. Using the same approximations that were used to arrive at (2.3, 2.4, and 2.5) of Chapter 2, we obtain the following equations for the vector potential on each member.

$$A_{1n}(z) = -\frac{j}{v}[C_{n1}\sin \beta z + C_{n2}\cos \beta z] \quad (3.45)$$

$$A_{21}(z) = -\frac{j}{v}[C_1\sin \beta z + C_2\cos \beta z] \quad (3.46)$$

$$A_{22}(z) = -\frac{j}{v}[C_3\sin \beta z + C_4\cos \beta z] \quad (3.47)$$

for $n = 1, 2, 3 \dots m+1$.

Using (2.6) and applying the boundary conditions on ϕ at each of the m side loads, we find, using (3.45, 3.46 and 3.47),

$$V_n = \phi_{1n}(\ell_n) - \phi_{1,n+1}(\ell_n) = (C_{n1} - C_{n+1,1})\cos \beta \ell_n - (C_{n2} - C_{n+1,2})\sin \beta \ell_n \quad (3.48)$$

for $n = 1, 2, 4, 5 \dots m$, and

$$V_3 = \phi_{22}(\ell_3) - \phi_{12}(\ell_3) = (C_3 - C_1)\cos \beta \ell_3 - (C_4 - C_2)\sin \beta \ell_3. \quad (3.49)$$

From the continuity of the vector potential, we obtain

$$A_{1n}(\ell_n) = A_{1,n+1}(\ell_n) \quad (3.50)$$

for $n = 1, 2, 4, 5 \dots m$, and

$$A_{21}(\ell_3) = A_{22}(\ell_3). \quad (3.51)$$

Combining (3.45, 3.46, 3.47, 3.50, and 3.51), we find

$$(C_{n1} - C_{n+1,1})\sin \beta \ell_n = - (C_{n2} - C_{n+1,2})\cos \beta \ell_n \quad (3.52)$$

for $n = 1, 2, 4, 5 \dots m$, and

...the side ...
...same ...
...effect ...
...of the ...
...already ...
...just one ...
...3. ...
...at ...
...equations ...
...
... $A_{21}(z) = -\frac{1}{2}C(z)$...
... $A_{22}(z) = \frac{1}{2}C(z)$...
...for $n = 1, 2, \dots$...
...Using (2.3) ...
...at each of the ...
...
... $V_n = A_{21}(z) - A_{22}(z)$...
...for $n = 1, 2, \dots$...
...From the ...
... $A_{21}(z) = A_{22}(z)$...
...for $n = 1, 2, \dots$...
... $A_{21}(z) = A_{22}(z)$...
...Combining (2.3) ...
... $C(z)$...
...for $n = 1, 2, \dots$...

Figure 3.4--The general side-loaded transmission-line antenna

Figure 2.4: A ladder network with series and shunt impedances.

$$(C_3 - C_1) \sin \beta \ell_3 = - (C_4 - C_2) \cos \beta \ell_3 . \quad (3.53)$$

Next we solve (3.48) and (3.52) for C_{n1} and C_{n2} in terms of $C_{n+1,1}$ and $C_{n+1,2}$. Observing that $C_{31} = C_{41}$, the result is

$$C_{n1} = C_{n+1,1} + V_n \cos \beta \ell_n \quad (3.54)$$

$$C_{n2} = C_{n+1,2} - V_n \sin \beta \ell_n \quad (3.55)$$

for $n = 1, 2, 4, 5 \dots m$. Solving (3.49) and (3.53) for C_1 and C_2 in terms of C_3 and C_4 , the result is

$$C_1 = C_3 - V_3 \cos \beta \ell_3 \quad (3.56)$$

$$C_2 = C_4 + V_3 \sin \beta \ell_3 . \quad (3.57)$$

Substituting (3.54) and (3.55) in (3.45) and (3.56) and (3.57) in (3.46), we can rewrite (3.45, 3.46, and 3.47) in the form

$$A_{1n}(z) = - \frac{j}{v} \left[\left\{ C_{m+1,1} + \sum_{\substack{p=n \\ p \neq 3}}^m V_p \cos \beta \ell_p \right\} \sin \beta z + \left\{ C_{m+1,2} - \sum_{\substack{p=n \\ p \neq 3}}^m V_p \sin \beta \ell_p \right\} \cos \beta z \right] \quad (3.58)$$

for $n = 1, 2, 4, 5 \dots m$, where $\sum_{p=n}^m V_p \cos \beta \ell_p = \sum_{p=n}^m V_p \sin \beta \ell_p = 0$ for $n = m+1$, and

$$A_{13}(z) = - \frac{j}{v} \left[\left\{ C_{m+1,1} + \sum_{p=4}^m V_p \cos \beta \ell_p \right\} \sin \beta z + \left\{ C_{m+1,2} - \sum_{p=4}^m V_p \sin \beta \ell_p \right\} \cos \beta z \right] \quad (3.59)$$

$$A_{21}(z) = - \frac{j}{v} \left[\left\{ C_3 - V_3 \cos \beta \ell_3 \right\} \sin \beta z + \left\{ C_4 + V_3 \sin \beta \ell_3 \right\} \cos \beta z \right] \quad (3.60)$$

$$A_{22}(z) = - \frac{j}{v} [C_3 \sin \beta z + C_4 \cos \beta z] . \quad (3.61)$$

For convenience, we define

$$(3.55) \quad (C_1 - C_2) \sin \alpha_1 = (C_1 - C_2) \cos \alpha_1$$

Next we solve (3.48) and (3.51) for C_1 and C_2 in terms of $C_{n+1,1}$ and $C_{n+1,2}$. Observing that $C_1 = C_2$, the results are

$$(3.56) \quad C_1 = C_{n+1,1} + V \cos \alpha_1$$

$$(3.57) \quad C_2 = C_{n+1,2} + V \sin \alpha_1$$

for $n = 1, 2, \dots, m$. Solving (3.49) and (3.52) for C_1 and C_2 in terms of C_3 and C_4 , the result is

$$(3.58) \quad C_1 = C_3 - V \cos \alpha_1$$

$$(3.59) \quad C_2 = C_4 + V \sin \alpha_1$$

Substituting (3.56) and (3.57) in (3.45) and (3.57) and (3.59)

in (3.46), we can rewrite (3.45), (3.46), and (3.47) in the form

$$A_n(z) = \frac{1}{V} \left\{ C_{m+1,1} + \sum_{p=1}^m V_p \cos \alpha_p \right\} \sin \alpha_1$$

$$(3.60) \quad \left\{ C_{m+1,2} + \sum_{p=1}^m V_p \sin \alpha_p \right\} \cos \alpha_1$$

$$\text{for } n = 1, 2, \dots, m \text{ where } \sum_{p=1}^m V_p \cos \alpha_p = \sum_{p=1}^m V_p \sin \alpha_p = 0$$

for $n = m+1$ and

$$A_{m+1}(z) = \frac{1}{V} \left\{ C_{m+1,1} + \sum_{p=1}^m V_p \cos \alpha_p \right\} \sin \alpha_1$$

$$(3.61) \quad \left\{ C_{m+1,2} + \sum_{p=1}^m V_p \sin \alpha_p \right\} \cos \alpha_1$$

$$(3.62) \quad A_{m+1}(z) = \frac{1}{V} \left\{ C_3 - V \cos \alpha_1 + C_4 + V \sin \alpha_1 \right\} \cos \alpha_1$$

$$(3.63) \quad A_{m+1}(z) = \frac{1}{V} \left\{ C_3 \sin \alpha_1 + C_4 \cos \alpha_1 \right\}$$

For convenience, we define

$$A_1(z) = A_{1n}(z)$$

$$\phi_1(z) = \phi_{1n}(z) \quad (3.62)$$

$$I_1(z) = I_{1n}(z)$$

for $l_{n-1} \leq z \leq l_n$, where $l_0 = 0$, $l_{m+1} = 2h$, and $n = 1, 2, 3, \dots, m+1$. Also, we define

$$\begin{aligned} A_2(z) &= \begin{cases} A_{21}(z) & 0 \leq z \leq l_3 \\ A_{22}(z) & l_3 \leq z \leq l_{m+1} \end{cases} \\ \phi_2(z) &= \begin{cases} \phi_{21}(z) & 0 \leq z \leq l_3 \\ \phi_{22}(z) & l_3 \leq z \leq l_{m+1} \end{cases} \\ I_2(z) &= \begin{cases} I_{21}(z) & 0 \leq z \leq l_3 \\ I_{22}(z) & l_3 \leq z \leq l_{m+1} \end{cases} \end{aligned} \quad (3.63)$$

Combining (2.6, 3.58, 3.61, 3.62, 3.63, and 3.12), we obtain

$$V_{m+1} = C_5 \cos 2\beta h - C_6 \sin 2\beta h, \quad (3.64)$$

where

$$C_5 = C_{n1} - C_3 \quad (3.65)$$

$$C_6 = C_{m2} - C_4. \quad (3.66)$$

Combining (2.6, 3.58, 3.60, 3.62, 3.63, and 3.16), we obtain

$$V_0 = C_5 + \sum_{p=1}^m V_p \cos \beta l_p. \quad (3.67)$$

Combining (3.58, 3.59, 3.60, 3.61, 3.62, 3.64, 3.65, 3.66 and 3.25), we find

$$I_1(z) + I_2(z) = -j \frac{2}{R_c} \left[\left\{ c_5 + \sum_{p=n}^m V_p \cos \beta \ell_p \right\} \sin \beta z + \left\{ c_6 - \sum_{p=n}^m V_p \sin \beta \ell_p \right\} \cos \beta z \right] \quad (3.68)$$

for $\ell_{n-1} \leq z \leq \ell_n$, where $\ell_0 = 0$, $\ell_{m+1} = 2h$ and $n = 1, 2, 3 \dots m+1$.

Comparing (3.64, 3.67, and 3.68) with (3.13, 3.17, and 3.26), we see that the two sets of equations are identical except for the designation of arbitrary constants. Thus, (3.32 and 3.34) hold regardless of which side of the antenna the side loads are placed.

If we now combine (3.58, 3.59, 3.60, 3.61, 3.62, 3.63, 3.35, and 3.36) with "d" defined in Chapter 2, we obtain

$$\int_0^{2h} I_T(z') K_d(z, z') dz' = j \frac{4\pi}{\eta} \left[- \left\{ \frac{C_{m+1,1} + C_3}{2} + \sum_{\substack{p=n \\ p \neq 3}}^m \frac{V_p}{2} \cos \beta \ell_p - \frac{V_3}{2} \cos \beta \ell_3 \right\} \sin \beta z - \left\{ \frac{C_{m+1,2} + C_4}{2} - \sum_{\substack{p=n \\ p \neq 3}}^m \frac{V_p}{2} \sin \beta \ell_p + \frac{V_3}{2} \sin \beta \ell_3 \right\} \cos \beta z \right] \quad (3.69)$$

for $\ell_{n-1} \leq z \leq \ell_n$, where $\ell_0 = 0$ and $n = 1, 2$.

$$\int_0^{2h} I_T(z') K_d(z, z') dz' = j \frac{4\pi}{\eta} \left[- \left\{ \frac{C_{m+1,1} + C_3}{2} + \sum_{p=4}^m \frac{V_p}{2} \cos \beta \ell_p - \frac{V_3}{2} \cos \beta \ell_3 \right\} \sin \beta z - \left\{ \frac{C_{m+1,2} + C_4}{2} - \sum_{p=4}^m \frac{V_p}{2} \sin \beta \ell_p + \frac{V_3}{2} \sin \beta \ell_3 \right\} \cos \beta z \right] \quad (3.70)$$

for $\ell_2 \leq z \leq \ell_3$.

$$I_1(z) = I_1(z) = -\frac{1}{R_0} \left\{ C_0 + \sum_{n=1}^{\infty} \frac{C_n}{p_n} \cos p_n z \right\} \quad (3.68)$$

$$I_1(z) = I_1(z) = -\frac{1}{R_0} \left\{ C_0 + \sum_{n=1}^{\infty} \frac{C_n}{p_n} \cos p_n z \right\} \quad (3.68)$$

for $n-1 \leq z \leq n$, where $C_0 = 0$ and $n = 1, 2, \dots$

Comparing (3.67), (3.68), and (3.69) with (3.13), (3.14), and (3.20), we see that the two sets of equations are identical.

except for the designation of arbitrary constants. Thus, (3.52 and 3.53) hold regardless of which side of the interface

the side loads are placed.

If we now compare (3.52), (3.53), (3.60), (3.61), (3.62), (3.63), (3.64), and (3.65) with (3.66) defined in Chapter 2, we obtain

$$I_T(z) = I_T(z) = \frac{1}{R_0} \left\{ C_0 + \sum_{n=1}^{\infty} \frac{C_n}{p_n} \cos p_n z \right\} \quad (3.69)$$

$$I_T(z) = I_T(z) = \frac{1}{R_0} \left\{ C_0 + \sum_{n=1}^{\infty} \frac{C_n}{p_n} \cos p_n z \right\} \quad (3.69)$$

for $n-1 \leq z \leq n$, where $C_0 = 0$ and $n = 1, 2, \dots$

$$I_T(z) = I_T(z) = \frac{1}{R_0} \left\{ C_0 + \sum_{n=1}^{\infty} \frac{C_n}{p_n} \cos p_n z \right\} \quad (3.70)$$

$$I_T(z) = I_T(z) = \frac{1}{R_0} \left\{ C_0 + \sum_{n=1}^{\infty} \frac{C_n}{p_n} \cos p_n z \right\} \quad (3.70)$$

for $n-1 \leq z \leq n$

$$\int_0^{2h} I_T(z') K_d(z, z') dz' = j \frac{4\pi}{\eta} \left[- \left\{ \frac{C_{m+1,1} + C_3}{2} + \sum_{p=n}^m \frac{V_p}{2} \cos \beta \ell_p \right\} \sin \beta z \right. \\ \left. - \left\{ \frac{C_{m+1,2} + C_4}{2} - \sum_{p=n}^m \frac{V_p}{2} \sin \beta \ell_p \right\} \cos \beta z \right] \quad (3.71)$$

for $\ell_{n-1} \leq z \leq \ell_n$, where $\ell_{m+1} = 2h$ and $n = 4, 5 \dots m+1$.

Again, if the set of equations (3.69, 3.70, and 3.71) can be solved for the ratios (3.38), then it is possible to evaluate the Υ_n for $n = 1, 2, 3 \dots m$, and (3.32) is a means of evaluating the input impedance of the multiply side-loaded antenna even if all the side loads are not on the same side. Since (3.68) and (3.26) are identical, we can use (3.43) to evaluate the Υ_n regardless of which side of the antenna the side loads are on, if we use the appropriate Z_{dn} .

The details of evaluating the Z_{dn} are given in Appendix B. It is shown there that the integral equations (3.69, 3.70, and 3.71) are identical with those that describe the multiply-fed linear antenna of Figure 3.5. It is further shown, that the multiply-fed linear antenna of Figure 3.5 is equivalent by linear superposition to the set of asymmetrically-driven linear antennas also shown in Figure 3.5. Using this equivalence, the Z_{dn} for $n = 1, 2, 3 \dots m$ can be evaluated in terms of the Z_{dak} for $k = 1, 2, 3 \dots m$. From Appendix B the equations for doing this are

$$Z_{dn} = \frac{Z_{dan}}{1 + \sum_{p=1}^{n-1} \frac{V_p}{V_n} \frac{Z_{dan}}{Z_{dap}} \frac{\sin \beta(\ell_{m+1} - \ell_n)}{\sin \beta(\ell_{m+1} - \ell_p)}} + \sum_{\substack{p=n+1 \\ p \neq 3}}^m \frac{V_p}{V_n} \frac{Z_{dan}}{Z_{dap}} \frac{\sin \beta \ell_n}{\sin \beta \ell_p} - \frac{V_3}{V_n} \frac{Z_{dan}}{Z_{da3}} \frac{\sin \beta \ell_n}{\sin \beta \ell_3} \quad (3.72)$$

for $n = 1, 2$.

$$\left\{ \begin{aligned} & \frac{1}{\sqrt{2}} \left[\frac{C_{n+1}}{2} + \sum_{p=n}^{\infty} \frac{C_p}{2} \cos \left(\frac{p-n}{2} \right) \right] \\ & \frac{1}{\sqrt{2}} \left[\frac{C_{n+1}}{2} + \sum_{p=n}^{\infty} \frac{C_p}{2} \cos \left(\frac{p-n}{2} \right) \right] \end{aligned} \right\} \quad (3.71)$$

for $n = 1, 2, \dots, \infty$, where $C_n = 2$ and $n = 1, 2, \dots, m+1$.
Again, as the set of equations (3.58), (3.59), and (3.61) can be
solved for the ratios (3.58), then it is possible to evaluate
the Y_n for $n = 1, 2, \dots, m$, and (3.52) is a system of equations
finding the input impedances of the multiply side-loaded antennas
even if all the side loads are not on the same side. Since
(3.58) and (3.59) are identical, we can use (3.58) to evaluate
the Y_n regardless of which side of the antenna the side loads
are on. If we use the approximate Z_{dn}
The details of evaluating the Z_{dn} are given in Appendix B.
It is shown there that the integral equations (3.59), (3.60), and
(3.61) are identical with those that describe the multiply-fed
linear antenna of Figure 3.5. It is further shown that the
multiply-fed linear antenna of Figure 3.5 is equivalent by
linear superposition to the set of asymmetrically-driven
linear antennas also shown in Figure 3.5. Using this equivalence,
hence, the Z_{dn} for $n = 1, 2, \dots, m$ can be evaluated in terms
of the Z_{dk} for $k = 1, 2, \dots, m$. From Appendix B the equations
for doing this are

$$\begin{aligned} Z_{dn} &= \frac{1}{\sqrt{2}} \left[\frac{C_{n+1}}{2} + \sum_{p=n}^{\infty} \frac{C_p}{2} \cos \left(\frac{p-n}{2} \right) \right] \\ &= \frac{1}{\sqrt{2}} \left[\frac{C_{n+1}}{2} + \sum_{p=n}^{\infty} \frac{C_p}{2} \cos \left(\frac{p-n}{2} \right) \right] \end{aligned}$$

Figure 3.5--Multiply-fed linear antenna and an equivalent set of asymmetrically-driven linear antennas

(3.73)

$$E_{\text{an}} = \frac{1}{2} \sum_{p=1}^n \frac{V_p}{Z_{\text{gap}}} \sin \left(\frac{p-1}{n} \pi \right) + \frac{1}{2} \sum_{p=1}^n \frac{V_p}{Z_{\text{gap}}} \sin \left(\frac{p-1}{n} \pi \right)$$

(3.74)

$$E_{\text{an}} = \frac{1}{2} \sum_{p=1}^n \frac{V_p}{Z_{\text{gap}}} \sin \left(\frac{p-1}{n} \pi \right)$$

(3.75)

$$E_{\text{an}} = \frac{1}{2} \sum_{p=1}^n \frac{V_p}{Z_{\text{gap}}} \sin \left(\frac{p-1}{n} \pi \right)$$

It is also shown in Appendix B that the integral equations (3.37) for the multiply side-loaded transmission-line antenna with all the side loads on the same side are identical with those that describe the multiply-fed linear antenna of Figure 3.5. The only difference between the antennas of Figures 3.5 and 3.6 is the polarity of the driving voltage at drive point three which is reversed. Again the multiply-fed linear antenna of Figure 3.6 is equivalent to the set of asymmetrically driven linear antennas also shown in Figure 3.6, and the E_{an} for $n = 1, 2, 3, \dots, m$ can be evaluated in terms of the E_{an} for $k = 1, 2, 3, \dots, m$. From Appendix B we find

(3.76)

$$E_{\text{an}} = \frac{1}{2} \sum_{p=1}^n \frac{V_p}{Z_{\text{gap}}} \sin \left(\frac{p-1}{n} \pi \right) + \frac{1}{2} \sum_{p=1}^n \frac{V_p}{Z_{\text{gap}}} \sin \left(\frac{p-1}{n} \pi \right)$$

Comparing (3.73, 3.74, 3.75, and 3.76) with (3.70), we see that shifting the side load three to the opposite side of the transmission line simply reverses the sign of every term in the

$$I_n = \sum_{p=1}^{n-1} I_{p2} + \sum_{p=n}^m I_{p1} \quad \text{for } n=1, 2, 3 \dots m+1$$

Figure 3.6--Multiply-fed linear antenna and an equivalent set of asymmetrically-driven linear antennas

$$I_m = \sum_{n=1}^m I_n \quad \text{for } n=1, 2, \dots, m-1$$

Figure 2. A linearly-driven linear antenna and an equivalent circuit.

denominator of Z_{dn} for $n = 1, 2, 3 \dots m$ that contains the factor, V_3 . If more than one side load were shifted to the opposite side of the transmission line, it would be necessary to insert minus signs in every term in the denominator of Z_{dn} for $n = 1, 2, 3 \dots m$ that contained as a factor the voltage across one of the shifted side loads. In some terms this might involve inserting two minus signs which would, of course, result in no change in sign.

Thus, for the general side-loaded transmission-line antenna, shifting the side loads from one side to the other affects only the equations for Z_{dn} for $n = 1, 2, 3 \dots m$ as described above but does not change the form of the equations (3.32, 3.34, and 3.43). Since the equivalent circuit of Figure 3.3 is based on these equations, the form of the equivalent circuit is not changed by shifting the side loads. We now have at our disposal a set of equations and an equivalent circuit for a completely general side-loaded transmission-line antenna.

3.4 Efficiency of the General Side-Loaded Transmission-Line Antenna

This discussion will consider only losses in the side loads and end loads as was done in Chapter 2. First, it is shown that the equivalent circuit of Figure 3.3 is valid for power calculations. Once this has been done, the method presented in Section 2.4 of Chapter 2 can be used to account for losses due to the finite conductivity of the wires or conductors of the antenna.

denominator of E_{in} for $n = 1, 2, 3, \dots$ that contains the factor V . It means that one side load was shifted to the opposite side of the transmission line. It would be necessary to insert minus signs in every term in the denominator of E_{in} for $n = 1, 2, 3, \dots$ that contained as a factor the voltage across one of the shifted side loads. In some cases this might involve inserting two minus signs which would, of course, result in no change in sign.

Thus, for the general side-loaded transmission-line antenna, shifting the side loads from one side to the other affects only the equations for E_{in} for $n = 1, 2, 3, \dots$ as described above and does not change the form of the equations (3.32, 3.33, and 3.34). Since the equivalent circuit of Figure 3.3 is based on these equations, the form of the equivalent circuit is not changed by shifting the side loads. We now have at our disposal a set of equations and an equivalent circuit for a completely general side-loaded transmission-line antenna.

3.4 Efficiency of the General Side-Loaded Transmission-Line Antenna

This discussion will consider only losses in the side loads and end loads as was done in Chapter 2. First, it is shown that the equivalent circuit of Figure 3.3 is valid for power calculations. Once this has been done, the method presented in Section 2.4 of Chapter 2 can be used to account for losses due to the finite conductivity of the wires or conductors of the antenna.

In order to prove that the equivalent circuit is valid for power calculations, the following steps are performed. First, an expression for efficiency is derived from the antenna structure. Second, an expression for efficiency is derived from the equivalent circuit. Third, it is observed that these expressions contain similar series, and that the expressions would give the same results for efficiency if the respective terms of the series were equal. Fourth, the terms are set equal and a set of necessary conditions is derived. Fifth, the necessary conditions are shown to follow from the antenna equations. This completes the proof that the equivalent circuit for the general side-loaded transmission-line antenna is valid for power calculations.

From Figure 3.1, we observe that the total power input and the power lost in the side loads and end load are given by the following equations.

$$\text{Total Power Input} = |V_O|^2 G_O, \quad (3.77)$$

where

$$G_O = \frac{R_O}{|Z_O|^2} \quad (3.78)$$

$$\text{Power Lost in the side loads and end load} = \sum_{n=1}^{m+1} |V_n|^2 G_n, \quad (3.79)$$

where

$$G_n = \frac{R_n}{|Z_n|^2} \quad (3.80)$$

and $n = 1, 2, 3 \dots m+1$. Thus, the expression for efficiency can be written,

$$\text{Efficiency} = 100 \left[1 - \sum_{n=1}^{m+1} \frac{|V_n|^2 G_n}{|V_O|^2 G_O} \right]. \quad (3.81)$$

in order to prove that the equivalent circuit is valid for power calculations, the following steps are performed. First, an expression for efficiency is derived from the antenna series circuit. Second, an expression for efficiency is derived from the equivalent circuit. Third, it is shown that these expressions contain similar terms, and that the expressions give the same results for efficiency at the respective terms of the series were equal. Fourth, the terms are set equal and a set of necessary conditions is derived. Fifth, the necessary conditions are shown to follow from the antenna equations. This completes the proof that the equivalent circuit for the general side-loaded transmission-line antenna is valid for power calculations.

From Figure 3.1, we observe that the total power input and the power lost in the side loads and end load are given by the

following equations:

$$\text{Total power input} = |V_0|^2 G_0$$

where

$$G_0 = \frac{R_0}{|Z_0|^2}$$

Power lost in the side loads and end load = $\sum_{n=1}^N |V_n|^2 G_n$

where

$$G_n = \frac{R_n}{|Z_n|^2}$$

and $n = 1, 2, \dots, N+1$. Thus, the expression for efficiency can be written

$$\text{Efficiency} = 100 \frac{|V_0|^2 G_0}{\sum_{n=1}^N |V_n|^2 G_n + |V_0|^2 G_0}$$

Next, we compute the efficiency from the equivalent circuit of Figure 3.3.

$$\text{Total Power Input} = |V_O|^2 G_O = \frac{|V_1|^2 R_{T1}'}{|Z_{O1}|^2} = \frac{|V_2|^2}{|Z_{O2}|^2} R_{T2}' \frac{R_{T1}'}{R_{T1}} \quad (3.82)$$

Continuing the process started in (3.82), we find in general

$$\text{Total Power Input} = \frac{|V_n|^2}{|Z_{On}|^2} R_{Tn}' \prod_{p=1}^{n-1} \frac{R_{Tp}'}{R_{Tp}} \quad (3.83)$$

For the special case where $n = m+1$, we find

$$\text{Total Power Input} = |V_{m+1}|^2 G_{m+1} \prod_{p=1}^m \frac{R_{Tp}'}{R_{Tp}} \quad (3.84)$$

$$\text{The Power Lost in the } n\text{th side load} = |V_n|^2 G_n \quad (3.85)$$

$$\text{The Power Lost in the end load} = |V_{m+1}|^2 G_{m+1} \quad (3.86)$$

Combining (3.83, 3.84, 3.85, and 3.86), we find for the efficiency,

$$\text{Efficiency} = 100 \left[1 - \sum_{n=1}^m |Z_{On}|^2 \frac{G_n}{R_{Tn}'} \prod_{p=1}^{n-1} \frac{R_{Tp}'}{R_{Tp}} - \prod_{p=1}^m \frac{R_{Tp}'}{R_{Tp}} \right] \quad (3.87)$$

If (3.81 and 3.87) are to give identical results, we can have

$$\left| \frac{V_n}{V_O} \right|^2 \frac{G_n}{G_O} = |Z_{On}|^2 \frac{G_n}{R_{Tn}'} \prod_{p=1}^{n-1} \frac{R_{Tp}'}{R_{Tp}} \quad (3.88)$$

for $n = 1, 2, 3 \dots m$, and

$$\left| \frac{V_{m+1}}{V_O} \right|^2 \frac{G_{m+1}}{G_O} = \prod_{p=1}^m \frac{R_{Tp}'}{R_{Tp}} \quad (3.89)$$

Solving for the square of the voltage ratios, we find

$$\left| \frac{V_n}{V_O} \right|^2 = G_O \frac{|Z_{On}|^2}{R_{Tn}'} \prod_{p=1}^{n-1} \frac{R_{Tp}'}{R_{Tp}} \quad (3.90)$$

for $n = 1, 2, 3 \dots m$, and

Next, we compute the efficiency from the equivalent circuit.

$$\text{Total Power Input} = \frac{V_o^2}{R_o} = \frac{(1.0)^2}{1} = 1.0 \text{ W} \quad (3.87)$$

Combining the powers scattered in (3.83), we find in general

$$\text{Total Power Input} = \frac{V_o^2}{R_o} = \frac{(1.0)^2}{1} = 1.0 \text{ W} \quad (3.87)$$

For the special case where $n = 1$, we find

$$\text{Total Power Input} = \frac{V_o^2}{R_o} = \frac{(1.0)^2}{1} = 1.0 \text{ W} \quad (3.87)$$

The power lost in the antenna load is $|V_o|^2 C_n$

$$\text{The power lost in the antenna load} = |V_o|^2 C_n \quad (3.88)$$

Combining (3.87), (3.88), and (3.89), we find for the efficiency

$$\text{Efficiency} = \frac{P_{\text{out}}}{P_{\text{in}}} = \frac{\frac{V_o^2}{R_o}}{\frac{V_o^2}{R_o} + |V_o|^2 C_n} = \frac{1}{1 + C_n R_o} \quad (3.89)$$

For $n = 1$, (3.81) and (3.87) are to give identical results, we can have

$$\frac{V_o^2}{R_o} = \frac{V_o^2}{R_o} = 1.0 \text{ W} \quad (3.88)$$

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For $n = 1$, (3.81) and (3.87) are to give identical results, we can have

$$\left| \frac{V_{m+1}}{V_o} \right|^2 = \frac{G_o}{G_{m+1}} \prod_{p=1}^m \frac{R_{Tp}}{R'_{Tp}} \quad (3.91)$$

For convenience, separate these voltage ratios as follows:

$$\left| \frac{V_n}{V_o} \right|^2 = \prod_{p=1}^n \frac{|V_p|^2}{|V_{p-1}|^2} \quad (3.92)$$

for $n = 1, 2, 3 \dots m$, and

$$\left| \frac{V_{m+1}}{V_o} \right|^2 = \prod_{p=1}^{m+1} \frac{|V_p|^2}{|V_{p-1}|^2} \quad (3.93)$$

Using (3.90) and (3.91), we find

$$\left| \frac{V_n}{V_{n-1}} \right|^2 = \frac{|Z_{on}|^2}{|Z_{o,n-1}|^2} \frac{R_{Tn-1}}{R'_{Tn}} \quad (3.94)$$

for $n = 2, 3, 4 \dots m$, and

$$\left| \frac{V_{m+1}}{V_m} \right|^2 = \frac{|Z_{m+1}|^2 R_{Tm}}{|Z_{om}|^2 R_{m+1}} \quad (3.95)$$

$$\left| \frac{V_1}{V_o} \right|^2 = \frac{G_o |Z_{o1}|^2}{R'_{T1}} \quad (3.96)$$

From transmission line theory, we can write

$$\left| \frac{V_n}{V_{n-1}} \right|^2 = \frac{|Z_{on}|^2}{|Z_{o,n-1}|^2} \frac{R_c^2}{d_6}, \quad (3.97)$$

where

$$d_6 = \left[R_c \cos \beta(\ell_n - \ell_{n-1}) - X'_{Tn} \sin \beta(\ell_n - \ell_{n-1}) \right]^2 + R_{Tn}^2 \sin^2 \beta(\ell_n - \ell_{n-1})$$

for $n = 2, 3, 4 \dots m$, and

$$\left| \frac{V_{m+1}}{V_m} \right|^2 = \frac{|Z_{m+1}|^2}{|Z_{om}|^2} \frac{R_c^2}{d_7}, \quad (3.98)$$

where

$$d_7 = \left[R_c \cos \beta(\ell_{m+1} - \ell_m) - X_{m+1} \sin \beta(\ell_{m+1} - \ell_m) \right]^2 + R_{m+1}^2 \sin^2 \beta(\ell_{m+1} - \ell_m)$$

$$\left| \frac{V_{m+1}}{V_m} \right| = \frac{1}{2} \left(1 + \frac{1}{2} \right)$$

for convenience, let

$$\left| \frac{V_{m+1}}{V_m} \right| = \frac{1}{2} \left(1 + \frac{1}{2} \right)$$

for $n = 1, 2, 3, \dots$

$$\left| \frac{V_{m+1}}{V_m} \right| = \frac{1}{2} \left(1 + \frac{1}{2} \right)$$

Using (3.10) and (3.11)

$$\left| \frac{V_{m+1}}{V_m} \right| = \frac{1}{2} \left(1 + \frac{1}{2} \right)$$

for $n = 1, 2, 3, \dots$

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$$\left| \frac{V_{m+1}}{V_m} \right| = \frac{1}{2} \left(1 + \frac{1}{2} \right)$$

$$\left| \frac{V_1}{V_0} \right|^2 = \frac{|Z_{01}|^2}{\left[R_C \sin \beta \ell_1 + X_{T1}' \cos \beta \ell_1 \right]^2 + R_{T1}'^2 \cos^2 \beta \ell_1} \quad (3.99)$$

(3.97, 3.98, and 3.99) are the set of necessary conditions.

Next, we start with the antenna equations (3.17, 3.30, and 3.31). Substituting (3.30 and 3.31) in (3.17), we find

$$\frac{V_1}{V_0} = \frac{1}{\cos \beta \ell_1 + \frac{\gamma_{m+1}}{\gamma_1 Z_1} + \sum_{p=2}^m \frac{\gamma_p Z_p}{Z_1 \gamma_1} \cos \beta \ell_p} \quad (3.100)$$

Using (C.36) in (3.100) and taking the square of the magnitude of the result, we obtain

$$\left| \frac{V_1}{V_0} \right|^2 = \frac{|Z_{01}|^2}{\left[R_C \sin \beta \ell_1 + X_{T1}' \cos \beta \ell_1 \right]^2 + R_{T1}'^2 \cos^2 \beta \ell_1} \quad (3.101)$$

Thus, we have established the necessary condition (3.99). To derive the necessary condition (3.98) from the antenna equations, we start by substituting (3.30 and 3.31) in (3.13). The result is

$$\frac{V_{m+1}}{V_m} = \frac{V_{m+1} \cos \beta \ell_{m+1} - \sin \beta \ell_{m+1}}{\gamma_m Z_m} \quad (3.102)$$

Using (C.16 and 3.31) in (3.102), we obtain

$$\begin{aligned} \frac{V_{m+1}}{V_m} = & \left[\frac{R_C \cos \beta \ell_m + j Z_{Tm} \sin \beta \ell_m}{-j Z_{om}} \right] \\ & \times \left[\left\{ \frac{R_C \sin \beta \ell_{m+1} - j Z_{m+1} \cos \beta \ell_{m+1}}{R_C \cos \beta \ell_{m+1} + j Z_{m+1} \sin \beta \ell_{m+1}} \right\} \cos \beta \ell_{m+1} - \sin \beta \ell_{m+1} \right] \end{aligned} \quad (3.103)$$

which after some rearrangement becomes

$$\frac{V_{m+1}}{V_m} = \frac{Z_{m+1}}{Z_{om}} \left[\frac{R_C \cos \beta \ell_m + j Z_{Tm} \sin \beta \ell_m}{R_C \cos \beta \ell_{m+1} + j Z_{m+1} \sin \beta \ell_{m+1}} \right] \quad (3.104)$$

Substituting (C.28) in (3.104) and taking the square of the magnitude, we find

$$(5.99) \quad \frac{|V_1|^2}{|V|^2} = \frac{R_1^2 \cos^2 \theta_1 + R_2^2 \cos^2 \theta_2}{R_1^2 + R_2^2}$$

(5.97, 5.98, and 5.99) are the set of necessary conditions

Next, we start with the antenna equations (5.14), (5.15), and

(5.21). Substituting (5.20) and (5.21) in (5.14), we find

$$(5.100) \quad \frac{|V_1|^2}{|V|^2} = \frac{R_1^2 \cos^2 \theta_1 + R_2^2 \cos^2 \theta_2}{R_1^2 + R_2^2}$$

Using (5.20) in (5.100) and taking the square of the magnitude

of the result, we obtain

$$(5.101) \quad \frac{|V_1|^2}{|V|^2} = \frac{R_1^2 \cos^2 \theta_1 + R_2^2 \cos^2 \theta_2}{R_1^2 + R_2^2}$$

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derive the necessary condition (5.99) from the antenna equations

we start by substituting (5.20 and 5.21) in (5.15). The result

is

$$(5.102) \quad \frac{|V_1|^2}{|V|^2} = \frac{R_1^2 \cos^2 \theta_1 + R_2^2 \cos^2 \theta_2}{R_1^2 + R_2^2}$$

Using (5.16 and 5.21) in (5.102), we obtain

$$(5.103) \quad \frac{|V_1|^2}{|V|^2} = \frac{R_1^2 \cos^2 \theta_1 + R_2^2 \cos^2 \theta_2}{R_1^2 + R_2^2}$$

$$\left[\frac{R_1 \sin \theta_1}{R_1 \cos \theta_1 + j R_2 \sin \theta_2} \right] \left[\frac{R_2 \sin \theta_2}{R_2 \cos \theta_2 + j R_1 \sin \theta_1} \right]$$

which after some rearrangement becomes

$$(5.104) \quad \frac{|V_1|^2}{|V|^2} = \frac{R_1^2 \cos^2 \theta_1 + R_2^2 \cos^2 \theta_2}{R_1^2 + R_2^2}$$

Substituting (5.20) in (5.104) and taking the square of the

magnitude, we find

$$\left| \frac{V_{m+1}}{V_m} \right|^2 = \frac{|Z_{m+1}|^2}{|Z_{om}|^2} \frac{R_c^2}{d_8}, \quad (3.105)$$

where

$$d_8 = \left[R_c \cos \beta(\ell_{m+1} - \ell_m) - X_{m+1} \sin \beta(\ell_{m+1} - \ell_m) \right]^2 + R_{m+1}^2 \sin^2 \beta(\ell_{m+1} - \ell_m)$$

which is identical with (3.98). Finally, to derive the necessary condition (3.92) from the antenna equations, we start with (3.30) and obtain the ratio

$$\frac{V_n}{V_{n-1}} = \frac{\gamma_n Z_n}{\gamma_{n-1} Z_{n-1}}. \quad (3.106)$$

Substituting (C.35) in (3.106) and taking the square of the magnitude, we find

$$\left| \frac{V_n}{V_{n-1}} \right|^2 = \frac{|Z_{on}|^2}{|Z_{o,n-1}|^2} \frac{R_c^2}{d_9}, \quad (3.107)$$

where

$$d_9 = \left[R_c \cos \beta(\ell_n - \ell_{n-1}) - X_{Tn} \sin \beta(\ell_n - \ell_{n-1}) \right]^2 + R_{Tn}^2 \sin^2 \beta(\ell_n - \ell_{n-1}) \quad (3.108)$$

which is identical with (3.97). This completes the derivation of the three necessary conditions from the antenna equations and proves that the equivalent circuit is valid for power calculations.

3.5 Comments on the Equivalent Circuit

From the similarity of the equivalent circuits for single side-loaded transmission-line antennas and general side-loaded transmission-line antennas, we might be led to the conclusion that the impedances Z_{on} for $n = 1, 2, 3 \dots m$ are independent of one another. If this were the case, we could match the

(3.105)

$$\left| \frac{V_{m+1}}{V_m} \right| = \frac{\left| \frac{Z_{m+1}}{Z_m} \right|}{\left| \frac{R_c}{R_c} \right|}$$

where

$$R_c = \left[R_c \cos^2 \theta_{m+1} - X_{m+1} \sin^2 \theta_{m+1} + R_{m+1} \sin^2 \theta_{m+1} \right] / \left[R_c \cos^2 \theta_m - X_m \sin^2 \theta_m + R_m \sin^2 \theta_m \right]$$

which is identical with (3.98). Finally, to derive the necessary condition (3.97) from the antenna equations, we start with (3.90) and obtain the ratio

(3.106)

$$\frac{V_n}{V_{n-1}} = \frac{Z_n}{Z_{n-1}}$$

Substituting (3.95) in (3.106) and taking the square of the

magnitude, we find

(3.107)

$$\left| \frac{V_n}{V_{n-1}} \right|^2 = \frac{\left| \frac{Z_n}{Z_{n-1}} \right|^2}{\left| \frac{R_c}{R_c} \right|^2}$$

where

(3.108)

$$R_c = \left[R_c \cos^2 \theta_n - X_n \sin^2 \theta_n + R_n \sin^2 \theta_n \right] / \left[R_c \cos^2 \theta_{n-1} - X_{n-1} \sin^2 \theta_{n-1} + R_{n-1} \sin^2 \theta_{n-1} \right]$$

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that the impedances Z_n for $n = 1, 2, 3, \dots, m$ are independent

of one another. If this were the case, we could match the

Z_n to the Z_{dn} for $n = 1, 2, 3 \dots m$ one at a time, choosing them in such a way as to set the poles of the input impedance where ever we desired. Observing equations (B.13, B.22, B.23, and B.24), we see, however, that this is not the case. Since each Z_{dn} for $n = 1, 2, 3 \dots m$ is a function of the voltages across all the other Z_{dn} for $n = 1, 2, 3 \dots m$, we have a completely coupled system, and a change in the impedance of any one side load will shift all the poles in the whole system.

Admitting this difficulty we do, never-the-less, have a more versatile system than was presented by the single side-loaded transmission-line antenna. In Chapter 4, we exploit some of these possibilities to obtain the desired input impedance characteristics described in Section 2.3 of Chapter 2.

Z_n to the Z_{in} for $n = 1, 2, \dots, m$ one at a time, choosing them in such a way as to set the poles of the input impedance where ever we desired. Observing equations (B.15), (B.16), (B.17), and (B.18), we see, however, that this is not the case. Since each Z_n for $n = 1, 2, \dots, m$ is a function of the voltages across all the other Z_n for $n = 1, 2, \dots, m$, we have a completely coupled system, and a change in the impedance of any one side load will shift all the poles in the whole system.

Admitting this difficulty we do, nevertheless, have a more versatile system than was presented by the single side-loaded transmission-line antenna. In Chapter 4, we exploit some of these possibilities to obtain the desired input impedance characteristics described in Section 2.3 of Chapter 2.

4. A NUMERICAL STUDY OF SINGLE AND DOUBLE SIDE-LOADED TRANSMISSION-LINE ANTENNAS

4.1 Introduction

In Chapters 2 and 3 we have developed the theoretical expressions for the input impedance and efficiency of the single side-loaded transmission-line antenna and the general side-loaded transmission-line antenna. We also presented, in a very general manner, some of the more favorable characteristics of the single side-loaded transmission-line antenna. These characteristics are an operating frequency at which the reactive part of the input impedance is zero, and the real part is large compared to the input reactance of the equivalent asymmetrically-driven linear antenna.

In this Chapter we first of all verify that these favorable characteristics do occur. This is done by calculations on several single side-loaded transmission-line antennas chosen to illustrate the properties under consideration here. A discussion of the practical limitations which might arise in an actual design situation is presented in connection with these calculations.

Having investigated the design possibilities of the single side-loaded transmission-line antenna, we investigate the possibility of more side loads using the equations developed for the general side-loaded transmission-line antenna. Because it greatly simplified the discussion, and also because there is

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1.1 Introduction

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a very general manner, some of the analytical characteristics

of the single side-loaded transmission-line antenna. These

characteristics are an operating frequency, radiation resistance

and part of the input impedance is zero, and the real part is

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operation is at the linear antenna.

In this chapter we first of all verify the results of

the characteristics do occur. This is done by calculations

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A discussion of the practical limitations of the antenna

in an actual design situation is presented in connection with

these calculations.

Having investigated the design possibilities of the single

side-loaded transmission-line antenna, we investigate the possi-

bility of more side loads using the equations developed for the

general side-loaded transmission-line antenna. Section 1.2

briefly summarizes the discussion, and the conclusions are

nothing to be gained practically by adding more side loads, we limit the discussion to double side-loaded transmission-line antennas. One further limitation which greatly simplifies the calculations is to consider only those double side-loaded antennas which have symmetrically located side loads. With these limitations another series of calculations is presented on a group of double side-loaded transmission-line antennas again chosen to illustrate the properties under consideration here. Along with the calculations we again discuss the practical limitations that would be encountered in an attempt to design such an antenna to meet a given set of specifications.

4.2 Description of the Antennas to be Considered and the Characteristics Desired

The frequency range of interest for the antennas to be considered is 10 to 40 kc, and the physical size must be such that they can be trailed behind aircraft. To fulfill these first two requirements it is necessary that the antenna be electrically quite short. ($\beta h < 1$). Some consideration is given to antennas which range from $0.5 \leq \beta h \leq 2.5$, but most of the discussion and calculations are confined to the electrically short antennas. The calculations show that it is inductive side loads that produce the desirable characteristics for electrically short antennas. In making the calculations we will assume that inductances with a Q of 100 are practically obtainable. Thus all efficiency considerations will be based on a Q of 100 for the side loads and end load.

The electrical characteristics that are desired are the same as those discussed in Section 2.3 of Chapter 2. In

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side loads that produce the desirable characteristics for

electrically short antennas. In making the calculations we

will assume that inductances with a Q of 100 are practically

obtainable. Thus all efficiency considerations will be based

on a Q of 100 for the side loads and end loads.

The electrical characteristics that are desired are the

antenna input impedance and the VSWR at the input.

review, they are an operating frequency at which the reactive part of the input impedance is zero and the real part is large compared to the input resistance of the equivalent center-fed linear antenna. For electrically short, thin, center-fed, linear antennas a fair approximation to the input resistance is given by King [1, page 192] as $20 \beta^2 h^2$. It is also desirable at this operating frequency that both the real and imaginary parts of the input impedance be slowly varying functions of frequency. The price we pay for these characteristics is the power lost in the side loads. We desire to keep this power lower or at least as low as the power that would be lost in any device used to balance out the reactive part of the input impedance of an equivalent center-fed linear antenna. If this can be done, we have an end-fed antenna that radiates equivalent to a center-fed antenna with as good or better efficiency and an operating point where the input impedance is real. The characteristic of slow variation of impedance with frequency would make this operating point easy to maintain in practice.

4.3 Calculations to Illustrate the Properties of the Single Side-Loaded Transmission-Line Antenna

In all calculations which follow in this section and the rest of the chapter the equivalent circuit will be used as a basis for the equations used to evaluate the input impedance and efficiency. In some cases it greatly simplifies the equations and thereby reduces the necessary calculations.

In this section we consider four single side-loaded transmission-line antennas. The first antenna has a short at the

review, they are an operating frequency at which the reactive
 part of the input impedance is zero and the real part is large
 compared to the input resistance of the equivalent circuit.
 linear antenna. For electrically short, thin, center-fed
 linear antenna a fair approximation to the input resistance
 is given by $R_{in} \approx 80 \log \left(\frac{4L}{\lambda} \right)$ ohms. It is also desired
 that at this operating frequency that both the real and
 imaginary parts of the input impedance be slowly varying func-
 tions of frequency. The price we pay for these characteristics
 is the power loss in the side lobes. We desire to keep this
 power loss or at least as low as the power that would be lost
 in any device used to balance out the reactive part of the
 input impedance of an equivalent center-fed linear antenna.
 If this can be done, we have an end-fed antenna that radiates
 equivalent to a center-fed antenna with as good or better
 efficiency and an operating point where the input impedance
 is real. The characteristic of slow variation of impedance
 with frequency would make this operating point easy to maintain
 in practice.

4. Calculations to illustrate the properties of the single side-loaded transmission-line antenna

In all calculations which follow in this section and the
 rest of the chapter the equivalent circuit will be used as a
 basis for the equations used to evaluate the input impedance
 and efficiency. In some cases it greatly simplifies the
 equations and thereby reduces the necessary calculations.
 In this section we consider four single side-loaded trans-
 mission-line antennas. The first is shown in Fig. 1.

end of the antenna, and the side load is adjusted to give a pole in the equivalent impedance Z_{O1} at $\beta h = 1$. We consider three cases for this antenna: (1) the side load located at the center of the antenna such that $\ell_1 = h$; (2) the side load is displaced toward the shorted end of the antenna such that $\ell_1 = 3h/2$; and (3) the side load is displaced toward the generator such that $\ell_1 = h/2$. Referring to Figure 2.4 the equations for the input impedance and efficiency are as follows:

$$X_{T1} = R_c \tan \beta \ell_2 \quad (4.1)$$

$$R_{T1} = 0 \quad (4.2)$$

$$X'_{T1} = \frac{16X_1(X_d^2 + R_d^2) + 4X_d(X_1^2 + R_1^2)}{(4R_d + R_1)^2 + (4X_d + X_1)^2} + X_{T1} \quad (4.3)$$

$$R'_{T1} = \frac{16R_1(X_d^2 + R_d^2) + 4R_d(X_1^2 + R_1^2)}{(4R_d + R_1)^2 + (4X_d + X_1)^2} \quad (4.4)$$

$$X_O = \frac{R_c}{2} \left[\frac{(R_c^2 - X_{T1}^2 - R_{T1}^2) \sin 2\beta \ell_1 + 2X_{T1}R_c \cos 2\beta \ell_1}{[R_c \cos \beta \ell_1 - X'_{T1} \sin \beta \ell_1]^2 + R_{T1}^2 \sin^2 \beta \ell_1} \right] \quad (4.5)$$

$$R_O = \frac{R_c^2 R'_{T1}}{[R_c \cos \beta \ell_1 - X'_{T1} \sin \beta \ell_1]^2 + R_{T1}^2 \sin^2 \beta \ell_1} \quad (4.6)$$

$$\text{Efficiency} = 100 \left[\frac{1}{1 + 4R_1(X_d^2 + R_d^2)/R_d(R_1^2 + X_1^2)} \right] \quad (4.7)$$

From King [1, page 192 and page 404] and Appendix A we evaluate the input impedance of the equivalent radiating linear antennas in the following manner assuming the antennas are electrically short.

$$R_d = \frac{20 \beta^2 \ell_1^2 + 20 \beta^2 \ell_2^2}{2} = 10 \beta^2 (\ell_1^2 + \ell_2^2) \quad (4.8)$$

$$x_d = - \frac{30}{\beta h} \left[2 \ln 2\ell_1/a + 2 \ln 2\ell_2/a - 6.78 \right] . \quad (4.9)$$

For three of the single side loaded antennas we assume a length to conductor radius ratio of $2h/a = 12000$. The ratio of conductor separation to the conductor radius is a constant for all four antennas. Its value is $b/a = 28$. The resulting characteristic impedance of the transmission line of the equivalent circuit is $R_c = 400$ ohms. A typical example of the above ratios is an antenna of the following dimensions: $2h = 1000$ feet, $a = 1$ inch and $b = 2$ feet 4 inches.

The side load Z_1 is evaluated in the following manner.

Set

$$x_1 = - 4x_d \Big|_{\beta h = 1} . \quad (4.10)$$

This makes Z_{o1} antiresonant at $\beta h = 1$. Assuming a Q of 100 at $\beta h = 1$, we find

$$R_1 = \frac{x_1}{100} \Big|_{\beta h = 1} . \quad (4.11)$$

Using the above equations and constants, the curves of Figures 4.1, 4.2, and 4.3 for antenna 1 were calculated using the CRC 102-A computer located in the Research Center at the University of New Mexico. All calculations were performed in this manner.

The second antenna is the same as the first except that the side load is adjusted to give a pole in the equivalent impedance Z_{o1} at $\beta h = 1.6$. We also calculate R_1 at the frequency for which $\beta h = 1.6$. Thus all the equations given above for antenna No. 1 are the same for antenna No. 2 except (4.10 and 4.11) which become

$$X_2 = \frac{1}{2} \left[\ln \frac{2\sqrt{a}}{b} + 2 \ln \frac{2\sqrt{a}}{b} + 0.78 \right] \quad (4.13)$$

For three of the single side loaded antennas we assume a length

to conductor radius ratio of $2\sqrt{a}/b = 1000$. The ratio of

conductor separation to the conductor radius is a constant

for all four antennas. Its value is $b/a = 28$. The resulting

characteristic impedance of the transmission line of the antenna

is $Z_0 = 400 \text{ ohms}$. A typical example of the above

ratios is an antenna of the following dimensions: $a = 100$

feet, $b = 1$ inch and $2\sqrt{a}/b = 2$ feet 4 inches.

The side load Z_1 is evaluated in the following manner:

Set

$$X_1 = \frac{1}{2} \left[\ln \frac{2\sqrt{a}}{b} + 2 \ln \frac{2\sqrt{a}}{b} + 0.78 \right] \quad (4.14)$$

This makes Z_1 antiresonant at $b/a = 1$. Assuming a Q of 100

at $b/a = 1$, we find

$$R_1 = \frac{Z_0}{100} \quad b/a = 1$$

Using the above equations and constants, the curves of figures

4.1, 4.2, and 4.3 for antenna 1 were calculated using the

CRC 102-A computer located in the Research Center at the

University of New Mexico. All calculations were performed in

this manner.

The second antenna is the same as the first except that

the side load is adjusted to give a pole in the equivalent

impedance Z_{01} at $b/a = 1.0$. We also calculate R_1 at the fre-

quency for which $b/a = 1.0$. Thus all the equations given

above for antenna 1 are the same for antenna 2 except

(4.10 and 4.11) which become

Equivalent Linear Antenna

At $\beta h = 1$ $X_1 = -4X_d$

$R_1 = \frac{X_1}{100}$ $\frac{b}{a} = 28$

$d = \sqrt{ab}$ $\frac{2h}{a} = 12000$

Case 1 $l_1 = h$

Case 2 $l_1 = 3h/2$

Case 3 $l_1 = h/2$

Figure 4.1--The input reactance for antenna 1

Figure 4-1--The input resistance for antenna 1

$$\text{At } \beta h \approx 1 \quad X_1 = -4X_d$$

$$R_1 = \frac{X_1}{100} \quad \frac{2h}{a} = 12000$$

$$\frac{b}{a} = 28 \quad d = \sqrt{ab}$$

$$\text{Case 1} \quad l_1 = h$$

$$\text{Case 2} \quad l_1 = 3h/2$$

$$\text{Case 3} \quad l_1 = h/2$$

Figure 4.2--The input resistance for antenna 1

Equivalent circuit diagram showing three cases:

- Case 1: $h = b$
- Case 2: $h = 2b/3$
- Case 3: $h = b/2$

Figure 4.3. The equivalent circuit for the antenna.

$$X_1 = -4X_d \Big|_{\beta h = 1.6} \quad (4.12)$$

$$R_1 = \frac{X_1}{100} \Big|_{\beta h = 1.6} \quad (4.13)$$

For the second antenna we consider only the case $\ell_1 = h$. The curves for antenna No. 2 are plotted in Figures 4.4, 4.5, and 4.6 along with the curves for first antenna case No. 1.

The third antenna is the same as antenna No. 1 except that the side load is adjusted to give a pole in the equivalent impedance Z_{o1} at $\beta h = 1.5$, and Z_2 is now an inductance chosen to produce an antiresonant point in Z_{T1} at $\beta h = 0.8$. We can again use the same equations that were used for antenna No. 1 except that (4.1, 4.2, and 4.7) now have the form

$$X_{T1} = \frac{R_c}{2} \left[\frac{(R_c^2 - X_2^2 - R_2^2) \sin 2\beta\ell_2 + 2X_2 R_c \cos 2\beta\ell_2}{[R_c \cos \beta\ell_2 - X_2 \sin \beta\ell_2]^2 + R_2^2 \sin^2 \beta\ell_2} \right] \quad (4.14)$$

$$R_{T1} = \frac{R_c^2 R_2}{[R_c \cos \beta\ell_2 - X_2 \sin \beta\ell_2]^2 + R_2^2 \sin^2 \beta\ell_2} \quad (4.15)$$

$$\text{Efficiency} = 100 \left[1 - \frac{(R_{o1}^2 + X_{o1}^2) R_1}{(R_1^2 + X_1^2) R_{T1}} - \frac{R_{T1}}{R_{T1}'} \right] \quad (4.16)$$

For the third antenna we calculate only the case where $\ell_1 = h$. The curves for antenna No. 3 are plotted in Figures 4.7, 4.8, and 4.9.

The fourth antenna is the same as the third except that we use a length to conductor radius ratio of $2h/a = 809$. An example of this antenna is one whose length is $2h = 34$ feet, conductor radius is $a = 1/2$ inch and conductor separation is $b = 1$ foot 2 inches. This antenna would not fit into the limitation established in Section 4.2, but it is included to

For the second antenna we consider only the case $\lambda_1 = \lambda_2$. The curves for antenna No. 2 are plotted in Figures 4.5, 4.6, and 4.7 along with the curves for first antenna case No. 1. The third antenna is the same as antenna No. 1 except

that the side load is substituted to give a hole in the equivalent impedance Z_1 at $\lambda_1 = \lambda_2$ and $\lambda_1 = \lambda_2$ as before. To produce an antiresonant point in Z_1 at $\lambda_1 = 0.5$ we can again use the same equations that were used for antenna No. 1 except that $\lambda_1 = 0.5$ and $\lambda_2 = 1$ now have the form

$$Z_1 = \frac{R_1 + jX_1 + \frac{R_2 + jX_2}{1 + j\frac{R_2 + jX_2}{Z_1}}}{1 + j\frac{R_2 + jX_2}{Z_1}} \quad (4.14)$$

$$Z_1 = \frac{R_1 + jX_1 + \frac{R_2 + jX_2}{1 + j\frac{R_2 + jX_2}{Z_1}}}{1 + j\frac{R_2 + jX_2}{Z_1}} \quad (4.15)$$

$$Z_1 = \frac{R_1 + jX_1 + \frac{R_2 + jX_2}{1 + j\frac{R_2 + jX_2}{Z_1}}}{1 + j\frac{R_2 + jX_2}{Z_1}} \quad (4.16)$$

For the third antenna we calculate only the case where $\lambda_1 = \lambda_2$. The curves for antenna No. 3 are plotted in Figures 4.8, 4.9, and 4.10. The fourth antenna is the same as the third except that we use a length to conductor radius ratio of $2b/a = 800$. An example of this antenna is one whose length is $2b = 24$ feet, conductor radius is $a = 1.5$ inch and conductor separation is $b = 1$ foot 2 inches. This antenna would not fit into the limitation established in Section 4.1. It is included to

Antenna 1 Case 1 and Antenna 2

$$\frac{2h}{a} = 12000 \quad \frac{b}{a} = 28 \quad d = \sqrt{ab}$$

Antenna 1 Case 1

$$\text{At } \beta h = 1 \quad X_1 = -4X_d$$

$$R_1 = \frac{X_1}{100}$$

Equivalent Linear Antenna

Antenna 2

$$\text{At } \beta h = 1.6 \quad X_1 = -4X_d$$

$$R_1 = \frac{X_1}{100}$$

Figure 4.4--The input reactance for antenna 1 Case 1 and antenna 2

Figure 4.4--The input resistance for antenna 1 Case 1 and antenna 2

Antenna 1 Case 1 and Antenna 2

$$\frac{2h}{a} = 12000 \quad \frac{b}{a} = 28 \quad d = \sqrt{ab}$$

Antenna 1 Case 1

$$\text{At } \beta h = 1 \quad X_1 = -4X_d \quad R_1 = \frac{X_1}{100}$$

Antenna 2

$$\text{At } \beta h = 1.6 \quad X_1 = -4X_d$$

$$R_1 = \frac{X_1}{100}$$

Figure 4.5--The input resistance for antenna 1 case 1 and antenna 2

Antenna 1 Case 1 and Antenna 2 Case 1

Figure 4.6--The efficiencies for antenna 1 case 1 and antenna 2

$$\frac{Z_p}{Z_0} = 1.000 \quad \frac{p}{s} = 0.8 \quad d = \sqrt{\lambda}$$

Antenna 1 Case 1

$$At \quad Q = 1 \quad X_1 = -4\lambda$$

$$R_1 = \frac{X_1}{100}$$

Antenna 2

$$At \quad p = 1.6 \quad X_1 = -4\lambda$$

$$R_1 = \frac{X_1}{100}$$

Figure 4-5- The characteristics for antenna 1 case 1

and antenna 2

Antenna 3 and Antenna 4

Equivalent Linear Antenna

$$\frac{b}{a} = 28 \quad d = \sqrt{ab}$$

$$\text{At } \beta h = 1.5 \quad X_1 = -4X_d$$

$$R_1 = \frac{X_1}{100}$$

$$\text{At } \beta h = 0.8 \quad X_2 = R_c \cot \beta h$$

$$R_2 = \frac{X_2}{100}$$

$$\text{Antenna 3} \quad \frac{2h}{a} = 12000$$

$$\text{Antenna 4} \quad \frac{2h}{a} = 809$$

Figure 4.7--The input reactance for antennas 3 and 4

Antenna 3 and Antenna 4

Figure 4-7- The input reactance for antennas 3 and 4

Antenna 3 and Antenna 4

Equivalent Linear Antenna

$$\frac{b}{a} = 28 \quad d = \sqrt{ab}$$

$$\text{At } \beta h = 1.5 \quad X_1 = -4X_d$$

$$R_1 = \frac{X_1}{100}$$

$$\text{At } \beta h = 0.8 \quad X_2 = R_c \cot \beta h$$

$$R_2 = \frac{X_2}{100}$$

$$\text{Antenna 3} \quad \frac{2h}{a} = 12000$$

$$\text{Antenna 4} \quad \frac{2h}{a} = 809$$

Figure 4.8--The input resistance for antennas 3 and 4

Equivalent linear antenna

Antenna 3: $\frac{Z_0}{a} = 1200$

Antenna 4: $\frac{Z_0}{a} = 800$

At $h=0.8$, $X_2 \approx R_0$ corner

At $h=1.2$, $X_1 \approx R_0$

$R_1 = 100$

$R_2 = 100$

$\frac{p}{8} = 28$, $d = \sqrt{ab}$

The diagram shows a linear antenna with a total impedance Z_0 at the input. It is divided into two sections with impedances Z_1 and Z_2 . A parameter p is associated with the coupling between the sections. The diagram also shows a load impedance Z_d and a distance h from the input to the load.

Antenna 3 and Antenna 4

$$\frac{b}{a} = 28 \quad d = \sqrt{ab}$$

$$\text{At } \beta h = 1.5 \quad X_1 = -4X_d$$

$$R_1 = \frac{X_1}{100}$$

$$\text{At } \beta h = 0.8 \quad X_2 = R_c \cot \beta h$$

$$R_2 = \frac{X_2}{100}$$

Equivalent Linear Antenna

$$\text{Antenna 3} \quad \frac{2h}{a} = 12000$$

$$\text{Antenna 4} \quad \frac{2h}{a} = 809$$

$$Z_{01} = \frac{4Z_d Z_1}{4Z_d + Z_1}$$

Figure 4.9--The efficiencies for antennas 3 and 4

illustrate the effect of changing the length to conductor radius ratio. All the equations are the same as for antenna three, and the curves are plotted in Figures 4.7, 4.8, and 4.9.

The values of the real part of the equivalent radiating linear antenna, R_d , are also plotted in Figures 4.2, 4.5, and 4.8 for comparison.

4.4 Discussion of the Properties of Single Side-Loaded Transmission-Line Antennas

Figures 4.1 through 4.9 give a general picture of the input impedances and efficiencies that can be obtained using single side-loaded transmission-line antennas. They are not necessarily the optimum results since this will depend on the Q of available side loads and the electrical length of the antenna being considered. They do, however, indicate the way in which the input impedance curves can be varied by varying the side load and end load impedances and the location of the side load impedance.

Figures 4.1 through 4.6 illustrate the properties of the input impedance of a single side-loaded antenna with the end load shorted. Within the range $\beta h < 1$ the input impedance has a single antiresonant point. Using this arrangement, it is possible to increase the value of the real part of the input impedance over that of the equivalent linear antenna, R_d , for certain frequency ranges, but the imaginary part of the input impedance is large in the same ranges. Antenna efficiencies ranging from 10% to 20% are possible in these same ranges using coils with a Q of 100. Also illustrated by Figures 4.1 through

illustrate the effect of changing the length of conductor
feeding points. The curves are the same as for antennas
three, and the curves are plotted in Figures 4.7, 4.8, and 4.9.
The values of the real part of the equivalent radiating
linear antenna, R_e , are also plotted in Figures 4.7, 4.8, and
4.9 for comparison.

4.4 Discussion of the Properties of Single-Side-Loaded
Transmission-Line Antennas

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input impedances and efficiency factors that can be obtained using
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varying the side load and load impedance and the location
of the side load impedance. Figures 4.1 through 4.9 illustrate the properties of the
input impedance of a single side-loaded antenna with the end
load shorted. Within the range $0 < l$ the input impedance has
a single antiresonant point. Using this arrangement, it is
possible to increase the value of the real part of the input
impedance over that of the equivalent linear antenna, R_e , for
certain frequency ranges, but the imaginary part of the input
impedance is large in the same ranges. Antenna efficiencies
ranging from 10% to 100% are possible in these same ranges using
coils with a Q of 100. Also illustrated by Figures 4.1 through

4.6 is the fact that the location of the antiresonant point can be shifted either by varying the side load impedance or by varying its location on the transmission line.

Figures 4.7 through 4.9 illustrate the double antiresonant input impedance that is obtained by adjusting the end load impedance to the proper value. For the particular cases illustrated there is a considerable frequency range ($0.7 \leq \beta h \leq 0.92$) where the input reactance is zero or near zero and the input resistance is about the same magnitude as the input resistance of the equivalent linear antenna, R_d . The variation of both the real and imaginary parts of the input impedance are relatively small with frequency also. Efficiencies in this range run from 2% to 10% for side load impedances with a Q of 100. It can also be seen from observing Figures 4.7 through 4.9 that the rather large change in the length to conductor radius ratio from antenna 3 to antenna 4 has very little effect on the location of the antiresonant points or the shape of the curves.

4.5 Some Properties of Double Side-Loaded Transmission-Line Antennas Derived from Approximations of the Theoretical Equations

Before going into detailed calculations on double side-loaded transmission-line antennas we obtain a rough picture of what to expect by considering some approximations of the theoretical equations developed in Chapter 3 and Appendix B. As mentioned in Section 3.5 of Chapter 3, the multiply side-loaded transmission-line antennas form coupled systems, and

4.5 is the fact that the location of the antiresonant point can be shifted either by varying the side load impedance or by varying the location on the transmission line.

Figures 4.7 through 4.9 illustrate the double antiresonant input impedance that is obtained by adjusting the end load impedance to the proper value. For the particular cases illustrated there is a considerable frequency range $0.5 < \omega/\omega_0 \leq 0.93$ where the input impedance is near or near zero and the input resistance is about the same magnitude as the input resistance of the equivalent linear antenna. The variations of both the real and imaginary parts of the input impedance are relatively small with frequency also. Efficiencies in this range run from 15 to 10% for side load impedances with a Q of 100. It can also be seen from observing Figures 4.7 through 4.9 that the rather large change in the length to conductor radius ratio from antenna 3 to antenna 4 has very little effect on the location of the antiresonant point or the shape of the curves.

4.5 Some Properties of Double Side-Loaded Transmission-Line Antennas Derived from Approximations of the Theoretical Equations

Before going into detailed calculations on double side-loaded transmission-line antennas we obtain a rough picture of what to expect by considering some approximations of the theoretical equations developed in Chapter 3 and Appendix B. As mentioned in Section 3.5 of Chapter 3, the multiply side-loaded transmission-line antenna form coupled systems, and

therefore gaining a rough picture of the double side-loaded transmission-line antenna will not be so straight forward as it was for the single side-loaded antenna.

Figure 4.10 illustrates a double side-loaded transmission-line antenna along with its equivalent circuit. For this antenna both side loads are on the same side and the end load $Z_3 = 0$. To simplify the calculations we set $\ell_1 = h - \ell$ and $\ell_2 = h + \ell$. This locates the two side loads symmetrically with respect to the center of the antenna. As a result of this choice $Z_{da1} = Z_{da2} = Z_{da}$, and B.13 becomes

$$Z_{d1} = \frac{Z_{da}}{1 + \frac{V_2}{V_1} \frac{\sin \beta(h-\ell)}{\sin \beta(h+\ell)}} \quad (4.17)$$

$$Z_{d2} = \frac{Z_{da}}{1 + \frac{V_1}{V_2} \frac{\sin \beta(h-\ell)}{\sin \beta(h+\ell)}} \quad (4.18)$$

Combining (3.30, 3.43, 4.17, and 4.18) and performing a considerable amount of algebra, we find

$$\frac{V_2}{V_1} = \frac{\gamma_2 Z_2}{\gamma_1 Z_1} = \frac{\left[\frac{4Z_{da}}{Z_1} + 1 \right] - qr}{r \left[\frac{4Z_{da}}{Z_2} + 1 \right] - q + jB}, \quad (4.19)$$

where

$$q = \frac{\sin \beta(h-\ell)}{\sin \beta(h+\ell)} \quad (4.20)$$

$$r = \frac{\cos \beta(h+\ell)}{\cos \beta(h-\ell)} \quad (4.21)$$

$$B = \frac{4Z_d}{R_c} \sin 2\beta\ell. \quad (4.22)$$

Using (4.17, 4.18, 4.19) and the equivalent circuit, we obtain

Therefore, assuming a single plane of the double side-loaded

transmission-line antenna will not be as straightforward as

it was for the single side-loaded antenna.

Figure 4.10 illustrates a double side-loaded transmission-

line antenna along with its equivalent circuit. For this

antenna both side loads are on the same side and the end load

$Z_L = 0$. To simplify the calculations we set $Z_1 = Z_2 = Z_L$ and

$Z_3 = Z_4$. This locates the two side loads symmetrically with

respect to the center of the antenna. As a result of this

choice $Z_1 = Z_2 = Z_L$ and $Z_3 = Z_4 = Z_0$.

$$Z_{in} = \frac{Z_0 \left[\frac{Z_L \sin \beta l}{\cos \beta l} + j Z_0 \tan \beta l \right]}{Z_0 \left[\frac{Z_L \sin \beta l}{\cos \beta l} + j Z_0 \tan \beta l \right] + Z_0 \left[\frac{Z_L \sin \beta l}{\cos \beta l} + j Z_0 \tan \beta l \right]} \quad (4.17)$$

$$Z_{in} = \frac{Z_0 \left[\frac{Z_L \sin \beta l}{\cos \beta l} + j Z_0 \tan \beta l \right]}{Z_0 \left[\frac{Z_L \sin \beta l}{\cos \beta l} + j Z_0 \tan \beta l \right] + Z_0 \left[\frac{Z_L \sin \beta l}{\cos \beta l} + j Z_0 \tan \beta l \right]} \quad (4.18)$$

Combining (4.17), (4.18), and (4.19) and performing a com-

parable amount of algebra, we find

$$Z_{in} = \frac{Z_0 \left[\frac{Z_L \sin \beta l}{\cos \beta l} + j Z_0 \tan \beta l \right]}{Z_0 \left[\frac{Z_L \sin \beta l}{\cos \beta l} + j Z_0 \tan \beta l \right] + Z_0 \left[\frac{Z_L \sin \beta l}{\cos \beta l} + j Z_0 \tan \beta l \right]} \quad (4.19)$$

where

$$d = \frac{\sin \beta(h-l)}{\sin \beta h} \quad (4.20)$$

$$e = \frac{\cos \beta(h-l)}{\cos \beta h} \quad (4.21)$$

$$f = \frac{\sin \beta h}{\sin \beta(h-l)} \quad (4.22)$$

Using (4.17), (4.18), and the equivalent circuit, we obtain

Figure 4.10--Double Side-loaded transmission-line antenna and the equivalent circuit (both side loads on the same side)

Figure 1. Double antenna system. (a) Double antenna system. (b) Double antenna system with mutual inductance. (c) Double antenna system with mutual inductance and a voltage source.

$$Z_{O1} = \frac{4Z_d \left\{ r \left[\frac{4Z_{da}}{Z_2} + 1 \right] - q + jB \right\}}{r \left[\frac{4Z_{da}}{Z_1} + 1 \right] \left[\frac{4Z_{da}}{Z_2} + 1 \right] + jB \left[\frac{4Z_{da}}{Z_1} + 1 \right] - q^2 r} \quad (4.23)$$

$$Z_{T1} = jR_c \frac{\left[\frac{4Z_{da}}{Z_1} + 1 \right] \left\{ q \left[\frac{4Z_{da}}{Z_2} + 1 \right] - jA \right\} - q^2 s + j \frac{4Z_{da}}{R_c} q}{r \left[\frac{4Z_{da}}{Z_1} + 1 \right] \left[\frac{4Z_{da}}{Z_2} + 1 \right] + jB \left[\frac{4Z_{da}}{Z_1} + 1 \right] - q^2 r} \quad (4.24)$$

$$Z'_{T1} = \frac{4Z_d \left\{ r \left[\frac{4Z_{da}}{Z_2} + 1 \right] + \cos 2\beta \ell \left[\frac{4Z_{da}}{Z_1} + 1 \right] - 2q \right\}}{r \left[\frac{4Z_{da}}{Z_1} + 1 \right] \left[\frac{4Z_{da}}{Z_2} + 1 \right] + jB \left[\frac{4Z_{da}}{Z_1} + 1 \right] - q^2 r} \quad (4.25)$$

$$+ \frac{jR_c \left\{ s \left(\left[\frac{4Z_{da}}{Z_1} + 1 \right] \left[\frac{4Z_{da}}{Z_2} + 1 \right] - q^2 \right) + \frac{4Z_{da} B}{R_c} \right\}}{r \left[\frac{4Z_{da}}{Z_1} + 1 \right] \left[\frac{4Z_{da}}{Z_2} + 1 \right] + jB \left[\frac{4Z_{da}}{Z_1} + 1 \right] - q^2 r},$$

where

$$A = \frac{4Z_{da}}{R_c} \cos 2\beta \ell \quad (4.26)$$

$$S = \frac{\sin \beta(h+\ell)}{\cos \beta(h-\ell)} \quad (4.27)$$

Observing (4.23 and 4.24), we see the result of the two side loads being a coupled system. The denominators of these two expressions are identical. This means that the poles of the input impedance are not produced partly by one side load and partly by the other side, but rather all the poles are produced by the system as a whole. To get some idea of the general nature of this impedance we make the following definitions and approximations.

(4.25)

$$\begin{bmatrix} \frac{1}{s} & 0 \\ 0 & \frac{1}{s} \end{bmatrix} \begin{bmatrix} X_1(s) \\ X_2(s) \end{bmatrix} = \begin{bmatrix} \frac{1}{s} & 0 \\ 0 & \frac{1}{s} \end{bmatrix} \begin{bmatrix} X_1(s) \\ X_2(s) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \frac{1}{s}$$

(4.26)

$$\begin{bmatrix} \frac{1}{s} & 0 \\ 0 & \frac{1}{s} \end{bmatrix} \begin{bmatrix} X_1(s) \\ X_2(s) \end{bmatrix} = \begin{bmatrix} \frac{1}{s} & 0 \\ 0 & \frac{1}{s} \end{bmatrix} \begin{bmatrix} X_1(s) \\ X_2(s) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \frac{1}{s}$$

(4.27)

$$\begin{bmatrix} \frac{1}{s} & 0 \\ 0 & \frac{1}{s} \end{bmatrix} \begin{bmatrix} X_1(s) \\ X_2(s) \end{bmatrix} = \begin{bmatrix} \frac{1}{s} & 0 \\ 0 & \frac{1}{s} \end{bmatrix} \begin{bmatrix} X_1(s) \\ X_2(s) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \frac{1}{s}$$

(4.28)

$$\begin{bmatrix} \frac{1}{s} & 0 \\ 0 & \frac{1}{s} \end{bmatrix} \begin{bmatrix} X_1(s) \\ X_2(s) \end{bmatrix} = \begin{bmatrix} \frac{1}{s} & 0 \\ 0 & \frac{1}{s} \end{bmatrix} \begin{bmatrix} X_1(s) \\ X_2(s) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \frac{1}{s}$$

(4.29)

$$\begin{bmatrix} \frac{1}{s} & 0 \\ 0 & \frac{1}{s} \end{bmatrix} \begin{bmatrix} X_1(s) \\ X_2(s) \end{bmatrix} = \begin{bmatrix} \frac{1}{s} & 0 \\ 0 & \frac{1}{s} \end{bmatrix} \begin{bmatrix} X_1(s) \\ X_2(s) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \frac{1}{s}$$

Observing (4.25) and (4.27), we see the result of the two sides loads being a coupled system. The denominator of these two expressions are identical. This means that the poles of the input impedances are not produced partly by one side load and partly by the other side, but rather all the poles are produced by the system as a whole. To get some idea of the general nature of this impedance we make the following definitions and approximations:

$$A = \frac{1}{s} \cos \theta$$

$$\ell = \frac{p}{h} \quad (4.28)$$

$$r \approx 1; s \approx \beta h(1+p); q \approx \frac{1-p}{1+p}; \cos 2\beta\ell \approx 1; \sin 2\beta\ell \approx 2\beta hp \quad (4.29)$$

$$Z_{da} = jX_d = -j \frac{K_{da}}{\beta h} \quad (4.30)$$

$$Z_1 = jX_1 = jK_1 \beta h \quad (4.31)$$

$$Z_2 = jX_2 = jK_2 \beta h \quad (4.32)$$

Substituting (4.28) through (4.32) in (4.25), the result is

$$X'_{T1} = R_c \beta h \left[\frac{\frac{4p}{1+p} \beta^4 h^4 - [2pc^2 + (1+p)(g+e) + \frac{4cp}{1+p}] \beta^2 h^2 + c(g+e) + (1+p)ge}{\left[\frac{4p}{(1+p)^2} + 2pc \right] \beta^4 h^4 - [g+e+2pgc] \beta^2 h^2 + ge} \right], \quad (4.33)$$

where

$$g = \frac{4K_{da}}{K_1}, \quad e = \frac{4K_{da}}{K_2} \quad \text{and} \quad c = \frac{4K_{da}}{R_c}. \quad (4.34)$$

The poles of (4.33) are located at

$$\beta^2 h^2 = \frac{g+e+2pgc \pm \sqrt{[g+e+2pgc]^2 - 4ge \left[\frac{4p}{(1+p)^2} + 2pc \right]}}{2 \left[\frac{4p}{(1+p)^2} + 2pc \right]}. \quad (4.35)$$

To insure positive real distinct roots for (4.35) we must have

$$R = [g+e+2pgc]^2 - 4ge \left[\frac{4p}{(1+p)^2} + 2pc \right] > 0. \quad (4.36)$$

To prove that this is so, consider the following reasonings:

$$R = g^2 + \left[2 - \frac{16p}{(1+p)^2} \right] ge + e^2 + 4pgc(g-e) + 4p^2 g^2 c^2$$

$$R > (g-e)^2 + 4pgc(g-e) + 4p^2 g^2 c^2$$

since $\frac{16p}{(1+p)^2} < 4$ for $0 < p < 1$ thus

$$R > [g-e + 2pgc]^2 \geq 0. \quad (4.37)$$

(4.28)

$$e = \frac{1}{1+p} \cos \theta \quad \text{and} \quad \sin \theta = \frac{1}{1+p} \sin \theta \quad (4.29)$$

(4.30)

(4.31)

(4.32)

Substituting (4.28) through (4.32) in (4.25), the result is

$$\frac{1}{(1+p)^2} \left[\frac{1}{2} \left(\frac{1}{1+p} \right)^2 \left(\frac{1}{1+p} \right)^2 \left(\frac{1}{1+p} \right)^2 \right] \quad (4.33)$$

where

$$\frac{1}{1+p} = \frac{1}{1+p} \quad \text{and} \quad \frac{1}{1+p} = \frac{1}{1+p} \quad (4.34)$$

The poles of (4.33) are located at

$$\frac{1}{1+p} = \frac{1}{1+p} \quad (4.35)$$

To insure positive real distinct roots for (4.35), we must have

$$R = \left[\frac{1}{1+p} \right] > 0 \quad (4.36)$$

To prove that this is so, consider the following reasoning:

$$R = \frac{1}{1+p} \quad (4.37)$$

Although the above is a good indication that we can expect two distinct poles and two distinct zeroes in Z_{T1}' and therefore in Z_o for the antenna of Figure 4.5, it is not a real proof since quite a number of rough approximations are involved.

Before presenting the computer calculations we consider one more double side-loaded antenna approximately. Figure 4.11 illustrates a double side-loaded transmission-line antenna with one side load on each side and the end load $Z_3 = 0$. Again to simplify the calculations we set $\ell_1 = h$ and $\ell_2 = \ell_1$. This locates the two side loads at the center of the antenna directly opposite each other. As a result of this choice $Z_{da1} = Z_{da2} = Z_d$, and (B.22 and B.23) become

$$Z_{d1} = \frac{Z_d}{1 - \frac{V_2}{V_1}} \quad (4.38)$$

$$Z_{d2} = \frac{Z_d}{1 - \frac{V_1}{V_2}} \quad (4.39)$$

Combining (3.30, 3.43, 4.38, and 4.39) and performing a considerable amount of algebra, we find

$$\frac{V_2}{V_1} = \frac{Z_2(2Z_d + Z_1)}{Z_1(2Z_d + Z_2)} \quad (4.40)$$

Using (4.38, 4.39, and 4.40) and the equivalent circuit, we obtain

$$Z_{o1} = \frac{2Z_1(2Z_d + Z_2)}{4Z_d + Z_1 + Z_2} \quad (4.41)$$

$$Z_{o2} = \frac{2Z_2(2Z_d + Z_1)}{4Z_d + Z_1 + Z_2} \quad (4.42)$$

Although the above is a good indication of the general nature of the problem, it is not sufficient for the purpose of this paper. In order to obtain a more detailed picture of the problem, it is necessary to consider the problem in more detail. The problem is to find the values of the parameters α_1 and α_2 such that the system is stable. The system is stable if the characteristic equation has all its roots in the left half of the complex plane. The characteristic equation is given by

$$s^2 + \alpha_1 s + \alpha_2 = 0$$

where s is the complex frequency. The roots of this equation are given by

$$s = \frac{-\alpha_1 \pm \sqrt{\alpha_1^2 - 4\alpha_2}}{2}$$

For the system to be stable, the real part of both roots must be negative. This requires that

$$\alpha_1 > 0 \quad \text{and} \quad \alpha_2 > 0$$

and that the discriminant is non-negative, i.e.,

$$\alpha_1^2 - 4\alpha_2 \geq 0$$

which can be written as

$$\alpha_2 \leq \frac{\alpha_1^2}{4}$$

Thus, the system is stable if and only if $\alpha_1 > 0$, $\alpha_2 > 0$, and $\alpha_2 \leq \frac{\alpha_1^2}{4}$.

$$\alpha_1 = \frac{2\alpha_2}{\alpha_1^2 - 4\alpha_2}$$

$$\alpha_2 = \frac{2\alpha_1}{\alpha_1^2 - 4\alpha_2}$$

Combining (1.30) and (1.31) and using (1.29), we find that the system is stable if and only if

$$\alpha_1 > 0 \quad \text{and} \quad \alpha_2 > 0 \quad \text{and} \quad \alpha_2 \leq \frac{\alpha_1^2}{4}$$

which can be written as

$$\alpha_2 \leq \frac{\alpha_1^2}{4}$$

Using (1.28), (1.29), and (1.30), the characteristic equation can be written as

$$s^2 + \alpha_1 s + \alpha_2 = 0$$

where s is the complex frequency. The roots of this equation are given by

$$s = \frac{-\alpha_1 \pm \sqrt{\alpha_1^2 - 4\alpha_2}}{2}$$

For the system to be stable, the real part of both roots must be negative. This requires that

$$\alpha_1 > 0 \quad \text{and} \quad \alpha_2 > 0$$

and that the discriminant is non-negative, i.e.,

$$\alpha_1^2 - 4\alpha_2 \geq 0$$

which can be written as

$$\alpha_2 \leq \frac{\alpha_1^2}{4}$$

Thus, the system is stable if and only if $\alpha_1 > 0$, $\alpha_2 > 0$, and $\alpha_2 \leq \frac{\alpha_1^2}{4}$.

Figure 4.11--Double side-loaded transmission-line antenna and the equivalent circuit (side loads on opposite sides)

Figure 4.11 - Double side-loaded transmission-line antenna and the equivalent circuit for side loads on opposite sides

$$Z'_{T1} = \frac{4Z_d(Z_1 + Z_2) + 4Z_1Z_2}{4Z_d + Z_1 + Z_2} + Z_{T1} . \quad (4.43)$$

The identical denominators of (4.41) and (4.42) again indicate the fact that the side loads are coupled together. To get a rough picture of the way Z'_{T1} varies with βh we make the following approximations.

$$Z_d = jX_d = -j\frac{K_d}{\beta h} \quad (4.44)$$

$$Z_1 = jX_1 = jK_1\beta h \quad (4.45)$$

$$Z_2 = jX_2 = jK_2\beta h . \quad (4.46)$$

Substituting (4.44, 4.45, and 4.46) in (4.34), the result is

$$X'_{T1} = \frac{4K_1K_2}{K_1+K_2} \beta h \left[\frac{\beta^2 h^2 - K_d(K_1+K_2)/K_1K_2}{\beta^2 h^2 - 4K_d/K_1+K_2} \right] + R_c \tan \beta h . \quad (4.47)$$

Since $\frac{4K_d}{K_1+K_2} > 0$, $\frac{K_d(K_1+K_2)}{K_1K_2} > 0$ and

$$\frac{\frac{K_d(K_1+K_2)}{K_1K_2}}{\frac{4K_d}{K_1+K_2}} = \frac{(K_1+K_2)^2}{4K_1K_2} = \frac{K_1^2 + K_2^2}{4K_1K_2} + \frac{1}{2} > 1 \quad (4.48)$$

as long as $K_1 \neq K_2$, we have proven the existence of one distinct zero and one distinct pole in X'_{T1} apart from $R_c \tan \beta h$. (4.48) also shows that the zero occurs at a higher frequency than the pole. The result of translating an impedance of this nature down the transmission line towards the generator is to produce another pole in the impedance seen at the generator. Thus we have shown that the antenna of Figure 4.6 will have two poles in its input impedance function. The proof is, of course,

$$(4.43)$$

The identical denominators of (4.41) and (4.42) again indicate the fact that the side loads are coupled together. To get a rough picture of the way Z_{II} varies with β we make the fol-

lowing approximations

$$\begin{aligned} Z_{II} &\approx Z_{II}^0 = \frac{K_B}{\beta} \\ Z_{II} &\approx Z_{II}^0 = \frac{K_B}{\beta} \\ Z_{II} &\approx Z_{II}^0 = \frac{K_B}{\beta} \end{aligned} \quad (4.44) \quad (4.45) \quad (4.46)$$

Substituting (4.44), (4.45), and (4.46) in (4.43) the result is

$$(4.47) \quad \left[\frac{K_B^2}{\beta^2} - \frac{K_B^2}{\beta^2} \right] \frac{K_B^2}{\beta^2} = \frac{K_B^2}{\beta^2} \frac{K_B^2}{\beta^2}$$

Since $\frac{K_B^2}{\beta^2} > 0$ and $\frac{K_B^2}{\beta^2} > 0$ and

$$(4.48) \quad \frac{K_B^2}{\beta^2} = \frac{K_B^2}{\beta^2} \frac{K_B^2}{\beta^2} = \frac{K_B^2}{\beta^2} \frac{K_B^2}{\beta^2}$$

as long as $K_B \neq 0$, we have proven the existence of one distinct

zero and one distinct pole in K_B apart from K_B can be (4.48)

also shows that the zero occurs at a higher frequency than the

poles. The result of transmitting an impedance of this nature

down the transmission line towards the generator is to produce

another pole in the impedance seen at the generator. Thus we

have shown that the input of Figure 4.3 will have two poles.

in its input impedance function. The proof is, of course,

based on the approximation of no loss in the system, but this is a rather good approximation for electrically short antennas. Having obtained good indication that we can obtain two distinct poles in the input impedance of an electrically short double side-loaded transmission-line antenna, we now present some detailed calculations performed on the CRC 102-A digital computer.

4.6 Calculations to Illustrate the Properties of the Double Side-Loaded Transmission-Line Antenna

In this section we consider four double side-loaded antennas. All four have the end load shorted. Antenna No. 5 has two side loads symmetrically located ($\ell_1 = h - \ell$, $\ell_2 = h + \ell$) with respect to the center of the antenna, and both are located on the same side of the antenna. The two side loads are equal and their value is adjusted so that the high frequency pole is at $\beta h = 1$, according to (4.35). The side loads are located such that $\ell = 3h/8$. Two other cases are also calculated and plotted. For case two $\ell = h/4$, and for case three $\ell = h/8$. The length to conductor radius ratio is $2h/a = 12000$, and the conductor separation to conductor radius ratio is $b/a = 28$. The resulting value for R_c is 400 ohms. Referring to Figure 4.5, the equations for the input impedance and efficiency are as follows:

$$X_{T2} = R_c \tan \beta(h - \ell) \quad (4.49)$$

$$R_{T2} = 0 \quad (4.50)$$

$$X'_{Tn} = X_{on} + X_{Tn} \quad R'_{Tn} = R_{on} + R_{Tn} \quad (4.51)$$

$$R_{on} = \frac{16R_n(R_{dn}^2 + X_{dn}^2) + 4R_{dn}(R_n^2 + X_n^2)}{(R_n + 4R_{dn})^2 + (X_n + 4X_{dn})^2}$$

based on the approximation of no loss in the system, but this is a rather good approximation for electrically short antennas. Having obtained good indication that we can obtain two distinct poles in the input impedance of an electrically short double side-loaded transmission-line antenna, we now present some of the calculations performed on the CMC 102-A digital computer.

4.6 Calculations to illustrate the properties of the double side-loaded transmission-line antenna

In this section we consider four double side-loaded antennas. All four have the end load shorted. Antenna No. 1 has two side loads symmetrically located ($L_1 = h - L_2 = h/2$) with respect to the center of the antenna, and both are located on the same side of the antenna. The two side loads are equal and their value is adjusted so that the high frequency pole is at $\beta h = 1.57$ according to (4.35). The side loads are located such that $L_1 = 3\lambda/8$. Two other cases are also calculated and plotted. For case two $L_1 = h/4$, and for case three $L_1 = h/8$. The length to conductor radius ratio is $2h/a = 12000$, and the conductor separation to conductor radius ratio is $b/a = 28$. The resulting value for R_0 is 400 ohms. Referring to Figure 4.5, the equations for the input impedance and efficiency are as follows:

$$Z_{in} = R_0 \tan^2(\beta h - L_1) + jX_{in}$$
$$R_{in} = R_0 \tan^2(\beta h - L_1) + R_{on} + R_{in}$$
$$X_{in} = X_0 \tan^2(\beta h - L_1) + X_{on} + X_{in}$$
$$R_{on} = \frac{R_0 (R_{in}^2 + X_{in}^2) + R_{in} (R_0^2 + X_0^2) + X_{in} (R_0^2 + X_0^2)}{(R_0 + R_{in})^2 + (X_0 + X_{in})^2}$$
$$X_{on} = \frac{X_0 (R_{in}^2 + X_{in}^2) + R_{in} (R_0^2 + X_0^2) + R_0 (R_{in}^2 + X_{in}^2)}{(R_0 + R_{in})^2 + (X_0 + X_{in})^2}$$

$$X_{on} = \frac{16X_n(R_{dn}^2 + X_{dn}^2) + 4X_{dn}(R_n^2 + X_n^2)}{(R_n + 4R_{dn})^2 + (X_n + 4X_{dn})^2} \quad (4.52)$$

$$X_{Tn} = \frac{R_c}{2} \left[\frac{(R_c^2 - X_{Tn+1}^2 - R_{Tn+1}^2) \sin 2\beta(\ell_{n+1} - \ell_n) + 2X_{Tn+1} R_c \cos 2\beta(\ell_{n+1} - \ell_n)}{[R_c \cos \beta(\ell_{n+1} - \ell_n) - X_{Tn+1} \sin \beta(\ell_{n+1} - \ell_n)]^2 + R_{Tn+1}^2 \sin^2 \beta(\ell_{n+1} - \ell_n)} \right] \quad (4.53)$$

$$R_{Tn} = \frac{R_{Tn+1}^2 R_o^2}{[R_c \cos \beta(\ell_{n+1} - \ell_n) - X_{Tn+1} \sin \beta(\ell_{n+1} - \ell_n)]^2 + R_{Tn+1}^2 \sin^2 \beta(\ell_{n+1} - \ell_n)} \quad (4.54)$$

for $n = 0, 1$ where $X_{To} = X_o$, $R_{To} = R_o$ and $\ell_o = 0$.

$$\text{Efficiency} = 100 \left[1 - \frac{(R_{o1}^2 + X_{o1}^2) R_1}{(R_1^2 + X_1^2) R_{T1}} - \frac{(R_{o2}^2 + X_{o2}^2) R_2}{(R_2^2 + X_2^2) R_{T2}} \frac{R_{T1}}{R_{T1}} \right] \quad (4.55)$$

From King [1, page 192, page 404], Appendix B and equations (4.17, 4.18, and 4.19) we evaluate Z_{d1} and Z_{d2} in the following manner.

$$R_{da} = \frac{20 \beta^2 (h - \ell)^2 + 20 \beta^2 (h + \ell)^2}{2} = 20 \beta^2 (h^2 + \ell^2) \quad (4.56)$$

$$X_{da} = - \frac{30}{\beta h} \left[2 \ln \frac{2(h + \ell)}{a} + 2 \ln \frac{2(h - \ell)}{a} - 6.78 \right] \quad (4.57)$$

$$\frac{4Z_{da}}{Z_n} = A_{nR} + jA_{nI}; \quad A_{nR} = \frac{4R_{da} R_n + 4X_{da} X_n}{R_n^2 + X_n^2}; \quad A_{nI} = \frac{4X_{da} R_n - 4R_{da} X_n}{R_n^2 + X_n^2} \quad (4.58)$$

for $n = 1, 2$.

$$N_R = 1 + A_{1R} - \frac{\sin \beta(h - \ell) \cos \beta(h + \ell)}{\sin \beta(h + \ell) \cos \beta(h - \ell)} \quad N_I = A_{1I} \quad (4.59)$$

$$D_R = \frac{\cos \beta(h + \ell)}{\cos \beta(h - \ell)} \left[1 + A_{2R} \right] - \frac{\sin \beta(h - \ell)}{\sin \beta(h + \ell)} - \frac{4X_{da}}{R_c} \sin 2\beta \ell \quad (4.60)$$

$$D_I = A_{2I} \frac{\cos \beta(h + \ell)}{\cos \beta(h - \ell)} + \frac{4R_{da}}{R_c} \sin 2\beta \ell \quad (4.61)$$

$$\frac{V_2}{V_1} = A_{3R} + jA_{3I}; \quad A_{3R} = \frac{N_R D_R + N_I D_I}{D_R^2 + D_I^2}; \quad A_{3I} = \frac{N_I D_R - N_R D_I}{D_R^2 + D_I^2} \quad (4.62)$$

$$\frac{V_1}{V_2} = A_{4R} + jA_{4I}; \quad A_{4R} = \frac{A_{3R}}{A_{3R}^2 + A_{3I}^2}; \quad A_{4I} = -\frac{A_{3I}}{A_{3R}^2 + A_{3I}^2} \quad (4.63)$$

$$R_{dn} = \frac{R_{da}(qA_{n+1,R}^{+1}) + qX_{da}A_{n+1,I}}{(qA_{n+1,R}^{+1})^2 + q^2A_{n+1,I}^2} \quad (4.64)$$

$$X_{dn} = \frac{X_{da}(qA_{n+1,R}^{+1}) - qR_{da}A_{n+1,I}}{(qA_{n+1,R}^{+1})^2 + q^2A_{n+1,I}^2} \quad (4.65)$$

for $n = 1, 2$ where q is defined by (4.20). The side loads Z_1 and Z_2 are evaluated in the following manner. Referring to (4.28, 4.34, and 4.35), we find

$$\beta^2 h^2 = g \left[\frac{1 + pc + \sqrt{(1+pc)^2 - \left[\frac{4p}{(1+p)^2} + 2pc \right]}}{\frac{4p}{(1+p)^2} + 2pc} \right] = 1 \quad (4.66)$$

Solving (4.66) for "g", we then find X_1 and X_2 from

$$X_1 = X_2 = -\frac{4X_{da}}{g} \Big|_{\beta h=1} \quad (4.67)$$

Assuming a Q of 100 at $\beta h = 1$, we find

$$R_1 = R_2 = \frac{X_1}{100} \Big|_{\beta h=1} \quad (4.68)$$

Using the above equations and constants, the curves of Figures 4.12, 4.13, and 4.14 were computed for the three cases of antenna 5.

Antenna 6 is the same as antenna 5 except that we use a conductor separation to conductor radius ratio of $b/a = 150$. The resulting value of R_c is 600 ohms. We calculate only case 2 using the same equations given above for antenna 5.

$$\frac{V_2}{V_1} = \frac{A_2 - jA_1}{A_1 - jA_2} = \frac{A_2 - jA_1}{A_1 - jA_2} \quad (4.52)$$

$$\frac{V_2}{V_1} = \frac{A_2 - jA_1}{A_1 - jA_2} = \frac{A_2 - jA_1}{A_1 - jA_2} \quad (4.53)$$

$$\frac{V_2}{V_1} = \frac{A_2 - jA_1}{A_1 - jA_2} = \frac{A_2 - jA_1}{A_1 - jA_2} \quad (4.54)$$

$$\frac{V_2}{V_1} = \frac{A_2 - jA_1}{A_1 - jA_2} = \frac{A_2 - jA_1}{A_1 - jA_2} \quad (4.55)$$

for $n = 1, 2$ where p is defined by (4.56). Referring to (4.58, 4.59) and (4.60) we find

$$\frac{V_2}{V_1} = \frac{A_2 - jA_1}{A_1 - jA_2} = \frac{A_2 - jA_1}{A_1 - jA_2} \quad (4.56)$$

Solving (4.56) for V_2/V_1 we then find V_2/V_1 and K_2 from (4.57) and (4.58). Assuming a Q of 100 at $\omega = 1$, we find

$$\frac{V_2}{V_1} = \frac{A_2 - jA_1}{A_1 - jA_2} = \frac{A_2 - jA_1}{A_1 - jA_2} \quad (4.58)$$

Using the above equations and constants, the curves of Figures 4.12, 4.13, and 4.14 were computed for the three cases of antenna 5.

Antenna 6 is the same as antenna 5 except that we use a conductor separation to spherical radius ratio of $b/a = 150$. The resulting value of K_2 is 600 ohms. We calculate only

Antenna 5

Equivalent Linear Antenna

Equivalent Set of Linear Antenna

$$\frac{2h}{a} = 12000 \quad \frac{b}{a} = 28$$

$$d = \sqrt{ab}$$

$$\text{At } \beta h = 1 \quad X_1 = -\frac{4X_{da}}{g}$$

$$R_1 = \frac{X_1}{100}$$

$$Z_1 = Z_2$$

$$\text{Case 1} \quad l = 3h/8 \quad g = 1.02$$

$$\text{Case 2} \quad l = h/4 \quad g = 1.08$$

$$\text{Case 3} \quad l = h/8 \quad g = 1.32$$

Figure 4.12--The input reactance for antenna 5

Figure 4.12 - The input impedance of a two-port network

Antenna 5

$$\frac{2h}{a} = 12000 \quad \frac{b}{a} = 28$$

$$d = \sqrt{ab}$$

$$\text{At } \beta h = 1 \quad X_1 = -\frac{4X_{da}}{g}$$

$$R_1 = \frac{X_1}{100}$$

$$Z_1 = Z_2$$

$$\text{Case 1} \quad l = 3h/8 \quad g = 1.02$$

$$\text{Case 2} \quad l = h/4 \quad g = 1.08$$

$$\text{Case 3} \quad l = h/8 \quad g = 1.32$$

Equivalent Linear Antenna

Equivalent Set of Linear Antennas

Figure 4.13--The input resistance for antenna 5

Figure 1.3 - The input resistance for linear antennas

Figure 4.14--The efficiency for antenna 5

Figure 2.14 - The efficiency for antenna

The curves for antenna 6 are plotted in Figures 4.15, 4.16, and 4.17 along with the curves of antenna 5 case two which is repeated in these Figures for comparison.

Antenna 7 is the same as antenna 5 except we use a length to conductor radius ratio of $2h/a = 809$, and again we consider only case No. 2. The curves for antenna seven are plotted in Figures 4.18, 4.19, and 4.20 along with the curves of antenna 5 case No. 2 which is repeated again for comparison.

Antenna 8 has two side loads located at the center of the antenna one on each side of the two wire line. ($\ell_1 = h, \ell_2 = h$). Using (4.47), we set the two side loads to produce a pole in X'_{T1} at $\beta h = \beta_0 h$. X_1 and X_2 cannot be made equal in this case. If they were ($K_1 = K_2 = K$), (4.47) would reduce to

$$X'_{T1} = 2K \beta h + R_c \tan \beta h, \quad (4.69)$$

and we would have no poles except from $\tan \beta h$. We consider three cases of antenna 8. For case one $\beta h_0 = .6$ and $X_1 = 2X_2$; for case two $\beta h_0 = .6$ and $X_1 = 4X_2$; and for case three $\beta h_0 = .8$ and $X_1 = 2X_2$. The length to conductor radius ratio is $2h/a = 12000$, and the conductor separation to conductor radius ratio is $b/a = 28$. The resulting value of R_c is 400 ohms. Referring to Figure 4.11 and (4.43), the equations for the input impedance and efficiency are as follows.

$$X'_{T1} = N_1/D_1 + R_c \tan \beta h \quad R'_{T1} = N_2/D_1, \quad (4.70)$$

where

$$\begin{aligned} N = & 16(X_1 + X_2)(R_d^2 + X_d^2) + 4X_d(R_1 + R_2)^2 + (X_1 + X_2)^2 + 4X_2(R_1^2 + X_1^2) \\ & + 4X_1(R_2^2 + X_2^2) + 16R_d(X_1R_2 + X_2R_1) - 16X_d(R_1R_2 - X_1X_2) \end{aligned} \quad (4.71)$$

Antenna 5 Case 2 and Antenna 6

$$\frac{2h}{a} = 12000 \quad d = \sqrt{ab}$$

$$l = h/4$$

$$\text{At } \beta h = 1 \quad X_1 = -\frac{4X_{da}}{g}$$

$$R_1 = \frac{X_1}{100}$$

Equivalent Linear Antenna

$$Z_1 = Z_2 \quad g = 1.08$$

$$\text{Antenna 5 Case 2} \quad \frac{b}{a} = 28$$

Equivalent Set of Linear Antennas

Antenna 6

$$\frac{b}{a} = 150$$

Figure 4.15--The input reactance for antenna 5 Case 2 and antenna 6

Antenna 3 Case 1 and Antenna 6

At $\theta = 0$, $V = V_0$
 $R_1 = 100$
 $X_1 = 100$

1	1	1	1
1	1	1	1

Equivalent circuit Antenna 3
 $Z_1 = 3.2$
 $X_1 = 1.08$

Antenna 3 Case 2
 $\frac{b}{a} = 28$

Antenna 6
 $\frac{b}{a} = 150$

Figure 1.15 - The input resistance for Antenna 3 Case 2

and Antenna 6

Antenna 5 Case 2 and Antenna 6

Equivalent Linear Antenna

Equivalent Set of Linear Antennas

$$\frac{2h}{a} = 12000 \quad d = \sqrt{ab}$$

$$l = h/4$$

$$\text{At } \beta h = 1 \quad X_1 = -\frac{4X_{da}}{g}$$

$$R_1 = \frac{X_1}{100}$$

$$Z_1 = Z_2 \quad g = 1.08$$

$$\text{Antenna 5 Case 2} \quad \frac{b}{a} = 28$$

$$\text{Antenna 6} \quad \frac{b}{a} = 150$$

Figure 4.16--The input resistance for antenna 5 Case 2 and antenna 6

Figure 4.17--The efficiencies for antenna 5 Case 2 and antenna 6

60
50
40
30
20
10

Antenna 2 Case 1 and Antenna 1 Case 2

Antenna 5 Case 2 and Antenna 7

Equivalent Linear Antenna

Equivalent Set of Linear Antennas

$$\frac{b}{a} = 28 \quad d = \sqrt{ab}$$

$$\text{At } \beta h = 1 \quad X_1 = -\frac{4X_{da}}{g}$$

$$R_1 = \frac{X_1}{100}$$

$$Z_1 = Z_2 \quad g = 1.08$$

Antenna 5 Case 2

$$\frac{2h}{a} = 12000$$

$$\text{Antenna 7} \quad \frac{2h}{a} = 809$$

Figure 4.18--The input reactance for antenna 5 case 2 and antenna 7

$$\frac{b}{a} = 28 \quad d = \sqrt{ab}$$

$$\text{At } \beta h = 1 \quad X_1 = -\frac{4X_{da}}{g}$$

$$R_1 = \frac{X_1}{100}$$

Equivalent Linear Antenna

$$Z_1 = Z_2 \quad g = 1.08$$

Antenna 5 Case 2

$$\frac{2h}{a} = 12000$$

Equivalent Set of Linear Antennas

$$\text{Antenna 7} \quad \frac{2h}{a} = 809$$

Figure 4.19--The input resistance for antenna 5 case 2 and antenna 7

.....

$$N_2 = 16(R_1 + R_2)(R_d^2 + X_d^2) + 4R_d(R_1 + R_2)^2 + (X_1 + X_2)^2 + 4R_2(R_1^2 + X_1^2) \\ + 4R_1(R_2^2 + X_2^2) + 16R_d(R_1R_2 - X_1X_2) + 16X_d(X_1R_2 + X_2R_1) \quad (4.72)$$

$$D_1 = (4R_d + R_1 + R_2)^2 + (4X_d + X_1 + X_2)^2 \quad (4.73)$$

X_0 and R_0 are given by (4.53 and 4.54) for $n = 0$.

$$\text{Efficiency} = 100 \frac{B_e}{A_e + B_e}, \quad (4.74)$$

where

$$A_e = 16(R_d^2 + X_d^2)(R_1 + R_2) + 4R_1(R_2^2 + X_2^2) + 4R_2(R_1^2 + X_1^2) \\ + 16X_d(R_1X_2 + R_2X_1) + 32 R_d R_1 R_2 \quad (4.75)$$

$$B_e = 4R_d[(R_1 - R_2)^2 + (X_1 - X_2)^2] \quad (4.76)$$

From King [1, page 192]

$$R_d = 20 \beta^2 h^2 \quad (4.77)$$

$$X_d = -\frac{60}{\beta h} \left[2 \ln \frac{2h}{a} - 3.39 \right] \quad (4.78)$$

From (4.43) we find for Z_1 and Z_2 :

Case No. 1

$$\text{At } \beta h = 0.6; X_2 = -4X_d/3; X_1 = 2X_2; R_1 = \frac{X_1}{100}; R_2 = \frac{X_1}{100} \quad (4.79)$$

Case No. 2

$$\text{At } \beta h = 0.6; X_2 = -4X_d/5; X_1 = 4X_2; R_1 = \frac{X_1}{100}; R_2 = \frac{X_2}{100} \quad (4.80)$$

Case No. 3

$$\text{At } \beta h = 0.8; X_2 = -4X_d/3; X_1 = 2X_2; R_1 = \frac{X_1}{100}; R_2 = \frac{X_2}{100} \quad (4.81)$$

Using the above equations, the curves of Figures 4.21, 4.22, and 4.23 were computed for the three cases.

$$R_1 = 10(R_1 - R_2)(R_1 + R_2) - 10(R_1 - R_2)^2 + 10(R_1 + R_2)^2$$

$$+ 10(R_1 - R_2)(R_1 + R_2) - 10(R_1 - R_2)^2 + 10(R_1 + R_2)^2 \quad (4.72)$$

$$D_1 = (R_1 - R_2)(R_1 + R_2) + 10(R_1 - R_2)^2 + 10(R_1 + R_2)^2 \quad (4.73)$$

X_0 and R_0 are given by (4.72 and 4.74) for $n = 0$.

$$D_0 = (R_1 - R_2)(R_1 + R_2) + 10(R_1 - R_2)^2 + 10(R_1 + R_2)^2 \quad (4.74)$$

where

$$R_1 = 10(R_1 - R_2)(R_1 + R_2) + 10(R_1 - R_2)^2 + 10(R_1 + R_2)^2$$

$$+ 10(R_1 - R_2)(R_1 + R_2) + 10(R_1 - R_2)^2 + 10(R_1 + R_2)^2 \quad (4.75)$$

$$R_2 = 10(R_1 - R_2)(R_1 + R_2) + 10(R_1 - R_2)^2 + 10(R_1 + R_2)^2 \quad (4.76)$$

from King [1, page 192]

$$R_2 = 10(R_1 - R_2)(R_1 + R_2) + 10(R_1 - R_2)^2 + 10(R_1 + R_2)^2 \quad (4.77)$$

$$X_2 = \frac{1}{2} \left[1 + \frac{R_2}{R_1} \right] \quad (4.78)$$

From (4.72) we find for R_1 and R_2

Case No. 1

$$\text{At } R_1 = 0.6: X_1 = -4X_2, R_1 = 2X_2, R_2 = 100, R_1 = 100, R_2 = 100 \quad (4.79)$$

Case No. 2

$$\text{At } R_1 = 0.6: X_1 = -4X_2, R_1 = 4X_2, R_2 = 100, R_1 = 100, R_2 = 100 \quad (4.80)$$

Case No. 3

$$\text{At } R_1 = 0.8: X_1 = -4X_2, R_1 = 2X_2, R_2 = 100, R_1 = 100, R_2 = 100 \quad (4.81)$$

Using the above equations, the curves of Figures 4.21, 4.22,

and 4.23 were computed for the three cases.

Antenna 8

Figure 4.21--The input reactance for antenna 8

Figure 4.21 - The input resistance for antenna B

Antenna 8

$$\frac{2h}{a} = 12000 \quad \frac{b}{a} = 28 \quad d = \sqrt{ab}$$

Case 1

Case 2

Case 3

$$X_1 = 2X_2$$

$$X_1 = 4X_2$$

$$X_1 = 2X_2$$

At $\beta h = 0.6$ At $\beta h = 0.6$ At $\beta h = 0.8$

Equivalent Set of Linear Antennas

$$X_2 = -\frac{4X_d}{3}$$

$$X_2 = -\frac{4X_d}{5}$$

$$X_2 = -\frac{4X_d}{3}$$

$$R_1 = \frac{X_1}{100}$$

$$R_1 = \frac{X_1}{100}$$

$$R_1 = \frac{X_1}{100}$$

$$R_2 = \frac{X_2}{100}$$

$$R_2 = \frac{X_2}{100}$$

$$R_2 = \frac{X_2}{100}$$

Figure 4.22--The input resistance for antenna 8

Antenna 8

Equivalent set of input parameters

Figure 4-23-1

Antenna 8

$$\frac{2h}{a} = 12000 \quad \frac{b}{a} = 28 \quad d = \sqrt{ab}$$

Case 1	Case 2	Case 3
$X_1 = 2X_2$	$X_1 = 4X_2$	$X_1 = 2X_2$
At $\beta h = 0.6$	At $\beta h = 0.6$	At $\beta h = 0.8$

Equivalent Set of Linear Antennas

Figure 4.23--The efficiency for antenna 8

$$\frac{Z_1}{Z_0} = \frac{12000}{28} = 428.57$$

Case	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0
Case 1	1.0	1.0	1.0
Case 2	1.0	1.0	1.0
Case 3	1.0	1.0	1.0

Equivalent set of linear antennas

Case	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0
Case 1	1.0	1.0	1.0
Case 2	1.0	1.0	1.0
Case 3	1.0	1.0	1.0

Figure 4.23 - The efficiency for antenna 3

4.7 Discussion of the Properties of Double Side-Loaded Transmission-Line Antennas

Figures 4.12 through 4.23 give a general picture of the input impedance and efficiencies that can be obtained using double side-loaded transmission-line antennas. Again the results are only representative of what can be done, and an optimum set of properties could only be obtained for a specific design situation.

Figures 4.12 through 4.14 illustrate the input impedance and the efficiency as a function of frequency for a double side-loaded transmission-line antenna with both side loads on the same side. It is clear from observing these Figures that operating points with the characteristics described in Section 4.2 are readily obtainable with an antenna of this type. The efficiencies for these operating points would range from 3% to 5% for the quality of coils used in the calculations ($Q = 100$). Figure 4.13 also illustrates the fact that the greater the separation of the side loads on the transmission line the smaller the minimum point in the input resistance between the poles. Thus one can design for a large range of input resistance simply by adjusting the separation of the side loads.

Figures 4.15, 4.16, and 4.17 show the effect of varying the conductor separation of a double side-loaded transmission line antenna with both side loads on the same side. The two antennas represented in these Figures are identical except that the conductor separation to conductor radius ratio of antenna 5, case 2 is 28 while that of antenna 6 is 150. It

4.7 Discussion of the Properties of Double Side-Loaded Transmission-Line Antennas

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and the efficiency as a function of frequency for a double

side-loaded transmission-line antenna with both side loads

on the same side. It is clear from observing these figures

that operating points with the characteristics described in

Section 4.2 are readily obtainable with an antenna of this type.

The efficiencies for these operating points would range from

25 to 55 for the quality of coils used in the calculations.

($Q = 100$). Figure 4.15 also illustrates the fact that the

greater the separation of the side loads on the transmission

line the smaller the minimum point in the input resistance

between the poles. Thus one can design for a large range of

input resistance simply by adjusting the separation of the

side loads.

Figures 4.15, 4.16, and 4.17 show the effect of varying

the conductor separation of a double side-loaded transmission

line antenna with both side loads on the same side. The two

antennas represented in these figures are identical except

that the conductor separation to conductor radius ratio of

antenna 2 is 2.5 while that of antenna 1 is 1.50. The

can be observed from the Figures that this rather large change in the conductor separation to conductor radius ratio has very little effect on the input resistance or efficiency of the antenna except to shift the frequency of the desirable operating point.

Figures 4.18, 4.19, and 4.20 show the effect of varying the length to conductor radius ratio of a double side-loaded transmission-line antenna with both side loads on the same side. Again the two antennas are identical except for one property, in this case the length to conductor radius ratio. For antenna 5, case 2, this ratio is 12000 and for antenna 7, the ratio is 809. In this case the poles are not only shifted slightly in frequency but the minimum point in the input resistance curve between the poles or antiresonant points is reduced.

Figures 4.21, 4.22, and 4.23 illustrate the input impedance and efficiency as a function of frequency for a double side-loaded transmission-line antenna with the side loads on opposite sides of the transmission line at the center of the antenna lengthwise. (Antenna 8). For this particular antenna the frequency separation of the poles can only be varied by varying the size of the side loads. Assuming inductive side loads on electrically short antennas, the separation of the poles is greater and thus the minimum value of the input resistance is smaller the greater the difference in the value of the two side loads. As pointed out above, if the side loads are equal in value, the poles disappear completely. Figure 4.21 also illustrates the fact that the high frequency pole is extremely sharp for this antenna, and its effect on the input reactance

curve is almost wiped out by the losses in the system. Thus it would be quite difficult to obtain an operating point where the input reactance was zero although quite small values of input reactance could be easily obtained.

In the range where the input impedance of antenna 8 has the desired characteristics the efficiency is considerably higher, for the same quality coils, than it is for any of the other antennas. In particular for case 2 where the antenna might be operated in the range from $0.6 \leq \beta h < 0.7$ the efficiency runs from 22% to 36%.

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it would be quite difficult to obtain an operating point

where the input resistance was zero although quite small values

of input resistance could be easily obtained.

is the range where the input impedance of antenna 8 has

the desired characteristic the efficiency is considerably

higher, for the same power output, than it is for any of the

other antennas. In particular for case 5 where the antenna

might be operated in the range from 0.6 to 0.7 the effi-

ciency runs from 25 to 35.

5. THE SIDE-LOADED TRANSMISSION-LINE ANTENNA AS A TRAILING WIRE ANTENNA

5.1 Introduction

In the previous chapters the electrical properties of a side-loaded transmission-line antenna have been presented, assuming that the antenna was straight and isolated in free space. In order to describe the electrical properties of the side-loaded transmission-line antenna used as a trailing wire antenna over the sea, we must take account of the curvature of the antenna and also the effects of the sea.

It was shown in Chapters 2 and 3 that the radiation properties and the input impedance of any side-loaded transmission-line antenna can be determined from the appropriate combination of the radiation properties and the input impedance of one or more asymmetrically or symmetrically fed cylindrical linear antennas. Further the antennas under consideration here as trailing wire antennas are in the range of several thousand feet long with wire diameters in the range of an inch. Thus in what follows we need to consider only thin antennas. Using the results obtained from these, the corresponding results for side-loaded transmission-line antennas can be calculated using the appropriate equations from chapters two and three.

5.2 The Method of Images

To account for the presence of the sea we assume the antenna is located over an infinite conducting plane and take

2.1 Introduction

In the previous chapters the electrical properties of a side-loaded transmission-line antenna have been presented assuming that the antenna was straight and isolated in free space. In order to describe the electrical properties of the side-loaded transmission-line antenna used as a trailing wire antenna over the sea, we must take account of the curvature of the antenna and also the effects of the sea. It was shown in Chapters 2 and 3 that the radiation properties and the input impedance of any side-loaded transmission-line antenna can be determined from the appropriate combination of the radiation properties and the input impedance of one or more asymmetrically or symmetrically fed cylindrical linear antennas. Further the antennas under consideration here as trailing wire antennas are in the range of several thousand feet long with wire diameters in the range of an inch. Thus in what follows we need to consider only this antenna. Using the results obtained from these, the corresponding results for side-loaded transmission-line antennas can be calculated using the appropriate equations from

chapters two and three.

2.2 The Method of Images

To account for the presence of the sea we assume the antenna is located over an infinite conducting plane and take

account of the presence of this plane by the method of images. Consider the horizontal and vertical antennas located a distance H above the sea as illustrated in Figure 5.1. Using the method of images, the sea is replaced by an image of the antenna located a distance H below the sea. For the horizontal antenna the current in the image is equal in magnitude but reversed in phase by 180° from the antenna to meet the condition of zero tangential electric fields along the plane of the sea. For the vertical antenna the condition of zero tangential electric field at the sea to air interface is met by requiring the current in the image to be equal in magnitude and phase to the current in the antenna for points located on the image and the antenna an equal distance from the sea to air interface. These images are illustrated in Figure 5.1.

5.3 Far Field Calculations

Once the correct images have been established, all that is necessary to calculate the far fields in the air resulting from the antenna in the air over the sea is to superimpose the field due to the antenna in the air and the field due to its image. This process is illustrated in Figure 5.2 for a vertical electric dipole and in Figure 5.3 for a horizontal electric dipole.

Referring to Stratton [4, page 435] the expressions for the far fields from an electric dipole are given by

$$\vec{E}_H \approx - \frac{\beta^2}{4\pi\epsilon_0 r_0} \hat{r}_0 \times (\hat{r}_0 \times \vec{p}_H) e^{-j\beta r_0} \quad (5.1)$$

account of the presence of this plane by the method of images. Consider the horizontal and vertical antennas located a distance h above the sea as illustrated in Figure 2.1. Using the method of images, the sea is replaced by an image of the antennas located a distance h below the sea. For the horizontal antennas the current in the image is equal in magnitude but reversed in phase by 180° from the antenna to meet the condition of zero tangential electric fields along the plane of the sea. For the vertical antennas the condition of zero tangential electric field at the sea to air interface is met by requiring the current in the image to be equal in magnitude and phase to the current in the antenna for poles located on the image and the antenna an equal distance from the sea to air interface. These images are illustrated in Figure 2.1.

2.2 Far Field Calculations

Once the correct images have been established, all that is necessary to calculate the far field in the air resulting from the antenna in the air over the sea is to superimpose the field due to the antenna in the air and the field due to its image. This process is illustrated in Figure 2.2 for a vertical electric dipole and in Figure 2.3 for a horizontal electric dipole.

Referring to equation (4, page 55) the expressions for the far fields from an electric dipole are given by

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r} \quad \text{and} \quad \vec{B} = \frac{\mu_0}{4\pi} \frac{q \times \hat{r}}{r^2}$$

Figure 5.1--The method of images

Figure 2.1-The method of images

Figure 5.2--Image of a horizontal electric dipole

Figure 5.3--Image of a vertical electric dipole

Figure 5.2--Image of a horizontal electric dipole

Figure 5.3--Image of a vertical electric dipole

$$\vec{H}_H \approx \frac{\omega\beta}{4\pi r_o} (\vec{p}_H \times \hat{r}_o) e^{-j\beta r_o} \quad (5.2)$$

$$\vec{E}_V \approx - \frac{\beta^2}{4\pi\epsilon_o r_o} \hat{r}_o \times (\hat{r}_o \times \vec{p}_V) e^{-j\beta r_o} \quad (5.3)$$

$$\vec{H}_V \approx \frac{\omega\beta}{4\pi r_o} (\vec{p}_V \times \hat{r}_o) e^{-j\beta r_o} \quad (5.4)$$

where $\hat{r}_o$ is a unit vector in the direction of $\vec{r}_o$, r_o is the magnitude of $\vec{r}_o$, $\vec{p}_H$ and $\vec{p}_V$ are the electric dipole moments of the horizontal and vertical antennas,

$$\beta = \frac{\omega}{v} \quad (5.5)$$

and we have suppressed the time variation $e^{j\omega t}$. Superimposing the field of the antenna and its image, we obtain for the horizontal dipole

$$\vec{E}_H = - \frac{\beta^2}{4\pi\epsilon_o r_o} \hat{r}_o \times (\hat{r}_o \times \vec{p}_H) e^{-j\beta r_o} (1 - e^{-j2\beta H \cos \theta}) \quad (5.6)$$

$$\vec{H}_H = \frac{\omega\beta}{4\pi r_o} (\vec{p}_H \times \hat{r}_o) e^{-j\beta r_o} (1 - e^{-j2\beta H \cos \theta}) \quad (5.7)$$

and for the vertical dipole

$$\vec{E}_V = - \frac{\beta^2}{4\pi\epsilon_o r_o} \hat{r}_o \times (\hat{r}_o \times \vec{p}_V) e^{-j\beta r_o} (1 + e^{-j2\beta H \cos \theta}) \quad (5.8)$$

$$\vec{H}_V = \frac{\omega\beta}{4\pi r_o} (\vec{p}_V \times \hat{r}_o) e^{-j\beta r_o} (1 + e^{-j2\beta H \cos \theta}) \quad (5.9)$$

For any antenna that can be reasonably approximated as an electric dipole, for far field calculations, (5.6 and 5.7) give the fields if the antenna is located a height H above a conducting plane and parallel to it. (5.8 and 5.9) give the fields for the same condition except that now the antenna is perpendicular to the conducting plane. For a specific antenna all that remains is to determine $\vec{p}_H$ and

$$\vec{H} = \frac{1}{4\pi r^2} (\vec{p} \times \vec{r}) e^{-jkr} \quad (5.2)$$

$$\vec{E} = -\frac{1}{4\pi r^2} \left[\vec{p} - (\vec{p} \cdot \vec{r}) \frac{\vec{r}}{r} \right] e^{-jkr} \quad (5.3)$$

$$\vec{E} = -\frac{1}{4\pi r^2} (\vec{p} \times \vec{r}) e^{-jkr} \quad (5.4)$$

where $\vec{r}_0$ is a unit vector in the direction of $\vec{r}$, r_0 is the magnitude of $\vec{r}$, and $\vec{p}$ and $\vec{q}$ are the electric dipole moments of the horizontal and vertical antennas.

$$\vec{E} = \frac{1}{4\pi r^2} \left[\vec{p} - (\vec{p} \cdot \vec{r}) \frac{\vec{r}}{r} \right] e^{-jkr} \quad (5.5)$$

and we have suppressed the time variation $e^{j\omega t}$. Superimposing the field of the antenna and its image, we obtain for the horizontal dipole

$$\vec{E} = \frac{1}{4\pi r^2} \left[\vec{p} - (\vec{p} \cdot \vec{r}) \frac{\vec{r}}{r} \right] e^{-jkr} + \frac{1}{4\pi r^2} \left[\vec{p} - (\vec{p} \cdot \vec{r}) \frac{\vec{r}}{r} \right] e^{-jkr} \quad (5.6)$$

$$\vec{H} = \frac{1}{4\pi r^2} (\vec{p} \times \vec{r}) e^{-jkr} + \frac{1}{4\pi r^2} (\vec{p} \times \vec{r}) e^{-jkr} \quad (5.7)$$

$$\vec{E} = -\frac{1}{4\pi r^2} \left[\vec{p} - (\vec{p} \cdot \vec{r}) \frac{\vec{r}}{r} \right] e^{-jkr} - \frac{1}{4\pi r^2} \left[\vec{p} - (\vec{p} \cdot \vec{r}) \frac{\vec{r}}{r} \right] e^{-jkr} \quad (5.8)$$

$$\vec{H} = \frac{1}{4\pi r^2} (\vec{p} \times \vec{r}) e^{-jkr} - \frac{1}{4\pi r^2} (\vec{p} \times \vec{r}) e^{-jkr} \quad (5.9)$$

For any antenna that can be reasonably approximated as an electric dipole, for far field calculations, (5.6) and (5.7) give the fields if the antenna is located a height h above a conducting plane and parallel to it. (5.8 and 5.9) give the fields for the same condition except that now the an-

tenna is perpendicular to the conducting plane. For a specific antenna all that remains is to determine $\vec{p}$ and

$\vec{p}_V$. The method for doing this for a symmetrically-fed thin linear antenna used as a trailing wire antenna will be presented in the next two sections.

5.4 The Horizontal and Vertical Moments of a Sloping Antenna

In this section we shall derive the equivalent horizontal and vertical dipole moments for an antenna tilted at an angle θ_0 to the vertical axis. Consider the center-fed cylindrical antenna of length $2h$ shown in Figure 5.4. The antenna lies in the x, z -plane, and we assume that the current distribution is sinusoidal such that

$$\vec{I}(s) = \begin{cases} \hat{s} \frac{I_0 \sin \beta(h-s)}{\sin \beta h} & 0 \leq s \leq h \\ \hat{s} \frac{I_0 \sin \beta(h+s)}{\sin \beta h} & -h \leq s \leq 0 \end{cases} \quad (5.10)$$

In order to find the electric dipole moment from this antenna, we write first an expression for the retarded Hertz vector caused by the antenna. Referring to Stratton [4, page 430, 431] and observing Figure 5.4, we find

$$\vec{\pi}(r) = \frac{1}{4\pi\epsilon_0} \int_{-h}^h \frac{\vec{P}_0(s) e^{-j\beta r} \hat{s} ds}{r}, \quad (5.11)$$

where

$$\vec{P}_0(s) = -\frac{j}{\omega} \vec{I}(s), \quad (5.12)$$

r is the magnitude of $\vec{r}$, we have suppressed the time factor $e^{j\omega t}$, $\frac{e^{-j\beta r}}{r}$ can be expanded in the form

$$\frac{e^{-j\beta r}}{r} = -j\beta \sum_{n=0}^{\infty} (2n+1) P_n(\cos \gamma) j_n(kS) h_n^{(1)}(\beta r_0) \quad \text{for } r_0 > S, \quad (5.13)$$

6. The method for doing this is as follows:

linear antenna used as a starting point with a dipole antenna

centered in the next two sections.

2.4 The Horizontal and Vertical Moments of a Dipole Antenna

In this section we shall derive the equivalent horizontal

and vertical dipole moments for a dipole antenna of length $2a$

centered at the origin. Consider the center-fed dipole antenna

shown in Figure 2.4. The antenna lies

in the xz -plane, and we assume that the current distribution

is sinusoidal such that

$$I(z) = \begin{cases} I_0 \sin k(a-z) & 0 \leq z \leq a \\ I_0 \sin k(a+z) & -a \leq z \leq 0 \end{cases}$$

In order to find the electric dipole moment from this antenna

we write first an expression for the vector potential A

caused by the antenna. Referring to Equation (1.10), page 10,

and observing Figure 2.4, we find that the vector potential

$$\vec{A}(x) = \frac{1}{4\pi\epsilon_0} \int_{-a}^a \frac{I(z) e^{-jkR}}{R} dz$$

where

$$R = \sqrt{x^2 + y^2 + z^2}$$

x is the magnitude of $\vec{r}$, we have expressed the line integral

as a volume integral in the form

$$\frac{1}{4\pi\epsilon_0} \int_{-a}^a \frac{I(z) e^{-jkR}}{R} dz = \frac{1}{4\pi\epsilon_0} \int_{-a}^a \frac{I(z) e^{-jkR}}{R} dz$$

Figure 5.4--Straight sloping antenna

$$\cos \gamma = \cos \theta \cos \theta_o + \sin \theta \sin \theta_o \cos \phi ,$$

and

$$r = [r_o^2 + s^2 - 2r_o s \cos \gamma]^{1/2} . \quad (5.14)$$

The electric dipole moment is found by approximating (5.13) by the first term of the series. Using this approximation, we find

$$\frac{e^{-j\beta r}}{r} = \frac{e^{-j\beta r_o}}{r_o} . \quad (5.15)$$

Using (5.15) in (5.11), we find

$$\vec{\pi}(r) = \frac{1}{4\pi\epsilon_o} \frac{e^{-j\beta r_o}}{r_o} \int_{-h}^h p_o(s) \hat{S} ds = \frac{1}{4\pi\epsilon_o} \frac{e^{-j\beta r_o}}{r_o} \vec{p} , \quad (5.16)$$

where $\vec{p}$ is defined as the equivalent electric dipole moment of the antenna. We further define $\vec{p}_V$ as the equivalent vertical dipole moment and $\vec{p}_H$ as the equivalent horizontal dipole moment, where

$$\vec{p} = \vec{p}_V + \vec{p}_H = p_V \hat{k} + p_H \hat{i} = p \cos \theta_o \hat{k} + p \sin \theta_o \hat{i} , \quad (5.17)$$

and $\hat{k}$ and $\hat{i}$ are unit vectors along the z and x axes, respectively. From (5.12, 5.16, and 5.17) we observe that

$$p_H = -j \frac{\sin \theta_o}{\omega} \int_{-h}^h I(s) ds \quad (5.18)$$

$$p_V = -j \frac{\cos \theta_o}{\omega} \int_{-h}^h I(s) ds . \quad (5.19)$$

Substituting (5.10) in (5.18 and 5.19), we obtain

$$\cos \theta = \cos \theta_0 \cos \theta' + \sin \theta_0 \sin \theta' \cos \phi$$

and

$$\mathbf{r} = \left[r_0^2 + z^2 - 2zr_0 \cos \theta' \right]^{1/2} \quad (5.10)$$

The electric dipole moment is found by approximating (5.15)

by the first term of the series. Using this approximation

we find

$$\frac{e^{-jkr}}{r} = \frac{e^{-jkr_0}}{r_0} \left[1 - \frac{jkr_0 z \cos \theta'}{r_0} + \dots \right] \quad (5.16)$$

Using (5.16) in (5.11) we find

$$\bar{\mathbf{E}}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \frac{e^{-jkr_0}}{r_0} \left\{ \mathbf{p}_0(a) \hat{\mathbf{z}} + \frac{1}{r_0} \frac{e^{-jkr_0}}{r_0} \mathbf{p}_0 \right\} \quad (5.16)$$

where $\mathbf{p}$ is defined as the equivalent electric dipole moment

of the antenna. We further define $\mathbf{p}_0$ as the equivalent

vertical dipole moment and $\mathbf{p}_H$ as the equivalent horizontal

dipole moment, where

$$\mathbf{p} = \mathbf{p}_V + \mathbf{p}_H = p_V \hat{\mathbf{z}} + p_H \hat{\mathbf{r}} = p \cos \theta_0 \hat{\mathbf{z}} + p \sin \theta_0 \hat{\mathbf{r}} \quad (5.17)$$

and $\hat{\mathbf{r}}$ and $\hat{\mathbf{z}}$ are unit vectors along the z and x axes, respec-

tively. From (5.15, 5.16, and 5.17) we observe that

$$\mathbf{p}_H = -j \int_{-h}^h \frac{\sin \theta_0}{r_0} I(z) dz \quad (5.18)$$

$$\mathbf{p}_V = -j \int_{-h}^h \frac{\cos \theta_0}{r_0} I(z) dz \quad (5.19)$$

Substituting (5.18) in (5.16 and 5.17) we obtain

$$p_H = -j \frac{2I_0}{\omega\beta} \frac{\sin \theta_0}{\sin \beta h} (1 - \cos \beta h) \quad (5.20)$$

$$p_V = -j \frac{2I_0}{\omega\beta} \frac{\cos \theta_0}{\sin \beta h} (1 - \cos \beta h) . \quad (5.21)$$

Using (5.20 and 5.21), we can compute the horizontal and vertical dipole moments for a center-fed cylindrical antenna of length $2h$ making an angle θ_0 with the vertical or z -axis.

5.5 The Horizontal and Vertical Moments of a Curved Antenna

Having developed the equivalent horizontal and vertical dipole moments for a straight antenna tilted at an angle θ_0 to the vertical or z -axis, we now apply the same methods to a curved antenna. Equations (5.10 - 5.17) will be exactly the same as in the case of the straight antenna but in performing the integrations over the antenna to find p_H and p_V , θ_0 will be a function of S . In order to determine what this function is, we must first decide on a specific family of curves.

To illustrate the method we choose the catenary of Figure 5.5. The equation of this curve is

$$x + a - x_3 = a \cosh \frac{z - z_3}{a} , \quad (5.22)$$

where $a > 0$ and we have set x_3 and z_3 such that

$$\int_{x_3}^0 \frac{ds(x)}{dx} = \int_0^{x_1} \frac{ds(x)}{dx} dx = h . \quad (5.23)$$

In order to determine $\theta_0(s)$, we first find

$$\cot \theta_0(s) = \frac{dz}{dx} . \quad (5.24)$$

From (5.22) we find

$$P_H = \frac{1}{2} \frac{I_0 \sin \theta_0}{\sin \theta} (1 - \cos \theta) \quad (5.20)$$

$$P_V = \frac{1}{2} \frac{I_0 \cos \theta_0}{\sin \theta} (1 - \cos \theta) \quad (5.21)$$

Using (5.20 and 5.21), we can compute the horizontal and vertical dipole moments for a center-fed cylindrical antenna of length l making an angle θ_0 with the vertical or z -axis.

5.5 The Horizontal and Vertical Moments of a Curved Antenna

Having developed the equivalent horizontal and vertical dipole moments for a straight antenna tilted at an angle θ_0 to the vertical or z -axis, we now apply the same methods to a curved antenna. Equations (5.10 - 5.17) will be exactly the same as in the case of the straight antenna but in performing the integrations over the antenna to find P_H and P_V , θ_0 will be a function of s . In order to determine what this function is, we must first decide on a specific family of curves.

To illustrate the method we choose the category of Figure 5.5. The equation of this curve is

$$x + a - x_0 = a \cosh \frac{z-z_0}{a} \quad (5.22)$$

where $a > 0$ and we have set x_0 and z_0 such that

$$\int_{x_0}^x \frac{ds(x)}{dx} dx = h \quad (5.23)$$

In order to determine $\theta_0(s)$, we first find

$$\cos \theta_0(s) = \frac{dz}{ds}$$

From (5.22) we find

Figure 5.5--Curved antenna

Figure 2.5--Curved antenna

$$\frac{dz}{dx} = \frac{a}{[(x+a-x_3)^2 - a^2]^{1/2}} \quad (5.25)$$

Evaluating the integrals in (5.23), we find

$$\int_{x_3}^0 \frac{ds(x)}{dx} dx = \int_{x_3}^0 \left[1 + \left(\frac{dz}{dx} \right)^2 \right]^{1/2} dx = \int_{x_3}^0 \frac{x+a-x_3}{[(x+a-x_3)^2 - a^2]^{1/2}} dx = \left[x_3^2 - 2ax_3 \right]^{1/2} = h \quad (5.26)$$

$$\int_0^{x_1} \frac{ds(x)}{dx} dx = \left[(x_1+a-x_3)^2 - a^2 \right]^{1/2} - \left[x_3^2 - 2ax_3 \right]^{1/2} = h \quad (5.27)$$

From (5.25, 5.26, and 5.27) we observe that

$$s = \left[(x+a-x_3)^2 - a^2 \right]^{1/2} - h \quad (5.28)$$

$$\frac{dz}{dx} = \frac{a}{s+h} \quad (5.29)$$

Substituting (5.29) in (5.24), we find

$$\tan \theta_0(s) = \frac{s+h}{a} \quad (5.30)$$

Using (5.30) and referring to (5.10), (5.18), and (5.19),

we find

$$P_H = -j \frac{I_0}{\omega \sin \beta h} \left[\int_{-h}^0 \frac{(s+h) \sin \beta (h+s)}{[(s+h)^2 + a^2]^{1/2}} ds + \int_0^h \frac{(s+h) \sin \beta (h-s)}{[(s+h)^2 + a^2]^{1/2}} ds \right] \quad (5.31)$$

$$P_V = -j \frac{I_0 a}{\omega \sin \beta h} \left[\int_{-h}^0 \frac{\sin \beta (h+s) ds}{[(s+h)^2 + a^2]^{1/2}} + \int_0^h \frac{\sin \beta (h-s) ds}{[(s+h)^2 + a^2]^{1/2}} \right] \quad (5.32)$$

The evaluation of (5.31) and (5.32) in closed form is not possible to the best of the author's knowledge. Assuming electrically short antennas and using the approximations

Evaluating the integral

$$\int_0^1 \frac{dx}{x^2 + 1} = \left[\arctan x \right]_0^1 = \arctan 1 - \arctan 0 = \frac{\pi}{4} - 0 = \frac{\pi}{4}$$

From 1.15, 1.16, and 1.17 we observe that

$$S = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\frac{dx}{dt} = \frac{1}{2}x$$

Substituting (1.15) in (1.16) we find

$$S = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$S = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

we find

$$P = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$P = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

The evaluation of the integral and the integral form is not

possible to find the integral form of the integral

electrically when the integral form of the integral

$\sin x \approx x$ and $\cos x \approx 1$, it is possible to evaluate (5.32). Since antennas up to a half wave length are under consideration these approximations are not very accurate, however, A method of graphical approximation which is simple to perform and is independent of the length of the antenna is presented in Appendix D. It is shown there that the catenaries of interest in the antenna problem can be closely approximated by three straight lines making angles of θ_1 , θ_2 and θ_3 with the z-axis. Using this approach, the integration over the curved antenna can be broken up into three integrations which can be done in closed form. The graphical process is illustrated in Figure 5.6. First we must choose the length $2h$ and the specific curve of the catenary family (which is determined by "a") that we desire to investigate. Using these values of h and "a," x_3 can be determined from (5.26). As illustrated in Figure 5.6 and Appendix D, we can determine θ_1 , θ_2 , θ_3 and x_2 graphically. Using (5.28), we find

$$s_2 = \left[(x_2^2 + a - x_3)^2 - a^2 \right]^{1/2} - L. \quad (5.33)$$

Returning to (5.31) and (5.32), we find

$$P_H = -j \frac{I_0}{\omega \sin \beta h} \left[\sin \theta_1 \int_0^h \sin \beta (h-s) ds + \sin \theta_2 \int_{s_2}^0 \sin \beta (h+s) ds + \sin \theta_3 \int_{-h}^{s_2} \sin \beta (h+s) ds \right] \quad (5.34)$$

$$P_V = -j \frac{I_0}{\omega \sin \beta h} \left[\cos \theta_1 \int_0^h \sin \beta (h-s) ds + \cos \theta_2 \int_{s_2}^0 \sin \beta (h+s) ds + \cos \theta_3 \int_{-h}^{s_2} \sin \beta (h+s) ds \right]. \quad (5.35)$$

and x_2 and $\cos \theta_1$, it is possible to evaluate (5.32).

Since antennas up to a half wave length are under consideration

these approximations are not very accurate. However, a method

of graphical approximation which is simple to perform and is

independent of the length of the antenna is presented in App-

endix B. It is shown there that the characteristics of interest

in the antenna problem can be closely approximated by three

straight lines making angles of θ_1 , θ_2 and θ_3 with the z-axis.

Using this approach, the integration over the curved antenna

can be broken up into three integrations which can be done in

closed form. The graphical process is illustrated in Figure

5.6. First we must choose the length $2h$ and the specific

curve of the antenna family (which is determined by "a")

that we desire to investigate. Using these values of h and

"a", x_2 can be determined from (5.26). As illustrated in

Figure 5.6 and Appendix B, we can determine θ_1 , θ_2 , and

x_2 graphically. Using (5.28), we find

$$a = \left[(x_2^2 + a^2 - x_1^2) / 2x_2 \right]^{1/2} \quad (5.33)$$

Returning to (5.31) and (5.32), we find

$$p_H = - \frac{1}{2\pi \sin \theta_1} \int_{-h}^h \sin \theta_1 (h-z) \sin \theta_2 \sin \theta_3 (h+z) dz \quad (5.34)$$

$$p_V = - \frac{1}{2\pi \sin \theta_1} \int_{-h}^h \cos \theta_1 (h-z) \cos \theta_2 \cos \theta_3 (h+z) dz \quad (5.35)$$

Figure 5.6--Straight line approximation of a curved antenna

Figure 2.6--Straight line approximation of a curved

antenna

Integrating (5.34 and 5.35), we find

$$P_H = -j \frac{I_o}{\beta \omega \sin \beta h} \left[\sin \theta_1 \left\{ 1 - \cos \beta h \right\} + \sin \theta_2 \left\{ \cos \beta (h + S_2) - \cos \beta h \right\} + \sin \theta_3 \left\{ 1 - \cos \beta (h + S_2) \right\} \right] \quad (5.36)$$

$$P_V = -j \frac{I_o}{\beta \omega \sin \beta h} \left[\cos \theta_1 \left\{ 1 - \cos \beta h \right\} + \cos \theta_2 \left\{ \cos \beta (h + S_2) - \cos \beta h \right\} + \cos \theta_3 \left\{ 1 - \cos \beta (h + S_2) \right\} \right]. \quad (5.37)$$

Since the integrals above were evaluated by straight line approximations, the method is not limited to the catenary family of curves, but would be applicable to any family of curves. From the simplicity of the integrations in (5.18, 5.19, 5.34, and 5.35) it is easy to see that the asymmetrically-fed thin linear antenna can be treated quite simply by the same methods used above for the symmetrical case.

5.6 Impedance Calculations

When a conducting plane such as the sea is present the driving point impedance of an antenna is, in general, different from its free space value. Therefore, to calculate the driving point impedance of a trailing wire antenna being flown over the sea we must account for the effect of the sea. If we assume the sea is a perfectly conducting plane, we can use the method of images. Using this method, the input impedance of the trailing wire antenna would be the sum of the self impedance of the antenna and the mutual impedance between the antenna and its image. In taking the sum, we must account for the relative phase of the antenna current and image current.

Since the curvature of the trailing wire antenna will not be great in most practical cases, we will assume that for

interesting is that

$$p_1 = \frac{1}{\cos \theta} \left[\frac{1}{\cos \theta} \right]$$

$$p_2 = \frac{1}{\cos \theta} \left[\frac{1}{\cos \theta} \right]$$

Since the integrals

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2.6 Impedance Calculations

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impedance calculations the trailing wire antenna can be approximated by a straight symmetrical or asymmetrical thin linear antenna. With this approximation the self impedance can be read directly from tables in King's book [1, page 176, 192] for long thin antennas symmetrically fed. For the asymmetrical case the same tables can be used applying the approximations presented by King [1, page 403, 404].

The mutual impedance between the antenna and its image is not as easily available even for straight symmetrically-fed thin linear antennas except in the special case where the antenna and its image are parallel. Since the results for parallel antennas are not directly applicable to sloping trailing wire antennas, an expression for the mutual impedance of two skew symmetrically-fed linear antennas is derived in Appendix E using the induced e.m.f. method. The expression obtained is quite long and time consuming to use. By considering the special case of a half wave length symmetrically-fed thin linear antenna making an angle of 45° with the vertical axis, the expression for the mutual impedance of the antenna and its image becomes quite simple, and useful information can be easily obtained. For the special case described above θ_A and βh of Appendix E both become $\pi/2$. Substituting these values in (E.34), we find

$$Z_{21} = 15 \left[E_i(-jX_2) + E_i(-jX_6) - 2E_i(-jX_9) + e^{j2\beta a} \{ E_i(-jX_4) - E_i(-jX_{10}) \} + e^{-j2\beta a} \{ E_i(-jX_8) - E_i(-jX_{12}) \} \right], \quad (5.38)$$

where

$$x_2 = \beta \left\{ [(a-h)^2 + (a+h)^2]^{1/2} + 2h \right\} \quad (5.39)$$

impedance calculations the trailing wire antenna can be approx-

imated by a straight wire antenna of symmetrical form.

antenna. With this approximation the self impedance can be

read directly from tables in King's book [1, page 176-182]

for long thin antennas symmetrically fed. For the asymmetrical

case the same tables can be used applying the spin extension

presented by King [1, page 207, 208].

The mutual impedance between the antenna and its image is

not as easily available even for isotropic symmetrically-fed

thin linear antennas except in the special case where the antenna

and its image are parallel. Since the results for parallel an-

tennas are not directly applicable to sloping trailing wire

antennas, an expression for the mutual impedance of two skew

symmetrically-fed linear antennas is derived in Appendix E

using the method of [2, 3]. The expression obtained is

rather long and tedious to use. It is considered the

special case of a half-wave length symmetrically-fed thin

linear antenna making an angle α with the vertical axis.

The expression for the mutual impedance of the antenna and its

image becomes quite simple, and useful information can be

easily obtained. For the special case described above α and

β of Appendix E both become $\pi/2$. Substituting these values

in (E.34), we find

$$Z_{12} = \frac{1}{2} \left[Z_{11} + Z_{22} - \frac{1}{2} (Z_{11} - Z_{22}) \cos 2\alpha \right] \\ + \frac{1}{2} (Z_{11} - Z_{22}) \sin 2\alpha \sin 2\beta$$

where

$$x_4 = \beta \left\{ \left[(a-h)^2 + (a+h)^2 \right]^{1/2} + 2a \right\} \quad (5.40)$$

$$x_6 = \beta \left\{ \left[(a-h)^2 + (a+h)^2 \right]^{1/2} - 2h \right\} \quad (5.41)$$

$$x_8 = \beta \left\{ \left[(a-h)^2 + (a+h)^2 \right]^{1/2} - 2a \right\} \quad (5.42)$$

$$x_9 = \sqrt{2} \beta (a-h) \quad (5.43)$$

$$x_{10} = (2 + \sqrt{2}) \beta (a-h) \quad (5.44)$$

$$x_{12} = (\sqrt{2} - 2) \beta (a-h) \quad (5.45)$$

$$a = H / \cos \theta_A / 2 \quad (5.46)$$

and H is the height of the antenna above the sea. Figure 5.7 is a plot of (5.38) as a function of " a ." It is seen that for H greater than or equal to the length of the antenna the mutual impedance Z_{21} is quite small compared with the self impedance of a half wave symmetrically-fed thin linear antenna, which from Kraus [5, page 261] is

$$Z_{11} = 73 + j 42.5 \text{ ohms.} \quad (5.47)$$

For an angle between the antenna and the vertical axis of 90° the value of Z_{12} has been computed in the literature for various spacings. Referring to the curves in Kraus [5, page 266] for side by side symmetrically-fed thin half wave length antennas, we find that for H greater than or equal to the length of the antenna $|R_{21}| \leq 16$ ohms and $|x_{21}| \leq 20$ ohms. Thus as a first approximation the driving point impedance of a trailing wire antenna over the sea can be estimated from the self impedance of the same antenna assumed to be straight and located in free space. If the antenna is to be flown very

$$\begin{aligned}
 x_1 &= \frac{1}{2} \left\{ (a-b) + \sqrt{(a-b)^2 - 4ac} \right\} \\
 x_2 &= \frac{1}{2} \left\{ (a-b) - \sqrt{(a-b)^2 - 4ac} \right\} \\
 x_3 &= \frac{1}{2} \left\{ (a-b) + \sqrt{(a-b)^2 - 4ac} \right\} \\
 x_4 &= \frac{1}{2} \left\{ (a-b) - \sqrt{(a-b)^2 - 4ac} \right\}
 \end{aligned}$$

$$\begin{aligned}
 x_{10} &= (1 + \sqrt{1 - 4ac}) / 2 \\
 x_{12} &= (1 - \sqrt{1 - 4ac}) / 2
 \end{aligned}$$

$$a = H/\cos \theta$$

and H is the height of the antenna above the earth. It is a plot of (5.38) as a function of θ . It is essential that H be greater than or equal to the distance to the antenna. The impedance Z_1 is quite small compared with the self impedance of a half wave symmetrical antenna. (See Kraus [5], page 101).

For an angle between the antenna and the vertical of 90° the value of Z_1 has been computed in the literature. Various spacings, referring to the curves in Kraus [5], for 200° for side by side antennas, and for 180° for end to end antennas, we find that for H greater than or equal to the length of the antenna, Z_1 is small and Z_2 is large.

Thus as a first approximation the direct point impedance of a trailing wire antenna over the earth can be estimated from the self impedance of the same antenna assumed to be a half wave antenna located in free space. All distances are in feet.

Figure 5.7--Mutual impedance of two symmetrically-fed half wave length thin dipole antennas

Figure 5.7-Mutual impedance of two asymmetrically-fed half-wave dipole antennas

close to the surface of the sea, equation (E.34) can be used to compute the effect of the sea on the driving point impedance.

Although the derivation in Appendix E was carried out for a symmetrically-fed thin linear antenna and its image, it can be quite easily extended to the asymmetrical case.

close to the surface of the sea, equation (E.54) can be used to compute the effect of the sea on the driving point impedance. Although the derivation in Appendix E was carried out for a symmetrically-fed thin linear antenna and its image, it can be quite easily extended to the asymmetrical case.

6. EXPERIMENTAL VERIFICATION OF THE THEORY

6.1 Introduction

This chapter contains a description of a series of measurements made on four different types of side-loaded transmission-line antennas and a presentation of the data obtained from these experiments. Also included for comparison are curves calculated from the theory developed in Chapters 2 and 3.

The application for which the antenna is being considered calls for frequencies in the range from 10 to 40 kc and antennas ranging from 1000 to 10,000 feet long. Also the antenna is to be dragged by a vehicle relatively small compared to the antenna in actual application. This latter condition makes it possible to analyze the antenna for the application as though it were located at least the length of the antenna above a conducting plane. In the experimental measurements, it was not possible to match the above conditions except by scaling down the antenna. The antennas used were in the range of a few feet long, and the frequency range was from 50 to 250 MC. Because the antennas were quite small, the measuring equipment was significant in size compared to that of the antenna creating a shielding problem. The details of these problems are discussed below. We mention them here only to indicate why the experimental work was so far removed

2.1. Introduction

This chapter contains a description of the apparatus

measurements made on two different types of antennas

theoretical and experimental results and a presentation of the data

obtained from these experiments. Also included for comparison

are curves calculated from the theory for the same

Chapters 3 and 4

The application for which the antenna is being considered

is for frequencies in the range from 10 to 100 Mc and

antennas ranging from 100 to 10,000 feet long. Also included

is to be discussed in a separate section the possibility of

using the antenna in actual applications. This latter

condition makes it possible to compare the antenna for the

application as though it were located at least 100 feet from

of the antenna above a conducting plane. In the experimental

measurements it was not possible to match the above conditions

except by scaling down the antenna. The antenna used

was in the range of a few feet long, and the frequency range

was from 50 to 250 Mc. Because the antennas were physically

the measuring equipment was significantly in size compared to

that of the antenna creating a shielding problem. The results

of these problems are discussed below. We mention them here

only to indicate why the experimental work was done.

from the application being considered. The comparison of theory and experiment is quite satisfactory to verify the theory even at these frequencies.

6.2 Description of the Experiments

The purpose of the experiments to be described was to measure the input impedance of and the radiation from a side-loaded transmission-line antenna to determine whether or not the theory and calculations of Chapters 2, 3, and 4 do actually predict the performances of such an antenna.

The first problem to be solved was one of physical size and space. As mentioned above the application for which the antenna is being considered calls for frequencies in the range from 10 to 40 kc and antennas ranging from 1000 to 10,000 feet long. Since no facilities were available for making measurements on antennas much longer than 5 or 10 feet, it was necessary to scale the length of the antenna down and the frequency up. Because of the availability of an admittance bridge with accessories operating in the frequency range from 50 to 250 MC, this frequency range was chosen as the modeling range. The approximate antenna length for this frequency range was 2 or 3 feet long which made the construction problem quite simple.

Another problem in carrying out the experiment was how to feed or drive the antenna. When used in the trailing wire application, the antenna would be much longer than the dimensions of the dragging vehicle making it possible to neglect the effects of the vehicle and analyze the antenna as though

from the application of the theory. The comparison of theory and experiment is made in the following table. The theory is shown in the first column and the experiment in the second column.

6.1. Description of the experiment

The purpose of the experiment was to measure the input impedance of the antenna at the frequency of 1000 kc. The antenna was a simple dipole antenna of length 10 feet. The antenna was connected to a bridge circuit which was used to measure the input impedance. The bridge circuit was a simple Wheatstone bridge. The antenna was connected to one of the terminals of the bridge. The other terminals of the bridge were connected to a standard impedance of 100 ohms. The bridge was balanced by adjusting the variable components of the bridge. The input impedance of the antenna was then determined from the balance of the bridge. The input impedance was measured at the frequency of 1000 kc. The results of the experiment are shown in the following table.

Frequency (kc)	Input Impedance (ohms)
1000	100

Another problem in carrying out the experiment was how to feed or drive the antenna. When used in the feeding wire application, the antenna would be much longer than the diameter of the feeding wire. This would make the antenna difficult to handle. The solution was to use a simple dipole antenna of length 10 feet. This antenna was easy to handle and could be used in the feeding wire application.

it were located at the appropriate height above a conducting plane. This process was described in Chapter 5. Since the experimental model was only a few feet long, the effects of the oscillators and measuring equipment as well as the experimenter himself could not be neglected. To eliminate this problem a conducting plane was built of brass screen measuring 8 feet by 10 feet. The measuring equipment was placed under the conducting plane, and the antenna was mounted directly above the conducting plane as shown in Figure 6.1. In this way, the effect of the experimenter and the measuring equipment are shielded from the antenna by the conducting plane and should not affect the impedance measurement. The conducting plane will affect the impedance measurement but this can be accounted for approximately using (E.33) of Appendix E which assumes the conducting plane is infinite.

The bridge used to measure the input impedance of the antenna was a General Radio Type 1602-B Admittance Meter which is a coaxial device and, therefore, has an unbalanced feed point to the load being measured. The side-loaded transmission-line antenna is basically a two wire transmission line, and, therefore, presents a balanced feed point. To match these two dissimilar systems a General Radio Type 874-UB Balun was used which is designed to impedance match an unbalanced system to a balanced system over a frequency range from 54 to 1000 MC. Due to the lack of accessories, the equipment used in the experiment was only operable from 60 to 300 MC, but this covered a major portion of the frequency range of interest. The balun is shown in Figure 6.1.

it were located at the appropriate height above a conducting plane. This process was described in Chapter 5. Since the experimental model was only a few feet long, the effects of the oscillators and measuring equipment as well as the experimenter himself could not be neglected. To eliminate this problem a conducting plane was built of brass screen measuring 8 feet by 10 feet. The measuring equipment was placed under the conducting plane, and the antenna was mounted directly above the conducting plane as shown in Figure 5.1. In this way, the effect of the experimenter and the measuring equipment are shielded from the antenna by the conducting plane and should not affect the impedance measurement. The conducting plane will affect the impedance measurement but this can be accounted for approximately using (5.33) of Appendix B, which assumes the conducting plane is infinite.

The bridge used to measure the input impedance of the antenna was a General Radio Type 1602-B Admittance Meter which is a coaxial device and, therefore, has an unbalanced feed point to the load being measured. The side-loaded transmission line antenna is basically a two wire transmission line, and, therefore, presents a balanced feed point. To match these two dissimilar systems a General Radio Type 87A-UB Balun was used which is designed to impedance match an unbalanced system to a balanced system over a frequency range from 24 to 1000 Mc. Due to the lack of accessories, the equipment used in this experiment was only operable from 60 to 500 Mc, but this covered a major portion of the frequency range of interest.

The balun is shown in Figure 5.1.

Figure 6.1--Arrangement of the experimental equipment

Impedance
matching
bain

Variable
50 ohm
airline

Figure 1-1-1

The next problem that was encountered was that of obtaining the appropriate side load impedances. For the trailing wire application, it would be desirable to keep the antenna as short as possible which would indicate the use of electrically short antennas probably in the range $\beta h = 0.2$ to $\beta h = 1.0$. In this range, the input reactance of the equivalent linear antenna is capacitive. This means that the side load must be inductive in order to take advantage of the resonant point of the antenna. Another requirement on the side load was that it be physically small compared with the dimensions of the antenna itself. These two requirements turned out to be almost mutually exclusive since any inductor with enough inductance to resonate with the input reactance of the equivalent linear antenna in the range from $\beta h = 0.2$ to $\beta h = 1.0$ and still be physically small compared to the dimensions of the antennas was not an inductance but rather an antiresonant circuit. This is due to the shunt capacitance which must necessarily be relatively large in a small closely wound inductor. This situation is, of course, true of any inductor, but in most cases the antiresonant frequency occurs far above the frequency range of interest. In the experiments being described here, all the available inductors had antiresonant frequencies lying in the frequency range of interest or just above.

One means of avoiding this difficulty was to abandon the electrically short antenna and make measurements on an antenna over the range from $\beta h = 0.5$ to $\beta h = 2.5$. In this range, the input reactance of the equivalent linear antenna is inductive

The next problem that was encountered was that of obtaining the optimum value of the coupling coefficient for the antenna with application, it would be desirable to keep the antenna as short as possible which would indicate the use of a relatively short antenna probably in the range $h = 0.5$ to $h = 1.0$. In this range, the input resistance of the equivalent linear antenna is resistive. This means that the load must be inductive in order to take advantage of the resonant point of the antenna. Another requirement on the side load is that it is essentially small compared with the dimensions of the antenna itself. These two requirements turned out to be almost mutually exclusive since any inductor with enough inductance to resonate with the input resistance of the equivalent linear antenna in the range from $h = 0.5$ to $h = 1.0$ will be physically small compared to the dimensions of the antenna and not an inductor but rather an antiresonant circuit. This is due to the short circuitance which must necessarily be relatively large in a small closely wound inductor. This situation is, of course, true of any inductor, but in most cases the antiresonant frequency occurs far above the frequency range of interest. In the experiments being described here, all the available inductors had antiresonant frequencies lying in the frequency range of interest or just above.

One means of avoiding this difficulty was to abandon the electrically short antenna and have measurement on an antenna over the range from $h = 0.5$ to $h = 2.5$. In this range, the input resistance of the equivalent linear antenna is inductive

above about $\beta h = 1.4$ making it possible to use a capacitive side load. Using values obtained from King [1, Section II.30] for the input impedance of the equivalent linear antenna, it was determined that a capacitance in the range from $0.5 \mu\mu f$ to $2 \mu\mu f$ would resonate with the input reactance of the equivalent linear antenna in the range of frequencies being considered. Since the voltage requirement on the side load was very small, it was not difficult to obtain suitably small capacitors with a capacitance in the range from $0.5 \mu\mu f$ to $2 \mu\mu f$.

The more straight forward way of attacking the side load reactance problem is to use inductors and take account of their shunt capacitance. This was done by measuring the impedance of the inductor on the bridge and from these measurements determining the inductance and capacitance of the equivalent antiresonant circuit. By choosing inductors whose antiresonant frequency was slightly above the range of interest, this made it possible to investigate the theory for electrically short antennas.

One further difficulty was a characteristic of the General Radio Type 1602-B Admittance Meter. In using the meter, it was observed that in situations where the imaginary part of the admittance was many times larger than the real part it was almost impossible to obtain an accurate measurement of the real part. Since this was the case over a large part of the region of interest, it was impossible to obtain good accuracy in measuring the real part of the antenna admittance with the equipment being used. Another contributing factor to this difficulty was the losses in the measuring equipment itself

 

which were larger than the losses in the antennas over part of the range of measurement.

So far, we have considered the equipment used and the problems involved in measuring the input impedance of the antenna. In order to obtain some information about radiation from the antenna, two measurements were made of the relative magnitude of the electric field parallel to the antenna. The equipment used was a NM-30A Radio Interference Field Intensity Meter made by Stoddart Aircraft Radio Co., Inc. One measurement was taken at a point 24 feet from the antenna as a function of frequency. The other measurement was taken at the fixed frequency of 120 MC as a function of distance from the antenna. Because of the limited extent of the ground plane and the fact that the measurements were made inside a room only 2 or 3 wavelengths long in its shortest dimensions at the low end of the frequency range, not too much significance can be attached to the results. By comparing them with similar measurements made on a transmission line of the same dimensions but with no side loads, they did give strong evidence that the side loads do make the transmission line a radiator and that this radiation is the greatest in the range of frequencies predicted by theory.

6.3 The Terminal or Feed Point Problem

In all the data that was taken in the first measurements on the antennas, the general character of the experimental curves seemed to agree with the theory, but they were always shifted in frequency by an amount that seemed too large to be

which were larger than the ones in the antenna... of the range of frequencies... as far as we have been able to determine... problems involved in measuring the... antenna. In order to obtain some... from the antenna, two... of the... equipment used was a... by... was taken at a point... of frequency. The other... of frequency of 120 Mc... antenna. Because of the... and the fact that the... 2 or 3 wavelengths long... and of the frequency... be attached to the... measurements made on a... along but with no... that the side loads... and that this radiation is... generated produced by...

6.3. The Terminal or Feed Point Problem

In all the data that was taken in the... on the antenna, the general... curves seemed to agree... of frequency by...

just experimental error. A search was made for some reason to explain this shift or some means to eliminate it. It was found that raising the feed point of the antenna off of the conducting plane by inserting a double coaxial transmission line between the balun and the antenna eliminated most of the shift and brought the experimental data into much closer agreement with the theory. This is illustrated in Figure 6.2. This suggested that the origin of the difficulty was at the feed point. By reviewing the derivation of the equation used to correct for the effect of the ground plane on the input impedance of the equivalent linear antenna, it was discovered that one of the terms in the reactive part of this mutual impedance resulted from the expression

$$\operatorname{Re} \int_0^{\infty} \frac{e^{-j\beta\sigma} \sin \beta\sigma}{\sigma} d\sigma = \frac{S_i(2\beta h)}{2}, \quad (6.1)$$

where the factor $\sin \beta\sigma$ is dependent on the fact that the currents I_1 and I_2 of Figure 6.3 are equal at the ground plane ($\sigma = 0$). Thus, the vector sum of these currents $I_T = I_1 - I_2$ is equal to zero at $\sigma = 0$. In order for this condition to be realized in the actual experiment, the balun would have to be perfectly balanced.

Due to the losses in the balun and due to the system used to tune it, it is practically impossible to obtain a perfect balance, and, thus, we would expect a slight unbalance in the currents I_1 and I_2 at $\sigma = 0$. The effect of this unbalance on expression (6.1) is as follows:

last experimental error. A search was made for some reason to explain this shift in some means to eliminate it. It was found that raising the feed point of the antenna off of the conducting plane by inserting a dielectric coaxial expansion line between the balun and the antenna eliminated most of the shift and brought the experimental data into much closer agreement with the theory. This is illustrated in Figure 2. This suggested that the origin of the difficulty was at the feed point. By revising the derivation of the equation used to correct for the effect of the ground plane on the input impedance of the equivalent linear antenna, it was discovered that one of the terms in the reactive part of this mutual impedance resulted from the expression

$$Z_{12} = \frac{j\omega \mu_0}{4\pi} \int_{-l}^l \int_{-l}^l \frac{e^{-jk_0 R}}{R} dz_1 dz_2 \quad (2.1)$$

where the factor $e^{-jk_0 R}$ is dependent on the fact that the currents I_1 and I_2 of Figure 1 are equal at the ground plane ($z = 0$). Thus, the vector sum of these currents $I_1 = I_2$ is equal to zero at $z = 0$. In order for this condition to be realized in the actual experiment, the balun would have to be perfectly balanced. Due to the losses in the balun and due to the system used to tune it, it is practically impossible to obtain a perfect balance, and, thus, we would expect a slight imbalance in the currents I_1 and I_2 at $z = 0$. The effect of this on balance on expression (2.1) is as follows:

Figure 6.2--The location of the double coaxial transmission line

Figure 6.1--The location of the double coaxial transmission line

line

Figure 6.3--The currents in the terminal or feed zone

Figure 6.7 - The currents in the antenna or feed zone

$$\begin{aligned}
\operatorname{Re} \int_0^h \frac{e^{-j\beta\sigma} \sin(\beta\sigma + a) d\sigma}{\sigma} &= \operatorname{Re} \cos a \int_0^h \frac{e^{-j\beta\sigma} \sin \beta\sigma d\sigma}{\sigma} \\
&+ \sin a \int_0^h \frac{e^{-j\beta\sigma} \cos \beta\sigma d\sigma}{\sigma} \\
&= \frac{\cos a S_i(2\beta h)}{2} + \sin a \int_0^h \frac{\cos^2 \beta\sigma}{\sigma} d\sigma \quad . \quad (6.2)
\end{aligned}$$

The second term in (6.2) clearly increases without limit due to the lower limit of the integral being zero. Equations (6.1) and (6.2) are based on the assumption that the currents I_1 and I_2 are superimposed and exist as a filament with zero diameter. As Figure 6.3 illustrates this is not actually the case, but rather they are distributed over two wires separated by a distance small with respect to wavelength. Thus in the experiment, we might expect the second term in (6.2) to be large for a small unbalance in the currents I_1 and I_2 at $\sigma = 0$ but not unbounded.

Since it was not possible to measure the unbalance in the currents I_1 and I_2 [the "a" of (6.2)] and since the expression (6.2) does not accurately represent the physical configuration in the region near $\sigma = 0$, it was not possible to calculate accurately the true effect of the ground plane on the input impedance of the equivalent linear antenna. To get some experimental check on the above concept, it was determined empirically what value of mutual impedance was required to make the theoretical curves fit the experimental

The second term in (6.2) clearly increases without limit due
 to the lower limit of the integral being zero. Equations
 (6.1) and (6.2) are based on the assumption that the currents
 I_1 and I_2 are unidirectional and exist as a filament with zero
 diameter. As Figure 6.5 illustrates this is not actually the
 case, but rather they are distributed over two wires separated
 by a distance small with respect to wavelength. Thus in the
 experiment, we might expect the second term in (6.2) to be
 large for a small unbalance in the currents I_1 and I_2 at
 $a = 0$ but not unbounded.
 Since it was not possible to measure the unbalance in
 the currents I_1 and I_2 at the "a" of (6.2) and since the
 expression (6.2) does not accurately represent the physical
 configuration in the region near $a = 0$, it was not possible
 to calculate accurately the true effect of the ground plane
 on the input impedance of the equivalent linear antenna. To
 get some experimental check on the above concept, it was
 determined empirically what value of mutual impedance was
 required to make the theoretical curves fit the experimental

$$\begin{aligned}
 \text{Re} \int_0^h e^{-\gamma z} \cos \alpha z \, dz &= \frac{1 - e^{-\gamma h} \cos \alpha h}{\gamma^2 + \alpha^2} \\
 + \text{Im} \int_0^h e^{-\gamma z} \sin \alpha z \, dz &= \frac{1 - e^{-\gamma h} \sin \alpha h}{\gamma^2 + \alpha^2}
 \end{aligned}
 \tag{6.2}$$

data.¹ The reactive part of this mutual impedance turned out to be much larger than the value predicted by the mutual impedance expression using the term (6.1). This confirmed the idea that a term of the type (6.2) was necessary. It might be mentioned that the remainder of the terms in the mutual impedance expression were small in comparison to (6.1).

Next, this same value of mutual impedance was used to calculate the theoretical curves for two completely different side-loaded transmission-line antennas. The curves calculated in this way were found to agree very well with the experimental data tending to confirm the above concept. The remainder of the experimental antennas were measured using a length of double coax between the balun and the antenna as illustrated in Figure 6.2. As mentioned above, this greatly reduces the mutual effects of the conducting plane and thus brings the theoretical curves and the experimental data into good agreement without the need for the correction term.

6.4 Description of the Antennas Measured and the Results Obtained

Four different antennas are described in this section. For all four antennas the end load is a short, and the wire

¹This was done using the susceptance plots rather than reactance plots. The reason for this is that the meter measures susceptance directly, and these susceptance values are relatively independent of the real part measurements. If reactance plots had been used, it would have been necessary to convert the susceptance to reactance using the measured real parts which as explained above would introduce a large random error. Thus, Figures 6.7, 6.8, 6.11, 6.12, 6.15 and 6.16 are susceptance and conductance plots.

date. The positive part of this model is shown in Figure 1.

As a first step, the model was fitted to the data shown in Figure 1.

and the results are shown in Figure 2. The model is shown in Figure 3.

that a value of $\alpha = 0.1$ was necessary. It might be noted

that the value of α is the same as the value of α in Figure 1.

experiments were made at $\alpha = 0.1$ and $\alpha = 0.2$.

that, this value of α is the same as the value of α in Figure 1.

estimate the theoretical curves for two different values of α .

side-by-side comparison of the experimental and theoretical curves is shown in Figure 4.

in this case, the theoretical curves are shown in Figure 5.

data leading to confirm the above concept. The results are shown in Figure 6.

the experimental curves were plotted against a family of curves.

double curve between the data and the theoretical curves is shown in Figure 7.

in Figure 8. As mentioned above, this curve is shown in Figure 9.

normal effects of the concentration of the solution on the rate of reaction.

theoretical curves and the experimental curves are shown in Figure 10.

need without the need for the concentration of the solution.

theoretical curves and the experimental curves are shown in Figure 11.

theoretical curves and the experimental curves are shown in Figure 12.

theoretical curves and the experimental curves are shown in Figure 13.

for all four solutions and for all four values of α , and the results are shown in Figure 14.

theoretical curves and the experimental curves are shown in Figure 15.

This was done using the same method as in Figure 14.

reaction rate. The reason for this is that the reaction rate is

independent of the initial concentration of the reactants. It is

also independent of the initial concentration of the reactants. It is

also independent of the initial concentration of the reactants. It is

used for the conductors is No. 12 enamel coated copper wire with radius $a = .0404$ inches. The first antenna is 22.5 inches long and has a wire separation of 1.25 inches. This gives a length to conductor radius ratio of 557 and a conductor separation to conductor radius ratio of 30.9. The resulting value of R_c is 414 ohms. The side load was 1.5 μ h inductor. Using admittance data measured on the G. R. Bridge, the equivalent circuit was computed to be an inductance of 1.5 μ h in parallel with a capacitance of 0.637 μ f. The calculated admittance curve is compared with the data points in Figure 6.4. It can be seen that the inductor has a pole of impedance at 162 MC.

Using the above data and the theoretical expressions developed in Chapter 2, the curves of Figures 6.5 and 6.6 were calculated. The mutual impedance of the ground plane for these curves was calculated using (6.1). It can be seen that the experimental data taken with the antenna connected directly to the balun does not match the theoretical curves. The experimental data taken with 8.5 inches of double coax inserted between the balun and the antenna, however, match the theoretical curves quite well especially for the imaginary part. Using the corrected value for mutual impedance as explained in Section 6.3, the theoretical curves of Figures 6.7 and 6.8 were calculated. These curves are seen to corroborate very well with the experimental data taken with the antenna connected directly to the balun.

The second antenna is 37.5 inches long and has a conductor separation of 1 inch. This gives a length to conductor radius ratio of 927 and a conductor separation to conductor radius

used for the condenser. The condenser was
with a volume of 100 ml. The condenser was
long and had a wide diameter. The condenser
length to diameter ratio was 1.5. The condenser
ratio to diameter was 1.5. The condenser was
of 1.5. The condenser was 1.5. The condenser
admittance data were calculated. The condenser
circuit was compared with the condenser
with a capacitance of 1.5. The condenser
curve is compared with the condenser
be seen that the condenser is 1.5. The condenser
Using this curve, the condenser is 1.5. The condenser
developed in Chapter 1. The condenser is 1.5. The condenser
calculated. The condenser is 1.5. The condenser
curves was calculated. The condenser is 1.5. The condenser
experimental data. The condenser is 1.5. The condenser
to the data does not agree. The condenser is 1.5. The condenser
experimental data. The condenser is 1.5. The condenser
between the data and the condenser. The condenser is 1.5. The condenser
and curves give well. The condenser is 1.5. The condenser
the corrected value for the condenser. The condenser is 1.5. The condenser
0.5. The theoretical curve for the condenser. The condenser is 1.5. The condenser
calculated. The condenser is 1.5. The condenser
the experimental data. The condenser is 1.5. The condenser
to the data. The condenser is 1.5. The condenser
The second curve. The condenser is 1.5. The condenser
separation of 1.5. The condenser is 1.5. The condenser

Figure 6.4--Susceptance curve of the 1.5 μh inductor

Figure 6.4--Successive curves of the 1.5V H Inductor

Figure 6.5--Input reactance of experimental antenna 1

X-Data taken with 8.5° of double coax

O-Data taken with the antenna connected directly to the balun

Figure 0.5- Impedance of experimental antenna

Figure 6.6--Input resistance of experimental antenna 1

log
10²

0

0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0

Figure 6.0

Figure 6.7--Input susceptance corrected for terminal zone effects experimental antenna 1

$\mu = 1.25$
 $\sigma = 0.05$
 $\tau = 1.25$

Figure 6. - Input resistance corrected for terminal
 loss effects experimental error.

Figure 6.8--Input conductance corrected for terminal zone effects experimental antenna 1

ratio of 24.8. The resulting value of R_c is 385 ohms. The side load was a 1 $\mu\mu\text{f}$ capacitor and can be considered a pure capacitance. The theoretical curves of Figures 6.9 and 6.10 were calculated using the uncorrected value for the mutual impedance. As with antenna No. 1, they are seen to agree quite well with the experimental data taken with 5 inches of double coax inserted between the balun and the antenna, but the agreement is poor for the experimental data taken with the antenna connected directly to the balun. Using the same corrected value for mutual impedance that was determined empirically using antenna No. 1, the theoretical curves of Figures 6.11 and 6.12 were calculated for antenna No. 2. They agree quite well with the experimental data taken with the antenna connected directly to the balun. From observing Figure 6.9, we see that the input reactance of this antenna, when relatively unaffected by the terminal or feed zone, has two poles. One of these poles is caused by the shorted length of transmission line stretching from the side load to the end of the antenna, and the other is the result of the side load resonating with the input impedance of the equivalent linear antenna, Z_d . Theoretically there is another pole in this reactance function at an even lower frequency. This is due to the fact that Z_d is itself equivalent to a series resonant circuit because βh exceeds 1.4 for this antenna. Because we could not make experimental measurements below 60 MC, the theoretical calculations were not extended below 50 MC and so this third pole does not appear.

ratio of 2.48. The resulting value of R_2 is 385 ohms. The side load was a 1 μ F capacitor and can be considered a pure capacitance. The theoretical curves of Figures 6.9 and 6.10

were calculated using the uncorrected value for the mutual

inductance. As with antenna No. 1, they are seen to agree

quite well with the experimental data taken with 5 inches of

double coax inserted between the balun and the antenna, but

the agreement is poor for the experimental data taken with the

antenna connected directly to the balun. Using the same

corrected value for mutual inductance that was determined empiri-

cally using antenna No. 1, the theoretical curves of Figures

6.11 and 6.12 were calculated for antenna No. 2. They agree

quite well with the experimental data taken with the antenna

connected directly to the balun. From observing Figure 6.9,

we see that the input resistance of this antenna, when re-

latively unaffected by the terminal or feed zone, has two poles.

One of these poles is caused by the shortened length of trans-

mission line stretching from the side load to the end of the

antenna, and the other is the result of the side load reso-

ning with the input impedance of the equivalent linear

antenna, Z_d . Theoretically there is another pole in this

resonance function at an even lower frequency. This is due

to the fact that Z_d is itself equivalent to a series resonant

circuit because it exceeds 1 μ for this antenna. Because we

could not make experimental measurements below 50 MC, the

theoretical calculations were not extended below 50 MC and

so this third pole does not appear.

Figure 6.9--Input reactance of experimental antenna 2

$\epsilon = 1$
 $\mu = 1$
 $\gamma = 0$

Data taken with 2 of
 3000 cm
 Data taken with the
 antenna connected
 directly to the balun

Figure 3.9--Input reactance of experimental antenna 2

Figure 6.10--Input resistance of experimental antenna 2

Figure 6.10-Input resistance of experimental tubes 2

Figure 6.11--Input susceptance corrected for terminal zone effects experimental antenna 2

Figure 9-1. Potential vs. distance for a system with a dielectric constant of 1.75.

Figure 6.12--Input conductance corrected for terminal zone effects experimental antenna 2

Figure 6.15 - Typical conductance curves for a central zone effect experimental antenna.

From Figure 6.11 we observe that one of the poles of the input susceptance of the antenna is reduced to merely a hump in the curve when the antenna is connected directly to the balun. This is due to the large mutual effect of the ground plane and the losses in the system.

The third antenna has the same physical dimensions as antenna No. 1 and, thus, the same value of R_c . It is a double side-loaded transmission-line antenna, however, with both side loads located at the center of the antenna, one on each side of the two wire transmission line. One of the side loads was the same as the side load used in antenna No. 1. Using the same method that was described in connection with antenna No. 1, it was determined that the other side load was equivalent to to a $0.965 \mu\text{h}$ inductance in parallel with a $0.153 \mu\mu\text{h}$ capacitance.

The theoretical curves of Figures 6.13 and 6.14 were calculated using the uncorrected value for the mutual impedance. They agree quite well with the experimental data taken with 5 inches of double coax inserted between the balun and the antenna, but the agreement is poor for the experimental data taken with the antenna connected directly to the balun. Using the same corrected value for mutual impedance that was determined empirically using antenna No. 1, the theoretical curves of Figures 6.15 and 6.16 were calculated for antenna No. 3. They agree quite well with the experimental data taken with the antenna connected directly to the balun.

For this antenna, the large mutual impedance of the ground plane shifts one of the poles out of the range of

From Figure 6.11 we observe that one of the poles of the

input susceptibility of the antenna is located at a distance

in the curve when the antenna is connected directly to the

balun. This is due to the fact that the antenna is connected

plane and the losses in the system.

The third antenna has the same physical dimensions as

antenna No. 1 and, thus, the same value of R . It is a

side-loaded transmission line antenna. However, the antenna

loads located at the center of the antenna. The antenna

of the two wire transmission line. One of the side loads was

the same as the side load used in antenna No. 1. The

same method that was described in connection with antenna

it was determined that the other side load was equivalent to

to a 0.905 ohm inductance in parallel with a 0.1 ohm resistance.

ance.

The theoretical curves for antenna No. 1 and antenna No. 2

calculated using the uncorrected values for the antenna

ance. They agree quite well with the experimental data

with 5 inches of double coax inserted between the balun and

the antenna, but the agreement is poor for the experimental

data taken with the antenna connected directly to the balun.

Using the same corrected values for antenna No. 1 and antenna

determined empirically using antenna No. 1, the theoretical

curves of Figures 6.12 and 6.13 were calculated for antenna

No. 2. They agree well with the experimental data taken

with the antenna connected directly to the balun.

For this antenna, the theoretical agreement is poor

ground plane antenna one of the poles of the antenna

Figure 6.13--Input reactance of experimental antenna 3

$\frac{1}{2} \times 10^{-3}$ m
 $\frac{1}{2} \times 10^{-3}$ m
 $\frac{1}{2} \times 10^{-3}$ m

X-axis taken with 2 or double
 C-axis taken with the antenna
 centered directly to the

1.00
 0.80
 0.60
 0.40
 0.20
 0.00
 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
 C

Figure 6-13-10: Results of experimental antenna

Figure 6.14--Input resistance of experimental antenna 3

$C_1 = 0.001 \mu F$ $L = 1.5 mH$

$C_2 = 0.001 \mu F$ $L = 0.001 mH$

$\omega = 1.5 \times 10^6$

$\omega = 0.5 \times 10^6$

$\omega = 1.5 \times 10^6$

X-data taken with
2" double coil

O-data taken with
the antenna
connected directly
to the balun

Figure 8-11- Impedance response of experimental antenna

Figure 6.15--Input susceptance corrected for terminal zone effects experimental antenna 3

Figure 6.15--Input admittance corrected for terminal loss effects experimental scheme

Figure 6.16--Input conductance corrected for terminal zone effects experimental antenna 3

Figure 6.16 - Input conductance corrected for parasitic effects experimental results

measurements. Even for the case where this effect is greatly reduced, however, the second pole is so sharp that the losses in the system reduce it to merely a hump in the reactance curve. That it really is a pole is clear from the hump in the curve of the real part of the input impedance in the same frequency range.

The fourth antenna, like the third, has the same physical dimensions as antenna No. 1 and, thus, the same value of R_c . It is also a double side-loaded transmission-line antenna, except that, both side loads are located on the same side of the two wire transmission line. They are symmetric with respect to the center of the antenna and separated by one fourth the length of the antenna. One of the side loads is a $0.965 \mu h$ inductance in parallel with a $0.153 \mu\mu f$ capacitance, and the other is a $2.34 \mu h$ inductance in parallel with a $0.140 \mu\mu h$ capacitance.

The theoretical curves of Figures 6.17 and 6.18 were calculated using the uncorrected value for mutual impedance. They agree quite well with the experimental data taken with 4 inches of double coax inserted between the balun and the antenna. Observing these curves, we see that in the region of $\beta h = 0.7$ the reactive part of the input impedance passes through zero with a relatively gentle slope and that the real part of the input impedance is four times larger than the real part of the input impedance of the equivalent radiating linear antenna, R_d , and has a relatively small slope with respect to frequency. This point therefore meets the requirements set up in Chapter 4 for a desirable operating point.

measurements. Even for the case where this effect is greatly reduced, however, the second point is so sharp that the losses in the system reduce it to nearly a hump in the resistance curve. That it really is a point is clear from the hump in the curve of the real part of the input impedance in the same frequency range.

The fourth antenna, like the third, has the same physical dimensions as antenna No. 1 and, thus, the same value of R_0 . It is also a double side-loaded transmission-line antenna except that both side loads are located on the same side of the two wire transmission line. They are symmetric with respect to the center of the antenna and separated by one fourth the length of the antenna. One of the side loads is a 0.955 μ h inductance in parallel with a 0.155 μ mf capacitance, and the other is a 2.34 μ h inductance in parallel with a 0.140 μ mf

capacitance. The theoretical curves of Figures 6.17 and 6.18 were calculated using the uncorrected value for mutual impedance. They agree quite well with the experimental data taken with 4 inches of double coax inserted between the balun and the antenna. Observing these curves, we see that in the region of $\beta x = 0.7$ the reactive part of the input impedance passes through zero with a relatively gentle slope and that the real part of the input impedance is four times larger than the real part of the input impedance of the equivalent radiating linear antenna, R_0 , and has a relatively small slope with respect to frequency. This point therefore meets the requirements set up in Chapter 4 for a desirable operating point.

Figure 6.17--Input reactance of experimental antenna 4

$C_1 = 0.140 \mu F$ $L_1 = 2.34 \mu H$
 $C_2 = 0.153 \mu F$ $L_2 = 0.965 \mu H$
 $h = 11.25$
 $a = 40.4$ $mils$ $b = 1.25$

X-Data taken with h of double coax

Figure 17 - Input reactance of experimental antenna

Figure 6,18--Input resistance of experimental antenna 4

Figure 6, 18--Input resistance of experimental antenna

The last experimental data to be presented are the relative field measurements described in Section 6.2. The antenna used was antenna 2, and the results are plotted in Figures 6.19 and 6.20. Comparing Figure 6.19 with Figures 6.11 and 6.12, we observe that in the region of $\beta h = 1.2$, where the real part of the input admittance is large and the imaginary part is passing through a hump caused by the side load resonating with the input impedance of the equivalent linear antenna, the relative magnitude of the field strength 24 feet from the antenna is a maximum. This is just what the theory would predict since this is precisely the region where the currents on the antenna should have a maximum unbalance, and therefore, we would expect maximum radiation. The dashed line in Figure 6.19 is the field strength measured with the antenna replaced by a two wire transmission line of precisely the same dimensions. Comparing the two curves gives a clear indication that inserting the side load does indeed enhance the radiating properties of the transmission line. The comparison above was made with Figures 6.11 and 6.12 rather than Figures 6.9 and 6.10 because the antenna was connected directly to the balun for the relative field intensity measurements.

Figure 6.20 shows the variation of the relative magnitude of the electric field intensity with distance from the antenna, r . For comparison, curves for a $1/r$ variation and a $1/r^2$ variation are plotted also. Since the measured curves compared quite closely with a $1/r^2$ variation in the region from 24 feet out to 48 feet, it would appear that we are measuring in a transition region between the near fields and the far fields.

The last experimental data to be presented are the relative

field measurements described in Section 3.2. The antenna

used was antenna 2, and the results are plotted in Figures 3.10

and 3.11. Comparing Figure 3.10 with Figures 3.11 and 3.12,

we observe that in the region of $z = 1.1$, where the real part

of the input admittance is large and the imaginary part is

passing through a sharp change, the side load resonance with

the input impedance of the equivalent linear antenna, the relative

field magnitude of the field strength is less than the antenna

is a maximum. This is just what the theory would predict since

this is precisely the region where the currents on the antenna

should have a maximum admittance, and therefore, we would expect

maximum radiation. The dashed line in Figure 3.10 is the first

strength measured with the antenna replaced by a two wire transmission

line of precisely the same dimensions. Comparing the

two curves gives a clear indication that the side load antenna

load does indeed enhance the radiating properties of the antenna

resistor line. The comparison above was made with Figures 3.11

and 3.12 rather than Figures 3.9 and 3.10 because the antenna

was connected directly to the balun for the relative field

intensity measurements.

Figure 3.10 shows the variation of the relative magnitude

of the electric field intensity with distance from the antenna

for comparison, curves for a $1/r$ variation and a $1/r^2$

variation are plotted also. Since the measured curves compared

quite closely with a $1/r^2$ variation in the region from 10 feet

out to 20 feet, it would appear that we are measuring in a

transmission region where the field is a function of distance

Figure 6.19--Relative magnitude of electric field as a function of the electrical length of the antenna for experimental antenna 2

Figure 6.19. Relative magnitude of electric field as a function of the electrical length of the antenna for experimental antenna.

Figure 6.20--Relative magnitude of electric field as a function of distance from the antenna for experimental antenna 2

Frequency 150 Mc

Distance from antenna to point of field measurement in feet
 1.5 2.0 2.5 3.0 3.5 4.0 4.5 5.0 5.5 6.0 6.5 7.0 7.5 8.0 8.5 9.0 9.5 10.0 10.5 11.0 11.5 12.0 12.5 13.0 13.5 14.0 14.5 15.0 15.5 16.0 16.5 17.0 17.5 18.0 18.5 19.0 19.5 20.0 20.5 21.0 21.5 22.0 22.5 23.0 23.5 24.0 24.5 25.0 25.5 26.0 26.5 27.0 27.5 28.0 28.5 29.0 29.5 30.0 30.5 31.0 31.5 32.0 32.5 33.0 33.5 34.0 34.5 35.0 35.5 36.0 36.5 37.0 37.5 38.0 38.5 39.0 39.5 40.0

Figure 6-20-Relative magnitude of electric field as a function of distance from the antenna for experimental antenna 2

Before making any strong arguments about this last data, however, it should be kept in mind that the measurements were made inside a room whose shortest dimension was only 2 or 3 wavelengths long.

Before making any strong statements about this last data.

However, it should be kept in mind that the measurements

were made inside a room whose greatest dimension was only

3 or 4 wavelengths long.

APPENDIX A

In this appendix, we show first of all that (2.48 and 2.49) are indeed the integral equations describing the asymmetrically-driven linear antenna of Figure 2.3. Next, we use an approximate method developed by King [1, Section III.29] to evaluate the input impedance of this asymmetrically-driven linear antenna in terms of the impedances of two center-fed linear antennas. Values for these latter impedances are available from tables in King's book. [1, Section II.30].

Referring to Figure 2.3 and using the same approximations that were used to arrive at (2.3, 2.4, and 2.5) of Chapter 2, we obtain the following equations for the vector potential on each member.

$$A_1(z) = -\frac{j}{V}[C_{11}\sin \beta z + C_{12}\cos \beta z] \quad (A.1)$$

$$A_2(z) = -\frac{j}{V}[C_{21}\sin \beta z + C_{22}\cos \beta z] \quad (A.2)$$

Using (2.6) and applying the boundary conditions on ϕ and the vector potential at $z = \ell_1$ in the same manner as was done in Chapter 2, we find

$$\frac{V_1}{2} = \phi_{11}(\ell_1) - \phi_{12}(\ell_1) = (C_{11} - C_{21})\cos \beta \ell_1 - (C_{12} - C_{22})\sin \beta \ell_1 \quad (A.3)$$

and

$$A_1(\ell_1) = A_2(\ell_1) \quad (A.4)$$

or

$$(C_{11} - C_{21})\sin \beta \ell_1 = - (C_{12} - C_{22})\cos \beta \ell_1 \quad (A.5)$$

In this appendix, we show that the linear antenna in terms of the input impedance of the antenna is available from tables in the literature. Referring to Figure 2, and using the same notation that were used to arrive at (2.1) and (2.2), we obtain the following equations for the vector potential on each member.

$$A_1(z) = -\frac{j}{4\pi} \int_{-L}^L \frac{e^{-jk_0 R}}{R} dz$$

$$A_2(z) = -\frac{j}{4\pi} \int_{-L}^L \frac{e^{-jk_0 R}}{R} dz$$

Using (2.3) and applying the boundary conditions at the vector potential at $z = \pm L$ in the same manner as in Chapter 2, we find

$$\frac{V_1}{Z} = \frac{1}{2} (A_1(L) - A_1(-L)) - \frac{1}{2} (A_2(L) - A_2(-L))$$

and

$$A_1(L) = A_2(L)$$

or

$$(C_{11} - C_{21}) \sin k_0 L = 0$$

Solving (A.3 and A.5) for C_{11} and C_{12} in terms of C_{21} and C_{22} and substituting the result back in (A.1), we obtain

$$A_1(z) = -\frac{j}{V} \left[\left\{ C_{21} + \frac{V_1}{2} \cos \beta \ell_1 \right\} \sin \beta z + \left\{ C_{22} - \frac{V_1}{2} \sin \beta \ell_1 \right\} \cos \beta z \right] \quad (A.6)$$

To obtain the integral representation of the vector potential on the surface of the antenna, we refer to (2.23) of Chapter 2 and write

$$A(z) = \frac{\mu_0}{4\pi} \int_0^{2h} I_T(z') K_d(z, z') dz' \quad , \quad (A.7)$$

where

$$I_T(z) = \begin{cases} I_{T1}(z) & \text{for } 0 \leq z \leq \ell_1 \\ I_{T2}(z) & \text{for } \ell_1 \leq z \leq \ell_2 \end{cases} \quad (A.8)$$

and

$$A(z) = \begin{cases} A_1(z) & \text{for } 0 \leq z \leq \ell_1 \\ A_2(z) & \text{for } \ell_1 \leq z \leq \ell_2 \end{cases} \quad (A.9)$$

Equating (A.2 and A.6) to (A.7), we find

$$\int_0^{2h} I_T(z') K_d(z, z') dz' = j \frac{4\pi}{\eta} \left[- \left\{ C_{21} + \frac{V_1}{2} \cos \beta \ell_1 \right\} \sin \beta z - \left\{ C_{22} - \frac{V_1}{2} \sin \beta \ell_1 \right\} \cos \beta z \right] \quad (A.10)$$

for $0 \leq z \leq \ell_1$, and

$$\int_0^{2h} I_T(z') K_d(z, z') dz' = j \frac{4\pi}{\eta} \left[- C_{21} \sin \beta z - C_{22} \cos \beta z \right] \quad (A.11)$$

for $\ell_1 \leq z \leq \ell_2$.

Solving (A.5) and (A.6) for C_{11} and C_{12} in terms of C_{21} and C_{22} and substituting the result back in (A.1), we obtain

$$A_1(z) = -\frac{1}{V} \left[C_{11} + \frac{V}{2} \cos \beta_1 \left\{ C_{21} + \frac{1}{2} \sin \beta_1 \right\} \cos \beta_2 \right]. \quad (A.9)$$

To obtain the integral representation of the vector

potential on the surface of the antenna, we refer to (2.22)

of Chapter 2 and write

$$A(z) = \frac{1}{V} \int_{-L}^L I_T(z') K(z, z') dz' \quad (A.7)$$

where

$$I_T(z) = \begin{cases} I_{T1}(z) & \text{for } 0 \leq z \leq L_1 \\ I_{T2}(z) & \text{for } L_1 \leq z \leq L_2 \end{cases} \quad (A.8)$$

and

$$A(z) = \begin{cases} A_1(z) & \text{for } 0 \leq z \leq L_1 \\ A_2(z) & \text{for } L_1 \leq z \leq L_2 \end{cases} \quad (A.9)$$

Evaluating (A.7) and (A.8) to (A.7), we find

$$\begin{aligned} & \int_{-L}^L I_T(z') K(z, z') dz' = \frac{1}{V} \left[C_{21} + \frac{1}{2} \cos \beta_1 \left\{ C_{21} + \frac{1}{2} \sin \beta_1 \right\} \cos \beta_2 \right] \\ & \left\{ C_{22} - \frac{1}{2} \sin \beta_1 \cos \beta_2 \right\} \end{aligned} \quad (A.10)$$

for $0 \leq z \leq L_1$, and

$$\int_{-L}^L I_T(z') K(z, z') dz' = \frac{1}{V} \left[C_{21} \sin \beta_2 + C_{22} \cos \beta_2 \right] \quad (A.11)$$

for $L_1 \leq z \leq L_2$

Comparing (A.10 and A.11) with (2.48 and 2.49), we see that they are identical except for the designation of the arbitrary constants which is not important.

Since (2.48 and 2.49) are identical with (A.10 and A.11) for the asymmetrically-fed linear antenna of Figure 2.3, the ratio (2.50) is equal to the input impedance of that antenna. Using King's approximate method we evaluate Z_d in the following manner. By means of a series of graphs King shows that in determining the distribution of current in the two parts of an asymmetrically-driven linear antenna each may be analyzed separately as if it were a base-driven antenna over an infinite, perfectly conducting plane without introducing appreciable error. The impedance of each of these base driven antennas is equal to one half the value of the impedance of a center-fed linear antenna of the same length, and the impedance of the asymmetric antenna is just the series combination of the impedances of the two base driven antennas. Thus for our situation in Figure 2.3, if Z_{d1} is the input impedance of a center-fed linear antenna of half length ℓ_1 and radius d , and Z_{d2} is the input impedance of a center-fed linear antenna of half length ℓ_2 then

$$Z_d = \frac{Z_{d1} + Z_{d2}}{2} . \quad (A.12)$$

Since values for Z_{d1} and Z_{d2} can be found in the tables in King's book [1, Section II.30], we have an approximate method for evaluating the impedance of an asymmetrically-driven linear antenna.

APPENDIX B

In this appendix, we show first of all that (3.37) are indeed the integral equations describing the multiply-fed linear antenna of Figure 3.2. Next, we use a superposition method presented by King [1, Section III.30] to evaluate the input impedances of the multiply-fed linear antenna in terms of the input impedances and certain current ratios of a group of asymmetrically-driven linear antennas. A method for evaluating the impedances of the asymmetrically-driven linear antennas is presented in Appendix A, and we evaluate the current ratios by assuming sinusoidal current distributions.

Referring to Figure 3.2 and using the same approximations that were used to arrive at (2.3, 2.4, and 2.5) of Chapter 2, we obtain the following equations for the vector potential on each member.

$$A_n(z) = -\frac{j}{v} \left[C_{n1} \sin \beta z + C_{n2} \cos \beta z \right], \quad (B.1)$$

where $n = 1, 2, 3 \dots m+1$. Using (2.6) and applying the boundary conditions on ϕ and the vector potential at $z = \ell_n$ for $n = 1, 2, 3 \dots m$ in the same manner as was done in Chapter 3, we find

$$\frac{V_n}{2} = (C_{n1} - C_{n+1,1}) \cos \beta \ell_n - (C_{n2} - C_{n+1,2}) \sin \beta \ell_n \quad (B.2)$$

$$(C_{n1} - C_{n+1,1}) \sin \beta \ell_n = - (C_{n2} - C_{n+1,2}) \cos \beta \ell_n \quad (B.3)$$

for $n = 1, 2, 3 \dots m$. Solving (B.2) and (B.3) for C_{n1} and C_{n2} in terms of $C_{n+1,1}$ and $C_{n+1,2}$ and substituting the result in (B.1), the result is

APPENDIX B

In this appendix, we show that (2.27) are indeed the integral equations describing the multipole linear antenna of Figure 2.2. Next, we use a superposition method presented by King [1, Section III.50] to evaluate the input impedances of the multipole-fed linear antenna in terms of the input impedances and current ratios of a group of asymmetrically-driven linear antennas. A method for evaluating the impedances of the asymmetrically-driven linear antenna is presented in Appendix A, and we evaluate the current ratios by assuming sinusoidal current distributions. Referring to Figure 2.2 and using the same approximations that were used to arrive at (2.3), (2.4), and (2.5) of Chapter 2, we obtain the following equations for the vector potential on each member

$$A_n(z) = \frac{1}{\sqrt{2}} \left[C_n \sin \beta z + C_n \cos \beta z \right] \quad (B.1)$$

where $n = 1, 2, \dots, m-1$. Using (2.6) and applying the boundary conditions on A and the vector potential at $z = 0$ for $n = 1, 2, \dots, m$ in the same manner as was done in Chapter 2, we find

$$\frac{V_n}{Z} = (C_{n-1} - C_{n+1}) \cos \beta z - (C_n - C_{n-2}) \sin \beta z \quad (B.2)$$

$$(C_{n-1} - C_{n+1}) \sin \beta z - (C_n - C_{n-2}) \cos \beta z \quad (B.3)$$

for $n = 1, 2, \dots, m$. Solving (B.2) and (B.3) for C_n and C_{n+1} in terms of C_{n-1} and C_{n-2} and substituting the result in (B.1), the result is

$$A_n(z) = -\frac{j}{v} \left[\left\{ C_{m+1,1} + \sum_{p=n}^m \frac{V_p}{Z} \cos \beta \ell_p \right\} \sin \beta z + \left\{ C_{m+1,2} - \sum_{p=n}^m \frac{V_p}{Z} \sin \beta \ell_p \right\} \cos \beta z \right] \quad (B.4)$$

for $n = 1, 2, 3 \dots m+1$,

$$\text{where } \sum_{p=n}^m \frac{V_p}{Z} \cos \beta \ell_p = \sum_{p=n}^m \frac{V_p}{Z} \sin \beta \ell_p = 0 \text{ for } n = m+1.$$

To obtain the integral representation of the vector potential on the surface of the antenna, we refer to (2.23) of Chapter 2 and write

$$A(z) = \frac{\mu_0}{4\pi} \int_0^{2h} I_T(z') K_d(z, z') dz' \quad , \quad (B.5)$$

where

$$I_T(z) = I_{Tn}(z) \quad (B.6)$$

$$A(z) = A_n(z) \quad (B.7)$$

for $\ell_{n-1} \leq z \leq \ell_n$, and where $n = 1, 2, 3 \dots m+1$, $\ell_0 = 0$ and $\ell_{m+1} = 2h$. Equating (B.4) to (B.5), we find

$$\int_0^{2h} I_T(z') K_d(z, z') dz' = j \frac{4\pi}{\eta} \left[- \left\{ C_{m+1,1} + \sum_{p=n}^m \frac{V_p}{Z} \cos \beta \ell_p \right\} \sin \beta z - \left\{ C_{m+1,2} - \sum_{p=n}^m \frac{V_p}{Z} \sin \beta \ell_p \right\} \cos \beta z \right] \quad (B.8)$$

for $\ell_{n-1} \leq z \leq \ell_n$, where $n = 1, 2, 3 \dots m+1$, $\ell_0 = 0$ and $\ell_{m+1} = 2h$. Comparing (B.8) and (3.37), we see that they are identical except for the designation of the arbitrary constants which is not important.

$$f_n(z) = \frac{1}{2\pi i} \int_{\Gamma_n} \frac{f(\zeta)}{\zeta - z} d\zeta$$
 for $n = 1, 2, 3, \dots$ and $z \in D_n$.
 where Γ_n is the boundary of D_n and D_n is the domain of definition of f .
 To obtain the integral representation of the function f on the surface of the sphere, we refer to [2].
 of Chapter 2 and write

$$f(z) = \frac{1}{2\pi} \int_0^{2\pi} f(e^{i\theta}) d\theta$$

where

$$f(e^{i\theta}) = f_n(e^{i\theta})$$

for $n = 1, 2, 3, \dots$ and where f_n is the function f on the boundary of D_n .
 Comparing (2.1) and (2.2) we find that f is identical except for the definition of the boundary domain, which is not important.

Since (B.8) are identical with (3.37) for the multiply-fed linear antenna of Figure 3.2, the ratios (3.38) are equal to the input impedances of that antenna. Rather than attacking the solution of (B.8) directly to obtain the ratios (3.38), we use the method presented by King and take advantage of the linearity of the system. Since the antenna is linear, we can use superposition and represent it by the superimposed set of asymmetrically-driven linear antennas illustrated in Figure B.1. Using this equivalence, we observe that

$$\begin{aligned}
 \frac{1}{Z_{dn}} &= - \frac{2I_n(\ell_n)}{V_n} = - 2 \left[\frac{\sum_{p=n}^{n-1} I_{p2}(\ell_n) + I_{n1}(\ell_n) + \sum_{p=n+1}^m I_{p1}(\ell_n)}{V_n} \right] \\
 &= \frac{1}{Z_{dan}} - \left[\sum_{p=1}^{n-1} \frac{2I_{n1}(\ell_n)}{V_n} \frac{I_{p2}(\ell_n)}{I_{n1}(\ell_n)} + \sum_{p=n+1}^m \frac{2I_{n1}(\ell_n)}{V_n} \frac{I_{p1}(\ell_n)}{I_{n1}(\ell_n)} \right] \\
 &= \frac{1}{Z_{dan}} \left[1 + \sum_{p=1}^{n-1} \frac{I_{p2}(\ell_n)}{I_{n1}(\ell_n)} + \sum_{p=n+1}^m \frac{I_{p1}(\ell_n)}{I_{n1}(\ell_n)} \right] \quad (B.9)
 \end{aligned}$$

for $n = 1, 2, 3 \dots m$. Assuming sinusoidal current distributions on the asymmetrically-driven linear antennas, we can write

$$I_{n1}(z) = - \frac{V_n}{2Z_{dan}} \frac{\sin \beta z}{\sin \beta \ell_n} \quad (B.10)$$

$$I_{n2}(z) = - \frac{V_n}{2Z_{dan}} \frac{\sin \beta(\ell_{m+1}-z)}{\sin \beta(\ell_{m+1}-\ell_n)} \quad (B.11)$$

for $n = 1, 2, 3 \dots m$. Substituting (B.10) and (B.11) in (B.9), we obtain

$$\frac{1}{Z_{dn}} = \frac{1}{Z_{dan}} \left[1 + \sum_{p=1}^{n-1} \frac{V_p}{V_n} \frac{Z_{dan} \sin \beta(\ell_{m+1}-\ell_n)}{Z_{dap} \sin \beta(\ell_{m+1}-\ell_p)} + \sum_{p=n+1}^m \frac{V_p}{V_n} \frac{Z_{dan} \sin \beta \ell_n}{Z_{dap} \sin \beta \ell_p} \right], \quad (B.12)$$

or

Since (8.8) are identical with (5.25) for the multiply-fed linear antenna of Figure 5.2, the ratios (5.28) are equal to the input impedances of that antenna. Rather than attacking the solution of (8.8) directly to obtain the ratios (5.28), we use the method presented by King and take advantage of the linearity of the system. Since the antenna is linear, we can use superposition and represent it by the superimposed set of asymmetrically-driven linear antennas illustrated in Figure 8.1. Using this equivalence, we observe that

$$\frac{1}{Z_{in}^{(n)}} = \frac{1}{V_n} \left[\sum_{p=1}^{n-1} \frac{I_p^{(n)}(V_n)}{I_n^{(n)}(V_n)} + \sum_{p=n+1}^M \frac{I_p^{(n)}(V_n)}{I_n^{(n)}(V_n)} \right] \quad (8.9)$$

for $n = 1, 2, 3, \dots, M$. Assuming sinusoidal current distributions on the asymmetrically driven linear antennas, we can write

$$I_n^{(n)}(z) = \frac{V_n}{Z_{in}^{(n)}} \sin \frac{\beta z}{2} \quad (8.10)$$

$$I_n^{(p)}(z) = \frac{V_p}{Z_{in}^{(p)}} \sin \frac{\beta(z - z_p)}{2} \quad (8.11)$$

for $n = 1, 2, 3, \dots, M$. Substituting (8.10) and (8.11) in (8.9), we obtain

$$\frac{1}{Z_{in}^{(n)}} = \frac{1}{V_n} \left[1 + \sum_{p=1}^{n-1} \frac{V_p}{V_n} \frac{\sin \frac{\beta(z - z_p)}{2}}{\sin \frac{\beta z}{2}} + \sum_{p=n+1}^M \frac{V_p}{V_n} \frac{\sin \frac{\beta(z - z_p)}{2}}{\sin \frac{\beta z}{2}} \right] \quad (8.12)$$

$$I_n = \sum_{p=1}^{n-1} I_{p2} + \sum_{p=n}^m I_{p1} \quad \text{for } n=1, 2, 3 \dots m+1$$

Figure B.1--Multiply-fed linear antenna and an equivalent set of asymmetrically driven linear antennas

$$I_n = \sum_{p=1}^{n-1} I_p + \sum_{p=1}^m I_p \quad \text{for } n=1, 2, 3, \dots, m+1$$

Figure 8.1-Multiplexed linear antenna and its equivalent set of symmetrically driven linear antennas

$$Z_{dn} = \frac{Z_{dan}}{1 + \sum_{p=1}^{n-1} \frac{V_p}{V_n} \frac{Z_{dan}}{Z_{dap}} \frac{\sin \beta(\ell_{m+1} - \ell_n)}{\sin \beta(\ell_{m+1} - \ell_p)} + \sum_{p=n+1}^m \frac{V_p}{V_n} \frac{Z_{dan}}{Z_{dap}} \frac{\sin \beta \ell_n}{\sin \beta \ell_p}} \quad (B.13)$$

for $n = 1, 2, 3 \dots m$. Since a method is presented in Appendix A for evaluating the Z_{dan} which are the input impedances of the set of asymmetrically-driven linear antennas illustrated in Figure B.1, we have a method for evaluating the input impedances of the multiply-fed linear antennas of Figure B.1.

As mentioned in Chapter 3, the above proofs are concerned with a side-loaded antenna that is not quite general. To complete our proof, we must show that (3.69, 3.70, and 3.71) are also a set of integral equations describing a multiply-fed linear antenna, namely the antenna of Figure B.2, and then show how the equations for evaluating the input impedance of this antenna differ from (B.13). Comparing Figure B.2 with Figure 3.2, we see that (B.1 and B.3) are the same for the antenna of Figure B.2. We must rewrite (B.2), however, in the form

$$\frac{V_n}{2} = (C_{n1} - C_{n+1,1}) \cos \beta \ell_n - (C_{n2} - C_{n+1,2}) \sin \beta \ell_n \quad (B.14)$$

for $n = 1, 2, 4, 5 \dots m$, and

$$-\frac{V_3}{2} = (C_{31} - C_{41}) \cos \beta \ell_3 - (C_{32} - C_{42}) \sin \beta \ell_3 \quad (B.15)$$

Solving (B.3, B.14, and B.15) for C_{n1} and C_{n2} in terms of $C_{n+1,1}$ and $C_{n+1,2}$ and substituting the result in (B.1), the result is

$$\frac{V_n}{Z_{in}} = \frac{1}{Z_{in}} \left[\sum_{m=1}^N C_m \sin \beta_m \right] \quad (B.1)$$

for $n = 1, 2, 3, \dots, M$. Since a method is presented in Appendix A for evaluating the Z_{in} which are the input impedances of the set of asymmetrically-driven linear antennas illustrated in Figure B.1, we have a method for evaluating the input impedances of the multiply-fed linear antennas of Figure B.1. As mentioned in Chapter 5, the above results are concerned with a side-loaded antenna that is not quite general. To complete our proof, we must show that (B.6), (B.7), and (B.8) are also a set of integral equations describing a multiply-fed linear antenna, namely the antenna of Figure B.2, and then show how the equations for evaluating the input impedance of this antenna differ from (B.1).

Comparing Figure B.2 with Figure B.1, we see that (B.1) and (B.2) are the same for the antenna of Figure B.2. We must rewrite (B.3), however, in the form

$$\frac{V_n}{Z_{in}} = (C_1 - C_{n+1}) \cos \beta_n - (C_{n+2} - C_{n+3}) \sin \beta_n \quad (B.10)$$

for $n = 1, 2, 3, \dots, M$, and

$$\frac{V_n}{Z_{in}} = (C_1 - C_{n+1}) \cos \beta_n - (C_{n+2} - C_{n+3}) \sin \beta_n \quad (B.11)$$

Solving (B.10), (B.11), and (B.12) for C_1 and C_{n+1} in terms of C_{n+2} and C_{n+3} and substituting the result in (B.1), the result is

Figure B.2--Multiply-fed linear antenna

Figure 1.3 - 1000

$$A_n(z) = -\frac{j}{v} \left[\left\{ C_{m+1,1} + \sum_{p=n}^m \frac{V_p}{2} \cos \beta \ell_p - \frac{V_3}{2} \cos \beta \ell_3 \right\} \sin \beta z \right. \\ \left. + \left\{ C_{m+1,2} - \sum_{\substack{p=n \\ p \neq 3}}^m \frac{V_p}{2} \sin \beta \ell_p + \frac{V_3}{2} \sin \beta \ell_3 \right\} \cos \beta z \right] \quad (B.16)$$

for $\ell_{n-1} \leq z \leq \ell_n$, where $\ell_0 = 0$ and $n = 1, 2$.

$$A_3(z) = -\frac{j}{v} \left[\left\{ C_{m+1,1} + \sum_{p=4}^m \frac{V_p}{2} \cos \beta \ell_p - \frac{V_3}{2} \cos \beta \ell_3 \right\} \sin \beta z \right. \\ \left. + \left\{ C_{m+1,2} - \sum_{p=4}^m \frac{V_p}{2} \sin \beta \ell_p + \frac{V_3}{2} \sin \beta \ell_3 \right\} \cos \beta z \right] \quad (B.17)$$

for $\ell_2 \leq z \leq \ell_3$.

$$A_n = -\frac{j}{v} \left[\left\{ C_{m+1,1} + \sum_{p=n}^m \frac{V_p}{2} \cos \beta \ell_p \right\} \sin \beta z \right. \\ \left. + \left\{ C_{m+1,2} - \sum_{p=4}^m \frac{V_p}{2} \sin \beta \ell_p \right\} \cos \beta z \right] \quad (B.18)$$

for $\ell_{n-1} \leq z \leq \ell_n$, where $\ell_{m+1} = 2h$ and $n = 4, 5, 6 \dots m+1$.

Combining (B.5, B.7, B.16, B.17, and B.18), we obtain

$$\int_0^{2h} I_T(z') K_d(z, z') dz' = j \frac{4\pi}{\eta} \left[- \left\{ C_{m+1,1} + \sum_{p=4}^m \frac{V_p}{2} \cos \beta \ell_p - \frac{V_3}{2} \cos \beta \ell_3 \right\} \sin \beta z \right. \\ \left. - \left\{ C_{m+1,2} - \sum_{p=4}^m \frac{V_p}{2} \sin \beta \ell_p + \frac{V_3}{2} \sin \beta \ell_3 \right\} \cos \beta z \right] \quad (B.19)$$

for $\ell_{n-1} \leq z \leq \ell_n$, where $\ell_0 = 0$ and $n = 1, 2$.

$$\int_0^{2h} I_T(z') K_d(z, z') dz' = j \frac{4\pi}{\eta} \left[- \left\{ C_{m+1,1} + \sum_{p=4}^m \frac{V_p}{2} \cos \beta \ell_p - \frac{V_3}{2} \cos \beta \ell_3 \right\} \sin \beta z \right. \\ \left. - \left\{ C_{m+1,2} - \sum_{p=4}^m \frac{V_p}{2} \sin \beta \ell_p + \frac{V_3}{2} \sin \beta \ell_3 \right\} \cos \beta z \right] \quad (B.20)$$

for $\ell_2 \leq z \leq \ell_3$.

$$A_n(z) = \frac{1}{2} \left(\sum_{p=0}^n c_{n+1-p} z^p + \sum_{p=0}^n c_p z^{n-p} \right) \quad (B.16)$$

$$A_n(z) = \frac{1}{2} \left(\sum_{p=0}^n c_{n+1-p} z^p + \sum_{p=0}^n c_p z^{n-p} \right) \quad (B.17)$$

for $n-1 \leq k \leq n$, where $k_0 = 0$ and $n = 1, 2, \dots$

$$A_n(z) = \frac{1}{2} \left(\sum_{p=0}^n c_{n+1-p} z^p + \sum_{p=0}^n c_p z^{n-p} \right) \quad (B.18)$$

$$A_n(z) = \frac{1}{2} \left(\sum_{p=0}^n c_{n+1-p} z^p + \sum_{p=0}^n c_p z^{n-p} \right) \quad (B.19)$$

for $k_0 \leq k \leq n$

$$A_n(z) = \frac{1}{2} \left(\sum_{p=0}^n c_{n+1-p} z^p + \sum_{p=0}^n c_p z^{n-p} \right) \quad (B.20)$$

$$A_n(z) = \frac{1}{2} \left(\sum_{p=0}^n c_{n+1-p} z^p + \sum_{p=0}^n c_p z^{n-p} \right) \quad (B.21)$$

for $n-1 \leq k \leq n$, where $k_0 = 0$ and $n = 1, 2, \dots$

Combining (B.16), (B.17), and (B.18), we obtain

$$A_n(z) = \frac{1}{2} \left(\sum_{p=0}^n c_{n+1-p} z^p + \sum_{p=0}^n c_p z^{n-p} \right) \quad (B.22)$$

$$A_n(z) = \frac{1}{2} \left(\sum_{p=0}^n c_{n+1-p} z^p + \sum_{p=0}^n c_p z^{n-p} \right) \quad (B.23)$$

for $n-1 \leq k \leq n$, where $k_0 = 0$ and $n = 1, 2, \dots$

$$A_n(z) = \frac{1}{2} \left(\sum_{p=0}^n c_{n+1-p} z^p + \sum_{p=0}^n c_p z^{n-p} \right) \quad (B.24)$$

$$A_n(z) = \frac{1}{2} \left(\sum_{p=0}^n c_{n+1-p} z^p + \sum_{p=0}^n c_p z^{n-p} \right) \quad (B.25)$$

$$A_n(z) = \frac{1}{2} \left(\sum_{p=0}^n c_{n+1-p} z^p + \sum_{p=0}^n c_p z^{n-p} \right) \quad (B.26)$$

for $n-1 \leq k \leq n$, where $k_0 = 0$ and $n = 1, 2, \dots$

$$\int_0^{2h} I_T(z') K_d(z, z') dz' = j \frac{4\pi}{\eta} \left[- \left\{ C_{m+1,1} + \sum_{p=n}^m \frac{V_p}{Z} \cos \beta \ell_p \right\} \sin \beta z \right. \\ \left. + \left\{ C_{m+1,2} - \sum_{p=4}^m \frac{V_p}{Z} \sin \beta \ell_p \right\} \cos \beta z \right] \quad (B.21)$$

for $\ell_{n-1} \leq z \leq \ell_n$, where $\ell_{m+1} = 2h$ and $n = 4, 5, 6 \dots m+1$.

Comparing (B.19, B.20, and B.21) with (3.69, 3.70, and 3.71), we see that they are identical except for the designation of the arbitrary constants which is not important.

To complete the discussion, we now indicate the form which (B.13) takes for the antenna of Figure B.2. Since the only difference in the antennas of Figures 3.2 and B.2 is the polarity of the driving voltage at the point $z = \ell_3$, we can do this by inspection and the result is

$$Z_{dn} = \frac{Z_{dan}}{1 + \sum_{p=1}^{n-1} \frac{V_p}{V_n} \frac{Z_{dan}}{Z_{dap}} \frac{\sin \beta(\ell_{m+1} - \ell_n)}{\sin \beta(\ell_{m+1} - \ell_p)} + \sum_{p=n+1}^m \frac{V_p}{V_3} \frac{Z_{dan} \sin \beta \ell_n}{Z_{dap} \sin \beta \ell_p} - \frac{V_3 Z_{dan} \sin \beta \ell_n}{V_n Z_{da3} \sin \beta \ell_3}} \quad (B.22)$$

for $n = 1, 2$.

$$Z_{d3} = \frac{Z_{da3}}{1 - \sum_{p=1}^2 \frac{V_p}{V_3} \frac{Z_{da3} \sin \beta(\ell_{m+1} - \ell_3)}{Z_{dap} \sin \beta(\ell_{m+1} - \ell_p)} - \sum_{\substack{p=n+1 \\ p \neq 3}}^m \frac{V_p}{V_3} \frac{Z_{da3} \sin \beta \ell_3}{Z_{dap} \sin \beta \ell_p}} \quad (B.23)$$

$$Z_{dn} = \frac{Z_{dan}}{d_1},$$

where

$$d_1 = 1 + \sum_{\substack{p=1 \\ p \neq 3}}^{n-1} \frac{V_p}{V_n} \frac{Z_{dan} \sin \beta(\ell_{m+1} - \ell_n)}{Z_{dap} \sin \beta(\ell_{m+1} - \ell_p)} - \frac{V_3}{V_n} \frac{Z_{dan} \sin \beta(\ell_{m+1} - \ell_n)}{Z_{da3} \sin \beta(\ell_{m+1} - \ell_3)} \\ + \sum_{p=n+1}^m \frac{V_p}{V_n} \frac{Z_{dan} \sin \beta \ell_n}{Z_{dap} \sin \beta \ell_p} \quad (B.24)$$

for $n = 4, 5 \dots m$.

$$I_T(e) = \frac{1}{2} \sum_{p=1}^n \left\{ \frac{1}{2} \left[\frac{1}{2} \left(\frac{1}{2} \right)^{p-1} \right] \right\} \cos \left(\frac{1}{2} \right) \quad (8.11)$$

for $n-1 \leq p \leq n$, where $m+1 = 2n$ and $n = 2n-1$.
Comparing (8.10), (8.11) and (8.12) with (8.13) and (8.14)

we see that they are identical except for the designation of
the arbitrary constant which is not important.

To complete the discussion, we now indicate the form
which (8.12) takes for the antenna of Figure 8.2. Since the

only difference in the antennas of Figures 8.1 and 8.2 is the
polarity of the driving voltage at the point $x = \frac{1}{2}$, we can
do this by inspection and the result is

$$E_{2n} = \frac{1}{2} \sum_{p=1}^n \frac{1}{2} \left[\frac{1}{2} \left(\frac{1}{2} \right)^{p-1} \right] \sin \left(\frac{1}{2} \right) \quad (8.15)$$

for a $\sin \left(\frac{1}{2} \right)$ distribution of current.

$$E_{2n} = \frac{1}{2} \sum_{p=1}^n \frac{1}{2} \left[\frac{1}{2} \left(\frac{1}{2} \right)^{p-1} \right] \sin \left(\frac{1}{2} \right) \quad (8.16)$$

$$d_1 = 1 + \frac{1}{2} \sum_{p=1}^n \frac{1}{2} \left[\frac{1}{2} \left(\frac{1}{2} \right)^{p-1} \right] \sin \left(\frac{1}{2} \right) \quad (8.17)$$

$$d_2 = \frac{1}{2} \sum_{p=1}^n \frac{1}{2} \left[\frac{1}{2} \left(\frac{1}{2} \right)^{p-1} \right] \sin \left(\frac{1}{2} \right) \quad (8.18)$$

APPENDIX C

In this appendix, we show first for the single side-loaded transmission-line antenna that the equations for the input impedance of the equivalent circuit of Figure 2.4 are identical to (2.55 and 2.56). This proves that the equivalent circuit of Figure 2.4 can be used to calculate the input impedance of the single side-loaded transmission-line antenna. Next, we prove in the same way that the equivalent circuit of Figure 3.3 can be used to calculate the input impedance of the general side-loaded transmission-line antenna. We start by repeating equations (2.41, 2.43, 2.54, and 2.56).

$$Z_o = jR_c \left[\frac{\gamma_2 + \gamma_1 Z_1 \cos \beta \ell_1}{1 - \gamma_1 Z_1 \sin \beta \ell_1} \right] \quad (C.1)$$

$$\gamma_2 = \frac{R_c \sin \beta \ell_2 - jZ_2 \cos \beta \ell_2}{R_c \cos \beta \ell_2 + jZ_2 \sin \beta \ell_2} \quad (C.2)$$

$$\gamma_1 Z_1 = - \frac{jZ_{o1}}{R_c} [\cos \beta \ell_1 + \gamma_2 \sin \beta \ell_1] \quad (C.3)$$

$$Z_{o1} = \frac{4Z_d Z_1}{4Z_d + Z_1} \quad (C.4)$$

Substituting (C.2) in (C.3), we find

$$\gamma_1 Z_1 = -j \frac{Z_{o1}}{R_c} \left[\frac{R_c \cos \beta(\ell_2 - \ell_1) + jZ_2 \sin \beta(\ell_2 - \ell_1)}{R_c \cos \beta \ell_2 + jZ_2 \sin \beta \ell_2} \right] \quad (C.5)$$

Rearranging (C.5), we obtain

$$\frac{R_c \cos \beta \ell_2 + jZ_2 \sin \beta \ell_2}{R_c \cos \beta(\ell_2 - \ell_1) + jZ_2 \sin \beta(\ell_2 - \ell_1)} = -j \frac{Z_{o1}}{R_c \gamma_1 Z_1} \quad (C.6)$$

APPENDIX C

In this appendix, we show first for the single side-
loaded transmission-line antenna that the equations for the
input impedance of the equivalent circuit of Figure 2.1 are
identical to (2.25 and 2.26). This proves that the equiva-
lent circuit of Figure 2.1 can be used to calculate the input
impedance of the single side-loaded transmission-line antenna.
Next, we prove in the same way that the equivalent circuit of
Figure 2.2 can be used to calculate the input impedance of
the general side-loaded transmission-line antenna. We start
by repeating equations (2.1), (2.2), and (2.3):

$$E_o = j\omega \left[\frac{Y_2 + Y_1 \cos \beta L}{Y_1 \sin \beta L} \right] V_1 \quad (C.1)$$

$$V_2 = \frac{R_o \sin \beta L - jX_o \cos \beta L}{R_o \cos \beta L + jX_o \sin \beta L} V_1 \quad (C.2)$$

$$V_{1/2} = -\frac{jX_o}{R_o} \left[\cos \beta L + Y_1 \sin \beta L \right] V_1 \quad (C.3)$$

$$Z_{ol} = \frac{R_o^2 Z_1}{jX_o + Z_1} \quad (C.4)$$

Substituting (C.2) in (C.3), we find

$$V_{1/2} = -\frac{jX_o}{R_o} \left[\frac{R_o \cos \beta L - jX_o \sin \beta L}{R_o \cos \beta L + jX_o \sin \beta L} + Y_1 \sin \beta L \right] V_1 \quad (C.5)$$

Rearranging (C.5), we obtain

$$\frac{R_o \cos \beta L - jX_o \sin \beta L}{R_o \cos \beta L + jX_o \sin \beta L} + Y_1 \sin \beta L = -\frac{jX_o}{R_o} \left[\frac{R_o \cos \beta L - jX_o \sin \beta L}{R_o \cos \beta L + jX_o \sin \beta L} + Y_1 \sin \beta L \right] \quad (C.6)$$

Next, we multiply (C.6) by $j\gamma_2$. Referring to (C.2), the result is

$$\frac{Z_2 \cos \beta \ell_2 + jR_c \sin \beta \ell_2}{R_c \cos \beta(\ell_2 - \ell_1) + jZ_2 \sin \beta(\ell_2 - \ell_1)} = \frac{Z_{o1} \gamma_2}{R_c \gamma_1 Z_1} \quad (C.7)$$

Leaving these expressions for the moment, we refer to Figure 2.4 and write

$$Z_o = R_c \left[\frac{Z_{o1} \cos \beta \ell_1 + Z_{T1} \cos \beta \ell_1 + jR_c \sin \beta \ell_1}{jZ_{o1} \sin \beta \ell_1 + jZ_{T1} \sin \beta \ell_1 + R_c \cos \beta \ell_1} \right] \quad (C.8)$$

$$Z_{T1} = R_c \left[\frac{Z_2 \cos \beta(\ell_2 - \ell_1) + jR_c \sin \beta(\ell_2 - \ell_1)}{jZ_2 \sin \beta(\ell_2 - \ell_1) + R_c \cos \beta(\ell_2 - \ell_1)} \right] \quad (C.9)$$

Breaking up (C.8) and using (C.9), we find

$$Z_{T1} \cos \beta \ell_1 + jR_c \sin \beta \ell_1 = R_c \left[\frac{Z_2 \cos \beta \ell_2 + jR_c \sin \beta \ell_2}{jZ_2 \sin \beta(\ell_2 - \ell_1) + R_c \cos \beta(\ell_2 - \ell_1)} \right] \quad (C.10)$$

$$jZ_{T1} \sin \beta \ell_1 + R_c \cos \beta \ell_1 = R_c \left[\frac{jZ_2 \sin \beta \ell_2 + R_c \cos \beta \ell_2}{jZ_2 \sin \beta(\ell_2 - \ell_1) + R_c \cos \beta(\ell_2 - \ell_1)} \right] \quad (C.11)$$

Comparing (C.6 and C.7) with (C.10 and C.11), we see that

$$Z_{T1} \cos \beta \ell_1 + jR_c \sin \beta \ell_1 = \frac{Z_{o1} \gamma_2}{\gamma_1 Z_1} \quad (C.12)$$

$$jZ_{T1} \sin \beta \ell_1 + R_c \cos \beta \ell_1 = -j \frac{Z_{o1}}{\gamma_1 Z_1} \quad (C.13)$$

Substituting (C.12 and C.13) in (C.8), we finally obtain

$$Z_o = jR_c \left[\frac{\gamma_2 + \gamma_1 Z_1 \cos \beta \ell_1}{1 - \gamma_1 Z_1 \sin \beta \ell_1} \right] \quad (C.14)$$

Comparing (C.1) with (C.14), we see that they are identical and thus the equivalent circuit is established for the single side-loaded transmission-line antenna.

For the general side-loaded transmission-line antenna, we refer to Figure C.1 which helps to organize the algebraic

Next, we multiply (C.5) by $\frac{1}{R_1}$ and referring to (C.7), the

result is

$$\frac{R_2 \cos \theta_2}{R_1 \cos \theta_1} = \frac{R_2 \sin \theta_2}{R_1 \sin \theta_1} \quad (C.8)$$

leaving these expressions for the moment, we refer to Figure

2.4 and write

$$\frac{R_2 \cos \theta_2}{R_1 \cos \theta_1} = \frac{R_2 \sin \theta_2}{R_1 \sin \theta_1} \quad (C.9)$$

$$\frac{R_2 \cos \theta_2}{R_1 \cos \theta_1} = \frac{R_2 \sin \theta_2}{R_1 \sin \theta_1} \quad (C.10)$$

Breaking up (C.8) and using (C.9), we find

$$\frac{R_2 \cos \theta_2}{R_1 \cos \theta_1} = \frac{R_2 \sin \theta_2}{R_1 \sin \theta_1} \quad (C.11)$$

$$\frac{R_2 \cos \theta_2}{R_1 \cos \theta_1} = \frac{R_2 \sin \theta_2}{R_1 \sin \theta_1} \quad (C.12)$$

Comparing (C.10) and (C.11) with (C.10) and (C.11), we see that

$$\frac{R_2 \cos \theta_2}{R_1 \cos \theta_1} = \frac{R_2 \sin \theta_2}{R_1 \sin \theta_1} \quad (C.13)$$

$$\frac{R_2 \cos \theta_2}{R_1 \cos \theta_1} = \frac{R_2 \sin \theta_2}{R_1 \sin \theta_1} \quad (C.14)$$

Substituting (C.12) and (C.13) in (C.8), we finally obtain

$$\frac{R_2 \cos \theta_2}{R_1 \cos \theta_1} = \frac{R_2 \sin \theta_2}{R_1 \sin \theta_1} \quad (C.15)$$

Comparing (C.1) with (C.15), we see that they are identical

and thus the equivalent circuit is established for the single

side-loaded transmission-line antenna

For the general side-loaded antenna, the antenna

is referred to Figure 2.1 which shows the equivalent

Figure C.1--The equivalent circuit for a general side-loaded transmission-line antenna

Figure C.1--The equivalent circuit for a general side-loaded transmission-line antenna

process. We start the derivation by repeating equations (3.32, 3.34, 3.43, and 3.44).

$$Z_o = jR_c \left[\frac{\gamma_{m+1} + \sum_{p=1}^m \gamma_p Z_p \cos \beta \ell_p}{1 - \sum_{p=1}^m \gamma_p Z_p \sin \beta \ell_p} \right] \quad (C.15)$$

$$\gamma_{m+1} = \left[\frac{R_c \sin \beta \ell_{m+1} - jZ_{m+1} \cos \beta \ell_{m+1}}{R_c \cos \beta \ell_{m+1} + jZ_{m+1} \sin \beta \ell_{m+1}} \right] \quad (C.16)$$

$$\gamma_k Z_k = -j \frac{Z_{ok}}{R_c} \left[\gamma_{m+1} \sin \beta \ell_k + \cos \beta \ell_k - \sum_{p=k+1}^m \gamma_p Z_p \sin \beta (\ell_p - \ell_k) \right] \quad (C.17)$$

$$Z_{ok} = \frac{4Z_k Z_{dk}}{4Z_{dk} + Z_k} \quad (C.18)$$

for $k = 1, 2, 3 \dots m$. Substituting (C.16) in (C.17), we find

$$\gamma_k Z_k = -j \frac{Z_{ok}}{R_c} \left[\frac{R_c \cos \beta (\ell_{m+1} - \ell_k) + jZ_{m+1} \sin \beta (\ell_{m+1} - \ell_k)}{R_c \cos \beta \ell_{m+1} + jZ_{m+1} \sin \beta \ell_{m+1}} - \sum_{p=k+1}^m \gamma_p Z_p \sin \beta (\ell_p - \ell_k) \right] \quad (C.19)$$

for $k = 1, 2, 3 \dots m$. Rearranging (C.19), we obtain

$$\frac{R_c \cos \beta (\ell_{m+1} - \ell_k) + jZ_{m+1} \sin \beta (\ell_{m+1} - \ell_k)}{R_c \cos \beta \ell_{m+1} + jZ_{m+1} \sin \beta \ell_{m+1}} = j \frac{R_c \gamma_k Z_k}{Z_{ok}} + \sum_{p=k+1}^m \gamma_p Z_p \sin \beta (\ell_p - \ell_k) \quad (C.20)$$

for $k = 1, 2, 3 \dots m$. For the special case where $k = m$, we invert (C.20). The result is

$$\frac{R_c \cos \beta \ell_{m+1} + jZ_{m+1} \sin \beta \ell_{m+1}}{R_c \cos \beta (\ell_{m+1} - \ell_m) + jZ_{m+1} \sin \beta (\ell_{m+1} - \ell_m)} = -j \frac{Z_{om}}{R_c \gamma_m Z_m} \quad (C.21)$$

Multiplying (C.20) by (C.21), we obtain

$$\begin{aligned} \frac{R_c \cos \beta (\ell_{m+1} - \ell_k) + jZ_{m+1} \sin \beta (\ell_{m+1} - \ell_k)}{R_c \cos \beta (\ell_{m+1} - \ell_m) + jZ_{m+1} \sin \beta (\ell_{m+1} - \ell_m)} &= \frac{\gamma_k Z_k Z_{om}}{\gamma_m Z_m Z_{ok}} \\ &- j \frac{Z_{om}}{R_c} \sum_{p=k+1}^m \frac{\gamma_p Z_p}{\gamma_m Z_m} \sin \beta (\ell_p - \ell_m) \end{aligned} \quad (C.22)$$

process. We have:

$$(3.21) \quad \frac{d}{dt} \left(\frac{1}{2} \int_{\mathbb{R}^n} |\nabla u|^2 dx \right) = \int_{\mathbb{R}^n} u \frac{d}{dt} \nabla u \cdot \nabla u dx$$

$$= \int_{\mathbb{R}^n} u \nabla u \cdot \nabla u dx - \int_{\mathbb{R}^n} u \nabla u \cdot \nabla u dx$$

$$= \int_{\mathbb{R}^n} u \nabla u \cdot \nabla u dx - \int_{\mathbb{R}^n} u \nabla u \cdot \nabla u dx$$

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$$\text{for } k = 1, \dots, n$$

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$$\text{for } k = 1, \dots, n$$

$$\text{for } k = 1, \dots, n$$

$$\frac{d}{dt} \left(\frac{1}{2} \int_{\mathbb{R}^n} |\nabla u|^2 dx \right) = \int_{\mathbb{R}^n} u \nabla u \cdot \nabla u dx$$

$$\text{for } k = 1, \dots, n$$

$$\text{invert (C.20): the matrix is invertible and$$

$$\frac{d}{dt} \left(\frac{1}{2} \int_{\mathbb{R}^n} |\nabla u|^2 dx \right) = \int_{\mathbb{R}^n} u \nabla u \cdot \nabla u dx$$

$$\text{Multiplying (C.20) by } u$$

$$\frac{d}{dt} \left(\frac{1}{2} \int_{\mathbb{R}^n} |\nabla u|^2 dx \right) = \int_{\mathbb{R}^n} u \nabla u \cdot \nabla u dx$$

for $k = 1, 2, 3 \dots m+1$. The equality is true also for $k = m$, but it becomes the trivial expression $1 = 1$. Multiplying through (C.21) by $j\gamma_{m+1}$ and referring to (C.16), we obtain

$$\frac{Z_{m+1} \cos \beta \ell_{m+1} + jR_c \sin \beta \ell_{m+1}}{R_c \cos \beta (\ell_{m+1} - \ell_m) + jZ_{m+1} \sin \beta (\ell_{m+1} - \ell_m)} = \frac{Z_{om} \gamma_{m+1}}{R_c \gamma_m Z_m} . \quad (C.23)$$

Leaving these expressions for the moment, we refer to Figure 3.3 and see that (C.8) through (C.11) hold again for the general side-loaded transmission-line antenna. It is possible for the present case, however, to continue the process, and we can write in general

$$Z_{Tn} \cos \beta \ell_n + jR_c \sin \beta \ell_n = R_c \frac{n_2}{d_2} , \quad (C.24)$$

where

$$\frac{n_2}{d_2} = \frac{Z_{o,n+1} \cos \beta \ell_{n+1} + Z_{Tn+1} \cos \beta \ell_{n+1} + jR_c \sin \beta \ell_{n+1}}{jZ_{o,n+1} \sin \beta (\ell_{n+1} - \ell_n) + jZ_{Tn+1} \sin \beta (\ell_{n+1} - \ell_n) + R_c \cos \beta (\ell_{n+1} - \ell_n)} . \quad (C.25)$$

$$jZ_{Tn} \cos \beta \ell_n + R_c \cos \beta \ell_n = R_c \frac{n_3}{d_3} ,$$

where

$$\frac{n_3}{d_3} = \frac{jZ_{o,n+1} \sin \beta \ell_{n+1} + jZ_{Tn+1} \sin \beta \ell_{n+1} + R_c \cos \beta \ell_{n+1}}{jZ_{o,n+1} \sin \beta (\ell_{n+1} - \ell_n) + jZ_{Tn+1} \sin \beta (\ell_{n+1} - \ell_n) + R_c \cos \beta (\ell_{n+1} - \ell_n)} .$$

$$jZ_{Tn} \sin \beta (\ell_n - \ell_k) + R_c \cos \beta (\ell_n - \ell_k) = R_c \frac{n_4}{d_4} , \quad (C.26)$$

where

$$\frac{n_4}{d_4} = \frac{jZ_{o,n+1} \sin \beta (\ell_{n+1} - \ell_k) + jZ_{Tn+1} \sin \beta (\ell_{n+1} - \ell_k) + R_c \cos \beta (\ell_{n+1} - \ell_k)}{jZ_{o,n+1} \sin \beta (\ell_{n+1} - \ell_n) + jZ_{Tn+1} \sin \beta (\ell_{n+1} - \ell_n) + R_c \cos \beta (\ell_{n+1} - \ell_n)}$$

and where $k = 1, 2, 3 \dots n-1$ for each n and $n = 1, 2, 3 \dots m$.

Letting $n = m$ in (C.24, C.25, and C.26), we obtain

for $k = 1, 2, \dots, m+1$. The equality is true also for $k = m$.

but it becomes the trivial expression $1 = 1$ multiplying

through (C.11) by $\frac{1}{\sin \theta}$ and referring to (C.10), we obtain

$$\frac{1}{\sin \theta} \left[\frac{1}{\sin \theta} \frac{d}{d\theta} \left(\sin \theta \frac{d}{d\theta} \right) + \frac{1}{\sin \theta} \frac{d}{d\theta} \left(\sin \theta \frac{d}{d\theta} \right) + \frac{1}{\sin \theta} \frac{d}{d\theta} \left(\sin \theta \frac{d}{d\theta} \right) \right] Y_{lm} = \frac{1}{\sin \theta} \frac{d}{d\theta} \left(\sin \theta \frac{d}{d\theta} \right) Y_{lm} \quad (C.12)$$

leaving these expressions for the moment. We refer to Figure

3.3 and see that (C.8) through (C.11) hold again for the

general side-loaded transmission-line antenna. It is possible

for the present case, however, to combine the process, and

we can write in general

$$\frac{1}{\sin \theta} \left[\frac{1}{\sin \theta} \frac{d}{d\theta} \left(\sin \theta \frac{d}{d\theta} \right) + \frac{1}{\sin \theta} \frac{d}{d\theta} \left(\sin \theta \frac{d}{d\theta} \right) + \frac{1}{\sin \theta} \frac{d}{d\theta} \left(\sin \theta \frac{d}{d\theta} \right) \right] Y_{lm} = \frac{1}{\sin \theta} \frac{d}{d\theta} \left(\sin \theta \frac{d}{d\theta} \right) Y_{lm} \quad (C.13)$$

where

$$\frac{1}{\sin \theta} \left[\frac{1}{\sin \theta} \frac{d}{d\theta} \left(\sin \theta \frac{d}{d\theta} \right) + \frac{1}{\sin \theta} \frac{d}{d\theta} \left(\sin \theta \frac{d}{d\theta} \right) + \frac{1}{\sin \theta} \frac{d}{d\theta} \left(\sin \theta \frac{d}{d\theta} \right) \right] Y_{lm} = \frac{1}{\sin \theta} \frac{d}{d\theta} \left(\sin \theta \frac{d}{d\theta} \right) Y_{lm} \quad (C.14)$$

where

$$\frac{1}{\sin \theta} \left[\frac{1}{\sin \theta} \frac{d}{d\theta} \left(\sin \theta \frac{d}{d\theta} \right) + \frac{1}{\sin \theta} \frac{d}{d\theta} \left(\sin \theta \frac{d}{d\theta} \right) + \frac{1}{\sin \theta} \frac{d}{d\theta} \left(\sin \theta \frac{d}{d\theta} \right) \right] Y_{lm} = \frac{1}{\sin \theta} \frac{d}{d\theta} \left(\sin \theta \frac{d}{d\theta} \right) Y_{lm} \quad (C.15)$$

where

$$\frac{1}{\sin \theta} \left[\frac{1}{\sin \theta} \frac{d}{d\theta} \left(\sin \theta \frac{d}{d\theta} \right) + \frac{1}{\sin \theta} \frac{d}{d\theta} \left(\sin \theta \frac{d}{d\theta} \right) + \frac{1}{\sin \theta} \frac{d}{d\theta} \left(\sin \theta \frac{d}{d\theta} \right) \right] Y_{lm} = \frac{1}{\sin \theta} \frac{d}{d\theta} \left(\sin \theta \frac{d}{d\theta} \right) Y_{lm} \quad (C.16)$$

and where $k = 1, 2, \dots, m+1$ for each n and $n = 1, 2, \dots, m$.

Letting $\theta = \pi$ in (C.12) and (C.13), we obtain

$$Z_{Tm} \cos \beta \ell_m + jR_c \sin \beta \ell_m = R_c \left[\frac{Z_{m+1} \cos \beta \ell_{m+1} + jR_c \sin \beta \ell_{m+1}}{jZ_{m+1} \sin \beta (\ell_{m+1} - \ell_m) + R_c \cos \beta (\ell_{m+1} - \ell_m)} \right] \quad (C.27)$$

$$jZ_{Tm} \sin \beta \ell_m + R_c \cos \beta \ell_m = R_c \left[\frac{jZ_{m+1} \sin \beta \ell_{m+1} + R_c \cos \beta \ell_{m+1}}{jZ_{m+1} \sin \beta (\ell_{m+1} - \ell_m) + R_c \cos \beta (\ell_{m+1} - \ell_m)} \right] \quad (C.28)$$

$$jZ_{Tm} \sin \beta (\ell_m - \ell_k) + R_c \cos \beta (\ell_m - \ell_k) = R_c \frac{n_5}{d_5}, \quad (C.29)$$

where

$$\frac{n_5}{d_5} = \frac{jZ_{m+1} \sin \beta (\ell_{m+1} - \ell_k) + R_c \cos \beta (\ell_{m+1} - \ell_k)}{jZ_{m+1} \sin \beta (\ell_{m+1} - \ell_m) + R_c \cos \beta (\ell_{m+1} - \ell_m)}$$

and where $k = 1, 2, 3 \dots m-1$. Comparing (C.27, C.28, and C.29) with (C.21, C.22, and C.23), we observe that

$$Z_{Tm} \cos \beta \ell_m + jR_c \sin \beta \ell_n = \frac{Z_{om} \gamma_{m+1}}{\gamma_m Z_m} \quad (C.30)$$

$$jZ_{Tm} \sin \beta \ell_m + R_c \cos \beta \ell_m = -j \frac{Z_{om}}{\gamma_m Z_m} \quad (C.31)$$

$$jZ_{Tm} \sin \beta (\ell_m - \ell_k) + R_c \cos \beta (\ell_m - \ell_k) = R_c \left[\frac{\gamma_k Z_k Z_{om}}{\gamma_m Z_m Z_{ok}} - jZ_{om} \sum_{p=k+1}^m \frac{\gamma_p Z_p}{\gamma_m Z_m} \sin \beta (\ell_p - \ell_k) \right], \quad (C.32)$$

where $k = 1, 2, 3 \dots m+1$. We now substitute (C.30, C.31, and C.32) back into (C.24, C.25, and C.26) for $n = m-1$. Taking the resulting set of m equations, we substitute them into (C.24, C.25, and C.26) for $n = m-2$. Continuing this process, we can write in general

$$Z_{Tn} \cos \beta \ell_n + jR_c \sin \beta \ell_n = Z_{on} \left[\sum_{p=n+1}^m \frac{\gamma_p Z_p}{\gamma_n Z_n} \cos \beta \ell_p + \frac{\gamma_{m+1}}{\gamma_n Z_n} \right] \quad (C.33)$$

$$jZ_{Tn} \sin \beta \ell_n + R_c \cos \beta \ell_n = jZ_{on} \left[\sum_{p=n+1}^m \frac{\gamma_p Z_p}{\gamma_n Z_n} \sin \beta \ell_p - \frac{1}{\gamma_n Z_n} \right] \quad (C.34)$$

$$jZ_{Tn} \sin \beta (\ell_n - \ell_k) + R_c \cos \beta (\ell_n - \ell_k) = R_c \frac{\gamma_k Z_k Z_{on}}{\gamma_n Z_n Z_{ok}} - jZ_{on} \sum_{p=k+1}^n \frac{\gamma_p Z_p}{\gamma_n Z_n} \sin \beta (\ell_p - \ell_k) \quad (C.35)$$

a set of $n+1$ equations where $k = 1, 2, 3 \dots n-1$ for each n , and $n = 1, 2, 3 \dots m$. For $n = 1$, we find

$$Z_{T1} \cos \beta \ell_1 + jR_c \sin \beta \ell_1 = Z_{o1} \left[\sum_{p=2}^m \frac{\gamma_p Z_p}{\gamma_1 Z_1} \cos \beta \ell_p + \frac{\gamma_{m+1}}{\gamma_1 Z_1} \right] \quad (C.36)$$

$$jZ_{T1} \sin \beta \ell_1 + R_c \cos \beta \ell_1 = jZ_{o1} \left[\sum_{p=2}^m \frac{\gamma_p Z_p}{\gamma_1 Z_1} \sin \beta \ell_p - \frac{1}{\gamma_1 Z_1} \right]. \quad (C.37)$$

Substituting (C.36 and C.37) in (C.8), we finally obtain

$$Z_o = jR_c \left[\frac{\gamma_{m+1} + \sum_{p=1}^m \gamma_p Z_p \cos \beta \ell_p}{1 - \sum_{p=1}^m \gamma_p Z_p \sin \beta \ell_p} \right]. \quad (C.38)$$

Comparing (C.38) and (3.32), we find they are identical and thus the equivalent circuit is established for the general side-loaded transmission-line antenna.

$$\begin{aligned} \text{The admittance } Y_{in} \text{ is given by} \\ Y_{in} = \frac{1}{Z_{in}} = \frac{1}{Z_0 + jZ_0 \tan \beta l} \\ = \frac{1}{Z_0} \frac{1}{1 + j \tan \beta l} \\ = \frac{1}{Z_0} \frac{1 - j \tan \beta l}{1 + \tan^2 \beta l} \\ = \frac{1}{Z_0} \frac{1 - j \tan \beta l}{\sec^2 \beta l} \\ = \frac{\cos^2 \beta l}{Z_0} (1 - j \tan \beta l) \end{aligned} \quad (C.25)$$

A set of $n-1$ equations where $k = 1, 2, \dots, n-1$ for each n and $n=1, 2, 3, \dots, n$. For $n=1$, we find

$$\begin{aligned} Y_{in} \cos \beta l - j \sin \beta l = \frac{1}{Z_0} \left[\frac{1}{\cos \beta l} + \frac{j \sin \beta l}{\cos^2 \beta l} \right] \quad (C.26) \\ Y_{in} \sin \beta l + j \cos \beta l = \frac{1}{Z_0} \left[\frac{j \sin \beta l}{\cos^2 \beta l} - \frac{1}{\cos \beta l} \right] \quad (C.27) \end{aligned}$$

Substituting (C.26) and (C.27) in (C.8), we finally obtain

$$\begin{aligned} \frac{1}{Z_0} \left[\frac{1}{\cos \beta l} + \frac{j \sin \beta l}{\cos^2 \beta l} \right] = \frac{1}{Z_0} \left[\frac{1}{\cos \beta l} + \frac{j \sin \beta l}{\cos^2 \beta l} \right] \quad (C.28) \end{aligned}$$

Comparing (C.26) and (C.27), we find they are identical and

thus the equivalent circuit is established for the general

side-loaded transmission-line antenna.

APPENDIX D

In Chapter 5 it was stated that the integrals which must be evaluated to find the far fields from an antenna curved in the shape of a catenary can only be evaluated by numerical methods. A rather simple way to avoid this very laborious task is to approximate the antenna by a series of straight lines. If this is done, the far field integrals are broken up into a series of integrals all of which are quite simple to evaluate. Another advantage of this method is that it can be applied to any curved antenna regardless of how complicated the equation of the curve is.

In Figures D.1, D.2, and D.3 below it is shown graphically that catenaries with a broad range of slopes can be closely approximated by three straight lines and that the intersection of two of these can be located at the center of the antenna which is very convenient for the dipole moment integrals. Using three straight lines to approximate the curved antenna reduces the problem to that of studying three straight antennas making angles θ_1 , θ_2 , and θ_3 with the vertical. The equations for doing this are presented in Chapter 5, and all that is necessary here is to show how the angles of tilt and the lengths of these three straight antennas can be determined. With the straight line approximations laid out as they are in Figure D.1, it can be seen that for antenna 1 we have

APPENDIX C

In Chapter 2 it was stated that the integrals which must be evaluated to find the far field from an antenna curved in the shape of a catenary can only be evaluated by numerical methods. A rather simple way to avoid this very laborious task is to approximate the antenna by a series of straight lines. If this is done, the far field integrals are broken up into a series of integrals all of which are quite simple to evaluate. Another advantage of this method is that it can be applied to any curved antenna regardless of how complicated the equation of the curve is. In Figures D.1, D.2, and D.3 below it is shown graphically that catenaries with a broad range of slopes can be closely approximated by three straight lines and that the intersection of two of these can be located at the center of the antenna which is very convenient for the dipole moment integrals. Using three straight lines to approximate the curved antenna reduces the problem to that of studying three straight antennas making angles θ_1 , θ_2 , and θ_3 with the vertical. The equations for doing this are presented in Chapter 5, and all that is necessary here is to show how the angles of tilt and the lengths of these three straight antennas can be determined. With the straight line approximations laid out as they are in Figure D.1 it can be seen that for antenna 1 we have

Assume $2h=1$; $a=1/2$; $x = \frac{1}{2} \cosh 2(z+0.44) - 1 = 0.207$

$$\tan \theta_1 = \frac{x_1}{z_1} = \frac{0.41}{0.28} = 1.462 ; \theta_1 = 0.96 ; s_2 = \frac{z_2}{\cos \theta_2} = -\frac{0.220}{0.795} = -0.278$$

Figure D.1--Straight line approximation for curved antenna 1

Assume $2h=1$; $a=1/10$; $x=\frac{1}{10} \cosh 10(z+0.231)-1=0.4$
 $\tan \theta_1 = \frac{x_1}{z_1} = \frac{0.500}{0.069} = 7.25$; $\theta_1 = 1.434$; $s_2 = \frac{z_2}{\cos \theta_2} = -\frac{0.104}{0.309} = -0.337$

$\tan \theta_2 = \frac{x_2}{z_2} = \frac{0.320}{0.104} = 3.075$; $\theta_2 = 1.257$

$\tan \theta_3 = \frac{x_3 - x_2}{z_3 - z_2} = \frac{0.080}{0.127} = 0.630$; $\theta_3 = 0.561$

Figure D.2--Straight line approximation for curved antenna 2

Figure 1

Figure 2

Figure 3

Figure 4

Figure 5

Figure 6

Assume $2h=1$; $a=\frac{1}{100}$; $x=\frac{1}{100} \cosh 100(z+0.046)-1=0.5$

$$\tan\theta_1 = \frac{x_1}{z_1} = \frac{0.5}{0.007} = 72.5 ; \theta_1 = 1.557 ; s_2 = \frac{z_2}{\cos\theta_2} = \frac{0.014}{-0.035} = -0.40$$

$$\tan\theta_2 = \frac{x_2}{z_2} = \frac{0.416}{0.014} = 29.7 ; \theta_2 = 1.537$$

Figure D.3--Straight line approximation for curved antenna 3

$$\tan \theta_1 = \frac{x_1}{s_1} = \frac{0.2}{0.07} = 2.857; \theta_1 = 70.7^\circ$$

$$\tan \theta_2 = \frac{x_2}{s_2} = \frac{0.014}{0.035} = 0.4; \theta_2 = 21.8^\circ$$

$$\tan \theta_3 = \frac{x_3}{s_3} = \frac{0.416}{0.014} = 29.7; \theta_3 = 88.1^\circ$$

Figure 2-5-4: Graph of the function $y = 0.014x^2$ for curved

distance

$$\tan \theta_1 = \frac{x_1}{z_1} , \quad (D.1)$$

for antenna 2 we have

$$\tan \theta_2 = \frac{x_2}{z_2} \quad (D.2)$$

$$S_2 = \frac{z_2}{\cos \theta_2} \quad (D.3)$$

and for antenna 3 we have

$$\tan \theta_3 = \frac{x_3}{z_3} . \quad (D.4)$$

Having chosen a value for "a" which determines the particular catenary to be studied, θ_1 , θ_2 , θ_3 and S_2 can be determined graphically using (D.1) through (D.4). The curves of Figures D.1, D.2, and D.3 are normalized such that $2h$ is set equal to 1. This has no effect on the angles θ_1 , θ_2 , and θ_3 but the length S_2 obtained from the graphical process must be multiplied by $2h$. This means that for each value of "a" the graphical work only has to be done once, and it is applicable to an antenna of any length.

(D.1)

for antenna 2 we have

(D.2)

(D.3)

and for antenna 3 we have

(D.4)

Having chosen a value for "a" which determines the partial capacity to be studied, θ_1 , θ_2 , and θ_3 can be determined graphically using (D.1) through (D.4). The curves of Figures D.1, D.2, and D.3 are normalized such that θ is set equal to 1. This has no effect on the angles θ_1 , θ_2 , and θ_3 but the length z obtained from the graphical process must be multiplied by $2a$. This means that for each value of "a" the graphical work only has to be done once, and it is applicable to an antenna of any length.

APPENDIX E

In order to find the mutual impedance between antenna 1 and 2 of Figure E.1, we must determine the field E_x at the surface of antenna 2 due to the current I_1 on antenna 1.

From Figure E.1 we observe that

$$E_x = E_z \cos \theta_A + E_\rho \sin \theta_A \quad . \quad (E.1)$$

Following Jordan [6, page 320-323], we find that for

$$I_1 = \begin{cases} \frac{I_o \sin \beta(h-z_o)}{\sin \beta h} & 0 \leq z_o \leq h \\ \frac{I_o \sin \beta(h+z_o)}{\sin \beta h} & -h \leq z_o \leq 0 \end{cases} \quad (E.2)$$

the fields E_z and E_ρ are given by

$$E_z = -j \frac{30 I_o}{\sin \beta h} \left\{ \frac{e^{-j\beta r_1}}{r_1} + \frac{e^{-j\beta r_2}}{r_2} - 2 \cos \beta h \frac{e^{-j\beta r_o}}{r_o} \right\} \quad (E.3)$$

$$E_\rho = j \frac{30 I_o}{\sin \beta h} \left\{ \frac{z-h}{\rho} \frac{e^{-j\beta r_1}}{r_1} + \frac{z+h}{\rho} \frac{e^{-j\beta r_2}}{r_2} - 2 \cos \beta h \frac{e^{-j\beta r_o}}{r_o} \right\}, \quad (E.4)$$

where

$$r_1 = [(z-h)^2 + \rho^2]^{1/2} \quad (E.5)$$

$$r_2 = [(z+h)^2 + \rho^2]^{1/2} \quad (E.6)$$

$$r_o = [z^2 + \rho^2]^{1/2} \quad (E.7)$$

and where we have assumed that the antennas are in the atmosphere and therefore

$$\sqrt{\frac{\mu}{\epsilon}} = 120 \pi \quad . \quad (E.8)$$

APPENDIX B

In order to find the mutual impedance between antennas

1 and 2 of Figure B.1, we must determine the field E_2 at the

surface of antenna 1 due to the current I_2 on antenna 2.

From Figure B.1 we observe that

$$E_x = E_1 \cos \theta + E_2 \sin \theta \quad (B.1)$$

Following Jordan [6, page 320-322], we find that for

$$I_2 = \begin{cases} \frac{I_0 \sin k(h-z)}{\sin kh} & 0 \leq z \leq h \\ \frac{I_0 \sin k(h+z)}{\sin kh} & -h \leq z \leq 0 \end{cases} \quad (B.2)$$

the fields E_x and E_z are given by

$$E_x = -\frac{1}{r_1} \frac{\partial}{\partial r_1} \left(\frac{e^{-jkr_1}}{r_1} \right) + \frac{e^{-jkr_2}}{r_2} - \frac{2 \cos kh}{r_2} \frac{e^{-jkr_2}}{r_2} \quad (B.3)$$

$$E_z = \frac{1}{r_1} \frac{\partial}{\partial r_1} \left(\frac{e^{-jkr_1}}{r_1} \right) + \frac{e^{-jkr_2}}{r_2} - \frac{2 \cos kh}{r_2} \frac{e^{-jkr_2}}{r_2} \quad (B.4)$$

where

$$r_1 = [(x-h)^2 + z^2]^{1/2} \quad (B.5)$$

$$r_2 = [(x+h)^2 + z^2]^{1/2} \quad (B.6)$$

$$r_0 = [z^2 + a^2]^{1/2} \quad (B.7)$$

and where we have assumed that the antennas are in the

atmosphere and therefore

$$\sqrt{\epsilon} = 1, \quad \mu = \mu_0 \quad (B.8)$$

Figure E.1--Coordinate systems for the two skew antennas

Figure 2. Linear regression lines for the two cases.

Referring to Figure E.1, we observe that

$$\frac{z-h}{\rho} = \frac{a-h + (a+S) \cos \theta_A}{(a+S) \sin \theta_A} \quad (\text{E.9})$$

$$\frac{z+h}{\rho} = \frac{a+h + (a+S) \cos \theta_A}{(a+S) \sin \theta_A} \quad (\text{E.10})$$

$$\frac{z}{\rho} = \frac{a + (a+S) \cos \theta_A}{(a+S) \sin \theta_A} \quad (\text{E.11})$$

Combining (E.1, E.3, E.4, E.9, E.10, and E.11), we find

$$E_x = j \frac{30 I_o}{\sin \beta h} \left\{ \frac{a-h}{a+S} \frac{e^{-j\beta r_1}}{r_1} + \frac{a+h}{a+S} \frac{e^{-j\beta r_2}}{r_2} - 2 \cos \beta h \frac{a}{a+S} \frac{e^{-j\beta r_o}}{r_o} \right\}. \quad (\text{E.12})$$

From Jordan [6, page 350] we find that the mutual impedance between antennas 1 and 2 of Figure E.1 is given by

$$Z_{21} = - \frac{1}{I_1(0) I_2(0)} \int_{-h}^h E_x(s) I_2(s) ds \quad (\text{E.13})$$

Assuming that

$$I_2 = \begin{cases} \frac{I_o \sin \beta(h-s)}{\sin \beta h} & 0 \leq s \leq h \\ \frac{I_o \sin \beta(h+s)}{\sin \beta h} & -h \leq s \leq 0 \end{cases}, \quad (\text{E.14})$$

we obtain from (E.12, E.13, and E.14)

$$Z_{21} = - j \frac{30}{\sin^2 \beta h} \left\{ \int_0^h \left[\frac{a-h}{a+S} \frac{e^{-j\beta r_1}}{r_1} + \frac{a+h}{a+S} \frac{e^{-j\beta r_2}}{r_2} - 2 \cos \beta h \frac{a}{a+S} \frac{e^{-j\beta r_o}}{r_o} \right] \sin \beta(h-s) ds + \int_{-h}^0 \left[\frac{a-h}{a+S} \frac{e^{-j\beta r_1}}{r_1} + \frac{a+h}{a+S} \frac{e^{-j\beta r_2}}{r_2} - 2 \cos \beta h \frac{a}{a+S} \frac{e^{-j\beta r_o}}{r_o} \right] \sin \beta(h+s) ds \right\}. \quad (\text{E.15})$$

Referring to Figure E.1, we observe that

$$E_1 = \frac{a - h + (a-h)\cos \theta}{a + (a-h)\cos \theta} \quad (E.9)$$

$$E_2 = \frac{a - h + (a-h)\cos \theta}{a + (a-h)\cos \theta} \quad (E.10)$$

$$E_3 = \frac{a - h + (a-h)\cos \theta}{a + (a-h)\cos \theta} \quad (E.11)$$

Combining (E.9), (E.10), (E.11), and (E.12), we find

$$E_1 = \frac{a - h + (a-h)\cos \theta}{a + (a-h)\cos \theta} \quad (E.12)$$

From Jordan [6, page 350] we find that the mutual impedance

between antennas 1 and 2 of Figure E.1 is given by

$$Z_{12} = \frac{1}{h} \int_0^h E_1(a) E_2(a) da \quad (E.13)$$

Assuming that

$$E_1 = \frac{1 - \sin \theta(a-h)}{\sin \theta a} \quad (E.14)$$

we obtain from (E.13), (E.14), and (E.15)

$$Z_{12} = \frac{1}{h} \int_0^h \left[\frac{1 - \sin \theta(a-h)}{\sin \theta a} \right] \left[\frac{1 - \sin \theta(a-h)}{\sin \theta a} \right] da \quad (E.15)$$

$$Z_{12} = \frac{1}{h} \int_0^h \left[\frac{1 - \sin \theta(a-h)}{\sin \theta a} \right] \left[\frac{1 - \sin \theta(a-h)}{\sin \theta a} \right] da$$

The integrals in (E.15) have been evaluated by Murray [7].

Following Murray's paper, we find

$$\int_0^h \left(\frac{a-h}{a+s} \right) \frac{e^{-j\beta r_1}}{r_1} \sin \beta(h-s) ds = \frac{a-h}{2j} \left[e^{j\beta(h+a)} \int_a^{a+h} \frac{e^{-j\beta(r_1+\sigma)}}{\sigma r_1} d\sigma \right. \\ \left. - e^{-j\beta(h+a)} \int_a^{a+h} \frac{e^{-j\beta(r_1-\sigma)}}{\sigma r_1} d\sigma \right], \quad (\text{E.16})$$

where

$$r_1 = \left[(a-h)^2 + 2(a-h)\sigma \cos \theta_A + \sigma^2 \right]^{1/2}. \quad (\text{E.17})$$

Evaluating the integrals, we find

$$\int_a^{a+h} \frac{e^{-j\beta(r_1+\sigma)}}{\sigma r_1} d\sigma = \frac{e^{-j\beta(a-h)\cos \theta_A}}{(a-h)} \left[e^{-j\frac{\beta(a-h)}{\alpha} \sin \theta_A} \int_{x_1}^{x_2} \frac{e^{-jx}}{x} dx \right. \\ \left. - e^{j\beta(a-h)\alpha \sin \theta_A} \int_{x_3}^{x_4} \frac{e^{-jx}}{x} dx \right], \quad (\text{E.18})$$

where

$$x_1 = \beta(a-h) \sin \theta_A \left\{ \frac{\left[(a-h)^2 + 2(a-h)a \cos \theta_A + a^2 \right]^{1/2} + a - (a-h) \cos \theta_A}{(a-h) \sin \theta_A} - \frac{1}{\alpha} \right\} \quad (\text{E.19})$$

$$x_2 = \beta(a-h) \sin \theta_A \left\{ \frac{\left[(a-h)^2 + 2(a-h)(a+h) \cos \theta_A + (a+h)^2 \right]^{1/2}}{(a-h) \sin \theta_A} \right. \\ \left. + \frac{(a+h) - (a-h) \cos \theta_A}{(a-h) \sin \theta_A} - \frac{1}{\alpha} \right\} \quad (\text{E.20})$$

$$x_3 = \beta(a-h) \sin \theta_A \left\{ \frac{\left[(a-h)^2 + 2(a-h)a \cos \theta_A + a^2 \right]^{1/2} + a - (a-h) \cos \theta_A}{(a-h) \sin \theta_A} + \alpha \right\} \quad (\text{E.21})$$

$$+ \frac{e^{-j\beta(a+h)}}{2j} \left[e^{j\beta(a-h)} \cos \theta_A \left\{ e^{j\beta(a-h)} \frac{\sin \theta_A}{\alpha} \left(E_i(-jX_6) - E_i(-jX_5) \right) \right. \right. \\ \left. \left. - e^{j\beta(a-h)} \alpha \sin \theta_A \left(E_i(-jX_8) - E_i(-jX_7) \right) \right\} \right], \quad (\text{E.28})$$

where

$$x_5 = \beta(a-h) \sin \theta_A \left\{ \frac{[(a-h)^2 + 2(a-h)a \cos \theta_A + a^2]^{1/2} - a + (a-h) \cos \theta_A}{(a-h) \sin \theta_A} + \frac{1}{\alpha} \right\} \quad (\text{E.29})$$

$$x_6 = \beta(a-h) \sin \theta_A \left\{ \frac{[(a-h)^2 + 2(a-h)(a+h) \cos \theta_A + (a+h)^2]^{1/2}}{(a-h) \sin \theta_A} \right. \\ \left. - \frac{(a+h) - (a-h) \cos \theta_A}{(a-h) \sin \theta_A} + \frac{1}{\alpha} \right\} \quad (\text{E.30})$$

$$x_7 = \beta(a-h) \sin \theta_A \left\{ \frac{[(a-h)^2 + 2(a-h)a \cos \theta_A + a^2]^{1/2} - a + (a-h) \cos \theta_A}{(a-h) \sin \theta_A} - \alpha \right\} \quad (\text{E.31})$$

$$x_8 = \beta(a-h) \sin \theta_A \left\{ \frac{[(a-h)^2 + 2(a-h)(a+h) \cos \theta_A + (a+h)^2]^{1/2}}{(a-h) \sin \theta_A} \right. \\ \left. - \frac{(a+h) - (a-h) \cos \theta_A}{(a-h) \sin \theta_A} - \alpha \right\}. \quad (\text{E.32})$$

Using the same system to evaluate the remainder of the integrals in (E.15), we obtain

$$Z_{21} = - \frac{15e^{j\beta(a+h)}}{\sin^2 \beta h} \left[e^{-j\beta(a-h)} \cos \theta_A \left\{ e^{-j\beta(a-h)} \frac{\sin \theta_A}{\alpha} \left(E_i(-jX_2) - E_i(-jX_1) \right) \right. \right. \\ \left. \left. - e^{j\beta(a-h)} \alpha \sin \theta_A \left(E_i(-jX_4) - E_i(-jX_3) \right) \right\} \right. \\ \left. + e^{-j\beta(a+h)} \cos \theta_A \left\{ e^{-j\beta(a+h)} \frac{\sin \theta_A}{\alpha} \left(E_i(-jY_2) - E_i(-jY_1) \right) \right. \right. \\ \left. \left. - e^{j\beta(a+h)} \alpha \sin \theta_A \left(E_i(-jY_4) - E_i(-jY_3) \right) \right\} \right. \\ \left. - 2 \cos \beta h e^{-j\beta a} \cos \theta_A \left\{ e^{-j\beta a} \frac{\sin \theta_A}{\alpha} \left(E_i(-jZ_2) - E_i(-jZ_1) \right) \right. \right. \\ \left. \left. - e^{j\beta a} \alpha \sin \theta_A \left(E_i(-jZ_4) - E_i(-jZ_3) \right) \right\} \right] \\ - \frac{15e^{-j\beta(a+h)}}{\sin^2 \beta h} \left[e^{j\beta(a-h)} \cos \theta_A \left\{ e^{j\beta(a-h)} \frac{\sin \theta_A}{\alpha} \left(E_i(-jX_6) - E_i(-jX_5) \right) \right. \right. \\ \left. \left. - e^{j\beta(a-h)} \alpha \sin \theta_A \left(E_i(-jX_8) - E_i(-jX_7) \right) \right\} \right]$$

$$\begin{aligned}
& - e^{-j\beta(a-h)} \alpha \sin \theta_A \left\{ E_i(-jX_8) - E_i(-jX_7) \right\} \\
& + e^{j\beta(a+h)} \cos \theta_A \left\{ e^{j\beta(a+h)} \frac{\sin \theta_A}{\alpha} \left\{ E_i(-jY_6) - E_i(-jY_5) \right\} \right. \\
& \quad \left. - e^{-j\beta(a+h)} \alpha \sin \theta_A \left\{ E_i(-jY_8) - E_i(-jY_7) \right\} \right\} \\
& - 2 \cos \beta h e^{j\beta a} \cos \theta_A \left\{ e^{j\beta a} \frac{\sin \theta_A}{\alpha} \left\{ E_i(-jZ_6) - E_i(-jZ_5) \right\} \right. \\
& \quad \left. - e^{-j\beta a} \alpha \sin \theta_A \left\{ E_i(-jZ_8) - E_i(-jZ_7) \right\} \right\} \Bigg] \\
& + \frac{15e^{j\beta(a-h)}}{\sin^2 \beta h} \left[e^{-j\beta(a-h)} \cos \theta_A \left\{ e^{-j\beta(a-h)} \frac{\sin \theta_A}{\alpha} \left\{ E_i(-jX_1) - E_i(-jX_9) \right\} \right. \right. \\
& \quad \left. - e^{j\beta(a-h)} \alpha \sin \theta_A \left\{ E_i(-jX_3) - E_i(-jX_{10}) \right\} \right\} \\
& \quad + e^{-j\beta(a+h)} \cos \theta_A \left\{ e^{-j\beta(a+h)} \frac{\sin \theta_A}{\alpha} \left\{ E_i(-jY_1) - E_i(-jY_9) \right\} \right. \\
& \quad \left. - e^{j\beta(a+h)} \alpha \sin \theta_A \left\{ E_i(-jY_3) - E_i(-jY_{10}) \right\} \right\} \\
& \quad - 2 \cos \beta h e^{-j\beta a} \cos \theta_A \left\{ e^{-j\beta a} \frac{\sin \theta_A}{\alpha} \left\{ E_i(-jZ_1) - E_i(-jZ_9) \right\} \right. \\
& \quad \left. - e^{j\beta a} \alpha \sin \theta_A \left\{ E_i(-jZ_3) - E_i(-jZ_{10}) \right\} \right\} \Bigg] \\
& + \frac{15e^{-j\beta(a-h)}}{\sin^2 \beta h} \left[e^{j\beta(a-h)} \cos \theta_A \left\{ e^{j\beta(a-h)} \frac{\sin \theta_A}{\alpha} \left\{ E_i(-jX_5) - E_i(-jX_{11}) \right\} \right. \right. \\
& \quad \left. - e^{-j\beta(a-h)} \alpha \sin \theta_A \left\{ E_i(-jX_7) - E_i(-jX_{12}) \right\} \right\} \\
& \quad + e^{j\beta(a+h)} \cos \theta_A \left\{ e^{j\beta(a+h)} \frac{\sin \theta_A}{\alpha} \left\{ E_i(-jY_5) - E_i(-jY_{11}) \right\} \right. \\
& \quad \left. - e^{-j\beta(a+h)} \alpha \sin \theta_A \left\{ E_i(-jY_7) - E_i(-jY_{12}) \right\} \right\} \\
& \quad - 2 \cos \beta h e^{j\beta a} \cos \theta_A \left\{ e^{j\beta a} \frac{\sin \theta_A}{\alpha} \left\{ E_i(-jZ_5) - E_i(-jZ_{11}) \right\} \right. \\
& \quad \left. - e^{-j\beta a} \alpha \sin \theta_A \left\{ E_i(-jZ_7) - E_i(-jZ_{12}) \right\} \right\} \Bigg] , \quad (E.33)
\end{aligned}$$

where

$$x_9 = \beta(a-h) \sin \theta_A \left\{ \frac{2 \cos \theta_A / 2 + 1 - \cos \theta_A}{\sin \theta_A} - \frac{1}{\alpha} \right\} \quad (E.34)$$

$$x_{10} = \beta(a-h) \sin \theta_A \left\{ \frac{2 \cos \theta_A/2 + 1 - \cos \theta_A}{\sin \theta_A} + \alpha \right\} \quad (\text{E.35})$$

$$x_{11} = \beta(a-h) \sin \theta_A \left\{ \frac{2 \cos \theta_A/2 - 1 + \cos \theta_A}{\sin \theta_A} + \frac{1}{\alpha} \right\} \quad (\text{E.36})$$

$$x_{12} = \beta(a-h) \sin \theta_A \left\{ \frac{2 \cos \theta_A/2 - 1 + \cos \theta_A}{\sin \theta_A} - \alpha \right\} . \quad (\text{E.37})$$

y_k is the same as x_k except that $a+h$ is substituted for $a-h$,

$k = 1, 2, \dots, 12$ and z_k is the same as x_k except that a is

substituted for $a-h$, $k = 1, 2, 3, \dots, 12$.

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