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Direct and Semi-Direct Product of Neutrosophic Extended Triplet Group

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Abstract: The object of this article is mainly to discuss the notion of neutrosophic extended triplet direct product (NETDP) and neutrosophic extended triplet semi-direct product (NETS-DP) of NET group. The purpose is to give a clear introduction that allows a solid foundation for additional studies into the field. We introduce neutrosophic extended triplet internal direct product (NETIDP) and neutrosophic extended triplet external direct products (NETEDP) of NET group. Then, we define NET internal and external semi-direct products for NET group by utilizing the notion of NET set theory of Smarandache. Moreover, some results related to NETDP and NETS-DPs are obtained.

Keywords: NET direct product; NET internal direct product; NET external direct product; NET semi-direct product; NET internal semi-direct product; NET external semi-direct product.

1. Introduction

Neutrosophy is a new branch of philosophy, presented by Florentin Smarandache [1] in 1980, which deals the interactions with different ideational spectra in our everyday life. A NET is an object of the structure $(x, e^{neut(x)}, e^{anti(x)})$, for $x \in N$, was firstly presented by Florentin Smarandache [2-4] in 2016. In this theory, the extended neutral and the extended opposites can similar or non-identical from the classical unitary element and inverse element respectively. The NETs are depend on real triads: (friend, neutral, enemy), (pro, neutral, against), (accept, pending, reject), and in general $(x, neut(x), anti(x))$ as in neutrosophy is a conclusion of Hegel's dialectics that is depend on x and $anti(x)$. This theory acknowledges every concept or idea x together along its opposite $anti(x)$ and along their spectrum of neutralities $neut(x)$ among them. Neutrosophy is the foundation of neutrosophic logic, neutrosophic set, neutrosophic probability, and neutrosophic statistics that are utilized or applied in engineering (like software and information fusion), medicine, military, airspace, cybernetics, and physics. Kandasamy and Smarandache [5] introduced many new neutrosophic notions in graphs and applied it to the case of neutrosophic cognitive and relational maps. The same researchers [6] were introduced the concept of neutrosophic algebraic structures for groups, loops, semi groups and groupoids and also their $N-$ algebraic structures in 2006. Smarandache and Mumtaz Ali [7] proposed neutrosophic triplets and by utilizing these they defined NTG and the application areas of NTGs. They also define NT field [8] and NT in physics [9]. Smarandache investigated physical structures of hybrid NT ring [10]. Zhang et al [11] examined the Notion of cancellable NTG and group coincide in 2017. Şahın and Kargın [12], [13] firstly introduced new structures called NT normed space and NT inner product respectively. Smarandache et al [14]

studied new algebraic structure called NT G-module which is constructed on NTGs and NT vector spaces. The above set theories have been applied to many different areas including real decision making problems [15-39]. Additionally, Abdel Basset et al applied neutrosophic set theory to artificial intelligence in medicine [43, 44, 46, 56], decision making [45, 48, 49, 52], programming [47], forecasting [50], IoT [51], chain management [53], TOPSIS technique [54], and importing field [55].

This paper deals with direct and semi-direct products of NETGs. We give basic definitions, notations, facts, and examples about NETs which play a significant role to define and build new algebraic structures. Then, the concept of NET internal and external direct and semi-direct products are given and their difference between the classical structures are briefly discussed. Finally, some results related to NET direct and semi-direct products are obtained.

2. Preliminaries

Since some properties of NETs are used in this work, it is important to have a keen knowledge of NETs. We will point out some few NETs and concepts of NET group, NT normal subgroup, and NT costs according to what needed in this work.

Definition 2.1 [7, 9] A NT has a form $\left(a, neut(a), anti(a)\right)$, for $\left(a, neut(a), anti(a)\right) \in N$, accordingly $neut(a)$ and $anti(a) \in N$ are neutral and opposite of a, that is different from the unitary element, thus: $a * neut(a) = neut(a) * a = a$ and $a * anti(a) = anti(a) * a = neut(a)$ respectively. In general, a may have one or more than one neut's and one or more than one anti's.

Definition 2.2 [3, 9] A NET is a NT, defined as definition 1, but where the neutral of a (symbolized by $e^{neut(a)}$ and called "extended neutral") is equal to the classical unitary element. As a consequence, the "extended opposite" of a, symbolized by $e^{anti(a)}$ is also same to the classical inverse element. Thus, a NET has a form $(a, e^{neut(a)}, e^{anti(a)})$, for $a \in N$, where $e^{neut(a)}$ and $e^{anti(a)}$ in N are the extended neutral and negation of a respectively, thus: $a * e^{neut(a)} = e^{neut(a)} * a = a$, which can be the same or non-identical from the classical unitary element if any and $a * e^{anti(a)} = e^{anti(a)} * a = e^{neut(a)}$. Generally, for each a ∈ N there are one or more $e^{neut(a)}$'s and $e^{anti(a)}$'s.

Definition 2.3 [7, 9] suppose $(N, *)$ is a NT set. Subsequently $(N, *)$ is called a NTG, if the axioms given below are holds.

(1) $(N, *)$ is well-defined, i.e. for and $(a, neut(a), anti(a)), (b, neut(b), anti(b) \in N$, one has $(a, neut(a), anti(a)) * (b, neut(b), anti(b)) \in N$.

(2) $(N, *)$ is associative, i.e. for anyone has
$(a, neut(a), anti(a)) * (b, neut(b), anti(b) * (c, neut(c), anti(c)) \in N$.

Theorem 2.4 [41] Let $(N, *)$ be a commutative NET relating to $*$ an $(a, neut(a), anti(a)), (b, neut(b), anti(b)) \in N$;

(i) $neut(a) * neut(b) = neut(a * b)$;

(ii) $anti(a) * anti(b) = anti(a * b)$;

Definition 2.5 [3, 9] Assume $(N, *)$ is a NET strong set. Subsequently $(N, *)$ is called a NETG, if the axioms given below are holds.

(1) $(N, *)$ is well-defined, i.e. for any $(a, neut(a), anti(a)), (b, neut(b), anti(b) \in N$, one has $(a, neut(a), anti(a)) * (b, neut(b), anti(b) \in N$.

(2) $(N, *)$ is associative,

i.e. for any $(a, neut(a), anti(a)), (b, neut(b), anti(b)), (c, neut(c), anti(c)) \in N$, one has

$$(a, neut(a), anti(a)) * \big((b, neut(b), anti(b)) * (c, neut(c), anti(c))\big)$$

$$= \big((a, neut(a), anti(a)) * (b, neut(b), anti(b))\big) * (c, neut(c), anti(c)).$$

Definition 2.6 [42] Assume that $(N_1, *)$ and $(N_2, \circ)$ are two NETG's. A mapping $f : N_1 \to N_2$ is called a neutro-homomorphism if:

(1) For any $(a, neut(a), anti(a)), (b, neut(b), anti(b) \in N_1$, we have

$$f\big((a, neut(a), anti(a)) * (b, neut(b), anti(b))\big)$$

$$= f\big((a, neut(a), anti(a))\big) * f\big((b, neut(b), anti(b))\big)$$

(2) If $(a, neut(a), anti(a))$ is a NET from N_1, Then

$$f\big(neut(a)\big) = neut\big(f(a)\big) \text{ and } f\big(anti(a)\big) = anti\big(f(a)\big).$$

Definition 2.8 [40] Assume that $(N_1, *)$ is a NETG and H is a subset of N_1. H is called a NET subgroup of N if itself forms a NETG under $*$. On other hand it means:

(1) $e^{neut(a)}$ lies in H.

(2) For any $(a, neut(a), anti(a)), (b, neut(b), anti(b) \in H$,

$$(a, neut(a), anti(a)) * (b, neut(b), anti(b) \in H.$$

(3) If $(a, neut(a), anti(a)) \in H$, then $e^{anti(a)} \in H$.

Definition 2.9 [40] A NET subgroup H of a NETG N is called a NT normal subgroup of N if $(a, neut(a), anti(a))H = H(a, neut(a), anti(a)), \forall (a, neut(a), anti(a)) \in N$ and we represent it as $H (N$.

3. Direct Products of NETG

In this section, we define NET internal and external direct products. Then, we give propositions and proof them.

Definition 3.1 Assume that we have two neutrosophic extended triplet groups H and K, and $N = H \times K$ is the NET cartesian product (NETCP) of H and K, in other words

$$N = \big((h_1, neut(h_1), anti(h_1)), (k_1, neut(k_1), anti(k_1))\big), \begin{pmatrix} (h_2, neut(h_2), anti(h_2)), \\ (k_2, neut(k_2), anti(k_2)) \end{pmatrix}$$

$$= \big(h_1 * h_2, neut(h_1 * h_2), anti(h_1 * h_2)\big), \big(k_1 * k_2, neut(k_1 * k_2), anti(k_1 * k_2)\big) \in H \times K.$$

Clearly N is closed under multiplication, it is obvious to see associativity and it has a neutral element denoted by $1_N = (1_H, 1_K)$ and the anti-neutrals of $\big((h, neut(h), anti(h)), (k, neut(k), anti(k))\big)$ is $\big(anti(h)), anti(k))\big)$, respectively.

Definition 3.2 Suppose that H, K are two NETGs. The NETG $N = H \times K$ with binary operation described componentwise as denoted in definition (3.1.1) is called the "neutrosophic extended triplet direct product" of H and K.

Example 3.3 Find the NET direct product of two NETG z_2 and z_3. Since $z_2 = \{0, 1\}$ and $z_3 = \{0, 1, 3\}$, the NETs z_2 is $(0, 0, 0), (1, 0, 1)$ and the NETs of z_3 is $(0, 0, 0), (1, 0, 2), (2, 0, 1)$. The NET direct products are

$$Z_2 \times Z_3 = \begin{cases} \left((0,0,0),(0,0,0)\right), \left((0,0,0),(1,0,2)\right), \left((0,0,0),(2,0,1)\right), \left((1,0,1),(0,0,0)\right), \\ \left((1,0,1),(1,0,2)\right), \left((1,0,1),(2,0,1)\right) \end{cases}.$$

Definition 3.4 If a NETG N contains neutrosophic triplet normal subgroups (NTNS-Gs) H and K as shown $N = HK$ and $H \cap K = \{1_N\}$, we call N is the "NETIDP" of H and K.

Example 3.5 Examine the NETG $(Z_6, +)$ and the following NET subgroups:

$H = \{(0,0,0),(2,0,4),(4,0,2)\}$

$K = \{(0,0,0),(3,0,3)\}$.

Note that $\begin{cases} (h, neut(h), anti(h)) * (k, neut(k), anti(k)) : (h, neut(h), anti(h)) \in H, \\ (k, neut(k), anti(k)) \in K \end{cases} = N.$

That means $\{(0,0,0),(2,0,4),(4,0,2) + (0,0,0),(3,0,3)\}$

$= \{(0,0,0),(1,0,5),(2,0,4),(3,0,3),(4,0,2),(5,0,1)\}$. So the first condition is met. Also the neutral for z_6 is 0_N and $H \cap K = 0_N = \{(0,0,0)\}$ so the second condition is met. Lastly Z_6 is an abelian so the third condition is met.

Table 1. The elements of NETG $(Z_6, +)$.

+	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	2	3	4	5	0
2	2	3	4	5	0	1
3	3	4	5	0	1	2
4	4	5	0	1	2	3
5	5	0	1	2	3	4

As can be seen, the formed NETs of z_6 is $\{(0,0,0),(1,0,5),(2,0,4),(3,0,3),(4,0,2),(5,0,1)\}$.

and also all classical internal direct products are usually not NETIDPs (some do not even contain either the neutral or anti-neutral elements).

Proposition 3.6 If N is the NETIDP of H and K, subsequently N is neutro-isomorphic to the NETDP $H \times K$.

Proof to put on that N is neutro-isomorphic to $H \times K$, we describe the succeeding map

$f : H \times K \to N,$

$$f\left((h, neut(h), anti(a)),(k, neut(k), anti(k))\right) = \left(h*k, neut(h*k), anti(h*k)\right) \qquad (1)$$

If $\left((h, neut(h), anti(h)) \in H, (k, neut(k), anti(k)) \in K,\right.$ then

$$\left((h*k, neut(h*k), anti(h*k)\right)$$
$$= \left((k*h, neut(k*h), anti(k*h)\right).$$

Actually, we've utilizing that both NETGs K and H are neutrosophic triplet normal that

$$\Big((h, neut(h), anti(h))(k, neut(k), anti(k)\big(h, neut(h), anti(h)\big)^{-1}\Big)\big((k, neut(k), anti(k)\big)^{-1} \in K,$$

$$\Big((h, neut(h), anti(h))(k, neut(k), anti(k)\big(h, neut(h), anti(h)\big)^{-1}\Big)\big((k, neut(k), anti(k)\big)^{-1} \in H$$

Implying that

$$\Big((h, neut(h), anti(h))(k, neut(k), anti(k)\big(h, neut(h), anti(h)\big)^{-1}\Big)\big((k, neut(k), anti(k)\big)^{-1}$$

$$\in K \cap H = \big\{1_N\big\}.$$

At the same time let us show that f is a NETG neutro-isomorphism.

1. This a NETG neutro-homomorphism onwards

$$f\Big((h, neut(h), anti(h)), (k, neut(k), anti(k)), (h^{'}, neut(h^{'}), anti(h^{'})), (k^{'}, neut(k^{'}), anti(k^{'}))\Big)$$

$$= f\Big((h * h^{'}, neut(h * h^{'}), anti(h * h^{'})), (k * k^{'}, neut(k * k^{'}), anti(k * k^{'}))\Big) \text{ by } (1).$$

$$= (h, neut(h), anti(h))\Big((h^{'} * k), neut(h^{'} * k), anti(h^{'} * k))\Big)(k^{'}, neut(k^{'}), anti(k^{'}))$$

$$= (h, neut(h), anti(h))\Big((k * h^{'}), neut(k * h^{'}), anti(k * h^{'}))\Big)(k^{'}, neut(k^{'}), anti(k^{'}))$$

$$= f\Big((h, neut(h), anti(h)), (k, neut(k), anti(k))\Big) f\begin{pmatrix} (h^{'}, neut(h^{'}), anti(h^{'})), \\ (k^{'}, neut(k^{'}), anti(k^{'})) \end{pmatrix}.$$

2. Let us show that the map f is injective. First we have to check that its neutro-kernel is trivial. Actually, if

$$f\Big((h, neut(h), anti(h)), (k, neut(k), anti(k))\Big) = 1_N \text{ Then}$$

$$\Big((h, neut(h), anti(h)), (k, neut(k), anti(k))\Big) = 1_N$$

$$\Rightarrow \big(h, neut(h), anti(h)\big) = \big(k, neut(k), anti(k)\big)^{-1}$$

$$\Rightarrow \big(h, neut(h), anti(h)\big) \in K$$

$$\Rightarrow \big(h, neut(h), anti(h)\big) \in H \cap K = \big\{1_N\big\}$$

We have then that $\big(h, neut(h), anti(h)\big) = \big(k, neut(k), anti(k)\big) = \big\{1_N\big\}$ which proves that

the neutro-kernel is $\big\{(1_N, 1_N)\big\}$.

3. Lastly it's obvious to see that f is subjective since $N = HK$. briefly record that the definitions of NETEDP and NETIDP are assuredly unlimited to two NETGs. We can totally

describe them for n NETGs as $H_1, ..., H_n$.

Definition 3.7 If $H_1, ..., H_n$ are random NETGs the NET external direct product of $H_1, ..., H_n$ is $N = H_1 \times H_2 \times ... \times H_n$ which is the NET cartesian product with componentwise multiplication.

Example 3.8 Let NETG $u(8) = \{1, 3, 5, 7\}$ and $u(12) = \{1, 5, 7, 11\}$ under multiplication modulo 8 and mudulo 12 respectively. Let's construct a NETG table for $u(12)$.

Table 2. The elements of NETG $u(12)$.

×	1	5	7	11
1	1	5	7	11
5	5	1	11	7
7	7	11	1	5
11	11	7	5	1

The NETs of $u(8)$ are $(1,1,1), (3,1,3), (5,1,5), (7,1,7)$ and the NETs of $u(12)$ are $(1,1,1), (5,1,5), (7,1,7), (11,1,11)$.

Now let's see the NET external direct products of
$$u(8) \times u(12) = \big((1,1,1),(1,1,1)\big), \big((1,1,1),(5,1,5)\big), \big((1,1,1),(7,1,7)\big), \big((1,1,1),(11,1,11)\big),$$
$$\big((3,1,3),(1,1,1)\big), \big((3,1,3),(5,1,5)\big), \big((3,1,3),(7,1,7)\big), \big((3,1,3),(11,1,11)\big), \big((5,1,5),(1,1,1)\big),$$
$$\big((5,1,5),(5,1,5)\big), \big((5,1,5),(7,1,7)\big), \big((5,1,5),(11,1,11)\big), \big((7,1,7),(1,1,1)\big), \big((7,1,7),(3,1,3)\big),$$
$$\big((7,1,7),(5,1,5)\big), \big((7,1,7),(7,1,7)\big), \big((7,1,7),(11,1,11)\big).$$

In general, all classical internal direct products are not NETEDPs (some do not even contain either the neutral or anti-neutral elements).

Definition 3.9 If N contains NETNS-Gs $H_1, \ldots, H_n$ as shown $N = H_1 \ldots H_n$ and every n can be symbolized as $(h, neut(h), anti(h)) \ldots (h_n, neut(h_n), anti(h_n))$ particularly, we call N is the neutrosophic extended triplet internal direct product of $H_1, \ldots, H_n$. There is a small distinction between neutrosophic extended triplet internal product as we see in the definition, since in this instance of two NET subgroups, the condition dedicated briefly record that each n can be symbolized particularly as $(h_1, neut(h_1), anti(h_1))(h_2, neut(h_2), anti(h_2))$, but alternately that the intersection of the two NET subgroups is $\{(1_N)\}$. The following proposition indicates the relation among those two points of view.

Proposition 3.10 Assume that $N = H_1 \ldots H_n$ thus every H_i is a NET normal subgroup of N.

The succeeding axioms are equivalent.

    I. N is the NETDP of the H_i.

    II. $H_1 H_2 \ldots H_{i-1} \cap H_i = \{1_N\}$, $\forall i = 1, \ldots, n.$

Proof Let's show I. $\Leftrightarrow$ II. Let's suppose that N is the NETIDP of the H_i, in other words all element in N can be inscribed particularly as a product of elements in H_i. Let's assume

$$\left(n,neut(n),anti(n)\right)\in H_1 H_2 \cdots H_{i-1}\cap H_i=\left\{\left(1_N\right)\right\}. \qquad \text{We} \qquad \text{obtain} \qquad \text{that}$$

$\left(n,neut(n),anti(n)\right)\in H_1 H_2\cdots H_{i-1},$ this is particularly expressed as

$$\left(n,neut(n),anti(n)\right)=\left(h_1,neut(h_1),anti(h_1)\right)\left(h_2,neut(h_2),anti(h_2)\right)\ldots$$
$$\left(h_{i-1},neut(h_{i-1}),anti(h_{i-1})\right)1_N H_i\cdots 1_N H_n,\left(h_j,neut(h_j),anti(h_j)\right)\in H_j.$$

Moreover, $\left(n,neut(n),anti(n)\right)\in H_i$ thus $\left(n,neut(n),anti(n)\right)=\left(1_N\right)H_1^n\ldots\left(1_N\right)H_{i-1}^n$ and we

have $\left(h_j,neut(h_j),anti(h_j)\right)=\left(1_N\right)$ for all j and $\left(n,neut(n),anti(n)\right)=\left(1_N\right).$

II. $\Rightarrow$ I. Conversely, let us assume that $\left(n,neut(n),anti(n)\right)\in N$ can be written either

$$\left(n,neut(n),anti(n)\right)=\left(h_1,neut(h_1),anti(h_1)\right)\left(h_2,neut(h_2),anti(h_2)\right)\ldots$$
$$\left(h_n,neut(h_n),anti(h_n)\right),\left(h_j,neut(h_j),anti(h_j)\right)\in H_j,$$

or

$$\left(n,neut(n),anti(n)\right)=\left(k_1,neut(k_1),anti(k_1)\right)\left(k_2,neut(k_2),anti(k_2)\right)\ldots$$
$$\left(k_n,neut(k_n),anti(k_n)\right),\left(k_j,neut(k_j),anti(k_j)\right)\in H_j.$$

Remember that whereby every H_j are NET normal subgroups, subsequently

$$\left(h_i,neut(h_i),anti(h_i)\right)\left(h_j,neut(h_j),anti(h_j)\right)$$

$$=\left(h_j,neut(h_j),anti(h_j)\right)\left(h_i,neut(h_i),anti(h_i)\right),\left(h_i,neut(h_i),anti(h_i)\right)\in H_i,$$
$$\left(h_j,neut(h_j),anti(h_j)\right)\in H_j.$$

In other words, we can do the succeeding manipulations.

$$\left(h_1,neut(h_1),anti(h_1)\right)\left(h_2,neut(h_2),anti(h_2)\right)\ldots\left(h_n,neut(h_n),anti(h_n)\right)$$

$$=\left(k_1,neut(k_1),anti(k_1)\right)\left(k_2,neut(k_2),anti(k_2)\right)\ldots\left(k_n,neut(k_n),anti(k_n)\right)$$

$$\Leftrightarrow\left(h_2,neut(h_2),anti(h_2)\right)\ldots\left(h_n,neut(h_n),anti(h_n)\right)$$

$$=\left(\left(h_1,neut(h_1),anti(h_1)\right)^{-1}\ldots\left(k_1,neut(h_1),anti(h_1)\right)\right)\left(k_2,neut(k_2),anti(k_2)\right)\ldots$$
$$\left(k_n,neut(k_n),anti(k_n)\right)$$

$$\Leftrightarrow\left(h_3,neut(h_3),anti(h_3)\right)\ldots\left(h_n,neut(h_n),anti(h_n)\right)$$

$$= \left(\left(h_1, neut(h_1), anti(h_1) \right)^{-1} \ldots \left(k_1, neut(h_1), anti(h_1) \right) \right) \left(\begin{matrix} \left(h_2, neut(h_2), anti(h_2) \right)^{-1} \\ \left(k_2, neut(k_2), anti(k_2) \right) \end{matrix} \right)$$

$$\left(k_3, neut(k_3), anti(k_3) \right) \ldots \left(k_n, neut(k_n), anti(k_n) \right)$$

and likewise and then so long as we achieve

$$\left(h_n, neut(h_n), anti(h_n) \right) \left(k_n, neut(k_n), anti(k_n) \right)^{-1} \tag{1}$$

$$= \left(h_1, neut(h_1), anti(h_1) \right)^{-1} \left(k_1, neut(k_1), anti(k_1) \right) \ldots \left(h_{n-1}, neut(h_{n-1}), anti(h_{n-1}) \right)^{-1}$$

$$\left(k_{n-1}, neut(k_{n-1}), anti(k_{n-1}) \right).$$

Until now the left hand side (1) refers to H_n although the right hand side refers to $H_1 \ldots H_{n-1}$,

we obtain such $\left(h_n, neut(h_n), anti(h_n) \right) \left(k_n, neut(k_n), anti(k_n) \right)^{-1} \in H_n \cap H_1 \ldots H_{n-1} = \{1_N\}$

signifying that $\left(h_n, neut(h_n), anti(h_n) \right) = \left(k_n, neut(k_n), anti(k_n) \right).$

We end this by repeating the procedure. Let's prove this for the conditions of two NETGs. We've noticed overhead that the NET cartesian product of two NETGs H and K endowed in relation to a NETG structure by taking in mind componentwise binary operation.

$$\left(h_1, neut(h_1), anti(h_1) \right), \left(k_1, neut(k_1), anti(k_1) \right)$$

$$= \left(h_1 * h_1, neut(h_1 * h_1), anti(h_1 * h_1) \right), \left(k_1 * k_1, neut(k_1 * k_1), anti(k_1 * k_1) \right) \in H \times K.$$

The preference of this binary operation of course decides the structures of $N = H \times K$, and exceptionally, we've noticed such the neutro-isomorphic duplicates of NETGs H and K in N are NETNS-Gs. Contrarily that one may describe a NETIDP, we have to suppose that we've two NETNS-Gs.

Now let's examine a further overall setting, thus the NET subgroup K doesn't need to be NET normal, for whatever we have to describe another binary operation on the NETCP $H \times K$. this'll take us to the definition of NETIS-DP and NETES-DP.

Remember that a neutro-auto orphism of a NETG H is an objective NETG neutro-homomorphism from $H \rightarrow H$. It's obvious to realize such the set of neutro-auto orphism of H shapes a NETG according to the composition of maps and identify element the neutrality map 1_H. We symbolize it by $Aut(1_H)$.

Proposition 3.11 Suppose that H and K are NETGs, and

$$\rho : K \rightarrow Aut(H), \left(k, neut(k), anti(k) \right) \mapsto \rho \left(k, neut(k), anti(k) \right) \qquad \text{are} \qquad \text{a} \qquad \text{NETG}$$

neutro-homomorphism. Subsequently the binary operation $\left(H \times K \right) \times \left(H \times K \right) \rightarrow \left(H \times K \right),$

$$\left((h,neut(h),anti(h)),(k,neut(k),anti(k))\right), \begin{pmatrix} (h^{'},neut(h^{'}),anti(h^{'})),(k^{'},neut(k^{'}), \\ anti(k^{'})) \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} (h,neut(h),anti(h))\rho((k,neut(k),anti(k))\left((h^{'},neut(h^{'}),anti(h^{'}))\right) \\ (k,neut(k),anti(k))(k^{'},neut(k^{'}),anti(k^{'})) \end{pmatrix}$$

endows $H \times K$ with a NETG structure, with neutral element $\left(1_H,1_K\right)$.

Proof let's realize such the closure property is holds.

1) Neutrality: Let's prove that $\left(1_H,1_K\right)$ is the neutral element. We have

$$\left((h,neut(h),anti(h)),(k,neut(k),anti(k))\right)\left(1_H,1_K\right)$$

$$=\left((h,neut(h),anti(h))\rho(k,neut(k),anti(k))(1_H),(k,neut(k),anti(k))\right)$$

$$=\left((h,neut(h),anti(h)),(k,neut(k),anti(k))\right) \text{For all } (h,neut(h),anti(h)) \in H,$$

$(k,neut(k),anti(k)) \in K$, Whereby $\rho(k,neut(k),anti(k))$ is a NETG neutro-homomorphism. We also have

$$\left(1_H,1_K\right)\left((h^{'},neut(h^{'}),anti(h^{'})),(k^{'},neut(k^{'}),anti(k^{'}))\right)$$

$$=\left(\rho1_H(h^{'},neut(h^{'}),anti(h^{'})),(k^{'},neut(k^{'}),anti(k^{'}))\right)$$

$$=\left((h^{'},neut(h^{'}),anti(h^{'})),(k^{'},neut(k^{'}),anti(k^{'}))\right)$$

2) Anti-neutrality : Let $\left((h,neut(h),anti(h)),(k,neut(k),anti(k))\right) \in H \times K$ and let's prove that

$$\left(\rho^{-1}{}_{(k,neut(k),anti(k))}\left((h,neut(h),anti(h))\right)^{-1},\left(k,neut(k),anti(k)\right)\right)^{-1}$$

is the anti-neutral of

$$\left((h,neut(h),anti(h)),(k,neut(k),anti(k))\right).$$

We have

$$\left((h,neut(h),anti(h)),(k,neut(k),anti(k))\right)\begin{pmatrix} \rho^{-1}{}_{(k,neut(k),anti(k))}\begin{pmatrix} h,neut(h) \\ ,anti(h) \end{pmatrix}^{-1} \\ \left(k,neut(k),anti(k)\right)^{-1} \end{pmatrix}$$

$$= \left(h, neut(h), anti(h) \right) \rho_{(k,neut(k),anti(k))} \begin{pmatrix} \rho^{-1}(k,neut(k),anti(k)) \\ \left(h,neut(h),anti(h) \right)^{-1}, 1_K \end{pmatrix}$$

$$= \left(\left(h,neut(h),anti(h) \right) \left(h,neut(h),anti(h) \right)^{-1}, 1_K \right) = \left(1_H, 1_K \right).$$

We also have

$$\left(\rho^{-1}{}_{(k,neut(k),anti(k)}(h,neut(h),anti(h))^{-1}, (k,neut(k),anti(k))^{-1} \right)$$
$$\left((h,neut(h),anti(h))(k,neut(k),anti(k)) \right)$$

$$= \begin{pmatrix} \rho^{-1}{}_{(k,neut(k),anti(k)}(h,neut(h),anti(h))^{-1} \rho(k,neut(k),anti(k))^{-1} \\ \left(h,neut(h),anti(h) \right), 1_K \end{pmatrix}$$

$$= \begin{pmatrix} \rho(k,neut(k),anti(k))^{-1}(h,neut(h),anti(h))^{-1} \rho(k,neut(k),anti(k))^{-1} \\ \left(h,neut(h),anti(h) \right)^{-1} \rho(k,neut(k),anti(k))^{-1}(h,neut(h),anti(h)), 1_K \end{pmatrix}.$$

Using that

$$\rho^{-1}\left(k,neut(k),anti(k) \right) = \rho\left(k,neut(k),anti(k) \right)^{-1}$$

Whereby ρ is a NETG neutro-homomorphism. Instantly

$$\begin{pmatrix} \rho(k,neut(k),anti(k))^{-1}(h,neut(h),anti(h))^{-1} \rho(k,neut(k),anti(k))^{-1} \\ \left(h,neut(h),anti(h) \right), 1_K \end{pmatrix}$$

$$= \left(\rho(k,neut(k),anti(k))^{-1}(h,neut(h),anti(h))^{-1}(h,neut(h),anti(h)), 1_K \right)$$

$$= \left(\rho(k,neut(k),anti(k))^{-1}(1_H), 1_K \right) = \left(1_H, 1_K \right)$$

using that $\rho(k,neut(k),anti(k))^{-1}$ is a NETG neutro-homomorphism for every

$\left(k,neut(k),anti(k) \right) \in K.$

3) Associativity : Lastly let's check that the following condition holds, we've

$$\begin{pmatrix} (h,neut(h),anti(h)),(k,neut(k),anti(k)),(h',neut(h'),anti(h')), \\ (k',neut(k'),anti(k')) \end{pmatrix}$$
$$\left((h'',neut(h''),anti(h'')),(k'',neut(k''),anti(k'')) \right)$$

$$= \begin{pmatrix} (h, neut(h), anti(h)), \rho(k, neut(k), anti(k)), (h', neut(h'), anti(h')), \\ (k, neut(k), anti(k)), (k', neut(k'), anti(k')) \end{pmatrix}$$
$$\big((h'', neut(h''), anti(h'')), (k'', neut(k''), anti(k'')) \big)$$

$$= \begin{pmatrix} (h, neut(h), anti(h))\rho_{(k, neut(k), anti(k))}, (h', neut(h'), anti(h')), \\ \rho_{(k, neut(k), anti(k))}(k', neut(k'), anti(k')) \end{pmatrix}$$
$$\begin{pmatrix} (h'', neut(h''), anti(h'')), (k, neut(k), anti(k)), (k', neut(k'), anti(k')), \\ (k'', neut(k''), anti(k'')) \end{pmatrix},$$

While conversely

$$((h, neut(h), anti(h)), (k, neut(k), anti(k))) \begin{pmatrix} (h', neut(h'), anti(h')), (k', neut(k') \\ , anti(k'))(h'', neut(h''), anti(h'')), \\ (k'', neut(k''), anti(k'')) \end{pmatrix}$$

$$= ((h, neut(h), anti(h)), (k, neut(k), anti(k)))$$
$$\begin{pmatrix} (h', neut(h'), anti(h')), \rho_{(k', neut(k'), anti(k'))}(h'', neut(h''), \\ anti(h'')), (k', neut(k'), anti(k'))(k'', neut(k''), anti(k'')) \end{pmatrix}$$

$$= \begin{pmatrix} (h, neut(h), anti(h))\rho_{(k, neut(k), anti(k))} \begin{pmatrix} (h', neut(h'), anti(h'))\rho^{(k', neut(k'),} \\ \qquad\qquad anti(k')) \\ ((h'', neut(h''), anti(h''))) \end{pmatrix}, \\ (k, neut(k), anti(k))((k', neut(k'), anti(k'))(k'', neut(k''), anti(k''))) \end{pmatrix}$$

Whereby K is a NETG, we have

$$\big((k, neut(k), anti(k))(k', neut(k'), anti(k')) \big)(k'', neut(k''), anti(k''))$$
$$= (k, neut(k), anti(k))\big((k', neut(k'), anti(k'))(k'', neut(k''), anti(k'')) \big).$$

Mark that by seeing at the first component

$$\rho_{(k, neut(k), anti(k))}(k', neut(k'), anti(k'))$$
$$= \rho_{(k, neut(k), anti(k))} \circ \rho_{(k', neut(k'), anti(k'))}$$

utilizing that ρ is a NETG neutro-homomorphism, therefore

$$(h, neut(h), anti(h))\rho_{(k, neut(k), anti(k))}\big((h', neut(h'), anti(h')) \big)$$
$$\rho_{(k, neut(k), anti(k))}(k', neut(k'), anti(k')\big((h'', neut(h''), anti(h'')) \big)$$
$$= (h, neut(h), anti(h))\rho_{(k, neut(k), anti(k))}\big((h', neut(h'), anti(h')) \big)$$
$$\rho_{(k, neut(k), anti(k))}\rho_{(k, neut(k), anti(k))} \begin{pmatrix} \rho_{(k', neut(k'), anti(k'))} \\ ((h'', neut(h''), anti(h''))) \end{pmatrix}.$$

Furthermore, $\rho_{(k,neut(k),anti(k))}$ is a NETG neutro-homomorphism, yielding

$$(h,neut(h),anti(h))\rho_{(k,neut(k),anti(k))}\big((h',neut(h'),anti(h'))\big)$$

$$\rho_{(k,neut(k),anti(k))}\Big(\rho_{(k',neut(k'),anti(k'))}\Big)\big((h'',neut(h''),anti(h''))\big)$$

$$=(h,neut(h),anti(h))\rho_{(k,neut(k),anti(k))}\begin{pmatrix} (h',neut(h'),anti(h')) \\ \rho_{(k',neut(k'),anti(k'))} \\ \big((h'',neut(h''),anti(h''))\big) \end{pmatrix}$$

which concludes the proof. Now let's define the first NET semi-direct product.

In general, the NET direct product is not enough because the operation between elements of the two NET subgroups is always commutative. On other hand, if N is a NETG, H is a NTNS-G, K is a NET subgroup (K need not be NT normal like in a NET direct product), $K \cap N = 1_N,$ then N must be a NET semi-direct product. (The operation between elements of H and K need not be commutative.) So, we can argue that the NET semi-direct product classifies all NETGs constructed in this way.

4. Semi-Direct Products of NETG

Definition 4.1 Suppose that H and K are two NETGs, and $\rho : K \to Aut(H)$ is a NETG neutro-homomorphism. The set $H \times K$ endowed in a relation to the binary operation

$$\big((h,neut(h),anti(h)),(k,neut(k),anti(k))\big)\big((h',neut(h'),anti(h')),(k',neut(k'),anti(k'))\big)$$

$$\to \begin{pmatrix} (h,neut(h),anti(h))\rho_{(k,neut(k),anti(k))}\big((h',neut(h'),anti(h'))\big), \\ (k,neut(k),anti(k))(k',neut(k'),anti(k')) \end{pmatrix}$$

is a NETG N called a "NET external semi-direct product of NETGs H and K" b ρ, symbolized by $N = H \chi_\rho K.$

Example 4.2 The NET set $L = H \times N$, where H, N are NETGs and $N \le AutH$ is the NETES-DP of H and N when equipped with the following operation, defined by the action

$$\theta : N \to AutH : \big((h_1,neut(h_1),anti(h_1)),(n_1,neut(n_1),anti(n_1))\big)$$

$$= \begin{pmatrix} (h_1,neut(h_1),anti(h_1))\theta_{(n_1,neut(n_1),anti(n_1))}\big((h_2,neut(h_2),anti(h_2))\big), \\ (n_1,neut(n_1),anti(n_1))(n_2,neut(n_2),anti(n_2)) \end{pmatrix}$$

$$= \begin{pmatrix} (h_1,neut(h_1),anti(h_1))(n_1,neut(n_1),anti(n_1))\big((h_2,neut(h_2),anti(h_2))\big), \\ (n_1,neut(n_1),anti(n_1))(n_2,neut(n_2),anti(n_2)) \end{pmatrix},$$

for all $(h_1, neut(h_1), anti(h_1)), (h_2, neut(h_2), anti(h_2)) \in H$ and all $(n_1, neut(n_1), anti(n_1)),$ $(n_2, neut(n_2), anti(n_2)) \in N.$

Definition 4.3 Let N be a NETG in a relation to NET subgroups H and $K.$ We say that N is the "NETIS-DP of H and K" if H is a NETNS-G of $N,$ thus $HK = N$ and $H \cap K = \{1_N\}.$ It is symbolized by $N = H \rtimes K.$

Example 4.4 Let's show that the dihedral NETG D_{2n} is the NETIS-DP of two of its NET subgroups : the NET subgroup of rotations of a regular $n-$gon, and the NET subgroup generated by a single reflection of the same regular $n-$gon. If $D_{2n} = <(a, neut(a), anti(a)), (x, neut(x), anti(x))>,$ where $(a, neut(a), anti(a))$ generates the NET subgroup $<(a, neut(a), anti(a))>$ of rotations and $(x, neut(x), anti(x))$ generates the NET subgroup $<(x, neut(x), anti(x))>,$ then we know that $(a, neut(a), anti(a))^n = 1_N$ and $(x, neut(x), anti(x))^2 = 1_N,$ where 1_N is the neutral symmetry. We know that $\{1_N\} = <(a, neut(a), anti(a))> \cap <(x, neut(x), anti(x))>;$ we also know that, if x is a reflection and a a rotation, then

$$(x, neut(x), anti(x))(a, neut(a), anti(a)) = (a, neut(a), anti(a))^{n-1}(x, neut(x), anti(x)).$$

Being D_{2n} the NETG of all symmetries of a regular $n-$gon, it contains all and only the rotations and reflections of the $n-$gon itself; this fact, combined with the fact that $\{1_N\} = <(a, neut(a), anti(a))> \cap <(x, neut(x), anti(x))>,$ allows us to deduce

$$\left| <(a, neut(a), anti(a))> \cap <(x, neut(x), anti(x))> \right| = \left| D_{2n} \right|.$$

Since $<(a, neut(a), anti(a))> \cap <(x, neut(x), anti(x))> \leq D_{2n},$ it follows $<(a, neut(a), anti(a))> \cap <(x, neut(x), anti(x))> = D_{2n}.$ Finally, we obtain

$$(x, neut(x), anti(x))(a, neut(a), anti(a))(x, neut(x), anti(x))^{-1}$$
$$= (a, neut(a), anti(a))^{n-1} \in <(a, neut(a), anti(a))>;$$

thus, $<(a, neut(a), anti(a))>$ is NT normal. Therefore

$$D_{2n} = <(a, neut(a), anti(a))> \rtimes <(x, neut(x), anti(x))>.$$

Lemma 4.5 Assume that N is a NETG with NET subgroups H and K. Assume that $N = HK$

and $H \cap K = \{1_N\}$. Subsequently all element $(n, neut(n), anti(n))$ of N can be inscribed

particularly in the form $(h, neut(h), anti(h))(k, neut(k), anti(k))$, for $(h, neut(h), anti(h)) \in H$
and $(k, neut(k), anti(k)) \in K$.

Proof Since $N = HK$, we know that $(n, neut(n), anti(n))$ can be written as
$(h, neut(h), anti(h))(k, neut(k), anti(k))$. Assume it can also be inscribed
$(h', neut(h'), anti(h'))(k', neut(k'), anti(k'))$. Then

$$(h, neut(h), anti(h))(k, neut(k), anti(k)) = (h', neut(h'), anti(h'))(k', neut(k'), anti(k'))$$

so

$$(h', neut(h'), anti(h'))^{-1}(h, neut(h), anti(h)) = (k', neut(k'), anti(k'))(k, neut(k), anti(k))^{-1}$$
$$\in H \cap K = \{1_N\}.$$

In case $(h, neut(h), anti(h)) = (h', neut(h'), anti(h'))$ and
$(k, neut(k), anti(k)) = (k', neut(k'), anti(k'))$.

The NETIDPs and NETEDPs were two sides of the similar objects, consequently are the

NETIS-DPs and NETES-PDs. If $N = H \chi_\rho K$ is the NETES-DP of NETGS H and K, subsequently

$\overline{H} = H \times \{1\}$ is a NETNS-G of N and it's obvious that N is the NETIS-DP of $H \times \{1\}$ and

$\{1\} \times K$. Because of this we can go from NETES-PDs to NETIS-PDs. The following conclusion goes

in the another way, from NET internal to external semi-direct products.

Proposition 4.6 Assume that N is a NETG with NET subgroups H and K, and N is the

NETIS-PDs of H and K. Then $N \square H \chi_\rho K$ where $\rho: K \to Aut(H)$ is stated by

$$\rho_{(k, neut(k), anti(k))}\big((h, neut(h), anti(h))\big) = (k, neut(k), anti(k))(h, neut(h), anti(h))$$
$$\big((k, neut(k), anti(k))\big)^{-1},$$
$$(h, neut(h), anti(h)) \in H, (k, neut(k), anti(k)) \in K.$$

Proof Note that $\rho_{(k, neut(k), anti(k))}$ refers to $Aut(H)$ where H is NET normal. By the lemma

4.5 all the element $(n, neut(n), anti(n))$ of N can be inscribed particularly in terms of
$$(h, neut(h), anti(h))(k, neut(k), anti(k)),$$

with $(h, neut(h), anti(h)) \in H$ and $(k, neut(k), anti(k)) \in K$. So that, the map $\varphi: H \chi_\rho K \to N$,

$$\varphi\big((h, neut(h), anti(h))(k, neut(k), anti(k))\big) = (h, neut(h), anti(h))(k, neut(k), anti(k))$$

is a bijection. It is just to prove such this bijection is a neutro-homomorphism. Stated

$$\big((h, neut(h), anti(h)), (k, neut(k), anti(k))\big)$$

and

$$\big((h',neut(h'),anti(h')),(k',neut(k'),anti(k'))\big) \text{ in } H\chi_\rho K.$$

We have

$$\varphi\left(\big((h,neut(h),anti(h)),(k,neut(k),anti(k))\big)\begin{pmatrix}(h',neut(h'),anti(h')),(k',neut(k'),\\ anti(k'))\end{pmatrix}\right)$$

$$=\varphi\left(\begin{pmatrix}(h,neut(h),anti(h))\rho_{(k,neut(k),anti(k))}((h',neut(h'),anti(h'))),\\ (k,neut(k),anti(k))(k',neut(k'),anti(k'))\end{pmatrix}\right)$$

$$=\varphi\begin{pmatrix}(h,neut(h),anti(h))(k,neut(k),anti(k))(h',neut(h'),anti(h'))\\ (k,neut(k),anti(k))^{-1},(k,neut(k),anti(k))(k',neut(k'),anti(k'))\end{pmatrix}$$

$$=(h,neut(h),anti(h))(k,neut(k),anti(k))(h',neut(h'),anti(h'))(k',neut(k'),anti(k'))$$

$$=\varphi\big((h,neut(h),anti(h)),(k,neut(k),anti(k))\big)\varphi\begin{pmatrix}(h',neut(h'),anti(h')),\\ (k',neut(k'),anti(k'))\end{pmatrix}.$$

Therefore φ is a NETG neutro-homomorphism, which ends the proof. Shortly, we obtain such all NETIS-DP is neutro-isomorphic to any NETES-DP, when φ is conjugation.

5. Conclusion

The most important point of this article is first to define the NETs and subsequently use these NETs to describe the NET internal and external direct and semi-direct products of NETG. As in classical group theory, in neutrosophic extended triplet group theory building blocks for finite NET groups is simple NET groups. One way to make this simple NETG to larger group is NET direct product. As an addition, we allow rise to a new field called NT Structures (such as neutrosophic extended triplet direct product and semi-direct product. Another researchers can work on the application of NETEDP and NETIDP and semi-direct product to NT vector spaces (representation of the NETG), module theory, number theory, analysis, geometry, zigzag products of graphs and topological spaces.

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