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$\mathcal{N}_{\alpha g^{\#}\psi}$-open map, $\mathcal{N}_{\alpha g^{\#}\psi}$-closed map and $\mathcal{N}_{\alpha g^{\#}\psi}$-homeomorphism in neutrosophic topological spaces

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Abstract: As a generalization of fuzzy sets and intuitionistic fuzzy sets, neutrosophic sets have been developed by Smarandache to represent imprecise, incomplete and inconsistent information existing in the real world. A neutrosophic set is characterized by a truth-value, an indeterminacy value, and a falsity-value. Salama introduced neutrosophic topological spaces by using Smarandache's neutrosophic sets. In this article, we introduce the concept of $\mathcal{N}_{\alpha g^{\#}\psi}$-open and $\mathcal{N}_{\alpha g^{\#}\psi}$-closed mappings in neutrosophic topological spaces and studied some of their related properties. Further the work is extended to $\mathcal{N}_{\alpha g^{\#}\psi}$- homeomorphism, $\mathcal{N}_{\alpha g^{\#}\psi}$-C homeomorphism and $\mathcal{T}_{\mathcal{N}_{\alpha g^{\#}\psi}}$-space in neutrosophic topological spaces and establishes some of their related attributes.

Keywords: $\mathcal{N}_{\alpha g^{\#}\psi}$ -open map, $\mathcal{N}_{\alpha g^{\#}\psi}$ -closed map, $\mathcal{T}_{\mathcal{N}_{\alpha g^{\#}\psi}}$ -space, $\mathcal{N}_{\alpha g^{\#}\psi}$ -homeomorphism, $\mathcal{N}_{\alpha g^{\#}\psi}$-C homeomorphism.

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1. Introduction

The first successful attempt towards containing non-probabilistic uncertainty, i.e. uncertainty which is not incite by randomness of an event, into mathematical modeling was made in 1965 by L. A. Zadeh [21] through his significant theory on fuzzy sets (FST).

A fuzzy set is a set where each element of the universe belongs to it but with some value or degree of belongingness which lies between 0 and 1 and such values are called membership value of an element in that set. This gradation concept is very well suited for applications involving vague data such as natural language processing or in artificial intelligence, handwriting and speech recognition etc. Although Fuzzy set theory is very successful in handling uncertainties arising from vagueness or partial belongingness of an element in a set, it cannot model all type of uncertainties pre-veiling in different real physical problems such as problems involving incomplete information.

Further generalization of this fuzzy set was introduced by K. Atanassov [10] in 1986, which is known as Intuitionistic fuzzy sets (IFS). In IFS, instead of one membership value, there is also a non-membership value devoted to each element. Further there is a restriction that the sum of these two values is less or equal to unity. In IFS the degree of non-belongingness is not independent but it is dependent on the degree of belongingness. Fuzzy set theory can be considered as a special case of an IFS where the degree of non-belongingness of an element is exactly equal to 1 minus the degree of belongingness. IFS have the expertise to handle vague data of both complete and incomplete in nature. In applications like expert systems, belief systems and information fusion etc., where degree of non-belongingness is equally important as degree of belongingness, intuitionistic fuzzy sets are quite useful.

There are of course several other generalizations of Fuzzy as well as Intuitionistic fuzzy sets like L-fuzzy sets and intuitionistic L- fuzzy sets, interval valued fuzzy and intuitionistic fuzzy sets etc that have been developed and applied in solving many practical physical problems. Recently a new theory has been introduced which is known as neutrosophic logic and sets. The term neutrosophy means knowledge of impartial thought and this impartial represents the main distinction between fuzzy and intuitionistic fuzzy logic and set. Neutrosophic logic was introduced by Smarandache [14] in 1995. It is a logic in which each proposition is calculated to have a degree of truth (T), a degree of indeterminacy (I) and a degree of falsity (F). A Neutrosophic set is a set where each element of the universe has a degree of truth, indeterminacy and falsity respectively and which lies between [0, 1]*, the non-standard unit interval

Unlike in intuitionistic fuzzy sets, where the included uncertainty is dependent of the degree of belongingness and degree of non-belongingness, here the uncertainty present, i.e. the indeterminacy factor, is independent of truth and falsity values. Neutrosophic sets are indeed more general than IFS as there are no constraints between the degree of truth, degree of indeterminacy and degree of falsity. All these degrees can individually vary within [0, 1]*.

Smarandache's neutrosophic concept have wide range of real time applications for the fields of [1,2,3,4,5,6,7&8] Information Systems, Computer Science, Artificial Intelligence, Applied Mathematics, decision making. Mechanics, Electrical & Electronic, Medicine and Management Science etc.

Salama and Alblowi[18] introduced the new concept of neutrosophic topological space in 2012. The neutrosophic closed sets and neutrosophic continuous functions were introduced by Salama, Smarandache and Valeri[19] in 2014. Arokiarani et al.[9] introduced the neutrosophic α-closed set in neutrosophic topological spaces.

Parimala et al.[14] studied the concept of neutrosophic $\alpha\psi$-closed sets and neutrosophic homeomorphisms[15] in neutrosophic topological spaces. Recently Vigneshwaran et al.[13] introduced the concept of $\mathcal{N}_{\alpha g^{\#}\psi}$-closed sets in neutrosophic topological spaces and studied some of its properties and also $\mathcal{N}_{\alpha g^{\#}\psi}$-continuous and $\mathcal{N}_{\alpha g^{\#}\psi}$-irresolute functions[12] were initiated and studied in neutrosophic topological spaces.

The focus of this article is to introduce the idea of $\mathcal{N}_{\alpha g^{\#}\psi}$ -open and $\mathcal{N}_{\alpha g^{\#}\psi}$- closed mappings in neutrosophic topological spaces and also the work is extended to $\mathcal{N}_{\alpha g^{\#}\psi}$ -homeomorphism, $\mathcal{N}_{\alpha g^{\#}\psi}$-C homeomorphism and $\mathcal{T}_{\mathcal{N}_{\alpha g^{\#}\psi}}$-space in neutrosophic topological spaces and obtain some of its basic properties.

2. Preliminaries

Definition 2.1.[17] A neutrosophic set $\mathcal{S}$ is an object of the following form $\mathcal{A}$={⟨s, $\mathcal{U}_{\mathcal{A}}$(s), $\mathcal{V}_{\mathcal{A}}$(s), $\mathcal{W}_{\mathcal{A}}$(s): s∈$\mathcal{S}$⟩} where $\mathcal{U}_{\mathcal{A}}$(s), $\mathcal{V}_{\mathcal{A}}$(s) and $\mathcal{W}_{\mathcal{A}}$(s) denote the degree of membership, the degree of indeterminacy and the degree of non membership for each element $s\epsilon\mathcal{S}$ to the set $\mathcal{A}$, respectively.

Definition 2.2. [17] Let $\mathcal{A}$ and $\mathcal{B}$ be Neutrosophic sets of the form

$\mathcal{A}$={⟨s, $\mathcal{U}_{\mathcal{A}}$(s), $\mathcal{V}_{\mathcal{A}}$(s), $\mathcal{W}_{\mathcal{A}}$(s): s$\epsilon\mathcal{S}$⟩} and
$\mathcal{B}$={⟨ s, $\mathcal{U}_{\mathcal{B}}$(s), $\mathcal{V}_{\mathcal{B}}$(s), $\mathcal{W}_{\mathcal{B}}$(s): s$\epsilon\mathcal{S}$⟩}. Then
(i)$\mathcal{A} \subseteq \mathcal{B}$ if and only if $\mathcal{U}_{\mathcal{A}}(s) \leq \mathcal{U}_{\mathcal{B}}(s)$, $\mathcal{V}_{\mathcal{A}}(s) \leq \mathcal{V}_{\mathcal{B}}(s)$ and $\mathcal{W}_{\mathcal{A}}(s) \geq \mathcal{W}_{\mathcal{B}}(s)$;
(ii)$\bar{\mathcal{A}}$= {⟨$\mathcal{W}_{\mathcal{A}}$(s), $\mathcal{V}_{\mathcal{A}}$(s), $\mathcal{U}_{\mathcal{A}}$(s): s$\epsilon\mathcal{S}$⟩};
(iii)$\mathcal{A} \cup \mathcal{B}$={ ⟨s, $\mathcal{U}_{\mathcal{A}}(s) \vee \mathcal{U}_{\mathcal{B}}(s)$, $\mathcal{V}_{\mathcal{A}}(s) \wedge \mathcal{V}_{\mathcal{B}}(s)$, $\mathcal{W}_{\mathcal{A}}(s) \wedge \mathcal{W}_{\mathcal{B}}(s)$: s$\epsilon\mathcal{S}$⟩};
(iv)$\mathcal{A} \cap \mathcal{B}$={ ⟨s, $\mathcal{U}_{\mathcal{A}}(s) \wedge \mathcal{U}_{\mathcal{B}}(s)$, $\mathcal{V}_{\mathcal{A}}(s) \vee \mathcal{V}_{\mathcal{B}}(s)$, $\mathcal{W}_{\mathcal{A}}(s) \vee \mathcal{W}_{\mathcal{B}}(s)$: s$\epsilon\mathcal{S}$⟩}.

Definition 2.3. [18] A neutrosophic topology in a nonempty set $\mathcal{X}$ is a family $\Im$ of neutrosophic sets in $\mathcal{X}$ satisfying the following axioms:

(i) 0_N, $1_N \in \Im$;
(ii)$\mathcal{U} \cap \mathcal{V} \in \Im$ for any $\mathcal{U}, \mathcal{V} \in \Im$;
(iii)$\cup (\mathcal{U})_i$ for any arbitrary family $(\mathcal{U})_i$: $i \in J \subseteq \Im$

Definition 2.4.[18] Let $\mathcal{P}$ be a neutrosophic set in neutrosophic topological space $\mathcal{X}$. Then
$\mathcal{N}$int($\mathcal{P}$)=∪{$\mathcal{D}$: $\mathcal{D}$ is a neutrosophic open set in $\mathcal{X}$ and $\mathcal{D} \subseteq \mathcal{P}$ } is called a neutrosophic interior of $\mathcal{P}$.
$\mathcal{N}$cl($\mathcal{P}$)=∩{ $\mathcal{E}$: $\mathcal{E}$ is a neutrosophic closed set in $\mathcal{X}$ and $\mathcal{E} \supseteq \mathcal{P}$ } is called a neutrosophic closure of $\mathcal{P}$.

Definition 2.5.[12] A subset $\mathcal{A}$ of a neutrosophic space $(\mathcal{X},\Im)$ is called a neutrosophic $\mathcal{N}_{\alpha g^{\#}\psi}$-closed set if $\mathcal{N}_{\alpha}cl(\mathcal{A}) \subseteq \mathcal{G}$ whenever $\mathcal{A} \subseteq \mathcal{G}$ and $\mathcal{G}$ is $\mathcal{N}_{g^{\#}\psi}$-open in $(\mathcal{X},\Im)$.

Definition 2.6. A function d: $(\mathcal{S},\Im) \rightarrow (\mathcal{T}, \xi)$ is called
(i) a $\mathcal{N}_{\alpha g^{\#}\psi}$-continuous[13] if $d^{-1}(\mathcal{A})$ is a $\mathcal{N}_{\alpha g^{\#}\psi}$ -closed set of $(\mathcal{S},\Im)$ for every neutrosophic closed set $\mathcal{A}$ of $(\mathcal{T},\xi)$.
(ii) a $\mathcal{N}_{\alpha g^{\#}\psi}$ -irresolute[13] if $d^{-1}(\mathcal{A})$ is a $\mathcal{N}_{\alpha g^{\#}\psi}$-closed set of $(\mathcal{S},\Im)$ for every $\mathcal{N}_{\alpha g^{\#}\psi}$-closed set $\mathcal{A}$ of $(\mathcal{T},\xi)$.

Definition 2.7.[15] A bijection g: $(\mathcal{S},\Im) \rightarrow (\mathcal{T},\xi)$ is called a homeomorphism if g and g^{-1} are neutrosophic continuous mappings.

All over this paper neutrosophic $\alpha g^{\#}\psi$-interior and neutrosophic $\alpha g^{\#}\psi$-closure is denoted by $\mathcal{N}_{\alpha g^{\#}\psi}$-i* and $\mathcal{N}_{\alpha g^{\#}\psi}$-c* respectively.

3. $\mathcal{N}_{\alpha g^{\#}\psi}$-open mapping

Definition 3.1. A mapping $d: (\mathcal{S},\Im) \rightarrow (\mathcal{T},\xi)$ is $\mathcal{N}_{\alpha g^{\#}\psi}$-open if image of every neutrosophic open set of $(\mathcal{S},\Im)$ is $\mathcal{N}_{\alpha g^{\#}\psi}$ -open set in $(\mathcal{T},\xi)$.

Theorem 3.2. Each neutrosophic open mapping is a $\mathcal{N}_{\alpha g^{\#}\psi}$-open mapping.

Proof: Let $\mathcal{A}$ be a neutrosophic open set in $(\mathcal{S},\Im)$. Since d is a neutrosophic open mapping, $d(\mathcal{A})$ is neutrosophic open in $(\mathcal{T},\xi)$. But every neutrosophic open set is a $\mathcal{N}_{\alpha g^{\#}\psi}$-open set. Therefore, $d(\mathcal{A})$ is a $\mathcal{N}_{\alpha g^{\#}\psi}$-open set in $(\mathcal{T},\xi)$. Hence, d is a $\mathcal{N}_{\alpha g^{\#}\psi}$-open mapping.

Let a $\mathcal{N}_{\alpha g^{\#}\psi}$-open mapping be not a neutrosophic open map by the following example.

Example 3.3. Let $\mathcal{S} = \{u, v, w\}$, $\mathfrak{I}$ ={0N, $\mathcal{D}_1$,$\mathcal{D}_2$, $\mathcal{D}_3$, $\mathcal{D}_4$ 1N} be a neutrosophic topology on ($\mathcal{S}$, $\mathfrak{I}$).

$\mathcal{D}_1$ =⟨s, (0.2,0.1,0.1), (0.2,0.1,0.1), (0.3,0.5,0.5)⟩

$\mathcal{D}_2$ =⟨s, (0.1,0.2,0.2), (0.4,0.3,0.3), (0.3,0.3,0.3)⟩

$\mathcal{D}_3$ =⟨s, (0.2,0.2,0.2), (0.2,0.1,0.1), (0.3,0.3,0.3)⟩

$\mathcal{D}_4$=⟨s, (0.1,0.1,0.1), (0.4,0.3,0.3), (0.3,0.5,0.5)⟩, and

let $\mathcal{T} = \{u, v, w\}$, ξ ={0N, $\mathcal{F}_1$,$\mathcal{F}_2$, $\mathcal{F}_3$, $\mathcal{F}_4$ 1N} be a neutrosophic topology on ($\mathcal{T}$, ξ).

$\mathcal{F}_1$=⟨t, (0.3,0.3,0.3), (0.2,0.1,0.1), (0.2,0.2,0.2)⟩

$\mathcal{F}_2$=⟨t, (0.2,0.2,0.2), (0.1,0.1,0.1), (0.3,0.3,0.3)⟩

$\mathcal{F}_3$=⟨t, (0.3,0.3,0.3), (0.1,0.1,0.1), (0.2,0.1,0.1)⟩

$\mathcal{F}_4$=⟨t, (0.2,0.2,0.2), (0.2,0.1,0.1), (0.3,0.3,0.3)⟩

Define $d : (\mathcal{S}, \mathfrak{I}) \rightarrow (\mathcal{T}, \xi)$ by $d(u) = u$, $d(v) = v$, $d(w) = w$.

$\mathcal{N}_{\alpha g^{\#}\psi}$ -open sets of $(\mathcal{T}, \xi)$ = ⟨s, (0.2,0.1,0.1), (0.2,0.1,0.1), (0.3,0.5,0.5)⟩.

Here $d(\mathcal{D}_1)$ is $\mathcal{N}_{\alpha g^{\#}\psi}$-open in $(\mathcal{T}, \xi)$. Therefore d is $\mathcal{N}_{\alpha g^{\#}\psi}$-open mapping. However, it is not a neutrosophic open mapping because $d(\mathcal{D}_1)$ is not neutrosophic open in $(\mathcal{T}, \xi)$.

Theorem 3.4. A mapping $d: (\mathcal{S}, \mathfrak{I}) \rightarrow (\mathcal{T}, \xi)$ is $\mathcal{N}_{\alpha g^{\#}\psi}$-open iff for every neutrosophic set $\mathcal{A}$ of $(\mathcal{S}, \mathfrak{I})$, $d(i^*(\mathcal{A}) \subseteq \mathcal{N}_{\alpha g^{\#}\psi} - (i^*(d(\mathcal{A}))$.

Proof: Necessity: Let d be a $\mathcal{N}_{\alpha g^{\#}\psi}$-open mapping and $\mathcal{A}$ is a neutrosophic open set in $(\mathcal{S}, \mathfrak{I})$. Now, $i^*(\mathcal{A}) \subseteq \mathcal{A}$ implies $d(i^*(\mathcal{A})) \subseteq d(\mathcal{A})$. Since d is a $\mathcal{N}_{\alpha g^{\#}\psi}$-open mapping, $d(i^*(\mathcal{A}))$ is $\mathcal{N}_{\alpha g^{\#}\psi}$-open set in $(\mathcal{T}, \xi)$ such that $d(i^*(\mathcal{A}) \subseteq d(\mathcal{A})$ therefore $d(i^*(\mathcal{A} \subseteq \mathcal{N}_{\alpha g^{\#}\psi}(i^* d(\mathcal{A}))$.

Sufficiency: Assume $\mathcal{A}$ is a neutrosophic open set of $(\mathcal{S}, \mathfrak{I})$. Then $d(\mathcal{A}) = d(i^*(\mathcal{A}) \subseteq \mathcal{N}_{\alpha g^{\#}\psi} - (i^*(d(\mathcal{A}))$. But $\mathcal{N}_{\alpha g^{\#}\psi} - (i^*(d(\mathcal{A}))) \subseteq d(\mathcal{A})$. So $d(\mathcal{A})$ = $\mathcal{N}_{\alpha g^{\#}\psi} - (i^*(\mathcal{A}))$ which implies $d(\mathcal{A})$ is a $\mathcal{N}_{\alpha g^{\#}\psi}$-open set of $(\mathcal{T}, \xi)$ and hence d is a $\mathcal{N}_{\alpha g^{\#}\psi}$-open.

Theorem 3.5. If $d: (\mathcal{S}, \mathfrak{I}) \rightarrow (\mathcal{T}, \xi)$ is a $\mathcal{N}_{\alpha g^{\#}\psi}$-open mapping then $i^*(d^{-1}(\mathcal{A}) \subseteq d^{-1}(\mathcal{N}_{\alpha g^{\#}\psi}\text{-} (i^*(\mathcal{A}))$ for every neutrosophic set $\mathcal{A}$ of $(\mathcal{T}, \xi)$.

Proof: Let $\mathcal{A}$ is a neutrosophic set of $(\mathcal{T}, \xi)$. Then $i^*\,(d^{-1}(\mathcal{A}))$ is a neutrosophic open set in $(\mathcal{S}, \mathfrak{I})$. Since d is $\mathcal{N}_{\alpha g^{\#}\psi}$-open $d(i^*\,(d^{-1}(\mathcal{A})))$ is $\mathcal{N}_{\alpha g^{\#}\psi}$- open in $(\mathcal{T}, \xi)$ and hence $d(i^*(d^{-1}(\mathcal{A}))) \subseteq \mathcal{N}_{\alpha g^{\#}\psi} - (i^*\,(d(d^{-1}(\mathcal{A}))) \subseteq \mathcal{N}_{\alpha g^{\#}\psi} - (i^*\,(\mathcal{A}))$. Thus $i^*\,(d^{-1}(\mathcal{A})) \subseteq d^{-1}(\mathcal{N}_{\alpha g^{\#}\psi} - (i^*\,(\mathcal{A})))$.

Theorem 3.6. A mapping $d: (\mathcal{S}, \mathfrak{I}) \rightarrow (\mathcal{T}, \xi)$ is $\mathcal{N}_{\alpha g^{\#}\psi}$-open iff for each neutrosophic set $\mathcal{F}$ of $(\mathcal{T}, \xi)$ and for each neutrosophic closed set $\mathcal{U}$ of $(\mathcal{S}, \mathfrak{I})$ containing $d^{-1}(\mathcal{F})$ there is a $\mathcal{N}_{\alpha g^{\#}\psi}$-closed set $\mathcal{A}$ of $(\mathcal{T}, \xi)$ such that $\mathcal{F} \subseteq \mathcal{A}$ and $d^{-1}(\mathcal{A}) \subseteq \mathcal{U}$.

Proof: Necessity: Assume d is a $\mathcal{N}_{\alpha g^{\#}\psi}$-open mapping. Let $\mathcal{F}$ be the neutrosophic closed set of $(\mathcal{T}, \xi)$ and $\mathcal{U}$ is a neutrosophic closed set of $(\mathcal{S}, \mathfrak{I})$ such that $d^{-1}\,(\mathcal{F}) \subseteq \mathcal{U}$. Then $\mathcal{A} = (d^{-1}(\mathcal{U}^c\,))^c$ is $\mathcal{N}_{\alpha g^{\#}\psi}$-closed set of $(\mathcal{T}, \xi)$ such that $d^{-1}\,(\mathcal{A}) \subseteq \mathcal{U}$.

Sufficiency: Assume $\mathcal{G}$ is a neutrosophic open set of $(\mathcal{S}, \mathfrak{I})$. Then $d^{-1}((d(\mathcal{G}))^c \subseteq \mathcal{G}^{\,c}$ and $\mathcal{G}^{\,c}$ is neutrosophic closed set in $(\mathcal{S}, \mathfrak{I})$. By hypothesis there is a $\mathcal{N}_{\alpha g^{\#}\psi}$-closed set $\mathcal{A}$ of $(\mathcal{T}, \xi)$ such that $(d(\mathcal{G}))^c \subseteq \mathcal{A}$ and $d^{-1}(\mathcal{A}) \subseteq \mathcal{G}^{\,c}$. Therefore $\mathcal{G} \subseteq (d^{-1}(\mathcal{A}))^c$. Hence $\mathcal{A}^c \subseteq d(\mathcal{G}) \subseteq d((d^{-1}\,(\mathcal{A}))^c) \subseteq \mathcal{A}^c$ which implies $d(\mathcal{G}) = \mathcal{A}^c$. Since $\mathcal{A}^c$ is $\mathcal{N}_{\alpha g^{\#}\psi}$-open set of $(\mathcal{T}, \xi)$. Hence $d(\mathcal{G})$ is $\mathcal{N}_{\alpha g^{\#}\psi}$-open in $(\mathcal{T}, \xi)$ and thus d is $\mathcal{N}_{\alpha g^{\#}\psi}$-open mapping.

Theorem 3.7. A mapping $d: (\mathcal{S}, \mathfrak{I}) \rightarrow (\mathcal{T}, \xi)$ is $\mathcal{N}_{\alpha g^{\#}\psi}$-open iff $d^{-1}\,(\mathcal{N}_{\alpha g^{\#}\psi}\text{-}(c^*(\mathcal{B})) \subseteq c^*(\,d^{-1}\,(\mathcal{B}))$ for every neutrosophic set $\mathcal{B}$ of $(\mathcal{T}, \xi)$.

Proof: Necessity: Assume d is a $\mathcal{N}_{\alpha g^{\#}\psi}$ -open mapping. For any neutrosophic set $\mathcal{B}$ of $(\mathcal{T}, \xi)$, $d^{-1}(\mathcal{B}) \subseteq c^*(d^{-1}(\mathcal{B}))$. Therefore by theorem 3.3 there exists a $\mathcal{N}_{\alpha g^{\#}\psi}$-closed set $\mathcal{F}$ in $(\mathcal{T}, \xi)$ such that

$\mathcal{B} \subseteq \mathcal{F}$ and $d^{-1}(\mathcal{F}) \subseteq c^*(d^{-1}(\mathcal{B}))$. Therefore we obtain that $d^{-1}(\mathcal{N}_{\alpha g^{\#}\psi}$- c^* $(\mathcal{B})) \subseteq d^{-1}(\mathcal{F}) \subseteq c^*(d^{-1}(\mathcal{B}))$.

Sufficiency: Assume $\mathcal{B}$ is a neutrosophic set of $(\mathcal{T}, \xi)$ and F is a neutrosophic closed set of $(\mathcal{S}, \Im)$ containing $d^{-1}(\mathcal{B})$. Put $\mathcal{W} = c^*(\mathcal{B})$, then $\mathcal{B} \subseteq \mathcal{W}$ and $\mathcal{W}$ is $\mathcal{N}_{\alpha g^{\#}\psi}$-closed and $d^{-1}(\mathcal{W}) \subseteq c^*(d^{-1}(\mathcal{B})) \subseteq \mathcal{F}$. Then by theorem 3.6, d is $\mathcal{N}_{\alpha g^{\#}\psi}$-open mapping.

Theorem 3.8. If $d: (\mathcal{S}, \Im) \to (\mathcal{T}, \xi)$ and $e: (\mathcal{T}, \xi) \to (\mathcal{V}, \omega)$ be two neutrosophic mappings and $eod: (\mathcal{S}, \Im) \to (\mathcal{V}, \omega)$ is $\mathcal{N}_{\alpha g^{\#}\psi}$-open. If $e: (\mathcal{T}, \xi) \to (\mathcal{V}, \omega)$is $\mathcal{N}_{\alpha g^{\#}\psi}$ -irresolute then d: $(\mathcal{S}, \Im)$→ $(\mathcal{T}, \xi)$ is $\mathcal{N}_{\alpha g^{\#}\psi}$-open mapping.

Proof: Let $\mathcal{H}$ be a neutrosophic open set in $(\mathcal{S}, \Im)$. Then $eod(\mathcal{H})$ is $\mathcal{N}_{\alpha g^{\#}\psi}$-open set of $(\mathcal{V}, \omega)$ because eod is $\mathcal{N}_{\alpha g^{\#}\psi}$-open mapping. Since e is $\mathcal{N}_{\alpha g^{\#}\psi}$-irresolute and $eod(\mathcal{H})$ is $\mathcal{N}_{\alpha g^{\#}\psi}$-open set of $(\mathcal{V}, \omega)$ therefore $e^{-}(eod(\mathcal{H})) = d(\mathcal{H})$ is $\mathcal{N}_{\alpha g^{\#}\psi}$-open set in $(\mathcal{T}, \xi)$. Hence d is $\mathcal{N}_{\alpha g^{\#}\psi}$-open mapping.

Theorem 3.9. If $d: (\mathcal{S}, \Im) \to (\mathcal{T}, \xi)$ is neutrosophic open and $e: (\mathcal{T}, \xi) \to (\mathcal{V}, \omega)$ is $\mathcal{N}_{\alpha g^{\#}\psi}$-open mappings then eod: $(\mathcal{S}, \Im) \to (\mathcal{V}, \omega)$ is $\mathcal{N}_{\alpha g^{\#}\psi}$-open.

Proof: Let $\mathcal{H}$ be a neutrosophic open set in $(\mathcal{S}, \Im)$. Then $d(\mathcal{H})$ is a neutrosophic open set of $(\mathcal{T}, \xi)$ because d is a neutrosophic open mapping. Since e is $\mathcal{N}_{\alpha g^{\#}\psi}$-open, $e(d(\mathcal{H})) = (eod)(\mathcal{H})$ is $\mathcal{N}_{\alpha g^{\#}\psi}$-open set of $(\mathcal{V}, \omega)$. Hence eod is $\mathcal{N}_{\alpha g^{\#}\psi}$-open mapping.

4. $\mathcal{N}_{\alpha g^{\#}\psi}$-closed mapping

Definition 4.1. A mapping $d: (\mathcal{S}, \Im) \to (\mathcal{T}, \xi)$ is $\mathcal{N}_{\alpha g^{\#}\psi}$-closed if image of every neutrosophic closed set of $(\mathcal{S}, \Im)$ is $\mathcal{N}_{\alpha g^{\#}\psi}$-closed set in $(\mathcal{T}, \xi)$.

Theorem 4.2. Each neutrosophic closed mapping is $\mathcal{N}_{\alpha g^{\#}\psi}$-closed mapping.

Proof: Let $\mathcal{A}$ be a neutrosophic closed set in $(\mathcal{S}, \Im)$. Since d is a neutrosophic closed mapping, $d(\mathcal{A})$ is neutrosophic closed in $(\mathcal{T}, \xi)$. But every neutrosophic closed set is a $\mathcal{N}_{\alpha g^{\#}\psi}$-closed set. Therefore, $d(\mathcal{A})$ is a $\mathcal{N}_{\alpha g^{\#}\psi}$-closed set in $(\mathcal{T}, \xi)$. Hence, d is a $\mathcal{N}_{\alpha g^{\#}\psi}$-closed mapping.

Let a $\mathcal{N}_{\alpha g^{\#}\psi}$ -closed mapping need not be a neutrosophic closed map by the following example.

Example 4.3. Let $\mathcal{S} = \{u, v, w\}$, $\Im$ ={0N, $\mathcal{D}_1$,$\mathcal{D}_2$, $\mathcal{D}_3$, $\mathcal{D}_4$ 1N} be a neutrosophic topology on $(\mathcal{S}, \Im)$.

$\mathcal{D}_1$ =⟨s, (0.2,0.1,0.1), (0.2,0.1,0.1), (0.3,0.5,0.5)⟩

$\mathcal{D}_2$ =⟨s, (10.1,0.2,0.2), (0.4,0.3,0.3), (0.3,0.3,0.3)⟩

$\mathcal{D}_3$ =⟨s, (0.2,0.2,0.2), (0.2,0.1,0.1), (0.3,0.3,0.3)⟩

$\mathcal{D}_4$=⟨s, (0.1,0.1,0.1), (0.4,0.3,0.3), (0.3,0.5,0.5)⟩, and

let $\mathcal{T} = \{u, v, w\}$, ξ ={0N, $\mathcal{F}_1$,$\mathcal{F}_2$, $\mathcal{F}_3$, $\mathcal{F}_4$ 1N} be a neutrosophic topology on $(\mathcal{T}, \xi)$.

$\mathcal{F}_1$=⟨t, (0.3,0.3,0.3), (0.2,0.1,0.1), (0.2,0.2,0.2)⟩

$\mathcal{F}_2$=⟨t, (0.2,0.2,0.2), (0.1,0.1,0.1), (0.3,0.3,0.3)⟩

$\mathcal{F}_3$=⟨t, (0.3,0.3,0.3), (0.1,0.1,0.1), (0.2,0.1,0.1)⟩

$\mathcal{F}_4$=⟨t, (0.2,0.2,0.2), (0.2,0.1,0.1), (0.3,0.3,0.3)⟩

Define $d: (\mathcal{S}, \Im) \to (\mathcal{T}, \xi)$ by $d(u) = u$, $d(v) = v$, $d(w) = w$.

$\mathcal{N}_{\alpha g^{\#}\psi}$-closed sets of $(\mathcal{T}, \xi)$ = ⟨s, (0.3,0.5,0.5), (0.2,0.1,0.1), (0.2,0.1,0.1)⟩.

Here $d(\mathcal{D}_1)^c$ is $\mathcal{N}_{\alpha g^{\#}\psi}$-closed in $(\mathcal{T}, \xi)$. Therefore d is $\mathcal{N}_{\alpha g^{\#}\psi}$-closed mapping. However, it is not a neutrosophic closed mapping because $d(\mathcal{D}_1)^c$ is not neutrosophic closed set in $(\mathcal{T}, \xi)$.

Theorem 4.4. A mapping $d: (\mathcal{S},\Im) \to (\mathcal{T},\xi)$ is $\mathcal{N}_{\alpha g^{\#}\psi}$-closed iff for each neutrosophic set $\mathcal{S}$ of $(\mathcal{T},\xi)$ and for each neutrosophic open set $\mathcal{U}$ of $(\mathcal{S},\Im)$ containing d^{-1} $(\mathcal{S})$ there is a $\mathcal{N}_{\alpha g^{\#}\psi}$-open set $\mathcal{A}$ of $(\mathcal{T},\xi)$ such that $\mathcal{S}\subseteq \mathcal{A}$ and $d^{-1}(\mathcal{A})\subseteq \mathcal{U}$.

Proof: Necessity: Assume d is a $\mathcal{N}_{\alpha g^{\#}\psi}$-closed mapping. Let $\mathcal{S}$ be the neutrosophic closed set of $(\mathcal{T},\xi)$ and $\mathcal{U}$ is a neutrosophic open set of $(\mathcal{S},\Im)$ such that $d^{-1}(\mathcal{S}) \subseteq \mathcal{U}$. Then $\mathcal{A} = \mathcal{T} - d^{-1}$ $(\mathcal{U})^c$ is $\mathcal{N}_{\alpha g^{\#}\psi}$-open set of $(\mathcal{T},\xi)$ such that $d^{-1}(\mathcal{A})\subseteq \mathcal{U}$.

Sufficiency: Assume $\mathcal{F}$ is a neutrosophic closed set of $(\mathcal{S},\Im)$. Then $(d(\mathcal{F}))^c$ is a neutrosophic set of $(\mathcal{T},\xi)$ and $\mathcal{F}^c$ is neutrosophic open set in $(\mathcal{S},\Im)$ such that $d^{-1}((d(\mathcal{F}))^c)\subseteq \mathcal{F}^c$. By hypothesis there is a $\mathcal{N}_{\alpha g^{\#}\psi}$-open set $\mathcal{A}$ of $(\mathcal{T},\xi)$ such that $(d(\mathcal{F}))^c \subseteq \mathcal{A}$ and $d^{-1}(\mathcal{A})\subseteq \mathcal{F}^c$. Therefore $\mathcal{F} \subseteq (d^{-1}(\mathcal{A}))^c$. Hence $\mathcal{A}^c \subseteq d(\mathcal{F}) \subseteq d((d^{-1}(\mathcal{A}))^c) \subseteq \mathcal{A}^c$ which implies $d(\mathcal{F}) = \mathcal{A}^c$. Since $\mathcal{A}^c$ is $\mathcal{N}_{\alpha g^{\#}\psi}$-closed set of $(\mathcal{T},\xi)$. Hence $d(\mathcal{F})$ is $\mathcal{N}_{\alpha g^{\#}\psi}$-closed in $(\mathcal{T},\xi)$ and thus d is neutrosophic $\mathcal{N}_{\alpha g^{\#}\psi}$-closed mapping.

Theorem 4.5. If $d: (\mathcal{S},\Im) \to (\mathcal{T},\xi)$ is neutrosophic closed and $e: (\mathcal{T},\xi) \to (\mathcal{V},\omega)$ is $\mathcal{N}_{\alpha g^{\#}\psi}$-closed. Then eod: $(\mathcal{S},\Im) \to (\mathcal{V},\omega)$ is $\mathcal{N}_{\alpha g^{\#}\psi}$-closed.

Proof: Let $\mathcal{H}$ be a neutrosophic closed set in $(\mathcal{S},\Im)$. Then $d(\mathcal{H})$ is neutrosophic closed set of $(\mathcal{T},\xi)$ because d is neutrosophic closed mapping. Now $eod(\mathcal{H}) = e(d(\mathcal{H}))$ is $\mathcal{N}_{\alpha g^{\#}\psi}$-closed set in $(\mathcal{V},\omega)$ because e is $\mathcal{N}_{\alpha g^{\#}\psi}$-closed mapping. Thus eod is $\mathcal{N}_{\alpha g^{\#}\psi}$-closed mapping.

Theorem 4.6. If $d: (\mathcal{S},\Im) \to (\mathcal{T},\xi)$ is $\mathcal{N}_{\alpha g^{\#}\psi}$-closed map, then $\mathcal{N}_{\alpha g^{\#}\psi}$- $(c^*(d(\mathcal{A}))\subseteq d(c^*(\mathcal{A}))$.

Proof: Obvious.

Theorem 4.7. Let $d: (\mathcal{S},\Im) \to (\mathcal{T},\xi)$ and $e: (\mathcal{T},\xi) \to (\mathcal{V},\omega)$are $\mathcal{N}_{\alpha g^{\#}\psi}$- closed mappings. If every $\mathcal{N}_{\alpha g^{\#}\psi}$- closed set of $(\mathcal{T},\xi)$ is neutrosophic α-closed then, eod: $(\mathcal{S},\Im) \to (\mathcal{V},\omega)$ is $\mathcal{N}_{\alpha g^{\#}\psi}$ -closed.

Proof: Let $\mathcal{H}$ be a neutrosophic closed set in $(\mathcal{S},\Im)$. Then $d(\mathcal{H})$ is $\mathcal{N}_{\alpha g^{\#}\psi}$ -closed set of $(\mathcal{T},\xi)$ because d is $\mathcal{N}_{\alpha g^{\#}\psi}$-closed mapping. By hypothesis $d(\mathcal{H})$ is neutrosophic α-closed set of $(\mathcal{T},\xi)$. Now $e(d(\mathcal{H})) = (eod)(\mathcal{H})$ is $\mathcal{N}_{\alpha g^{\#}\psi}$ -closed set in $(\mathcal{V},\omega)$ because e is $\mathcal{N}_{\alpha g^{\#}\psi}$ -closed mapping. Thus eod is $\mathcal{N}_{\alpha g^{\#}\psi}$-closed mapping.

Theorem 4.8. Let $d: (\mathcal{S},\Im) \to (\mathcal{T},\xi)$ be a objective mapping, then the following statements are equivalent:

(a) d is a neutrosophic $\mathcal{N}_{\alpha g^{\#}\psi}$-open mapping.

(b) d is a neutrosophic $\mathcal{N}_{\alpha g^{\#}\psi}$-closed mapping.

(c) d^{-1} is $\mathcal{N}_{\alpha g^{\#}\psi}$-continuous mapping.

Proof: (a)⇒(b): Let us assume that d is a $\mathcal{N}_{\alpha g^{\#}\psi}$-open mapping. By definition, $\mathcal{H}$ is a neutrosophic open set in $(\mathcal{S},\Im)$, then $d(\mathcal{H})$ is a $\mathcal{N}_{\alpha g^{\#}\psi}$-open set in $(\mathcal{T},\xi)$. Here, $\mathcal{H}$ is neutrosophic closed set in $(\mathcal{S},\Im)$, then $\mathcal{S} - \mathcal{H}$ is a neutrosophic open set in $(\mathcal{S},\Im)$. By assumption, $d(\mathcal{S} - \mathcal{H})$ is a $\mathcal{N}_{\alpha g^{\#}\psi}$-open set in $(\mathcal{T},\xi)$. Hence, $\mathcal{T}-d(\mathcal{S} - \mathcal{H})$ is a $\mathcal{N}_{\alpha g^{\#}\psi}$ -closed set in $(\mathcal{T},\xi)$. Therefore, d is a $\mathcal{N}_{\alpha g^{\#}\psi}$-closed mapping.

(b) ⇒(c): Let $\mathcal{H}$ be a neutrosophic closed set in $(\mathcal{S},\Im)$. By (b), $d(\mathcal{H})$ is a $\mathcal{N}_{\alpha g^{\#}\psi}$-closed set in $(\mathcal{T},\xi)$. Hence, $d(\mathcal{H}) = (d^{-1})^{-1}(\mathcal{H})$, so d^{-1} is a $\mathcal{N}_{\alpha g^{\#}\psi}$-closed set in $(\mathcal{T},\xi)$. Hence, d^{-1} is $\mathcal{N}_{\alpha g^{\#}\psi}$ -continuous.

(c) ⇒(a): Let $\mathcal{H}$ be a neutrosophic open set in $(\mathcal{S},\Im)$. By (c), $(d^{-1})^{-1}(\mathcal{H}) = d(\mathcal{H})$ is a $\mathcal{N}_{\alpha g^{\#}\psi}$-open mapping.

5. $\mathcal{N}_{\alpha g^{\#}\psi}$-homeomorphism

Definition 5.1. A bijection $d:(\mathcal{S},\mathfrak{I}) \to (\mathcal{T},\xi)$ is called a $\mathcal{N}_{\alpha g^{\#}\psi}$-homeomorphism if d and d^{-1} are $\mathcal{N}_{\alpha g^{\#}\psi}$ -continuous.

Theorem 5.2. Each neutrosophic homeomorphism is a $\mathcal{N}_{\alpha g^{\#}\psi}$-homeomorphism.

Proof: Let d be neutrosophic homeomorphism, then d and d^{-1} are neutrosophic continuous. But every neutrosophic continuous function is $\mathcal{N}_{\alpha g^{\#}\psi}$-continuous. Hence, d and d^{-1} is $\mathcal{N}_{\alpha g^{\#}\psi}$ -continuous. Therefore, d is a $\mathcal{N}_{\alpha g^{\#}\psi}$-homeomorphism.

Let a $\mathcal{N}_{\alpha g^{\#}\psi}$-homeomorphism need not be a neutrosophic homeomorphism by the following example.

Example 5.3. Let $\mathcal{S} = \{u, v, w\}$, $\mathfrak{I} = \{0_N, \mathcal{D}_1, \mathcal{D}_2, \mathcal{D}_3, \mathcal{D}_4\ 1_N\}$ be a neutrosophic topology on $(\mathcal{S},\mathfrak{I})$.

$\mathcal{D}_1$ =⟨s, (0.2,0.1,0.1), (0.2,0.1,0.1), (0.3,0.5,0.5)⟩

$\mathcal{D}_2$ =⟨s, (0.1,0.2,0.2), (0.4,0.3,0.3), (0.3,0.3,0.3)⟩

$\mathcal{D}_3$ =⟨s, (0.2,0.2,0.2), (0.2,0.1,0.1), (0.3,0.3,0.3)⟩

$\mathcal{D}_4$=⟨s, (0.1,0.1,0.1), (0.4,0.3,0.3), (0.3,0.5,0.5)⟩, and

let $\mathcal{T} = \{u, v, w\}$, $\xi = \{0_N, \mathcal{F}_1, \mathcal{F}_2, \mathcal{F}_3, \mathcal{F}_4\ 1_N\}$ be a neutrosophic topology on $(\mathcal{T}, \xi)$.

$\mathcal{F}_1$=⟨t, (0.3,0.3,0.3), (0.2,0.1,0.1), (0.2,0.2,0.2)⟩

$\mathcal{F}_2$=⟨t, (0.2,0.2,0.2), (0.1,0.1,0.1), (0.3,0.3,0.3)⟩

$\mathcal{F}_3$=⟨t, (0.3,0.3,0.3), (0.1,0.1,0.1), (0.2,0.1,0.1)⟩

$\mathcal{F}_4$=⟨t, (0.2,0.2,0.2), (0.2,0.1,0.1), (0.3,0.3,0.3)⟩

Define $d : (\mathcal{S},\mathfrak{I}) \to (\mathcal{T},\xi)$ by $d(u) = u$, $d(v) = v$, $d(w) = w$.

$\mathcal{N}_{\alpha g^{\#}\psi}$ -closed sets of $(\mathcal{S},\mathfrak{I})$ =$\mathcal{A}$=⟨s, (0.3,0.3,0.3), (0.1,0.1,0.1), (0.2,0.1,0.1)⟩

Here $d^{-1}(\mathcal{F}_3)^c$ is $\mathcal{N}_{\alpha g^{\#}\psi}$-closed in $(\mathcal{S},\mathfrak{I})$. Therefore d is $\mathcal{N}_{\alpha g^{\#}\psi}$-continuous and d^{-1} is $\mathcal{N}_{\alpha g^{\#}\psi}$ -continuous if $(\mathcal{D}_3)^c$ is a $\mathcal{N}_{\alpha g^{\#}\psi}$-closed set in $(\mathcal{S},\mathfrak{I})$, then the image $d(\mathcal{D}_3)^c$=$(\mathcal{F}_4)^c$ is neutrosophic closed in $(\mathcal{T},\xi)$. Hence, d and d^{-1} are $\mathcal{N}_{\alpha g^{\#}\psi}$-continuous then it is a $\mathcal{N}_{\alpha g^{\#}\psi}$- homeomorphism. However, $\mathcal{A}$ is neutrosophic closed in $(\mathcal{T},\xi)$ but it is not neutrosophic closed in $(\mathcal{S},\mathfrak{I})$. Therefore it is not neutrosophic continuous. Therefore it is not neutrosophic homeomorphism.

Theorem 5.4. Let $d:(\mathcal{S},\mathfrak{I}) \to (\mathcal{T},\xi)$ be a bijective mapping. If d is $\mathcal{N}_{\alpha g^{\#}\psi}$-continuous, then the following statements are equivalent:

(a) d is a $\mathcal{N}_{\alpha g^{\#}\psi}$-closed mapping.

(b) d is a $\mathcal{N}_{\alpha g^{\#}\psi}$-open mapping.

(c) d^{-1} is a $\mathcal{N}_{\alpha g^{\#}\psi}$-homeomorphism.

Proof: (a) ⇒(b): Assume that d is a bijective mapping and a $\mathcal{N}_{\alpha g^{\#}\psi}$-closed mapping. Hence, d^{-1} is a $\mathcal{N}_{\alpha g^{\#}\psi}$-continuous mapping. We know that each neutrosophic open set in $(\mathcal{S},\mathfrak{I})$ is a $\mathcal{N}_{\alpha g^{\#}\psi}$-open set in $(\mathcal{T},\xi)$. Hence, d is a $\mathcal{N}_{\alpha g^{\#}\psi}$-open mapping.

(b) ⇒(c): Let d be a bijective and neutrosophic open mapping. Further, d^{-1} is a $\mathcal{N}_{\alpha g^{\#}\psi}$ -continuous mapping. Hence, d and d^{-1} are $\mathcal{N}_{\alpha g^{\#}\psi}$-continuous. Therefore, d is a $\mathcal{N}_{\alpha g^{\#}\psi}$ -homeomorphism.

(c) ⇒(a): Let d be a $\mathcal{N}_{\alpha g^{\#}\psi}$-homeomorphism, then d and d^{-1} are $\mathcal{N}_{\alpha g^{\#}\psi}$-continuous. Since each neutrosophic closed set in $(\mathcal{S},\mathfrak{I})$ is a $\mathcal{N}_{\alpha g^{\#}\psi}$-closed set in $(\mathcal{T},\xi)$, hence d is a $\mathcal{N}_{\alpha g^{\#}\psi}$-closed mapping.

Definition 5.5. Let $(\mathcal{S},\mathfrak{I})$ be a neutrosophic topological spaces said to be a neutrosophic

$\mathcal{T}_{\mathcal{N}_{\alpha g^{\#}\psi}}$ -space if every $\mathcal{N}_{\alpha g^{\#}\psi}$-closed set is neutrosophic closed in$(\mathcal{S},\mathfrak{I})$.

Theorem 5.6. Let $d: (\mathcal{S}, \mathfrak{I}) \to (\mathcal{T}, \xi)$ be a $\mathcal{N}_{\alpha g^{\#}\psi}$-homeomorphism, then d is a neutrosophic homeomorphism if $(\mathcal{S}, \mathfrak{I})$ and $(\mathcal{T}, \xi)$ are $\mathcal{T}_{\mathcal{N}_{\alpha g^{\#}\psi}}$-space.

Proof: Assume that $\mathcal{H}$ is a neutrosophic closed set in $(\mathcal{T}, \xi)$, then d^{-1} $(\mathcal{H})$ is a $\mathcal{N}_{\alpha g^{\#}\psi}$ -closed set in $(\mathcal{S}, \mathfrak{I})$. Since $(\mathcal{S}, \mathfrak{I})$ is an $\mathcal{T}_{\mathcal{N}_{\alpha g^{\#}\psi}}$-space, d^{-1} $(\mathcal{H})$ is a neutrosophic closed set in $(\mathcal{S}, \mathfrak{I})$. Therefore, d is neutrosophic continuous. By hypothesis, d^{-1} is $\mathcal{N}_{\alpha g^{\#}\psi}$ -continuous. Let $\mathcal{G}$ be a neutrosophic closed set in $(\mathcal{S}, \mathfrak{I})$. Then, $(d^{-1})^{-1}(\mathcal{G}) = d(\mathcal{G})$ is a neutrosophic closed set in $(\mathcal{T}, \xi)$, by presumption. Since $(\mathcal{T}, \xi)$ is a $\mathcal{T}_{\mathcal{N}_{\alpha g^{\#}\psi}}$-space, $d(\mathcal{G})$ is a neutrosophic closed set in $(\mathcal{T}, \xi)$. Hence, d^{-1} is neutrosophic continuous. Hence, d is a neutrosophic homeomorphism.

Theorem 5.7. Let $d: (\mathcal{S}, \mathfrak{I}) \to (\mathcal{T}, \xi)$ be a neutrosophic topological space, then the following are equivalent if $(\mathcal{T}, \xi)$ is a $\mathcal{T}_{\mathcal{N}_{\alpha g^{\#}\psi}}$ -space:

(a) d is $\mathcal{N}_{\alpha g^{\#}\psi}$-closed mapping.

(b) If $\mathcal{H}$ is a neutrosophic open set in $(\mathcal{S}, \mathfrak{I})$, then $d(\mathcal{H})$ is $\mathcal{N}_{\alpha g^{\#}\psi}$-open set in $(\mathcal{T}, \xi)$.

(c) $d(i^*$ $(\mathcal{H}))\subseteq c^*(\, i^*(d(\mathcal{H})))$ for every neutrosophic set $\mathcal{H}$ in $(\mathcal{S}, \mathfrak{I})$.

Proof: (a) ⇒ (b): Obvious.

(b) ⇒ (c): Let $\mathcal{H}$ be a neutrosophic set in $(\mathcal{S}, \mathfrak{I})$. Then, $i^*(\mathcal{H})$ is a neutrosophic open set in $(\mathcal{S}, \mathfrak{I})$. Then, $d(i^*(\mathcal{H}))$ is a $\mathcal{N}_{\alpha g^{\#}\psi}$-open set in $(\mathcal{T}, \xi)$. Since $(\mathcal{T}, \xi)$ is a $\mathcal{T}_{\mathcal{N}_{\alpha g^{\#}\psi}}$ -space, so $d(i^*(\mathcal{H}))$ is a neutrosophic open set in $(\mathcal{T}, \xi)$. Therefore, $d(i^*(\mathcal{H})) = i^*(d(i^*(\mathcal{H})))\subseteq c^*(i^*$ $(\mathrm{d}(\mathcal{H})))$.

(c) ⇒ (a): Let $\mathcal{H}$ be a neutrosophic closed set in $(\mathcal{S}, \mathfrak{I})$. Then, $\mathcal{H}^c$ is a neutrosophic open set in $(\mathcal{S}, \mathfrak{I})$. From, $d(i^*(\mathcal{H}^c))\subseteq$ $c^*(\, i^*(d(\mathcal{H}^c)))$. Hence, $d(\mathcal{H}^c)\subseteq$ $c^*(int(d(\mathcal{H}^c)))$. Therefore, $d(\mathcal{H}^c)$ is $\mathcal{N}_{\alpha g^{\#}\psi}$-open set in $(\mathcal{T}, \xi)$. Therefore, $d(\mathcal{H})$ is a $\mathcal{N}_{\alpha g^{\#}\psi}$ -closed set in $(\mathcal{S}, \mathfrak{I})$. Hence, d is a neutrosophic closed mapping.

Theorem 5.8. Let $d: (\mathcal{S}, \mathfrak{I}) \to (\mathcal{T}, \xi)$ and e : $(\mathcal{T}, \xi)$ → $(\mathcal{V}, \omega)$ be $\mathcal{N}_{\alpha g^{\#}\psi}$-closed, where $(\mathcal{S}, \mathfrak{I})$ and $(\mathcal{V}, \omega)$ are two neutrosophic topological spaces and $(\mathcal{T}, \xi)$ a $\mathcal{T}_{\mathcal{N}_{\alpha g^{\#}\psi}}$-space, then the composition eod is $\mathcal{N}_{\alpha g^{\#}\psi}$-closed.

Proof: Let $\mathcal{H}$ be a neutrosophic closed set in $(\mathcal{S}, \mathfrak{I})$. Since d is $\mathcal{N}_{\alpha g^{\#}\psi}$-closed and $d(\mathcal{H})$ is a $\mathcal{N}_{\alpha g^{\#}\psi}$-closed set in $(\mathcal{T}, \xi)$, by assumption, $d(\mathcal{H})$ is a neutrosophic closed set in $(\mathcal{T}, \xi)$. Since e is $\mathcal{N}_{\alpha g^{\#}\psi}$-closed, then $e(d(\mathcal{H}))$ is $\mathcal{N}_{\alpha g^{\#}\psi}$-closed in $(\mathcal{V}, \omega)$ and $e(d(\mathcal{H})) = eod$ $(\mathcal{H})$. Therefore, eod is $\mathcal{N}_{\alpha g^{\#}\psi}$- closed.

Theorem 5.9. Let $d: (\mathcal{S}, \mathfrak{I}) \to (\mathcal{T}, \xi)$ and e : $(\mathcal{T}, \xi) \to (\mathcal{V}, \omega)$be two neutrosophic topological spaces, then the following hold:

(a) If eod is $\mathcal{N}_{\alpha g^{\#}\psi}$-open and d is neutrosophic continuous, then e is $\mathcal{N}_{\alpha g^{\#}\psi}$-open.

(b) If eod is neutrosophic open and e is $\mathcal{N}_{\alpha g^{\#}\psi}$-continuous, then d is $\mathcal{N}_{\alpha g^{\#}\psi}$-open.

Proof: Obvious

6. $\mathcal{N}_{\alpha g^{\#}\psi}$–C Homeomorphism

Definition 6.1. A bijection $d: (\mathcal{S}, \mathfrak{I}) \to (\mathcal{T}, \xi)$ is called a $\mathcal{N}_{\alpha g^{\#}\psi}$-C homeomorphism if d and d^{-1} are $\mathcal{N}_{\alpha g^{\#}\psi}$-irresolute mappings.

Theorem 6.2. Each $\mathcal{N}_{\alpha g^{\#}\psi}$-C homeomorphism is a $\mathcal{N}_{\alpha g^{\#}\psi}$-homeomorphism.

Proof: Let us assume that $\mathcal{H}$ is a neutrosophic closed set in $(\mathcal{T},\xi)$. This shows that $\mathcal{H}$ is a $\mathcal{N}_{\alpha g^{\#}\psi}$-closed set in $(\mathcal{T},\xi)$. By assumption, $d^{-1}(\mathcal{H})$ is a $\mathcal{N}_{\alpha g^{\#}\psi}$-closed set in $(\mathcal{S},\mathfrak{I})$. Hence, d is a $\mathcal{N}_{\alpha g^{\#}\psi}$-continuous mapping. Hence, d and d^{-1} are $\mathcal{N}_{\alpha g^{\#}\psi}$-continuous mappings. Hence d is a $\mathcal{N}_{\alpha g^{\#}\psi}$-homeomorphism.

Let a $\mathcal{N}_{\alpha g^{\#}\psi}$-homeomorphism need not be a $\mathcal{N}_{\alpha g^{\#}\psi}$-C homeomorphism by the following example.

Example 6.3. Let $\mathcal{S} = \{u, v, w\}$, $\mathfrak{I}$ ={0N, $\mathcal{D}_1$,$\mathcal{D}_2$, $\mathcal{D}_3$, $\mathcal{D}_4$ 1N} be a neutrosophic topology on $(\mathcal{S},\mathfrak{I})$.

$\mathcal{D}_1$ =⟨s, (0.2,0.1,0.1), (0.2,0.1,0.1), (0.3,0.5,0.5)⟩

$\mathcal{D}_2$ =⟨s, (0.1,0.2,0.2), (0.4,0.3,0.3), (0.3,0.3,0.3)⟩

$\mathcal{D}_3$ =⟨s, (0.2,0.2,0.2), (0.2,0.1,0.1), (0.3,0.3,0.3)⟩

$\mathcal{D}_4$=⟨s, (0.1,0.1,0.1), (0.4,0.3,0.3), (0.3,0.5,0.5)⟩, and

let $\mathcal{T} = \{u, v, w\}$, ξ ={0N, $\mathcal{F}_1$,$\mathcal{F}_2$, $\mathcal{F}_3$, $\mathcal{F}_4$ 1N} be a neutrosophic topology on $(\mathcal{T},\xi)$.

$\mathcal{F}_1$=⟨t, (0.3,0.3,0.3), (0.2,0.1,0.1), (0.2,0.2,0.2)⟩

$\mathcal{F}_2$=⟨t, (0.2,0.2,0.2), (0.1,0.1,0.1), (0.3,0.3,0.3)⟩

$\mathcal{F}_3$=⟨t, (0.3,0.3,0.3), (0.1,0.1,0.1), (0.2,0.1,0.1)⟩

$\mathcal{F}_4$=⟨t, (0.2,0.2,0.2), (0.2,0.1,0.1), (0.3,0.3,0.3)⟩

Define $d : (\mathcal{S},\mathfrak{I}) \to (\mathcal{T},\xi)$ by d(u) = u, d(v) = v, d(w) = w.

Assume $\mathcal{N}_{\alpha g^{\#}\psi}$-closed sets of $(\mathcal{S},\mathfrak{I})$ =$\mathcal{A}$=⟨s, (0.3,0.3,0.3), (0.1,0.1,0.1), (0.2,0.1,0.1)⟩ is $\mathcal{N}_{\alpha g^{\#}\psi}$ -continuous then it is $\mathcal{N}_{\alpha g^{\#}\psi}$ -homeomorphism. However, it is not a $\mathcal{N}_{\alpha g^{\#}\psi}$ -C homeomorphism because it is not $\mathcal{N}_{\alpha g^{\#}\psi}$- irresolute.

Theorem 6.4. If $d: (\mathcal{S},\mathfrak{I}) \to (\mathcal{T},\xi)$ is a $\mathcal{N}_{\alpha g^{\#}\psi}$-C homeomorphism, then $\mathcal{N}_{\alpha g^{\#}\psi}$-$c^*(d^{-1}(\mathcal{H}))\subseteq d^{-1}(\mathcal{N}_\alpha(c^*(\mathcal{H})))$ for each neutrosophic topological space $\mathcal{H}$ in $(\mathcal{T},\xi)$.

Proof: Let $\mathcal{H}$ be a neutrosophic topological space in $(\mathcal{T},\xi)$. Then, $\mathcal{N}_\alpha(c^*(\mathcal{H}))$ is a neutrosophic α-closed set in $(\mathcal{T},\xi)$, and every neutrosophic α-closed set is a $\mathcal{N}_{\alpha g^{\#}\psi}$-closed set in $(\mathcal{T},\xi)$. Assume d is $\mathcal{N}_{\alpha g^{\#}\psi}$ -irresolute, d^{-1} $(\mathcal{N}_\alpha(c^*(\mathcal{B}))$ is a $\mathcal{N}_{\alpha g^{\#}\psi}$-closed set in $(\mathcal{S},\mathfrak{I})$, then $\mathcal{N}_{\alpha g^{\#}\psi}$ $c^*(d^{-1}(\mathcal{N}_\alpha(c^*(\mathcal{H})))) = d^{-1}(\mathcal{N}_\alpha(c^*(\mathcal{H})))$. Here, $\mathcal{N}_{\alpha g^{\#}\psi}$-$c^*(d^{-1}(\mathcal{H}))\subseteq \mathcal{N}_{\alpha g^{\#}\psi}$-$c^*(d^{-1}(\mathcal{N}_\alpha(c^*(\mathcal{H})))) = d^{-1}(\mathcal{N}_\alpha(c^*(\mathcal{H})))$. Therefore, $\mathcal{N}_{\alpha g^{\#}\psi}$- $c^*(d^{-1}(\mathcal{H}))\subseteq d^{-1}(\mathcal{N}_\alpha(c^*(\mathcal{H})))$ for every neutrosophic set $\mathcal{H}$ in $(\mathcal{T},\xi)$.

Theorem 6.5. Let $d: (\mathcal{S},\mathfrak{I}) \to (\mathcal{T},\xi)$ be a $\mathcal{N}_{\alpha g^{\#}\psi}$-C homeomorphism, then $\mathcal{N}_\alpha(c^*(d^{-1}(\mathcal{H}))) = d^{-1}(\mathcal{N}_\alpha(c^*(\mathcal{H})))$ for each neutrosophic set $\mathcal{H}$ in $(\mathcal{T},\xi)$.

Proof: Since d is a $\mathcal{N}_{\alpha g^{\#}\psi}$-C homeomorphism, then d is a $\mathcal{N}_{\alpha g^{\#}\psi}$-irresolute mapping. Let $\mathcal{H}$ be a neutrosophic set in $(\mathcal{T},\xi)$. Clearly, $\mathcal{N}_\alpha(c^*(\mathcal{H}))$ is a $\mathcal{N}_{\alpha g^{\#}\psi}$-closed set in $(\mathcal{S},\mathfrak{I})$. Then $\mathcal{N}_\alpha(c^*(\mathcal{H}))$ is a $\mathcal{N}_{\alpha g^{\#}\psi}$-closed set in $(\mathcal{S},\mathfrak{I})$. Since $d^{-1}(\mathcal{H})\subseteq d^{-1}(\mathcal{N}_\alpha(c^*(\mathcal{H})))$, then$\mathcal{N}_\alpha(c^*(d^{-1}(\mathcal{H})))\subseteq\mathcal{N}_\alpha(c^*(d^{-1}(\mathcal{N}_\alpha(c^*(\mathcal{H}))))) = d^{-1}(\mathcal{N}_\alpha(c^*(\mathcal{H})))$. Therefore, $\mathcal{N}_\alpha(c^*(d^{-1}(\mathcal{H})))\subseteq d^{-1}(\mathcal{N}_\alpha(c^*(\mathcal{H})))$. Let d be a $\mathcal{N}_{\alpha g^{\#}\psi}$-C homeomorphism. d^{-1} is a $\mathcal{N}_{\alpha g^{\#}\psi}$ -irresolute mapping. Let us consider neutrosophic set $d^{-1}(\mathcal{H})$ in $(\mathcal{S},\mathfrak{I})$, which implies $\mathcal{N}_\alpha(c^*(d^{-1}(\mathcal{H})))$ is a $\mathcal{N}_{\alpha g^{\#}\psi}$-closed set in $(\mathcal{S},\mathfrak{I})$. Hence, $\mathcal{N}_{\alpha g^{\#}\psi}$-$c^*(d^{-1}(\mathcal{H}))$ is a $\mathcal{N}_{\alpha g^{\#}\psi}$-closed set in $(\mathcal{S},\mathfrak{I})$. This implies that $(d^{-1})^{-1}(\mathcal{N}_\alpha(c^*(d^{-1}(\mathcal{H})))) = d(\mathcal{N}_\alpha(c^*(d^{-1}(\mathcal{H}))))$ is a $\mathcal{N}_{\alpha g^{\#}\psi}$-closed set in $(\mathcal{T},\xi)$. This proves $\mathcal{H} = (d^{-1})^{-1}(d^{-1}(\mathcal{H}))\subseteq (d^{-1})^{-1}(\mathcal{N}_\alpha(c^*(d^{-1}(\mathcal{H})))$ = $d(\mathcal{N}_\alpha(c^*(d^{-1}(\mathcal{H}))))$. Therefore, $\mathcal{N}_\alpha(c^*(\mathcal{H}))\subseteq \mathcal{N}_\alpha(c^*(d(\mathcal{N}_\alpha(c^*(d^{-1}(\mathcal{H}))))))= d(\mathcal{N}_\alpha(c^*(d^{-1}(\mathcal{H}))))$, since d^{-1} is a $\mathcal{N}_{\alpha g^{\#}\psi}$-irresolute mapping. Hence, $d^{-1}(\mathcal{N}_\alpha(c^*(\mathcal{H})))\subseteq d^{-1}(d(\mathcal{N}_\alpha(c^*(d^{-1}(\mathcal{H})))))$ = $\mathcal{N}_\alpha(c^*(d^{-1}(\mathcal{H})))$. That is, $d^{-1}(\mathcal{N}_\alpha(c^*(\mathcal{H})))\subseteq\mathcal{N}_\alpha(c^*(d^{-1}(\mathcal{H})))$. Hence, $\mathcal{N}_\alpha(c^*(d^{-1}(\mathcal{H}))) = d^{-1}(\mathcal{N}_\alpha(c^*(\mathcal{H})))$.

Theorem 6.6. If $d:(\mathcal{S},\mathfrak{I}) \to (\mathcal{T},\xi)$ and $e:(\mathcal{T},\xi) \to (\mathcal{V},\omega)$ are $\mathcal{N}_{\alpha g^{\#}\psi}$-C homeomorphisms, then eod is a $\mathcal{N}_{\alpha g^{\#}\psi}$-C homeomorphism.

Proof: Let d and e to be two $\mathcal{N}_{\alpha g^{\#}\psi}$-C-homeomorphisms. Assume $\mathcal{H}$ is a $\mathcal{N}_{\alpha g^{\#}\psi}$ -closed set in $(\mathcal{V},\omega)$. Then, $e^{-1}(\mathcal{H})$ is a $\mathcal{N}_{\alpha g^{\#}\psi}$-closed set in $(\mathcal{T},\xi)$. Then, by hypothesis, d^{-1} $(e^{-1}(\mathcal{H}))$ is a $\mathcal{N}_{\alpha g^{\#}\psi}$-closed set in $(\mathcal{S},\mathfrak{I})$. Hence, eod is a $\mathcal{N}_{\alpha g^{\#}\psi}$ -irresolute mapping. Now, let $\mathcal{G}$ be a $\mathcal{N}_{\alpha g^{\#}\psi}$ -closed set in $(\mathcal{S},\mathfrak{I})$. Then, by presumption, $d(e)$ is a $\mathcal{N}_{\alpha g^{\#}\psi}$-closed set in $(\mathcal{T},\xi)$. Then, by hypothesis, $e(d(\mathcal{G}))$ is a $\mathcal{N}_{\alpha g^{\#}\psi}$-closed set in $(\mathcal{V},\omega)$. This implies that eod is a $\mathcal{N}_{\alpha g^{\#}\psi}$-irresolute mapping. Hence, eod is a $\mathcal{N}_{\alpha g^{\#}\psi}$-C-homeomorphism.

7. Conclusions

In this paper, the new concept of a neutrosophic homeomorphism and a $\mathcal{N}_{\alpha g^{\#}\psi}$ -homeomorphism in neutrosophic topological spaces was discussed. Furthermore, the work was extended as the $\mathcal{N}_{\alpha g^{\#}\psi}$ -C homeomorphism, $\mathcal{N}_{\alpha g^{\#}\psi}$ -open and $\mathcal{N}_{\alpha g^{\#}\psi}$ -closed mapping and neutrosophic $\mathcal{T}_{\mathcal{N}_{\alpha g^{\#}\psi}}$-space. Further, the study demonstrated $\mathcal{N}_{\alpha g^{\#}\psi}$-C homeomorphisms and also derived some of their related attributes.

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