[Neutrosophic Sets and Systems](https://digitalrepository.unm.edu/nss_journal)

[Volume 29](https://digitalrepository.unm.edu/nss_journal/vol29) Article 14

10-15-2019

Neutrosophic Shortest Path Problem (NSPP) in a Directed Multigraph

Siddhartha Sankar Biswas

NSS
Neutrosophic Sets and Systems, Vol. 29, 2019
University of New Mexico

Neutrosophic Shortest Path Problem (NSPP) in a Directed Multigraph

Siddhartha Sankar Biswas

Department of Computer Science & Engineering, School of Engineering Sciences & Technology
Jamia Hamdard (Deemed University), New Delhi – 110062, INDIA.
Email : ssbiswas1984@gmail.com

Abstract: One of the important non-linear data structures in Computer Science is graph. Most of the real life network, be it a road transportation network, or airlines network or a communication network etc., cannot be exactly transformed into a graph model, but into a Multigraphs model. The Multigraph is a topological generalization of the graph where multiple links (or edges/arcs) may exist between two nodes unlike in graph. The existing algorithms to extract the neutrosophic shortest path in a graph cannot be applied to a Multigraphs. In this paper a method is developed to extract the neutrosophic shortest path in a directed Multigraph and then the corresponding algorithm is designed. The classical Dijkstra's algorithm is applicable to graphs only where all the link weights are crisp, but we borrow this concept to apply to Multigraphs where the weights of the links are neutrosophic numbers (NNs). This new method may be useful in many application areas of computer science, communication networks, transportation networks, etc. in particular in those type of networks which cannot be modelled into graphs but into Multigraphs.

Keywords: Multiset, NN, neutrosophic-min-weight arc-set, neutrosophic shortest path estimate, neutrosophic relaxation.

1. Introduction

Graph Theory [4, 13, 51] is used in huge volume of applications in various branches of Engineering, mainly in Information Technology, Computer Science, Communication Engineering, Transportation Engineering, Space Engineering, Oceanography, and also in Mathematical Sciences, Social Science, Medical Science, Economics, Optimization, Decision Sciences, etc. The Multigraph [45, 51] is an important generalization of the data structure graph in which multiple links (or edges/arcs) may exist to connect a pair of nodes. For instance, consider a communication system in an Adhoc Network or a MANET where there are many multipaths or multiroute facilities. For another example, it is common that two neighbor routers in a network may share more than one direct connections existing in the topology between them, for the purpose of reducing the bandwidth compared to the case where a single connection be used. In fact there are a number of real life instances of communication network system, airlines network, road transportation network, etc. which cannot be transformed into graphs model, but can be well transformed into multigraphs model for the purpose of various analysis and decision makings. In real life situation, in many of these type of directed multigraphs another issue is that the weights of the links are not always crisp rather neutrosophic numbers (NNs). Throughout in this paper, those multigraphs are under consideration which are not having any loop.

The NSPP problem is solved by Broumi in [32-36], but there is no work reported in the existing literature on solving neutrosophic shortest path problem (NSPP) in a multigraph. In this paper we solve the NSPP problem for a multigraph where the arc-weights are neutrosophic numbers (NNs). It is known that the very popular Dijkstra's algorithm is applicable to graphs only where the weights of the links are crisp numbers, but is not applicable to multigraphs even having crisp weights for its liunks. In this paper we extend this philosophy of Dijkstra's algorithm to apply to the case of directed multigraphs having the weights of the links as neutrosophic numbers (NNs). This problem is not solved so far in any literature, but the SPP in a multigraph having weights of the links as fuzzy or intuitionistic fuzzy numbers are solved (for example, see [46-49]). But it has been well justified in length in the pioneering works [27,28,29] about the cases where fuzzy theory fails, and intuitionistic fuzzy theory can offer soft solutions; in fact the works [27,28,29] expos the major drawbacks of the fuzzy set theory. And then in the work [8,49] it is further justified that neutrosophic theory generalizes the intuitionistic fuzzy theory. An intuitionistic fuzzy set can be viewed as a special case of a neutrosophic set, but the converse is not necessarily true. The era of improvement of various models are like:- Crisp Set → Fuzzy Set (and various types of higher order Fuzzy Sets) → IFS→ NS. And hence, by heredity the same is true for the corresponding notion of numbers too, i.e. Crisp Number → Fuzzy Number → IFN→ NN. Consequently, it is now obvious to the soft-computing researchers that the application of neutrosophic theory can surely provide better solutions [35] for ill-defined or imprecise problems.

2. Preliminaries

In this section some relevant literatures are recollected from the work of Smarandache [8-12], Salama [1, 2] and also few works of other authors [5, 14, 15, 19]. In his pioneer work, Smarandache introduced the concepts of neutrosophic trio components T, I, and F which represent respectively the membership value, indeterminacy value, and non-membership value, where $]^-0,1^+[$ stands for a non-standard unit interval.

2.1 Basic Preliminaries of the Neutrosophic Theory

This subsection contains some elements of basic notions on the theory of neutrosophic sets, in particular about the single valued neutrosophic sets out of the existing literatures.

Definition 2.1.1 Let X be a non-null set. A neutrosophic set A of the universe X is an object having the form $A = \{< x : T_A(x), I_A(x), F_A(x)> , x \in X\}$, where the trio functions $T, I, F : X \rightarrow]^-0,1^+[$ define the truth-membership function, indeterminacy-membership function, and falsity-membership function respectively of the element $x \in X$ to the set A along with the following condition:

$^-0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+$.

The trio functions $T_A(x)$, $I_A(x)$ and $F_A(x)$ are three real standard (or nonstandard) subsets of non-standard unit interval $]^-0,1^+[$.

Application of the general model of NSs as defined above to the practical problems and issues may require complex computations, and consequently the authors [14, 15] suggested the notion of a

SVNS as a particular instance of a NS which can be used in real problems of scientific and engineering areas.

Definition 2.1.2 Let T, I, F be three real standard or nonstandard subsets of the non-standard unit interval $]^{-}0,1^{+}[$, with the following:

Sup_T = t_sup, inf_T = t_inf

Sup_I = i_sup, inf_I = i_inf

Sup_F = f_sup, inf_F = f_inf

n-sup = t_sup + i_sup + f_sup

n-inf = t_inf + i_inf + f_inf,

Then T, I, F are called neutrosophic trio components.

Definition 2.1.3 The NS 0_N in X is defined as follows:

(i) $0_N = \{<x, (0,0,1)> : x \in X\}$

(ii) $0_N = \{<x, (0,1,1)> : x \in X\}$

(iii) $0_N = \{<x, (0,1,0)> : x \in X\}$

(iv) $0_N = \{<x, (0,0,0)> : x \in X\}$

The NS 1_N in X is defined as follows:

(i) $1_N = \{<x, (1,0,0)> : x \in X\}$

(ii) $1_N = \{<x, (1,0,1)> : x \in X\}$

(iii) $1_N = \{<x, (1,1,0)> : x \in X\}$

(iv) $1_N = \{<x, (1,1,1)> : x \in X\}$

Definition 2.1.4 Let X be a non-null set. A single valued neutrosophic set A (SVNS A) is an object having the form $A = \{< x : T_A(x), I_A(x), F_A(x)> , x \in X\}$, where $T_A(x), I_A(x), F_A(x) \in [0,1]$ define the truth-membership function, indeterminacy-membership function, and falsity-membership function respectively of the element $x \in X$ Therefore a SVNS A could be expressed as $A = \{< x : T_A(x), I_A(x), F_A(x)> , x \in X\}$ where $T_A(x), I_A(x), F_A(x) \in [0,1]$.

Definition 2.1.5 Let $A_1 = (T_1, I_1, F_1)$ and $A_2 = (T_2, I_2, F_2)$ be two single valued neutrosophic numbers. Then, the operations for SVNNs are defined as below:

(i) $A_1 \oplus A_2 = <T_1 + T_2 - T_1T_2, I_1I_2, F_1F_2>$.

(ii) $A_1 \otimes A_2 = <T_1T_2, I_1 + I_2 - I_1I_2, F_1 + F_2 - F_1F_2>$.

(iii) $kA_1 = < 1- (1-T_1)^k, I_1^k, F_1^k >$ where $k > 0$.

(iv) $A_1^k = < T_1^k, 1- (1-I_1)^k, 1- (1-F_1)^k>$ where $k > 0$.

Definition 2.1.6

The neutrosophic zero 0_N may be defined as follow:

$0_N = \{<x, (0,1,1)> : x \in X\}$ To compare two single valued neutrosophic numbers, one can use score function.

Definition 2.1.7 Let $A_1 = (T_1, I_1, F_1)$ be a single valued neutrosophic number. Then, the score function $s(A_1)$, accuracy function $a(A_1)$ and the certainty function $c(A_1)$ of the SVNN A_1 are defined as below :

(i) $s(A_1) = \dfrac{2 + T_1 - I_1 - F_1}{3}$

(ii) $a(A_1) = T_1 - F_1$

(iii) $c(A_1) = T_1$

Definition 2.1.8 Suppose that $A_1 = (T_1, I_1, F_1)$ and $A_2 = (T_2, I_2, F_2)$ be two single valued neutrosophic numbers. Then we define a ranking method as follows:

(i) if $s(A_1) > s(A_2)$, then the SVNN A_1 is neutrosophic greater than the SVNN A_2 denoted by the notation $A_1 \succ A_2$.

(ii) if $s(A_1) = s(A_2)$ but $a(A_1) > a(A_2)$, then the SVNN A_1 is neutrosophic greater than the SVNN A_2 denoted by the notation $A_1 \succ A_2$.

(iii) if $s(A_1) = s(A_2)$ but $a(A_1) = a(A_2)$ and $c(A_1) > c(A_2)$, then the SVNN A_1 is neutrosophic greater than the SVNN A_2 denoted by the notation $A_1 \succ A_2$.

(iv) if $s(A_1) = s(A_2)$ and $a(A_1) = a(A_2)$ and $c(A_1) > c(A_2)$, then the SVNN A_1 is neutrosophic equal to the SVNN A_2 denoted by the notation $A_1 = A_2$.

However for simple cases, the following ranking method may be followed for easy applications:

(i) if $s(A_1) > s(A_2)$, then the SVNN A_1 is neutrosophic greater than the SVNN A_2 denoted by the notation $A_1 \succ A_2$.

(ii) if $s(A_1) < s(A_2)$, then the SVNN A_1 is neutrosophic less than the SVNN A_2 denoted by the notation $A_1 \prec A_2$.

(iii) if $s(A_1) = s(A_2)$, then the SVNN A_1 is neutrosophic equal to the SVNN A_2 denoted by the notation $A_1 = A_2$.

For a deep study on the Theory of Nneutrosophic Sets introduced by Smarandache, his main work [8-12] could be viewed. The notion of a neutrosophic numbers (NNs) is important to quantify an imprecise or ill-defined quantity. In this paper although, we shall use the very basic neutrosophic operations viz. neutrosophic addition $\oplus$, neutrosophic subtraction $\ominus$, and ranking of neutrosophic numbers, etc.

If we can rank n number of neutrosophic numbers, we then easily by soft-compute find out the min NN and max NN of these n number of NNs. If $A_1, A_2, A_3, \ldots, A_n$ be n neutrosophic numbers sorted in neutrosophic ascending order i.e. if $A_1 \prec A_2 \prec A_3 \prec \ldots\ldots. \prec A_n$, then A_1 and A_n can be regarded respectively as the neutrosophic-min NN and neutrosophic-max NN of these n NNs.

2.2 *Multisets: Some Preliminaries*

We present some basic preliminaries of the notion of multigraphs [45, 51]. Mathematically, a multigraph G is an ordered pair (V, E) consisting of two sets V and E, where V or V (G) is a set of vertices (or, nodes), and E or E(G) is the set of links or edges or arcs. In multigraphs, although multiple links (or edges or arcs) may exist between a pair of nodes (vertices), but in our work here we consider only those multigraphs that has no loop. The multigraphs could be classified by two types: undirected multigraphs and directed multigraphs. For any undirected multigraph if the edge (i, j) and the edge (j, i) exist, then it is obvious that they are identical unlike in the case of the directed multigraphs. A rigorous theoretical study on the algebra of multigraphs has been done in the work [45]. Figure 1 below shows a directed multigraph G = (V, E), in which the set V = {A, B, C, D} and the set E = {AB_1, AB_2, BA, AD, AC, CB, BD, DB}.

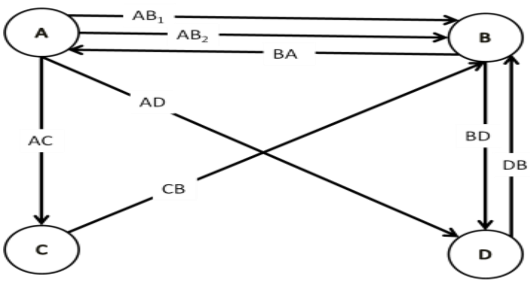

Figure 1: A directed multigraph G

A multigraph H = (W, F) is called a submultigraph of the multigraph G = (V, E) if W ⊆ V and F ⊆ E. The Figure 2 below shows a submultigraph H of the multigraph G (of Figure 1).

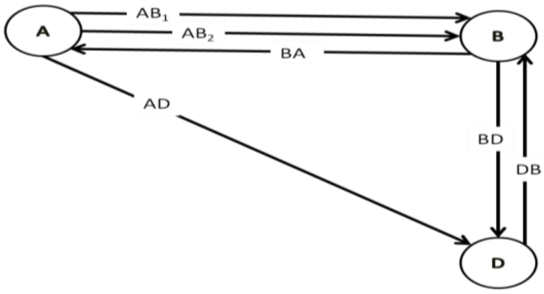

Figure 2: A submultigraph H of the directed multigraph G

It is observed that in many real life cases of various networks, be it in a communication network or road transportation network, or any such network topologies, the weights of the links are not always crisp but neutrosophic numbers. For an example, see the Figure-3 below which shows a public road transportation network multigraph for a traveler in which case the cost implication for traveling each link have been available to him as a neutrosophic number (NN). The NN of an arc in such a multigraphs is called neutrosophic weight (nw) of the arc.

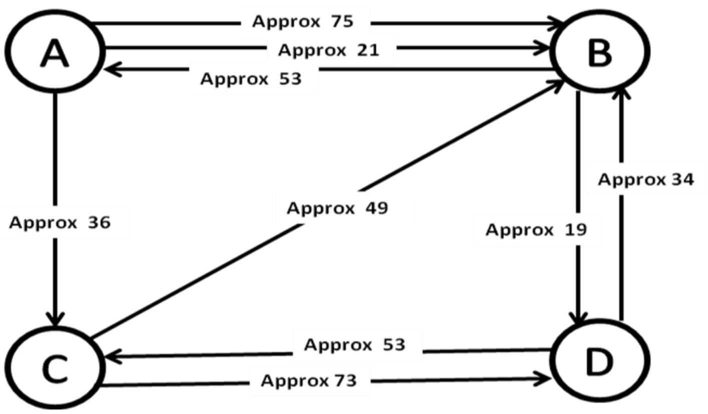

Figure 3: A directed multigraph G with neutrosophic weights of arcs.

In our work here we consider this type (as viewed in Figure 3) of real life instances of directed multigraphs of a network and then develop a soft-computing method to extract the neutrosophic shortest path from a given source node to a pre-decided destination node.

3. Neutrosophic Shortest Path in a Directed Multigraph

A good amount of work has been done on the notion of neutrosophic graph and its application by several authors [6, 7, 16, 17, 19-44, 49]. The Neutrosophic Shortest Path Problem (NSPP) has been solved for graphs by Broumi [32-36], but for the case of a directed multigraph no attempt has been reported so far in the literature for extracting a neutrosophic shortest path. In our proposed method here, we solve the NSPP for multigraphs using the style of Dijkstra's Algorithm but by soft-computing exercises. And for doing this, first of all we define the terms: Neutrosophic-Min-Weight arc-set, Neutrosophic shortest path estimate (d[v]) of a vertex, Neutrosophic relaxation of an arc, etc. In the context of the theory of multigraphs, and then develop few sub algorithms.

3.1. Neutrosophic-Min Weight Arc-set of a directed multigraph

Consider a directed multigraph G in which the links are of having neutrosophic weights. Consider two adjacent nodes u and v, and suppose that there exist n number of links arcs from the node u to the node v in G, n being a non-negative integer. Let W_{uv} denotes the ordered set consisting of the elements which are the arcs connecting the nodes u and v, but keyed & sorted in non-descending order by the values of the respective neutrosophic weights (where sorting is done by using a suitable and pre-choosen ranking method of neutrosophic numbers).

$\therefore \quad W_{uv} = \{ (uv_1 , w_{1uv}), \quad (uv_2 , w_{2uv}), \quad (uv_3 , w_{3uv}), \ldots\ldots , (uv_n , w_{nuv}) \}.$

Here uv_i is the arc-i from node u to nodex v and w_i is the neutrosophic weight of this arc, for i = 1, 2, 3,……, n. If two or more neutrosophic weights here happen to be neutrosophic equal then they may be placed at random at the corresponding place of non-descending array in this set with no loss of generality in our analysis.

Without any confusion, we may denote the multiset $\{ w_{1uv}, w_{2uv}, w_{3uv}, \ldots\ldots , w_{nuv} \}$ also using the same notational name W_{uv}. Suppose that w_{uv} be the neutrosophic-min value of the members of the multiset $W_{uv} = \{w_{1uv}, w_{2uv}, w_{3uv}, \ldots. , w_{nuv} \}$. Obviously, $w_{uv} = w_{1uv}$, because the multiset W_{uv} is already sorted.

Now construct the set $W = \{< (u,v), w_{uv} > : (u,v) \in E \}$. Then W is called the neutrosophic-min-weight arc-set of the multigraph G. Suppose that the sub algorithm NMWA(G) returns the neutrosophic-min-weight arc-set W.

3.2. Neutrosophic Shortest Path Estimate d[v] of a vertex v in a directed multigraph

Suppose that during the execution the node s is the source vertex and the currently traversed vertex is u. There is, in general, no single value of neutrosophic weight for link between the vertex u and the neighbor vertex v, rather there are multiple neutrosophic weights as there are multiple arcs between the vertex u and the neighbor vertex v. Using the value of w_{uv} from the neutrosophic-min weight multiset w of the directed multigraph G, one could now soft-compute the neutrosophic shortest path estimate i.e. d[v] of any vertex v as mentioned below:-

(Neutrosophic shortest path estimate of the vertex v) =
(Neutrosophic shortest path estimate of the vertex u) $\oplus$ (Neutrosophic-min of all the neutrosophic weights corresponding to the links from the vertex u to the vertex v).
or, d[v] = d[u] $\oplus$ w_{uv} .

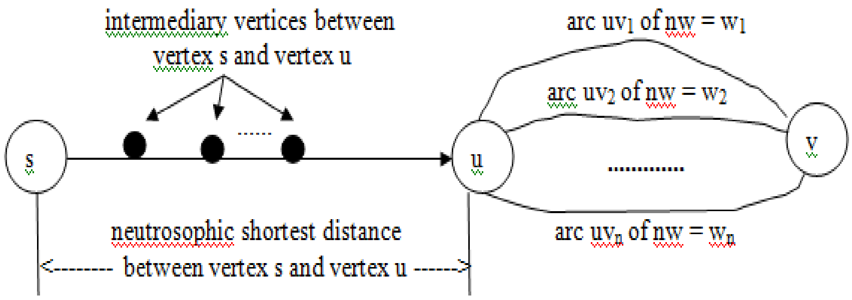

Figure 4: Neutrosophic estimation procedure for d[v]

3.3. Neutrosophic Relaxation of an Arc

In this subsection we present the next step which is 'relaxation' as introduced in the classical Dijkstra's algorithm. In our proposed method here we extend the notion of relaxation to the case of neutrosophic weighted arcs. By the term 'neutrosophic relaxation' in our work we mean the relaxation process of an arc for which the arc-weight is a neutrosophic number (as particular cases, it could be crisp or fuzzy or intuitionistic fuzzy number too as all of them could be viewed as NN). First of all we do initialization of the multigraph along with its starting vertex and neutrosophic shortest path estimate for each vertices of the multigraph G. The corresponding algorithm is called 'NEUTROSOPHIC-INITIALIZATION-SINGLE-SOURCE' as presented below:

NEUTROSOPHIC-INITIALIZATION-SINGLE-SOURCE (G, s)

1. For each vertex v ∈ V[G]
2. d[v] = ∞
3. v.π = NIL
4. d[s] = 0

After doing the neutrosophic initialization, the process of neutrosophic relaxation of each arc starts. The following sub-algorithm NEUTROSOPHIC-RELAX will play the role to update d[v] i.e. the neutrosophic shortest distance value between the starting vertex s and the vertex v (which is a neighbor of the currently traversed vertex u).

NEUTROSOPHIC-RELAX (u, v, W)

1. IF d[v] $\succ$ d[u] $\oplus$ w_{uv}

2. THEN d[v] ← d[u] $\oplus$ w_{uv}
3. v.π ← u

Where, $w_{uv} \in W$ is the neutrosophic-min weight of the arcs from vertex u to vertex v, and v.π denotes the parent node of vertex v.

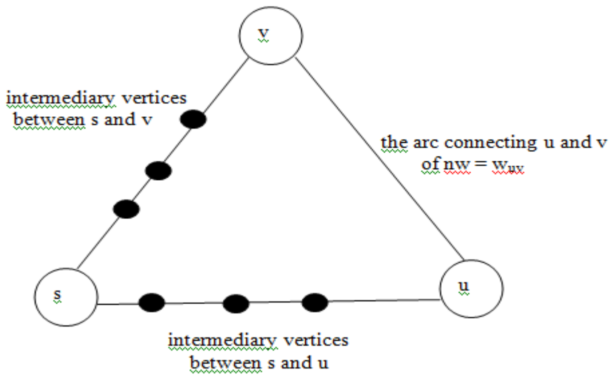

Figure 5: Diagram showing how the NEUTROSOPHIC-RELAX algorithm works.

3.4. Neutrosophic Shortest Path Algorithm (NSPA)

In this subsection we develop the main algorithm to extract the single source neutrosophic shortest path in a directed multigraph. Let us name this Neutrosophic Shortest Path Algorithm by the title NSPA. In our proposed algorithm we call the sub algorithms developed so far in this work, and also the sub algorithm EXTRACT-NEUTROSOPHIC-MIN (Q) which extracts the node u with the minimum key by using the neutrosophic ranking of NN method, and then it updates Q.

NSPA (G, s)

```
NEUTROSOPHIC-INITIALIZATION-SINGLE-SOURCE (G, s)
W ← NMWA (G)
S ← ∅
Q ← V [G]
WHILE   Q ≠ ∅
          DO    u ← EXTRACT-NEUTROSOPHIC-MIN (Q)
                S ← S ∪ {u}
                FOR   each vertex v ∈ Adj[u]
                           DO NEUTROSOPHIC-RELAX (u, v, W)
```

Example 3.1

Let us consider the directed Multigraph G (as in Figure 6) with neutrosophic weights of its links. The problem is to solve the single-source neutrosophic shortest paths problem over this multigraph taking the node A as the source and the node D as the destination.

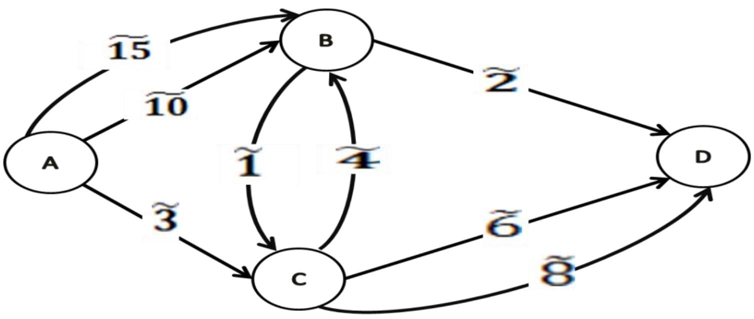

Figure 6: A directed multigraph G with neutrosophic weighted arcs.

It is clear that if the NSPA algorithm is applied to solve this NSPP, it will yield the following results:

1. $w_{AB} = \tilde{10}$, $w_{AC} = \tilde{3}$, $w_{CB} = \tilde{4}$, $w_{CD} = \tilde{6}$, and $w_{BD} = \tilde{2}$; and then

2. S = {A, C, B, D}, i.e. the extracted neutrosophic shortest path from starting the source node A to the destination node D is:

   A → C → B → D.

3. d–values i.e. Neutrosophic shortest distance estimate-values of each node

   From the starting node an up to the destination node D will be:

$$d[A] = 0 , d[C] = NN \tilde{3} , \quad d[B] = NN \tilde{7} , d[D] = NN \tilde{9} .$$

Here all operations are to be carried out using Definition 2.1.5. The method for ranking of n number of neutrosophic numbers is already mentioned earlier (Definition 2.1.7 and 2.1.8), and the concept of the 'neutrosophic shortest distance' is to be understood accordingly with the help of this ranking method.

Thus the result finally is A → C → B → D with minimum cost of NN $\tilde{9}$.

4. Conclusion

Multigraph is a very useful generalization of the mathematical model graph. In real life environment there are many problems of network (viz. road transportation network, communication network, circuit systems, airlines network etc.) Which cannot be mathematically modeled into 'graphs' but can be very appropriately modeled into 'multigraphs' only. And besides that, many of the directed multigraphs have the weights of the links which are not always crisp but neutrosophic number (NN). The important problem NSPP has been solved by Broumi [32-36] while it is for graphs, but not for multigraphs. In this work we have considered the NSPP for those networks which are multigraphs, and we have proposed a method to extract the neutrosophic shortest path in a directed multigraph from a given source node to one pre-choosen destination node. It is claimed by us that that our proposed method and the corresponding algorithms developed for NSPP on directed multigraphs can play an important role in many real life application areas in the fields of computer science, communication network, road transportation systems, etc. in particular for those type of networks that cannot be mathematically modeled into 'graphs' but into the multigraphs.

References

1. A.A.Salama, The Concept of Neutrosophic Set and Basic Properties of Neutrosophic Set Operations, WASET 2012 PARIS, FRANC, International University of Science, Engineering and Technology, 2012.
2. A. A. Salama and S. A. Alblowi, Neutrosophic Set and Neutrosophic Topological Spaces, IOSR Journal of Math.Vol.(3) ISSUE4 PP31-35, 2012.
3. A.Q.Ansari, R. Biswas and S. Aggarwal. Neutrosophic classifier: An extension of fuzzy classifier, Applied Soft Computing 13 (2013) pp 563–573.
4. Bela Bollobas, Modern Graph Theory, Springer; 2002.
5. C. Ashbacher, Introduction to Neutrosophic Logic, American Research Press Rehoboth. 2002.
6. D. Nagarajan, M. Lathamaheswari, Said Broumi, J. Kavikumar: Blockchain Single and Interval Valued Neutrosophic Graphs, Neutrosophic Sets and Systems, vol. 24, 2019, pp. 23-35. DOI: 10.5281/zenodo.2593909

7. D. Nagarajan, M.Lathamaheswari, S. Broumi, J. Kavikumar: Dombi Interval Valued Neutrosophic Graph and its Role in Traffic Control Management, Neutrosophic Sets and Systems, vol. 24, 2019, pp. 114-133. DOI: 10.5281/zenodo.2593948

8. F. Smarandache, Neutrosophic set a generalisation of the intuitionistic fuzzy sets. Int. J. Pure. Applic. Math., 24(2005) 287–297.

9. F. Smarandache, Neutrosophy: Neutrosophic Probability, Set, and Logic: Analytic Synthesis & Synthetic Analysis. Ameri. Res. Press: Rehoboth. DE. USA. (1998).

10. F. Smarandache. Neutrosophy and Neutrosophic Logic, First International Conference on Neutrosophy, Neutrosophic Logic , Set, Probability, and Statistics University of New Mexico, Gallup, NM 87301, USA, 2002.

11. F. Smarandache. A Unifying Field in Logics. Neutrosophy: Neutrosophic Probability, Set and Logic. Rehoboth: American Research Press. 1998.

12. F. Smarandache, Neutrosophic set, a generalisation of the intuitionistic fuzzy sets. Int. J. Pure Appl. Math. 24 (2005) 287-297.

13. Frank Harary, Graph Theory, Addison Wesley Publishing Company, 1995

14. H. Wang, F. Y. Smarandache, Q. Zhang and R. Sunderraman. Single valued neutrosophic sets. Multispace and Multistructure 4 (2010) 410–413.

15. H. Wang, Y. Zhang, R. Sunderraman and F. Smarandache, Single valued neutrosophic sets, Fuzzy Sets, Rough Sets and Multivalued Operations and Applications, 3(1), 33-39, (2011).

16. K. Sinha, P. Majumdar: On Single Valued Neutrosophic Signed Digraph and its applications, Neutrosophic Sets and Systems, vol. 22, 2018, pp. 171-179. DOI: 10.5281/zenodo.2160012

17. K. Sinha, P. Majumdar: Entropy based Single Valued Neutrosophic Digraph and its applications, Neutrosophic Sets and Systems, vol. 19, 2018, pp. 119-126. http://doi.org/10.5281/zenodo.1235197

18. L.A.Zadeh, Fuzzy Sets, Inform. And Control, Vol.8(1965) 338-353.

19. Muhammad Akram, Nabeela Ishfaq , Florentin Smarandache, Said Broumi: Application of Bipolar Neutrosophic sets to Incidence Graphs, Neutrosophic Sets and Systems, vol. 27, 2019, pp. 180-200. DOI: 10.5281/zenodo.3275595

20. Naeem Jan, Lemnaouar Zedam, Tahir Mahmood, Kifayat Ullah, Said Broumi, Florentin Smarandache: Constant single valued neutrosophic graphs with Applications, Neutrosophic Sets and Systems, vol. 24, 2019, pp. 77-89. DOI: 10.5281/zenodo.2593932

21. Nasir Shah, Said Broumi: Irregular Neutrosophic Graphs, Neutrosophic Sets and Systems, vol. 13, 2016, pp. 47-55. doi.org/10.5281/zenodo.570846

22. Nasir Shah: Some Studies in Neutrosophic Graphs, Neutrosophic Sets and Systems, vol. 12, 2016, pp. 54-64. doi.org/10.5281/zenodo.571148

23. Nasir Shah, Asim Hussain: Neutrosophic Soft Graphs, Neutrosophic Sets and Systems, vol. 11, 2016, pp. 31-44. doi.org/10.5281/zenodo.571574

24. R. Dhavaseelan, S. Jafari, M. R. Farahani, S. Broumi: On single-valued co-neutrosophic graphs, Neutrosophic Sets and Systems, vol. 22, 2018, pp. 180-187. DOI: 10.5281/zenodo.2159886

25. Ranjan Kumar, S Edalatpanah, Sripati Jha, S.Broumi, Ramayan Singh, Arindam Dey: A Multi Objective Programming Approach to Solve Integer Valued Neutrosophic Shortest Path Problems, Neutrosophic Sets and Systems, vol. 24, 2019, pp. 134-154. DOI: 10.5281/zenodo.2595968

26. Ranjan Kumar, S A Edaltpanah, Sripati Jha, Said Broumi, Arindam Dey: Neutrosophic Shortest Path Problem, Neutrosophic Sets and Systems, vol. 23, 2018, pp. 5-15. DOI: 10.5281/zenodo.2155343

27. Ranjit Biswas, Is 'Fuzzy Theory' An Appropriate Tool For Large Size Problems?. in the book-series of *SpringerBriefs in Computational Intelligence*. *Springer.* Heidelberg, 2016.

28. Ranjit Biswas, Is 'Fuzzy Theory' An Appropriate Tool For Large Size Decision Problems?, Chapter-8 in Imprecision and Uncertainty in Information Representation and Processing, in the series of *STUDFUZZ. Springer*. Heidelberg, 2016..

29. Ranjit Biswas, Intuitionistic Fuzzy Theory for Soft-Computing: More Appropriate Tool Than Fuzzy Theory, *International Journal of Computing and Optimization*. Vol. 6(1), 2019 pp 13-56.

30. S. A. Alblowi, A. A. Salama & Mohmed Eisa, New Concepts of Neutrosophic Sets, International Journal of Mathematics and Computer Applications Research (IJMCAR),Vol. 4, Issue 1, 59-66, 2014.

31. S. Broumi, M. Talea, F Smarandache, and Assia Bakali, Single valued neutrosophic graphs : Degree, Order and Size, Proceedings of the IEEE International Conference on Fuzzy Systems (FUZZ) 2016, pp 2444-2451.

32. S. Broumi, M. Talea, Assia Bakali, F Smarandache, and Kishore Kumar P.K., Shortest Path Problem on Single Valued Neutrosophic Graphs, Int. Symp. On Networks, Computers and Communications (ISNCC-2017), Marrakech, Morocco, May 16-18, 2017.

33. S. Broumi, Assia Bakali, M. Talea and F Smarandache, Applying Dijkstra Algorithm for solving Neutrosophic Shortest Path Problems, Proceedings of the 2016 Iternational Conference on Advanced Mechatronic Systems Melbourne, Australia, Nov 30 – December 3, 2016, pp 412-416.

34. S. Broumi, A. Dey, M. Talea, A. Bakali, F. Smarandache, D. Nagarajan, M. Lathamaheswari and Ranjan Kumar(2019), "Shortest Path Problem using Bellman Algorithm under Neutrosophic Environment," Complex & Intelligent Systems ,pp-1-8, https://doi.org/10.1007/s40747-019-0101-8

35. S. Broumi, M.Talea, A. Bakali, F. Smarandache, D.Nagarajan, M. Lathamaheswari and M.Parimala, Shortest path problem in fuzzy, intuitionistic fuzzy and neutrosophic environment: an overview, Complex & Intelligent Systems ,2019,pp 1-8, https://doi.org/10.1007/s40747-019-0098-z

36. S.Broumi, D. Nagarajan, A. Bakali, M. Talea, F. Smarandache, M. Lathamaheswari, The shortest path problem in interval valued trapezoidal and triangular neutrosophic environment, Complex & Intelligent Systems , 2019,pp 1-12, https://doi.org/10.1007/s40747-019-0092-5

37. S. Broumi, Mohamed Talea, Assia Bakali, Prem Kumar Singh, Florentin Smarandache: Energy and Spectrum Analysis of Interval Valued Neutrosophic Graph using MATLAB, Neutrosophic Sets and Systems, vol. 24, 2019, pp. 46-60. DOI: 10.5281/zenodo.2593919

38. S. Broumi, A. Dey, A. Bakali, M. Talea, F. Smarandache, L. H. Son, D. Koley: Uniform Single Valued Neutrosophic Graphs, Neutrosophic Sets and Systems, vol. 17, 2017, pp. 42-49. http://doi.org/10.5281/zenodo.1012249

39. S. Broumi, Assia Bakali, Mohamed Talea, Florentin Smarandache: Isolated Single Valued Neutrosophic Graphs, Neutrosophic Sets and Systems, vol. 11, 2016, pp. 74-78. doi.org/10.5281/zenodo.571458

40. S. Satham Hussain , R. Jahir Hussain , F. Smarandache: Domination Number in Neutrosophic Soft Graphs , Neutrosophic Sets and Systems, vol. 28, 2019, pp. 228-244. DOI: 10.5281/zenodo.3382548

41. S. Satham Hussain, R. Jahir Hussain, F. Smarandache: On Neutrosophic Vague Graphs, Neutrosophic Sets and Systems, vol. 28, 2019, pp. 245-258. DOI: 10.5281/zenodo.3382550

42. S. Satham Hussain, R. Jahir Hussain, Young Bae Jun, F. Smarandache: Neutrosophic Bipolar Vague Set and its Application to Neutrosophic Bipolar Vague Graphs, Neutrosophic Sets and Systems, vol. 28, 2019, pp. 69-86. DOI: 10.5281/zenodo.3387802
43. S. Satham Hussain , R. Jahir Hussain , F. Smarandache: Domination Number in Neutrosophic Soft Graphs , Neutrosophic Sets and Systems, vol. 28, 2019, pp. 228-244. DOI: 10.5281/zenodo.3382548
44. S. Satham Hussain, R. Jahir Hussain, Young Bae Jun, F. Smarandache: Neutrosophic Bipolar Vague Set and its Application to Neutrosophic Bipolar Vague Graphs, Neutrosophic Sets and Systems, vol. 28, 2019, pp. 69-86. DOI: 10.5281/zenodo.3387802
45. Siddhartha Sankar Biswas, Basir Alam and M.N. Doja, A Theoretical Characterization of the Data Structure 'Multigraphs', Journal of Contemporary Applied Mathematics, Vol.2(2) December'2012, page 88-106.
46. Siddhartha Sankar Biswas, Bashir Alam and M. N. Doja, Generalization of Dijkstra's Algorithm For Extraction of Shortest Paths in Directed Multigraphs, Journal of Computer Science, Vol.9 (3) 2013: pp 377-382, ISSN 1549-3636, doi: 10.3844/jcssp.2013.377.382.
47. Siddhartha Bashir Alam, Siddhartha Sankar Biswas and M. N. Doja, Fuzzy Shortest Path in A Directed Multigraph, European Journal of Scientific Research, Vol.101 (3) 2013: pp 333-339, ISSN 1450-216X / 1450-202X.
48. Siddhartha Sankar Biswas, Bashir Alam and M. N. Doja, An Algorithm For Extracting Intuitionistic Fuzzy Shortest Path in A Graph, Applied Computational Intelligence and Soft Computing , Vol.2(2) 2013 (Hindawi Publishing Corporation), http://dx.doi.org/10.1155/2013/970197.
49. Siddhartha Sankar Biswas, "Real Time Neutrosophic Graphs For Communication Networks", to appear as a book-chapter in the book entitled: Neutrosophic Sets in Decision Analysis and Operations Research, Edited by Prof. Dr. Florentin Smarandache and Dr. Mohamed Abdel-Baset, IGI Global, USA, 2019**.**
50. T. Chalapathi, R. V M S S Kiran Kumar: Neutrosophic Graphs of Finite Groups, Neutrosophic Sets and Systems, vol. 15, 2017, pp. 22-30. doi.org/10.5281/zenodo.570943
51. V. K. Balakrishnan, Graph Theory, McGraw-Hill; 1997.

Received: June 21, 2019. Accepted: October 19, 2019