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The DC Design of Reliable Multistable Circuits

Richard H. Williams

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THE DC DESIGN OF RELIABLE
MULTISTABLE CIRCUITS

By

Richard H. Williams

A Thesis

Submitted in Partial Fulfillment of the
Requirements for the Degree of
Master of Science in Electrical Engineering

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1957

This thesis, directed and approved by the candidate's committee, has been accepted by the Graduate Committee of the University of New Mexico in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

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DEAN

5-13/1957

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R. H. Williams

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CHAPTER I

INTRODUCTION

Today, almost all the fields of engineering are expanding in an exponential manner. Electronic engineering, along with the other fields, is being applied to a fantastic diversity of problems. Some of the products which are the result of this engineering are prodigious in their complexity, and more complex products are envisioned! This exponential growth coupled with increasing product complexity has caused many electronic engineers to become alarmed about, or at least alerted to, the inherent reliability of their products.

The concept of product reliability may be a nebulous thing to a designer who has built, perhaps, one "breadboard" model. There is nothing nebulous about the effect, however, to the business man, the military man, or the maintenance man who have to work with the product. Equipment malfunction costs time and money; perhaps it will even cost lives.

That the electronic engineer is becoming alerted to his product's reliability--or unreliability--is evidenced by an increasing amount of literature devoted to the subject. The one general rule which stands out in all this literature is that each design engineer is obligated to choose each circuit, each component, and each fabrication detail for the most reliable operation consistent with its intended environment. A great amount of excellent work has been done in this direction, notably with components such as vacuum tubes. However, more has to be done.

The reliability of a piece of equipment is a measure of its ability to function as intended. Equipment may malfunction in two different ways. It may fail catastrophically as a vacuum tube does when its heater opens; it may fail gradually as a vacuum tube does when its transconductance slowly falls to an unacceptable level over a period of time. It is not the intent of this paper to study catastrophic component failures, but rather to illustrate a technique whereby gradual failure of circuits can be minimized.

All manufactured materials have a certain variance from an intended nominal value. For reliable circuit design, the engineer must consider these variances just as thoroughly as he must consider the circuit's theory of operation. It is with this idea in mind that the multistable circuit is here re-examined.

1.1 The Multistable Circuit

The term multistable circuit is generic. It describes a general type of circuit which possesses two or more stable states. A multistable circuit which has N stable states is said to be a multistable circuit of order N , where $N \geq 2$. The concept of several different stable states implies switching action, and multistable circuits do, in fact, operate in the switching mode. The best analogy of a multistable circuit is the simple mechanical switch. A multistable circuit of order two may be thought of as a single-pole, double-throw switch. A multistable circuit of order three may be thought of as a single-pole, triple-throw switch, and so on. In each case, when one tube is "ON" the other $N - 1$ tubes are "OFF".

When a multistable circuit is in one of its N stable states, it is a DC circuit; all transients from switching have disappeared and no changing voltages or currents are present. When a multistable circuit is changing from one stable state to another it is a transient problem. This paper is limited to a study of the reliability of a stable state once it has been achieved. It is not concerned with the mechanism of transition from one stable state to another.

This paper is further limited to those multistable circuits which employ vacuum tubes and which operate on the principle that one tube is required for each of the N stable states. Thyatron ring counters are not considered here, and neither are those special multistable circuits which employ N tubes for $N + 1$ stable states.*

The general multistable circuit with which this paper is concerned is illustrated in Figure 1. With regard to this circuit: when $N = 2$, the common binary is formed; when $N = 3$, the ternary; when $N = 4$, the quaternary; when $N = 5$, the quinary; and so forth.

---

*In this case the "perching effect"¹ or nonlinear cathode resistance² is employed. As an example of this case, a multistable circuit of order three may use two tubes. The three stable states are: (a) either tube conducting with the other biased "OFF", (b) vice versa, and (c) both tubes conducting at the same time.

¹R. S. Mackay, and R. MacIntyre, "Ternary Counters," IRE Transactions on Electronic Computers, Vol. EC-4, No. 4, December 1955, pp. 144-149.

²Richard Weissman, "High-Speed Counter Uses Ternary Notations," Electronics, Vol. 25, No. 10, October 1952, pp. 118-121.

FIGURE 1.- THE NTHNARY

1.2 The Nthnary

It is apropos at this time to define a new term, the nthnary. An nthnary is a multistable circuit of order N , and its general form is illustrated in Figure 1. The term, nthnary, is introduced because of the cumbersomeness of the term, multistable circuit of order N , and also, because it is more specific. When an nthnary of a particular order is referred to, then it will be called by its particular name, e.g., binary, ternary, etc.

1.3 The Scope of the Paper

A simple method of designing nthnaries will be developed which will:

- (a) yield a functionally adequate circuit for each practical value of N ; and
- (b) take into account the tolerance limits which can be applied to resistors and vacuum tubes.

Furthermore, an analysis will be made and information given about other factors of nthnary design which have a real bearing on its reliability such as the use of cathode resistance for self-bias. Finally, some experimental data will be included to substantiate the analytical conclusions of this paper.

It should be realized at this point that this paper deals with a general circuit in a general fashion. It is intended that this paper will be a guide for circuit designers who have to design an nthnary in a semi-routine fashion. A circuit designer who has a very special requirement

1.1. The rate of reaction was found to be directly proportional to the concentration of the solution. This was determined by measuring the time taken for a fixed amount of reactant to be consumed at different concentrations. The results are shown in the table below.

Concentration of solution (M)	Time taken (s)
0.1	120
0.2	60
0.3	40
0.4	30
0.5	24

1.2. The rate of reaction was also found to be directly proportional to the surface area of the reactant. This was determined by measuring the time taken for a fixed amount of reactant to be consumed at different surface areas. The results are shown in the table below.

Surface area (cm ²)	Time taken (s)
10	120
20	60
30	40
40	30
50	24

2. The effect of temperature on the rate of reaction

2.1. The rate of reaction was found to be directly proportional to the temperature. This was determined by measuring the time taken for a fixed amount of reactant to be consumed at different temperatures. The results are shown in the table below.

2.2. The rate of reaction was also found to be directly proportional to the concentration of the solution. This was determined by measuring the time taken for a fixed amount of reactant to be consumed at different concentrations. The results are shown in the table below.

2.3. The rate of reaction was also found to be directly proportional to the surface area of the reactant. This was determined by measuring the time taken for a fixed amount of reactant to be consumed at different surface areas. The results are shown in the table below.

2.4. The rate of reaction was also found to be directly proportional to the concentration of the solution. This was determined by measuring the time taken for a fixed amount of reactant to be consumed at different concentrations. The results are shown in the table below.

2.5. The rate of reaction was also found to be directly proportional to the surface area of the reactant. This was determined by measuring the time taken for a fixed amount of reactant to be consumed at different surface areas. The results are shown in the table below.

2.6. The rate of reaction was also found to be directly proportional to the concentration of the solution. This was determined by measuring the time taken for a fixed amount of reactant to be consumed at different concentrations. The results are shown in the table below.

2.7. The rate of reaction was also found to be directly proportional to the surface area of the reactant. This was determined by measuring the time taken for a fixed amount of reactant to be consumed at different surface areas. The results are shown in the table below.

2.8. The rate of reaction was also found to be directly proportional to the concentration of the solution. This was determined by measuring the time taken for a fixed amount of reactant to be consumed at different concentrations. The results are shown in the table below.

2.9. The rate of reaction was also found to be directly proportional to the surface area of the reactant. This was determined by measuring the time taken for a fixed amount of reactant to be consumed at different surface areas. The results are shown in the table below.

2.10. The rate of reaction was also found to be directly proportional to the concentration of the solution. This was determined by measuring the time taken for a fixed amount of reactant to be consumed at different concentrations. The results are shown in the table below.

2.11. The rate of reaction was also found to be directly proportional to the surface area of the reactant. This was determined by measuring the time taken for a fixed amount of reactant to be consumed at different surface areas. The results are shown in the table below.

2.12. The rate of reaction was also found to be directly proportional to the concentration of the solution. This was determined by measuring the time taken for a fixed amount of reactant to be consumed at different concentrations. The results are shown in the table below.

of an nthnary (such as an extremely high-frequency triggering rate) will not find all his answers here.

1.4 The Organization of the Paper

Chapter II contains a search of the literature along with an explanation of the operation of the nthnary. Chapter II also contains a development of a simplified equivalent nthnary circuit that is used in the subsequent analysis.

In Chapter III the equivalent nthnary circuit is analyzed, and the nodal voltages that are important in circuit design are derived in terms of the circuit resistors and the supply voltage.

Chapter IV contains a development of nthnary design criteria which takes into account the expected tolerance limits of resistors and vacuum tubes. Chapter IV also contains a proof that the reliability of the nthnary can be made independent of supply voltage changes.

Chapter V contains a derivation of equations that indicate to a circuit designer how he might choose the nominal value of resistors for best nthnary reliability. A figure of merit for vacuum tubes is defined that will indicate the best tube types for nthnary applications. Also discussed briefly in this chapter are the effects of grid current and the tolerance limits which may be expected of practical resistors and tubes.

Chapter VI contains the author's conclusions as he derived them from the material in this paper. As evidence in support of his conclusions he refers to experimental data contained in the appendix.

CHAPTER II

ANALYSIS OF THE NTHNARY

2.1 The Search of the Literature

As was stated previously, the nthnary is a generic term embracing the binary, ternary, quaternary, etc. A search of the literature will reveal a preponderance of articles about the binary, a smattering about the ternary, and nil about the quaternary and circuits of higher order. Since this analysis is concerned with the general family, much of the following work is based on techniques originally applied to the binary but which are here reapplied to the nthnary.

It is a common technique to replace a vacuum tube with its equivalent static plate resistance for the purposes of analysis.* This technique may be quite readily applied to the binary and, indeed, has been so done a great many times. However, for the purposes of reliability studies, the application of this technique by Pressman,² Ritchie,³ and

---

*Martin in his book Electronic Circuits¹ presents a lucid explanation of how this equivalent resistance is derived.

¹T. L. Martin, Electronic Circuits, Prentice-Hall, Inc., New York, 1955, pp. 3-23.

²R. Pressman, "How to Design Bistable Multivibrators," Electronics, Vol. 26, No. 4, April 1953, pp. 164-168.

³D. K. Ritchie, "The Optimum DC Design of Flip-Flops," Proc. of the IRE, Vol. 41, No. 11, November 1953, pp. 1614-1617.

Paivinen and Auerbach⁴ are among the best because their analysis takes into account the tolerances of resistors, vacuum tubes and supply voltages. This same technique, including the effect of component and supply voltage variation, has been applied to the nthnary by Beckwith.⁵ The equivalent circuit technique that Beckwith employed is quite similar to what is used in this paper, except that this paper's analysis was aimed at producing design criteria which are simpler, to some degree, to use and to understand.

Paivinen and Auerbach also present an analysis of the binary in which they prove that "the self-biased flip-flop is independent of supply voltage variations when no diodes are used in the grid divider."⁶

The technical literature today contains many articles which give the maximum, or statistical, variations of components such as resistors and tubes. Use will be made of some of these articles in Chapter V to determine tolerance constants which may be used in practical circuit design.

2.2 The Operation of the Nthnary

The operation of the nthnary may be understood with the aid of Figure 1. By the nature of this circuit, it is known that at any particular

---

⁴J. O. Paivinen, and I. L. Auerbach, "Design of Triode Flip-Flops for Long-Term Stability," IRE Transactions on Electronic Computers, Vol. EC-2, No. 2, June 1953, pp. 14-26.

⁵H. Beckwith, "Flip-Flop Counter has Expanded Range," Electronics, Vol. 28, No. 1, January 1955, pp. 149-151.

⁶Paivinen and Auerbach, loc. cit.

time only one tube is conducting, or "ON", and that the other $N - 1$ tubes are nonconducting, or "OFF". Since the circuit is ideally symmetrical, it does not matter which of the N tubes is "ON". If it is assumed that V_1 is "ON", then the plate voltage of V_1 will be lower than the plate voltage of the other $N - 1$ tubes which are "OFF". It can be further noted from Figure 1 that the plate of each tube is direct-coupled to the grid of each of the other $N - 1$ tubes by divider networks. Therefore, for proper nthnary operation the following two conditions must balance:

- (a) the conducting tube must be held "ON" when its grid is direct-coupled to the relatively high plate potential of each of the $N - 1$ "OFF" tubes; and
- (b) each nonconducting tube must be held "OFF" when its grid is direct-coupled to not only the relatively low plate potential of the one "ON" tube, but also to the relatively high plate potential of each of the other $N - 2$ "OFF" tubes.

One might intuitively examine this balance and see that the paralleling action of the divider networks will soon work a hardship on the reliability, or stability, of the nthnary as N becomes large.

Figure 1 shows that a capacitor is associated with the grid circuit of each tube. These capacitors are used for sequencing or "steering". If these capacitors were not present and V_1 was turned "OFF" with a negative pulse on its grid, then in the ideal case, each of the other $N - 1$ tubes would have an equal probability of turning "ON". However, if a capacitor is connected from the plate of V_1 to the grid of V_2 then when V_1 is turned

the first two cases, the results are as follows: Case I - a linear relationship between the logarithm of the rate of reaction and the concentration of the reactants. Case II - a linear relationship between the logarithm of the rate of reaction and the concentration of the reactants.

the case of a linear relationship between the logarithm of the rate of reaction and the concentration of the reactants, the rate of reaction is proportional to the concentration of the reactants.

operator has followed the following procedure: (a) The concentration of the reactants was varied in a series of experiments.

directed towards the study of the effect of the concentration of the reactants on the rate of reaction. In each of the three cases, the rate of reaction was found to be proportional to the concentration of the reactants.

relationship is shown in Figure 1. The rate of reaction is proportional to the concentration of the reactants.

Each case is shown in Figure 1. The rate of reaction is proportional to the concentration of the reactants. The rate of reaction is proportional to the concentration of the reactants.

the rate of reaction is proportional to the concentration of the reactants. The rate of reaction is proportional to the concentration of the reactants.

some experiments were performed and it was found that the rate of reaction is proportional to the concentration of the reactants.

was also found that the rate of reaction is proportional to the concentration of the reactants. The rate of reaction is proportional to the concentration of the reactants.

"OFF" a larger positive impulse from the plate of V1 is coupled to the grid of V2 than to any of the other $N - 2$ "OFF" tubes which are only resistance coupled to the plate of V1. This action causes V2 to turn "ON" in a decisive manner. The negative impulse from the plate of V2 then suppresses the turning "ON" of any of the other $N - 1$ tubes. In an identical manner, when V2 is turned "OFF", V3 is favored to turn "ON" because of the capacitor connected from the plate of V2 to the grid of V3. It is by such a technique that the nthary is made to sequence or "steer". The capacitors, of course, do not enter into a discussion of the DC stability and in the following work they will be neglected.

2.3 The Equivalent Circuit of the Nthary

The circuit in Figure 1 is too complicated to be conveniently used in an analysis directly. It must therefore be simplified. The first step is to redraw Figure 1 with the triodes replaced by their equivalent resistances as is done in Figure 2. V1 has some finite static plate resistance, r_p ; V2, V3, ... V_N are assumed to have a resistance of infinite ohms since they are all nonconducting. Next, we define the following voltages with respect to ground:

- $(E_p)_c$ -- The plate voltage of the "ON" triode;
- $(E_g)_c$ -- The grid voltage of the "ON" triode;
- $(E_p)_{nc}$ -- The plate voltage of the $N - 1$ "OFF" triodes;
- $(E_g)_{nc}$ -- The grid voltage of the $N - 1$ "OFF" triodes; and
- E_k -- The cathode voltage common to all N triodes.

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It can be noticed from Figure 2 that $N - 1$ points are at potential $(E_p)_{nc}$. For the purposes of analysis, these $N - 1$ points may be considered to be electrically as one point since if they were wired together no resultant current would flow to change the existing potentials. By the same reasoning, the $N - 1$ points which are all at potential $(E_g)_{nc}$ may be considered electrically common. Using the above two simplifications and by careful rearranging, the circuit in Figure 2 can be transformed into a simpler circuit which is illustrated in Figure 3, which, in turn, can be redrawn as shown in Figure 4. Figure 4 is the basic simplified nthnary from which all of the subsequent DC analysis is based. As stated previously, N may have the values of 2, 3, 4, ... ∞ .* If N is set equal to two, the expression

$$\frac{R_1}{(N - 2)(N - 1)}$$

becomes infinity and two separate simple circuits remain. A cursory examination will reveal that these two circuits are the simplification of the binary. When N is set equal to a number greater than two, the complex four-mesh circuit remains.

---

*Obviously, a circuit for which N equals one is trivial in a discussion of multistable circuits. If N equals one is substituted in the expressions in Figure 4, a single triode, zero biased, with a plate load resistor of R_p is all that would remain.

FIGURE 2.- THE NTHNARY WITH VACUUM TUBES REPLACED BY EQUIVALENT RESISTANCES

STATIONARY AIR CONDITIONING UNIT

FIGURE 3.- NTHNARY EQUIVALENT CIRCUIT

FIGURE 4 :- NTHNARY SIMPLIFIED EQUIVALENT CIRCUIT

CHAPTER III

A SOLUTION OF THE NTHNARY'S EQUIVALENT CIRCUIT

In this chapter the simplified equivalent circuit of the nthnary will be solved for the five nodal voltages which are important in practical circuit design. These voltages are the plate and grid voltages of the conducting, or "ON", tube; the plate and grid voltages of the nonconducting, or "OFF", tubes; and the cathode voltage.

3.1 An Approximation

The solution of a four-mesh network can be very difficult. The results are often complex and cumbersome. Often, however, for a given practical application, an approximation can be assumed which is not too restrictive, which allows a great simplification of labor, and which also yields a greater clarity of results. In the following solution of the nthnary it has been found convenient to use the approximation that

$$R_p \ll \frac{R_1}{N - 1} \quad . \quad (3-1)$$

Actually, this approximation is quite easy to use in a practical design. It specifies a condition which allows the greatest change of plate voltage from the "ON" to the "OFF" condition and thereby aids in designing reliably stable circuits.* Equation (3-1) states that as N approaches

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*The larger plate voltage change will, to some extent, lower the maximum allowable triggering rate of the nthnary because the stray capacities will have to be charged and discharged over larger voltage extremes.

infinity, R_p will approach zero. This would seem to say that a theoretical limit is placed on the validity of the approximation. However, one can observe from Figure 4 that as N approaches infinity the equivalent circuit itself becomes trivial since almost all the equivalent resistors approach "short circuits". Actually, it will be shown later that other factors of a more practical nature limit the feasibility of the nthary much before N reaches infinity.

3.2 Solution for $(E_p)_c$ and E_k

Referring to Figure 4, the equivalent resistance to the left of point A is

$$\overleftarrow{R_A} = \frac{R_p(r_b + R_k)}{R_p + r_b + R_k} \cdot$$

The equivalent resistance to the right of point A is

$$\overrightarrow{R_A} > \frac{R_1}{N - 1} \cdot$$

Also,

$$R_p > \frac{R_p(r_b + R_k)}{R_p + r_b + R_k} \cdot$$

Then, using equation (3-1), we may reason that

$$\overleftarrow{R_A} = \frac{R_p(r_b + R_k)}{R_p + r_b + R_k} < R_p \ll \frac{R_1}{N - 1} < \overrightarrow{R_A}$$

or,

$$\overleftarrow{R_A} \ll \overrightarrow{R_A} ,$$

and the portion of the circuit to the right of point A may be neglected in an analysis of the first mesh.

The current in the first mesh is

$$I_1 = \frac{E_{bb}}{R_p + r_b + R_k} .$$

Therefore,

$$E_k = E_{bb} \frac{R_k}{R_k + r_b + R_p} \quad (3-2)$$

and

$$(E_p)_c = E_{bb} \frac{R_k + r_b}{R_k + r_b + R_p} . \quad (3-3)$$

3.3 Solution for $(E_p)_{nc}$ and $(E_g)_c$

Again referring to Figure 4, the equivalent resistance to the right of point D is

$$R_D = \frac{R_p}{N-1} \left(\frac{R_1}{N-1} + R_2 \right) ,$$

$$\rightarrow \frac{R_p}{N-1} + \frac{R_1}{N-1} + R_2$$

which may be reduced to

$$\xrightarrow{R_D} = \frac{R_p}{N-1}$$

by using the approximation in equation (3-1). The equivalent resistance to the left of point D is

$$\xleftarrow{R_D} > \frac{R_1}{(N-1)(N-2)} \cdot$$

By multiplying both sides of equation (3-1) by $\frac{1}{N-1}$ we have,

$$\frac{R_p}{N-1} \ll \frac{R_1}{(N-1)^2} \cdot$$

Also, it follows by inspection that

$$\frac{R_1}{(N-1)(N-2)} > \frac{R_1}{(N-1)^2} \cdot$$

Therefore, it can be reasoned that

$$\xrightarrow{R_D} = \frac{R_p}{N-1} \ll \frac{R_1}{(N-1)^2} < \frac{R_1}{(N-1)(N-2)} < \xleftarrow{R_D}$$

or

$$\xrightarrow{R_D} \ll \xleftarrow{R_D} ,$$

and the portion of the circuit to the left of point D may be neglected in an analysis of the fourth mesh.

The current in the fourth mesh is

$$I_4 = \frac{E_{bb}}{\frac{R_p}{N-1} + \frac{R_1}{N-1} + R_2} = \frac{E_{bb}}{\frac{R_1}{N-1} + R_2} .$$

Therefore,

$$(E_g)_c = E_{bb} \frac{R_2}{\frac{R_1}{N-1} + R_2} \quad (3-4)$$

and

$$(E_p)_{nc} = E_{bb} \frac{\frac{R_1}{N-1} + R_2}{\frac{R_1}{N-1} + R_2} = E_{bb} . \quad (3-5)$$

3.4 Solution for $(E_g)_{nc}$

The Thevenin equivalent circuit of the portion of the circuit in Figure 4 to the left of point B has the following values:

$$R_B = \frac{R_1}{N-1} \quad (3-6)$$

and

$$E_B = (E_p)_c . \quad (3-7)$$

Also, the Thevenin equivalent circuit of the portion of the circuit in Figure 4 to the right of point C has these following values:

$$\overrightarrow{R_C} = \frac{R_1}{(N-1)(N-2)} \quad (3-8)$$

and

$$\overrightarrow{E_C} = E_{bb} \quad (3-9)$$

Figure 4 may now be redrawn as illustrated in Figure 5 using the values of equations (3-6), (3-7), (3-8), and (3-9). The equations for the solution of the two-mesh circuit in Figure 5 are:

$$(E_p)_c = I_2 \left(\frac{R_1}{N-1} + \frac{R_2}{N-1} \right) - I_3 \frac{R_2}{N-1} ,$$

$$-E_{bb} = -I_2 \frac{R_2}{N-1} + I_3 \left(\frac{R_2}{N-1} + \frac{R_1}{(N-1)(N-2)} \right) .$$

Solving for I_2 by determinants:

$$I_2 = \frac{\begin{vmatrix} (E_p)_c & -\frac{R_2}{N-1} \\ -E_{bb} & \frac{R_2}{N-1} + \frac{R_1}{(N-1)(N-2)} \end{vmatrix}}{\begin{vmatrix} \frac{R_1}{N-1} + \frac{R_2}{N-1} & -\frac{R_2}{N-1} \\ -\frac{R_2}{N-1} & \frac{R_2}{N-1} + \frac{R_1}{(N-1)(N-2)} \end{vmatrix}} ,$$

Also, the first two equations of the system of the problem (1) are

$$x_1 + x_2 + \dots + x_n = 1, \quad (2)$$

where x_i is the probability of the event A_i occurring.

From (2) we have $x_1 = 1 - x_2 - \dots - x_n$. The system of equations (1) and (2) can be written as

$$x_1 + x_2 + \dots + x_n = 1, \quad (3)$$

$$x_1 + x_2 + \dots + x_n = 1, \quad (4)$$

where x_i is the probability of the event A_i occurring.

x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{16}	x_{17}	x_{18}	x_{19}	x_{20}	x_{21}	x_{22}	x_{23}	x_{24}	x_{25}	x_{26}	x_{27}	x_{28}	x_{29}	x_{30}	x_{31}	x_{32}	x_{33}	x_{34}	x_{35}	x_{36}	x_{37}	x_{38}	x_{39}	x_{40}	x_{41}	x_{42}	x_{43}	x_{44}	x_{45}	x_{46}	x_{47}	x_{48}	x_{49}	x_{50}	x_{51}	x_{52}	x_{53}	x_{54}	x_{55}	x_{56}	x_{57}	x_{58}	x_{59}	x_{60}	x_{61}	x_{62}	x_{63}	x_{64}	x_{65}	x_{66}	x_{67}	x_{68}	x_{69}	x_{70}	x_{71}	x_{72}	x_{73}	x_{74}	x_{75}	x_{76}	x_{77}	x_{78}	x_{79}	x_{80}	x_{81}	x_{82}	x_{83}	x_{84}	x_{85}	x_{86}	x_{87}	x_{88}	x_{89}	x_{90}	x_{91}	x_{92}	x_{93}	x_{94}	x_{95}	x_{96}	x_{97}	x_{98}	x_{99}	x_{100}
x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{16}	x_{17}	x_{18}	x_{19}	x_{20}	x_{21}	x_{22}	x_{23}	x_{24}	x_{25}	x_{26}	x_{27}	x_{28}	x_{29}	x_{30}	x_{31}	x_{32}	x_{33}	x_{34}	x_{35}	x_{36}	x_{37}	x_{38}	x_{39}	x_{40}	x_{41}	x_{42}	x_{43}	x_{44}	x_{45}	x_{46}	x_{47}	x_{48}	x_{49}	x_{50}	x_{51}	x_{52}	x_{53}	x_{54}	x_{55}	x_{56}	x_{57}	x_{58}	x_{59}	x_{60}	x_{61}	x_{62}	x_{63}	x_{64}	x_{65}	x_{66}	x_{67}	x_{68}	x_{69}	x_{70}	x_{71}	x_{72}	x_{73}	x_{74}	x_{75}	x_{76}	x_{77}	x_{78}	x_{79}	x_{80}	x_{81}	x_{82}	x_{83}	x_{84}	x_{85}	x_{86}	x_{87}	x_{88}	x_{89}	x_{90}	x_{91}	x_{92}	x_{93}	x_{94}	x_{95}	x_{96}	x_{97}	x_{98}	x_{99}	x_{100}

FIGURE 5.- SIMPLIFIED NTHNARY FOR THE SOLUTION OF $(E_g)_{nc}$

$$I_2 = \frac{(E_p)_c \left(R_2 + \frac{R_1}{N-2} \right) - E_{bb} R_2}{\frac{R_1}{N-1} \left(R_2 + \frac{R_1}{N-2} + \frac{R_2}{N-2} \right)} .$$

The grid voltage of the nonconducting tube is

$$\begin{aligned} (E_g)_{nc} &= (E_p)_c - I_2 \frac{R_1}{N-1} , \\ &= (E_p)_c - \frac{(E_p)_c \left(R_2 + \frac{R_1}{N-1} \right) - E_{bb} R_2}{R_2 + \frac{R_1}{N-2} + \frac{R_2}{N-2}} , \\ &= \frac{R_2 \left[(E_p)_c + E_{bb} (N-2) \right]}{R_1 + R_2 (N-1)} . \end{aligned}$$

And, by using equation (3-3), we have

$$(E_g)_{nc} = E_{bb} \frac{R_2 \left(\frac{R_k + r_b}{R_p + R_k + r_b} + N-2 \right)}{R_1 + R_2 (N-1)} . \quad (3-10)$$

3.5 Summary

With reference to Figure 4 and using the approximation that

$$R_p \ll \frac{R_1}{N-1} \quad (3-1)$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

The first row of the matrix is

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

and by using equation (3-2), we have

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

(or)

It follows that

is a solution to (3-1) and using the equation that

we have

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

the five important nodal voltages of the nthary are:

$$E_k = E_{bb} \frac{R_k}{R_p + R_k + r_b} , \quad (3-2)$$

$$(E_p)_c = E_{bb} \frac{R_k + r_b}{R_p + R_k + r_b} , \quad (3-3)$$

$$(E_g)_c = E_{bb} \frac{R_2}{\frac{R_1}{N-1} + R_2} , \quad (3-4)$$

$$(E_p)_{nc} = E_{bb} , \quad (3-5)$$

$$(E_g)_{nc} = E_{bb} \frac{R_2 \left(\frac{R_k + r_b}{R_p + R_k + r_b} + N - 2 \right)}{R_1 + R_2(N-1)} . \quad (3-10)$$

CHAPTER IV

CRITERIA FOR RELIABLE NTHNARY OPERATION

When a circuit is to be analyzed from the standpoint of reliable operation, the theory of operation must first be understood. Then this theory must be made to yield certain criteria of operation which can be expressed analytically. These criteria must then be adjusted to include the expected component tolerances which occur in a practical circuit.

4.1 Criterion for Operation of the "OFF" Tubes

The bias voltage of an "OFF" tube is the difference between the cathode voltage and the grid voltage of the "OFF" tube. In order to ensure that an "OFF" tube will always be nonconducting, this bias voltage must always be equal to, or greater than, the grid cutoff voltage of the tube. This criterion can be expressed analytically as follows:

$$E_k - |E_{co}| \geq (E_g)_{nc} \quad , \quad (4-1)$$

where $|E_{co}|$ is the magnitude of the cutoff voltage of the tube.

Using equations (3-2) and (3-10), equation (4-1) may be rewritten as

$$\frac{E_{bb} R_k}{R_p + R_k + r_b} - |E_{co}| \geq E_{bb} \frac{R_2 \left(\frac{R_k + r_b}{R_p + R_k + r_b} + N - 2 \right)}{R_1 + R_2(N - 1)} \quad (4-2)$$

Let B be the ratio

$$B = \frac{E_{pk}}{|E_{co}|} = \frac{(E_p)_{nc} - E_k}{|E_{co}|}, \quad (4-3)$$

where B is always a positive number and is a constant for a given tube type over a wide operating range, and where E_{pk} is the plate-to-cathode voltage of the tube. Using equations (3-5) and (3-2), and rearranging equation (4-3),

$$|E_{co}| = \frac{E_{bb}}{B} - \frac{1}{B} \left(\frac{E_{bb} R_k}{R_k + R_p + r_b} \right). \quad (4-4)$$

Substituting equation (4-4) into equation (4-2),

$$\frac{E_{bb} R_k}{R_p + R_k + r_b} \left(1 + \frac{1}{B} \right) - \frac{E_{bb}}{B} \geq \frac{E_{bb} R_2 \left(\frac{R_k + r_b}{R_p + R_k + r_b} + N - 2 \right)}{R_1 + R_2 (N - 1)},$$

and then rearranging,

$$R_k \left(1 + \frac{1}{B} \right) - \frac{1}{B} (R_k + R_p + r_b) \geq \frac{\left[\frac{R_k + r_b + (N - 2)(R_p + R_k + r_b)}{N - 1} \right] R_2 (N - 1)}{R_1 + R_2 (N - 1)},$$

and finally,

$$\frac{R_k - \frac{R_p + r_b}{B}}{R_k + \left(\frac{N - 2}{N - 1} \right) R_p + r_b} \geq \frac{\frac{R_2}{R_1}}{\frac{1}{N - 1} + \frac{R_2}{R_1}}. \quad (4-5)$$

Equation (4-5) expresses analytically, in terms of the component values, the criterion for operation of the "OFF" tubes.

4.2 Criterion for Operation of the "ON" Tube

It was previously stated that circuits of the nthary type generally operate the "ON" tube at zero bias. If the bias tends to be positive, it is limited to zero volts by the clamping action of the grid-to-cathode "diode" of the tube. We may then state as a criterion that the bias of the "ON" tube should never tend to be negative. Analytically this may be written as follows:

$$(E_g)_c \geq E_k \quad . \quad (4-6)$$

Using equations (3-2) and (3-4), equation (4-6) may be rewritten as

$$\frac{\frac{E_{bb} R_2}{R_1}}{\frac{1}{N-1} + R_2} \geq \frac{E_{bb} R_k}{R_k + R_p + r_b} \quad ,$$

and by further rearranging,

$$\frac{\frac{R_2}{R_1}}{\frac{1}{N-1} + \frac{R_2}{R_1}} \geq \frac{R_k}{R_k + R_p + r_b} \quad . \quad (4-7)$$

Equation (4-7) expresses analytically, in terms of the component values, the criterion for operation of the "ON" tube.

4.3 Effect of Component and Supply Voltage Variations

Proper operation of the nthnary is expressed by the simultaneous solution of equations (4-5) and (4-7). It is now necessary to adjust these equations to include the effects of the expected variations which occur in the components employed in a practical circuit.

4.3.1 Supply Voltage Variations. It is important to note that equations (4-5) and (4-7) are independent of supply voltage variations, provided that the ratio B is a constant for the tubes employed. It is also important to note that the nthnary discussed in this paper employs a cathode resistor for self-bias. The effect of this cathode resistor on nthnary reliability is to make the circuit independent of supply voltage variations. A proof of this statement follows:

The ratio B, for an "OFF" tube, has been defined as

$$B = \frac{(E_p)_{nc} - E_k}{|E_{co}|}, \quad (4-3)$$

where $|E_{co}|$ is the rated cutoff grid voltage for the tube. However, now let the ratio B' be

$$B' = \frac{(E_p)_{nc} - E_k}{|(E_g)_{nc} - E_k|}, \quad (4-8)$$

where $|(E_g)_{nc} - E_k|$ is the magnitude of the actual bias of an "OFF" tube, and where

$$|(E_g)_{nc} - E_k| \geq |E_{co}|$$

A study of the effect of the concentration of the solution on the rate of reaction was made. The results are shown in Table I. It is seen that the rate of reaction increases with increasing concentration of the solution. This is to be expected, since the rate of reaction is proportional to the concentration of the reactants.

The effect of the temperature on the rate of reaction was also studied. The results are shown in Table II. It is seen that the rate of reaction increases with increasing temperature. This is to be expected, since the rate of reaction is proportional to the temperature.

The effect of the catalyst on the rate of reaction was also studied. The results are shown in Table III. It is seen that the rate of reaction increases with increasing concentration of the catalyst. This is to be expected, since the rate of reaction is proportional to the concentration of the catalyst.

The effect of the solvent on the rate of reaction was also studied. The results are shown in Table IV. It is seen that the rate of reaction increases with increasing concentration of the solvent. This is to be expected, since the rate of reaction is proportional to the concentration of the solvent.

The effect of the pressure on the rate of reaction was also studied. The results are shown in Table V. It is seen that the rate of reaction increases with increasing pressure. This is to be expected, since the rate of reaction is proportional to the pressure.

as specified by equation (4-1). Then, using equations (3-2), (3-5), and (3-10) the following may be written:

$$E_k = a E_{bb} , \quad (4-9)$$

where

$$a = \frac{R_k}{R_k + R_p + r_b} ;$$

$$(E_p)_{nc} = E_{bb} ; \quad (4-10)$$

and

$$(E_g)_{nc} = \beta E_{bb} , \quad (4-11)$$

where

$$\beta = \frac{R_2 \left(\frac{R_k + r_b}{R_p + R_k + r_b} + N - 2 \right)}{R_1 + R_2(N - 1)} .$$

Substituting equations (4-9), (4-10), and (4-11) into equation (4-8),

$$B' = \frac{1 - a}{|\beta - a|} . \quad (4-12)$$

Equation (4-12) states that if

$$|(E_g)_{nc} - E_k| \geq |E_{co}| ,$$

as given in (1-10) and (1-11). Then, using (1-10) and (1-11), we have

Let H be a Hilbert space and let T be a linear operator on H . Then, we have

where $\| \cdot \|$ is the norm on H .

Let $\{e_n\}_{n=1}^{\infty}$ be an orthonormal basis for H . Then, we have

where $\langle \cdot, \cdot \rangle$ is the inner product on H .

Let $\{f_n\}_{n=1}^{\infty}$ be a sequence of functions in $L^2(\mathbb{R})$. Then, we have

where $\| \cdot \|_2$ is the L^2 norm.

Let $\{g_n\}_{n=1}^{\infty}$ be a sequence of functions in $L^2(\mathbb{R})$. Then, we have

where $\| \cdot \|_2$ is the L^2 norm.

Let $\{h_n\}_{n=1}^{\infty}$ be a sequence of functions in $L^2(\mathbb{R})$. Then, we have

where $\| \cdot \|_2$ is the L^2 norm.

Let $\{i_n\}_{n=1}^{\infty}$ be a sequence of functions in $L^2(\mathbb{R})$. Then, we have

where $\| \cdot \|_2$ is the L^2 norm.

Let $\{j_n\}_{n=1}^{\infty}$ be a sequence of functions in $L^2(\mathbb{R})$. Then, we have

where $\| \cdot \|_2$ is the L^2 norm.

Let $\{k_n\}_{n=1}^{\infty}$ be a sequence of functions in $L^2(\mathbb{R})$. Then, we have

where $\| \cdot \|_2$ is the L^2 norm.

Let $\{l_n\}_{n=1}^{\infty}$ be a sequence of functions in $L^2(\mathbb{R})$. Then, we have

where $\| \cdot \|_2$ is the L^2 norm.

Let $\{m_n\}_{n=1}^{\infty}$ be a sequence of functions in $L^2(\mathbb{R})$. Then, we have

where $\| \cdot \|_2$ is the L^2 norm.

Let $\{n_n\}_{n=1}^{\infty}$ be a sequence of functions in $L^2(\mathbb{R})$. Then, we have

where $\| \cdot \|_2$ is the L^2 norm.

Let $\{o_n\}_{n=1}^{\infty}$ be a sequence of functions in $L^2(\mathbb{R})$. Then, we have

where $\| \cdot \|_2$ is the L^2 norm.

Let $\{p_n\}_{n=1}^{\infty}$ be a sequence of functions in $L^2(\mathbb{R})$. Then, we have

where $\| \cdot \|_2$ is the L^2 norm.

Let $\{q_n\}_{n=1}^{\infty}$ be a sequence of functions in $L^2(\mathbb{R})$. Then, we have

where $\| \cdot \|_2$ is the L^2 norm.

and if B is a constant for a given tube type, then the reliability of the "OFF" tube is independent of supply voltage variations.

Now considering the "ON" tube, equation (4-6) may be rewritten as

$$(E_g)_c - E_k \geq 0 \quad . \quad (4-13)$$

Using equation (3-4) the following may be written:

$$(E_g)_c = \gamma E_{bb} \quad , \quad (4-14)$$

where

$$\gamma = \frac{R_2}{\frac{R_1}{N-1} + R_2} \quad .$$

Substituting equations (4-14) and (4-9) into equation (4-13),

$$E_{bb}(\gamma - a) \geq 0 \quad . \quad (4-15)$$

Equation (4-15) states that when γ equals a the reliability of the "ON" tube will also be independent of supply voltage variations. However, in a practical circuit, if γ (or $(E_g)_c$) tends to be larger than a (or E_k), then the grid-to-cathode "diode" of the tube clamps γ (or $(E_g)_c$) to a value equal to a (or E_k). Therefore, in a practical circuit, γ is a non-linear function and is always equal to a provided that

$$(E_g)_c \geq E_k \quad .$$

The above proofs have shown that if an nthnary employs a cathode resistor for self-bias, and if the nthnary conforms to the inequalities expressed in equations (4-1) and (4-6), then the nthnary is independent of supply voltage variations.

4.3.2 Definition of Component Tolerances. The components in the nthnary in which variations may be expected are resistors and tubes. The maximum expected variation from nominal, or the tolerance, of these components may be expressed as follows:

- (a) the resistor tolerance, expressed as a fraction of the nominal value, is $\pm\Delta r$, where Δr is a positive number; and
- (b) the tube's static plate resistance tolerance, expressed as a fraction of the nominal value, is $\pm\Delta r_b$, where Δr_b is a positive number.

4.3.3 Resistor and Tube Variations -- "OFF" Tubes. The inequality expressed in equation (4-5) is the criterion for operation of the "OFF" tubes in an nthnary. Equation (4-5) can be adjusted to be a criterion for reliable nthnary operation if tolerances are added minimizing the quantity on the left and maximizing the quantity on the right.

It is obvious that if R_p and r_b are placed at their positive tolerance limit, the quantity on the left will be made smaller. It is also obvious that if R_2 is placed at its positive tolerance limit and R_1 placed at its negative tolerance limit, the quantity on the right will be maximized. The tolerance limit which should be assigned to R_k to minimize

The above results are shown in Table I. The results are given for the case of a constant α and for the case of a variable α . The results are given for the case of a constant α and for the case of a variable α .

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the quantity on the left is not obvious. Therefore the quantity on the left will be differentiated with respect to R_k ,

$$\frac{\partial}{\partial R_k} \left[\frac{R_k - \frac{R_p + r_b}{B}}{R_k + \left(\frac{N-2}{N-1}\right)R_p + r_b} \right] = \frac{\left(R_k + \left(\frac{N-2}{N-1}\right)R_p + r_b\right) - \left(R_k - \frac{R_p + r_b}{B}\right)}{\left(R_k + \left(\frac{N-2}{N-1}\right)R_p + r_b\right)^2},$$

$$= \frac{\left(\frac{N-2}{N-1}\right)R_p + r_b + \frac{R_p + r_b}{B}}{\left(R_k + \left(\frac{N-2}{N-1}\right)R_p + r_b\right)^2}.$$

The algebraic sign of the derivative is positive, therefore, the quantity on the left in equation (4-5) will change magnitude in the same direction as changes in R_k . Since it is necessary to minimize the quantity on the left, the resistor R_k shall be placed at its negative tolerance limit. Equation (4-5) may then be adjusted for the most adverse tolerance build-up as shown in equation (4-16).

$$\frac{R_k(1 - \Delta r) - \frac{R_p(1 + \Delta r) + r_b(1 + \Delta r_b)}{B}}{R_k(1 - \Delta r) + \left(\frac{N-2}{N-1}\right)R_p(1 + \Delta r) + r_b(1 + \Delta r_b)} \geq \frac{\frac{R_2(1 + \Delta r)}{R_1(1 - \Delta r)}}{\frac{1}{N-1} + \frac{R_2(1 + \Delta r)}{R_1(1 - \Delta r)}}.$$

$$\frac{R_k(1 - \Delta r) - \frac{R_p(1 + \Delta r) + r_b(1 + \Delta r_b)}{B}}{R_k(1 - \Delta r) + \left(\frac{N-2}{N-1}\right)R_p(1 + \Delta r) + r_b(1 + \Delta r_b)} \geq \frac{\frac{R_2}{R_1}}{\frac{1 - 2\Delta r}{N-1} + \frac{R_2}{R_1}}. \quad (4-16)$$

The function $f(\text{"OFF"})$ is defined as,

$$f(\text{"OFF"}) = \frac{R_k(1 - \Delta r) - \frac{R_p(1 + \Delta r) + r_b(1 + \Delta r_b)}{B}}{R_k(1 - \Delta r) + \left(\frac{N - 2}{N - 1}\right) R_p(1 + \Delta r) + r_b(1 + \Delta r_b)} \quad (4-17)$$

Also, the maximum nominal value of the ratio $\frac{R_2}{R_1}$ that will satisfy equation (4-16) is defined as $\left[\frac{R_2}{R_1}\right]_{\max}$. Then, using the above two definitions, equation (4-16) may be arranged into a form more convenient for practical computation.

$$\left[\frac{R_2}{R_1}\right]_{\max} = \frac{f(\text{"OFF"}) \left(\frac{1 - 2\Delta r}{N - 1}\right)}{1 - f(\text{"OFF"})} \quad (4-18)$$

4.3.4 Resistor and Tube Variations -- "ON" Tube. The inequality expressed in equation (4-7) is the criterion for operation of the "ON" tube in an nthnary. This equation can be adjusted to be a criterion for reliable operation if tolerances are added minimizing the quantity on the left and maximizing the quantity on the right.

It is obvious that the quantity on the left will be minimized if R_2 is placed at its negative tolerance limit and R_1 is placed at its positive tolerance limit. Also, the quantity on the right will be maximized if R_k is placed at its positive tolerance limit and R_p and r_b are

placed at their negative tolerance limits. Then equation (4-7) may be rewritten and arranged as follows:

$$\frac{\frac{R_2(1 - \Delta r)}{R_1(1 + \Delta r)}}{\frac{1}{N - 1} + \frac{R_2(1 - \Delta r)}{R_1(1 + \Delta r)}} \geq \frac{R_k(1 + \Delta r)}{R_k(1 + \Delta r) + R_p(1 - \Delta r) + r_b(1 - \Delta r_b)} ,$$

$$\frac{\frac{R_2}{R_1}}{\frac{1 + 2\Delta r}{N - 1} + \frac{R_2}{R_1}} \geq \frac{R_k(1 + \Delta r)}{R_k(1 + \Delta r) + R_p(1 - \Delta r) + r_b(1 - \Delta r_b)} . \quad (4-19)$$

Then, by using a process similar to that which we used in the case of the "OFF" tubes, $f(\text{"ON"})$ is defined as,

$$f(\text{"ON"}) = \frac{R_k(1 + \Delta r)}{R_k(1 + \Delta r) + R_p(1 - \Delta r) + r_b(1 - \Delta r_b)} , \quad (4-20)$$

and $\left[\frac{R_2}{R_1} \right]_{\min}$ is defined as the minimum nominal value of the ratio $\frac{R_2}{R_1}$ that will satisfy equation (4-19). To simplify equation (4-19) we write,

$$\left[\frac{R_2}{R_1} \right]_{\min} = \frac{f(\text{"ON"}) \left(\frac{1 + 2\Delta r}{N - 1} \right)}{1 - f(\text{"ON"})} . \quad (4-21)$$

4.3.5 The Approximation. The approximation expressed in equation (3-1) was used in deriving all the above work. However, since tolerances have been added to all the resistance values, it is necessary also to adjust equation (3-1) as follows:

The first of these is the fact that the
 

$$R_p(1 + \Delta r) \ll \frac{R_1(1 - \Delta r)}{N - 1} ,$$

$$R_p(1 + 2\Delta r)(N - 1) \ll R_1 . \quad (4-22)$$

4.4 Summary

The DC design of a reliable nthary is expressed by the simultaneous solution of the following criteria:

$$\left[\frac{R_2}{R_1} \right]_{\max} = \frac{f(\text{"OFF"}) \left(\frac{1 + 2\Delta r}{N - 1} \right)}{1 - f(\text{"OFF"})} , \quad (4-18)$$

and

$$\left[\frac{R_2}{R_1} \right]_{\min} = \frac{f(\text{"ON"}) \left(\frac{1 + 2\Delta r}{N - 1} \right)}{1 - f(\text{"ON"})} ; \quad (4-21)$$

where

$$f(\text{"OFF"}) = \frac{R_k(1 - \Delta r) - \frac{R_p(1 + \Delta r) + r_b(1 + \Delta r_b)}{B}}{R_k(1 - \Delta r) + \left(\frac{N - 2}{N - 1} \right) R_p(1 + \Delta r) + r_b(1 + \Delta r_b)} , \quad (4-17)$$

$$f(\text{"ON"}) = \frac{R_k(1 + \Delta r)}{R_k(1 + \Delta r) + R_p(1 - \Delta r) + r_b(1 - \Delta r_b)} ; \quad (4-20)$$

and where $\left[\frac{R_2}{R_1} \right]_{\max}$ and $\left[\frac{R_2}{R_1} \right]_{\min}$ are the maximum and minimum nominal ratios of the plate-to-grid divider resistors. The actual nominal value of the ratio must be, of course, somewhere in the range between the maximum and the minimum.

The above criteria were derived based on the assumption that

$$R_p(1 + 2\Delta r)(N - 1) \ll R_L . \quad (4-22)$$

The quantities Δr and Δr_b are fractions that express the tolerances expected in practical resistors and tubes, respectively. No tolerance fraction has been provided for the supply voltage because it has been proven that the reliability of the nthnary is independent of supply voltage changes if a cathode resistor is used for self-bias.

CHAPTER V

DESIGN CONSIDERATIONS

The simultaneous solution of equations (4-16) and (4-19) describes an nthnary that will operate reliably when resistor and tube variations are within their predicted tolerance range. However, the concept of design of reliable circuits embraces more than a consideration of component tolerance. It must also consider that the nominal design values of components can sometimes be chosen so as to have a real effect on circuit reliability. Therefore, in this chapter we shall examine the nthnary design criteria for relations which indicate how we should choose the nominal values of R_k , R_p , r_b , B and N so as to enhance nthnary reliability. Also discussed in this chapter are the effects of grid current and the tolerance limits which may be expected of practical resistors and tubes.

5.1 A Simultaneous Solution

The criteria for proper nthnary operation are stated for the nominal case in equations (4-5) and (4-7). By combining these two equations, we have

$$\frac{R_k - \frac{R_p + r_b}{B}}{R_k + \left(\frac{N-2}{N-1}\right) R_p + r_b} \geq \frac{R_k}{R_k + R_p + r_b} \quad (5-1)$$

THEORY OF THE

 

Equation (5-1) may be rearranged as follows,

$$R_k^2 + R_p R_k \left(1 - \frac{1}{B}\right) + R_k r_b \left(1 - \frac{1}{B}\right) - \frac{1}{B}(R_p + r_b)^2 \geq R_k^2 \\ + \left(\frac{N-2}{N-1}\right) R_p R_k + r_b R_k .$$

Most common tube types used in nthnary applications have values of B sufficiently large that we may write

$$1 \gg \frac{1}{B} ,$$

therefore,

$$R_k^2 + R_p R_k + R_k r_b - \frac{1}{B}(R_p + r_b)^2 \geq R_k^2 + \left(\frac{N-2}{N-1}\right) R_p R_k + R_k r_b .$$

If the function

$$R_k^2 + \left(\frac{N-2}{N-1}\right) R_p R_k + R_k r_b$$

is subtracted from both sides of this inequality, we will have the difference, δ , where

$$\delta = R_p R_k \left(\frac{1}{N-1}\right) - \frac{1}{B}(R_p + r_b)^2 \geq 0 . \quad (5-2)$$

If we examine equation (5-2), we may reason that if the function on the left side were equal to zero, then an nthnary could be mechanized only if the components used had a plus or minus zero tolerance. If the

inequality expressed in equation (5-2) was made large, then a circuit could be mechanized with components having a relatively large tolerance. Therefore, we may further reason that nominal values of R_k , R_p , r_b , B and N should be chosen so as to maximize the difference, δ , expressed in equation (5-2).

Actually, the difference, δ , is a relation of three somewhat independent functions. These three functions are:

- (a) application -- the design requirements of each application determine the value, or order, of N required;
- (b) tube type -- the values of r_b and B are functions of the tube type that the circuit designer chooses; and
- (c) tube load -- the tube load consists of its plate resistor, R_p , and its cathode resistor, R_k .

We shall now examine the effect of each of these functions on the magnitude of the difference, δ .

5.1.1 The Application. It is readily apparent by inspection of equation (5-2), that, with everything else constant, as the order of N is increased, the magnitude of the difference, δ , will decrease. Therefore, the reliability of the nthary varies inversely as the order of N .

5.1.2 The Tube Type. Equation (5-2) may be rewritten as

$$\delta = R_p R_k \left(\frac{1}{N - 1} \right) - \frac{r_b^2}{B} \left(\frac{R_p}{r_b} + 1 \right)^2 \geq 0 . \quad (5-3)$$

The ratio

$$\frac{r_b^2}{B}$$

in equation (5-3) is a function of the tube type the designer chooses. Tubes with a small static plate resistance and a large value for the ratio B will tend to make the difference, δ , large, hence will tend to increase nthnary reliability. It is therefore possible to define a figure of merit, F, that will aid designers in their choice of a tube type for nthnary applications.

Let

$$F = \frac{B}{r_b^2} \times 10^6 , \quad (5-4)$$

where r_b is the tube's static plate resistance in ohms and B is as defined in equation (4-3). The ratio of B to r_b^2 in equation (5-4) is the inverse of the ratio in equation (5-3) for the arbitrary reason that the author would rather have the larger values of F indicate the better tubes for nthnary use.

$$(1.1) \quad \left(\frac{1}{2} \right)^2 = \frac{1}{4}$$

The figure of merit, F , for several tube types has been listed in Table I. Data for the computation of F were obtained in each case from the tube characteristics published in a tube manual.¹ As for example, for the tube type GL-5670, the tube manual states that:²

Plate Current Cutoff (1), Each Section	<u>Minimum</u>	<u>Maximum</u>
$E_f = 6.3$ volts, $E_b = 150$ volts		
$E_c = -10$ volts, $R_L = 0.25$ meg		
Initial	- - - -	45 micro-amperes

Therefore,

$$B = \frac{150 \text{ volts}}{10 \text{ volts}} = 15 \text{ .}$$

The value of r_b was computed from the relation

$$r_b = \frac{E_{pk}}{I_b}$$

where E_{pk} is the plate voltage required for the tube to conduct 10* milliamperes static plate current (I_b) when the grid bias is zero volts.

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¹Tube Manual, Electronic Tubes, 5-Star Types, GL-5670, ET-T1083A, 8-56; GL-5751, ET-T1092, 8-54; 5814-A, ET-T1095A, 8-56; GL-6021, ET-T1101, 8-54; GL-6072, ET-T1102, 8-54; GL-6111, ET-T1104, 8-54; GL-6112, ET-T1105, 8-54; GL-6201, ET-T1110, 8-54; 6414, ET-T1317, 5-56; 6829, ET-T1250, 1-56, Tube Department, General Electric Company, Schenectady 5, N. Y.

²Ibid., GL-5670, ET-T1083A, 8-56.

*A value of I_b equals 10 ma is used unless otherwise noted in Table I.

TABLE I

COMPARISON OF THE FIGURE OF MERIT, F,
FOR VARIOUS TWIN TRIODE TUBES

Tube Type				$F = \frac{B}{r_b^2} \times 10^6$
Designation	Description	B	r_b , ohms	
GL-5670	Similar to 2C51	15.0	9,200	0.18
GL-5751	Similar to 12AX7	23.8	32,000 $I_b = 5 \text{ ma}$	0.02
5814-A	Similar to 12AU7	8.3	9,200	0.10
GL-6021	Subminiature	15.4	8,400	0.22
GL-6072	Similar to 12AY7	24.8	22,000 $I_b = 5 \text{ ma}$	0.05
GL-6111	Subminiature	11.1	6,400	0.27
GL-6112	Subminiature	35.8	27,000 $I_b = 2 \text{ ma}$	0.05
GL-6201	Similar to 12AT7	12.5	11,800	0.09
6414	Computer type	20.0	10,000	0.20
6829	Similar to 5965 Computer type	20.0	7,500	0.36

For the tube GL-5670, when

$$I_b = 10 \text{ ma}$$

$$E_c = 0 \text{ volts}$$

E_{pk} is read from the average plate characteristic to be 92 volts.³

Therefore,

$$r_b = \frac{92 \text{ volts}}{10 \text{ ma}} = 9,200 \text{ ohms} .$$

The figure of merit for the GL-5670 tube is then computed using equation (5-4).

$$F = \frac{B}{r_b^2} \times 10^6 = \frac{15}{(9.2)^2} = 0.18 .$$

Since figures of merit which are relatively large indicate tube types which tend to make nthnary circuits more reliable, it is apparent from Table I that the tube 6829 is the best choice of the ten tube types listed.*

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³Ibid.

* A word of caution should be expressed here regarding the values listed in Table I. The data for calculating B are presented in the tube manual under different test conditions for each different tube type. For the best cross-comparison the test conditions should be the same. However, until better data for cross-comparison are available, the author feels that the results are valid to the extent that they show that tubes designed specifically for switching applications (6414 and 6829) are significantly better for nthnary applications than are most ordinary "receiving" tube types.

For the 1950-51 season

1950-51

1951-52

1952-53

1953-54

1954-55

The amount of water for the 1950-51 season is shown in Table 1.

1955-56

1956-57

1957-58

1958-59

1959-60

1960-61

1961-62

1962-63

A series of studies should be expressed here regarding the

values listed in Table 1. The data for caloric value is presented

in the table under different test conditions for each different

type of fuel. The test conditions for each fuel are shown

in the table. However, the test conditions for cross-comparison are available

only for the fuel listed in the table. The test conditions for the other fuels

are shown in the table. The test conditions for the other fuels are shown

in the table. The test conditions for the other fuels are shown

in the table. The test conditions for the other fuels are shown

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in the table. The test conditions for the other fuels are shown

5.1.3 The Tube Load. The nominal value of the static plate resistance is usually determined graphically from the average plate characteristic published in a tube manual for the tube type. In order to make this graphical determination of r_b at any particular operating point, a load line for the tube load, R_L , is required, where

$$R_p + R_k = R_L \quad (5-5)$$

Using equation (5-5), we may rewrite equation (5-2) as follows:

$$\begin{aligned} \delta &= R_p(R_L - R_p) \left(\frac{B}{N-1} \right) - R_p^2 - 2R_p r_b - r_b^2 \geq 0 \\ \delta &= -R_p^2 \left(1 + \frac{B}{N-1} \right) + R_p \left[\left(\frac{R_L B}{N-1} \right) - 2r_b \right] - r_b^2 \geq 0 \quad (5-6) \end{aligned}$$

Then, by taking the first and second partial derivatives of the difference, δ , with respect to R_p , we have

$$\frac{\partial \delta}{\partial R_p} = -2 \left(1 + \frac{B}{N-1} \right) R_p + \frac{R_L B}{N-1} - 2r_b \quad ;$$

$$\frac{\partial^2 \delta}{\partial R_p^2} = -2 \left(1 + \frac{B}{N-1} \right) \quad .$$

The first partial derivative shows that the difference, δ , has a critical value at

$$R_p = \frac{R_L B - 2(N-1)r_b}{2(N-1+B)} \quad (5-7)$$

2.1. The first part of the proof is devoted to the case of a function f which is not identically zero. In this case we shall show that the function f is not identically zero on any interval (a, b) of the real line. To this end we shall assume that f is identically zero on some interval (a, b) and derive a contradiction.

Let f be a function which is not identically zero on any interval (a, b) of the real line. We shall show that f is not identically zero on any interval (a, b) of the real line. To this end we shall assume that f is identically zero on some interval (a, b) and derive a contradiction.

$$\begin{aligned}
 & \text{Let } f \text{ be a function which is not identically zero on any interval } (a, b) \text{ of the real line.} \\
 & \text{We shall show that } f \text{ is not identically zero on any interval } (a, b) \text{ of the real line.} \\
 & \text{To this end we shall assume that } f \text{ is identically zero on some interval } (a, b) \text{ and derive a contradiction.}
 \end{aligned}$$

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$$\begin{aligned}
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 & \text{We shall show that } f \text{ is not identically zero on any interval } (a, b) \text{ of the real line.} \\
 & \text{To this end we shall assume that } f \text{ is identically zero on some interval } (a, b) \text{ and derive a contradiction.}
 \end{aligned}$$

$$\begin{aligned}
 & \text{Let } f \text{ be a function which is not identically zero on any interval } (a, b) \text{ of the real line.} \\
 & \text{We shall show that } f \text{ is not identically zero on any interval } (a, b) \text{ of the real line.} \\
 & \text{To this end we shall assume that } f \text{ is identically zero on some interval } (a, b) \text{ and derive a contradiction.}
 \end{aligned}$$

The above partial derivative shows that the function f is not identically zero on any interval (a, b) of the real line. To this end we shall assume that f is identically zero on some interval (a, b) and derive a contradiction.

Let f be a function which is not identically zero on any interval (a, b) of the real line. We shall show that f is not identically zero on any interval (a, b) of the real line. To this end we shall assume that f is identically zero on some interval (a, b) and derive a contradiction.

The second partial derivative shows that the critical value is a maximum.

It is very often true in nthnary design that

$$R_L B \gg 2(N - 1)r_b ,$$

$$\text{and } B \gg (N - 1) .$$

If this is the case, equation (5-7) can be rewritten as

$$R_p \approx \frac{R_L}{2} .$$

It is, therefore, apparent that for maximum nthnary reliability, the nominal values of R_k and R_p should be chosen so that

$$R_k \approx R_p \approx \frac{R_L}{2} . \quad (5-8)$$

By inspection of equation (5-2), we may note that the difference, δ , varies directly as R_k . Then, since equation (5-8) states that R_k and R_p are to be approximately equal for maximum nthnary reliability, it must follow that the difference, δ , also varies directly as R_L .

5.2 Effects of Grid Current

The value of the static plate resistance of a tube at "zero bias" varies considerably with the value of the current conducted through the grid of the tube. This variation is great enough so that it must be considered at the time of nthnary design.

Figure 6 shows the plate characteristic of the 6829 for various values of grid current and zero-volt bias.⁴ Using the 40,000-ohm load line shown, it may be easily computed and seen that the nominal static plate resistance may vary between 5,900 ohms and 11,500 ohms, depending upon the magnitude of the grid current.

However, when the design of an nthnary is started, there is not enough information with which to compute the grid current precisely. Therefore, this initial phase of the design process must begin with an educated guess and proceed by trial and error. After a value of grid current has been assumed, the nthnary design can proceed. When the design has been completed, the value of the grid current must be calculated to ascertain if it corresponds with the assumed value. It is up to the designer to judge each practical case and pursue the result to the degree of precision he feels necessary.

After an nthnary design has been completed, the magnitude of the grid current, I_g , in the "ON" tube may be computed using equation (5-10) which is derived as follows:

Referring to Figure 4, the Thevenin equivalent circuit at the grid of the "ON" tube, to the left of point E, is

$$(E_g)_c = E_{bb} \frac{R_2}{\frac{R_1}{N-1} + R_2}, \quad (3-4)$$

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⁴Ibid., 6829, ET-T1250, 1-56.

FIGURE 6.- PLATE CHARACTERISTIC FOR TUBE TYPE 6829

$$\overleftarrow{R_E} = \frac{R_2 \frac{R_1}{N-1}}{R_2 + \frac{R_1}{N-1}} . \quad (5-9)$$

An equivalent circuit may be drawn as illustrated in Figure 7 where $(E_g)_c$ and $\overleftarrow{R_E}$ are as defined in equations (3-4) and (5-9), where E_k is the tube's cathode voltage as defined in equation (3-2), and where the diode is the equivalent grid-to-cathode "diode" of the "ON" tube. If,

$$(E_g)_c \leq E_k ,$$

then

$$I_g = 0 ;$$

$$(E_g)_c > E_k ,$$

then

$$I_g = \frac{(E_g)_c - E_k}{\overleftarrow{R_E}} . \quad (5-10)$$

5.3 Practical Resistor Tolerance

In any study of reliable circuit design, some mention should be made of the practical tolerances available for the components which the circuit employs. The design engineer must also bear in mind that there may be a considerable difference between a rated tolerance and an actual tolerance.

As, for example, carbon or fixed composition resistors with a rated tolerance of 5 per cent are guaranteed to this rated tolerance only at room temperature and at the time of shipment from the factory

FIGURE 7.- EQUIVALENT CIRCUIT USED FOR
COMPUTING GRID CURRENT

GRID-TO-CATHODE
DIODE OF TUBE

FIGURE 3. EQUIVALENT CIRCUIT USED FOR
COMPUTING GRID CURRENT

unless special packaging techniques are used.⁵ A maximum tolerance of a 5-per cent carbon composition resistor will include the effects of handling and storage, soldering, voltage, temperature and humidity. F. A. Paul⁶ has made a study of these effects on various types of commercial resistors, the results of which are shown in Table II. The maximum tolerances shown in Table II reflect the additive effect of rating, handling and storage, and the environments. They are not root-mean-square sums and, therefore, the cautious design engineer need not feel that he must impose an additional safety factor. In fact, Paul states that the maximum tolerance includes about a 50-per cent safety factor for the "average" circuit.

5.4 Practical Tolerances for the r_b of Tubes

A study of tolerances which may be assigned to any vacuum tube characteristic is always a statistical study. The many variables involved preclude the straightforward approach that can be taken with passive components like resistors. Figure 8 illustrates how a statistical median control is exercised over tube quality. "If the median of a sample lot falls outside the range in which the average of individual production lots is allowed to vary on any specified characteristic, the entire lot is rejected."⁷

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⁵Resistors, Fixed, Composition (Insulated), Military Specification, MIL-R-11B, 9 March 1956, U. S. Government Printing Office.

⁶F. A. Paul, "Factors Affecting Resistor Tolerances," Electronic Design, Vol. 4, No. 11, April 1, 1956, pp. 36-37.

⁷Brochure, No. ETD-548C, Five-Star Tubes, High-Reliability Receiving Types, Tube Department, General Electric Company, Schenectady 5, N. Y.

TABLE II

COMPARISON OF TOLERANCES FOR VARIOUS RESISTORS

Resistor Type	Rated Tolerance (per cent)	Maximum Tolerance (per cent)
Carbon composition	5	20
Carbon composition	20	35
Deposited carbon	1	4
Metal film	1	5
Wirewound	1	1.25
Wirewound	0.1	0.35

FIGURE 8.- ILLUSTRATION OF MEDIAN CONTROL OF VACUUM TUBE CHARACTERISTIC

FIGURE 8.1. ADJUSTMENT OF MEDIAN CONTROL
 VALUE OF CHARACTERISTIC UNDER TEST

Two tube types which are commonly used in nthnary circuits are the 2C51 and the 5814. As an example of how median control is employed with these tubes, pertinent limits from the MIL-E-1 tube specification⁸ are listed in Table III.

An article by The Applications Engineers of the Advisory Group on Electron Tubes contains several frequency distribution curves for some common tube types at various intervals of aging.⁹

Most tube specifications do not define the tolerance limits of the static plate resistance directly; rather, they define a maximum and a minimum value of the static plate current for a given operating condition. Table III illustrates this specification for the 5814WA and the 5670. From this maximum and minimum value of static plate current, I_b , we may compute the tolerance of r_b at the given operating condition as shown in the following derivation:

$$r_b(1 \pm \Delta r_b) = \frac{I_b(1 \pm \Delta I_b)}{E_{pk}},$$

$$r_b \pm r_b \Delta r_b = \frac{I_b}{E_{pk}} \pm \frac{I_b \Delta I_b}{E_{pk}},$$

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⁸Electron Tubes and Crystal Rectifiers, Military Specification, MIL-E-1C, 3 October 1955, U. S. Government Printing Office.

⁹Applications Engineers of the Advisory Group on Electron Tubes, The, "Tube Characteristic Changes and Equipment Reliability," Tele-Tech & Electronic Industries, Vol. 15, No. 6, June 1956, pp. 96-97 & 346-348.

TABLE III

EXAMPLES OF MEDIAN CONTROL SPECIFICATIONS
FOR THE 5814WA AND THE 5670 TUBE TYPES

Tube Type	Quantity	Min.	Lower Acceptable Lot Median	Bogey	Upper Acceptable Lot Median	Max.
5814WA*	I_b (ma)	6.5	9.0	10.5	12.0	14.5
	g_m	1750	2000	2200	2400	2650
5670**	I_b (ma)	5.9	7.0	8.2	9.4	10.5
	g_m	4500	4880	5500	6120	6500

*Per MIL-E-1/238A, 3 May 1954, Test Conditions:
 $E_{ff} = 12.6$ volts AC, $E_{pk} = 250$ volts DC, $E_c = -8.5$ volts DC.

**Per MIL-E-1/5, 13 January 1953, Test Conditions:
 $E_{ff} = 6.3$ volts, $E_{pk} = 150$ volts DC, $R_k = 240$ ohms.

STATEMENT OF INVESTMENT OPERATIONS
FOR THE YEAR AND THE FISCAL YEAR

Description of Investment		Cost	Market Value	Gain or Loss
U.S. Government Bonds				
1. U.S. Treasury Bonds	100,000	100,000	105,000	5,000
2. U.S. Treasury Notes	50,000	50,000	52,000	2,000
3. U.S. Treasury Bills	25,000	25,000	25,000	0
Corporate Bonds				
4. General Electric	10,000	10,000	11,000	1,000
5. IBM	10,000	10,000	10,000	0
6. Ford Motor	10,000	10,000	9,000	(1,000)
7. Coca-Cola	10,000	10,000	10,000	0
8. American Express	10,000	10,000	11,000	1,000
9. United States Steel	10,000	10,000	9,000	(1,000)
10. General Motors	10,000	10,000	10,000	0
Common Stocks				
11. IBM	10,000	10,000	11,000	1,000
12. General Electric	10,000	10,000	10,000	0
13. Ford Motor	10,000	10,000	9,000	(1,000)
14. Coca-Cola	10,000	10,000	10,000	0
15. American Express	10,000	10,000	11,000	1,000
16. United States Steel	10,000	10,000	9,000	(1,000)
17. General Motors	10,000	10,000	10,000	0
Total		300,000	310,000	10,000

Investment in U.S. Government Bonds

Investment in Corporate Bonds

Investment in Common Stocks

where E_{pk} is the plate-to-cathode voltage given in the operating condition. Therefore,

$$r_b = \frac{I_b}{E_{pk}}, \text{ and } \pm r_b \Delta r_b = \pm \frac{I_b \Delta I_b}{E_{pk}}.$$

The tolerance is then

$$\pm \Delta r_b = \pm \Delta I_b ;$$

but,

$$\pm \Delta I_b = + \frac{I_b]_{\max} - I_b}{I_b} - \frac{I_b - I_b]_{\min}}{I_b} ;$$

finally,

$$\pm \Delta r_b = + \frac{I_b]_{\max} - I_b}{I_b} - \frac{I_b - I_b]_{\min}}{I_b} . \quad (5-11)$$

Equation (5-11) was employed to compute Δr_b for the several tube types¹¹ listed in Table IV. We may then assume that for each tube type, the tabulated value of Δr_b would be a reasonable tolerance for some operating point other than the one specified.

---

¹¹General Electric Tube Manual, 5-Star Types, loc. cit.

where μ is the mean of the random variable X and σ^2 is the variance of X .

The probability density function of X is given by

$$f(x) = \frac{1}{\sigma \sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

where μ is the mean and σ^2 is the variance.

The standard deviation of X is given by

$$\sigma = \sqrt{\sigma^2}$$

where σ^2 is the variance of X .

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where σ^2 is the variance of X .

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where σ^2 is the variance of X .

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$$\sigma = \sqrt{\sigma^2}$$

TABLE IV

VALUES OF Δr_b FOR VARIOUS TRIODE TUBE TYPES*

Tube Type	$\pm \Delta r_b$	Operating Conditions				
		E_f volts	E_b volts	R_k ohms (bypassed)	E_c volts	I_c μa
GL-5670	0.28	6.3	150	240	--	--
GL-5751	0.63	12.6	250	--	-3.0	--
5814-A	0.38	12.6	250	--	-8.5	--
GL-6021	0.31	6.3	100	150	--	--
GL-6072	0.36	12.6	250	--	-4.0	--
GL-6111	0.29	6.3	100	220	--	--
GL-6112	0.38	6.3	100	1500	--	--
GL-6201	0.33	12.6	250	200	--	--
6414	0.20	6.3	100	--	0	200
6829	0.29	6.3	100	--	0	200

*Computed with equation (5-11),

$$\pm \Delta r_b = + \frac{I_b \big]_{\max} - I_b}{I_b} - \frac{I_b - I_b \big]_{\min}}{I_b} .$$

CHAPTER VI

CONCLUSIONS

It was the author's intention that this paper analyze a particular multistable circuit of order N , herein called the nthnary, with two purposes in mind. First, it was intended that DC design criteria would be developed which could be used to design functionally adequate nthnaries. Second, it was intended that the design criteria take into account the expected tolerances of resistors and tubes. This second purpose was supposed to make any nthnary thus designed more reliable because the effect of "gradual failure" would be minimized.

It has been quite well known for several years that nthnaries could be built and that they will work. Therefore, the author will limit his concluding remarks to those aspects of this paper that pertain to the reliability of nthnary operation.

6.1 Criticism of the Technique of Analysis

The simplest binary circuit that employs self-bias is composed of two triode tubes, two plate resistors, four resistors in the plate-to-grid dividers, and a cathode resistor. There is, then, a total of nine components required to mechanize the simplest nthnary. Each of these nine components has associated with it a tolerance that defines a maximum variance from a nominal value. The technique used to analyze the nthnary in this paper considers that when each component tolerance is in its most unfavorable extreme the nthnary must still be able to function.

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This condition of having all tolerances in their most unfavorable extreme is called maximum tolerance build-up and is very rarely encountered in a practical situation. The greater the number of components to be considered, the more improbable is the condition of maximum tolerance build-up if the variances occur at random. Even with such a small number of components as the nine that make up the simplest nthnary, the probability of maximum tolerance build-up is remote. In this light, it might appear as if the nthnary design criteria should be employed to reflect the most probable tolerances that will occur in the components.

On the other hand, a design engineer who assumes that all the components to be used in his design will be within the expected tolerance range is either naive, or has an excellent quality control organization at his disposal. In this light, it would be best to design to the maximum tolerance build-up (not because it will be expected to occur often enough to make the effort worthwhile), but because this tends to minimize the effects of the relatively more frequent occurrence of components that are out of tolerance.

It is not an easy chore, by any means, to choose resistors with an expected tolerance of ± 20 per cent and tubes that have an expected tolerance of ± 30 per cent on their static plate resistance, and then design a circuit for these components that will operate under the conditions of maximum tolerance build-up. It is rather for the circuit designer to carefully weigh the limitations of the available components against their requirements and expected environments and to make a judicious compromise between reality and reliability.

6.2 Optimum Tube Load, R_L

Regarding the tube load, R_L , which is the sum of the plate resistor and the cathode resistor, three very significant factors affecting circuit reliability have been demonstrated.

First, it was proven in Chapter IV that, if a cathode resistor is used for self-bias, the nthary is independent of supply voltage changes. The reader can review this proof and demonstrate to himself that this independence is not true if a second voltage supply is used for a bias source. If fixed bias were employed, the nthary design criteria would most certainly have to contain factors for the tolerance expected of both supplies. Therefore, the author strongly recommends that, if nthary reliability is an important factor, a cathode resistor be used for self-bias. Experimentally, this independence of supply voltage changes was demonstrated with the binary design contained in the Appendix. This binary remained bistable over the supply voltage range from approximately 70 volts to something greater than 500 volts.

The second significant conclusion regarding the tube load pertains to the optimum ratio of the plate resistor to the cathode resistor. For any given circuit design, with any given value of R_L , the circuit is most reliable when the cathode resistor is approximately equal to the plate resistor. The specific relation is given by equations (5-5) and (5-7).

The third significant conclusion is that, with all other values constant, the reliability of the nthary increases as the magnitude of R_L increases. This conclusion has its greatest meaning to the designer who requires a high triggering rate from his nthary. A high triggering rate

requires a low value of R_p (and, therefore, R_L). This situation is much akin to the gain-bandwidth problems encountered with the conventional amplifier. In this case, gain is directly analagous to stability, or reliability, and bandwidth is analogous to maximum triggering rate.

6.3 Reliability versus the Value of N

The Appendix of this paper contains examples of the design of a binary, ternary, and a quaternary. The same tube type and the same R_L were used as initial conditions for each design. Table V compares the results of these designs. If the binary is to be mechanized according to the given conditions, the maximum allowable resistor tolerance must be 10 per cent. The comparable maximum allowable resistor tolerance for the ternary design is ± 1 per cent. The tube's static plate resistance tolerance is ± 30 per cent for both the binary and the ternary. However, if the quaternary design is to conform to the design criteria, and the maximum resistor tolerance is ± 1 per cent, the tolerance on r_p must be less than ± 15 per cent.

Qualitatively, it was shown in Chapter V that the reliability of the nthnary decreased as the value of N increased. Quantitatively, the computations in the Appendix show that the use of ± 1 per cent resistors, plus selection of tubes, are going to be required if reliable nthnaries of order greater than three are to be designed.

Based on the analysis in this paper, and based on his experience, the author has concluded that:

- (a) binaries, ($N = 2$), can be designed quite easily and can be made very reliable;

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TABLE V

COMPARISON OF BINARY, TERNARY AND QUATERNARY DESIGNS*

Common Conditions: $R_L = 40,000$ ohms

tube type = 6829

 $r_b = 7,500$ ohms $B = 20$

N	R_k , ohms	R_p , ohms	Δr , %	Δr_b , %
2	20,000	20,000	10	30
3	22,000	18,000	1	30
4	24,000	16,000	1	< 15

*Data from Appendix

STATEMENT OF REVENUE, EXPENSE AND RESERVE

For the year ending 1900

Prepared by the Board of Directors

Page 1 of 1

Item	Amount	Percentage	Total
Revenue	100.00	100.00%	100.00
Expenses	25.00	25.00%	25.00
Reserve	75.00	75.00%	75.00
Total	100.00	100.00%	100.00

- (b) ternaries, ($N = 3$), can be designed which are reasonably reliable, but not without a significant amount of care, and not without expensive precision resistors; and
- (c) quaternaries, ($N = 4$), and circuits of higher order of N should be avoided if reliability is a consideration.

If it is required in a particular application to have a circuit with four or more stable states, then the author recommends that binaries and/or ternaries be cascaded and pulse feedback be employed, to give the desired number of stable states. Figure 9 illustrates, with block diagrams, several ways stable states greater than three may be obtained by cascading binaries and/or ternaries.

6.4 Fabrication Details

The author stated in Chapter I that if reliable operation is a specific design intent, then careful attention must be focused on fabrication details as well as the theory of operation and the choice of components. A complete discussion of fabrication details is beyond the scope of this paper, but the author feels that a few concluding remarks in this direction would be in order since by proper circuit fabrication the expected tolerances of components can be significantly reduced.

Regarding the vacuum tube, more uniform characteristics and a longer useful life can be expected if:

- (a) the maximum bulb temperature is kept as low as possible;
- (b) the heater voltage is regulated within one or two per cent of the rated value;

 

FIGURE 9.- METHODS OF OBTAINING STABLE STATES GREATER THAN THREE BY CASCADING BINARIES AND/OR TERNARIES

FIGURE 9. - METHOD OF OBTAINING STABLE STATES GREATER THAN THREE BY CASCADING BINARIES AND/OR TERNARIES

- (c) the heater current during the warm-up period never exceeds the rated operating heater current; and
- (d) vibration and shock isolators are employed to protect the tube mechanically.

Regarding resistors, more uniform characteristics and a longer useful life can be expected if:

- (a) the resistors are cooled by either placing them on a heat sink or by moving cool air around them; and
- (b) a nonhygroscopic varnish or coating is placed on the resistor body to seal out moisture and corrosive atmospheres.

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APPENDIX

 

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APPENDIX

EXAMPLES OF NITHNARY DESIGN

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A.3 The Ternary, $N = 3$	76
A.4 The Quaternary, $N = 4$	81

APPENDIX

EXAMPLES OF NTHNARY DESIGN

The purpose of this Appendix is to illustrate how nthnaries can be designed using the material that has been formulated in the previous chapters.

A.1 The Tube and Its Load

Table I shows that the best tube, of those types listed, for nthnary applications is the 6829. This tube type will be used in all the following examples.

Regarding the nthnaries to be designed, let us assign the following quantities the values indicated in Table VI.

Using the 40,000-ohm load line drawn in Figure 6 we shall, as an estimate, assume that the grid current, I_g , will be 0.2 ma. Then, from Figure 6,

$$r_b = 7500 \text{ ohms} .$$

A.2 The Binary, $N = 2$

Using the values assigned in Section A.1, we calculate from equation (5-7)

$$\begin{aligned} R_p &= \frac{R_L B - 2(N - 1)r_b}{2(N - 1 + B)} = \frac{(40)(20) - 2(7.5)}{2(20)} \times 10^3 \\ &= 19.6K \approx 20k \text{ ohms} , \end{aligned}$$

TABLE VI

VALUES ASSIGNED FOR EXAMPLE
NTHNARY DESIGNS

Quantity	Value	Reason
E_{bb}	300 volts	assumed
R_L	40,000 ohms	assumed
B	20	from Table I
Δr	0.10	assumed
Δr_b	0.30	from Table IV

TABLE VI

VALUES ASSIGNED FOR EXAMPLES
STANDARD DEVIATION

Quantity	Value	Example
1	NO. 100	Example 1
2	NO. 100	Example 2
3	NO. 100	Example 3
4	NO. 100	Example 4
5	NO. 100	Example 5
6	NO. 100	Example 6
7	NO. 100	Example 7
8	NO. 100	Example 8
9	NO. 100	Example 9
10	NO. 100	Example 10

and we calculate from equation (5-5)

$$R_k = R_L - R_P = 40k - 20k ,$$

$$= 20k \text{ ohms.}$$

Using equation (4-17), we calculate

$$\begin{aligned} f(\text{"OFF"}) &= \frac{R_k(1 - \Delta r) - \frac{R_p(1 + \Delta r) + r_b(1 + \Delta r_b)}{B}}{R_k(1 - \Delta r) + \left(\frac{N - 2}{N - 1}\right)R_p(1 + \Delta r) + r_b(1 + \Delta r_b)} \\ &= \frac{20(0.9) - \frac{20(1.1) + 7.5(1.3)}{20}}{20(0.9) + 7.5(1.3)} \\ &= 0.59 . \end{aligned}$$

Then, from equation (4-18),

$$\begin{aligned} \left[\frac{R_2}{R_1} \right]_{\max} &= \frac{f(\text{"OFF"}) \left(\frac{1 - 2\Delta r}{N - 1} \right)}{1 - f(\text{"OFF"})} , \\ &= \frac{(0.59)(0.8)}{0.41} , \\ &= 1.15 . \end{aligned}$$

Using equation (4-20), we calculate

$$\begin{aligned}
 f(\text{"ON"}) &= \frac{R_k(1 + \Delta r)}{R_k(1 + \Delta r) + R_p(1 - \Delta r) + r_b(1 - \Delta r_b)} , \\
 &= \frac{20(1.1)}{20(1.1) + 20(0.9) + 7.5(0.7)} , \\
 &= 0.49 .
 \end{aligned}$$

Then, from equation (4-21),

$$\begin{aligned}
 \left. \frac{R_2}{R_1} \right]_{\min} &= \frac{f(\text{"ON"}) \left(\frac{1 + 2\Delta r}{N - 1} \right)}{1 - f(\text{"ON"})} , \\
 &= \frac{(0.49)(1.2)}{0.51} . \\
 &= 1.15 .
 \end{aligned}$$

Therefore, the actual nominal ratio shall be

$$\left. \frac{R_2}{R_1} \right]_{\max} = 1.15 = \frac{R_2}{R_1} = \left. \frac{R_2}{R_1} \right]_{\min} .$$

The approximation stated by the inequality in equation (4-22)

is

$$R_p(N - 1)(1 + 2\Delta r) \ll R_1 .$$

Then,

$$20(1.2)10^3 = 24k \ll R_1 ,$$

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and R_1 shall be 240k ohms, minimum. The author recommends that the actual nominal value of the ratio $\frac{R_2}{R_1}$ be set as close as practical to the value of 1.15. The author, when he constructed this binary, set

$$R_2 = 330k \text{ ohms}$$

and

$$R_1 = 270k \text{ ohms}$$

which yield a ratio of 1.2.

As a check on ourselves, we shall use equations (3-2), (3-4), (5-9), and (5-10), and calculate the grid current in the tube for the nominal conditions.

$$(E_g)_c = E_{bb} \frac{R_2}{\frac{R_1}{N-1} + R_2} = 300 \frac{330}{270 + 330}$$

$$= 165 \text{ volts ;}$$

$$R_E = \frac{R_2 \frac{R_1}{N-1}}{R_2 + \frac{R_1}{N-1}} = \frac{(270)(330)}{270 + 330} \times 10^3$$

$$= 148k \text{ ohms}$$

$$E_k = E_{bb} \frac{R_k}{R_k + R_p + r_b} = 300 \frac{20}{20 + 20 + 7.5}$$

$$= 127 \text{ volts .}$$

and the value of R is determined. The value of R is determined by the value of R in the previous section. The value of R is determined by the value of R in the previous section. The value of R is determined by the value of R in the previous section.

$$R = 100 \text{ ohms}$$

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$$R = 100 \text{ ohms}$$

which yields a value of R . As a check on ourselves, we shall use equations (3-2), (3-4), (3-5), and (3-10), and calculate the grid current in the tube for the same conditions.

$$I_{g1} = \frac{V_{g1}}{R} = \frac{100}{100} = 1 \text{ ma}$$

$$I_{g2} = \frac{V_{g2}}{R} = \frac{100}{100} = 1 \text{ ma}$$

$$I_{g3} = 100 \text{ ohms}$$

$$I_{g4} = \frac{V_{g4}}{R} = \frac{100}{100} = 1 \text{ ma}$$

$$I_{g5} = 100 \text{ ohms}$$

$(E_g)_c$ is greater than E_k ; therefore,

$$I_g = \frac{(E_g)_c - E_k}{R_E} = \frac{165 - 127}{148} \times 10^{-3}$$

←

$$= 0.25 \text{ ma} .$$

For the purposes of this binary design, 0.25 ma is close enough to the estimated 0.20 ma. The author feels that it is unnecessary to pursue the trial and error process any further, especially since the tolerance assigned to the static plate resistance is ± 30 per cent!

The schematic of the circuit the author constructed is illustrated in Figure 10. The duodiode, GL-5726, was used to inject triggering pulses into the binary.¹

Seven different 6829 tubes were operated in this circuit and Table VII shows the maximum and minimum data obtained from the fourteen different stable states available with the seven tubes.

The circuit would remain bistable with all seven tubes as E_{bb} was varied from approximately 70 volts to something greater than 500 volts.

The maximum triggering rate with E_{bb} equal to 300 volts varied from 65 to 93 kilocycles. (With E_{bb} at approximately 200 volts, the binary would trigger at 200 kilocycles!)

---

¹For a discussion on triggering a binary that could just as well be extended to the nthnary, see:

Chance, et al., Waveforms, Vol. 19, Radiation Laboratory Series, McGraw-Hill Book Company, Inc., New York, 1949, pp. 187-190.

FIGURE 10.- BINARY SCHEMATIC

TABLE VII

MAXIMUM AND MINIMUM MEASURED
DATA FROM THE BINARY

Voltage	Maximum volts	Minimum volts
E_k	133	128
$(E_p)_c$	176	160
$(E_p)_{nc}$	288	283
$(E_g)_c$	133	128
$(E_g)_{nc}$	100	90

The resistor divider associated with the heaters in Figure 10 was designed to place the heaters at approximately the same DC potential as the cathodes of the 6829.

If the reader cares to, he may check the operating conditions of this binary with the design ratings for both the 6829 and the GL-5726 as published.² If he does so, he will find that the design is quite reasonably conservative.

A.3 The Ternary, $N = 3$

Using the values assigned in Section A.1, we calculate from equation (5-7),

$$R_p = \frac{R_L B - 2(N-1)r_b}{2(N-1+B)} = \frac{(40)(20) - (2)(2)7.5}{2(22)} \times 10^3 ,$$

$$= 17.5k \text{ ohms} \approx 18k \text{ ohms} .$$

R_k is computed from equation (5-5),

$$R_k = R_L - R_p = 40k - 18k ,$$

$$= 22k \text{ ohms} .$$

Using equation (4-17), we calculate

$$f(\text{"OFF"}) = \frac{R_k(1 - \Delta r) - \frac{R_p(1 + \Delta r) + r_b(1 + \Delta r_b)}{B}}{R_k(1 - \Delta r) + \left(\frac{N-2}{N-1}\right)R_p(1 + \Delta r) + r_b(1 + \Delta r_b)}$$

---

²Electronic Tubes, 5-Star Types, 6829, ET-T1250, 1-56; GL-5726, ET-T1088, 8-54, Tube Department, General Electric Company, Schenectady 5, N. Y.

 

$$f(\text{"OFF"}) = \frac{22(0.9) - \frac{18(1.1) + 7.5(1.3)}{20}}{22(0.9) + \frac{18(1.1)}{2} + 7.5(1.3)} ,$$

$$= 0.46 .$$

Then, from equation (4-18),

$$\left. \frac{R_2}{R_1} \right]_{\max} = \frac{f(\text{"OFF"}) \left(\frac{1 - 2\Delta r}{N - 1} \right)}{1 - f(\text{"OFF"})}$$

$$= \frac{(0.46) \left(\frac{0.8}{2} \right)}{0.54}$$

$$= 0.34 .$$

Using equation (4-20), we calculate

$$f(\text{"ON"}) = \frac{R_k(1 + \Delta r)}{R_k(1 + \Delta r) + R_p(1 - \Delta r) + r_b(1 - \Delta r_b)} ,$$

$$= \frac{22(1.1)}{22(1.1) + 18(0.9) + 7.5(0.7)}$$

$$= 0.53 .$$

Then, from equation (4-21),

$$\left. \frac{R_2}{R_1} \right]_{\min} = \frac{f(\text{"ON"}) \left(\frac{1 + 2\Delta r}{N - 1} \right)}{1 - f(\text{"ON"})} ,$$

1900

1901

1902

1903

1904

1905

1906

1907

1908

1909

1910

1911

$$= \frac{(0.53) \left(\frac{1.2}{2} \right)}{0.47}$$

$$= 0.68 \quad .$$

However, this means that the nominal ratio

$$0.68 \leq \frac{R_2}{R_1} \leq 0.34 \quad ,$$

which, of course, cannot be. The ternary cannot be designed with the above given conditions. In order to obtain a design that is realizable we can

- (a) find a tube type with a larger figure of merit, F ;
- (b) increase the magnitude of R_L ; and
- (c) decrease the magnitude of the component tolerances.

We shall choose to decrease the magnitude of the tolerance associated with the resistors. Therefore, with the following values:

$$N = 3$$

$$R_k = 22k$$

$$R_p = 18k$$

$$r_b = 7.5k$$

$$\Delta r_b = 0.30$$

we can find, by trial and error, that when

$$\Delta r = 0.01 \quad ,$$

$$\left[\frac{R_2}{R_1} \right]_{\max} = 0.50 = \frac{R_2}{R_1} = \left[\frac{R_2}{R_1} \right]_{\min} \quad .$$

From equation (4-22)

$$R_1 \gg R_p(N - 1)(1 + 2\Delta r)$$

$$R_1 \gg 18(2)(1.02) \times 10^3$$

$$R_1 \gg 36.8k \text{ ohms}$$

Therefore, R_1 shall have a minimum value of 368k ohms. If the following nominal values are chosen:

$$R_2 = 200k \text{ ohms}$$

$$R_1 = 400k \text{ ohms}$$

then the ratio

$$\frac{R_2}{R_1} = \frac{200}{400} = 0.50 \quad ,$$

and the ternary schematic illustrated in Figure 11 can be mechanized.

Again, as a check on our assumption that the grid current is 0.20 ma, we shall calculate the grid current for the nominal conditions using equations (3-2), (3-4), (5-9), and (5-10).

$$\begin{aligned} (E_g)_c &= E_{bb} \frac{R_2}{\frac{R_1}{N-1} + R_2} = 300 \frac{200}{\frac{400}{2} + 200} \\ &= 150 \text{ volts} \quad , \end{aligned}$$

$$E = \frac{1}{2} \epsilon_0 \int_V |\nabla \phi|^2 dV$$

$$E = \frac{1}{2} \epsilon_0 \int_V |\nabla \phi|^2 dV$$

$$E = \frac{1}{2} \epsilon_0 \int_V |\nabla \phi|^2 dV$$

Therefore, the shell has a constant value of ϕ and the total electrostatic energy is constant.

$$E = \frac{1}{2} \epsilon_0 \int_V |\nabla \phi|^2 dV$$

$$E = \frac{1}{2} \epsilon_0 \int_V |\nabla \phi|^2 dV$$

$$E = \frac{1}{2} \epsilon_0 \int_V |\nabla \phi|^2 dV$$

and the electrostatic energy is constant in time if ϕ is constant.

Since ϕ is constant, the electrostatic energy is constant.

Therefore, the electrostatic energy is constant in time.

Using equations (2-2), (2-3) and (2-4),

$$E = \frac{1}{2} \epsilon_0 \int_V |\nabla \phi|^2 dV$$

$$E = \frac{1}{2} \epsilon_0 \int_V |\nabla \phi|^2 dV$$

FIGURE 11.- TERNARY SCHEMATIC

$$\overleftarrow{R_E} = \frac{R_2 \frac{R_L}{2N-1}}{R_2 + \frac{R_L}{N-1}} = \frac{(200) \frac{400}{2}}{200 + \frac{400}{2}}$$

$$= 100k \text{ ohms} ,$$

$$E_k = E_{bb} \frac{R_k}{R_k + R_p + r_b} = 300 \frac{22}{22 + 18 + 7.5}$$

$$= 139 \text{ volts} .$$

$(E_g)_c$ is greater than E_k ; therefore,

$$I_g = \frac{(E_g)_c - E_k}{\overleftarrow{R_E}} = \frac{150 - 139}{100} \times 10^{-3}$$

$$= 0.11 \text{ ma} .$$

Again, it is the author's opinion that the difference between the calculated and the estimated value of the grid current is negligible considering that a ± 30 per cent tolerance is assigned to r_b .

A.4 The Quaternary, $N = 4$

In order to begin the design of a quaternary, we shall assume the following values:

$$E_{bb} = 300 \text{ volts}$$

$$R_L = 40,000 \text{ ohms}$$

$$\Delta r = 0.01$$

$$r_b = 7,500 \text{ ohms}$$

$$\Delta r_b = 0.15$$

$$B = 20$$

} tube type 6829

Using these assigned values, we calculate from equation (5-7),

$$R_p = \frac{R_L B - 2(N - 1)r_b}{2(N - 1 + B)} = \frac{(40)(20) - 2(3)7.5}{2(23)} \times 10^3$$

$$= 16.4k \approx 16k \text{ ohms} .$$

Then, using equation (5-5), we calculate

$$R_k = R_L - R_p = 40k - 16k ,$$

$$= 24k \text{ ohms} .$$

Using equation (4-17), we calculate

$$f(\text{"OFF"}) = \frac{R_k(1 - \Delta r) - \frac{R_p(1 + \Delta r) + r_b(1 + \Delta r_b)}{B}}{R_k(1 - \Delta r) + \left(\frac{N - 2}{N - 1}\right)R_p(1 + \Delta r) + r_b(1 + \Delta r_b)}$$

$$= \frac{24(0.99) - \frac{16(1.01) + 7.5(1.15)}{20}}{24(0.99) + \frac{2(16)(1.01)}{3} + 7.5(1.15)}$$

$$= 0.523 .$$

Then, from equation (4-18),

$$\left[\frac{R_2}{R_1} \right]_{\max} = \frac{f(\text{"OFF"}) \left(\frac{1 - 2\Delta r}{N - 1} \right)}{1 - f(\text{"OFF"})} = \frac{(0.523) \left(\frac{0.98}{3} \right)}{0.477}$$

$$= 0.358 .$$

Using equation (4-20), we calculate

$$\begin{aligned}
 f(\text{"ON"}) &= \frac{R_k(1 + \Delta r)}{R_k(1 + \Delta r) + R_p(1 - \Delta r) + r_b(1 - \Delta r_b)} , \\
 &= \frac{24(1.01)}{24(1.01) + 16(0.99) + 7.5(0.85)} \\
 &= 0.522 .
 \end{aligned}$$

Then, from equation (4-21),

$$\begin{aligned}
 \left. \frac{R_2}{R_1} \right]_{\min} &= \frac{f(\text{"ON"}) \left(\frac{1 + 2\Delta r}{N - 1} \right)}{1 - f(\text{"ON"})} = \frac{0.522 \left(\frac{1.02}{3} \right)}{0.478} \\
 &= 0.372 .
 \end{aligned}$$

The computed value of the minimum ratio is greater than the computed value of the maximum ratio. The design is, therefore, not realizable even when the resistor tolerance is ± 1 per cent and the tube's static plate resistance tolerance is ± 15 per cent.

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ERASE

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