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Single-Valued Neutrosophic Hyperrings and Single-Valued Neutrosophic Hyperideals

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Abstract. In this paper, we introduced the concepts of Single-valued neutrosophic hyperring and Single-valued neutrosophic hyperideal. The algebraic properties and structural characteristics of the single-valued neutrosophic hyperrings and hyperideals are investigated and verified.

Keywords: Hyperring, Hyperideal, Single-valued neutrosophic set, Single-valued neutrosophichyperring and Single-valued neutrosophichyperideal.

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1 Introduction

Hyperstructure theory was introduced by Marty in 1934 [16]. The concept of hyperring and the general form of hyperring for introducing the notion of hyperring homomorphism was developed by Corsini [11]. Vougiouklis [31] coined different type of hyperrings called H_v-ring, H_v-subring, and left and right H_v-ideal of a H_v-ring, all of which are generalizations of the corresponding concepts related to hyperrings introduced by Corsini [11].

In general fuzzy sets [34] the grade of membership is represented as a single real number in the interval [0,1]. The uncertainty in the grade of membership of the fuzzy set model was overcome using the interval-valued fuzzy set modelintroduced by Turksen [29]. In 1986, Atanassov [8] introduced intuitionistic fuzzy sets which is a generalization of fuzzy sets. This model was equivalent to interval valued fuzzy sets in [32]. Intuitionistic fuzzy sets can only handle incomplete information, and not indeterminate information which commonly exists in real-life [32]. To overcome these problems, Smarandache introduced the neutrosophic model. Some new trends of neutrosophic theory were introduced in [1,2,3,4,5,6,7] .Wang et al. [32] introduced the concept of single-valued neutrosophic sets (SVNSs), whereas Smarandache introduced plithogenic set as generalization of neutrosophic set model in [13].

The theory of hyperstructures are widely used in various mathematical theories. The study on fuzzy algebra began by Rosenfeld [17], and this was subsequently expanded to other fuzzy based models such as intuitionistic fuzzy sets, fuzzy soft sets and vague soft sets. Some of the recent works related to fuzzy soft rings and ideal, vague soft groups, vague soft rings and vague soft ideals can be found in [21; 22; 23; 26, 27]. Research on fuzzy algebra led to the development of fuzzy hyperalgebraic theory. The concept of fuzzy ideals of a ring introduced by Liu [15]. The generalization of the fuzzy hyperideal introduced by Davvaz[12]. The concepts of fuzzy γ-ideal was then introduced by Bharathi

and Vimala [10], and the fuzzy γ-ideal was subsequently expanded in [33]. The hypergroup and hyperring theory for vague soft sets were developed by Selvachandran et al. in [18,19,20,24,25]
In this paper we develop the theory of single-valued neutrosophic hyperrings and single-valued neutrosophic hyperideals to furter contribute to the development of the body of knowledge in neutrosophic hyperalgebraic theory.

2 Preliminaries

Let X be a space of points (objects) with a generic element in X denoted by x.
Definition 2.1. [32] A SVNS $A$is a neutrosophic set that is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$, where $T_A(x), I_A(x), F_A(x) \in [0,1]$. This set A can thus be written as

$$A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle : x \in U\}. \quad (1)$$

The sum of $T_A(x), I_A(x)$ and $F_A(x)$ must fulfill the condition $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$. For a SVNS A in U, the triplet $\big(T_A(x), I_A(x), F_A(x)\big)$ is called a single-valued neutrosophic number (SVNN). Let $x = (T_x, I_x, F_x)$ to represent a SVNN .

Definition 2.2. [32] Let A and B be two SVNSs over a universe U.
(i) A is contained in B, if $T_A(x) \leq T_B(x), I_A(x) \leq I_B(x)$, and $F_A(x) \geq F_B(x)$, for all $x \in U$. This relationship is denoted as $A \subseteq B$.
(ii) A and B are said to be equal if $A \subseteq B$ and $B \subseteq A$.
(iii) $A^c = \langle x, \big(F_A(x), 1 - I_A(x), T_A(x)\big)\rangle$, for all $x \in U$.
(iv) $A \cup B = \big(x, (\max(T_A, T_B), \max(I_A, I_B), \min(F_A, F_B))\big)$, for all $x \in U$.
(v) $A \cap B = \big(x, (\min(T_A, T_B), \min(I_A, I_B), \max(F_A, F_B))\big)$, for all $x \in U$.

Definition 2.3. [16] A *hypergroup* $\langle H, \circ\rangle$ is a set H with an associative hyperoperation $(\circ) : H \times H \to P(H)$ which satisfies $x \circ H = H \circ x = H$ for all x in H (reproduction axiom) .

Definition 2.4.[12] A hyperstructure $\langle H, \circ\rangle$ is called an H_v*-group* if the following axioms hold:
(i) $x \circ (y \circ z) \cap (x \circ y) \circ z \neq \emptyset$ for all $x, y, z \in H$, (H_v-semigroup)
(ii) $x \circ H = H \circ x = H$ for all x in H.

Definition 2.5.[16] A subset K of H is called a *subhypergroup* if $\langle K, \circ\rangle$ is a hypergroup.

Definition 2.6.[11]A H_v*-ring* is a multi-valued system $(R, +, \circ)$ which satisfies the following axioms:
(i) $(R, +)$ is a H_v-group,
(ii) $(R, \circ)$ is a H_v-semigroup,
(iii) The hyperoperation "$\circ$" is weak distributive over the hyperoperation "+", that is for each $x, y, z \in R$ the conditions $x \circ (y + z) \cap \big((x \circ y) + (x \circ z)\big) \neq \phi$ and $(x + y) \circ z \cap \big((x \circ z) + (y \circ z)\big) \neq \phi$ holds true.

Definition 2.7. [11]A nonempty subset R' of R is a *subhyperring* of $(R, +, \circ)$ if $(R', +)$ is a subhypergroup of $(R, +)$ and for all $x, y, z \in R'$, $x \circ y \in P^*(R')$, where $P^*(R')$ is the set of all non-empty subsets of R'.
Definition 2.8. [11] Let R be a H_v-ring. A nonempty subset I of R is called a *left* (respectively *right*) H_v*-ideal* if the following axioms hold:
(i) $(I, +)$ is a H_v-subgroup of $(R, +)$,
(ii) $R \circ I \subseteq I$(resp. $I \circ R \subseteq I$).
If I is both a left and right H_v-ideal of R, then I is said to be a H_v*-ideal* of R.

3 Single-Valued NeutrosophicHyperrings

Throughout this section, we denote the hyperring$(R, +, \circ)$by R.

Definition 3.1. Let A be a SVNS over R. A is called a single-valued neutrosophic hyperringover R, if ,

(i) $\forall \quad a, b \in R, \min\{T_A(a), T_A(b)\} \leq \inf\{T_A(c): c \in a + b\}, \max\{I_A(a), I_A(b)\} \geq \sup\{I_A(c): c \in a + b\}$ and $\max\{F_A(a), F_A(b)\} \geq \sup\{F_A(c): c \in a + b\}$

(ii) $\forall \quad x, a \in R$, there exists $b \in R$ such that $a \in x + b$ and $\min\{T_A(x), T_A(a)\} \leq T_A(b)$, $\max\{I_A(x), I_A(a)\} \geq I_A(b)$ and $\max\{F_A(x), F_A(a)\} \geq F_A(b)$

(iii) $\forall \quad x, a \in R$, there exists $c \in R$ such that $a \in c + x$ and $\min\{T_A(x), T_A(a)\} \leq T_A(c)$, $\max\{I_A(x), I_A(a)\} \geq I_A(c)$ and $\max\{F_A(x), F_A(a)\} \geq F_A(c)$

(iv) $\forall \ a, b \in R, \min\{T_A(a), T_A(b)\} \leq \inf\{T_A(c): c \in a \circ b\}, \max\{I_A(a), I_A(b)\} \geq \sup\{I_A(c): c \in a \circ b\}$ and $\max\{F_A(a), F_A(b)\} \geq \sup\{F_A(c): c \in a \circ b\}$

Example 3.2. The family of t-level sets of SVNSs over R is a subhyperring of R is given below:

$$A_t = \{a \in R: T_A(a) \geq t, I_A(a) \geq t, F_A(a) \leq t\}, \text{for all } t \in [0, 1].$$

Then A is a single-valued neutrosophic hyperring over R.

Theorem 3.3. A is a SVNS over R. Then A is a single-valued neutrosophichyperring over R iff A is single-valued neutrosophic semi hyper group over $(R, \circ)$ and also a single-valued neutrosophic hypergroup over $(R, +)$.

Proof. This is obvious by Definition 3.1. ■

Theorem 3.4. Let A and B be single-valued neutrosophic hyperrings over R. Then$A \cap B$ is a single-valued neutrosophichyperring over R if it is non-null.

Proof. Let A and B are single-valued neutrosophic hyperrings over R.By Definition 3.1, $A \cap B = \{\langle a, T_{A\cap B}(a), I_{A\cap B}(a), F_{A\cap B}(a)\rangle: a \in R\}$, where $T_{A\cap B}(a) = \min(T_A(a), T_B(a))$, $I_{A\cap B}(a) = \max(I_A(a), I_B(a))$, $F_{A\cap B}(a) = \max(F_A(a), F_B(a))$. Then for all $a, b \in R$, we have the following. We only prove all the four conditions for the truth membership terms T_A, T_B. The proof for the I_A, I_B and F_A, F_B membership functions obtained in a similar manner.

(i)
$$\begin{aligned}\min\{T_{A\cap B}(a), T_{A\cap B}(b)\} &= \min\{\min(T_A(a), T_B(a)), \min(T_A(b), T_B(b))\}\\ &\leq \min\{\min(T_A(a), T_A(b)), \min(T_B(a), T_B(b))\}\\ &\leq \min\{\inf\{T_A(c): c \in a + b\}, \inf\{T_B(c): c \in a + b\}\}\\ &\leq \inf\{\min(T_A(c), T_B(c)): c \in a + b\}\\ &= \inf\{T_{A\cap B}(c): c \in a + b\}\end{aligned}$$

Similarly, $\max\{I_{A\cap B}(a), I_{A\cap B}(b)\} \geq \sup\{I_{A\cap B}(c): c \in a + b\}$ and $\max\{F_{A\cap B}(a), F_{A\cap B}(b)\} \geq \sup\{F_{A\cap B}(c): c \in a + b\}$.

(ii) $\forall x, a \in R$, there exists $b \in R$ such that $a \in x + b$. Then it follows that:

$$\begin{aligned}\min\{T_{A\cap B}(a), T_{A\cap B}(b)\} &= \min\{\min(T_A(a), T_B(a)), \min(T_A(b), T_B(b))\}\\ &\leq \min\{\min(T_A(a), T_A(b)), \min(T_B(a), T_B(b))\}\\ &\leq \min(T_A(c), T_B(c))\\ &= T_{A\cap B}(c)\end{aligned}$$

Similarly, $\max\{I_{A\cap B}(a), I_{A\cap B}(b)\} \geq I_{A\cap B}(c)$ and $\max\{F_{A\cap B}(a), F_{A\cap B}(b)\} \geq F_{A\cap B}(c)$.

(iii) It can be easily verified that $\forall \ x, a \in R$, there exists $c \in R$ such that $a \in c + x$ & $\min\{T_{A\cap B}(x), T_{A\cap B}(a)\} \leq T_{A\cap B}(c)$, $\max\{I_{A\cap B}(x), I_{A\cap B}(a)\} \geq I_{A\cap B}(c)$ and $\max\{F_{A\cap B}(x), F_{A\cap B}(a)\} \geq$

$F_{A\cap B}(c)$.

(iv) $\forall\ a \in R, \min\{T_{A\cap B}(a), T_{A\cap B}(b)\} \leq \inf\{T_{A\cap B}(c): c \in a \circ b\}, \max\{I_{A\cap B}(a), I_{A\cap B}(b)\} \geq \sup\{I_{A\cap B}(c): c \in a \circ b\}$ and $\max\{F_{A\cap B}(a), F_{A\cap B}(b)\} \geq \sup\{F_{A\cap B}(c): c \in a \circ b\}$.

Hence, $A \cap B$ is single-valued neutrosophichyperring over R. ■

Theorem 3.5. Let A be a single-valued neutrosophic hyperring over R. Then for every $t \in [0,1], A_t \neq \emptyset$ is a subhyperring over R.

Proof. Let A be a single-valued neutrosophichyperring over R. $\forall\ t \in [0,1]$, let $a, b \in A_t$. Then $T_A(a), T_A(b) \geq t, I_A(a), I_A(b) \leq t$and $F_A(a), F_A(b) \leq t$. Since A is a single-valued neutrosophic sub hyper group of $(R, +)$, we have the following:

$$\inf\{T_A(c): c \in a + b\} \geq \min\{T_A(a), T_A(b)\} \geq \min\{t, t\} = t,$$

$$\sup\{I_A(c): c \in a + b\} \leq t,$$

and

$$\sup\{F_A(c): c \in a + b\} \leq t.$$

This implies that $c \in A_t$ and then for every $c \in a + b$, we obtain $a + b \subseteq A_t$. As such, for every $c \in A_t$, we obtain $c + A_t \subseteq A_t$. Now let $a, c \in A_t$. Then $T_A(a), T_A(c) \geq t, I_A(a), I_A(c) \leq t$and $F_A(a), F_A(c) \leq t$.

A is a single-valued neutrosophic subhypergroup of $(R, +)$, there exists $b \in R$ such that $a \in c + b$ and $T_A(b) \geq \min\big(T_A(a), T_A(c)\big) \geq t$, $I_A(b) \leq \max\big(I_A(a), I_A(c)\big) \leq t$, $F_A(b) \leq \max\big(F_A(a), F_A(c)\big) \leq t$, and this implies that $b \in A_t$. Therefore, we obtain $A_t \subseteq c + A_t$. As such, we obtain $c + A_t = A_t$. As a result, A_t is a subhypergroup of $(R, +)$.

Let $a, b \in A_t$,then $T_A(a), T_A(b) \geq t, I_A(a), I_A(b) \leq t$ and $F_A(a), F_A(b) \leq t$.Since A is a single-valued neutrosophic subsemihypergroup of $(R, \circ)$, then for all $a, b \in R$, we have the following:

$$\inf\{T_A(c): c \in a \circ b\} \geq \min\{T_A(a), T_A(b)\} = t,$$

$$\sup\{I_A(c): c \in a \circ b\} \leq \max\big(I_A(a), I_A(b)\big) = t,$$

and

$$\sup\{F_A(c): c \in a \circ b\} \leq \max\big(F_A(a), F_A(b)\big) = t.$$

This implies that $c \in A_t$ and consequently $a \circ b \in A_t$. Therefore, for every $a, b \in A_t$ we obtain $a \circ b \in P^*(R)$. Hence A_t is a subhyperring over R.

Theorem 3.6. Let A be a single-valued neutrosophic set over R.Then the following statements are equivalent:

(i) $A$is a single-valued neutrosophic hyperring over R.

(ii) $\forall t \in [0,1]$, a non-empty A_t is a sub hyperring over R.

Proof.

$(i) \Longrightarrow (ii)$ $\forall\ t \in [0,1]$, by Theorem 3.5, A_t is sub hyperring over R.

$(ii) \Longrightarrow (i)$ Assume that A_t is a subhyperring over R. Let $a, b \in A_t$ and therefore $a + b \subseteq A_{t_0}$. Then for every $c \in a + b$ we have $T_A(c) \geq t_0, I_A(c) \leq t_0$and $F_A(c) \leq t_0$, which implies that:

$$\min\big(T_A(a), T_A(b)\big) \leq \inf\{T_A(c): c \in a + b\},$$

$$\max\big(I_A(a), I_A(b)\big) \geq \sup\{I_A(c): c \in a + b\},$$

and

$$\max(F_A(a), F_A(b)) \geq \sup\{F_A(c): c \in a + b\}$$

Therefore, condition (i) of Definition 3.1 has been verified.

Next, let $x, a \in A_{t_1}$ for every $t_1 \in [0, 1]$ which means that there exists $b \in A_{t_1}$ such that $a \in x \circ b$. Since$b \in A_{t_1}$, we have $T_A(b) \geq t_1, I_A(b) \leq t_1$ and $F_A(b) \leq t_1$, and thus we have

$$T_A(b) \geq t_1 = \min(T_A(a), T_A(c)),$$

$$I_A(b) \leq t_1 = \max(I_A(a), I_A(c)),$$

and

$$F_A(b) \leq t_1 = \max(F_A(a), F_A(c)).$$

Therefore, condition (ii) of Definition 3.1 has been verified. Compliance to condition (iii) of Definition 3.1 can be proven in a similar manner. Thus, A is a single-valued neutrosophic subhypergroup of $(R, +)$. Now since A_t is a subsemihypergroup of the semihypergroup $(R, \circ)$, we have the following. Let $a, b \in A_{t_2}$ and therefore we have $a \circ b \in A_{t_2}$. Thus for every $c \in a \circ b$, we obtain $T_A(c) \geq t_2, I_A(c) \leq t_2$ and $F_A(c) \leq t_2$, and therefore it follows that:

$$\min(T_A(a), T_A(b)) \leq \inf\{T_A(c): c \in a \circ b\},$$

$$\max(I_A(a), I_A(b)) \geq \sup\{I_A(c): c \in a \circ b\},$$

and

$$\max(F_A(a), F_A(b)) \geq \sup\{F_A(c): c \in a \circ b\},$$

which proves that condition (iv) of Definition 3.1 has been verified. Hence A is a single-valued neutrosophic hyperring over R.

4 Single-Valued Neutrosophic Hyperideals

Definition 4.1. Let A be a SVNS over R. Then A is single-valued neutrosophic left (resp. right) hyperideal over R, if ,

(i) $\forall \; a, b \in R, \min\{T_A(a), T_A(b)\} \leq \inf\{T_A(c): c \in a + b\}, \max\{I_A(a), I_A(b)\} \geq \sup\{I_A(c): c \in a + b\}$ and $\max\{F_A(a), F_A(b)\} \geq \sup\{F_A(c): c \in a + b\}$

(ii) $\forall \; x, a \in R$, there exists $b \in R$ such that $a \in x + b$ and $\min\{T_A(x), T_A(a)\} \leq T_A(b)$, $\max\{I_A(x), I_A(a)\} \geq I_A(b)$ and $\max\{F_A(x), F_A(a)\} \geq F_A(b)$

(iii) $\forall \; x, a \in R$, there exists $c \in R$ such that $a \in c + x$ and $\min\{T_A(x), T_A(a)\} \leq T_A(c)$, $\max\{I_A(x), I_A(a)\} \geq I_A(c)$ and $\max\{F_A(x), F_A(a)\} \geq F_A(c)$

(iv) $\forall \; a, b \in R, T_A(b) \leq \inf\{T_A(c): c \in a \circ b\}$ (resp. $T_A(a) \leq \inf\{T_A(c): c \in a \circ b\}$), $I_A(b) \geq \sup\{I_A(c): c \in a \circ b\}$ (resp. $I_A(a) \geq \sup\{I_A(c): c \in a \circ b\}$) and $F_A(b) \geq \sup\{F_A(c): c \in a \circ b\}$ (resp. $F_A(a) \geq \sup\{F_A(c): c \in a \circ b\}$)

A is a single-valued neutrosophic left (resp. right) hyperidealof R. From conditions (i), (ii) and (iii) A is a single-valued neutrosophic subhypergroup of $(R, +)$.

Definition 4.2. Let A be a SVNS over R. Then A is a single-valued neutrosophic hyper ideal over R, if the following conditions are satisfied:

(i) $\forall \; a, b \in R, \min\{T_A(a), T_A(b)\} \leq \inf\{T_A(c): c \in a + b\}, \max\{I_A(a), I_A(b)\} \geq \sup\{I_A(c): c \in a + b\}$ and $\max\{F_A(a), F_A(b)\} \geq \sup\{F_A(c): c \in a + b\}$

(ii) $\forall \; x, a \in R$, there exists $b \in R$ such that $a \in x + b$ and $\min\{T_A(x), T_A(a)\} \leq T_A(b)$, $\max\{I_A(x), I_A(a)\} \geq I_A(b)$ and $\max\{F_A(x), F_A(a)\} \geq F_A(b)$

(iii) $\forall \; x, a \in R$, there exists $c \in R$ such that $a \in c + x$ and $\min\{T_A(x), T_A(a)\} \leq T_A(c)$, $\max\{I_A(x), I_A(a)\} \geq I_A(c)$ and $\max\{F_A(x), F_A(a)\} \geq F_A(c)$

(iv) $\forall \; a, b \in R, \max(T_A(a), T_A(b)) \leq \inf\{T_A(c): c \in a \circ b\}, \max(I_A(a), I_A(b)) \geq \sup\{I_A(c): c \in a \circ b\}$ and $\max(F_A(a), F_A(b)) \geq \sup\{F_A(c): c \in a \circ b\}$

From conditions (i), (ii) and (iii) A is a single-valued neutrosophic sub hyper group of $(R,+)$. Condition (iv) indicate both single-valued neutrosophic left hyperideal and single-valued neutrosophic right hyperideal. Hence A is a single-valued neutrosophic hyper ideal of R.

Theorem 4.3. Let A be a non-null SVNS over R. A is a single-valued neutrosophic hyperideal over R iff A is a single-valued neutrosophic hyper group over $(R,+)$ and also $A$is both a single-valued neutrosophic left hyper ideal and a single-valued neutrosophic right hyper ideal of R.

Proof. This is straight forward by Definitions 4.1 and 4.2.

Theorem 4.4. Let $A$and B be two single-valued neutrosophic hyper ideals over R. Then $A \cap B$ is a single-valued neutrosophichyperideal over R if it is non-null.

Proof. Let A and B are single-valued neutrosophic hyper ideals over R.By Definition 4.2, $A \cap B = \{\langle a, T_{A\cap B}(a), I_{A\cap B}(a), F_{A\cap B}(a)\rangle : a \in R\}$, where $T_{A\cap B}(a) = \min\big(T_A(a), T_B(a)\big), I_{A\cap B}(a) = \max\big(I_A(a), I_B(a)\big)$ and$F_{A\cap B}(a) = \max\big(F_A(a), F_B(a)\big)$. Then $\forall\ a, b \in R$, we have the following. We only prove all the four conditions for the truth membership terms T_A, T_B. The proof for the I_A, I_B and F_A, F_B membership functions obtained in a similar manner.

(i)
$$\begin{aligned}\min\{T_{A\cap B}(a), T_{A\cap B}(b)\} &= \min\{\min\big(T_A(a), T_B(a)\big), \min\big(T_A(b), T_B(b)\big)\}\\ &\le \min\{\min\big(T_A(a), T_A(b)\big), \min\big(T_B(a), T_B(b)\big)\}\\ &\le \min\{\inf\{T_A(c): c \in a+b\}, \inf\{T_B(c): c \in a+b\}\}\\ &\le \inf\{\min\big(T_A(c), T_B(c)\big) : c \in a+b\}\\ &= \inf\{T_{A\cap B}(c): c \in a+b\}\end{aligned}$$

Similarly, it can be proven that $\max\{I_{A\cap B}(a), I_{A\cap B}(b)\} \ge \sup\{I_{A\cap B}(c): c \in a+b\}$ and $\max\{F_{A\cap B}(a), F_{A\cap B}(b)\} \ge \sup\{F_{A\cap B}(c): c \in a+b\}$.

(ii) $\forall x, a \in R$, there exists $b \in R$ such that $a \in x + b$. Then:

$$\begin{aligned}\min\{T_{A\cap B}(a), T_{A\cap B}(b)\} &= \min\{\min\big(T_A(a), T_B(a)\big), \min\big(T_A(b), T_B(b)\big)\}\\ &\le \min\{\min\big(T_A(a), T_A(b)\big), \min\big(T_B(a), T_B(b)\big)\}\\ &\le \min\big(T_A(c), T_B(c)\big)\\ &= T_{A\cap B}(c)\end{aligned}$$

Similarly, $\max\{I_{A\cap B}(a), I_{A\cap B}(b)\} \ge I_{A\cap B}(c)$ and $\max\{F_{A\cap B}(a), F_{A\cap B}(b)\} \ge F_{A\cap B}(c)$.

(iii) $\forall\ x, a \in R$, there exists $c \in R$ such that $a \in c + x$ and $\min\{T_{A\cap B}(x), T_{A\cap B}(a)\} \le T_{A\cap B}(c)$, $\max\{I_{A\cap B}(x), I_{A\cap B}(a)\} \ge I_{A\cap B}(c)$ and $\max\{F_{A\cap B}(x), F_{A\cap B}(a)\} \ge F_{A\cap B}(c)$.

(iv) $\forall\ a \in R, \max\{T_{A\cap B}(a), T_{A\cap B}(b)\} \le \inf\{T_{A\cap B}(c): c \in a \circ b\}, \min\{I_{A\cap B}(a), I_{A\cap B}(b)\} \ge \sup\{I_{A\cap B}(c): c \in a \circ b\}$ and $\min\{F_{A\cap B}(a), F_{A\cap B}(b)\} \ge \sup\{F_{A\cap B}(c): c \in a \circ b\}$.

Hence, it is verified that $A \cap B$ is a single-valued neutrosophichyperideal over R.

5. Conclusion

We developed hyperstructure for the SVNS model through several hyperalgebraic structures such as hyperrings and hyperideals. The properties of these structures were studied and verified. The future work is on the development of hyperalgebraic theory for Plithogenic sets which is the generalization of neutrosophic set and also planned to develop some real life applications.

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